Show $W(t+T)-W(T)$ is a B.m. if $W(t)$ is a B.m.

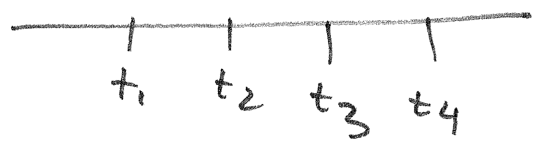
Denote $W(t+T)-W(T)=V(t)$

a) $V(0)=0$. Indeed

$$W(0)=W(T)-W(T)=0$$

b) $V_{t_2}-V_{t_1}$ and $V_{t_4}-V_{t_3}$ are independent for any t_1, t_2, t_3, t_4

s.t.:

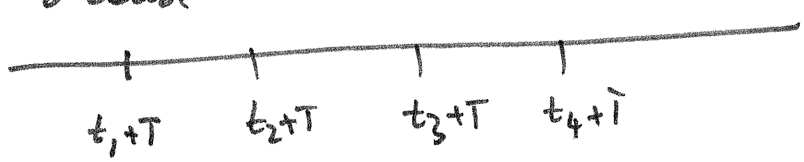


Indeed:

$$V_{t_2}-V_{t_1} = W(t_2+T)-W(T)-W(t_1+T)+W(T)=W(t_2+T)-W(t_1+T)$$

$$V_{t_4}-V_{t_3} = W(t_4+T)-W(t_3+T) \text{ in the same way}$$

Now, because



(t_1+T, t_2+T) and (t_3+T, t_4+T) are non-overlapping

$W(t_4+T)-W(t_3+T)$ & $W(t_2+T)-W(t_1+T)$ are independent,

hence $V_{t_4}-V_{t_3}$ & $V_{t_2}-V_{t_1}$ are independent.

c) V_t-V_s is $N(0, t-s)$. Indeed, $V_t-V_s=W(t+T)-W(T)$
$-W(s+T)+W(s) = W(t+T)-W(s+T) \sim N(0, t+T-s-T)$
$= N(0, t-s)$.

d) $t \to V_t$ is continuous because $t \to W(t+T)$ is.