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# Understanding and Interpreting Confidence Intervals 

from Section 3.2 in Lock, Robin et al. Statistics, Unlocking the Power of Data. Wiley Pub. 2013

## 1. Is a Television Set Necessary?

The percent of Americans saying that a television set is a necessity has dropped dramatically in recent years. In a nationwide survey conducted in May 2010 of 1484 people ages 18 and older living in the continental United States, only $42 \%$ say that a television set is a necessity rather than a luxury. The article goes on to say "the margin of error is plus or minus 3.0 percentage points." Use the information from this article to find an interval estimate for the proportion of people 18 and older living in the continental United States who believe that a television set is a necessity.

## Interval Estimate Based on a Margin of Error

An interval estimate gives a range of plausible values for a population parameter. One common form of an interval estimate is

$$
\text { Point estimate } \pm \text { margin of error }
$$

where the margin of error is a number that reflects the precision of the sample statistic as a point estimate for this parameter
2. Suppose the results of an election poll show the proportion supporting a particular candidate to be $\hat{p}=0.54$.

We would like to know how close the true $p$ is to $p$. Two possible margins of error are shown below. In each case , indicate whether we can be reasonably sure that this candidate will win the majority of votes and win the election.
a. Margin of error is 0.02
b. Margin of error is 0.10

## Confidence Intervals

This "range of plausible values" interpretation for interval estimate can be refined within the notion of confidence interval.

## Confidence Interval

A confidence interval for parameter in an interval computed from sample data by a method that will capture the parameter for a specified proportion of all samples.

The success rate (proportion of all samples whose intervals contain the parameter) is known as the confidence level.

## 95\% Confidence Interval Using the Standard Error

If we can estimate the standard error SE and if the sampling distribution is relatively symmetric and bell-shaped, a 95\% confidence interval can be estimated using

## Statistic $\pm 2 \cdot$ SE

3. Each of the three values listed below is one of the samples shown in the dotplot in the figure. Find a $95 \%$ confidence interval using the sample proportion and the fact that the standard error is about 0.03. In each case, also locate the sample proportion on the sampling distribution and indicate whether the $95 \%$ confidence interval captures the true population proportion.

a) $\hat{p}=0.26$
b) $\hat{p}=0.32$
c) $\hat{p}=0.20$
4. Construct an interval estimate for $p$ using $\hat{p}=0.37$ with margin of error 0.02.
5. Assume a sampling distribution is symmetric and bell-shaped use the following information to construct a $95 \%$ confidence interval if $\hat{p}=0.32$ and the standard error is 0.04 .
6. A $95 \%$ confidence interval for a proportion is 0.72 to 0.79 . Is the value given a plausible value of $p$ ?
a) $p=0.85$
b) $p=0.75$
c) $\quad p=0.07$
7. The CDC classifies an adult as "obese" if the Body Mass Index (BMI) $\geq 30$. Based on the data from a 2010 survey, a $95 \%$ confidence interval for the proportion of all adults living in the US that were obese in 2010, $p_{2010}$ is $(0.276,0.278)$. Based on the data from a 2000 survey, a $95 \%$ confidence interval for the proportion of all adults living in the US that were obese in 2000, $p_{2000}$ is (0.195, 0.199).
a) Interpret both confidence intervals
b) Do you think the sample size for the 2000 survey was smaller or larger than the sample size for the 2010 survey? Why?
8. A random sample of $n=1483$ adults in the US were asked whether they consider a car a necessity or a luxury and we find that a $95 \%$ confidence interval for the proportion saying that it is a necessity is 0.83 to 0.89 . Explain the meaning of this confidence interval in the appropriate context.
