

15. (25 pts) Let $W(t)$ be a Bm, and

$$U(t) = \frac{1}{t}W^2(t).$$

a) Is $U(t)$ a Bm?

b) Compute its mean.

c) Compute $Cov[U(t), U(s)] = E[U(t)U(s)] - E[U(t)]E[U(s)]$. Are $U(t)$ and $U(s)$ independent. Can U be a Bm?

HINT: Here use that two normally distributed rvs, X and Y , with mean 0, verify:

$$E[X^2Y^2] = E[X^2]E[Y^2] + 2(E[XY])^2.$$