Test 4
MATH 45021 - Euclidian Geometry
Summer I - 2014

Print Name Signature Date

1. Let $\triangle ABC$ be a right triangle and D be its centroid. If $AB = 3$ and $BC = 4$, find BD.

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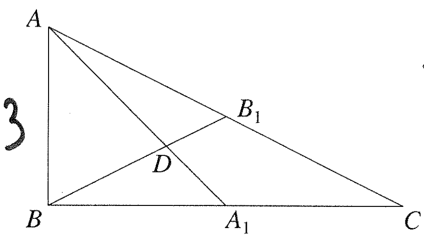

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D centroid $\Rightarrow BA_1 = A_1C$
$AB_1 = B_1C$ and
$AD = 2 \cdot DA_1$

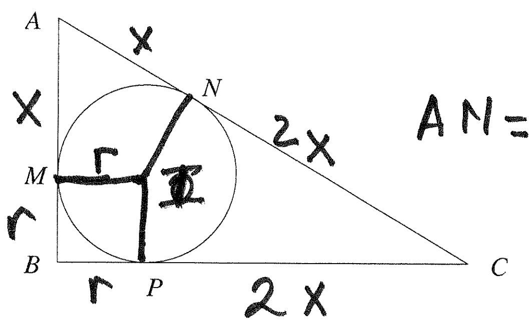
~~In $\triangle AA_1B$ Pythagoras $\Rightarrow AA_1^2 = 9 + 2^2 = 13$~~
~~$\Rightarrow AA_1 =$~~ (I thought we needed AD)

$\cancel{BB_1} =$ In $\triangle ABC$ BB_1 is median $\Rightarrow BB_1 = \frac{AC}{2}$

$= \frac{\sqrt{9+16}}{2} = \frac{5}{2} \Rightarrow BD = \frac{2}{3} BB_1 = \frac{2}{3} \cdot \frac{5}{2} = \frac{5}{3}$

(If you forgot that $BB_1 = \frac{AC}{2}$ you can remember by constructing $AA' \parallel BC$, $|AA'| \equiv |BC|$
$\Rightarrow AA'CB$ rectangle & $|BA'| \equiv |AC|$, $BA' \cap AC = B_1$

2. In a right triangle with hypotenuse 6, the inscribed circle splits the hypotenuse in two segments, one being twice as long as the other. Find the perimeter of the triangle.



$$AN = \frac{NC}{2}$$

Because $MI \perp AB$

$PI \perp BC \implies MIPB$ rectangle

$\angle B = 90^\circ$

But $|MI| = |IP| \implies MIPB$ is a square.

perimeter $= x + x + 2x + 2x + r + r = 6x + 2r$

Also: $(2x + x)^2 = (x+r)^2 + (2x+r)^2$

$$9x^2 = x^2 + 2xr + r^2 + 4x^2 + r^2 + 4xr$$

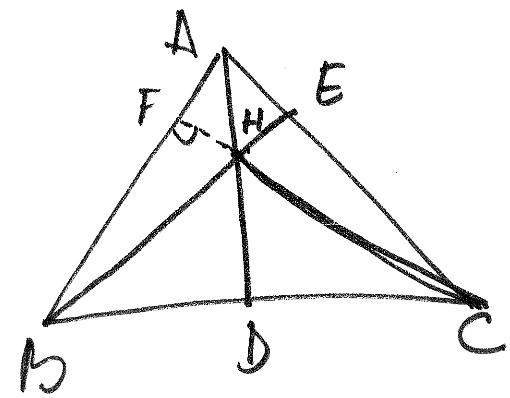
$$4x^2 - 6xr + 2r^2 = 0$$

$$2x^2 - 3xr + r^2 = 0$$

$$(2x - r)(x - r) = 0$$

$$\underline{x = \frac{r}{2}} \text{ or } \underline{x = r}$$

3. What is the orthocenter of $\triangle BCH$ where H is the orthocenter of the $\triangle ABC$? Give complete explanations.



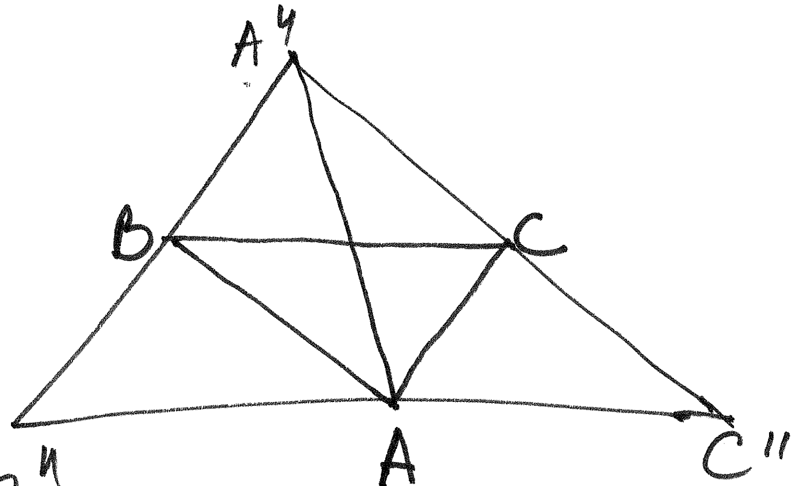
Let AD, BE, CF heights in $\triangle ABC$.
H is orthocenter.
Then in $\triangle BHC$: $HD \perp BC \Rightarrow$ HD height
$CH \perp AB$ because $CH = CF$
$\Rightarrow AB \perp CH \Rightarrow$ BA height.

$CA \perp BE \Rightarrow CA \perp BH \Rightarrow$ CA height

Therefore the orthocenter of $\triangle BHC$ is

$CA \cap BA \cap HD = \{A\}$.

4. Let $\triangle A"B"C"$ be the anti complimentary triangle of $\triangle ABC$. Identify the
a) centroid
b) circumcenter
d) center of the nine-point circle of $\triangle A"B"C"$.
Give complete explanations.

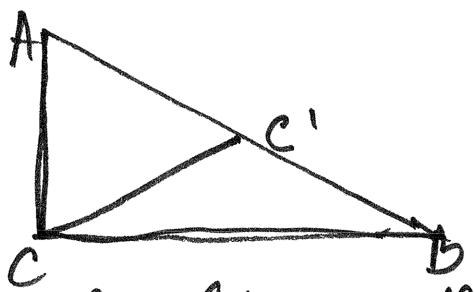


d) The 9 point circle of $\Delta A"B"C"$ contains the middles of $A"B"$, $B"C"$, $A"C"$ hence A, B, C. Therefore it has to be the circumcircle for ΔABC $\Rightarrow N" = O$
center of the 9-point circle.

Because $A"A$, $C"B$, $B"C$ are medians in $\Delta A"B"C"$ the centroid of $A"B"C"$ is $AA" \cap B"C \cap C"B = G"$.
But $BC" \cap AC = B'$ with $|AB'| \equiv |B'C|$ (by similar triangles, for example) $\Rightarrow BB'$ is median in ΔABC.
Similarly $CB"$ & $AA"$ are medians $\Rightarrow G =$
$= AA" \cap BC" \cap CB" = G"$. $\boxed{G = G"}$

b) let $O"$ be the circumcenter of $\Delta A"B"C"$. Then $O"C \perp A"C"$
$\Rightarrow O"C \perp AB$. Similarly $O"B \perp AC$, etc
$\Downarrow$
$O"C$ is height in ΔABC
So $O"C$, $O"B$, $O"A$ are heights in $\Delta ABC \Rightarrow O" = H$

5. Let $\triangle ABC$ have a right angle C. Prove that the median CC' is the Euler line. Is there any other type of triangle in which CC' will be the Euler line?



The Euler line is the line containing O, H & G

CC' is median, therefore $G \in CC'$

Moreover, because $\triangle ABC$ is a right triangle AB is diameter in its circumcircle $\Rightarrow$ C' is its center. So $C' = O \Rightarrow O \in CC'$

$\Rightarrow CC'$ is Euler line.

In an isosceles $\triangle$, with $|AC| = |BC|$ this is also true because the median is also the height, therefore $H \in CC'$, $G \in CC'$!

And, of course, equilateral $\triangle$s.