$E(X \mid \text{first choice is } D_1)$

$= 1 + [E(X \mid \text{2nd choice is } D_2)\, P(D_2)$

$+ E(X \mid \text{2nd choice is } D_3)\, P(D_3))$

$= 1 + [5 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2}) = 5$

$E(X \mid \text{first choice is } D_2) = 2 + \frac{7}{2}$

---

1 2 3

$$E(\underline{X}) = \sum x\, P(X=x)$$

$$= 3\,P(X=3) + 4\,P(X=4) + 5\,P(X=5) + \dots$$

$$= 3\cdot\frac{1}{3} + 4\cdot\frac{1}{9} + 5\left(\frac{1}{9} + \frac{1}{27}\right) + 6\left(\frac{2}{27} + \frac{1}{3^4}\right)$$

$$6\left(\frac{2}{3^3} + \frac{1}{3^4}\right)$$

$$+ 7\left(\frac{1}{3^5} + \frac{3}{3^4} + \frac{1}{3^3}\right) + \dots$$

$$= \frac{1}{3}\cdot 3 + \frac{1}{3^2}(4+5) + \frac{1}{3^3}(5 + 2\cdot 6 + 7)$$

$$+ \frac{1}{3^4}(6 + 3\cdot 7 + 3\cdot 8 + 9)$$

$$+ \frac{1}{3^5}(7 + 4\cdot 8 \quad)$$

$$E(X) = E(X \mid \text{first choice is } D_1)\cdot P(D_1)$$

$$+ E(X \mid \text{first ch } D_2)\, P(D_2)$$

$$+ E(X \mid \text{f ch } D_3)\, P(D_3)$$

$$1/3 \cdot \frac{1}{2}$$

$\mathcal{Y} = \{\emptyset, \Omega, B^c, B\}$

$$E(1_A \mid B) = 1 \cdot P(A \mid B) + 0\,P(A^c \mid B)$$
$$= P(A \mid B)$$

$$1_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \in A^c \end{cases}$$

$$\left(E(X \mid \mathcal{Y})\right)^2 \le E(X^2 \mid \mathcal{Y})$$
$$|E(X \mid \mathcal{Y})| \le E(|X| \mid \mathcal{Y})$$

$$(\Omega, \mathcal{F}, P) \xrightarrow{X} \mathbb{R}$$

$$X^{-1}((0,1)) \in \mathcal{F}$$
$$X^{-1}([1,2)) \in \mathcal{F}$$

$\sigma(X)$ = smallest σ-field that contains $X^{-1}(B)$ when B is a Borel set in $\mathbb{R}$.

$$E(S_n \mid \mathcal{F}_p) = S_p + (n-p)\mu \qquad p < n$$

$$\shortparallel$$

$$E(X_1 + X_2 + X_3 + \cdots + X_n \mid \sigma(X_1, X_2, \ldots X_p))$$

$$= E(\underbrace{X_1 + X_2 + X_3 + \cdots + X_p} + \underbrace{X_{p+1} + X_{p+2} + \cdots + X_n} \mid \mathcal{F}_p)$$

linearity

$$= E(X_1 + X_2 + \cdots + X_p \mid \mathcal{F}_p) + E(X_{p+1} + X_{p+2} + \cdots + X_n \mid \mathcal{F}_p)$$

$X_1, X_2, \ldots, X_p$ are $\mathcal{F}_p$-measurable

$$= \underline{X_1 + X_2 + \cdots + X_p} + E(X_{p+1} + X_{p+2} + \cdots + X_n \mid \mathcal{F}_p)$$

$$= S_p + E(X_{p+1} + X_{p+2} + \cdots + X_n \mid \mathcal{F}_p)$$

linearity

$$= S_p + E(X_{p+1} \mid \mathcal{F}_p) + E(X_{p+2} \mid \mathcal{F}_p) \cdots + E(X_n \mid \mathcal{F}_p)$$

$$= S_p + \underset{\mu}{\underset{\shortparallel}{E(X_{p+1})}} + \underset{\mu}{\underset{\shortparallel}{E(X_{p+2})}} + \cdots + \underset{\mu}{\underset{\shortparallel}{E(X_n)}}$$

$$= S_p + (n-p)\mu.$$