JUMP DIFFUSION OPTION WITH TRANSACTION COSTS

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Abstract. We develop a method for pricing a long and a short position in an European option on jump diffusion process, when the jump component of the stock return represents "non-systematic" risk, inclusive of transaction costs. We compute the total transaction costs and the turnover for different options, transaction costs, and revision intervals.

1. Introduction

The option pricing theory developed by Black and Scholes assumes perfect frictionless markets. It relays on the arbitrage argument by which investors can use a replicating portfolio of a long position in the risky asset and a short position in bonds to exactly reproduce the return structure of the option. But this portfolio must be continuously adjusted, meaning that the weights on the portfolio must be continuously changed in order to eliminate all the risk from the total position (short a call option, long in risky asset and short in bonds).

There are two problems with this model. First, it assumed a perfect market, and therefore that the rebalancing is costless. But if we introduce transaction costs, the constant rebalancing used in the Black–Scholes setup will be infinitely costly (no matter how small the transaction costs are) since the diffusion processes have infinite variation. Second, for the arbitrage theory to be true, Black and Scholes assumed that the underlying stock price follows a stochastic process with a continuous path. In many cases (jump-diffusion, Markovian diffusion, stochastic volatility-stochastic interest rate) we can not apply their technique.

There are several papers addressing the first problem and quite a few papers dealing with different models which relax Black and Scholes model. Among the papers addressing the effect of transaction costs on option prices, we can cite Leland's (1985), in which he uses a continuous-time framework and shows that there is an alternative to the Black and Scholes replicating theory in the presence of proportional transaction costs. The strategy can be used to replicate option returns inclusive of transaction costs, with accuracy that increases as the revision period becomes shorter. Merton (1990) uses a discrete-time framework and derives the current option value when there are proportional transaction costs on the underlying asset. Boyle and Vorst (1992) use a similar approach, but they extend the analysis to several periods and construct portfolios that replicate a long and short European call. They showed that the zero-transaction-cost option values lies between the short call price and the long call price (as it should) and as the transaction costs tend to zero, the two processes converge to the Cox-Ross-Rubinstein option price. Moreover, they proved that when the number of periods (n) is large and the transaction rate as a fraction of the amount traded (k) is low, the initial value of the hedge portfolio under the dynamic strategy that replicates a long call option (resp. a short call option) at the maturity date and is self-financing inclusive of transaction costs, is approximately equal to the Black–Scholes value, but with modified variance given by

$$\sigma^2\left(1+\frac{2k\sqrt{n}}{\sigma\sqrt{T}}\right), \ \left(\text{resp. } \sigma^2\left(1-\frac{2k\sqrt{n}}{\sigma\sqrt{T}}\right)\right),$$

where T is the time to option maturity and σ is the volatility of the risky asset.

Regarding the second problem, we will consider the jump-diffusion process case (that is, we assume that the underlying asset price follows a jump-diffusion process). These processes were incorporated in the option valuation for the first time by Merton (1976) as an alternative to the Black and Scholes model, then by Bates (1991), Madan and Chang (1996). Amin (1993) developed a discrete time model to value the option when the underlying process is a jump-diffusion process. Beginning with a discrete time model which converges weakly to the diffusion component of the jump diffusion process, they can superimpose jumps (rare events) on the existing local price changes. Using the fact that

$$0 = \lambda E_Y[V_Y(i+1)] + (1-\lambda)V_{\pm}(i+1),$$

where $V_{\pm}(i+1)$ is the value of the portfolio at the time $i+1$ if the risky asset undergoes a local price change, $V_Y(i+1)$ is the value of the portfolio if the stock had a jump in price, Y is the capital gain return on the stock when the rare event (jump) occurs, E_Y is the expected value with respect to Y, and λ is the probability that a jump will occur; they computed the option price at date i ($C(i)$):

$$C(i) = \max\left[F(i), \frac{E_{\tilde{Q}(i)}[C(i+1)]}{r}\right],$$

where $F(i)$ is the payoff if the option is exercised at the current time and date.

The paper is organized as follows: Section 2 reviews Merton's (1976) results about option pricing when the returns follow a jump diffusion model, without transaction costs, Section 3 presents the option pricing using jump diffusion model inclusive of transaction costs, and Section 4 presents some empirical results about the total transaction costs and turnover.

2. The Option Pricing when Underlying Stock Returns are Discontinuous: A Review

Merton (1976) studied option pricing in the case that the stock price of the underlying asset can be described as a jump diffusion process. The underlying stock price returns are a mixture of continuous time processes and a Poisson process; it can be described by

$$\frac{dS}{S} = (\alpha - \lambda k)dt + \sigma dZ + dq \tag{1}$$

where α is the instantaneous expected return on the stock; σ^2 is the instantaneous variance of the returns, conditional on no arrivals of important new information (i.e., the Poisson event does not occur); $q(t)$ is the independent Poisson process; dq and Z are assumed to be independent; λ is the mean number of arrivals per unit of time, $k \equiv E(Y-1)$ where $Y-1$ is the random variable percentage change in the stock price if the Poisson event occurs; E_y is the expectation operator over the random variable y.

Equation (1) can be written in a more explicit form as:

$$\frac{dS}{S} = (\alpha - \lambda k)dt + \sigma dZ,$$

if the Poisson event does not occur, and

$$\frac{dS}{S} = (\alpha - \lambda k)dt + \sigma dZ + Y - 1$$

if the Poisson event does occur.

Then he assumes that the jump represents pure non-systematic risk and proved that the differential equation that is verified by the option price, when the dynamics of the stock price is as given above, is

$$\frac{1}{2}\sigma^2 S^2 C_{SS} + (r - \lambda k)SC_S + C_\tau - rC + \lambda E(C(SY,\tau) - C(S,\tau)) \tag{2}$$

subject to the boundary conditions

$$C(0,\tau) = 0$$

$$C(S, 0) = \max[0, S - P],$$

where $C(S, \tau)$ is the option price written as a function of the time until expiration and S, P is the exercise price, r is the risk free rate, and the subscripts represent partial derivatives with respect to the corresponding variable.

It is important to note that even though the jumps represent non-systematic risk, the jump component does affect the equilibrium option price.

Define $W(S, \tau; P, \sigma^2, r)$ to be the Black–Scholes formula for the no-jump (continuous) case. Then

$$C(S, \tau) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} E_n \left[W(S X_n e^{-\lambda k Z}, \tau; P, r) \right], \tag{3}$$

where $X_0 \equiv 1$ and X_n is a random variable which has the same distribution as the product on n independent random variables distributed identically to Y in (1).

3. Option Pricing Method with Positive Transaction Costs

Consider a portfolio formed with one option and $-N$ shares of stock. The assumptions of our model are:

- The portfolio is revised every Δt units of time, where Δt is a non-infinitesimal, fixed time-step; note that we do not consider $\Delta t \to 0$. For example, the portfolio may be rehedged every day at 9:00 am.
- The random walk is given in discrete time by

  $$dS = (\alpha - \lambda k) S\, dt + \sigma S Z \sqrt{dt} + S\, dq,$$

  where α, λ, k, σ, dq are as in Equation (1).
- Transaction costs for buying or selling the asset are proportional to the monetary value of the transaction. Thus, if ν shares are bought ($\nu > 0$) or sold ($\nu < 0$ at price S, then the transaction costs are $\rho|\nu|S$, where ρ is a constant depending on the individual investor. A more complex cost structure can be incorporated into the model with only a small amount of effort: a fixed cost for each transaction, a cost proportional to the number of shares traded (besides the cost proportional to the value of the assets traded).
- The source of the jump is a firm (or even industry) specific information, hence the jump component will represent "non-systematic" risk, i.e. the jump component is uncorrelated with the market.

Theorem: *A long position in an European option on jump diffusion inclusive of transaction costs can be priced using Merton's formula with a modified variance*

$$\hat{\sigma}^2 = \sigma^2 - \frac{2}{\Delta t}\rho E\left(\left|\frac{\Delta S}{S}\right|\right).$$

Since S follows Equation (1), the option C will follow an equation of the same type (when there are jumps in S there are jumps in C), but with different parameters.

$$\frac{dC}{C} = (\alpha_C - \lambda k_C)dt + \sigma_C\, dZ + dq_C \tag{4}$$

where α_C, k_C, and σ_C are defined as in Equation (1) but for the option price C, i.e. α_C is the instantaneous expected return on the option; σ_C^2 is the instantaneous variance of the return, conditional on the Poisson event not occurring); $q_C(t)$ is the independent Poisson process with parameter λ, $k_C \equiv E(Y_C - 1)$ where $Y - 1$ is the random variable percentage change in the option price if the Poisson event occurs.

Looking at C as a function of S and time to the expiration τ, $C(S, \tau)$ since the Poisson process is an increasing process, therefore with bounded variation, we can apply Ito formula and obtain:

$$dC = C_\tau d\tau + C_S dS + \frac{1}{2}C_{SS}(dS)^2. \tag{5}$$

Using (1) we get

$$dC = \left[C_\tau + (\alpha - \lambda k)SC_S + \frac{1}{2}C_{SS}S^2\sigma^2\right]d\tau + SC_S\sigma dZ + SC_S dq. \tag{6}$$

which implies that

$$(\alpha_C - \lambda k_C)C = C_t + (\alpha - \lambda k)S + \frac{1}{2}C_{SS}\sigma^2, \tag{7}$$

and

$$\sigma_C C = SC_S\sigma. \tag{8}$$

Observe now that if the percentage increase in S is Y then the percentage increase in C is $Y_C = \frac{C(SY,t)}{C(S,t)}$. Hence

$$\begin{aligned} k_C = E_{Y_C}[Y_C - 1] &= E_{Y_C}\left[\frac{C(SY,t) - C(S,t)}{C(S,t)}\right] \\ &= \frac{E_Y[C(SY,t) - C(S,Y)]}{C(S,t)}. \end{aligned}$$

Therefore

(9)
$$\alpha_C C = C_t + (\alpha - \lambda k)SC_S + \frac{1}{2}S^2 C_{SS}\sigma^2 + \lambda E_Y[C(SY,t) - C(S,Y)]$$

and
$$\sigma_C C = SC_S\sigma.$$

Return now to our hedge portfolio. Inspection of return dynamics shows that the only source of uncertainty in the return is the jump component of the stock. But by hypothesis, such components represent only "non-systematic" risk, and therefore the "beta" of this portfolio is zero. If the Capital Asset Pricing model holds, then the expected return on all zero-beta securities must equal the riskless rate. Therefore, over the small interval of time Δt
$$(\alpha_C C)\Delta t - (N\alpha S)\Delta t = r(C - NS)\Delta t$$

If we introduce transaction costs, then we have that
$$(\alpha_C C)\Delta t - (N\alpha S)\Delta t - E(\rho S|\nu|) = r(C - NS)\Delta t. \tag{10}$$

Substituting (9) into (11) we get

(11)
$$\begin{aligned}(C_\tau + (\alpha - \lambda k)SC_S + \frac{1}{2}S^2 C_{SS}\sigma^2 + \lambda E_Y[C(SY,t) - C(S,Y)])\Delta t \\ - (N\alpha S)\Delta t - E(\rho S|\nu|) = r(C - NS)\Delta t.\end{aligned}$$

In the "no-jump" case, Black and Scholes derive the number of shares of stock to be bought, for each option sold, that will create a riskless hedge. Namely $N = C_S = \Phi(d_1)$. In the jump case, there is no such riskless mix. However, there is a mix which eliminates all systematic risk, and in that sense, is a hedge. The number of shares for this hedge is equal to C_S (see Appendix B) which can be obtained by differentiating formula (3). Note: while in both cases, the appropriate number of shares is equal to the derivative of the option pricing function with respect to the stock price, the formulas for the number of shares are different since the formulas for the option pricing are different.

Substituting $N = C_S$ into (11) and dividing by Δt we get:
$$\begin{aligned}C_t + (\alpha - \lambda k)SC_S + \frac{1}{2}S^2 C_{SS}\sigma^2 + \lambda E_Y[C(SY,t) - C(S,Y)] - C_S\alpha S \\ - \frac{1}{\Delta t}\rho S|\nu| = r(C - C_S S),\end{aligned}$$

which is equivalent to
$$C_\tau - \lambda kSC_S + \frac{1}{2}S^2 C_{SS}\sigma^2 + \lambda E_Y[C(SY,t) - C(S,Y)] - \frac{1}{\Delta t}\rho S|\nu| = r(C - C_S S),$$

or

$$C_t + (r - \lambda k)SC_S + \frac{1}{2}s^2 C_{SS}\sigma^2 + \lambda E_Y[C(SY,t) - C(S,Y)] - rC \\ - \frac{1}{\Delta t}\rho S|\nu| = 0. \tag{12}$$

Let us look now at the term $\frac{1}{\Delta t}\rho S|\nu|$.

$$\nu = C_S(S + \Delta St + \Delta t) - C_S(S,\tau)$$

$$= C_{SS}(S,\tau)\Delta S + C_{St}(S,\tau)\Delta t + \\ \frac{1}{2}C_{SSS}(S,\tau)\Delta S^2 + C_{StS}(S,\tau)\Delta S\Delta t + \cdots$$

The dominant term is $C_{SS}(S,\tau)\Delta S$. This way,

$$\nu \approx C_{SS}(S,\tau)\Delta S$$

and

$$\frac{1}{\Delta t}\rho S|C_{SS}(S,\tau)\Delta S| = \frac{1}{\Delta t}\rho|C_{SS}(S,\tau)\tau^2\frac{\Delta S}{S}| \\ = \frac{1}{\Delta t}\rho C_{SS}(S,\tau)S^2\left|\frac{\Delta S}{S}\right|, \tag{13}$$

(see Appendix A for the proof of the fact that the term $C_{SS}(S,\tau)S^2$ is positive). Substituting (13) into (12) we obtain the differential equation that C must satisfy, inclusive of transaction costs

$$\frac{1}{2}C_{SS}S^2\left(\sigma^2 - \frac{2}{\Delta t}\rho E\left(\left|\frac{\Delta S}{S}\right|\right)\right) + C_t + (r-\lambda k)SC_S - rC \\ + \lambda E_Y[C(SY,\tau) - C(S,Y)] = 0. \tag{14}$$

With the notation

$$\hat{\sigma}^2 = \sigma^2 - \frac{2}{\Delta t}\rho E\left(\left|\frac{\Delta S}{S}\right|\right) \tag{15}$$

the equation for the value of the option is identical to Merton's value with the exception that the actual variance σ^2 is replaced by the modified variance $\hat{\sigma}^2$. This is one way of valuing a long position on an option with transaction costs.

For a short option position we change all the signs in the above analysis with the exception of the transaction cost term, which must always be a drain on the portfolio. We then find that the option is valued using the new variance

$$\hat{\sigma}^2 = \sigma^2 + \frac{2}{\Delta t}\rho E\left(\left|\frac{\Delta S}{S}\right|\right). \tag{16}$$

The results (15) and (16) show that a long position in a single call or put has an apparent volatility that is less than the actual volatility. This is because when the asset price rises the owner of the option must sell some assets to remain delta-hedged; however, the effect of the bid-ask spread on the underlying is to reduce the price at which the asset is sold and so the effective increase in the asset price is less than the actual increase. The converse is true for a short option position.

4. Estimating Turnover and Transaction Costs of Replicating Strategies

We saw in the previous section that
(i) The strategy $[N = C_S]$ with initial cost $C[S_0; P, r, \sigma, \lambda, k, T]$ eliminates the systematic risk, when there are no transaction costs.
(ii) The strategy $[N = \hat{C}_S]$ with initial cost $\hat{C}[S_0; P, r, \hat{\sigma}, \lambda, k, T]$ eliminates the systematic risk inclusive of transaction costs.

It follows that the difference between the two initial option prices,

$$Z = \hat{C}_0 - C_0$$

is a measure of the total transaction costs associated with the hedge strategy.

The turnover estimates are given by:

$$\text{Turnover} = \frac{\mathrm{Z}}{\rho \mathrm{S}_0}.$$

Note that Z is computed as the difference between two Merton option values, with only the volatility adjusted. Since the volatility adjustment depends on the transaction costs rate ρ, and the revision interval Δt, these parameters as well as the environment parameters $(r, \sigma^2, \lambda, k)$ and the option parameters (P, T) will importantly affect the total transaction cost, Z as well as the turnover.

In Table 1 we present the transaction costs, Z, and the turnover for a variety of options, transaction costs, and revision period assumptions. As for the environment we use a special case when the random variable Y has a log-normal distribution. If δ^2 is the variance of the logarithm of Y then

$$C(S, \tau) = \sum_{n=0}^{\infty} \frac{e^{-\lambda'\tau}(\lambda'\tau)^n}{n!} f_n(S, \tau),$$

where $\lambda' = \lambda(1+k)$ and $f_n(S,\tau)$ is the value option, conditional on knowing that exactly n Poisson jumps will occur during the life of the option.

Table 2 shows how the total transaction costs change with the change of lambda. Observe that larger lambda gives larger total transaction costs.

P	Δt	ρ	C	$\hat{C}$	Z	Turnover
90.0	0.2500	0.0025	73.6125	73.5806	0.0319	12.7757%
	0.2500	0.0100		73.4894	0.1231	12.3078%
	0.2500	0.0400		73.1960	0.4165	10.4137%
	0.5000	0.0025		73.5892	0.0233	9.3317%
	0.5000	0.0100		73.5217	0.0908	9.0842%
	0.5000	0.0400		73.2892	0.3233	8.0823%
	1.0000	0.0025		73.5955	0.0170	6.8092%
	1.0000	0.0100		73.5457	0.0668	6.6783%
	1.0000	0.0400		73.3665	0.2460	6.1491%
100.0	0.2500	0.0025	70.9369	70.8948	0.0421	16.8530 %
	0.2500	0.0100		70.7736	0.1634	16.3382%
	0.2500	0.0400		70.3689	0.5681	14.2015%
	0.5000	0.0025		70.9062	0.0308	12.3030%
	0.5000	0.0100		70.8166	0.1203	12.0313%
	0.5000	0.0400		70.5005	0.4365	10.9118%
	1.0000	0.0025		70.9145	0.0224	8.9737%
	1.0000	0.0100		70.8486	0.0883	8.8301%
	1.0000	0.0400		70.6072	0.3297	8.2428%
110.0	0.2500	0.0025	68.3435	68.2903	0.0532	21.2821%
	0.2500	0.0100		68.1361	0.2074	20.7401%
	0.2500	0.0400		67.6059	0.7376	18.4410%
	0.5000	0.0025		68.3047	0.0388	15.5292%
	0.5000	0.0100		68.1911	0.1524	15.2436%
	0.5000	0.0400		67.7816	0.5620	14.0489%
	1.0000	0.0025		68.3152	0.0283	11.3230%
	1.0000	0.0100		68.2318	0.1117	11.1723%
	1.0000	0.0400		67.9216	0.4220	10.5490%

TABLE 1. *Results of a sample computation for* S_0*=100, and* T*=1 year (12 month),* r*=0.1,* σ*=0.2,* λ*=0.1,* k*=0.3*

λ	Δt	ρ	C	$\hat{C}$	Z	Turnover
0.1000	0.2500	0.0025	70.9369	70.8948	0.0421	16.8530%
	0.2500	0.0100	70.9369	70.7736	0.1634	16.3382%
	0.2500	0.0400	70.9369	70.3689	0.5681	14.2015%
	0.5000	0.0025	70.9369	70.9062	0.0308	12.3030%
	0.5000	0.0100	70.9369	70.8166	0.1203	12.0313%
	0.5000	0.0400	70.9369	70.5005	0.4365	10.9118%
	1.0000	0.0025	70.9369	70.9145	0.0224	8.9737%
	1.0000	0.0100	70.9369	70.8486	0.0883	8.8301%
	1.0000	0.0400	70.9369	70.6072	0.3297	8.2428%
0.1500	0.2500	0.0025	71.2080	71.1584	0.0496	19.8547%
	0.2500	0.0100	71.2080	71.0148	0.1932	19.3199%
	0.2500	0.0400	71.2080	70.5256	0.6825	17.0613%
	0.5000	0.0025	71.2080	71.1714	0.0367	14.6652%
	0.5000	0.0100	71.2080	71.0643	0.1438	14.3763%
	0.5000	0.0400	71.2080	70.6812	0.5268	13.1711%
	1.0000	0.0025	71.2080	71.1810	0.0271	10.8233%
	1.0000	0.0100	71.2080	71.1014	0.1067	10.6671%
	1.0000	0.0400	71.2080	70.8072	0.4009	10.0220%
0.2000	0.2500	0.0025	71.5105	71.4532	0.0573	22.9091%
	0.2500	0.0100	71.5105	71.2868	0.2237	22.3650%
	0.2500	0.0400	71.5105	70.7092	0.8012	20.0312%
	0.5000	0.0025	71.5105	71.4677	0.0427	17.0999%
	0.5000	0.0100	71.5105	71.3425	0.1680	16.7999%
	0.5000	0.0400	71.5105	70.8891	0.6213	15.5335%
	1.0000	0.0025	71.5105	71.4786	0.0319	12.7544%
	1.0000	0.0100	71.5105	71.3846	0.1259	12.5888%
	1.0000	0.0400	71.5105	71.0345	0.4759	11.8986%
0.3000	0.2500	0.0025	72.2001	72.1276	0.0725	29.0039%
	0.2500	0.0100	72.2001	71.9154	0.2847	28.4682%
	0.2500	0.0400	72.2001	71.1558	1.0443	26.1069%
	0.5000	0.0025	72.2001	72.1247	0.0754	30.1467%
	0.5000	0.0100	72.2001	71.9044	0.2957	29.5670%
	0.5000	0.0400	72.2001	71.1200	1.0801	27.0021%
	1.0000	0.0025	72.2001	72.1582	0.0419	16.7466%
	1.0000	0.0100	72.2001	72.0344	0.1657	16.5714%
	1.0000	0.0400	72.2001	71.5669	0.6332	15.8294%

TABLE 2. *Results of a sample computation for* S_0=*100, and* T=*1 year (12 month),* r=*0.1,* σ=*0.2,* P=*100,* k=*0.3 and different values of* λ

5. Conclusion and Direction for Future Research

We developed a method for computing the price of an option when the underlying asset returns follow a jump-diffusion model, inclusive of transaction costs. The formula holds when the jump component of the model represents non-systematic risk. The zero-transaction costs option price lies between the long option and short option price inclusive of transaction costs.

Observe that if ρ is 0 we get the same equation Merton obtained. Moreover, in the case of $\lambda = 0$ we obtained the same equation Leland obtained, since the infinite sum in Appendix C reduces to one term that does not depend on λ.

In the future we would like to look at the errors in hedging with and without transaction costs, compute their expected value, compare them, and look at their limiting value.

Appendix A

We will prove that $C_{SS}(S,\tau)S^2$, is positive. In fact, by (3),

$$C(S,\tau) = \sum_{n=0}^{\infty} P_n(\tau) E_n(W(\nu_n, \tau; P, \sigma^2, r)),$$

where

$$P_n(\tau) = \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}$$

and

$$V_n = S X_n e^{-\lambda k \tau},$$

hence

$$S^2 C_{SS}(S,\tau) = \sum_{n=0}^{\infty} P_n(\tau) E_n\left\{V_n^2 W_{V_n V_n}(V_n, \tau; P, \sigma^2, r)\right\}.$$

But for any Black–Scholes formula,

$$\begin{aligned} V_n^2 W_{V_n V_n}(V_n, \tau; P, \sigma^2, r) &= \frac{V_n N'(d_1)}{\sigma(T-\tau)^{1/2}} \\ &= \frac{V_n e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}\sigma(T-\tau)} > 0. \end{aligned}$$

Appendix B

We will show that $N = C_S$ is a hedge strategy which eliminates the systematic risk:

The value of our portfolio is:

$$\Pi = C - NS$$

and

$$\Delta\Pi = \Delta C - N\Delta S. \tag{17}$$

If we rewrite (6) as

$$\Delta C = \left[C_t + (\alpha - \lambda k)SC_S + \frac{1}{2}C_{SS}S^2\sigma^2\right]\Delta t + SC_S\sigma\Delta Z + SC_S\Delta q,$$

and if we substitute it in (17) we obtain:

$$\Delta\Pi = \Delta t\left(\frac{1}{2}\sigma^2 S^2 C_{SS} + C_\tau\right) + S(\alpha - \lambda k)(C_S - N)\Delta t + \\ \sigma S(C_S - N)\Delta Z + S(C_S - N)\Delta q.$$

Observe that if we take $N = C_S$, then we eliminate the random component in the random walk, therefore our strategy must be $N = C_S$.

Appendix C

We will compute $E\left(\left|\frac{\Delta S}{S}\right|\right)$.

$$\begin{aligned} E\left(\left|\frac{\Delta S}{S}\right|\right) &= \sum_{n=0}^{\infty} E\left(|(\alpha - \lambda k)\Delta t + \sigma\Delta Z + n\Delta t|\right) P(q(t) = n \\ &= \sum_{n=0}^{\infty} E\left(|(\alpha - \lambda k)\Delta t + \sigma\Delta Z + n\Delta t)|\right) \frac{e^{-\lambda\Delta t}(\lambda\Delta t)^n}{n!} \\ &= \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty}\frac{e^{-\lambda\Delta t}(\lambda\Delta t)^n}{n!}\int_{-\infty}^{\infty}|\sigma\sqrt{\Delta t}x + (n - \lambda k)\Delta t|e^{-\frac{x^2}{2}}dx. \end{aligned}$$

We know that this integral can not be computed symbolically (it does not have a formula) but only numerically.

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