Section 4.3 & Chapter 5 Suggestions

Lab Day. Two options:

1. M&M Experiment – See description after 5.4 below.

2. Interactive Software. The java applet for this activity is available on the students' Blackboard homepage. They can access it on the Course Content page in Blackboard by clicking on the "Interactive software (Exponential Graphs) link, which is the third link after the folders. When they click on it, a dialogue box will come up asking, "Do you want to run or save "**exponential.exe** (787 KB) from **learn.kent.edu**? They should click on "Run." A warning will appear that "the publisher couldn't be verified…" They should click on "Run" and the applet will appear.
The first activity uses E3: $y = Ca^x$ Sliders so click on that one and Enjoy!
Students will need the "Interactive Software Activity" pages which are also on the Blackboard Course Content page.

Sec. 4.3 Inverse Functions **(2 days)**

Day 1
Goals for students

- Understand that an inverse relation undoes the original
- Use composition of functions to verify that functions are inverses of each other
- Understand why the line $y = x$ is a line of symmetry for the graphs of relations and their inverses
- Determine if a function is one-to-one

You might start by asking students to work on this in small group:

> *Suppose that 0^o Celsius corresponds to 32^o Fahrenheit and that 100^o Celsius corresponds to 212^o Fahrenheit.*
> *a) Write a linear function with Celsius as input and Fahrenheit as output*
> *b) Write a linear function with Fahrenheit as input and Celsius as output.*
> *c) Using your first equation, find the degrees F for a temperature of 20 ^oC*
> *d) Now substitute this answer in your second equation. What did you notice?*
> *e) Repeat c) and d) above for several different initial values. Did the same pattern occur? Will this always happen? Explain*

Discuss the notion of inverse relation. You might want to introduce the idea of inverses as "undoing" each other, and then reinforce the idea with other examples:

- *What operation undoes the operation of adding 3? of multiplying by 3?*
- *How can we write these using function notation?*

Let $f(x) = x + 3$ and $g(x) = x - 3$. Find $f(10)$, then $g(f(10))$.

I then usually give the definition of inverse function, both informally ("one function undoes the other, you get the same thing back") and more formally (p. 279). Page 121 in the handbook provides opportunity for students to sharpen their algebraic skills and work with this definition.

You might then develop the idea that the inputs and the outputs are interchanged when we're dealing with inverse relations:

Using the previous example:

Note that $f(10)=13$ *so that the point* $(10,13)$ *is on the graph of the function f.*
Note also that $g(13)=10$*, so that the point* $(13,10)$ *is on the graph of the function g. Notice that the x coordinate and y coordinate are interchanged.*

You might then do a few more, plot the points on a graph (or let students do it), then ask students to work on the handout, "Sketch inverse graphs," available on the Instructor site. Try to pull from them where the line of symmetry might be.

I usually then ask my students to hand graph the function $y=x^2$ and try to sketch in its inverse.

- *What happens when you do this?*
- *Is there anything "wrong" with the result?*

Usually someone will note that the result is not a function because it doesn't pass the vertical line test.

- *How can we tell before we interchange the xs and ys if the result will be a function?*

Having students come up with the Horizontal Line test by themselves is quite cool.

Define one-to-one functions. That's certainly enough for one day.

Suggested Homework: ***MLP*** HW 4.3 and in HB, finish p. 121, 1-11 (odds minimum)

Day 2
Goals for students:

- Given a function, find it's inverse
- Given a function in context, find and interpret its inverse

Build upon yesterday's lesson and emphasize again examples verifying that functions are inverses of each other. Finding an inverse function, given a function, will follow quite naturally and is usually quite easy for students at this point.
Should spend some time discussing domains and ranges and also restricting the domain of a function to allow the inverse to be a function

Suggested Homework: ***MLP:*** HW 4.3 and in HB: p. 121: 13 – 27 odds (minimum)

Sec. 5.4 . Exponential and Log Models

- Given a real world scenario, student can write an exponential model
- Student understands, and can explain, the difference between a growth (or decay) *factor* and a growth (or decay) *rate*.

- Given an exponential model, student can identify the growth (or decay) factor, rate, and initial value.
- Student can recognize exponential data given a data set in a table.

The purpose here is to introduce students to exponential growth in context. You might start with a problem for them to discuss in small groups:

> *Suppose over Christmas break you had a choice of two jobs. Company A will pay you $1,000,000 for 30 days work. Company B will be willing to pay you a penny on the first day, two cents on the second, and continue doubling the amount until the end of the month. On day 30 you would receive the final doubled amount (not the sum of all days). Which job should you take and why?*

Create, with the students, the math model to represent the above scenario. Then you might continue with problems from the book, like #18, 20 on p. 367 OR problems from the FMVI e-book. After students feel (somewhat) comfortable writing exponential models, you might want to look at the corresponding data set and note that there's a constant *percent* change and this is what characterizes exponential functions. Page 134 in the handbook has tables for this development. As you work with students on these tables, you might compare exponential growth to linear.

Suggested Homework: ***MLP:*** HW 5.4; Finish p. 134 HB; Perhaps also problems from FMVI e-book

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M & M Experiment

Goals for students

- Use exponential functions to predict *decay* over time
- Work with numerical and graphical representations of decreasing exponential function

The purpose of the experiment is for students to have a hands-on experience with exponential decay. Marc's and Giant Eagle usually have good prices for large packs of M & Ms. For one class, 5-6 packs should do it. (We've used this activity for a few semesters now and the students really like it!) The handout is on page 135 in the handbook.

After students record the data, collect all totals from each group, typing the toss # in L1 and the total number of candies with Ms showing in L2. You might have a couple students do the adding on their calculators while a representative from each group indicates the number of Ms..

Before looking at the scatterplot, you might ask them to conjecture a "theoretical" formula for the data.

Make a scatterplot on the overhead calculator; discuss. Type in the "theoretical" formula and comment on the fit. You might then use the regression feature of the calculator and compare with the theoretical formula.

Time permitting, you might do a few modeling problems with exponential decay, clarifying the difference between decay factor and decay rate. (You might make them up or do #1 – 3 on the homework together.)

Suggested homework: Handbook: p. 142, 1 – 12

Section 5.5 Exponential Functions and Investing

Goals for students:

- Understand the derivation of and use the compound interest formulas
- Understand the meaning of the number *e*

I realize that this section is out of order as per the day by day schedule. I didn't realize when I was creating the schedule that this section does NOT require students to solve an exponential equation. Since it's a modeling section, it follows nicely after section 3.4 and the M & M experiment.

Be sure to intuitively derive the compound interest formulas, beginning with the one for annual interest, then intuiting the one for multiple compoundings per year.

For discussing and "discovering" the number *e*, you might want to discuss this hypothetical situation:

> *Suppose that you have $1.00 and you have a friend you will give you 100% interest. We want to see how much money you have after 1 year if you vary the number of compoundings.*

Using the TABLE feature of the graphing calculator is convenient here, after asking students to do the first couple calculations by hand. You also might look at the graph. Students will inevitably notice that this value seems to hover around 2.7.

Suggested Homework: ***MLP:*** HW 5.5

Section 5.1 Exponential Functions

Goals for students:

- Continue working with exponential functions in context, finding and interpreting function values, given an exponential function
- Graph various exponential functions involving transformations by hand

Since students by now have had some experience using exponential functions and modeling, the emphasis on this section should be graphs of exponential functions. Page 141 in the handbook has several graphing problems. You might ask students to work on these in class, refreshing the transformations from Section 4.1.

Suggested Homework:	***MLP:*** HW 5.1 and/or finish those problems indicated with a checkmark on p. 141 in the handbook.

Section 5.2 Logarithmic Functions

Goals for students:

- Explain the meaning of logarithm
- Find simple logarithms (base 10 and otherwise) without the use of a calculator

You might begin with the introductory activity on p. 143 in the handbook. Then reinforce that *"A logarithm is an exponent."* The problems on p. 144 HB are also appropriate for classroom use.

Suggested Homework:	***MLP:*** HW 5.2 and/or finish problems on pp. 144-45 in Handbook (HB)

Section 5.2 Logarithmic Functions Day 2

Goals for students

- Understand and work with basic properties of logarithms
- Understand and work with natural logarithms
- Understand the derivation of and use the properties of logarithms

You might use today to allow students to work on pp. 145-46 in the handbook and then derive the following basic properties from them:

1. $\log_a 1 = 0$ for any base, a >0, where a is any real number
2. $\log_a a = 1$ for any base, a >0, where a is any real number
3. $\log_a a^M = M$ for any base, a >0, where a is any real number

Suggested in-class activity for Properties of Logs:
Let x = 100 and y = 1000. Find log xy. Make a conjecture about how this value relates to log x and log y? Repeat for x = 10000 and y = 10, then for x = 2 and x = 3. (Use your calculator for this one.). Does your conjecture still hold? Will it always hold? Explain.

Then run through the verification.

Repeat for $\log \frac{x}{y}$.

Then show that $\log x^4 = \log\ x \cdot x \cdot x \cdot x = \log x + \log x + \log x + \log x = 4 \cdot \log x$. Then generalize.

Forgive the traditional side of me here, but if we want the students to be proficient with these properties, they need to do more than the few problems that the text provides. The MML HW 5.2B and 5.2C Laws of Logarithms assignments, however, have several problems taken from the Algebra for Calc book and should be sufficient.

Suggested Homework:	***MLP:*** HW 5.2B and 5.2C Laws of Logarithms. (25 problems total.) You might consider asking students to work through (i.e. write out) the examples on pp. 152-154 in the handbook. Problems #57-72 on p. 156 in the handbook are also good.

Section 5.3 Day 1**:** ***Exponential equations***

Goals for students:

- Solve exponential equations in context

You might start with the simple exponential equations in example 6 on p. 345, then reinforce by asking students to look at those p. 166 in the handbook.

Follow up with example 11 on p. 349, then perhaps #44 or #46 on p. 351. The applications supply ample opportunity for students to solve exponential equations.

Suggested Homework:	***MLP:*** HW 5.3A

Section 5.3 Day 2 Logarithmic equations

Goals for students:

- Solve logarithmic equations

Suggested Homework:	***MLP:*** HW 5.3B You might supplement with the problems on p. 166 in handbook, part C, lower right