QUALIFYING EXAMINATION IN PROBABILITY

August 18, 2003

Directions

1. Answer questions completely: a fully-done problem is far more revealing of your understanding than two half-done attempts.
2. State your reasons for your claims. You may cite and use standard results. However, you have to demonstrate that *you* know why things are so; you cannot assume that we know that you know. That's the purpose of this examination! An exercise that states a standard result of the theory requires that you supply a proof.

1. Let F be an increasing function on the real line. Then the set of discontinuities of F is countable.

2. A point x is said to belong to the support of the distribution function if for every $a > 0$ we have $F(x+a) - F(x-a) > 0$. Prove that the support of any distribution function is a closed set.

3. Count the number of subsets of an n-point set.

4. If F is a Borel field generated by a countable collection of pairwise disjoint sets , then each member of F is just the union of a subcollection of these sets.

5. If two random variables are equal almost everywhere, then they have the same probability measure.

6. The random variable X is independent of itself if and only if it is constant with probability one.