Table Number:__________ Group Name: _________________

Group Members:_____________________ _____________________ _____________________

A Review of Slope, Intercepts, and Linear Functions

Review: HOW TO FIND SLOPE

You may remember from your algebra class how to find the slope of the line between two points.

Given $(-3,4)$ and $(2,-1)$. We find the slope by taking the difference in the *y*-coordinates $(y_2 - y_1)$ divided by the difference in the *x*-coordinates $(x_2 - x_1)$ like this: $\frac{-1-4}{2-(-3)} = \frac{-1+-4}{2+3} = \frac{-5}{5} = -1$.

Find the slope of the line between each of the following pairs of points.

1. $(5,-3)$ and $(-4,8)$
2. $(-2.5,-7)$ and $(4,-3.5)$
3. $\left(\frac{1}{10},-9\right)$ and $\left(-\frac{1}{5},8\right)$

SLOPE IN CONTEXT

Now, suppose we had two quantities that are changing together, such that the change in one quantity depends upon the other. The two quantities might be number of toppings ordered on a pizza and the cost of the pizza. We can write them like this: (number of pounds of topings on a pizza, total cost of the pizza). Clearly the total cost of the pizza **depends upon** the number of toppings ordered. We say that the cost of the pizza is the ***dependent variable*** and the number of toppings is the ***independent variable***. Conventionally, we write the independent variable first, then the dependent variable.

Identify the dependent and independent quantity in each pair below.

4. Cost of chicken, the number of pounds of chicken purchased.
   Independent: __________________ Dependent:__________________

5. Number of credit hours a student takes during a semester, the total tuition in dollars a student pays that semester.
   Independent: __________________ Dependent:________________

6. The number of gallons of gas in your car's gas tank while on a road trip; the number of miles driven on the road trip.

   Independent: __________________ Dependent: __________________

Consider the scenario in the introduction above: (number of toppings on a pizza, cost of the pizza).

7. What would the point $(1,7.75)$ mean?

8. What would the point $(3,9.25)$ mean?

9. Find the slope between these two points. Show your work below and write your answer here:__________

What does the slope mean in this context? Take a guess. *(Hint: Recall that slope is* $\frac{(y_2 - y_1)}{(x_2 - x_1)}$*, which we might now think of as* $\frac{(\text{Change in the dependent quantity})}{(\text{Change in the independent quantity})}$.

10. Write your interpretation in a full sentence below.

Please note that a slope is a rate of ***change.*** So a correct interpretation of the above slope would be:

$\frac{(\text{Change in the cost of the pizza})}{(\text{Change in the number of toppings on the pizza})}$ or*, For each **additional** topping on a pizza, the cost of pizza **changes** (or increases by) $0.75*. Another correct statement would be, *For each **additional** topping on a pizza, the pizza costs $0.75 **more.*** It is NOT correct to say, *The pizza costs $0.75 per topping.* No it doesn't. It costs $0.75 **more** per topping. You need to include a statement of **change** in your interpretation.

Given each pair of quantities below and two sets of points, find the slope between the points and interpret it in the given context.

11. (age of baby in weeks, weight of baby in pounds) $(4,9),(7,10.5)$ [Note: For the first 6 months]

12. (age of child in years, height of child in inches) $(7,59.5),(10,67)$ [Note: For ages 6 – 12 years]

13. (number of cars sold in a month, gross monthly income) $(4,2900),(7,3950)$

14. (number of minutes spent walking, distance walked) $(15,3960),(21,5544)$

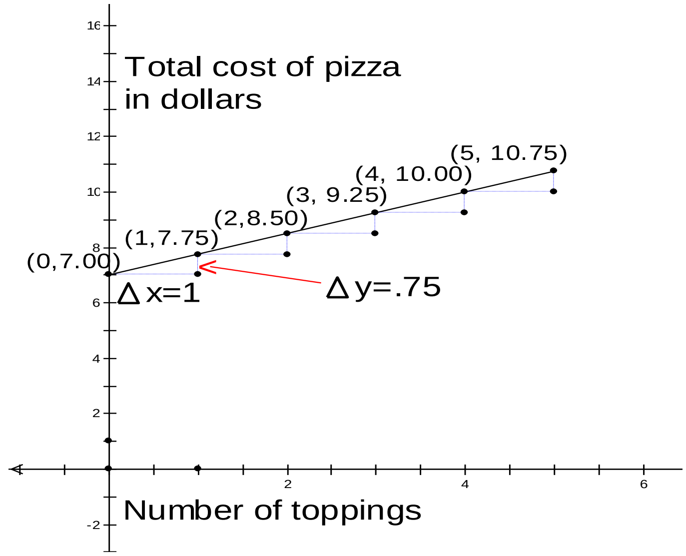
15. (number of pizza toppings ordered, cost of pizza) $(2,8.5),(6,11.5)$

16. (number of tons of landscaping rock ordered, cost of the order) $(1.5,210.5),(3,371)$

17. (number of hours a candle is burning, height of candle in inches) $(1.5,6.7),(3.5,2.3)$

18. (seconds spent walking toward each other, distance in feet two friends are apart) $(8,68),(11,41)$

You might notice that we are referring to the slope of **straight line. You may also remember that the slope between any two points on a line is the same.** Verify this fact all possible combination of two points using the points on the graph below.



We say that the rate of change of the cost per toppings on a pizza is **constant**. This means that for **any** added topping, the cost will **ALWAYS increase** \$0.75. For any two additional toppings, the cost will increase 2 times \$0.75; for any three additional toppings, the cost will increase 3 times \$0.75. In other words, the change in the cost is always \$0.75 times the number of additional toppings:

$$\text{Change in cost} = (0.75)\cdot\text{Change in number of toppings} \text{ or}$$

$$\Delta \text{cos}t = (0.75)\Delta\text{number of toppings} \text{ or}$$

$$\Delta y = (0.75)\Delta \text{x} .$$

In general, when there is a constant rate of change, *m*, and *y* is the dependent variable and *x* is the independent variable, it is always the case that,

$$\Delta y = m\Delta \text{x}$$

Suppose we wanted now to predict the cost of pizza for **any** number of toppings. We know that $\frac{(y_2 - y_1)}{(x_2 - x_1)} = 0.75$. We can multiply both sides of this equation by $(x_2 - x_1)$ to obtain

$$(y_2 - y_1) = 0.75(x_2 - x_1) .$$

Let's put in specific values for (x_1, y_1) , say $(1,7.75)$, and rewrite (x_2, y_2) as (x, y) so that it represents ***any*** point, (x, y). Now we have

$$(y - 7.75) = 0.75(x - 1) .$$

Solving this equation for y gives

$$y = 0.75x + 7 \quad (1)$$

19. Substitute in the following values of x into (1), and interpret in words the meaning of your values:
    a. $x = 6$

    b. $x = 4$

    c. $x = 8$

    d. $x = 2.5$

    e. $x = 0$

Your answer for e) should be $y = 7$. Interpret the meaning in a full sentence.

INTERCEPT OF A LINEAR FUNCTION

When the input value, $x = 0$, the output value is called the ***initial value*** or the ***y-intercept,*** or the ***vertical intercept.*** In the above scenario, this is the cost of a pizza with no toppings.

20. Find the initial value, (the y-intercept) for each of the following linear functions and interpret its meaning in the context of the problem.
    a. Gallons of gas remaining $= 12 - \left(\frac{\text{miles driven}}{30}\right)$ or $y = 12 - \left(\frac{x}{30}\right)$

    b. Miles from home $= 5 + (12) \cdot \text{hours spent bicycling}$ or $y = 5 + (12) \cdot x$, assuming one is travelling on a straight path away from home.

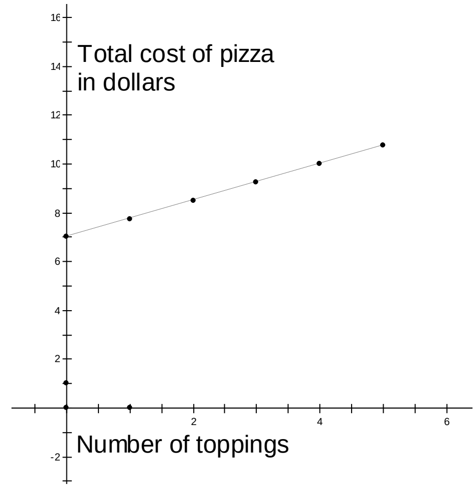
    c. Distance apart $= 140 - 9 \cdot \text{number of seconds spent walking}$

    d. Monthly income $= 350 \cdot \text{number of cars sold} + 1500$

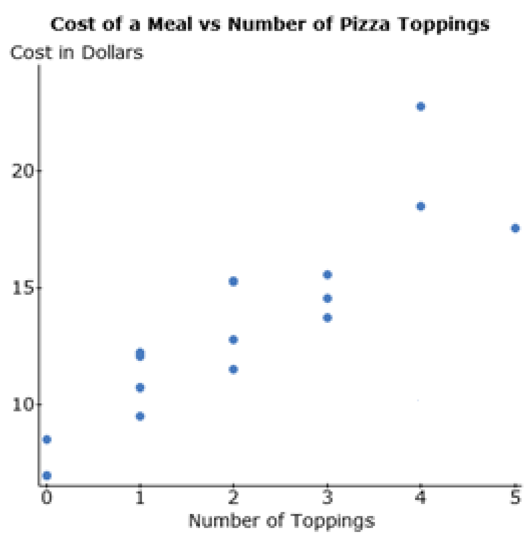
    e. Cost of a pizza $= 0.75 \cdot \text{number of toppings} + 7$

LINEAR FUNCTION VS LINE OF BEST FIT (REGRESSION LINE)

In the pizza example above, each of the points fit the equation of the line *exactly* and thus each point was on the line. Each number of topping had ***exactly one*** cost.



When data is collected, this is often not the case. Recall that given a scatterplot, points often have a ***linear trend***, but not all points fit on a line exactly. For example, if we were to relate the total cost of a meal that includes pizza and related that to the number of toppings ordered, we might obtain a scatterplot like the following.



Notice that the scatterplot has a linear trend, but not all points lie perfectly on a straight line. We will let technology construct a line of best fit, also called the regression line.

Interpretation of slope and y-intercept is the same as above, with the following exceptions:

- While interpreting slope of a regression line, we use the words, *on average.*
- The y-intercept does not always have a meaningful interpretation in context.