1. Prove that
\[ \left| \frac{a - b}{1 - \overline{ab}} \right| = 1 \]
if either \(|a| = 1\) or \(|b| = 1\). What exception must be made if \(|a| = |b| = 1\)?

2. Find the real and imaginary parts of \(z^z\).

3. Show that any four distinct points can be carried by a linear transformation to positions 1, \(-1\), \(k\), \(-k\), where the value of \(k\) depends on the points.

4. Map the common part of the discs \(|z| < 1\) and \(|z - 1| < 1\) conformally onto the unit disk \(|z| < 1\).

5. Compute
   (a) \(\int_{|z|=1} e^z z^{-n} dz\),
   (b) \(\int_{|z|=2} z^n(1 - z)^m dz\), where \(n, m \in \mathbb{Z}\).

6. (a). Show that there does not exist an entire function \(f\) whose only zero occurs at \(1/2\) and is such that \(|f(z)| = 1\) for every \(z, |z| = 1\).
   (b). Find all functions \(f\) that are analytic in an open set containing the closed unit disk \(|z| \leq 1\), such that the only zero of \(f\) inside the disk occurs at \(z = 1/2\) and is such that \(|f(z)| = 1\) for every \(z, |z| = 1\).

7. Explain how many zeros the polynomial \(P(z) = z^8 + 4z^6 + 3z^4 + z^2 + 5\) has in the first quadrant.

8. Let \(f\) be analytic in the unit disk \(|z| < 1\). Show that \(|f(z)| \leq 1\) for every \(|z| < 1\) implies
\[ \frac{|f'(z)|}{1 - |f(z)|^2} \leq \frac{1}{1 - |z|^2}, \]
for every \(z, |z| < 1\).

9. (a). Suppose that we are given a sequence of entire functions \((f_n)\) such that \(f_n(0) \to 1\). What, if anything, can we say about a limit function \(f\)?
   (b). Suppose that we are now given a sequence of analytic functions \((f_n)\) such that \(f_n(0) \to 1\), where each \(f_n : D = \{|z| < 1\} \to \{|z| \leq 2\}\). What, if anything, can we say about a limit function \(f\)?
   (c). Suppose that we are now given a sequence of analytic functions \((f_n)\) such that \(f_n(0) \to 1\), where each \(f_n : D \to D\). What, if anything, can we say about
a limit function $f$?

10. (a). Expand $\frac{2z+3}{z+1}$ in powers of $z - 1$.
(b). What is the radius of convergence?