Expected Value/Mean of a Probability Distribution Practice Set

1. Recall that a roulette wheel has 38 slots. Eighteen are red, 18 are black, and 2 are green. You can bet on 6 different numbers. If any of them comes up, you receive $6 back for each $1 bet. What is the expected loss on a $1 bet? Please round the answer to two decimal places.

   a. $0.01
   b. $0.08
   c. $0.07
   d. $0.05
   e. $0.04

   $$
   \begin{array}{ccc}
   \chi & P(\chi) & \text{expected loss} \\
   1 & \frac{3}{38} & 0.1319 \\
   3 & \frac{3}{38} & 0.1319 \\
   4 & \frac{11}{18} & 0.6111 \\
   5 & \frac{2}{38} & 0.0526 \\
   6 & \frac{2}{38} & 0.0526 \\
   \end{array}
   $$

   $$
   -1 \left( \frac{11}{18} \right) + 5 \left( \frac{2}{38} \right) = -0.15 + 0.19 = -0.1
   $$

2. In a gambling game, you receive a payoff of $82 if you roll a sum of 4, and $7 if you roll a sum of 7 on two dice. Otherwise, you receive no payoff. What is the average payoff per play?

   a. $8
   b. $9
   c. $6
   d. $12
   e. $5

   $$
   \begin{array}{ccc}
   \chi & P(\chi) & \text{sum of 4} \frac{3}{36} = \frac{1}{12} \\
   2 & \frac{1}{12} & \text{sum of 7} \frac{6}{36} = \frac{1}{6} \\
   7 & \frac{1}{6} & \text{Ave Payoff:} \frac{82}{12} + \frac{7}{6} = 8
   \end{array}
   $$

3. In a carnival game, you roll 2 dice. If the sum is 5, you receive a $6 payoff. If the sum is 10, you receive a $13 payoff.

   What is the expected payoff?

   a. $2.05
   b. $1.35
   c. $1.75
   d. $1.55
   e. $1.85

   $$
   \begin{array}{ccc}
   \chi & P(\chi) & \text{sum of 5} \frac{4}{36} = \frac{1}{9} \\
   5 & \frac{1}{9} & \text{sum of 10} \frac{3}{36} = \frac{1}{12} \\
   13 & \frac{1}{12} & 6 \cdot \frac{1}{9} + 13 \cdot \frac{1}{12} + 0 \cdot \frac{2}{3} = 1.75
   \end{array}
   $$

4. In a gambling game, you pick 1 card from a standard deck. If you pick an ace, you win $10. If you pick a picture card (J, Q, or K), you win $5. Otherwise, you win nothing. How much should a carnival booth charge you to play this game if they want an average profit of $0.60 per game? (Hint: first find the average payout)

   a. $2.12
   b. $2.62
   c. $2.72
   d. $2.22
   e. $2.52

   $$
   \begin{array}{ccc}
   \chi & P(\chi) & \text{ace} \frac{4}{52} = \frac{1}{13} \\
   10 & \frac{1}{13} & \text{pic} \frac{12}{52} = \frac{3}{13} \\
   5 & \frac{3}{13} & 10 \cdot \frac{1}{13} + 5 \cdot \frac{3}{13} = 1.92
   \end{array}
   $$

   $$
   \frac{1.92}{0.60} = 3.2
   $$
5. You have a job working for a mathematician. She pays you each day according to what card you select from a bag. Two of the cards say $220, five of them say $100, and three of them say $50. What is your expected (average) daily pay?

\[
\begin{array}{c|c}
\text{\$} & P(X) \\
\hline
220 & \frac{1}{5} \\
100 & \frac{1}{2} \\
50 & \frac{3}{10} \\
\end{array}
\]

\[
\frac{220}{5} + \frac{100}{2} + \frac{50}{3} = \frac{440 + 500 + 150}{15} = \frac{1100}{15} = \frac{220}{3} \approx 109.67
\]

\$109 per day

6. An apartment complex has 20 air conditioners. Each summer, a certain number of them have to be replaced.

<table>
<thead>
<tr>
<th>Number of Air Conditioners Replaced</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.21</td>
</tr>
<tr>
<td>1</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.13</td>
</tr>
<tr>
<td>3</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.11</td>
</tr>
<tr>
<td>5</td>
<td>0.07</td>
</tr>
</tbody>
</table>

What is the expected number of air conditioners that will be replaced in the summer?

\[
0 \cdot (0.21) + 1 \cdot (0.37) + 2 \cdot (0.13) + 3 \cdot (0.11) + 4 \cdot (0.11) + 5 \cdot (0.07) = 17.5
\]

7. In a gambling game, you receive a payoff of $46 if you roll a sum of 10, and $7 if you roll a sum of 7 on two dice. Otherwise, you receive no payoff. What is the average payoff per play?

\[
\begin{array}{c|c}
\text{\$} & P(X) \\
\hline
46 & \frac{3}{36} = \frac{1}{12} \\
7 & \frac{6}{36} = \frac{1}{6} \\
0 & \frac{3}{36} = \frac{1}{12} \\
\end{array}
\]

\[
\frac{46 \cdot \frac{1}{12}}{12} + \frac{7 \cdot \frac{1}{6}}{6} = \frac{6}{5}
\]

\$\frac{6}{5}$