Math 45021 | Euclidean Geometry | Spring 2017

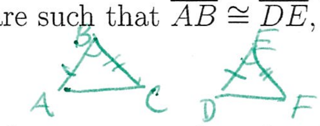
March 6, 2017 | Exam 1 | Matt Alexander

Name: Key | Score: ____ /50

> Please follow the directions provided in each section. You may assume that the parallel postulate holds for all questions on this exam.

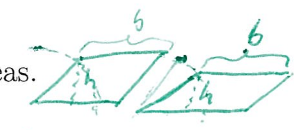
True or False (1 pt each): *For each of the following circle T if the statement is true or F if the statement is false.*

1. T F Suppose $\triangle ABC$ and $\triangle DEF$ are such that $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, and $\angle B \cong \angle E$. This implies $\triangle ABC \cong \triangle DEF$.



SAS

2. T F If two parallelograms have equal perimeters, then they have equal areas.



3. T F A triangle with side lengths of 5, 12, and 16 is a right triangle.

$5^2+12^2=25+144=169$ $16^2=256$ ✗

4. T F Every rhombus is a parallelogram.

5. T F Any two triangles with the same area are congruent.

6. T F If A, B, C, D are on the same circle such that $\angle ABD$ and $\angle ACD$ subtend the same arc, then $\angle ABD \cong \angle ACD$.

7. T F If three pairs of corresponding parts of two triangles are congruent, then the triangles are congruent. SSA

8. T F Suppose γ is a real number and $\triangle ABC$ and $\triangle DEF$ are such that $\frac{AB}{DE} = \frac{BC}{EF} = \gamma$, and $\angle B \cong \angle E$. This implies $\triangle ABC \sim \triangle DEF$. SAS for sim △s

9. T F Any two polygons with equal area can be cut into smaller polygons and reformed into the other. Equidecomposability

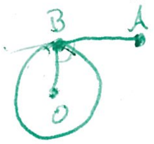
10. T F Two triangles whose sides are three pairs of parallel lines are similar.

11. T F A triangle with area 4 can be decomposed into a circle with radius 2. Different areas

12. T F Any point that is the intersection of two tangent lines to the circle is equidistant from every point on the circle. Center is only point equidistant to all points on a circle.

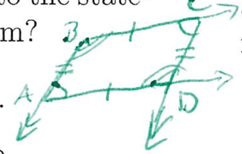
Multiple Choice (2 pts each): *Circle the letter of the best response.*

13. In a circle with center O, line $\overline{AB}$ is tangent and line $\overline{BO}$ is a radius. What must be true about $\overline{AB}$ and $\overline{BO}$?



   (a) They have the same length.
   (b) They are parallel.
   (c) They are perpendicular.
   (d) They have the same arc length.

14. Given a quadrilateral $ABCD$. Which of the following is **not** equivalent to the statement: $ABCD$ is a parallelogram?



   (a) $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.
   (b) $\angle A \cong \angle C$ and $\angle B \cong \angle D$.
   (c) $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$.
   (d) $\overline{AC}$ and $\overline{BD}$ are perpendicular. rhombus/kite

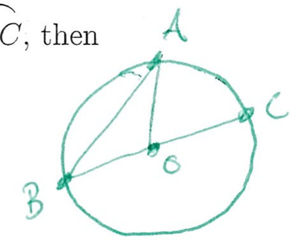
15. Given a convex quadrilateral $ABCD$ with $100° < \angle A + \angle B < 250°$ then Sum of all = 360°

   (a) $0° < \angle C + \angle D < 110°$
   (b) $110° < \angle C + \angle D < 260°$
   (c) $30° < \angle C + \angle D < 180°$
   (d) $100° < \angle C + \angle D < 250°$

16. Given the following numbers, which set **cannot** be the side lengths of a triangle?

   (a) 3, 4, 5
   (b) 5, 12, 13
   (c) 7, 9, 17 $7+9=16<17$ X
   (d) 8, 8, 12

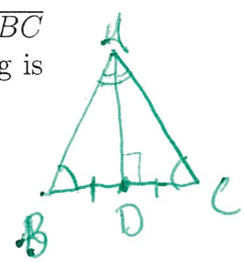
17. Given A, B, C on the circle with center O such that $\angle ABC$ subtends $\widehat{AC}$, then



   (a) $2\angle ABC = \angle AOC$
   (b) $\widehat{AC} = \angle ABC$ X
   (c) $2\,\widehat{AC} = \angle AOC$ X
   (d) $\angle ABC = \angle AOC$ X

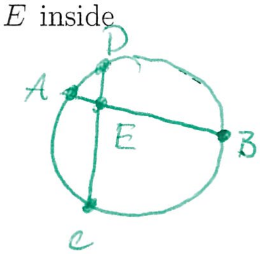
$\frac{1}{2}\angle AOC = \angle ABC$
$\widehat{AC} = \angle AOC$

18. Given $\triangle ABC$ with D the midpoint of $\overline{BC}$ and $\angle B \cong \angle C$. Which of the following is **not** true?



   (a) $\angle DAB \cong \angle DAC$ ✓
   (b) $\overline{AD} \perp \overline{BC}$ ✓
   (c) $\overline{AD} \cong \overline{AC}$ X
   (d) $\overline{BD} \cong \overline{DC}$ ✓

19. Given two chords $\overline{AB}$ and $\overline{CD}$ on the same circle which intersect at a point E inside the circle.



   (a) $\angle AED = \frac{1}{2}(\widehat{AD} + \widehat{BC})$ X
   (b) $\angle BAD \cong \angle CDA$ X
   (c) $AE \cdot AD = DE \cdot BE$ X
   (d) $\triangle ACE \sim \triangle DBE$ ✓

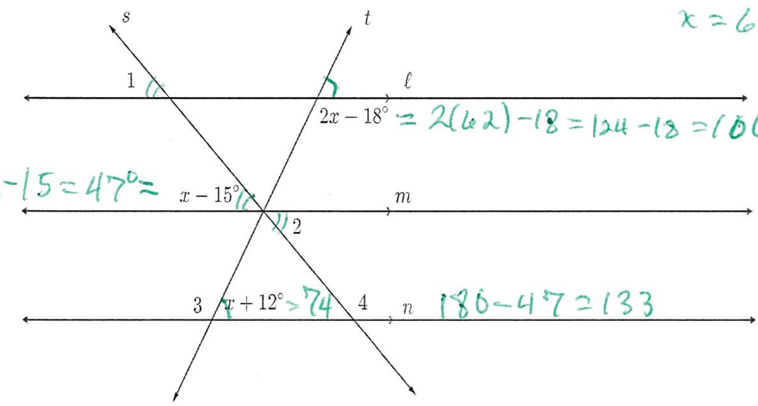
20. A rectangle is **not** a

   (a) Parallelogram
   (b) Trapezoid
   (c) Rhombus  X
   (d) Convex Quadrilateral

Find the angle measures ($\frac{1}{2}$ pt each): *Find the indicated values and angles.*

21. Find the value of x and the measures of angles 1-4 if the angles formed are related as shown and lines ℓ, m, n are all parallel.

$2x-18+x+12=180$
$3x-6=180$
$3x=186$
$x=62$



$x =$ 62

$\angle 1 =$ 47°

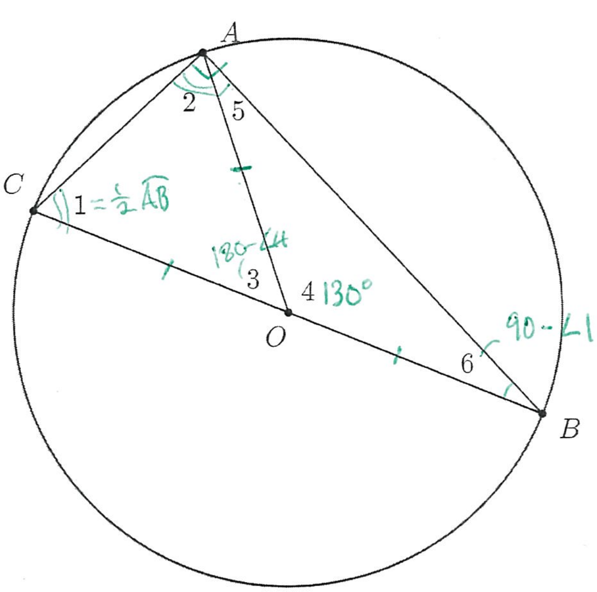
$\angle 2 =$ 47°

$\angle 3 =$ 106°

$\angle 4 =$ 133°

22. Given the following diagram and $\overset{\frown}{AB} = 130°$, write the angle measurement of each numbered angle.

$\overset{\frown}{AB} = \angle AOB = 130°$



$\angle 1 =$ 65°

$\angle 2 =$ 65°

$\angle 3 =$ 50°

$\angle 4 =$ 130°

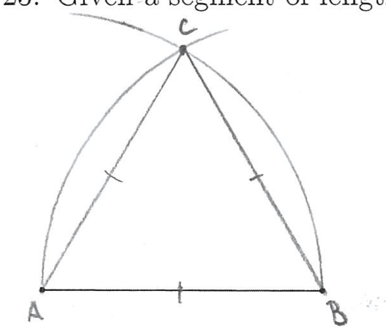
$\angle 5 =$ 25°

$\angle 6 =$ 25°

Constructions (2 pts each): Pick 2 of 3

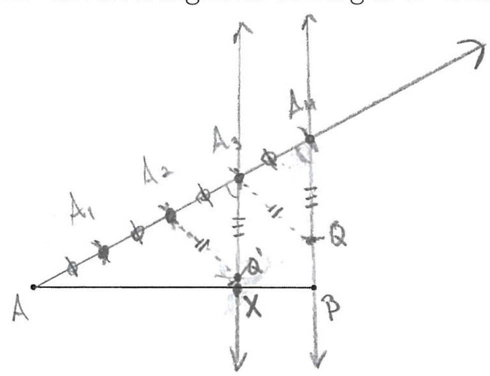
Create the drawings with a compass and straightedge. Show each step performed clearly.

23. Given a segment of length a. Construct an equilateral triangle.



Draw two circles of radius a with centers A and B. Then they intersect at a point C. $\triangle ABC$ is equilateral since $AC = CB = AB$.

24. Given a segment of length a. Construct a segment with length $\frac{3}{4} \cdot a$.



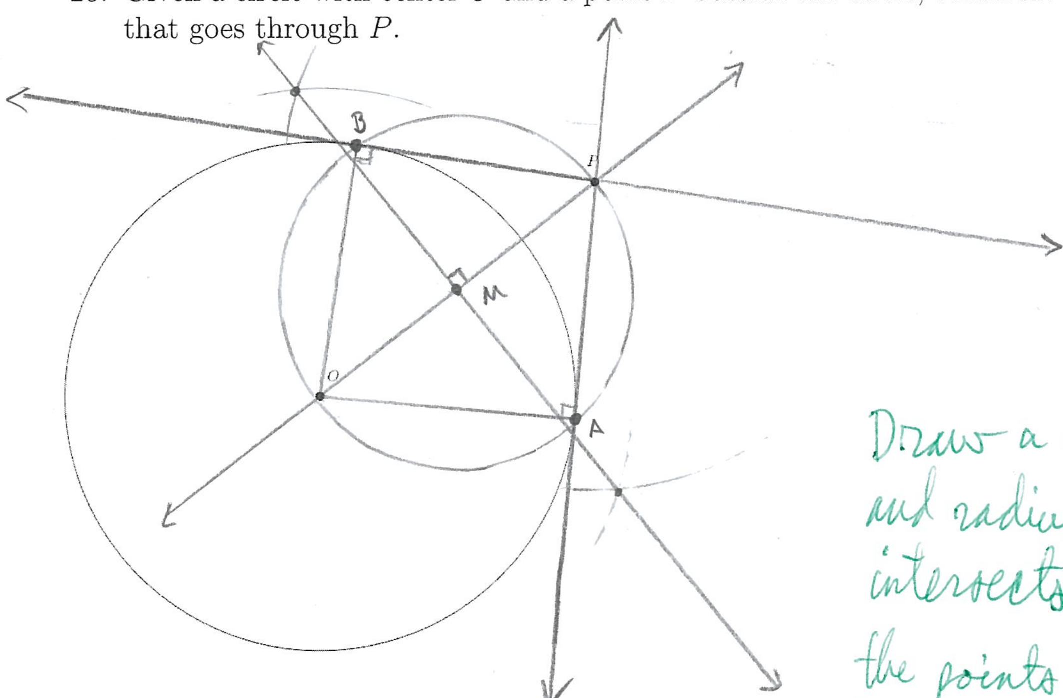
Draw any ray starting at A

Copy a segment of any length 4 times along the ray.

Draw a line parallel to $\overleftrightarrow{A_4B}$ through ~~B~~ A_3.

This line intersects $\overleftrightarrow{AB}$ at X and $\overline{AX} = \frac{3}{4} AB$ by similar triangles.

25. Given a circle with center O and a point P outside the circle, construct the line tangent to the circle that goes through P.



Draw the line $\overleftrightarrow{OP}$

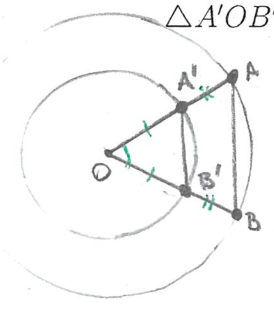
Construct the perpendicular bisector to $\overline{OP}$. Call M the midpoint of $\overline{OP}$.

Draw a circle with center M and radius OM. This circle intersects the original circle at the points A & B.

$\overleftrightarrow{AP}$ & $\overleftrightarrow{BP}$ are tangent since $\overset{\frown}{OP} = 180°$.

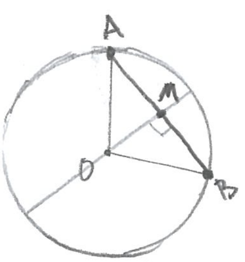
Proofs (4 pts each)**: Choose 3 of 5** *Justify the following statements with a proof.*

26. Let two circles have the same center O with radii $r_1 > r_2$. Let two segments $\overline{OA}$ and $\overline{OB}$ be radii of the larger circle that intersect the smaller circle at points A' and B' respectively. Show that $\triangle A'OB' \sim \triangle AOB$.



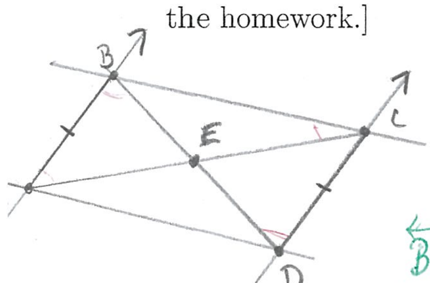
pf) $OA = OB = r_1$ and $OA' = OB' = r_2$. Further $\angle A'OB' \cong \angle AOB$.
So $\frac{OA}{OA'} = \frac{OB}{OB'}$ and $\angle A'OB' \cong \angle AOB$ implies $\triangle A'OB' \sim \triangle AOB$ by SAS for similar triangles. $\square$

27. Let K be a circle with center O, and $\overline{AB}$ be a chord with midpoint M. Assuming $O \neq M$, prove that $\overline{OM}$ is perpendicular to $\overline{AB}$.



pf) $\triangle AOB$ is isoceles since $OA = OB$. M is the midpoint of the base of an isoceles triangle so OM is a median. By previous homework the median ~~of an iso triangle~~ to the base of an isoceles triangle is also an altitude. Hence $\overleftrightarrow{OM} \perp \overleftrightarrow{AB}$ $\square$

28. Let $ABCD$ be a quadrilateral with $\overleftrightarrow{AB} || \overleftrightarrow{CD}$ and $AB \cong CD$. Prove $ABCD$ is a parallelogram. [Clearly you must show this directly and may not use the equivalences that we showed in class or the homework.]

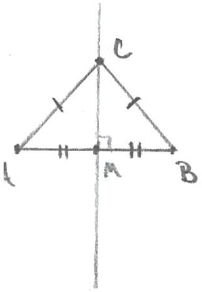


To show ABCD is a parallelogram I only need to show $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$ (since we are given $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$).

pf) Let E be the intersection of the diagonals $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$. $\overleftrightarrow{BD}$ is a transversal of $\overleftrightarrow{AB}$; $\overleftrightarrow{DC}$ so $\angle ABE \cong \angle EDC$.
Also, $\overleftrightarrow{AC}$ is a transversal of $\overleftrightarrow{AB}$; $\overleftrightarrow{DC}$ so $\angle ECD \cong \angle EAB$.
Since we are given $\overline{AB} \cong \overline{DC}$ then by ASA $\triangle AEB \cong \triangle CED$.
So, $\overline{BE} \cong \overline{DE}$ and $\overline{EC} \cong \overline{AE}$. By vertical angles $\angle BEC \cong \angle AED$. Hence $\triangle BEC \cong \triangle DEA$ by SAS. Then $\angle ECB \cong \angle EAD$. As these are alternate interior angles of $\overleftrightarrow{AC}$, $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$

Cont.

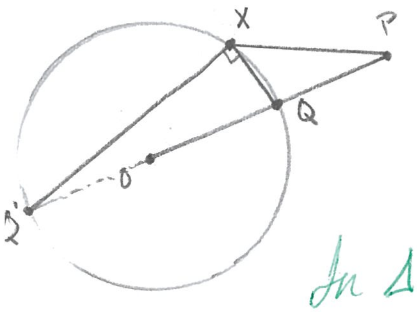
29. Let P be the set of all points that are equidistant to the endpoints of a segment $\overline{AB}$. Prove that the set P of all such points forms a line that is the perpendicular bisector of the segment $\overline{AB}$.



Pf) Let M be the midpoint of $\overline{AB}$. Then $M \in P$.
Let C be another point in P. [We may construct C by intersecting two circles of the same radius centered at A and B.]
Then $\triangle ACB$ is iso with $\overline{CM}$ a median. Hence $\overleftrightarrow{CM} \perp \overleftrightarrow{AB}$.

Now suppose C' is any other point in P. By the same argument for C $\overleftrightarrow{C'M} \perp \overleftrightarrow{AB}$. Since $\overleftrightarrow{CM}$ and $\overleftrightarrow{C'M}$ are perpendicular to the same line they must be parallel. Since both lines contain M they must be the same line by ~~p~~ Playfair. Hence every point of P falls on the same line $\overleftrightarrow{CM}$ which is the perp. bisector of $\overline{AB}$. $\square$

30. Let P be a point outside of a circle with center O, and let $\overline{OP}$ intersect the circle at Q. Prove that Q is the point on the circle closest to P.



Pf) Let X be any other point on the circle and let Q' be the antipodal point to Q. If $X = Q'$ then it is clear that $PQ' > PQ$ by the sum of the parts.

In $\triangle Q'XQ$ $\angle Q'XQ = 90°$ since $\overline{Q'Q}$ is a diameter (i.e. $\widehat{Q'Q} = 180$).
Thus $\angle XQQ' < 90°$ and so $\angle XQP > 90°$ as a supplementary pair.
There can be only one obtuse angle in a triangle thus $\angle PXQ < \angle PQX$.
Hence $PQ < QX$ since the greater angle of a triangle is opposite the greater side.
Thus $PQ < QX$ for any other X on the circle and so Q is the closest point on the circle to P. $\square$

The End.