Chapter 9
Learning to Reason About Center

> *A statistician sees group features such as the mean and median as indicators of stable properties of a variable system—properties that become evident only in the aggregate.*
> *(Konold & Pollatsek, 2002, p. 262)*

Snapshot of a Research-Based Activity on Center

Pairs of students are given a set of 10 small Post-It® notes and a number line that goes from 17 to 25. They are asked to think about a group of college students whose mean age is 21, and construct a dot plot, using the Post-It® notes, of an age distribution with a mean of 21. Most of the students quickly figure out that they can stack all 10 Post-It® notes at the 21 point. But then, they are told that one of the values is 24, so they have to figure out where the other values are on the number line (e.g., move one note to 18, to balance out the 24, being 3 units above the average of 21). Students are instructed to move one Post-It® note to 17, and arrange the rest of the Post-It® notes so the mean is still 21. Finally, they move all of the Post-It® notes so that none are at 21 but the mean age is 21. Students use different strategies to do this, such as making sure that every note above the mean has a value equally placed below the mean. Other students may balance one note that is four units above the mean with two notes that are two units below the mean, etc. Different results are compared and discussed.

Students are then instructed to draw dot plots of 10 points that have the same mean of 21, and then draw deviations from the mean for their graphs. They consider how the deviations balance each other out, so that if one value is moved producing a deviation of -3, another value must be moved to have a deviation of $+3$. This leads to a discussion of what a mean "means" in terms of these deviations all cancelling each other out to be zero.

Rationale for This Activity

Although most students have learned how to calculate means before entering their statistics course, research studies reveal that few understand what a mean really is or what it tells us about data. Research also shows that students often have difficulty understanding how the mean and median differ and why they behave differently (e.g., in the presence of outliers). This activity helps students to build a conceptual

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understanding of what the mean and median actually mean and how they are affected by the different values in a data set. This lesson also introduces the idea of deviation early in the course, as a way to understand the idea of a mean and what it represents. This deviation idea, which begins to connect ideas of center and variability, is revisited when learning about variability and the standard deviation, and again when considering residuals in a regression analysis. This activity helps students develop a conceptual and a procedural understanding of the mean. By having the students physically manipulate data values on a number line, they are better able to see and reason about the idea of deviation from the mean and the balancing of these deviations.

The Importance of Understanding Center

> The idea of data as a mixture of signal and noise is perhaps the most fundamental concept in statistics.
>
> (Konold & Pollatsek, 2002, p. 259)

Understanding the idea of center of a distribution of data as a signal amidst noise (variation) is a key component in understanding the concept of distribution, and is essential for interpreting data graphs and analyses. While students develop informal ideas of center in the earlier units as they graph and describe distributions of data, they later encounter the idea of center more formally as they learn about different measures of center, how to compute them, what information they provide, and how we use them. However, it is impossible to consider center without also considering spread, as both ideas are needed to find meaning in analyzing data.

The Place of Center in the Curriculum

Traditional textbooks first introduce center, thenintroduce spread, and then move on to the next topic. However, it may be more helpful to study these topics together because they are so interrelated (Konold & Pollatsek, 2002; Shaughnessy, 1997).

It is hard to imagine a situation where one would summarize a data set using only a measure of center without talking about the spread of the data or how much variability there is around that measure of center. However, there are many instances, particularly in the media, in which only measures of center are provided for a data set, in some cases, leading to incorrect conclusions. When comparing groups or making inferences we need to examine center and spread together: the signal, and the noise around the signal. While these ideas are introduced in early units on exploring data, these concepts re-appear when looking at theoretical models such as the normal distribution and sampling distributions. Later on, the ideas of center (and spread) are revisited when making statistical inferences about samples of data.

Review of the Literature Related to Reasoning About Center

> (The knowledge of) computational rules not only does not imply any real understanding of the basic underlying concept, but also may actually inhibit the acquisition of more adequate (relational) understanding.
>
> (Pollatsek, Lima, & Well, 1981, p. 202)

Understanding Means

How students understand ideas of center has been of central interest in the research literature. Research on the concept of average or mean was at first the most common topic studied on learning statistics in the school level (see Konold & Higgins, 2003; Shaughnessy, 1992, 2003). The studies suggested that the concept of the average is quite difficult to understand by children, college students, and even elementary school preservice and in-service teachers (Russell, 1990; Groth & Bergner, 2006).

Early research typically focused on the single idea of center rather than looking at the interrelated concepts of center and spread, and on procedural understanding. These studies focused primarily on the mean, either as a simple average of a single data set or a weighted mean. An early interest was on students' understanding of the mean as a *balance model* (e.g., Hardiman, Well, & Pollatsek, 1984; Strauss, 1987), which is a common method taught in a statistics course. A balance model illustrates how values are placed on a balance beam at distances from the mean so that the deviations from the mean are equal. Hardiman et al. (1984) tested whether improving students' knowledge of such balance rules through experience with a balance beam promoted deep understanding of the mean. Forty-eight college students enrolled in psychology classes participated in the study which involved pretest, training, and posttest of paper and pencil items. Students who were given the balance training performed significantly better on the posttest problems.

Other studies identified several characteristics of the mean and then examined students' understanding of these characteristics (Goodchild, 1988; Mevarech, 1983; Strauss & Bichler, 1988; Leon & Zawojewski, 1993). Mevarech (1983), for example, found that high school students made mistakes in solving problems about means because they believed that means have the same properties as simple numbers, and that it is helpful to provide students corrective-feedback instruction as they solve problems involving reasoning about the mean. Strauss and Bichler (1988) found that fourth- through eighth-grade students had a difficult time understanding seven properties of the mean. Leon and Zawojewski (1993) looked at school and college students' understanding of four components of the mean. Using different testing formats, they found that some properties of the mean are better understood than others. The two properties, "the mean is a data point located between the extreme values of a distribution," and "the sum of the deviations about the mean equals zero" were better understood by students in this study than the two properties, "when the mean is calculated, any value of zero must be taken into account," and "the average value is representative of the values that were averaged."

Gal, Rothschild, and Wagner (1989, 1990) found that sixth-grade students are generally unable to use the mean to compare two different-sized sets of data. Later work showed that students had difficulty working backward from a mean to a data set that could produce such a mean (Cai, 1998). Study by Mokros and Russell (1995) expanded on this task by having students manipulate data values to produce a given mean and studying how students reasoned during this process.

Earlier research has also concentrated on understanding weighted means. Pollatsek et al. (1981) reported data from interviews of college students indicating difficulties they had in understanding the need to weight data in computing a mean. While mathematically sophisticated college students can easily compute the mean of a group of numbers, this study indicated that a surprisingly large proportion of these students do not understand the concept of the weighted mean, which is a concept that they often encounter (e.g., grade point averages). When asked to calculate a mean in a context that required a weighted mean, most subjects answered with the simple or unweighted mean of the two means given in the problem, even though these two means were from different-sized groups of scores. Callingham (1997) found that the same problem in a study of preservice and in-service teachers. As a result of their study, Pollatsek et al. (1981) wrote that "for many students dealing with the mean is a computational rather than a conceptual act" (p. 191). They concluded that knowledge of "computational rules not only does not imply any real understanding of the basic underlying concept, but may actually inhibit the acquisition of more adequate (relational) understanding" (p. 202).

What students remember about the mean? In general, it appears that many students who complete college statistics classes are unable to understand the idea of the mean. Mathews and Clark (2003) analyzed audio-taped clinical interviews with eight college freshmen immediately after they completed an elementary statistics course with a grade of "A." The point of these interviews was neither to see how quickly isolated facts could be recalled, nor was the point to see how little students remember. Rather, the goal was to determine as precisely as possible the conceptions of mean, standard deviation, and the Central Limit Theorem, which the most successful students had shortly after having completed a statistics course. The results are alarming since these top students demonstrated a lack of understanding of the mean, and could only state how to find it, arithmetically. Interviewing along the same lines, a larger (n=17) and more diverse sample of college students from four distinct campuses, Clark et al. (2003) found overall the same disappointing results. These researchers call, therefore, for pedagogical reform that will dis-equilibrate the process image of statistical concepts that students bring with them to college in order to enable them to encapsulate the process of statistical concepts into objects that are workable entities (Sfard, 1991). For example, they recommend creating situations in which students have to determine and reflect which measure of center is more appropriate.

Understanding Medians

Difficulties in determining the medians of data sets have also been documented by research. Elementary school teachers have difficulty determining the medians of data sets presented graphically (Bright & Friel, 1998). Only about one-third of twelfth grade students in the United States who took the NAEP test were able to determine the median when given a set of unordered data (Zawojewski & Shaughnessy, 2000).

Measures of Center as Typical Values

The *typical value* interpretation of the arithmetic mean has received a great deal of attention in curriculum materials and in research literature (Konold & Pollatsek, 2002). The following is an example of a problem set in a *typical value* context:

> The numbers of comments made by 8 students during a class period were 0, 5, 2, 22, 3, 2, 1, and 2. What was the typical number of comments made that day? (Konold & Pollatsek, 2002, p. 268)

Several studies have provided insights about students' thinking in regard to typical value problems. Mokros and Russell (1995) studied the characteristics of fourth through eighth graders' constructions of "average" as a *representative* number summarizing a data set. Twenty-one students were interviewed, using a series of open-ended problems that called on children to construct their own notion of mathematical representativeness. They reported that students may respond to *typical value* problems by: (i) locating the most frequently occurring value; (ii) executing an algorithm; (iii) examining the data and giving a reasonable estimate; (iv) locating the midpoint of the data; or (v) looking for a point of balance within the data set. These approaches illustrate the ways in which school students are (or are not) developing useful, general definitions for the statistical concept of average, even after they have mastered the algorithm for the mean.

Levels of Reasoning About Measures of Center

In an investigation of the development of school students' thinking in regard to measures of center, Watson and Moritz (1999) placed a structure on the categories of thinking documented by Mokros and Russell (1995). Continuing this line of research, Watson and Moritz (2000c, 2000d) also used the SOLO taxonomy (Structure of Observed Learning Outcomes, see Biggs and Collis, 1982) to rank students' responses to interview questions about averages. They observed movement from Unistructural, to Multistructural, and finally to Relational levels of reasoning as students developed from thinking about centers first from "mosts and middles" and finally to the mean as "representative" of a data set. Jones, Thornton, Langrall, Mooney, Perry, and Putt (2000) and Mooney (2002) found that the ability to be thoughtful and critical about applying formal measures to typical value problems marks a relatively sophisticated level of statistical reasoning.

Measures of Center as "Signals in Noisy Processes"

A rich spectrum of student reasoning about center is identified by Konold and Pollatsek (2002): mean as typical value, mean as fair share, mean as data reducer, and finally, mean as signal amid noise. These researchers suggest that students should be given more opportunities to work with statistical problems set in contexts that involve searching for "signals in noisy processes." The following item is an example of a data analysis problem that involves detecting a signal in a noisy process:

> A small object was weighed on the same scale separately by nine students in a science class. The weights (in grams) recorded by each student were 6.2, 6.0, 6.0, 15.3, 6.1, 6.3, 6.2, 6.15, 6.2. What would you give as the best estimate of the actual weight of this object? (Konold & Pollatsek, 2002, p. 268)

In the case of the repeated measures problem above, the arithmetic mean of the weights that are bunched closely together could be viewed as a signal that estimates the true weight of the object. The measurement of the object can be viewed as a noisy process that contains variation stemming from various possible sources. Konold and Pollatsek (2002) acknowledge the possible cognitive complexity in using repeated measurement problems with students, pointing out that the mean as a reliable indicator of signal was not universally accepted by scientists during the early development of the discipline of statistics (Stigler, 1986). Hence, they call for more research on students' thinking in such contexts in order to help advise instruction.

Patterns of thinking about average in different contexts were investigated by Groth (2005) who studied fifteen high school students. He used problems set in two different contexts: determining the typical value within a set of incomes and determining an average set in a signal-versus-noise context. Analysis of the problem-solving clinical interview sessions showed that some students attempted to incorporate formal measures, while others used informal estimating strategies. Students displayed varying amounts of attention to both data and context in formulating responses to both problems. Groth pointed out the need for teachers to be conscious of building students' statistical intuitions about data and context and informal estimates of center, and connecting them to formal measures without implying that the formal measures should replace intuition.

Selecting an Appropriate Measure of Center

Another focus of research has been on the challenge of choosing an appropriate measure of center to represent a data set. The National Assessment of Education

Progress (NAEP) data confirm that school students frequently make poor choices in selecting measures of center to describe data sets (Zawojewski & Shaughnessy, 2000). Choosing an appropriate measure of center was also a challenge for students enrolled in an Advanced Placement high school statistics course (Groth, 2002). Similar results were found by Callingham (1997) who administered an item containing a data set structured, so that the median would be a better indicator of center than the mean, to a group of preservice and in-service teachers. Callingham reports that most of them calculated the mean instead of the more appropriate median.

In a study on statistical reasoning about comparing and contrasting mean, median, and mode of preservice elementary school teachers, Groth and Bergner (2006) described four levels, basing these on the SOLO Model:

- *Unistructural*-level: responses did not contain any strategy other than definition-telling when asked to compare and contrast the three measures.
- *Multistructural*-level: responses included definition-telling along with a vague notion that the mean, median, and the mode are all tools that can be used to analyze a set of data; responses did not reflect an understanding that each of the measures is intended to measure what is central or typical to data sets.
- *Relational*-level: responses differ from Multistructural responses in that they included recognition of the fact that the mean, median, and mode all measure the center of the data or what is typical about the data in some manner.
- *Extended abstract*-level: responses include all of the characteristics of those classified at the relational level, but go beyond relational-level responses in that they included discussions of when one measure of center might be more useful than another.

Groth and Bergner's (2006) study illustrated that attaining a deep understanding of these seemingly easy statistical concepts is a nontrivial matter, and that there are complex conceptual and procedural ideas that need to be carefully developed.

Using the History of Measures of Center to Suggest Instruction

The history of statistics can be a source of inspiration for instructional design. Bakker and Gravemeijer (2006) systematically examined examples of how measures of center were described and used, starting in ancient historical periods, and from countries such as India and Greece, in contexts involving mathematics and science. Based on their analysis of these examples, Bakker and Gravemeijer (2006) formulate hypotheses about how students in grades 7 and 8 (12–14-years old) could be supported in learning to reason with mean and median. The following ideas stemming from the historical phenomenology were found to be most fruitful for helping young children understand center.

1. Estimation of large numbers can challenge students to develop and use intuitive notions of mean.
2. Students may use the midrange as a precursor to more advanced notions of average.
3. Repeated measurement may be a useful instructional activity for developing understanding of the mean (cf., Petrosino, Lehrer, & Schauble, 2003).
4. To support students' understanding of the median, it is helpful to let them visually estimate the median in a dot plot and look for a value for which the areas on the left and right are the same.
5. Skewed distributions can be used to make the usefulness of the median a topic of discussion.

Such a historical study can help to "unpack" and distinguish different aspects and levels of understanding of statistical concepts and help instructional designers to look through the eyes of students. Note that some of these hypotheses are in accordance with the results of the research studies described above.

Implications of the Research: Teaching Students to Reason About Center

What has been striking over 25 years of research is the difficulty encountered by students of all ages and teachers in understanding concepts related to center. Although students may be able to compute simple arithmetic means, they need help in understanding what means actually mean. Activities can help students develop meaningful models such as balancing of data values by manipulating deviations from the mean to sum to zero.

The research suggests that careful attention be paid to developing the concepts of measures of center, focusing on mean and median rather than mode. These ideas should be first introduced informally as students estimate and reason about typical value for data sets, both large and small, prior to formally studying these topics in a unit on measure of center. Students may be asked to make and test conjectures about typical values using real data sets. The research also suggests that students have opportunities to explore the characteristics of the mean and median and how they are affected by different types of data sets and distributions. Developing an understanding of deviation may be an important part of understanding the mean as a balance point, so activities helping students to see and reason about deviations may help them better understand the mean. The literature suggests both visual, interactive activities as well as explorations with real data utilizing technology to produce measures of center, especially for data sets where values are changed (e.g., outliers are added or removed). Finally, the idea of the center as a signal in a noisy process should be developed, examining trends in repeated measurements. This also suggests that ideas of center be introduced along with ideas of spread or variability, and that these ideas are repeatedly connected as students explore and interpret data.

Progression of Ideas: Connecting Research to Teaching

Introduction to the Sequence of Activities to Develop Reasoning About Center

The idea of typical value as a summary measure of data set, shown graphically, is first introduced in early units whenever students make or examine graphs of numerical data. While students may intuitively look at the mode or peak of a graph or look at the middle value on the horizontal axis, they can be guided to think about the mean and median as typical values by looking at different graphs where mode or middle scale value do not seem to represent good "typical" values. This will help motivate the need for examining different measures of center and when to use them. These informal examinations and estimates should include estimates of spread of the data as well, as students respond to questions such "what is a typical value for these data" and "how spread out are the data?," learning to connect ideas of center and variability from the beginning of the course.

When formal measures of center are introduced, students are guided to explore their properties using physical and then computer manipulations of data. Properties of the mean and median can be explored and examined in this way. It is helpful for students to actually work backwards, starting with a given value of mean or median to reason about how different data sets may be constructed and altered to produce those given values. This can be done first for mean and then for median. Students then conjecture what typical values they might find for different types of variable, taking into account the shapes and characteristics of graphs of these variables. These conjectures can then be tested using real data and technology, and discussions can examine which measures are more useful summaries for each variable and why.

When students begin to study formal measures of variability, they see the relationship between mean and standard deviation, and between medial and Interquartile Range, and how it makes sense to use these pairings when summarizing different types of distributions (e.g., means and standard deviations for symmetric distributions, medians and IQR for very skewed distributions, and distributions with outliers). The idea of examining center at the same time as variability as a way to compare groups is then encountered as students learn about and compare boxplots. When the Normal distribution is introduced, students will see that the mean has special properties and use in relation with stand deviations and z-scores.

The mean is again examined when learning about samples and how the mean stabilizes as sample size gets very large, and the role of the mean in the Central Limit Theorem. As students move from sampling to statistical inference, they again encounter the mean, distinguishing between using the mean in an inference based on a large sample from using the mean as a summary measure of a single data set (when a median might be a better typical value given the shape of the distribution). The measures of center are also encountered in the unit on covariation when students look at trends by examining medians of sequential boxplots, and later as centers of distributions of the two variables. Table 9.1 shows a suggested series of ideas and activities that can be used to guide the development of students' reasoning about models and modeling.

Table 9.1 Sequence of activities to develop reasoning about center[1]

Milestones: ideas and concepts	Suggested activities
Informal ideas prior to formal study of center	
• Idea of center as a typical or representative value for a graph of a variable (e.g., dot)	• Distinguishing Distributions (Lesson 1, Distribution Unit, Chapter 8)
• The mean as somewhere in between the highest and lowest value, but not necessarily the middle value or the midpoint of the horizontal scale	• What does the Mean Mean Activity (Lesson 1: "Reasoning about Measures of Center")
Formal ideas of center	
• Properties of the mean as a balance point and the value for which all deviations from that value sum to zero	• What does the Mean Mean Activity (Lesson 1)
• How the mean is affected by extreme values	• What does the Mean Mean Activity (Lesson 1)
• The median as the middle value in a data set	• What does the Median Mean Activity (Lesson 1)
• Properties of the median: under what conditions it changes or stays the same	• What does the Median Mean Activity (Lesson 1)
• Comparing properties of the mean and median	• Means and Medians Activity (Lesson 1)
• The idea of a typical value	• What is Typical Activity (Lesson 2: "Choosing Appropriate Measures")
• Understanding why and how to use appropriate measures of center for a sample of data for a particular variable	• Choosing an Appropriate Measure of Center Activity (Lesson 2)
Building on formal ideas of center in subsequent topics	
• How center and spread are used together to compare groups	• Activities in Lessons 1, 2, 3 and 4, Comparing Groups Unit (Chapter 11)
• The idea and role of mean in normal distribution	• What is Normal Activity (Lesson 3, Statistical Models Unit, Chapter 7)
• Recognize stability of measures of center as sample size increases. When sample grows, see how measures of center predict center of larger population, and how it stabilizes (varies less)	• Sampling activities in Lessons 1, 2, and 3, Samples and Sampling Unit (Chapter 12)
• Role of mean in making inferences	• Activities in Lessons 1, 2, 3, 4, and 5 (Statistical Inference Unit, Chapter 13)
• Role of mean in bivariate distribution	❖ An activity involving fitting and interpreting the regression line to bivariate data. (The symbol ❖ indicates that this activity is not included in these lessons.)

[1] See page 391 for credit and reference to authors of activities on which these activities are based.

Introduction to the Lessons

There are two lessons on reasoning about measures of center. They begin with the physical activity described earlier where students manipulate Post-It® notes on a number line to develop an understanding of mean, and then median. Students use a *Fathom* demo to contrast how the mean and median behave for different types of data sets. Students make and test conjectures about typical values, testing these using software to generate graphs and statistics. The last activity has students compare features and uses of different measures of center when summarizing sample data.

Lesson 1: Reasoning About Measures of Center

While students have heard of means and medians before they enter an introductory high school or college statistics course, this lesson helps them develop a conceptual understanding of the mean and median. There are three parts to the lesson: an activity where students move dots on a plot to explore properties of the mean, a similar activity with the median, and then *Fathom* demos to further illustrate the properties of these concepts. Student learning goals for this lesson include:

1. Develop a conceptual understanding of the mean.
2. Understand the idea of deviations (differences from the mean) and how they balance out to zero.
3. Understand how these deviations cause the mean to be influenced by extreme values.
4. Develop an understanding of the median as a middle value that is resistant to extreme values.
5. Understand the differences between mean and median in their interpretation and properties.
6. Understand how to select appropriate measures of center to represent a sample of data.

Description of the Lesson

In the first activity described at the beginning of the chapter, (*What does the Mean Mean* activity) students are told that the average age (mean) for students in the class is 21 years and consider what we know about the distribution of students' ages for this class (e.g., "Are they all about 21 years old?"), and explain their answer first in a small group and then to the class. They also explain where this value of 21 came from and how it was produced. Students make conjectures about the ages of these 10 students and use 10 Post-It® notes to form a series of dot plots on a given number line so that the average is 21 years. Students move one Post-It® note to 24 years, and later one to 17, and figure out how to move *one or more* of the other Post-It® notes to keep the mean at 21 years, discussing their strategies and reasoning with their group and then the class. The term *deviation* is introduced to represent the distance of each data value from the mean and students examine deviations for their plots under different conditions, seeing how they have to balance to zero.

In the second activity (*What does the Median Mean?*), students reduce their Post-It® notes to 9 and arrange them on the same number line used earlier so that the *median* is 21 years. Again, they are given different constraints (e.g., change one of the values that is currently higher than 21 years) and they determine if and how the median is affected. Finally, students discuss and summarize what would they have to do with a data value in the plot in order to change the median.

In the final activity of this lesson (*Means and Medians*), students observe and discuss two *Fathom* Demos: *The Meaning of Mean* and *Mean and Median* to further understand properties of these measures. The lesson ends with a wrap-up discussion about use interpretation, and properties of the mean and median.

Lesson 2: Choosing Appropriate Measures

This lesson introduces the idea of choosing an appropriate measure of center to describe a distribution. It has students predict typical values for variables that have different distributions. The lesson then has students find the actual mean and median for those variables using computer software and examine the distributional features that made their prediction closer to either the mean or median. It also introduces the idea of outlier influence on these measures of center. Student learning goals for this lesson include:

1. Deeper conceptual understanding of mean and median.
2. Understand when it is better to use each as a summary measure for a distribution of data.
3. How to generate these statistics using *Fathom* Software.

Description of the Lesson

Students are first asked how we can describe the typical college students taking an introductory statistics course, and in what ways do students in this class differ? They discuss how people use the words: typical, average, and normal in an everyday sense and how these words are used as statistical terms: mean, median, center, and average.

In the *What is Typical* activity, students consider a set of variables that were measured on their first day of class Student Survey. Working in pairs, they predict what they might expect as a *typical value* for all students enrolled in their statistics class this term. They are reminded that a typical value is a single number that summarizes the class data for each variable. They write their prediction in the "First Prediction" column of the table shown below (Table 9.2).

Next, they generate dot plots of the data using *Fathom* software to see if their original predictions seemed reasonable. Based on the graphs, they are allowed to make revised predictions for the typical value for each of the variables, which are written in the table above in the "Revised Prediction" column.

Table 9.2 Predicting and verifying typical values in the *What is Typical Activity*

Attribute from Student Survey	First Prediction	Revised Prediction	Statistics from Fathom	
			Mean	Median
Age				
Number of statistics courses you are taking this semester				
Credits registered for this semester				
Total college credits completed				
Cumulative GPA				
Hours a week you study				
Number of emails you send each day				
Number of emails you receive each day				

Students use *Fathom* to find the mean and median for each of these variables and complete the last two columns of the table above. They discuss how close were their revised predictions to the "typical" values produced by *Fathom* and for which attributes were their predictions most accurate. They also consider what results were most surprising to them and why, and whether in general, were their revised predictions closer to the means or medians of these variables.

Students are asked to discuss:

- Which measures of center were closest to their intuitive ideas of "typical" values?
- What information do means and medians provide about a distribution?
- How to decide whether to use the mean or median to summarize a data set?
- In statistics, what do they think is meant by the word "typical"?

In *Choosing an Appropriate Measure of Center* activity, students suggest conditions where the mean and median provide similar information and when they give different information for the same data set. This leads to a discussion of which measure is more appropriate for each variable and why, and how to choose the best measure of center for a data set.

Students are asked if it is all right to compute a mean or median without first looking at a graph of data and then why that is not a good idea. They reason about what information is missing if all they are given is a measure of center, including what they know and not know if all they were given were measures of center. This provides a segue to discussion on spread (the next unit) and reinforces the connection between center and spread. In a wrap-up discussion, students are asked to imagine a variable that could be measured in two different settings that might yield data sets that have the same mean and different amounts of spread, one with a little spread and one with a lot of spread, and explain their reasoning.

Summary

The two lessons in the unit on measures of center are closely connected to ideas of distribution and variability, so that the ideas of mean and median are always connected to these concepts and contexts. The intent of the lessons is to help students build a conceptual understanding of mean and median as well as the idea of center of a distribution, through physical manipulations of data values, making and testing conjectures about typical values, and discussing the use and properties of these two measures. While the concepts may seem simple, and not worth two full lessons, we believe that that these lessons provide important foundations for and connections to subsequent units in the course.