PROBLEMS

1.1. There are six conditions in the definition of congruent triangles, labeled (1), (2), (3), (4), (5), (6). Make a list of all (20) three-element subsets of $\{(1), \ldots, (6)\}$ and, for each one, tell whether it as hypothesis implies $\triangle ABC \cong \triangle DEF$.

***1.2.** In our proof of SSS we choose G such that $\angle GBC \cong \angle E$, $\angle GCB \cong \angle F$ and such that G was on the same side of $\overleftrightarrow{BC}$ as A. Find a proof of SSS with G on the opposite side of $\overleftrightarrow{BC}$.

***1.3.** Prove SSA for right triangles: If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$ and if $\angle B = 90°$ and $\angle E = 90°$ then $\triangle ABC \cong \triangle DEF$.

***1.4.** Prove SAA: If $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.

1.5. Find the mistake in the following "proof" that all triangles are isosceles, see Fig. 1.17. Let $\triangle ABC$ be any triangle. We will prove that $\overline{AB} \cong \overline{AC}$. Draw

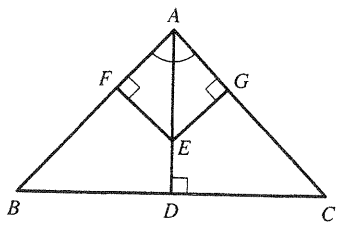

Figure 1.17: All triangles are isosceles

the perpendicular bisector of $\overline{BC}$ and the angle bisector of $\angle A$. Let these two lines intersect at the point E. From E draw lines $\overline{EF}$ and $\overline{EG}$ perpendicular to $\overline{AB}$ and $\overline{AC}$.

Now, $\triangle BED \cong \triangle CED$ by SAS, so $\overline{BE} \cong \overline{CE}$. Also, $\triangle AEF \cong \triangle AEG$ by SAA (see exercise 4), so $\overline{EF} \cong \overline{EG}$. Since $\overline{EF} \cong \overline{EG}$ and $\overline{BE} \cong \overline{CE}$, $\triangle BEF \cong \triangle CEG$ by SSA for right triangles (see exercise 3). From $\triangle AEF \cong \triangle AEG$ we conclude that $\overline{AF} \cong \overline{AG}$; from $\triangle BEF \cong \triangle CEG$ we conclude that $\overline{BF} \cong \overline{CG}$; and by addition we get $\overline{AB} \cong \overline{AC}$.

1.6. Let ℓ be a line and P a point on ℓ. Construct a line that contains P and that is perpendicular to ℓ.

1.7. Let ℓ be a line and P a point not on ℓ. Construct a line that contains P and that meets ℓ at a 45° angle. Construct a line that contains P and that meets ℓ at a 30° angle. (To do this exercise you need to use the fact that the sum of the angles in any triangle is 180°.)

***1.8.** Given triangles $\triangle ABC$ and $\triangle DEF$ such that $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$ and $\angle A > \angle D$, prove that $BC > EF$.

1.9. Given BC and $A_1, \ldots, A_n$, prove that $BC < BA_1 + A_1A_2 + A_2A_3 + \ldots + A_nC$.

1.10. This exercise extends the definition of congruence from triangles to more general polygons and explores some consequences. Let $2n$ points ($n \geq 3$, a whole number) $A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ be given. Think of these points as vertices and $A_1A_2 \ldots A_n$ as the polygon formed by the union of the segments