Math 45021 | Euclidean Geometry | Spring 2017
Quiz 2 | | Matt Alexander
Name: Key | | Quiz Score: ____ /20

1. Mark as true or false the following statements. Suppose $\triangle ABC$ and $\triangle DEF$ are such that:

A) **T** F There can be at most one obtuse angle in a triangle.

B) T **F** There exists a triangle with side lengths of 7, 9, and 38.

C) **T** F A triangle with side lengths of 3, 4, and 5 is a right triangle.

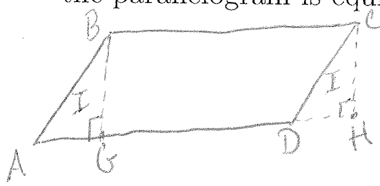
D) T **F** Two triangles with equal perimeter have equal area.

2. Suppose the interior angle of a regular polygon (a polygon in which all angles have the same measure) is 135°. How many sides must the polygon have?

The sum of all interior angles of an n-sided polygon is $(n-2)180$.
So each angle has a measure of $\frac{n-2}{n}(180)$.
Set equal to 135 and solve $\frac{n-2}{n}(180) = 135$

$n[(180)-135] = 360 \Rightarrow 45n = 360 \Rightarrow n = 8$ $\square$

3. Prove that the area of a parallelogram $ABCD$ is equal to its base times its height. [Hint: Show that the parallelogram is equidecomposible to the rectangle with the same base and height.]



Let $\overline{BG}$ be perp to $\overleftrightarrow{AD}$ with G on $\overleftrightarrow{AD}$.
or Let G be the unique point on $\overleftrightarrow{AD}$ such that the distance from B to $\overleftrightarrow{AD}$ is BG.
Let H be the point on $\overleftrightarrow{AD}$ such that $\overleftrightarrow{CH} \perp \overleftrightarrow{AD}$.
Then I claim that BCHG is a rectangle as $\overleftrightarrow{GH} \parallel \overleftrightarrow{BC}$ and $\overleftrightarrow{BG} \parallel \overleftrightarrow{CH}$ since they are perp to the same line. Further $\angle BGH$ & $\angle CHG$ are 90° so all angles must be 90°. So area(BCHG) = GH·BG.
Now I want to show ABCD decomposes into BCHG by showing $\triangle ABG \cong \triangle DCH$. Notice both ABCD & BGHC are parallelograms so $AB \cong DC$ and $BG \cong CH$. Then by Hypotenuse-leg congruence (SSA for right triangles) $\triangle ABG \cong \triangle DCH$. So then ABCD and BCHG are equidecomposible.
Therefor area(ABCD) = area(BCHG) = GH·BG = AD·BG the base × height. $\square$

GH = BC = AD