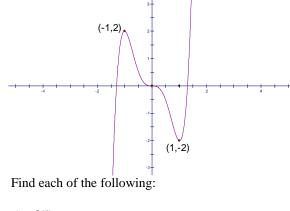
Final Exam Review Sheet

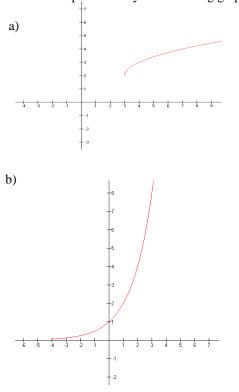
- 1. Determine whether each of the following defines *y* as a function of *x*. Identify domain and range.
 - a) $\{(2,7), (-2,-7), (7,-2), (0,2), (1,-4)\}$ b) $y = x^2$ c) $y = x^3$ d) $x = y^2$ e) $x = y^3$
- 2. A graph of a function, *f*, is shown.

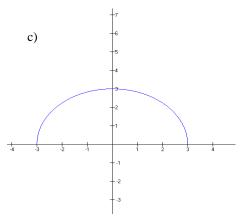


- a) *f*(1)
- b) f(-1)
- c) f(0)
- d) For how many values of x does f(x) = 1. Approximate these values of x.
- 3. Given that $f(x) = 2x^2 + 2x$, find:
 - a) f(-4)
 - b) f(0)
 - c) f(-x)
 - d) f(x-3)
 - e) f(x+h)

f)
$$\frac{f(x+h) - f(x)}{h}$$

- 4. Find the domain of each of the following:
 - a) $f(x) = \frac{2x+1}{x-3}$ b) $g(x) = \sqrt{2x+15}$ c) $h(x) = \frac{2x}{\sqrt{x+4}}$ d) $j(x) = \frac{1}{x^2 - 6x + 5}$ e) $k(x) = \ln(3x + 4)$
- Find the domain and range of each of the functions represented by the following graphs.





- According to the U.S. Bureau of the Census, workers' compensation payments in Florida rose from \$362 million in 1980 to \$1.976 billion in 1990. Find the average rate of change in payments.
- #7 11: Write an equation of the line:
- 7. through (2,4) and (-1,3).
- 8. a) parallel to
 - b) perpendicular to
 - 3x + 6y = 12 and passing through the point (-1,4)
- 9. with undefined slope that passes through (-5,10)
- 10. with zero slope that passes through (-6, 22).
- 11. If a town starts with an initial population of

100000 and 350 people leave each year, what is

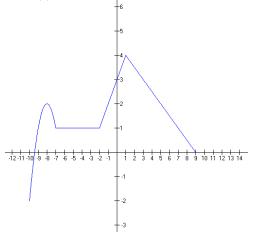
the linear equation that relates the population (P) to the number of years since the town began (n)?

12. Given the function f defined piecewise as

$$f(x) = \begin{cases} 4, & \text{for } x \le -2, \\ x+1, & \text{for } -2 < x < 3, \text{ follows:} \\ -x, & \text{for } x \ge 3 \end{cases}$$

Find
a) $f(5)$ b) $f(-2)$ c) $f(2.5)$

- 13. Sketch the graph of f as defined in # 14 above.
- 14. Using the graph below, determine the intervals on which the function is (a) increasing, (b) decreasing, and (c) constant.

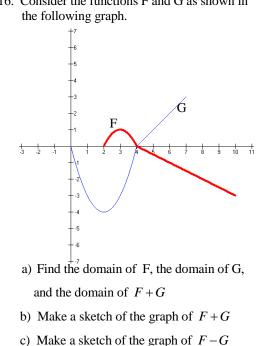


15. A farmer has 360 yards of fencing with which to enclose two adjacent rectangular plots, one for corn and the other for soybeans. Suppose the width of each plot is x.



- a) Express the total area of the two plots as a function of *x*.
- b) Find the domain of the function.

16. Consider the functions F and G as shown in the following graph.



- 17. Given the functions *f* and *g* defined by the following: $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$.
 - a) Find the domain of f + gb) Find f + g.
- 18. Given the functions *f* and *g* defined by the following: $f(x) = \frac{4}{x+1}$ and $g(x) = \frac{1}{6-x}$.
 - a) Find the domain of f + gb) Find f + g.

- 19. For f(x) = 1 + 2x and $g(x) = \sqrt{x}$ a) Find $(f \circ g)(x)$ and $(g \circ f)(x)$ b) Find the domain of $(f \circ g)(x)$ and $(g \circ f)(x)$
- 20. Given that f(x) = 2x+1 and $g(x) = x^2 3x + 5$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.
- 21. Determine whether the graph of $xy x^2 = 3$ is symmetric with respect to the x-axis, the y-axis, or origin.
- 22. Test algebraically whether the following functions are even, odd, or neither.
 - a) $f(x) = x^3 3x + 1$ b) g(x) = x - |x|c) $h(x) = \sqrt{x^2 + 1}$
- 23. Write an equation for a function that has the shape of a) $y = \sqrt{x}$ but shifted right 2 units and down 3 units.
 - b) $y = x^2$ but stretched vertically by a factor of 2 reflected through the x- axis and shifted up 5.
 - c) $y = \frac{1}{r}$ but shrunk horizontally by a factor of 3 and shifted up 2 units.

24. Express in terms of *i* :

a)
$$\sqrt{-13}$$
 b) $-\sqrt{-49}$

a) (5+2i)(4+i)b) (4-2i)-(8+3i)c) $\frac{2-i}{3+i}$ d) i^{18}

26. Find the zeros of

a) $f(x) = 3x^2 + 2x - 7$

b) $g(x) = x^2 + 5x + 8$

27. Complete the square on the function f given by

 $f(x) = 2x^2 - 6x + 1.$

- a) Name the vertex of f
- b) Name the axis of symmetry
- c) Determine if there is a maximum or

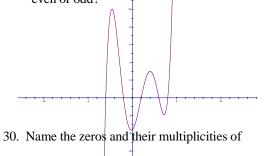
minimum function value and find that value

28. A ball is thrown directly upward from a height of 30 feet with an initial velocity of 60 ft/sec. If the height of the stone *t* seconds after it has been thrown is given by the function

 $s(t) = -16t^2 + 60t + 30,$

determine the time at which the ball reaches its maximum height and find the maximum height.

29. The graph below represents a polynomial function. Is the degree of the leading term even or odd?



 $g(x) = x^{3}(x+3)^{2}(x-4)(x+1)^{4}$. Then

indicate whether the graph would cross or be tangent to the x- axis at each zero. Also indicate the end behavior and explain your reasoning.

31. Sketch a graph by hand of $h(x) = -\frac{1}{2}(x-1)(x+2)^2(x-2)$. Be sure to

show all steps, including naming the zeros, multiplicities, end behavior, and showing the sign chart.

- 32. Use synthetic division to divide $P(x) = x^{4} - 3x^{3} - 2x^{2} + 10x - 12$ by D(x) = x - 3and write the quotient and remainder.
- 33. Use the Factor Theorem to determine if x+3 is a factor of $P(x) = 3x^4 + 8x^3 + 27$.
- 34. List all the possible rational zeros of each of the following:
 - a) $f(x) = 20x^3 12x^2 3x + 2$ b) $h(x) = 2x^3 + 7x^2 + 6x - 12$
- 35. Find all the zeros of these, then write in factored form:

a)
$$f(x) = 18x^3 - 21x^2 + 8x - 1$$

b) $g(x) = 3x^3 - 2x^2 + 3x - 2$
c) $h(x) = x^3 - 4x^2 + 9x - 10$
d) $m(x) = x^4 - 7x^3 + 18x^2 - 19x + 7$
e) $q(x) = 2x^3 - 5x^2 - 28x + 15$

36. Find a polynomial function of degree 3 with the following zeros:

a)
$$-3$$
, 0, $\frac{1}{2}$
b) -8 , $3i$, $-3i$
c) -5 , $\sqrt{3}$, $-\sqrt{3}$

37. Name the vertical and horizontal asymptotes of the following rational functions.

a)
$$h(x) = \frac{1}{x-8}$$

b) $m(x) = \frac{x-2}{x+5}$
c) $n(x) = \frac{x+3}{x^2+5x+5}$

d)
$$p(x) = \frac{x^3 + 2x^2 - 3x}{x^2 - 25}$$

38. Find the zeros of each of the functions in # 48.

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39. Solve each of the following inequalities:

a)
$$x^{2}-2x \ge 8$$

b) $x^{3}+x^{2}-4x-4 \le 0$
c) $x^{5}+24 \ge 3x^{3}+8x^{2}$
d) $\frac{-2}{5-x} < 0$
e) $\frac{x+1}{x-2} \le \frac{x+3}{x-1}$

40. Which of the following functions are one-to-one?

a)
$$f(x) = x^3$$

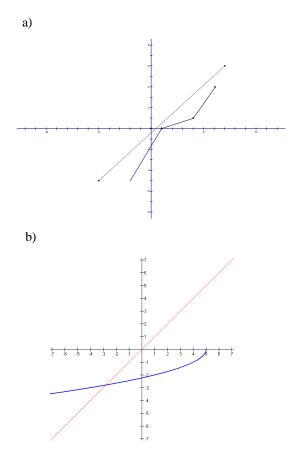
b) $f(x) = 2x + 7$
c) $f(x) = (x+4)^2, x \ge -4$
d) $f(x) = (x+4)^2$

41. Find f^{-1} for each of these.

a)
$$f(x) = \frac{5+2x}{3}$$

b) $f(x) = \frac{-3x+1}{4}$
c) $f(x) = \sqrt{x+2}$

- 42. Verify that $f(x) = \frac{1-x}{x}$ and $g(x) = \frac{1}{x+1}$ are inverses of each other by using composition of functions.
- 43. Sketch the inverse of the function represented by each graph below.



- 44. Without using your graphing calculator, sketch a graph of each of these.
 a) f(x) = 3^{x-4} + 2
 - b) $f(x) = 1 e^{-x}$
 - c) $f(x) = 4 2^{-x}$
- 45. How much money will be in an account in 20 years if it compounds 7.5% interest quarterly and you deposit \$5,000 now?
- 46. Evaluate each of these. a) $\log_2 32$ b) $\log_{\frac{1}{4}} 16$ c) $\log_{\frac{1}{3}} 81$ d) $\log_8 4$ e) $\log_{27} 9$
- 47. Sketch an accurate graph of $f(x) = \log_3(x+5)$. Name the domain, range, intercept, asymptote.
- 48. Express in terms of sums and differences of logarithms.

a)
$$\ln \frac{x^3 y^2}{z}$$
 b) $\ln \frac{6}{e^2}$ c)
 $\log_a \sqrt{\frac{a^6 b^8}{m^2 n^5}}$

49. Write as a single logarithm:

$$\frac{2}{3} \left[\ln(x^2 - 9) - \ln(x + 3) \right] + \ln(x + y)$$

50. Simplify

a)
$$\log_p p^3$$
 b) $2^{\log_2 50}$ c) $\ln e^{6-4x}$

- 51. Solve each of these. a) $\log_{16} 2 = x$ b) $\log_3(x-2) + \log_3(x+4) = 3$ c) $4^{x+5} = 64^x$ d) $e^{2x} = 10.4$ e) $\ln x - \ln(x-4) = \ln 3$
- 52. The population of a country doubled in 8 years. What was the exponential growth rate?
- 53. Suppose \$8,000 is invested at rate *k*, compounded continuously, and grows to \$11,466.64 in 6 years.
 - a) Find the interest rate
 - b) Find the exponential growth function.
 - c) Find the balance after 10 years.
 - d) Find the doubling time.
- 54. Average cell phone prices have fallen sharply since their introduction to the market in 1983. In 1984, the average price was \$3395, and in 2002, it was only \$145. Assuming the average price of a cell phone decreased according to the exponential model:
 - a) Find the value of *k* and write an exponential function that describes the average price of a cell phone after time *t*, in years, where *t* is the number of years since 1984.
 - b) Estimate the price of a cell phone in 2004.
 - c) At what time *t* was the price half the original price?

55. Find the distance between each pair of points.

A) (-3,7) and (2,11)

- B) $\left(-\frac{3}{5},-4\right)$ and $\left(-\frac{3}{5},\frac{2}{3}\right)$
- 56. Find the vertex, focus, and directrix, then sketch each parabola:

a)
$$y^2 = -2x$$

- b) $(x+2)^2 = -6(y-1)$
- c) $y^2 + y x 4 = 0$
- 57. Find the center and radius of the circle: $x^{2} + y^{2} + 6x - 10y = 0$
- 58. Find the center, vertices, and the foci of each ellipse. Then sketch the graph.
 - a) $\frac{x^2}{25} + \frac{y^2}{36} = 1$
 - b) $16x^2 + 9y^2 = 144$

c)
$$\frac{(x+3)^2}{25} + \frac{(y-5)^2}{36} = 1$$

d) $x^2 + 2y^2 - 10x + 8y + 29 = 0$

59 . Find the center, vertices, and the foci of each hyperbola . Then sketch the graph.

a)
$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

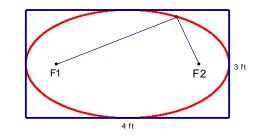
b) $25x^2 - 16y^2 = 400$
c) $\frac{(y+3)^2}{4} - \frac{(x+1)^2}{16} = 1$
d) $9x^2 - 4y^2 + 54x + 8y + 41 = 0$

60. Classify each of the following as a parabola, circle, ellipse, or hyperbola. Explain your reasoning.

a)
$$y^{2} = 16 - 4x^{2}$$

b) $x^{2} = 4y^{2} + 36$
c) $x - 4 + 4y = y^{2}$
d) $3x^{2} + 3y^{2} = 75$
e) $x^{2} + 8y^{2} - 14x - 2y + 9 = 0$
f) $x^{2} - 5y^{2} + 5x - 9y + 10 = 0$

61. A spotlight has a parabolic cross section that is 6 ft wide at the opening and 4.5 ft deep at the vertex. How far is the focus from the vertex? 62. A carpenter is cutting a 3 ft by 4 ft elliptical sign from a 3 ft by 4 ft piece of plywood. The ellipse will be drawn using a string attached at the foci of the ellipse. How far from the ends of the board should the string be attached?



63. If
$$a_n = (2n+3)^2$$
, find a_6

64. Evaluate

a)
$$\sum_{i=1}^{5} \frac{i-1}{i+3}$$

b)
$$\sum_{k=0}^{7} (-1)^k 4^{k+1}$$

65. Write in sigma notation:

a) $7+14+21+28+35+\cdots$

b) $4-9+16-25+\cdots$

- 66. Find the 17^{th} term of the arithmetic sequence 7,4,1,...
- 67. Find a_{20} when $a_1 = 14$ and d = -3
- 68. Find *d* when $a_1 = 5$ and $a_{15} = 26$
- 69. Find the sum of the odd numbers from 1 to 199, inclusive.

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- 70. A formation of marching band has 10 marchers in the front row, 12 in the second row, 14 in the third row, and so on, for 8 rows. How many marchers are there altogether?
- 71. Find the indicated term of each geometric sequence:
 - a) $3, 9, 27, \dots; 8^{\text{th}}$ term
 - b) $3, 3\sqrt{2}, 6, \dots; 11^{\text{th}} \text{ term}$
 - c) $-8, -0.8, -0.08, \dots; 5^{\text{th}}$ term
 - d) $-1, 1, -1, 1, \dots; 61^{st}$ term
- 72. Find the sum of the first 10 terms of the geometric series $16-8+4-\cdots$.
- 73. Find the sum, if it exists:

a) $7+3+\frac{9}{7}+\cdots$

- b) $100-10+1-\frac{1}{10}+\cdots$
- c) $7+35+105+\cdots$.
- 74. To create a college fund, a parent makes a sequence of 18 yearly deposits of \$1500 each in a savings account on which interest is compounded annually at 3.5%. Find the amount of the annuity.

- 75. A ball is dropped from a height of 20 ft and always rebounds $\frac{1}{4}$ of the distance fallen.
 - a) How high does it rebound the 5^{th} time?
 - b) Find the total sum of the rebound heights of the ball.