$$E(X) = \int_{\mathbb{R}} x f(x)\,dx$$

$$E(X) = E(1_A) = 1 \cdot P(1_A = 1) + 0\,P(1_A = 0)$$

$$= P(A)$$

$$\{1_A = 1\} = \{\omega \mid 1_A(\omega) = 1\} = \{\omega \mid \omega \in A\}$$
$$= A$$

$$1_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}$$

$$(\Omega, \mathcal{F}, P)$$

$$E(X) = \int X\,dP$$

$$\int f(x)\boxed{dx}$$

$$\int_0^1 1\,dx = 1$$

$$\int_0^2 dx = 2 \qquad \int_0^3 dx = 3$$

$$\int_{-1}^{1} dx = 2$$

<u>Lebesgue measure</u>

If X is a Borel r.v. $\int \ldots dP$

is the same as $\int dx$ it gives me the length of the set I am integrating.

Ex

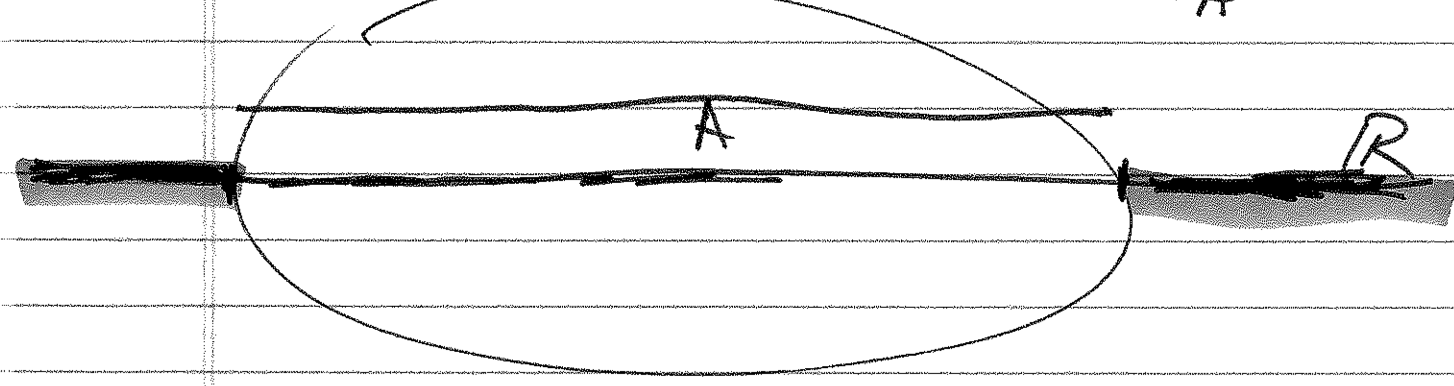
$$\int_A dP = 1+1+.5 = P(A)$$

? length of A

$$A = (1,2) \cup (3,4) \cup (100, 100.5)$$

$$E(1_A) = \int 1_A \, dP = \int_A 1 \, dP$$

$$= \int_A dP = P(A)$$



$$E(e^{tX}) = \int e^{tX} \, dP$$

$$\frac{d}{dt} E(e^{tX}) = \frac{d}{dt} \int e^{tX} \, dP = \int \frac{de^{tX}}{dt} \, dP$$

$$\int x e^{tX} \, dP$$

e^{tX} $\quad t=0$

for $t=0$ I get $\int X \cdot 1 \, dP = \int X \, dP$

$$= E(X)$$

$E(X|Y)$ is a function in y

$$\boxed{E(X|Y)}(y) = E(X|Y=y)$$

it is a r.v. on $\mathbb{R}$

$$E(X|Y=0) = E(X|Y)(0)$$
$$E(X|Y=5) = E(X|Y)(5)$$
$$E(X|Y=10) = E(X|Y)(10)$$
$$E(X|Y=15) = E(X|Y)(15)$$

$$E(X|Y=0) = \sum_x x \, P(X=x \mid Y=0)$$

$$= 0 \cdot P(X=0 \mid Y=0)$$

$$+ 1 P(X=1 \mid Y=0) + 5 P(X=5 \mid Y=0)$$

$$+ 10 P(X=10 \mid Y=0) + 6 P(X=6 \mid Y=0)$$

$$+ 15 P(X=15 \mid Y=0) + 16 P(X=16 \mid Y=0)$$

$$P(X=6 \mid Y=0) = \frac{P(X=6 \quad Y=0)}{P(Y=0)} = 0$$

$$P(X=1 \mid Y=0) = \frac{P(X=1, Y=0)}{P(Y=0)} = \frac{1/8}{\frac{1}{4}}$$

$$= \frac{1}{2}$$