Chapter 3
Quadratic, Piecewise-Defined, and Power Functions

Toolbox Exercises

1. $\left(\frac{2}{3}\right)^{-2}=\left(\frac{3}{2}\right)^{2}=\frac{3^2}{2^2}=\frac{9}{4}$

2. $\left(\frac{3}{2}\right)^{-3}=\left(\frac{2}{3}\right)^{3}=\frac{2^3}{3^3}=\frac{8}{27}$

3. $10^{-2}\times10^{0}=\frac{1}{10^2}\times1=\frac{1}{100}$

4. $8^{-2}\times8^{0}=\frac{1}{8^2}\times1=\frac{1}{64}$

5. $\left(2^{-1}\right)^{3}=2^{-1\times3}=2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

6. $\left(4^{-2}\right)^{2}=4^{-2\times2}=4^{-4}=\frac{1}{4^4}=\frac{1}{256}$

7. $|-6|=6$

8. $|7-11|=|-4|=4$

9. **a.** $\sqrt{x^3}=\sqrt[2]{x^3}=x^{\frac{3}{2}}$

b. $\sqrt[4]{x^3}=x^{\frac{3}{4}}$

c. $\sqrt[5]{x^3}=x^{\frac{3}{5}}$

d. $\sqrt[6]{27y^9}$

$=\left(27y^9\right)^{\frac{1}{6}}$

$=\left(3^3y^9\right)^{\frac{1}{6}}$

$=3^{\frac{3}{6}}y^{\frac{9}{6}}$

$=3^{\frac{1}{2}}y^{\frac{3}{2}}$

e. $27\sqrt[6]{y^9}=27y^{\frac{9}{6}}=27y^{\frac{3}{2}}$

10. **a.** $a^{\frac{3}{4}}=\sqrt[4]{a^3}$

b. $-15x^{\frac{5}{8}}=-15\sqrt[8]{x^5}$

c. $(-15x)^{\frac{5}{8}}=\sqrt[8]{(-15x)^5}$

11. $\left(4x^2y^3\right)\left(-3a^2x^3\right)$

$=-12a^2x^{2+3}y^3$

$=-12a^2x^5y^3$

12. $2xy^3\left(2x^2y+4xz-3z^2\right)$

$=\left(2xy^3\right)\left(2x^2y\right)+\left(2xy^3\right)(4xz)$

$-\left(2xy^3\right)\left(3z^2\right)$

$=4x^3y^4+8x^2y^3z-6xy^3z^2$

13. $(x-7)(2x+3)$

$x(2x)+x(3)-7(2x)-7(3)$

$2x^2+3x-14x-21$

$2x^2-11x-21$

Copyright ©2013 Pearson Education. Inc.

14. $(k-3)^2$

$(k-3)(k-3)$

$k^2-3k-3k+9$

k^2-6k+9

or

$(k-3)^2$

$(k)^2-2(k)(3)+(3)^2$

k^2-6k+9

15. $(4x-7y)(4x+7y)$

$16x^2+28xy-28xy-49y^2$

$16x^2-49y^2$

or

$(4x-7y)(4x+7y)$

$(4x)^2-(7y)^2$

$16x^2-49y^2$

16. Remove common factor $3x$

$3x^2-12x=3x(x-4)$

17. Remove common factor $12x^3$

$12x^5-24x^3=12x^3(x^2-2)$

18. Difference of two squares

$9x^2-25m^2=(3x+5m)(3x-5m)$

19. Find two numbers whose product is 15 and whose sum is -8.

$x^2-8x+15=(x-5)(x-3)$

20. Find two numbers whose product is -35 and whose sum is -2.

$x^2-2x-35=(x-7)(x+5)$

21.

To factor by grouping, first multiply the 2nd degree term by the constant term:

$3x^2(-2)=-6x^2$.

Then, find two terms whose product is $-6x^2$ and whose sum is $-5x$, the middle term. $(-6x)$ and (1x)

$3x^2-5x-2$

$=3x^2-6x+1x-2$

$=(3x^2-6x)+(1x-2)$

$=3x(x-2)+1(x-2)$

$=(3x+1)(x-2)$

22.

To factor by grouping, first multiply the 2nd degree term by the constant term:

$8x^2(5)=40x^2$.

Then, find two terms whose product is $40x^2$ and whose sum is $-22x$, the middle term. $(-2x)$ and $(-20x)$

$8x^2-22x+5$

$=8x^2-2x-20x+5$

$=(8x^2-2x)+(-20x+5)$

$=2x(4x-1)+(-5)(4x-1)$

$=(2x-5)(4x-1)$

Copyright ©2013 Pearson Education. Inc.

23.

$6n^2 + 18 + 39n$

$= 6n^2 + 39n + 18$

$= 3\left(2n^2 + 13n + 6\right)$

To factor by grouping, first remove the common factor 3, then multiply the 2nd degree term by the constant term:

$2n^2(6) = 12n^2$.

Then, find two terms whose product is $12n^2$ and whose sum is $13n$, the middle term. $(1n)$ and $(12n)$

$3\left(2n^2 + 13n + 6\right)$

$= 3\left(2n^2 + 1n + 12n + 6\right)$

$= 3\left[\left(2n^2 + 1n\right) + \left(12n + 6\right)\right]$

$= 3\left[n\left(2n+1\right) + 6\left(2n+1\right)\right]$

$= 3\left(n+6\right)\left(2n+1\right)$

24. $3y^4 + 9y^2 - 12y^2 - 36$

$= 3\left[y^4 + 3y^2 - 4y^2 - 12\right]$

$= 3\left[\left(y^4 + 3y^2\right) + \left(-4y^2 - 12\right)\right]$

$= 3\left[y^2\left(y^2 + 3\right) + \left(-4\right)\left(y^2 + 3\right)\right]$

$= 3\left(y^2 - 4\right)\left(y^2 + 3\right)$

$= 3\left(y-2\right)\left(y+2\right)\left(y^2+3\right)$

25. $18p^2 + 12p - 3p - 2$

$= \left(18p^2 + 12p\right) + \left(-3p - 2\right)$

$= 6p\left(3p+2\right) + \left(-1\right)\left(3p+2\right)$

$= \left(6p-1\right)\left(3p+2\right)$

26. $5x^2 - 10xy - 3x + 6y$

$5x(x-2y) - 3(x-2y)$

$\left(x-2y\right)\left(5x-3\right)$

27. **a.** Imaginary. The number has a non-zero real part and an imaginary part.

b. Pure imaginary. The real part is zero.

c. Real. The imaginary part is zero.

d. Real. $2 - 5i^2 = 2 - 5\left(-1\right) = 7$

28. **a.** Imaginary. The number has a non-zero real part and an imaginary part.

b. Real. The imaginary part is zero.

c. Pure imaginary. The real part is zero.

d. Imaginary. The number has a non-zero real part and an imaginary part.
$2i^2 - i = 2\left(-1\right) - i = -2 - i$

29. $a + bi = 4 + 0i$. Therefore,
$a = 4, b = 0$.

30. $a + 3i = 15 - bi$
Therefore, $a = 15$ and
$-b = 3$ or $b = -3$.

31. $a + bi = 2 + 4i$
Therefore, $a = 2$ and $b = 4$.

Copyright ©2013 Pearson Education. Inc.

Section 3.1 Skills Check

1. **a.** Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since a = 2 > 0, the graph opens up and is therefore concave up.

c. Since the graph is concave up, the vertex point is a minimum.

2. Not quadratic. The equation does not fit the form $f(x) = ax^2 + bx + c, a \neq 0.$ The highest exponent is 1.

3. Not quadratic. The equation does not fit the form $f(x) = ax^2 + bx + c, a \neq 0.$ The highest exponent is 3.

4. **a.** Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since a = 1 > 0, the graph opens up and is therefore concave up.

c. Since the graph is concave up, the vertex point is a minimum.

5. **a.** Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since a = –5 < 0, the graph opens down and is therefore concave down.

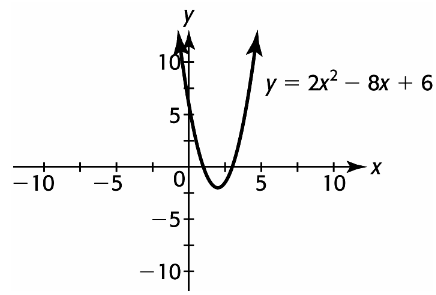
c. Since the graph is concave down, the vertex point is a maximum.

6. **a.** Yes. The equation fits the form $f(x) = ax^2 + bx + c, a \neq 0.$

b. Since a = –2 < 0, the graph opens down and is therefore concave down.

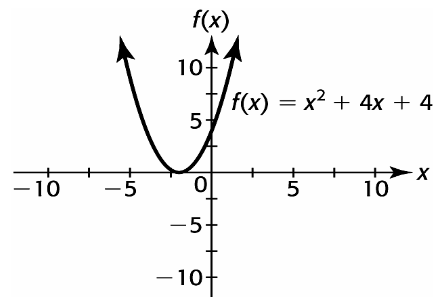
c. Since the graph is concave down, the vertex point is a maximum.

7. **a.**



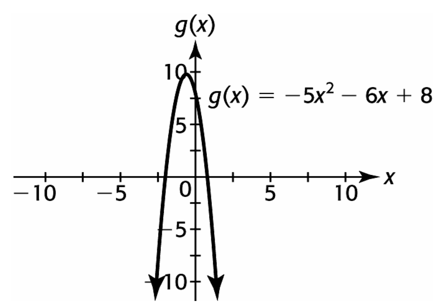
b. Yes.

8. **a.**



b. Yes.

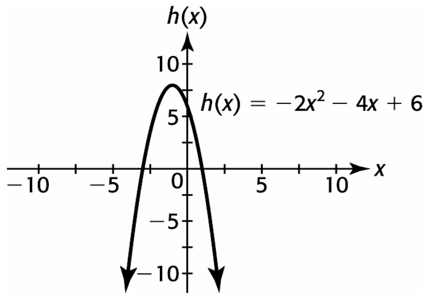
9. **a.**



b. Yes.

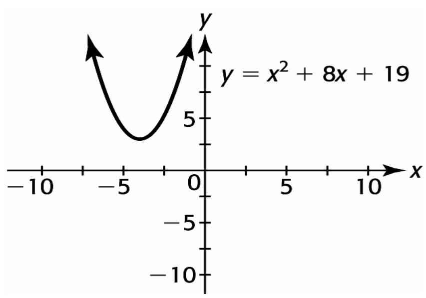
Copyright ©2013 Pearson Education. Inc.

10. a.



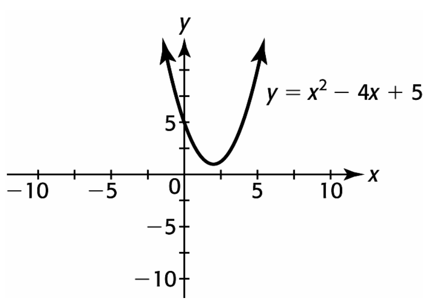
b. Yes.

11. a.



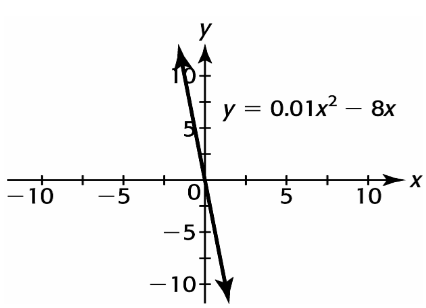
b. Yes.

12. a.



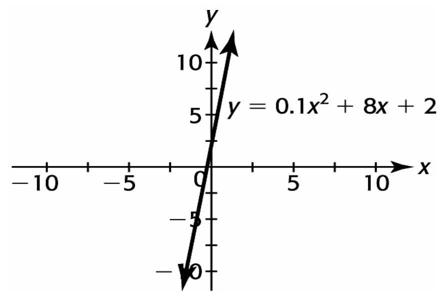
b. Yes.

13. a.



b. No. The complete graph will be a parabola.

14. a.



b. No. The complete graph will be a parabola.

15.
Using the given vertex point $(2,-4)$ and the vertex form of the parabola equation, then $y = a(x-2)^2 - 4$. To solve for the value of a, plug in the other given point $(4,0)$ to get $0 = a(4-2)^2 - 4$. Then $0 = a(2)^2 - 4 = 4a - 4$, so that $a = 1$. The final equation of the parabola in vertex form is $f(x) = 1(x-2)^2 - 4$.

16.
Using the given vertex point $(3,5)$ and the vertex form of the parabola equation, then $y = a(x-3)^2 + 5$. To solve for the value of a, plug in the other given point $(1,1)$ to get $1 = a(1-3)^2 + 5$. Then $1 = a(-2)^2 + 5 = 4a + 5$, so that $a = -1$. The final equation of the parabola in vertex form is $f(x) = -1(x-3)^2 + 5$.

17. The value of a is larger for y_1 since the graph is vertically stretched, causing it to appear more narrow than y_2.

Copyright ©2013 Pearson Education. Inc.

18. The value of $|a|$ is larger for y_2 since the graph is vertically stretched, causing it to appear more narrow than y_1. The fact that a may be a negative value has no effect on the width of the graph, only on its direction.

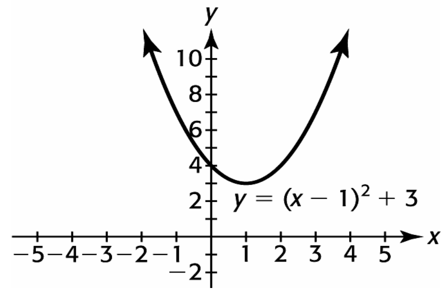
19. Since $(-1,-7)$ and $(3,-7)$ are symmetric points on the graph, if it were drawn, the axis of symmetry of the parabola would be $x = 1$. The point (1, 13) would then be the vertex of the parabola. Using the vertex form of the parabola equation, $y = a(x-1)^2 + 13$. To solve for the value of a, plug in the another given point from the table, $(5,-67)$, to get $-67 = a(5-1)^2 + 13$.

Then,
$-67 = a(4)^2 + 13$
$-67 = 16a + 13$ so that $a = -5$. The final equation of the parabola in vertex form is
$f(x) = -5(x-1)^2 + 13$
$f(x) = -5x^2 + 10x + 8$

20. Since $(-3,-12)$ and $(-1,-12)$ are symmetric points on the graph, if it were drawn, the axis of symmetry of the parabola would be $x = -2$. The point $(-2,-15)$ would then be the vertex of the parabola. Using the vertex form of the parabola equation, $y = a(x+2)^2 - 15$. To solve for the value of a, plug in the another given point $(-6,33)$ to get
$33 = a(-6+2)^2 - 15$. Then
$33 = a(-4)^2 - 15$
$33 = 16a - 15$ so that $a = 3$. The final equation of the parabola in vertex form is
$f(x) = 3(x+2)^2 - 15$
$f(x) = 3x^2 + 12x - 3$.

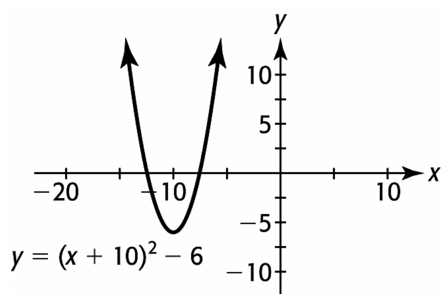
21. a. $(1,3)$

b.



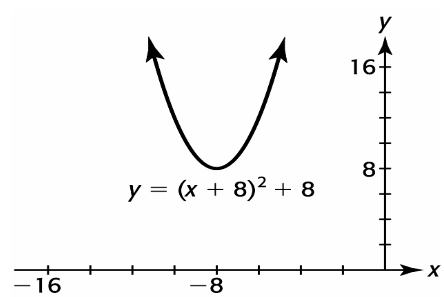
22. a. $(-10,-6)$

b.



23. a. $(-8,8)$

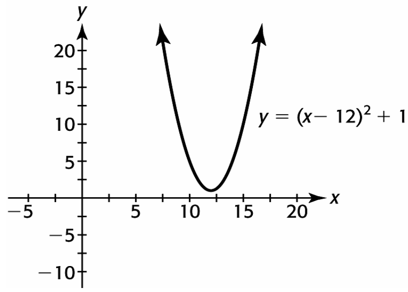
b.



Copyright ©2013 Pearson Education. Inc.

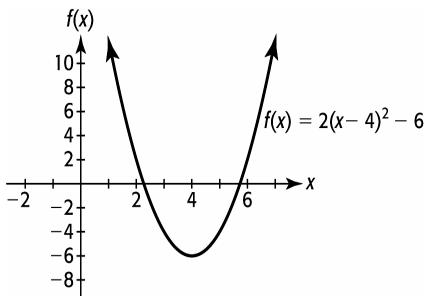
24. a. $(12,1)$

b.



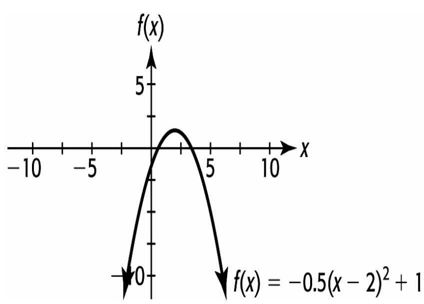
25. a. $(4,-6)$

b.



26. a. (2, 1)

b.



27. a. $y = 12x - 3x^2$

$y = -3x^2 + 12x$

$a = -3, b = 12, c = 0$

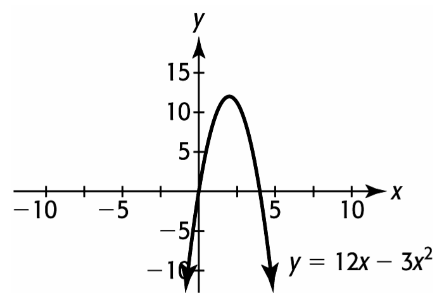
$h = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$

$k = f(h) = f(2) = 12(2) - 3(2)^2 =$

$24 - 3(4) = 24 - 12 = 12$

The vertex is $(2,12)$.

b.



28. a. $y = 3x + 18x^2$

$y = 18x^2 + 3x$

$a = 18, b = 3, c = 0$

$h = \frac{-b}{2a} = \frac{-3}{2(18)} = \frac{-3}{36} = -\frac{1}{12}$

$k = f(h)$

$= f\left(-\frac{1}{12}\right)$

$= 3\left(-\frac{1}{12}\right) + 18\left(-\frac{1}{12}\right)^2$

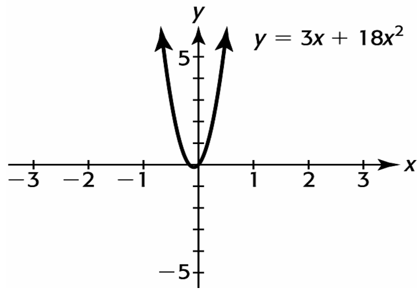
$= -\frac{3}{12} + 18\left(\frac{1}{144}\right)$

$= -\frac{1}{4} + \frac{1}{8} = -\frac{2}{8} + \frac{1}{8} = -\frac{1}{8}$

The vertex is $\left(-\frac{1}{12}, -\frac{1}{8}\right)$.

Copyright ©2013 Pearson Education. Inc.

b.



29. a. $y = 3x^2 + 18x - 3$

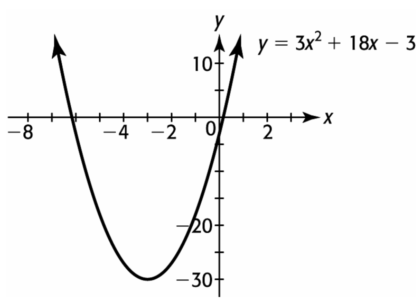
$a = 3, b = 18, c = -3$

$$h = \frac{-b}{2a} = \frac{-18}{2(3)} = \frac{-18}{6} = -3$$

$$\begin{aligned} k &= f(h) \\ &= f(-3) \\ &= 3(-3)^2 + 18(-3) - 3 \\ &= 3(9) - 54 - 3 \\ &= 27 - 54 - 3 = -30 \end{aligned}$$

The vertex is $(-3, -30)$.

b.



30. a. $y = 5x^2 + 75x + 8$

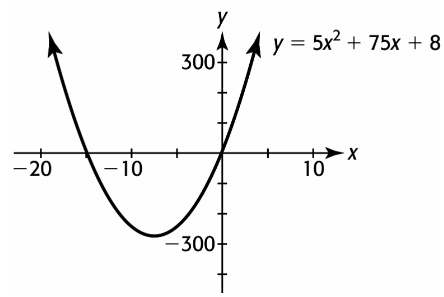
$a = 5, b = 75, c = 8$

$$h = \frac{-b}{2a} = \frac{-75}{2(5)} = \frac{-75}{10} = -7.5$$

$$\begin{aligned} k &= f(h) \\ &= f(-7.5) \\ &= 5(-7.5)^2 + 75(-7.5) + 8 \\ &= 5(56.25) - 562.5 + 8 \\ &= -273.25 \end{aligned}$$

The vertex is $(-7.5, -273.25)$.

b.

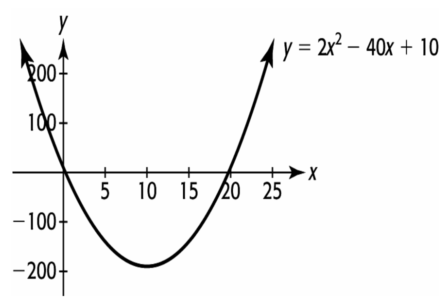


31. a. $y = 2x^2 - 40x + 10$

$a = 2, b = -40, c = 10$

$$h = \frac{-b}{2a} = \frac{-(-40)}{2(2)} = \frac{40}{4} = 10$$

b.



c. The vertex is located at $(10, -190)$

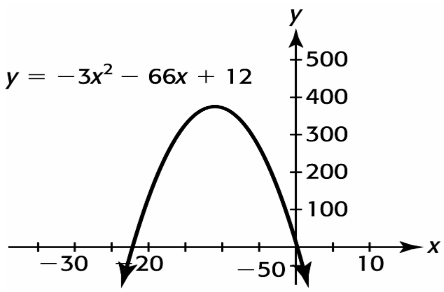
Copyright ©2013 Pearson Education. Inc.

32. a. $y = -3x^2 - 66x + 12$

$a = -3, b = -66, c = 12$

$$h = \frac{-b}{2a} = \frac{-(-66)}{2(-3)} = \frac{66}{-6} = -11$$

b.



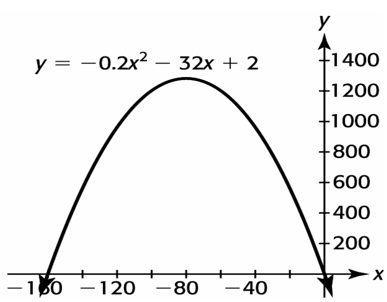
c. The vertex is located at $(-11, 375)$

33. a. $y = -0.2x^2 - 32x + 2$

$a = -0.2, b = -32, c = 2$

$$h = \frac{-b}{2a} = \frac{-(-32)}{2(-0.2)} = \frac{32}{-0.4} = -80$$

b.



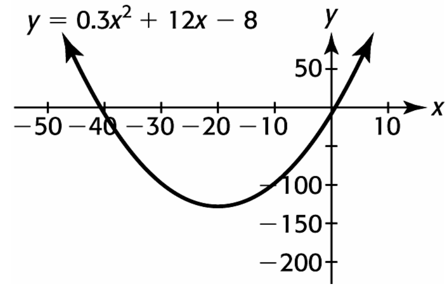
c. The vertex is located at $(-80, 1282)$

34. a. $y = 0.3x^2 + 12x - 8$

$a = 0.3, b = 12, c = -8$

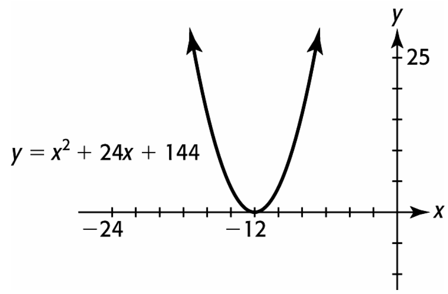
$$h = \frac{-b}{2a} = \frac{-(12)}{2(0.3)} = \frac{-12}{0.6} = -20$$

b.



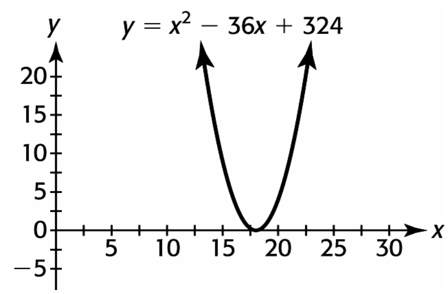
c. The vertex is located at $(-20, -128)$

35.



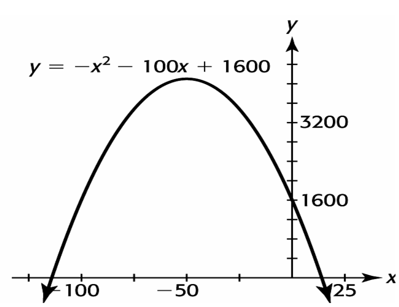
The vertex is located at $(-12, 0)$

36.



The vertex is located at (18, 0)

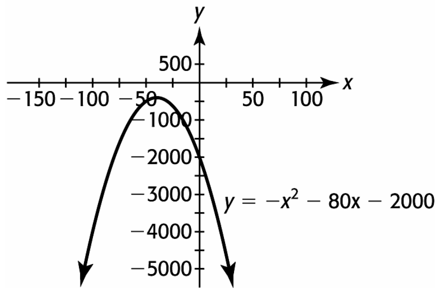
37.



The vertex is located at $(-50, 4100)$

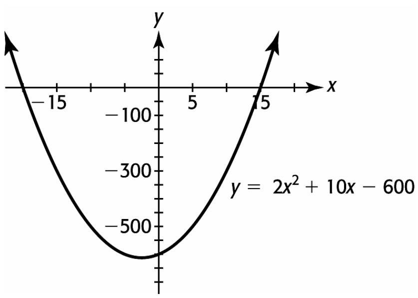
Copyright ©2013 Pearson Education. Inc.

38.



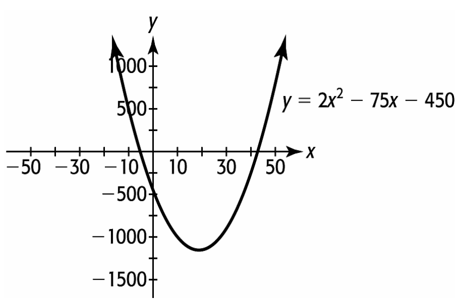
The vertex is located at $(-40, -400)$

39.



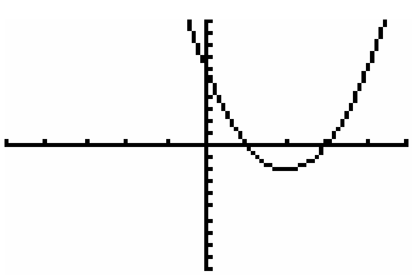
The vertex is located at $(-2.5, -612.5)$

40.



The vertex is located at $(18.75, -1153.125)$

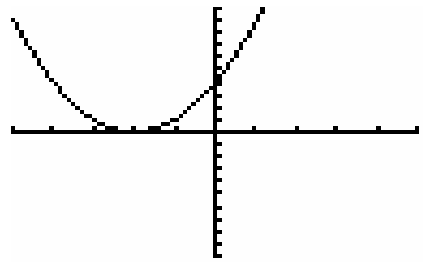
41. $y = 2x^2 - 8x + 6$



[–5, 5] by [–10, 10]

The x-intercepts are $(1,0), (3,0)$.

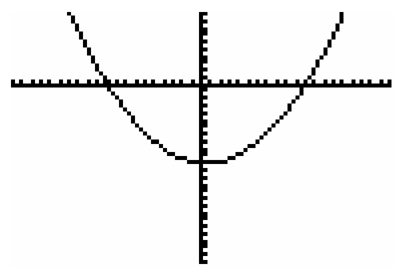
42. $y = x^2 + 4x + 4$



[–5, 5] by [–10, 10]

The x-intercept is $(-2,0)$.

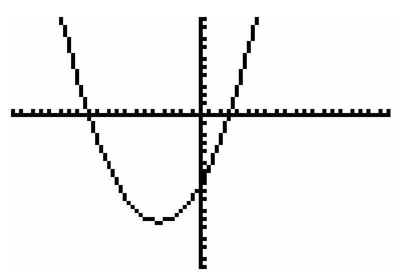
43. $y = x^2 - x - 110$



[–20, 20] by [–250, 100]

The x-intercepts are $(11,0), (-10,0)$.

44. $y = x^2 + 9x - 36$

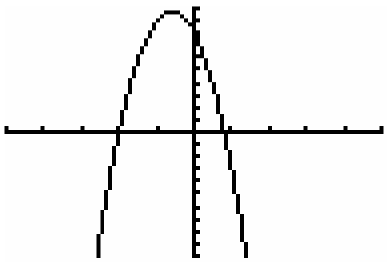


[–20, 20] by [–80, 50]

The x-intercepts are $(-12,0), (3,0)$.

Copyright ©2013 Pearson Education. Inc.

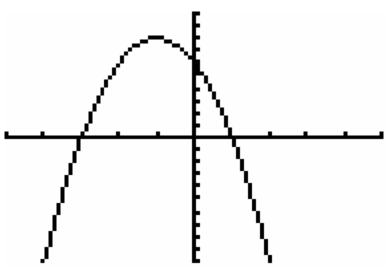
45. $y = -5x^2 - 6x + 8$



[–5, 5] by [–10, 10]

The x-intercepts are $(-2,0),(0.8,0)$.

46. $y = -2x^2 - 4x + 6$

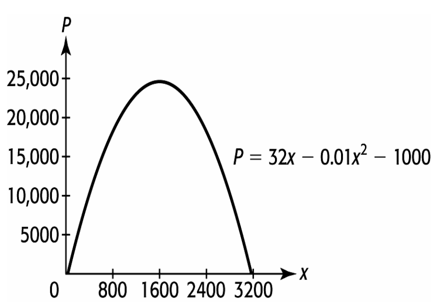


[–5, 5] by [–10, 10]

The x-intercepts are $(1,0),(-3,0)$.

Section 3.1 Exercises

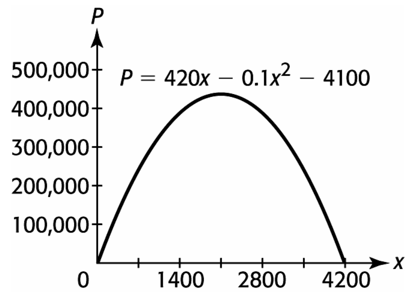
47. a.



b. For x between 1 and 1600, the profit is increasing.

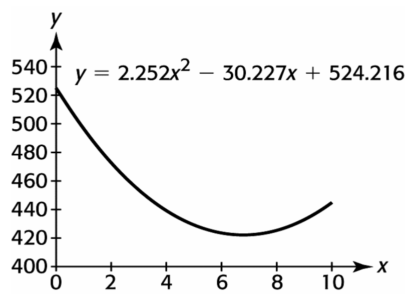
c. For x greater than 1600, the profit decreases.

48. a.



b. The graph is concave down.

49. a.



b. In 2015, $x = 2015 - 2000 = 15$.

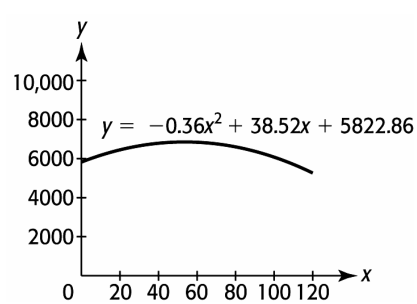
$y = 2.252x^2 - 30.227x + 524.216$

$y = 2.252(15)^2 - 30.227(15) + 524.216$

$y = 577.511$

In 2015, the number of juvenile arrests for property crimes is estimated to be 577.511 thousand or 577,511.

50. a.



b. In 2010, $x = 2010 - 1990 = 20$.

$y = -0.36x^2 + 38.52x + 5822.86$

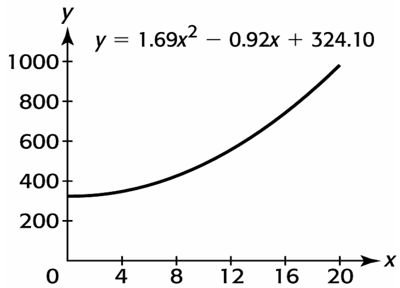
$y = -0.36(20)^2 + 38.52(20) + 5822.86$

$y = 6449.3$

Copyright ©2013 Pearson Education. Inc.

In 2010, the world population is estimated at 6449.3 million or 6.4493 billion people.

51. a.



b. In 2015, $x = 2015 - 1998 = 17$.

$y = 1.69x^2 - 0.92x + 324.10$

$y = 1.69(17)^2 - 0.92(17) + 324.10$

$y = 796.87$

The projected global spending (in billions of dollars) on travel and tourism for 2015 is $796.87 billion.

c. It is an extrapolation since it is outside the range 1998 to 2009 in the given model.

52. a. The model $S(t) = 100 + 96t - 16t^2$ is a quadratic function. The graph of the model is a concave down parabola.

b. $t = \frac{-b}{2a} = \frac{-96}{2(-16)} = \frac{-96}{-32} = 3$

$S = S(t) = S(3)$

$= 100 + 96(3) - 16(3)^2$

$= 100 + 288 - 144$

$= 244$

The vertex is $(3, 244)$.

c. The ball reaches its maximum height of 244 feet in 3 seconds.

53. a. $S(t) = 30 + 39.2t - 9.8t^2$

$t = \frac{-b}{2a} = \frac{-39.2}{2(-9.8)} = \frac{-39.2}{-19.6} = 2$

$S = S(t) = S(2)$

$= 30 + 39.2(2) - 9.8(2)^2$

$= 30 + 78.4 - 39.2$

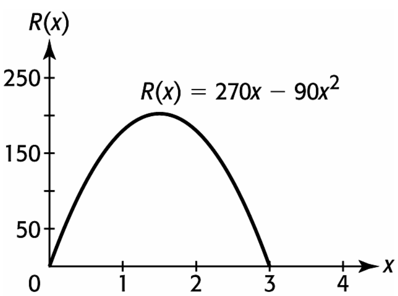
$= 69.2$

The vertex is $(2, 69.2)$.

b. The ball reaches its maximum height of 69.2 meters in 2 seconds.

c. The function is increasing until $t = 2$ seconds. The ball rises for two seconds, at which time it reaches its maximum height. After two seconds, the ball falls.

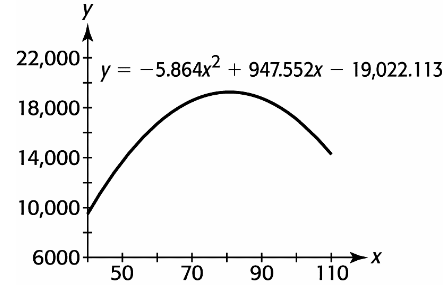
54. a.



b. $x = \frac{-b}{2a} = \frac{-270}{2(-90)} = \frac{-270}{-180} = 1.5$

1.5 lumens yields the maximum rate of photosynthesis.

55. a.

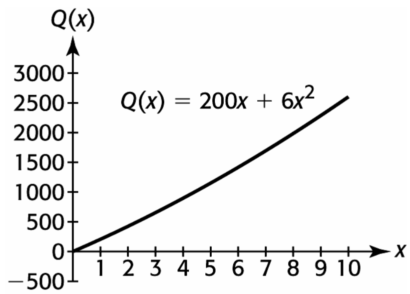


Copyright ©2013 Pearson Education. Inc.

b. Since $x = 40$ to $x = 80$ is for the years 1940 to 1980, it appears that the model is increasing in this interval.

c. After 1980, when the data include the employed only, the membership shown by the model appears to decline.

56.



57. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

a. $P(x) = 40x - 3000 - 0.01x^2$

$$x = \frac{-b}{2a} = \frac{-40}{2(-0.01)} = \frac{-40}{-0.02} = 2000$$

b. $P = P(x) = P(2000)$

$$\begin{aligned} &= 40(2000) - 3000 - 0.01(2000)^2 \\ &= 80{,}000 - 3000 - 0.01(4{,}000{,}000) \\ &= 80{,}000 - 3000 - 40{,}000 \\ &= 37{,}000 \end{aligned}$$

Producing and selling 2000 units (MP3 players) yields a maximum profit of \$37,000.

58. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

a. $P(x) = 840x - 75.6 - 0.4x^2$

$$x = \frac{-b}{2a} = \frac{-840}{2(-0.4)} = \frac{-840}{-0.8} = 1050$$

b. $P = P(x) = P(1050)$

$$\begin{aligned} &= 840(1050) - 75.6 - 0.4(1050)^2 \\ &= 882{,}000 - 75.6 - 441{,}000 \\ &= 440{,}924.4 \end{aligned}$$

Producing and selling 1050 units (mowers) yields a maximum profit of \$440,924.40.

59. Note that the maximum revenue occurs at the vertex of the quadratic function, since the function is concave down.

a. $R(x) = 1500x - 0.02x^2$

$$x = \frac{-b}{2a} = \frac{-1500}{2(-0.02)} = \frac{-1500}{-0.04} = 37{,}500$$

b. $R = R(x) = R(37{,}500)$

$$\begin{aligned} &= 1500(37{,}500) - 0.02(37{,}500)^2 \\ &= 56{,}250{,}000 - 28{,}125{,}000 \\ &= 28{,}125{,}000 \end{aligned}$$

Selling 37,500 units (pens) yields a maximum annual revenue of \$28,125,000.

60. Note that the maximum revenue occurs at the vertex of the quadratic function, since the function is concave down.

a. $R(x) = 300x - 0.01x^2$

$$x = \frac{-b}{2a} = \frac{-300}{2(-0.01)} = \frac{-300}{-0.02} = 15{,}000$$

b. $R = R(x) = R(15{,}000)$

$$\begin{aligned} &= 300(15{,}000) - 0.01(15{,}000)^2 \\ &= 4{,}500{,}000 - 2{,}250{,}000 \\ &= 2{,}250{,}000 \end{aligned}$$

Selling 15,000 units (radios) yields a maximum monthly revenue of \$2,250,000.

Copyright ©2013 Pearson Education. Inc.

61. a. Yes. $A = x(100 - x) = 100x - x^2$. Note that A fits the form $f(x) = ax^2 + bx + c, a \neq 0$.

b. The maximum area will occur at the vertex of graph of function A.

$$x = \frac{-b}{2a} = \frac{-100}{2(-1)} = \frac{-100}{-2} = 50$$

$$\begin{aligned} A &= A(x) = A(50) \\ &= 100(50) - (50)^2 \\ &= 5000 - 2500 \\ &= 2500 \end{aligned}$$

The maximum area of the pen is 2500 square feet.

62.

$$A = (12{,}500 - x)x$$
$$A = 12{,}500x - x^2$$
$$x = \frac{-b}{2a} = \frac{-12{,}500}{2(-1)} = \frac{-12{,}500}{-2} = 6250$$
$$A = A(x) = A(6250) =$$
$$12{,}500(6250) - (6250)^2 =$$
$$39{,}062{,}500$$

The maximum area is $39{,}062{,}500$ square feet.

63. a.

$$y = -0.0584x^2 + 1.096x + 24.3657$$
$$x = \frac{-b}{2a} = \frac{-1.096}{2(-0.0584)} = \frac{-1.096}{-0.1168} = 9.38$$
$$y = -0.0584(9.38)^2 + 1.096(9.38) + 24.3657$$
$$y = 29.51.$$

Thus the vertex is (9.38, 29.51).

b. 1990 + 9.38 = the year 2000.

c. The maximum percent usage is 29.5%.

64. a. $y = 0.592x^2 - 3.277x + 48.493$

The number of visitors reaches a minimum during the given time frame because the coefficient of x^2 is positive, so the parabola will open upward.

b.

$$x = \frac{-b}{2a} = \frac{-(-3.277)}{2(0.592)} = \frac{3.277}{1.184} = 2.7677$$
$$y = 0.592x^2 - 3.277x + 48.493$$
$$y = 0.592(2.7677)^2 - 3.277(2.7677) + 48.493$$
$$= 43.958 \approx 44$$

Thus the vertex is (2.768, 43.958).

c. When x is $2.768 \approx 3$, the year is 2003. In 2003, the minimum number of international visitors to the U.S., in millions, is approximately 43.958 million.

65. a. Since $a = 0.114 > 0$ in the equation $y = 0.114x^2 - 2.322x + 45.445$, the graph is concave up, and the vertex is a minimum.

b.

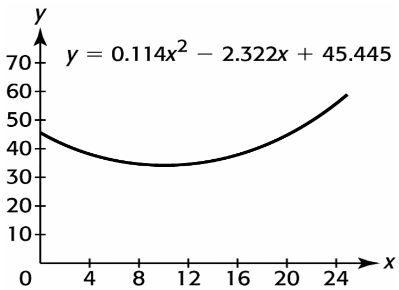
$$y = 0.114x^2 - 2.322x + 45.445$$
$$x = \frac{-b}{2a} = \frac{-(-2.322)}{2(0.114)} = \frac{2.322}{0.228} = 10.184$$
$$y = 0.114(10.184)^2 - 2.322(10.184) + 45.445$$
$$y = 33.621.$$

Thus the vertex is (10.184, 33.621).

When x is 10.184, it would be in the 11th year after 1990. Therefore, during the year 2001, 33.621 million people in the U.S. lived below the poverty level.

Copyright ©2013 Pearson Education. Inc.

c.



66. a. $y = -0.026x^2 + 0.951x + 18.161$

$$x = \frac{-b}{2a} = \frac{-(0.951)}{2(-0.026)} = \frac{-0.951}{-0.052}$$

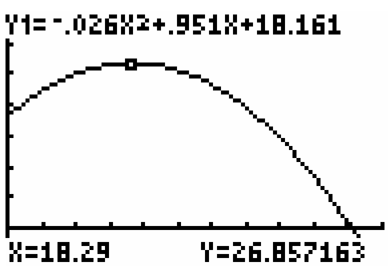
$= 18.29 \approx$ into the 19th year

Thus, the year is 1989 (1970 + 19).

b. $y = -0.026(18.29)^2 + 0.951(18.29) +$ 18.161

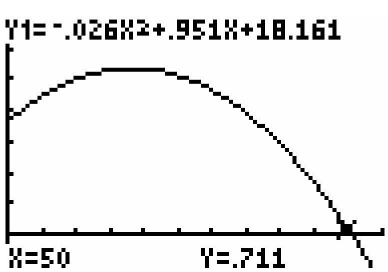
$y = 26.857 \approx 26.9$

Thus the maximum number of abortions in the year 1989 is 26.9 per 1000.



[0, 55] by [–5, 35]

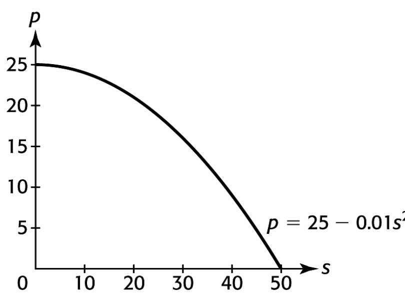
c. The model no longer applies when the number of abortions becomes negative.



[0, 55] by [–5, 35]

The number of abortions becomes negative in 2020 when $x = 50$.

67. a.

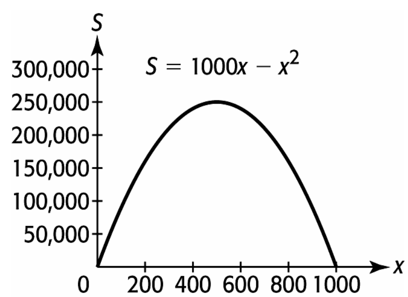


b. Decreasing. The graph is falling as s increases.

c. $(0, 25)$

d. When the wind speed is zero, the amount of particulate pollution is 25 ounces per cubic yard.

68. a.



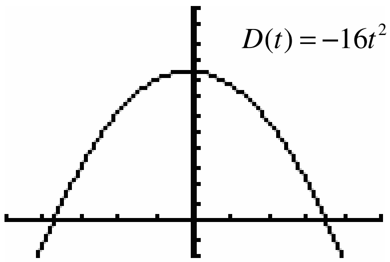
b. Increasing. The graph is rising as x increases to a value of 500. When x is greater than 500 the graph is decreasing.

c. $(1000, 0)$

d. The sensitivity to the drug drops to zero when the dosage is 1000. This could indicate that the person is overdosed on the drug and no longer able to detect sensitivity to the drug.

Copyright ©2013 Pearson Education. Inc.

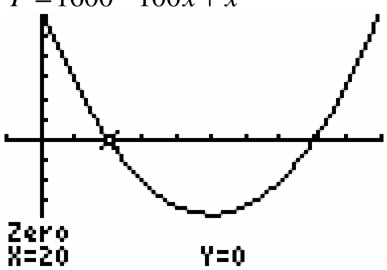
69.



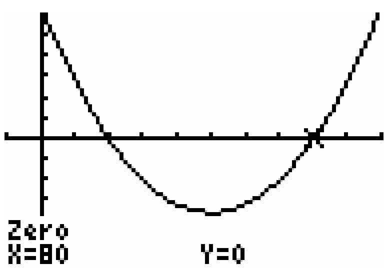
[–5, 5] by [–50, 300]

The t-intercepts are approximately $(-3.75, 0)$ and $(3.5, 0)$. The $(3.5, 0)$ makes sense because time is understood to be positive. In the context of the question, the tennis ball will hit the pool in 3.5 seconds.

70. $P = 1600 - 100x + x^2$



[–10, 100] by [–1500, 1500]



[–10, 100] by [–1500, 1500]

Break-even occurs when the number of units produced and sold is 20 or 80.

71. a. $y = -16t^2 + 32t + 3$

b. $y = -16t^2 + 32t + 3$

$$t = \frac{-b}{2a} = \frac{-(32)}{2(-16)} = \frac{-32}{-32} = 1$$

$$y = -16(1)^2 + 32(1) + 3 = 19$$

The maximum height of the ball is 19 feet when $t = 1$ second.

72. a. $y = -16t^2 + 48t + 4$

b. $y = -16t^2 + 48t + 4$

$$t = \frac{-b}{2a} = \frac{-(48)}{2(-16)} = \frac{-48}{-32} = 1.5$$

$$y = -16(1.5)^2 + 48(1.5) + 4 = 40$$

The maximum height of the ball is 40 feet in 1.5 seconds.

73. a.

Rent	Number of Apartments Rented	Total Revenue
\$1200	100	\$120,000
\$1240	98	\$121,520
\$1280	96	\$122,880
\$1320	94	\$124,080

b. Yes. Revenue = Rent multiplied by Number of Apartments Rented. $(1200 + 40x)$ represents the rent amount, while $(100 - 2x)$ represents the number of apartments rented.
$R = 120{,}000 + 1600x - 80x^2$

c. $$x = \frac{-b}{2a} = \frac{-(1600)}{2(-80)} = \frac{-1600}{-160} = 10$$

The maximum occurs when $x = 10$. Therefore the most profitable rent to charge is $1200 + 40(10) = \$1600$.

74. a.

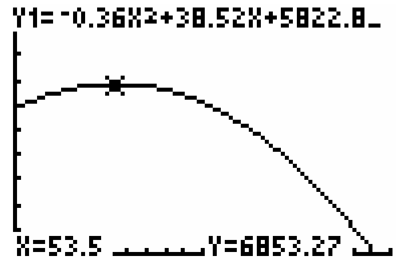
Cost per person	Number of Skaters	Total Revenue
$12.00	60	$720
$11.50	66	$759
$11.00	72	$792
$10.50	78	$819

b. Yes. Revenue = Cost multiplied by Number of Skaters. $(12-0.5x)$ represents the cost per skater, while $(60+6x)$ represents the number of skaters. $R = 720 + 42x - 3x^2$

c. $x = \dfrac{-b}{2a} = \dfrac{-(42)}{2(-3)} = \dfrac{-42}{-6} = 7$

The maximum occurs when $x = 7$. Therefore the most profitable number of skaters is $60 + 6(7) = 102$.

75. a.



[0, 200] by [0, 10,000]

The vertex is $(53.5, 6853.3)$.

b. The model predicts that in 2044 (1990 + 54), the population of the world will by 6853.3 million, or 6,853,300,000.

c. The model predicts a population increase from 1990 until 2044. After 2044, based on the model, the population of the world will decrease.

Section 3.2 Skills Check

1. $x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x - 5 = 0, x + 2 = 0$

$x = 5, x = -2$

2. $x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x - 6 = 0, x - 3 = 0$

$x = 6, x = 3$

3. $x^2 - 11x + 24 = 0$

$(x-8)(x-3) = 0$

$x - 8 = 0, x - 3 = 0$

$x = 8, x = 3$

4. $x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x + 5 = 0, x - 2 = 0$

$x = -5, x = 2$

5. $2x^2 + 2x - 12 = 0$

$2(x^2 + x - 6) = 0$

$2(x+3)(x-2)$

$x + 3 = 0, x - 2 = 0$

$x = -3, x = 2$

Copyright ©2013 Pearson Education. Inc.

6.

$2s^2 + s - 6 = 0$

Note that $2s^2(-6) = -12s^2$. Look for two terms whose product is $-12s^2$ and whose sum is the middle term, $1s$. $(4s)$ and $(-3s)$

$2s^2 + 4s - 3s - 6 = 0$

$(2s^2 + 4s) + (-3s - 6) = 0$

$2s(s+2) - 3(s+2) = 0$

$(2s-3)(s+2) = 0$

$2s - 3 = 0, s + 2 = 0$

$s = \frac{3}{2}, s = -2$

7.

$0 = 2t^2 - 11t + 12$

$2t^2 - 11t + 12 = 0$

Note that $2t^2(12) = 24t^2$. Look for two terms whose product is $24t^2$ and whose sum is the middle term, $-11t$.

$(-8t)$ and $(-3t)$

$2t^2 - 8t - 3t + 12 = 0$

$(2t^2 - 8t) + (-3t + 12) = 0$

$2t(t-4) - 3(t-4) = 0$

$(2t-3)(t-4) = 0$

$2t - 3 = 0, t - 4 = 0$

$t = \frac{3}{2}, t = 4$

8.

$6x^2 - 13x + 6 = 0$

Note that $6x^2(6) = 36x^2$. Look for two terms whose product is $36x^2$ and whose sum is the middle term, $-13x$. $(-9x)$ and $(-4x)$

$6x^2 - 9x - 4x + 6 = 0$

$(6x^2 - 9x) + (-4x + 6) = 0$

$3x(2x-3) - 2(2x-3) = 0$

$(3x-2)(2x-3) = 0$

$3x - 2 = 0, 2x - 3 = 0$

$x = \frac{2}{3}, x = \frac{3}{2}$

9.

$6x^2 + 10x = 4$

$6x^2 + 10x - 4 = 0$

$2(3x^2 + 5x - 2) = 0$

Remove the common factor, 2.

Note that $3x^2(-2) = -6x^2$. Look for two terms whose product is $-6x^2$ and whose sum is the middle term, $5x$.

$(6x)$ and $(-1x)$

$2(3x^2 + 6x - 1x - 2) = 0$

$2\left[(3x^2 + 6x) + (-1x - 2)\right] = 0$

$2\left[3x(x+2) + (-1)(x+2)\right] = 0$

$2(3x-1)(x+2) = 0$

$3x - 1 = 0, x + 2 = 0$

$x = \frac{1}{3}, x = -2$

Copyright ©2013 Pearson Education. Inc.

10.

$10x^2 + 11x = 6$

$10x^2 + 11x - 6 = 0$

Note that $10x^2(-6) = -60x^2$. Look for two terms whose product is $-60x^2$ and whose sum is the middle term, $11x$.

$(15x)$ and $(-4x)$

$10x^2 + 15x - 4x - 6 = 0$

$(10x^2 + 15x) + (-4x - 6) = 0$

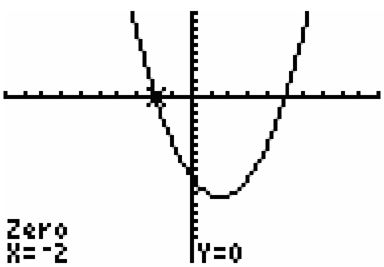
$5x(2x+3) + (-2)(2x+3) = 0$

$(2x+3)(5x-2) = 0$

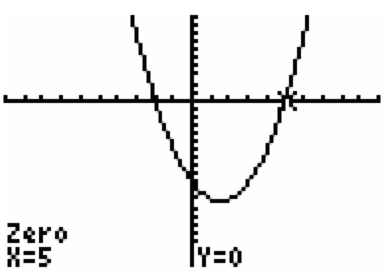
$2x + 3 = 0, 5x - 2 = 0$

$x = -\frac{3}{2}, x = \frac{2}{5}$

11. $y = x^2 - 3x - 10$



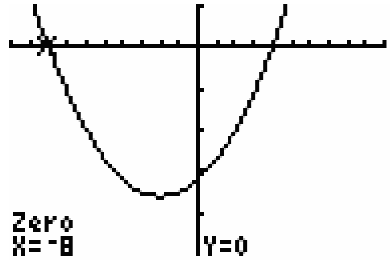
[–10, 10] by [–20, 10]



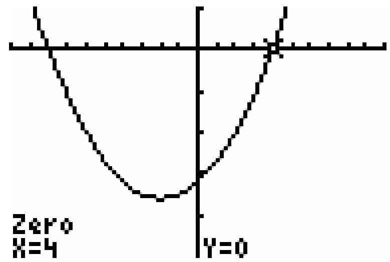
[–10, 10] by [–20, 10]

The x-intercepts are $(-2, 0)$ and $(5, 0)$.

12. $y = x^2 + 4x - 32$



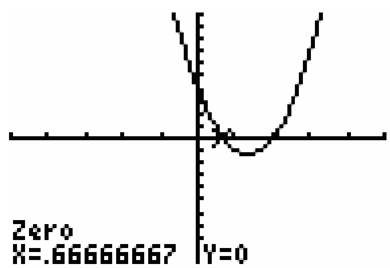
[–10, 10] by [–50, 10]



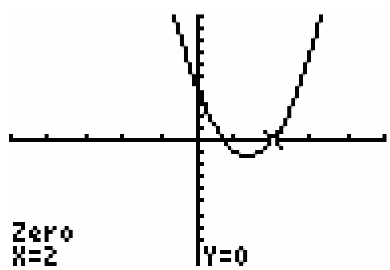
[–10, 10] by [–50, 10]

The x-intercepts are $(-8, 0)$ and $(4, 0)$.

13. $y = 3x^2 - 8x + 4$



[–5, 5] by [–10, 10]

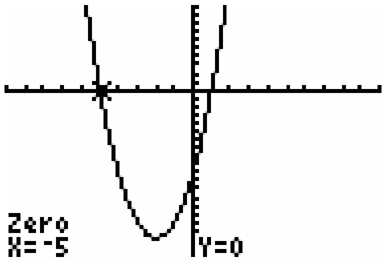


[–5, 5] by [–10, 10]

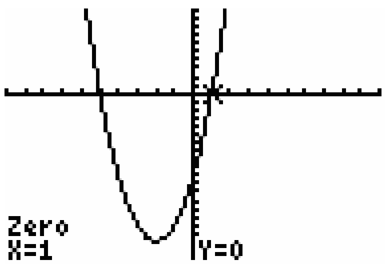
The x-intercepts are $\left(\frac{2}{3}, 0\right)$ and $(2, 0)$.

Copyright ©2013 Pearson Education. Inc.

14. $y = 2x^2 + 8x - 10$



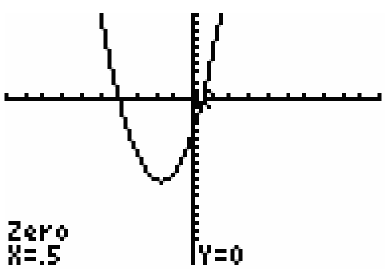
[–10, 10] by [–20, 10]



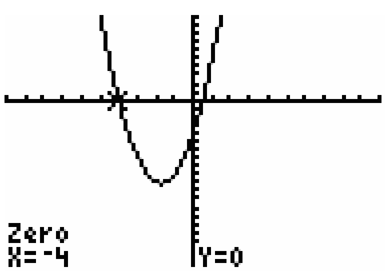
[–10, 10] by [–20, 10]

The x-intercepts are $(1,0)$ and $(-5,0)$.

15. $y = 2x^2 + 7x - 4$



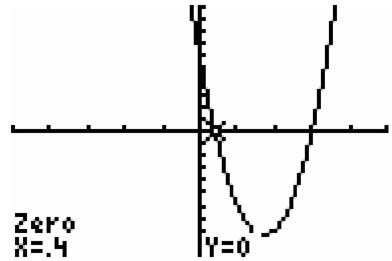
[–10, 10] by [–20, 10]



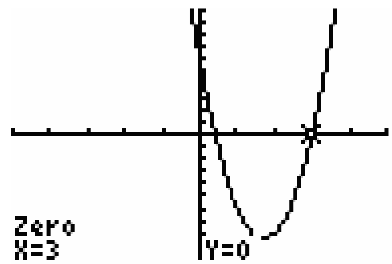
[–10, 10] by [–20, 10]

The x-intercepts are $(-4,0)$ and $(0.5,0)$.

16. $y = 5x^2 - 17x + 6$



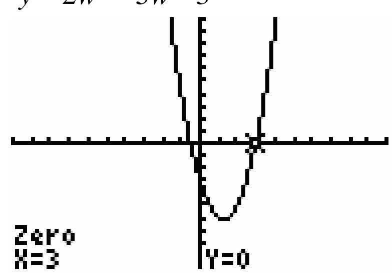
[–5, 5] by [–10, 10]



[–5, 5] by [–10, 10}

The x-intercepts are $\left(\frac{2}{5},0\right)$ and $(3,0)$.

17. $y = 2w^2 - 5w - 3$



[–10, 10] by [–10, 10]

Since $w = 3$ is an x-intercept, then $w - 3$ is a factor.

$2w^2 - 5w - 3 = 0$

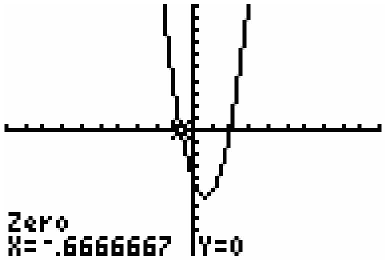
$(w-3)(2w+1) = 0$

$w - 3 = 0, 2w + 1 = 0$

$w = 3, w = -\frac{1}{2}$

Copyright ©2013 Pearson Education. Inc.

18. $y = 3x^2 - 4x - 4$



[–10, 10] by [–10, 10]

Since $x = -\frac{2}{3}$ is an x-intercept, then

$x + \frac{2}{3}$ is a factor.

Clearing fractions yields $3x + 2$ as a factor.

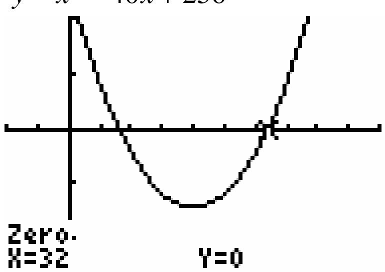
$3x^2 - 4x - 4 = 0$

$(3x + 2)(x - 2) = 0$

$3x + 2 = 0, x - 2 = 0$

$x = -\frac{2}{3}, x = 2$

19. $y = x^2 - 40x + 256$



[–10, 50] by [–250, 200]

Since $x = 32$ is an x-intercept, then $x - 32$ is a factor.

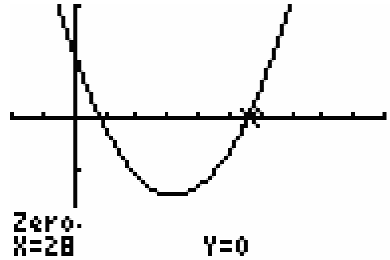
$x^2 - 40x + 256 = 0$

$(x - 32)(x - 8) = 0$

$x - 32 = 0, x - 8 = 0$

$x = 32, x = 8$

20. $y = x^2 - 32x + 112$



[–10, 50] by [–250, 200]

Since $x = 28$ is an x-intercept, then $x - 28$ is a factor.

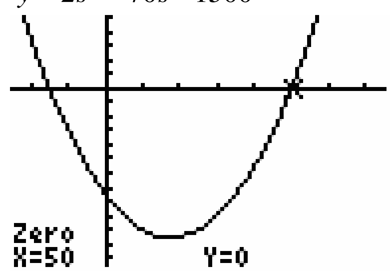
$x^2 - 32x + 112 = 0$

$(x - 28)(x - 4) = 0$

$x - 28 = 0, x - 4 = 0$

$x = 28, x = 4$

21. $y = 2s^2 - 70s - 1500$



[–25, 75] by [–2500, 1000]

Since $x = 50$ is an x-intercept for $2s^2 - 70s - 1500 = 0$, then $s - 50$ is a factor.

$(s - 50)(2s + 30) = 0$

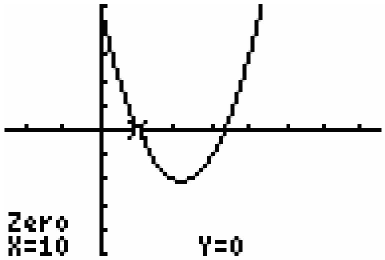
$2(s - 50)(s + 15) = 0$

$s - 50 = 0, s + 15 = 0$

$s = 50, s = -15$

Copyright ©2013 Pearson Education. Inc.

22. $y = 3s^2 - 130s + 1000$



[–25, 75] by [–1000, 1000]

Since $x = 10$ is an x-intercept for $3s^2 - 130s + 1000 = 0$, then $s - 10$ is a factor.

$(s-10)(3s-100) = 0$

$s - 10 = 0, 3s - 100 = 0$

$s = 10, s = \dfrac{100}{3}$

23. $4x^2 - 9 = 0$

$4x^2 = 9$

$\sqrt{4x^2} = \pm\sqrt{9}$

$2x = \pm 3$

$x = \pm\dfrac{3}{2}$

24. $x^2 - 20 = 0$

$x^2 = 20$

$\sqrt{x^2} = \pm\sqrt{20}$

$x = \pm\sqrt{20} = \pm\sqrt{(4)(5)} = \pm 2\sqrt{5}$

25. $x^2 - 32 = 0$

$x^2 = 32$

$\sqrt{x^2} = \pm\sqrt{32}$

$x = \pm\sqrt{32} = \pm\sqrt{(16)(2)} = \pm 4\sqrt{2}$

26. $5x^2 - 25 = 0$

$5x^2 = 25$

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

27. $x^2 - 4x - 9 = 0$

$\left(\dfrac{-4}{2}\right)^2 = (-2)^2 = 4$

$x^2 - 4x + 4 - 4 - 9 = 0$

$(x^2 - 4x + 4) + (-4 - 9) = 0$

$(x-2)^2 - 13 = 0$

$(x-2)^2 = 13$

$\sqrt{(x-2)^2} = \pm\sqrt{13}$

$x - 2 = \pm\sqrt{13}$

$x = 2 \pm \sqrt{13}$

28.

$x^2 - 6x + 1 = 0$

$\left(\dfrac{-6}{2}\right)^2 = (-3)^2 = 9$

$x^2 - 6x + 9 - 9 + 1 = 0$

$(x^2 - 6x + 9) + (-9 + 1) = 0$

$(x-3)^2 - 8 = 0$

$(x-3)^2 = 8$

$\sqrt{(x-3)^2} = \pm\sqrt{8}$

$x - 3 = \pm\sqrt{(2^2)(2)}$

$x = 3 \pm 2\sqrt{2}$

Copyright ©2013 Pearson Education. Inc.

29. $x^2-3x+2=0$

$$\left(\frac{-3}{2}\right)^2=\frac{9}{4}$$

$$x^2-3x+\frac{9}{4}-\frac{9}{4}+2=0$$

$$\left(x^2-3x+\frac{9}{4}\right)+\left(-\frac{9}{4}+2\right)=0$$

$$\left(x-\frac{3}{2}\right)^2+\left(-\frac{9}{4}+\frac{8}{4}\right)=0$$

$$\left(x-\frac{3}{2}\right)^2-\frac{1}{4}=0$$

$$\left(x-\frac{3}{2}\right)^2=\frac{1}{4}$$

$$\sqrt{\left(x-\frac{3}{2}\right)^2}=\pm\sqrt{\frac{1}{4}}$$

$$x-\frac{3}{2}=\pm\frac{1}{2}$$

$$x=\frac{3}{2}\pm\frac{1}{2}$$

$$x=\frac{3}{2}+\frac{1}{2},x=\frac{3}{2}-\frac{1}{2}$$

$$x=2,x=1$$

30. $2x^2-9x+8=0$

$$2\left(x^2-\frac{9}{2}x\right)+8=0$$

$$\left(-\frac{\left(\frac{9}{2}\right)}{2}\right)^2=\left(-\frac{9}{4}\right)^2=\frac{81}{16}$$

$$2\left(x^2-\frac{9}{4}x+\frac{81}{16}-\frac{81}{16}\right)+8=0$$

$$2\left(x^2-\frac{9}{4}x+\frac{81}{16}\right)+\left[2\left(-\frac{81}{16}\right)+8\right]=0$$

$$2\left(x-\frac{9}{4}\right)^2+\left(-\frac{162}{16}+\frac{128}{16}\right)=0$$

$$2\left(x-\frac{9}{4}\right)^2-\frac{34}{16}=0$$

$$2\left(x-\frac{9}{4}\right)^2=\frac{17}{8}$$

$$\sqrt{\left(x-\frac{9}{4}\right)^2}=\pm\sqrt{\frac{17}{16}}$$

$$x-\frac{9}{4}=\pm\frac{\sqrt{17}}{4}$$

$$x=\frac{9}{4}\pm\frac{\sqrt{17}}{4}$$

$$x=\frac{9\pm\sqrt{17}}{4}$$

31. $x^2-5x+2=0$

$$a=1,b=-5,c=2$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(2)}}{2(1)}$$

$$x=\frac{5\pm\sqrt{25-8}}{2}$$

$$x=\frac{5\pm\sqrt{17}}{2}$$

Copyright ©2013 Pearson Education. Inc.

32.

$$3x^2-6x-12=0$$
$$a=3, b=-6, c=-12$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(-12)}}{2(3)}$$
$$=\frac{6\pm\sqrt{36+144}}{6}$$
$$=\frac{6\pm\sqrt{180}}{6}$$
$$=\frac{6\pm\sqrt{(2^2)(3^2)(5)}}{6}$$
$$=\frac{6\pm(2)(3)\sqrt{5}}{6}$$
$$=\frac{6\pm6\sqrt{5}}{6}$$
$$=\frac{\cancel{6}\left(1\pm\sqrt{5}\right)}{\cancel{6}}$$
$$=1\pm\sqrt{5}$$

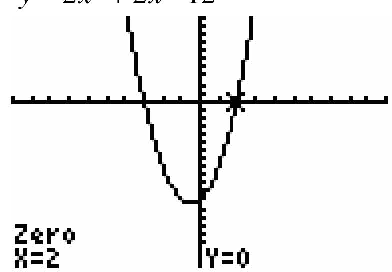
33. $5x+3x^2=8$

$$3x^2+5x-8=0$$
$$a=3, b=5, c=-8$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(5)\pm\sqrt{(5)^2-4(3)(-8)}}{2(3)}$$
$$x=\frac{-5\pm\sqrt{25+96}}{6}$$
$$x=\frac{-5\pm\sqrt{121}}{6}$$
$$x=\frac{-5\pm11}{6}$$
$$x=\frac{-5+11}{6}, x=\frac{-5-11}{6}$$
$$x=1, x=-\frac{16}{6}=-\frac{8}{3}$$

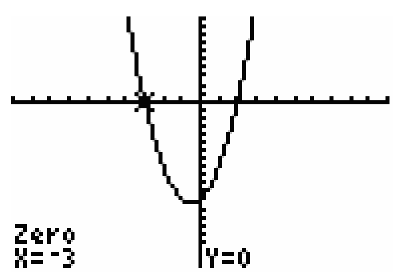
34.

$$3x^2-30x-180=0$$
$$3\left(x^2-10x-60\right)=0$$
$$a=1, b=-10, c=-60$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(-60)}}{2(1)}$$
$$x=\frac{10\pm\sqrt{100+240}}{2}$$
$$x=\frac{10\pm\sqrt{340}}{2}$$
$$x=\frac{10\pm\sqrt{(4)(85)}}{2}$$
$$x=\frac{10\pm2\sqrt{85}}{2}=\frac{\cancel{2}(5\pm\sqrt{85})}{\cancel{2}}$$
$$x=5\pm\sqrt{85}$$

35. $y=2x^2+2x-12$



[–10, 10] by [–20, 10]

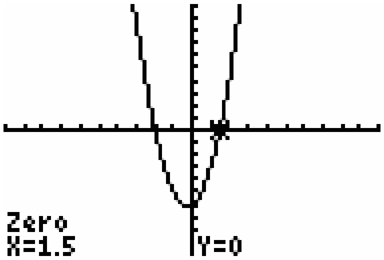


[–10, 10] by [–20, 10]

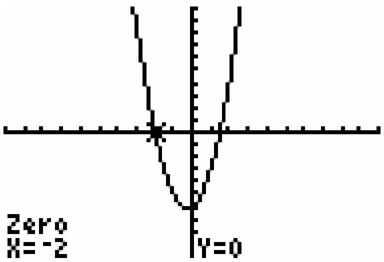
The solutions are $x=2$, $x=-3$.

Copyright ©2013 Pearson Education. Inc.

36. $y = 2x^2 + x - 6$



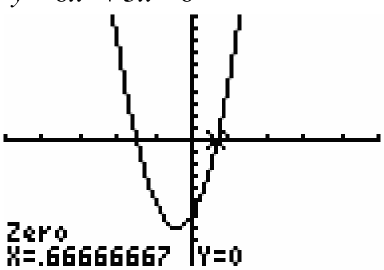
[–10, 10] by [–10, 10]



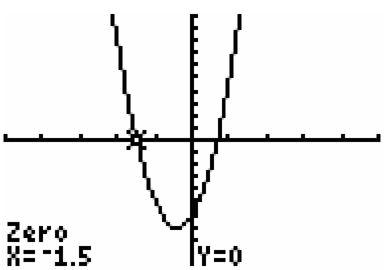
[–10, 10] by [–10, 10]

The solutions are $x = -2,\ x = 1.5.$

37. $y = 6x^2 + 5x - 6$



[–5, 5] by [–10, 10]

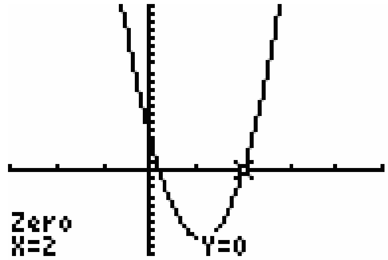


[–5, 5] by [–10, 10]

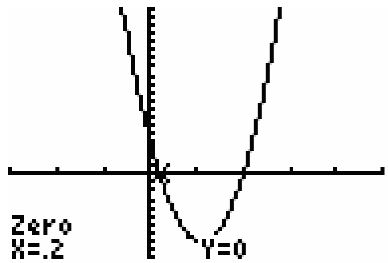
The solutions are $x = \frac{2}{3},\ x = -\frac{3}{2}.$

38. $10x^2 = 22x - 4$

$10x^2 - 22x + 4 = 0$



[–3, 5] by [–10, 20]



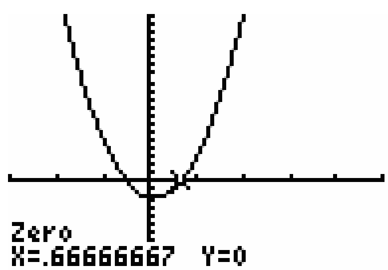
[–3, 5] by [–10, 20]

The solutions are $x = 0.2 = \frac{1}{5},\ x = 2.$

39. $4x + 2 = 6x^2 + 3x$

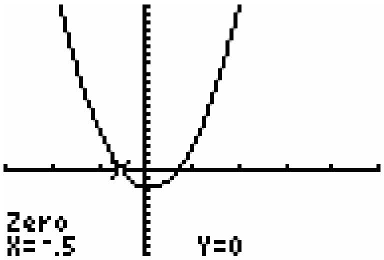
$0 = 6x^2 + 3x - 4x - 2$

$6x^2 - x - 2 = 0$



[–3, 5] by [–10, 20]

Copyright ©2013 Pearson Education. Inc.



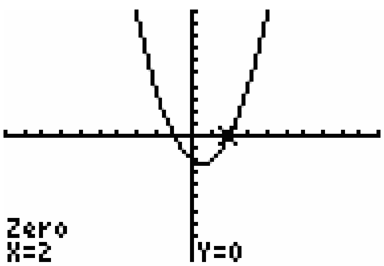
[–3, 5] by [–10, 20]

The solutions are $x = \frac{2}{3}, x = -\frac{1}{2}$.

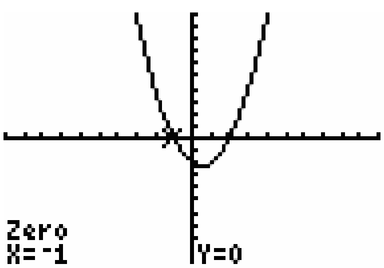
40. $(x-3)(x+2) = -4$

$x^2 + 2x - 3x - 6 = -4$

$x^2 - x - 2 = 0$



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

The solutions are $x = 2,\ x = -1$.

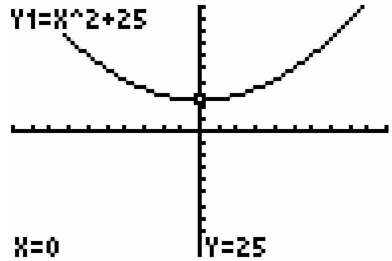
41. $x^2 + 25 = 0$

$x^2 = -25$

$\sqrt{x^2} = \pm\sqrt{-25}$

$x = \pm 5i$

Graphical check



[–10, 10] by [–100, 100]

Note that the graph has no *x*-intercepts.

42. $2x^2 + 40 = 0$

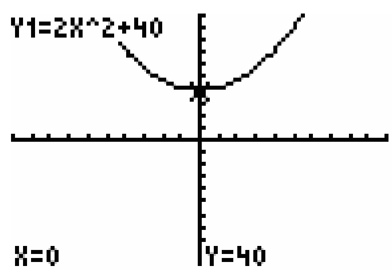
$2x^2 = -40$

$x^2 = -20$

$\sqrt{x^2} = \pm\sqrt{-20}$

$x = \pm 2i\sqrt{5}$

Graphical check



[–10, 10] by [–100, 100]

Note that the graph has no *x*-intercepts.

43. $(x-1)^2 = -4$

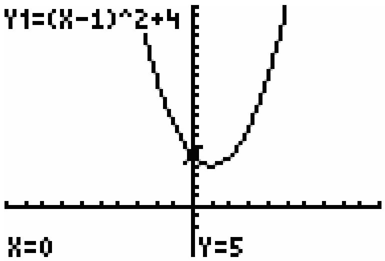
$\sqrt{(x-1)^2} = \pm\sqrt{-4}$

$x - 1 = \pm 2i$

$x = 1 \pm 2i$

Copyright ©2013 Pearson Education. Inc.

Graphical check



[–10, 10] by [–5, 20]

Note that the graph has no x-intercepts.

44. $(2x+1)^2+7=0$

$(2x+1)^2=-7$

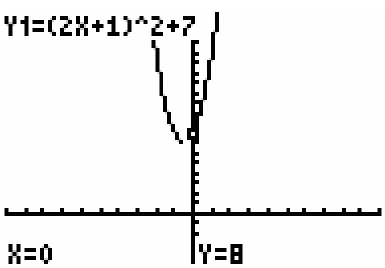
$\sqrt{(2x+1)^2}=\pm\sqrt{-7}$

$2x+1=\pm i\sqrt{7}$

$2x=-1\pm i\sqrt{7}$

$x=\dfrac{-1\pm i\sqrt{7}}{2}$

Graphical check



[–10, 10] by [–5, 20]

Note that the graph has no x-intercepts.

45. $x^2+4x+8=0$

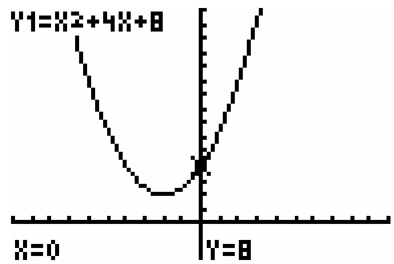
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-4\pm\sqrt{4^2-4(1)(8)}}{2(1)}$

$x=\dfrac{-4\pm\sqrt{-16}}{2}$

$x=\dfrac{-4\pm 4i}{2}=-2\pm 2i$

Graphical check



[–10, 10] by [–5, 30]

Note that the graph has no x-intercepts.

46. $x^2-5x+7=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

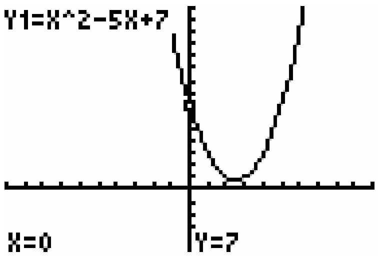
$x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(1)(7)}}{2(1)}$

$x=\dfrac{5\pm\sqrt{-3}}{2}$

$x=\dfrac{5\pm i\sqrt{3}}{2}$

Copyright ©2013 Pearson Education. Inc.

Graphical check



[–10, 10] by [–5, 15]

Note that the graph has no x-intercepts.

47. a. Since the graph shows two x-intercepts at $(-2,0)$ and $(3,0)$, by definition the value of the discriminant is positive.

b. And, there are two real solutions.

c. And, the solutions for $f(x)=0$ are $x=-2$ and $x=3$.

48. a. Since the graph shows two x-intercepts at $(-4,0)$ and $(2,0)$, by definition the value of the discriminant is positive.

b. And, there are two real solutions.

c. And, the solutions for $f(x)=0$ are $x=-4$ and $x=2$.

Section 3.2 Exercises

49. Let $S(t)=228$ and solve for t.

$228=100+96t-16t^2$

$16t^2-96t+128=0$

$16\left(t^2-6t+8\right)=0$

$16(t-4)(t-2)=0$

$t-4=0, t-2=0$

$t=4, t=2$

The ball is 228 feet high after 2 seconds and 4 seconds.

50. Let $D(t)=44$ and solve for t.

$44=-16t^2-4t+200$

$16t^2+4t-156=0$

$4\left(4t^2+t-39\right)=0$

$4(4t+13)(t-3)=0$

$4t+13=0, t-3=0$

$t=-\dfrac{13}{4}, t=3$

Since t represents time and time is not negative, the only solution in the given physical context is $t = 3$. The ball is 44 feet high after 3 seconds.

51. Let $P(x)=0$ and solve for x.

$-12x^2+1320x-21{,}600=0$

$-12\left(x^2-110x+1800\right)=0$

$-12(x-90)(x-20)=0$

$x-90=0, x-20=0$

$x=90, x=20$

Producing and selling either 20 or 90 of the Ipod Nano's produces a profit of zero dollars. Therefore 20 Ipods or 90 Ipods represent the break-even point for manufacturing and selling this product.

Copyright ©2013 Pearson Education. Inc.

52. Let $P(x)=0$ and solve for x.

$-15x^2+180x-405=0$

$-15\left(x^2-12x+27\right)=0$

$-15(x-9)(x-3)=0$

$x-9=0, x-3=0$

$x=9, x=3$

Producing and selling either 3 tons or 9 tons produces a profit of zero dollars. Therefore 3 tons or 9 tons represent the break-even point for producing and selling Coffee Roasters coffee beans.

53. a. $P(x)=R(x)-C(x)$

$P(x)=550x-\left(10{,}000+30x+x^2\right)$

$P(x)=550x-10{,}000-30x-x^2$

$P(x)=-x^2+520x-10{,}000$

b. Let $x=18$.

$P(18)=-(18)^2+520(18)-10{,}000$

$P(18)=-324+9360-10{,}000$

$p(18)=-964$

When 18 refrigerators are produced and sold, there is a loss of \$964.

c. Let $x=32$.

$P(32)=-(32)^2+520(32)-10{,}000$

$P(32)=-1024+16{,}640-10{,}000$

$p(32)=5616$

When 32 refrigerators are produced and sold, there is a profit of \$5616.

d. Let $P(x)=0$ and solve for x.

$-x^2+520x-10{,}000=0$

$-1\left(x^2-520x+10{,}000\right)=0$

$-1(x-500)(x-20)=0$

$x-500=0, x-20=0$

$x=500, x=20$

To break even on this product, the company should manufacture and sell either 20 or 500 refrigerators.

54. a. $P(x)=R(x)-C(x)$

$P(x)=266x-\left(2000+46x+2x^2\right)$

$P(x)=266x-2000-46x-2x^2$

$P(x)=-2x^2+220x-2000$

b. Let $x=55$

$P(55)=-2(55)^2+220(55)-2000$

$P(55)=-6050+12{,}100-2000$

$p(55)=4{,}050$

When 55 home theater systems are produced and sold, there is a profit of \$4,050.

c. Let $P(x)=0$ and solve for x.

$-2x^2+220x-2000=0$

$-2\left(x^2-110x+1000\right)=0$

$-2(x-100)(x-10)=0$

$x-100=0, x-10=0$

$x=100, x=10$

To break even on this product, the company should manufacture and sell either 10 or 100 home theater systems.

55. a. Let $p=0$ and solve for s.

$0=25-0.01s^2$

$-25=-0.01s^2$

$\frac{-25}{-0.01}=\frac{-0.01}{-0.01}s^2$

$2500=s^2$

$s=\pm\sqrt{2500}=\pm 50$

When s is 50 or –50, $p = 0$.

b. When $p = 0$, the pollution in the air above the power plant is zero.

c. Only the positive solution, $s = 50$, makes sense because wind speed must be positive.

Copyright ©2013 Pearson Education. Inc.

56. a. Let $v = 0.02$ and solve for r.

$0.02 = 2\left(0.01 - r^2\right)$

$\dfrac{0.02}{2} = \dfrac{2\left(0.01 - r^2\right)}{2}$

$0.01 = 0.01 - r^2$

$r^2 = 0$

$r = 0$

A distance of 0 cm produces a velocity of 0.02 cm/sec.

b. Let $v = 0.015$ and solve for r.

$0.015 = 2\left(0.01 - r^2\right)$

$\dfrac{0.015}{2} = \dfrac{2\left(0.01 - r^2\right)}{2}$

$0.0075 = 0.01 - r^2$

$r^2 = 0.01 - 0.0075$

$r^2 = 0.0025$

$r = \pm\sqrt{0.0025} = \pm 0.05$

A distance of 0.05 cm produces a velocity of 0.015 cm/sec. Since r is a distance, only $r = 0.05$ makes sense in the physical context of the question.

c. Let $v = 0$ and solve for r.

$0 = 2\left(0.01 - r^2\right)$

$\dfrac{0}{2} = \dfrac{2\left(0.01 - r^2\right)}{2}$

$0 = 0.01 - r^2$

$r^2 = 0.01$

$r = \pm\sqrt{0.01} = \pm 0.1$

A distance of 0.1 cm produces a velocity of 0 cm/sec. Since r is a distance, only $r = 0.1$ makes sense in the physical context of the question.

57. a. $0 = 100x - x^2$

$0 = x(100 - x)$

$x = 0, 100 - x = 0$

$x = 0, x = 100$

Dosages of 0 ml and 100 ml give zero sensitivity.

b. When x is zero, there is no amount of drug in a person's system , and therefore no sensitivity to the drug. When x is 100 ml the amount of drug in a person's system is so high that the person may be overdosed on the drug and therefore has no sensitivity to the drug.

58. a. Let $s = 20$

$K^2 = 16(20) + 4$

$K^2 = 324$

$K = \pm\sqrt{324} = \pm 18$

If K is positive, then $K = 18$.

b. Let $s = 60$

$K^2 = 16(60) + 4$

$K^2 = 964$

$K = \pm\sqrt{964} = \pm\sqrt{(4)(241)} = \pm 2\sqrt{241}$

If K is positive, then $K = 2\sqrt{241}$.

c. $\dfrac{f(b) - f(a)}{b - a}$

$= \dfrac{f(60) - f(20)}{60 - 20}$

$= \dfrac{2\sqrt{241} - 18}{40}$

$= 0.3262087348 \approx 0.326$

The average rate of change of the function between 20 mph and 60 mph is 0.326 per one unit increase in wind speed.

Copyright ©2013 Pearson Education. Inc.

59. Equilibrium occurs when demand is equal to supply. Solve

$$109.70 - 0.10q = 0.01q^2 + 5.91$$
$$0 = 0.01q^2 + 0.10q - 103.79$$
$$a = 0.01, b = 0.10, c = -103.79$$
$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$q = \frac{-0.10 \pm \sqrt{(0.10)^2 - 4(0.01)(-103.79)}}{2(0.01)}$$
$$q = \frac{-0.10 \pm \sqrt{0.01 + 4.1516}}{0.02}$$
$$q = \frac{-0.10 \pm \sqrt{4.1616}}{0.02}$$
$$q = \frac{-0.10 \pm 2.04}{0.02}$$
$$q = \frac{-0.10 + 2.04}{0.02}, q = \frac{-0.10 - 2.04}{0.02}$$
$$q = 97, q = -107$$

Since q represents the quantity of trees in hundreds at equilibrium, q must be positive. A q-value of –107 does not make sense in the physical context of the question. Selling 9700 trees, ($q = 97$), creates an equilibrium price. The equilibrium price is given by:

$$p = 109.70 - 0.10q$$
$$p = 109.70 - 0.10(97)$$
$$p = 109.70 - 9.70$$
$p = 100.00$ or \$100.00 per tree.

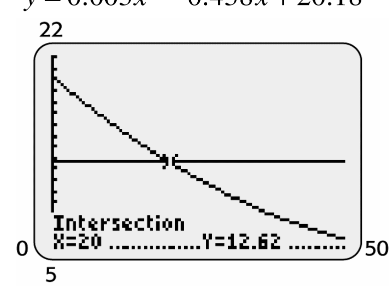
60. Equilibrium occurs when demand is equal to supply. Solve

$$7000 - 2x = 0.01x^2 + 2x + 1000$$
$$0 = 0.01x^2 + 4x - 6000$$
$$a = 0.01, b = 4, c = -6000$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(0.01)(-6000)}}{2(0.01)}$$
$$x = \frac{-4 \pm \sqrt{16 + 240}}{0.02}$$
$$x = \frac{-4 \pm \sqrt{256}}{0.02}$$
$$x = \frac{-4 \pm 16}{0.02}$$
$$x = \frac{-20}{0.02}, x = \frac{12}{0.02}$$
$$x = -1000, x = 600$$

Since x represents the quantity at equilibrium, x must be positive. An x-value of –1000 does not make sense in the physical context of the question. 600 watches creates an equilibrium price. The equilibrium price is given by:

$$p = 7000 - 2x$$
$$p = 7000 - 2(600)$$
$$p = 7000 - 1200$$
$p = 5800$ or \$5800 per watch.

61. a. $y = 0.003x^2 - 0.438x + 20.18$



b.

Let $y = 12.62$ and solve for x.

$$12.62 = 0.003x^2 - 0.438x + 20.18$$
$$0.003x^2 - 0.438x + 7.56 = 0$$
$$0.003\left(x^2 - 146x + 2520\right) = 0$$
$$0.003(x - 20)(x - 126) = 0$$
$$x = 20, x = 126$$

12.62% of the U.S. population is foreign born in 1920 and 2026.

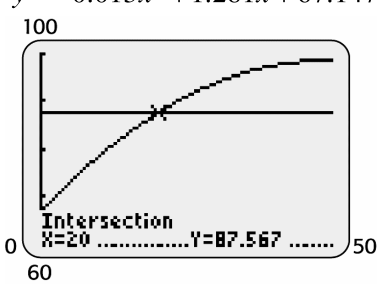
62. $y = -0.36x^2 + 38.52x + 5822.86$

$$6581 = -0.36x^2 + 38.52x + 5822.86$$
$$0 = -0.36x^2 + 38.52x - 758.14$$
$$x \approx 26, x \approx 81$$

Thus the world population will first reach 6581 million in 2016 (1990 + 26).

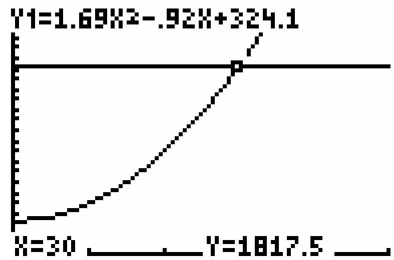
63. a. The solution $x = 20$, where x is the number of years after 1970, means that in 1990, energy consumption in the U.S. was 87.567 quadrillion BTU's.

b. $y = -0.013x^2 + 1.281x + 67.147$



c. Yes, if a second solution is found, it would show that after the year 2020, when the maximum BTU's are consumed, the consumption would again reach 87.567 about 28.5 years later, during the year 2049.

64.

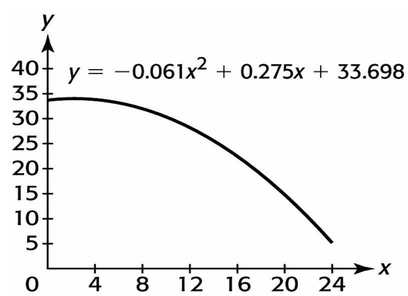


[0, 50] by [0, 2400]

$$y = 1.69x^2 - 0.92x + 324.10$$
$$1817.5 = 1.69x^2 - 0.92x + 324.10$$
$$0 = 1.69x^2 - 0.92x - 1493.4$$
$$x = 30$$

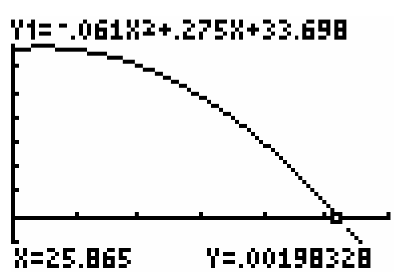
Thus spending is projected to reach \$1817.5 billion in the year 2020, (1990 + 30).

65. a.



b. According to the model and the graph above, 21.248 years after 1990, into the 22nd year (2012), the percent of high school students who smoke will be 12.

66.



[0, 25] by [0, 40]

Copyright ©2013 Pearson Education. Inc.

Since $x = 25.865$ causes the function to be equal to zero, and the model is decreasing, anything greater than 25.865 causes the function to be negative, making certain that the model is no longer valid. When $x =$ 25.865, the model would be in the 26^{th} years after 1990, or in the year 2016.

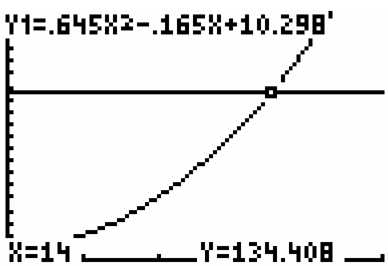
67. a. To determine the increase in the amount of federal funds, in billions, spent on child nutrition programs between 2000 and 2010, calculate $\underset{=N(10)-N(0)}{N(2010-2000)-N(2000-2000)}$.

$$\begin{aligned} N(10) &= 0.645(10)^2 - 0.165(10) + 10.298 \\ &= 64.5 - 1.65 + 10.298 \\ &= 73.148 \\ N(0) &= 0.645(0)^2 - 0.165(0) + 10.298 \\ &= 0 - 0 + 10.298 = 10.298 \end{aligned}$$

$$\begin{aligned} N(10) - N(0) &= 73.148 - 10.298 \\ &= 62.85 \end{aligned}$$

There is a increase of approximately 62.85 billion dollars.

b.



[0, 20] by [9, 200]

In 2014 (2000 + 14), the amount of funds spent will be 134.408 billion dollars.

c.

$$\begin{aligned} N(20) &= 0.645(20)^2 - 0.165(20) \\ &\quad + 10.298 \\ &= 258 - 3.3 + 10.298 \\ &= 264.998 \end{aligned}$$

Assuming the model continues to be valid until 2020, the amount spent in that year would be 264.998 billion dollars, an extrapolation, beyond the current year, of the function.

Copyright ©2013 Pearson Education. Inc.

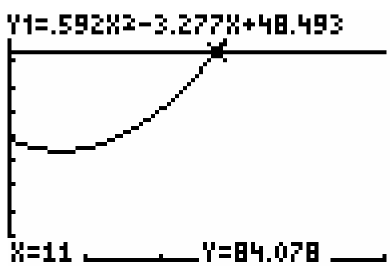
68. a. To determine the increase in the number of millions of visitors to the United States between 2000 and 2008, calculate $V(2008-2000)-V(2000-2000)=V(8)-V(0)$.

$$V(8)=0.592(8)^2-3.277(8)+48.493$$
$$=37.888-26.216+48.493$$
$$=60.165$$
$$V(0)=0.592(0)^2-3.277(0)+48.493$$
$$=0-0+48.493$$
$$=48.493$$

$$V(8)-V(0)=60.165-48.493$$
$$=11.672$$

There is a increase of 11.672 million visitors.

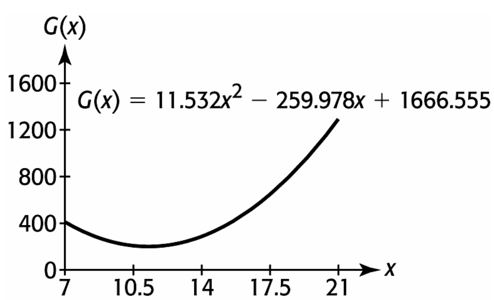
b.



[0, 20] by [0, 100]

In 2011, the number of millions of visitors to the United States will exceed 84 million.

69. a.

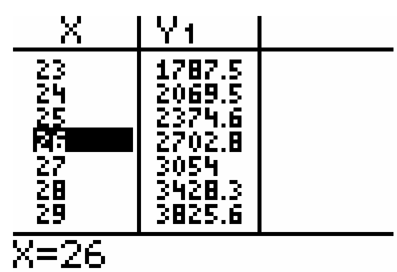


b.

The price of gold in 2020 corresponds to

$$G(30)=11.532(30)^2-259.978(30)+1666.555$$
$$=10{,}378.8-7799.34+1666.555$$
$$=4246.015 \text{ or } \$4246.02 \text{ U.S. dollars}$$

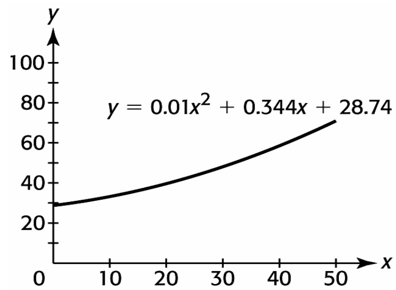
c.



Thus, 26 years after 1990, in the year 2016, the price of gold will reach \$2702.80.

Copyright ©2013 Pearson Education. Inc.

70. a.



b.

X	Y1
40	58.5
45	64.47
50	70.94
55	77.91
60	85.38
65	93.35
70	101.82

X=55

In 2015, 55 years after 1960, about 77.91% of the people who smoked after age 19 will have quit.

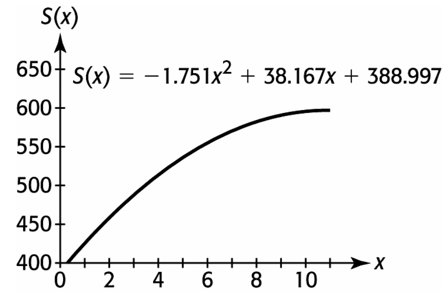
c.

X	Y1
18	38.172
19	38.886
20	39.62
21	40.374
22	41.148
23	41.942
24	42.756

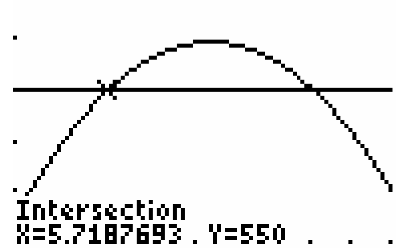
X=21

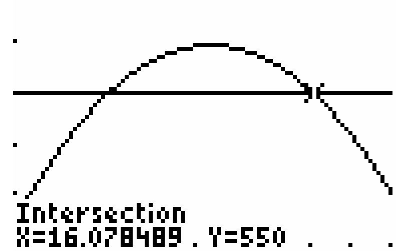
In 1981 (when $x = 21$), about 40% of people who smoked after age nineteen will have quit.

71. a.

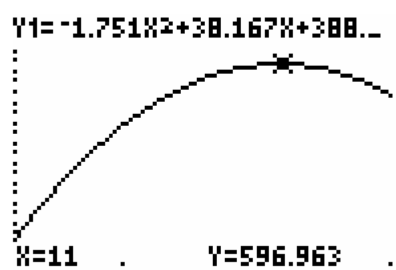


b. According to the complete model below in the window [1, 20] by [400, 645], the sales will be 550 billion dollars in the years following $x = 5.7$ (2006) and $x = 16.078$ (2017).





c.

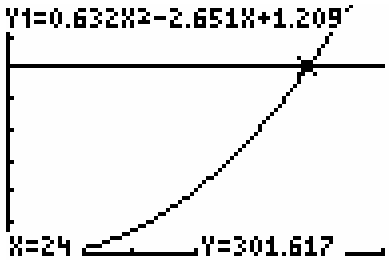


[1, 15] by [400, 645]

No, the model indicates the drop in sales to be nearer the year 2011, than to 2008.

Copyright ©2013 Pearson Education. Inc.

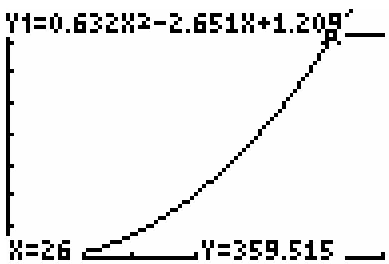
72. a.



[0, 30] by [0, 400]

The number of U.S. subscribers reached 301,617,000 in 2009 (1985 + 24).

b.



[0, 30] by [0, 400]

The number of U.S. subscribers is estimated to reach 359,515,000 in 2011 (1985 + 26).

c. The number exceeds the U.S. population, so the model will become invalid.

73. $y = -0.36x^2 + 38.52x + 5822.86$

$$-0.36x^2 + 38.52x + 5822.86 = 6702$$

$$-0.36x^2 + 38.52x - 879.14 = 0$$

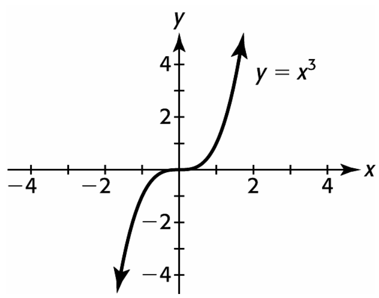
$$x \approx 33$$

Thus the world population will first reach 6702 million in the year 2023, 33 years after 1990.

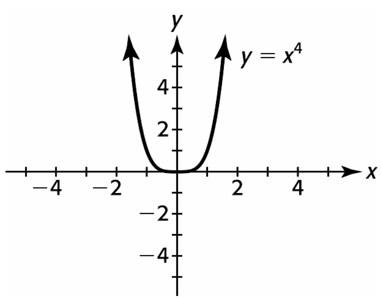
Copyright ©2013 Pearson Education. Inc.

Section 3.3 Skills Check

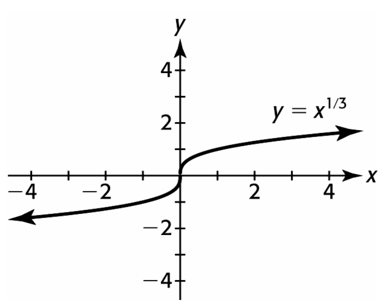
1.



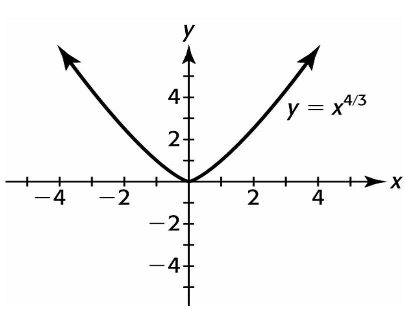
2.



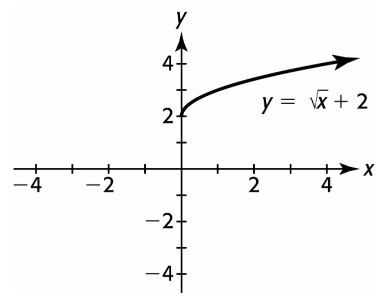
3.



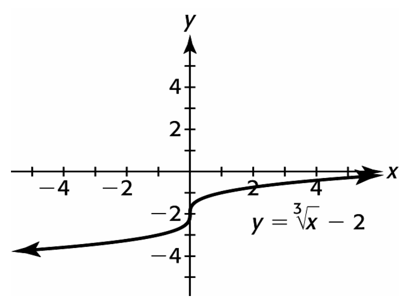
4.



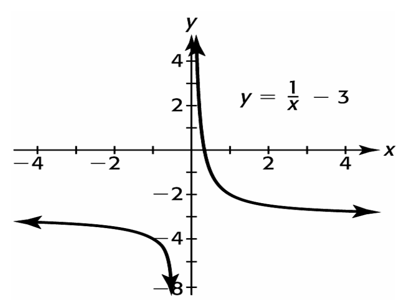
5.



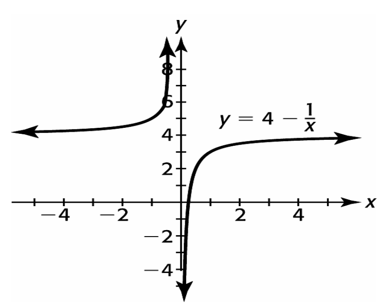
6.



7.

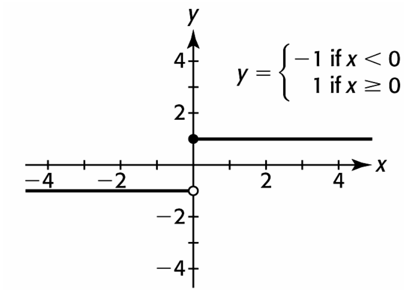


8.

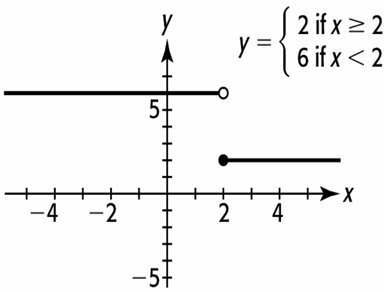


Copyright ©2013 Pearson Education. Inc.

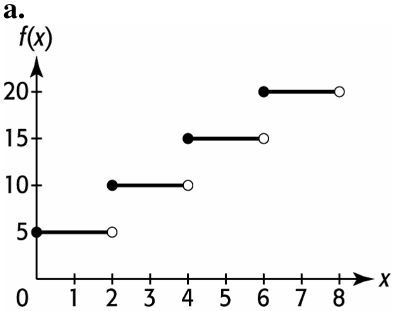
9.



10.

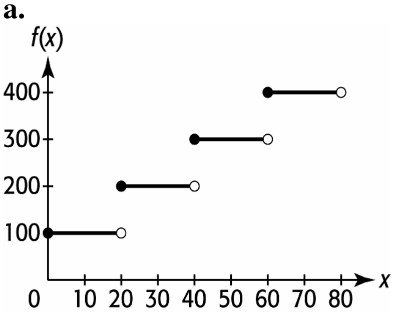


11. a.



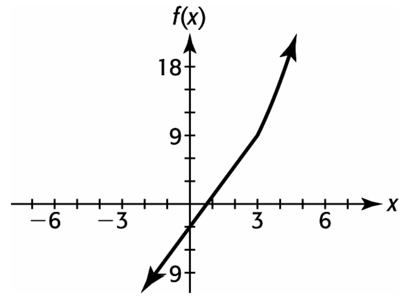
b. It is a piecewise, or step, function.

12. a.



b. It is a piecewise, or step, function.

13. a.

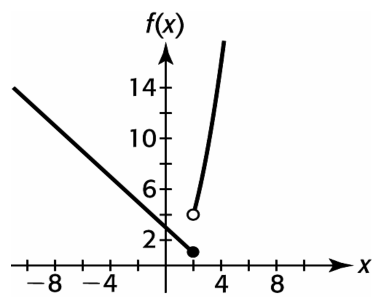


b. $f(2) = 4(2) - 3 = 8 - 3 = 5$

$f(4) = (4)^2 = 16$

c. Domain: $(-\infty, \infty)$

14. a.



b. $f(2) = 3 - 2 = 1$

$f(3) = (3)^2 = 9$

c. Domain: $(-\infty, \infty)$

15. a. $f(0.2) = [\![0.2]\!] = 0$

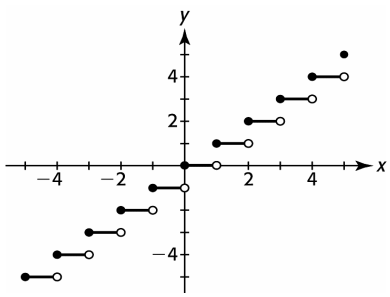
$f(3.8) = [\![3.8]\!] = 3$

$f(-2.6) = [\![-2.6]\!] = -3$

$f(5) = [\![5]\!] = 5$

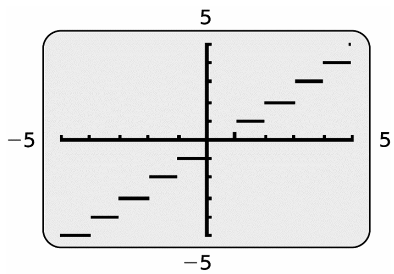
Copyright ©2013 Pearson Education. Inc.

b.



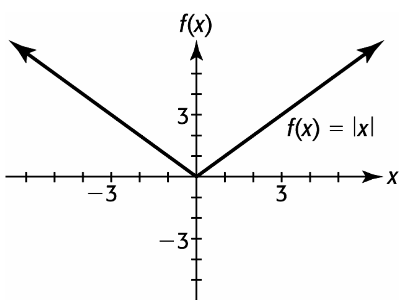
Each step should have a solid dot on its left end and an open dot on its right end.

16. a.



Each step should have a solid dot on its left end and an open dot on its right end.

17. a.

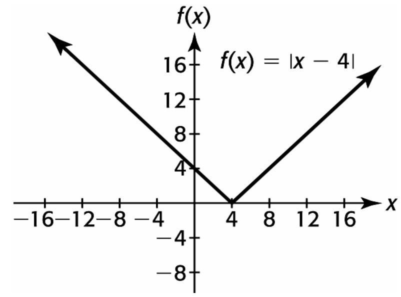


b. $f(-2) = |-2| = 2$

$f(5) = |5| = 5$

c. Domain: $(-\infty, \infty)$

18. a.



b. $f(-2) = |-2-4| = |-6| = 6$

$f(5) = |5-4| = |1| = 1$

c. Domain: $(-\infty, \infty)$

19. a. $f(-1) = 5$, since $x \le 1$.

b. $f(3) = 6$, since $x > 1$.

20. a. $f(-1) = 4$, since $x \ge -1$.

b. $f(3) = 4$, since $x \ge -1$.

21. a. $f(-1) = (-1)^2 - 1 = 0$, since $x \le 0$.

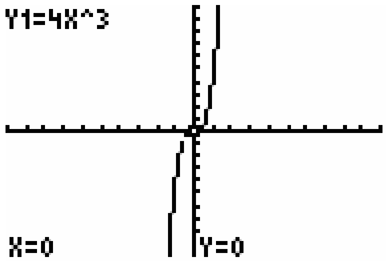
b. $f(3) = (3)^3 + 2 = 29$, since $x > 0$.

22. a. $f(-1) = 3(-1) + 1 = -2$, since $x < 3$.

b. $f(3) = (3)^2 = 9$, since $x \ge 3$.

Copyright ©2013 Pearson Education. Inc.

23. The function is increasing for all values of x.

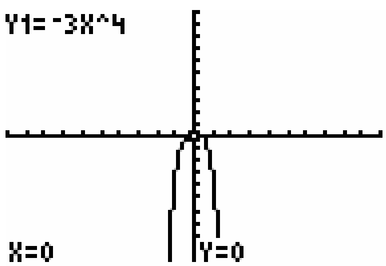


[–10, 10] by [–10, 10]

a. increasing

b. increasing

24. The function is decreasing when $x > 0$ and increasing when $x < 0$.

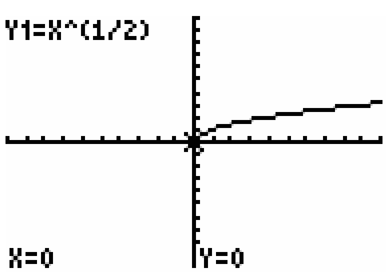


[–10, 10] by [–10, 10]

a. increasing

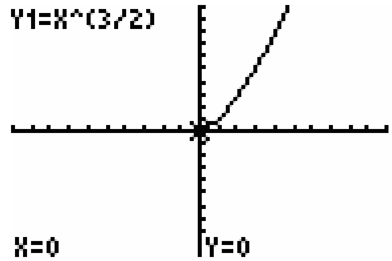
b. decreasing

25. Concave down.



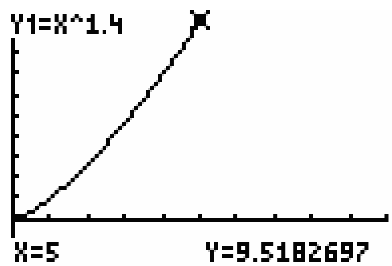
[–10, 10] by [–10, 10]

26. Concave up.



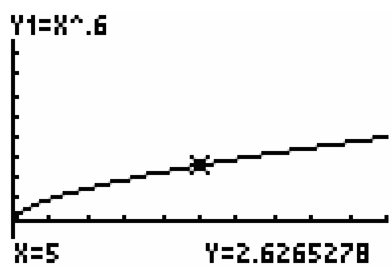
[–10, 10] by [–10, 10]

27. Concave up.



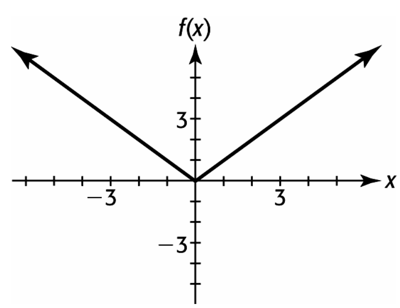
[0, 10] by [–2, 10]

28. Concave down.



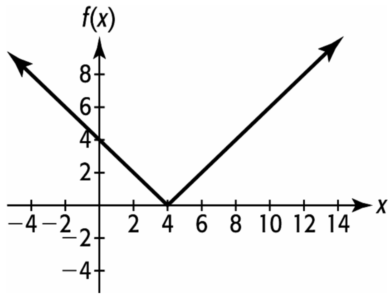
[0, 10] by [–2, 10]

29.

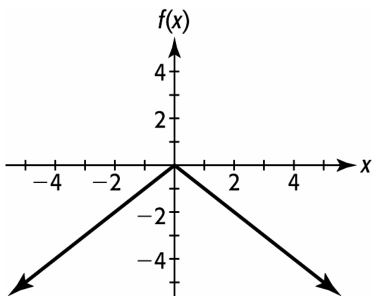


Copyright ©2013 Pearson Education. Inc.

30.



31.



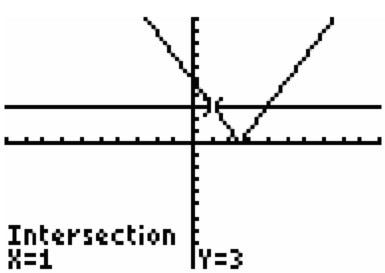
32. They are the same.

33. They are the same.

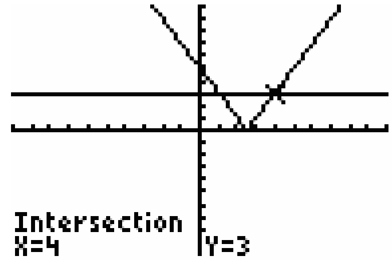
34. $|2x-5|=3$

$2x-5=3$	$2x-5=-3$
$2x=8$	$2x=2$
$x=4$	$x=1$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]

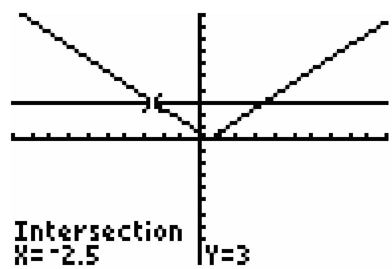


[–10, 10] by [–10, 10]

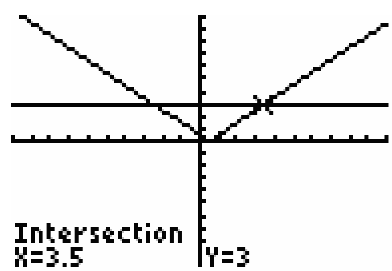
35. $\left|x-\frac{1}{2}\right|=3$

$x-\frac{1}{2}=3$	$x-\frac{1}{2}=-3$
$x=3+\frac{1}{2}$	$x=-3+\frac{1}{2}$
$x=\frac{7}{2}$	$x=-\frac{5}{2}$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

Copyright ©2013 Pearson Education. Inc.

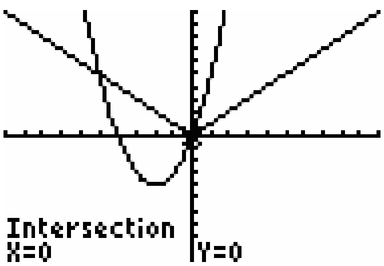
36.

$$|x| = x^2 + 4x$$

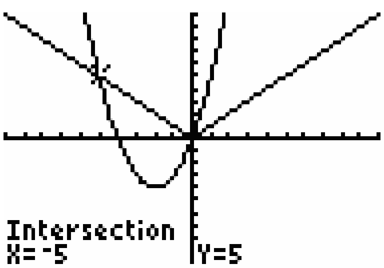
$$\begin{array}{l|l} & x = -\left(x^2 + 4x\right) \\ x = x^2 + 4x & x = -x^2 - 4x \\ x^2 + 3x = 0 & x^2 + 5x = 0 \\ x(x+3) = 0 & x(x+5) = 0 \\ x = 0, x = -3 & x = 0, x = -5 \end{array}$$

Note that -3 does not check. The solutions are $x = 0, x = -5$.

Using the intersection of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

37.

$$|3x - 1| = 4x$$

$$\begin{array}{l|l} 3x - 1 = 4x & 3x - 1 = -4x \\ -x = 1 & 7x = 1 \\ x = -1 & x = \dfrac{1}{7} \end{array}$$

Note that –1 does not check.

$x = -1$

$|3(-1) - 1| = 4(-1)$

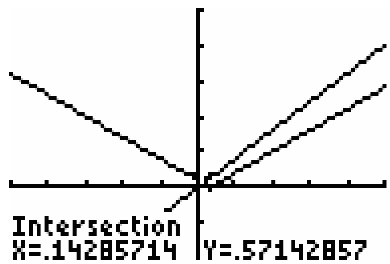
$|-3 - 1| = -4$

$|-4| = -4$

$4 \neq -4$

The only solution is $x = \dfrac{1}{7}$.

Using the intersection of graphs method to check the solution graphically yields,



[–5, 5] by [–10, 25]

38.

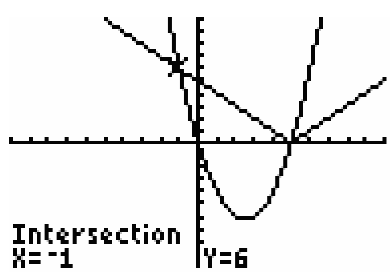
$$|x - 5| = x^2 - 5x$$

$$\begin{array}{l|l} & x - 5 = -\left(x^2 - 5x\right) \\ x - 5 = x^2 - 5x & x - 5 = -x^2 + 5x \\ x^2 - 6x + 5 = 0 & x^2 - 4x - 5 = 0 \\ (x-5)(x-1) = 0 & (x-5)(x+1) = 0 \\ x = 5, x = 1 & x = 5, x = -1 \end{array}$$

Note that 1 does not check.

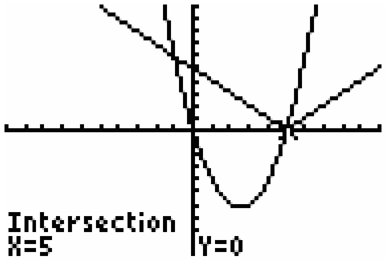
The solutions are $x = 5, x = -1$.

Using the intersection of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]

Copyright ©2013 Pearson Education. Inc.



[–10, 10] by [–10, 10]

39. $S = kT^{\frac{2}{3}}$

$64 = k(64)^{\frac{2}{3}} = k(16)$

$\frac{64}{16} = k = 4$

$S = 4T^{\frac{2}{3}}$

$S = 4(8)^{\frac{2}{3}} = 4(4) = 16$

40. $y = k\sqrt{x}$

$16 = k\sqrt{4} = k(2)$

$\frac{16}{2} = k = 8$

$24 = 8\sqrt{x}$

$\frac{24}{8} = \sqrt{x} = 3$

$x = 9$

41. $y = \frac{k}{x^4}$

$5 = \frac{k}{(-1)^4}$

$k = 5$

$y = \frac{5}{(.5)^4} = 80$

42. $S = \frac{k}{\sqrt{T}}$

$4 = \frac{k}{\sqrt{4}} = \frac{k}{2}$

$k = 8$

$S = \frac{8}{\sqrt{16}} = \frac{8}{4} = 2$

Section 3.3 Exercises

43. a. $M(x) = \begin{cases} 7.10 + 0.06747x & x \le 1200 \\ 88.06 + 0.05788x & x > 1200 \end{cases}$

b. For 960 kWh, $x = 960$

$M(960) = 7.10 + 0.06747(960)$

$M(960) = \$71.87.$

c. For 1580 kWh, $x = 1580 - 1200 = 380$

$M(380) = 88.06 + 0.05788(380)$

$M(380) = \$110.05.$

44.

$$P(x) = \begin{cases} 2.38 & 0 < x \le 1 \\ 2.77 & 1 < x \le 2 \\ 3.16 & 2 < x \le 3 \\ 3.55 & 3 < x \le 4 \\ 3.94 & 4 < x \le 5 \end{cases}$$

45. a.

$$P(w) = \begin{cases} 0.44 & 0 < w \le 1 \\ 0.64 & 1 < w \le 2 \\ 0.84 & 2 < w \le 3 \\ 1.04 & 3 < w \le 4 \end{cases}$$

b. $p(1.2) = 0.64$. The cost of mailing a 1.2-ounce first class letter is \$0.64.

c. Domain: $(0, 4]$

d. $P(2) = 0.64$

$p(2.01) = 0.84$

e. The cost of mailing a 2-ounce first class letter is \$0.64, and the cost for a 2.01-ounce letter is \$0.84.

Copyright ©2013 Pearson Education. Inc.

46. a.

$$T(x)=\begin{cases} 0.00+0.10x & 0\le x\le 16{,}750 \\ 1{,}675.00+0.15(x-16{,}750) & 16{,}750<x\le 68{,}000 \\ 9{,}362.50+0.25(x-68{,}000) & 68{,}000<x\le 137{,}300 \end{cases}$$

b. $T(42{,}000)=1{,}675.00+0.15(42{,}000-16{,}750)=5{,}462.50$

c.
$$\begin{aligned} T(65{,}000) &= 1{,}675.00+0.15(65{,}000-16{,}750) \\ &= 1{,}675.00+0.15(48{,}250) \\ &= 8{,}912.50 \end{aligned}$$

The total tax on \$65,000 is \$8,912.50.

d. The statement is incorrect.

$$\begin{aligned} T(68{,}000) &= 1{,}675.00+0.15(68{,}000-16{,}750) \\ &= 1{,}675.00+0.15(51{,}250)=9{,}362.50 \end{aligned}$$

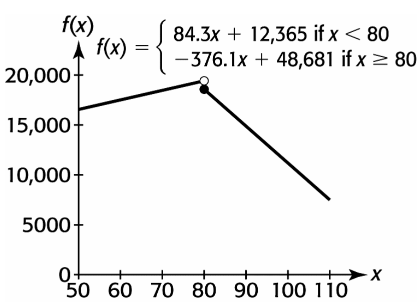
The total tax on \$68,000 is \$9,362.50.

$$\begin{aligned} T(68{,}000+1) &= 9{,}362.50+0.25(68{,}001-68{,}000) \\ &= 9{,}362.50+0.25(1) \\ &= 9{,}362.75 \end{aligned}$$

The total tax on \$68,001 is \$9,362.75.

The difference in the tax bills is \$0.25. Only the extra dollar is taxed at the 25% rate.

47. a. Using the given function, where x is the number of years from 1900,

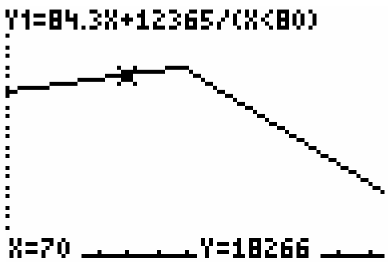


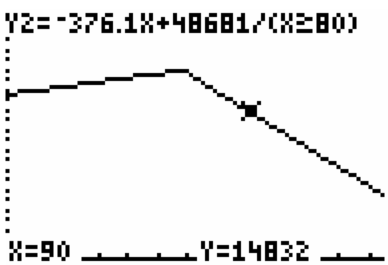
Copyright ©2013 Pearson Education. Inc.

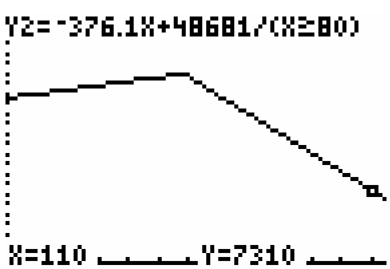
$f(1970) = f(1970 - 1900) = f(70)$,
where $x < 80$
$f(70) = 84.3(70) + 12,365 = 18,266$

$f(1990) = f(1990 - 1900) = f(90)$
$f(2010) = f(2010 - 1900) = f(110)$
where $x \geq 80$
$f(90) = -376.1(90) + 48,681 = 14,832$
$f(110) = -376.1(110) + 48,681 = 7,310$

b. graph using the interval [50, 112] by [0, 25000]



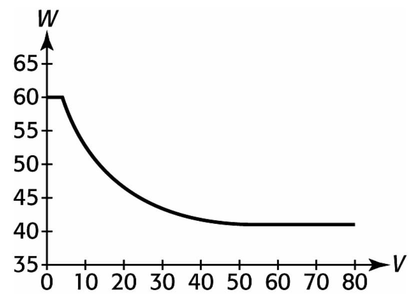




48. a. $W(20) = 0.644(20) - 9.518\sqrt{20} + 76.495$
$W(20) = 46.8 \approx 47$ degrees

b. $W(65) = 41$
$W(65) = 41$ degrees

c.



d. Domain: $0 \leq V \leq 80$, Range: $41 \leq W \leq 60$

49. a. Since x is raised to a power, this is classified as a power function.

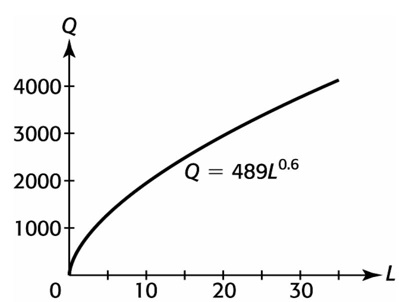
b. Since x is the number of years after 1960, $F(35)$ represents the number of female physicians 35 years after 1960, or in the year, 1995. Thus,
$F(x) = 0.623x^{1.552}$
$F(35) = 0.623(35)^{1.552} = 155.196$

Therefore, in 1995, there were 155.196 thousand (155,196) female physicians.

c. For the year 2020,
$x = 2020 - 1960 = 60$
$F(60) = 0.623(60)^{1.552} = 358.244$

According to the model, in the year 2020, there will be 358.244 thousand (358,244) female physicians.

50. a.



b. $Q = 489L^{0.6}$
$Q = 489(32)^{0.6} = 3912$

Copyright ©2013 Pearson Education. Inc.

If the company employs 32 taxi drivers then 3912 miles are driven per day.

c. The function is increasing. As the number of drivers increases, it is reasonable to expect the number of miles to increase.

51. a. For the year 2000,

$t = 2000 - 1999 = 1$

$f(t) = 61.925t^{0.041}$

$f(1) = 61.925(1)^{0.041} = 61.925$

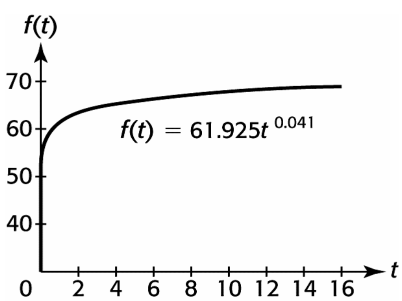
For the year 2008,

$t = 2008 - 1999 = 9$

$f(9) = 61.925(9)^{0.041} = 67.763$

Thus in 2000, the total percent of individuals aged 16 to 24 enrolled in college as of October, was 61.925%, and in the year 2008, it was 67.763%.

b.



c. Yes, but very far in the future.

52. a. Let $x = 160$ $(1960 - 1800)$

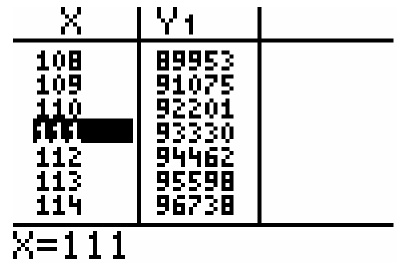
$y = 165.6(160)^{1.345}$

$= 152,616.5572 \approx 152,617$

In 1960 the U.S. population is predicted to by 152, 617 thousand people.

b. The graph of the function is concave up since the exponent is in the interval $(1, \infty)$.

c.

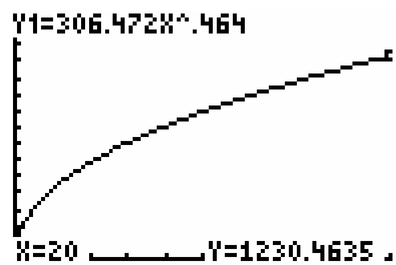


X	Y1
108	89953
109	91075
110	92201
111	93330
112	94462
113	95598
114	96738

X=111

The predicted population is 93.33 million in 1911 (1800 + 111).

53. a.

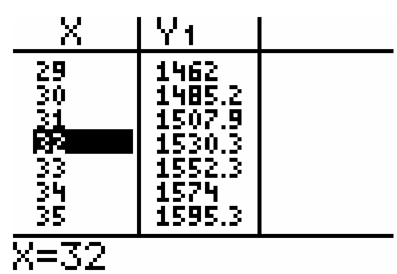


[0, 20] by [0, 1500]

According to the model, the function is increasing for the years 1990 to 2010.

b. According to the model, the function is concave down.

c.



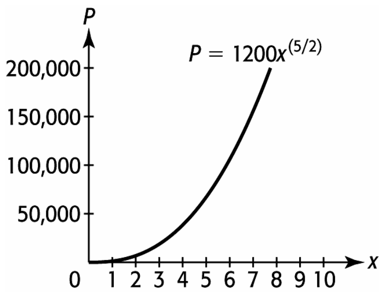
X	Y1
29	1462
30	1485.2
31	1507.9
32	1530.3
33	1552.3
34	1574
35	1595.3

X=32

Approximately 32 years after 1990, in the year 2022, the personal consumption expenditures will reach $1532.35.

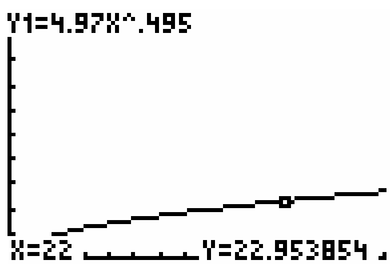
Copyright ©2013 Pearson Education. Inc.

54. a.



b. Considering the graph in part a), the graph is concave up.

55. a.

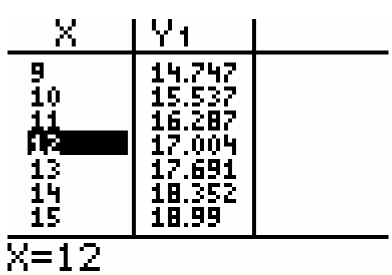


[0, 30] by [0, 100]

Although concave down, the model indicates that the percent of the United States adult population with diabetes is projected to increase.

b. From the model above and the Trace feature of the calculator, the projected percent for the year 2022 is approximately 23%.

c. Using the Table feature of the calculator, when $x = 12$ years from 2000, in the year 2012, the projection is to be 17%.



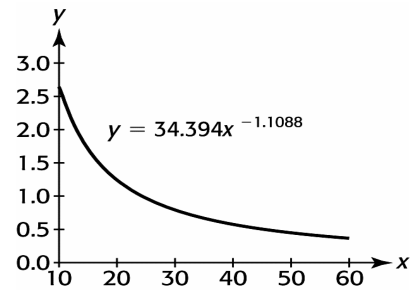
X	Y1
9	14.747
10	15.537
11	16.287
12	17.004
13	17.691
14	18.352
15	18.99

X=12

56. $B(8) = 6(8+1)^{\frac{3}{2}} = 6(27) = 162$

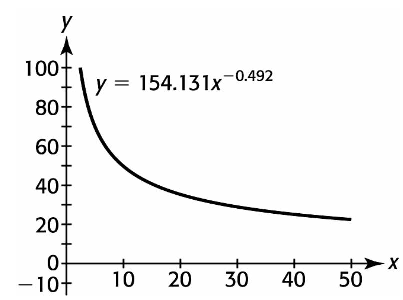
On May 8th the number of bushels of tomatoes harvested is 162.

57. a.



b. As shown in the model using the Trace feature of the calculator, the purchasing power of a 1983 dollar in 2020 is 0.367 dollars, or 37 cents.

58. a.



b. Trust in the government is decreasing. The graph is going down.

c. Since 1998 corresponds to $x = 38$, the percentage of people who say they trust the government is approximately 25.7%.

Copyright ©2013 Pearson Education. Inc.

59. Inverse variation format, $y = k/x$:

$W = \dfrac{k}{d^2}$ in this application,

where d is the radius of Earth.

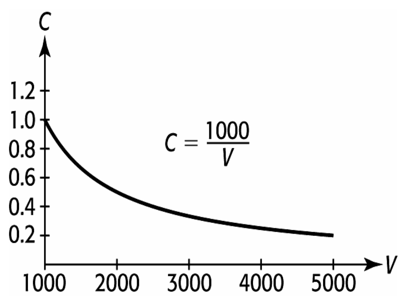
$$180 = \frac{k}{4000^2}$$

$k = 180(4000^2)$ Then, since 1000 miles above Earth makes $d = 5000$,

$$W = \frac{180(4000^2)}{(5000)^2} = 115.2$$

Thus, a 180-pound man would weigh 115.2 pounds 1000 miles above the surface of Earth.

60. a.



b. Decrease.

61. Direct variation format, $y = kx$:

$$S = k(1+r)^3$$
$$6298.56 = k(1+.08)^3$$
$$6298.56 = k(1.259712)$$
$$k = 5000$$

Then find r, given

$$5955.08 = 5000(1+r)^3$$
$$(1+r)^3 = \frac{5955.08}{5000} = 1.191016$$
$$1+r = \sqrt[3]{1.191016} = 1.06$$
$$r = 1.06 - 1 = .06 = 6\%$$

62. Direct variation format, $y = kx$:

$$S = k(1+r)^4$$
$$17{,}569.20 = k(1+.10)^4$$
$$17{,}569.20 = k(1.4641)$$
$$k = 12000$$

Then find r, given

$$24{,}883.20 = 12000(1+r)^4$$
$$(1+r)^4 = \frac{24{,}883.20}{12000} = 2.0736$$
$$1+r = \sqrt[4]{2.0736} = 1.20$$
$$r = 1.20 - 1 = .20 = 20\%$$

63. Inverse variation format, $y = k/x$:

$$P = \frac{S}{(1+r)^3}$$
$$8396.19 = \frac{10000}{(1+r)^3}$$
$$(1+r)^3 = \frac{10000}{8396.19} = 1.191016401$$
$$(1+r) = \sqrt[3]{1.191016401} = 1.06$$
$$r = .06 = 6\%$$

Then find P when $S = 16{,}500$

$$P = \frac{16{,}500}{(1+.06)^3} = 13{,}853.72$$

Thus, when r = 6% and the future value is \$16,500, the amount to be invested, the present value, is \$13,853.72.

Copyright ©2013 Pearson Education. Inc.

64. Inverse variation format, $y = k/x$:

$$P = \frac{S}{(1+r)^4}$$

$$1525.29 = \frac{2000}{(1+r)^4}$$

$$(1+r)^4 = \frac{2000}{1525.29} = 1.311226062$$

$$(1+r) = \sqrt[4]{1.311226062} = 1.07$$

$$r = .07 = 7\%$$

Then find P when $S = 8000$

$$P = \frac{8000}{(1+.07)^4} = 6103.16$$

Thus, when r = 7% and the future value is $8,000, the amount to be invested, the present value, is $6103.16.

Copyright ©2013 Pearson Education. Inc.

Section 3.4 Skills Check

1.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for $a,\ b,$ and c:

$1 = a(0)^2 + b(0) + c = 0a + 0b + c$

$10 = a(3)^2 + b(3) + c = 9a + 3b + c$

$15 = a(-2)^2 + b(-2) + c = 4a - 2b + c$

so that $a = 2, b = -3, c = 1$ and the quadratic equation is $y = 2x^2 - 3x + 1$.

2.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for $a,\ b,$ and c:

$-3 = a(0)^2 + b(0) + c = 0a + 0b + c$

$37 = a(4)^2 + b(4) + c = 16a + 4b + c$

$30 = a(-3)^2 + b(-3) + c = 9a - 3b + c$

so that $a = 3, b = -2, c = -3$ and the quadratic equation is $y = 3x^2 - 2x - 3$.

3.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for $a,\ b,$ and c:

$30 = a(6)^2 + b(6) + c = 36a + 6b + c$

$-3 = a(0)^2 + b(0) + c = 0a + 0b + c$

$7.5 = a(-3)^2 + b(-3) + c = 9a - 3b + c$

so that $a = 1, b = -0.5, c = -3$ and the quadratic equation is $y = x^2 - 0.5x - 3$.

4.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for $a,\ b,$ and c:

$-22 = a(6)^2 + b(6) + c = 36a + 6b + c$

$23 = a(-3)^2 + b(-3) + c = 9a - 3b + c$

$2 = a(0)^2 + b(0) + c = 0a + 0b + c$

so that $a = 1/3, b = -6, c = 2$ and the quadratic equation is $y = \dfrac{x^2}{3} - 6x + 2$.

5.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for $a,\ b,$ and c:

$6 = a(0)^2 + b(0) + c = 0a + 0b + c$

$\dfrac{22}{3} = a(2)^2 + b(2) + c = 4a + 2b + c$

$\dfrac{99}{2} = a(-9)^2 + b(-9) + c = 81a - 9b + c$

so that $a = 1/2, b = -1/3, c = +6$ and the quadratic equation is $y = \dfrac{x^2}{2} - \dfrac{x}{3} + 6$.

6.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for $a,\ b,$ and c:

$7 = a(0)^2 + b(0) + c = 0a + 0b + c$

$8.5 = a(2)^2 + b(2) + c = 4a + 2b + c$

$12.25 = a(-3)^2 + b(-3) + c = 9a - 3b + c$

so that $a = 1/2, b = -1/4, c = +7$ and the quadratic equation is $y = \dfrac{x^2}{2} - \dfrac{x}{4} + 7$.

Copyright ©2013 Pearson Education. Inc.

7. Three points of this function, a parabola, can be located at (0, 48), its starting point with $t = 0$, at (1, 64), the height of 64 feet after 1 second, and at (3, 0), the height at ground level after 3 seconds. Solving the system of three equations as in problems 1 through 6 produces the quadratic equation:
$h = -16t^2 + 32t + 48$.

8. Three points of this function, a parabola, can be located at (0, 256), its starting point with $t = 0$, at (2, 192), the height of 192 feet after 2 second, and at (4, 0), the height at ground level after 4 seconds. Solving the system of three equations as in problems 1 through 6 produces the quadratic equation:
$h = -16t^2 + 256$.

9.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for a, b, and c:

$6 = a(-1)^2 + b(-1) + c = 1a - 1b + c$

$3 = a(2)^2 + b(2) + c = 4a + 2b + c$

$10 = a(3)^2 + b(3) + c = 9a + 3b + c$

so that $a = 2, b = -3, c = 1$ and the quadratic equation is $y = 2x^2 - 3x + 1$.

10.

Given the quadratic model: $y = ax^2 + bx + c$, the following system of equations can be determined from the given points and then solved for a, b, and c:

$-4 = a(-2)^2 + b(-2) + c = 4a - 2b + c$

$1 = a(3)^2 + b(3) + c = 9a + 3b + c$

$4 = a(2)^2 + b(2) + c = 4a + 2b + c$

so that $a = -1, b = 2, c = 4$ and the quadratic equation is $y = -x^2 + 2x + 4$.

11. Choose any three points from the table and solve using the technique in the first 10 exercises, producing the quadratic equation:
$y = 3x^2 - 2x$.

12.

x	$f(x)$	first differences	second differences
0	0		
2	399	399	
4	1601	1202	803
6	3600	1999	797
8	6402	2802	803
10	9998	3596	794

Since the second differences are approximately equal, $f(x)$ is approximately quadratic.

x	$g(x)$	first differences	second differences
0	2.0		
2	0.8	–1.2	
4	1.2	0.4	1.6
6	3.2	2.0	1.6
8	6.8	3.6	1.6
10	12.0	5.2	1.6

Since the second differences are constant, $g(x)$ is exactly quadratic.

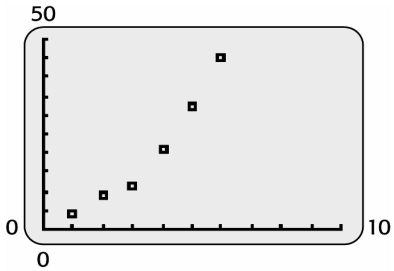
x	$h(x)$	first differences	second differences
0	0		
2	110	110	
4	300	190	80
6	195	–105	–295
8	230	35	140
10	290	60	25

Since the second differences are extremely variable, $h(x)$ is not quadratic.

13. The x-values are not equally spaced.

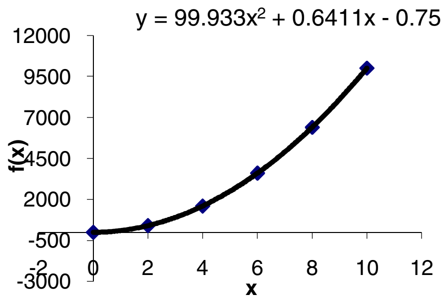
Copyright ©2013 Pearson Education. Inc.

14. a.

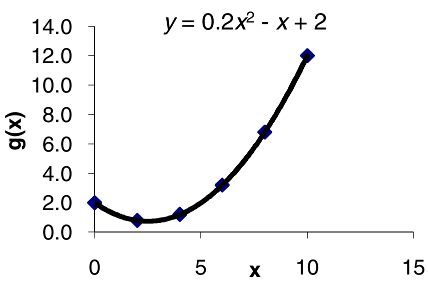


b. Quadratic Model.

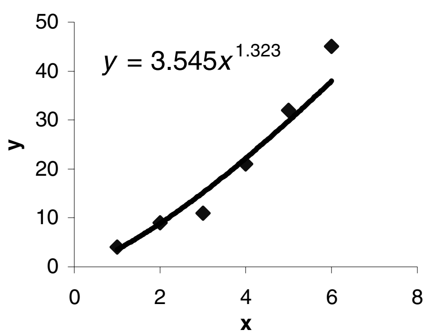
15.



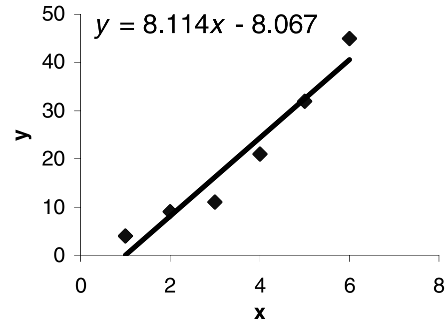
16.



17. a.

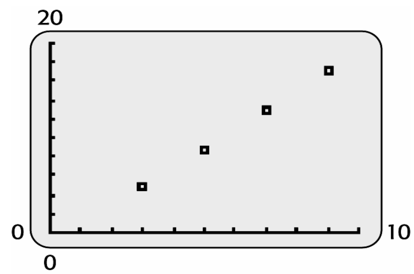


b.



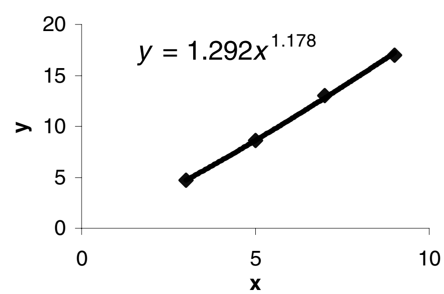
c. The power function is a better fit.

18. a.

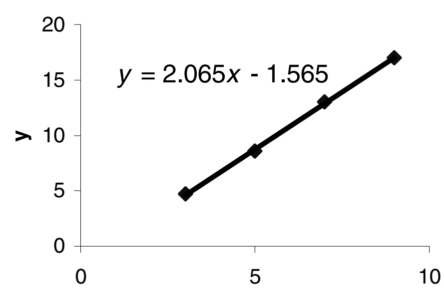


b. The scatter plot suggests that a linear model will fit the data reasonably well.

19. a.



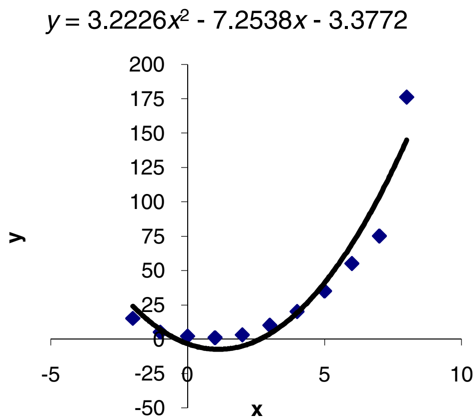
b.



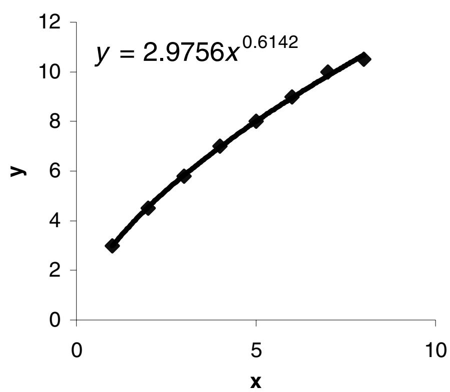
c. Both models appear to be good fits.

Copyright ©2013 Pearson Education. Inc.

20.

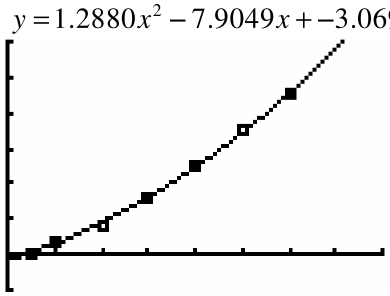


21.



22. Quadratic model:

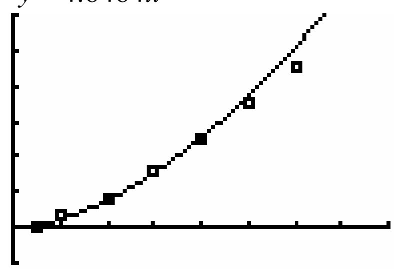
$y = 1.2880x^2 - 7.9049x + -3.0693$



[0, 8] by [–20, 120]

Power model:

$y = 4.6464x^{1.7214}$



[0, 8] by [–20, 120]

Both models seem to fit the data reasonably well.

Section 3.4 Exercises

23. a. $y = 3.688x^2 + 459.716x + 2985.640$

b. In 1997, $x = 1997 - 1960 = 37$

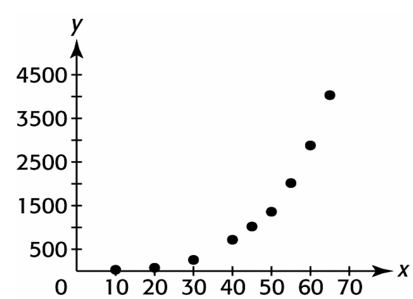
$$y = 3.688(37)^2 + 459.716(37)$$
$$+2985.640 = \$25,044$$

In 2014, $x = 2014 - 1960 = 54$

$$y = 3.688(54)^2 + 459.716(54)$$
$$+2985.640 = \$38,564$$

c. Since the median annual income shown in the table, fell in 2009, the extrapolation for 2014 may not be reliable.

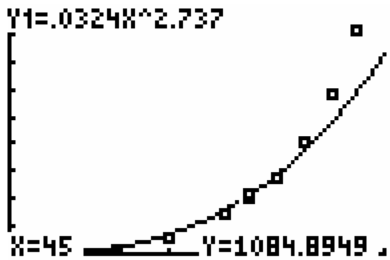
24. a.



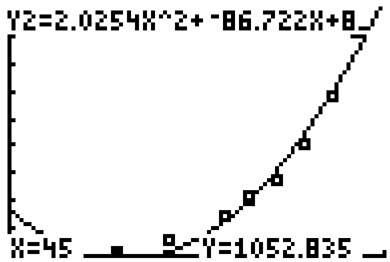
b. Power model:

$y = 0.0324x^{2.737}$

Copyright ©2013 Pearson Education. Inc.



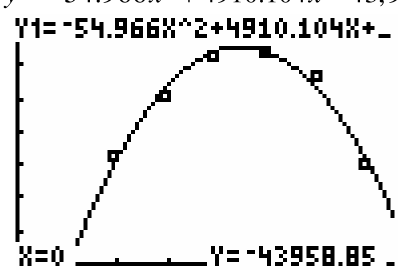
Quadratic model:
$y = 2.0254x^2 - 86.722x + 853.890$



c. The quadratic model appears better.

d. For the year 2020, $x = 2020 - 1950$
$= 70$
Quadratic model:
$y = 2.0254(70)^2 - 86.722(70)$
$+853.890 = \$4707.8$ billion

25. a. $y = -54.966x^2 + 4910.104x - 43,958.85$



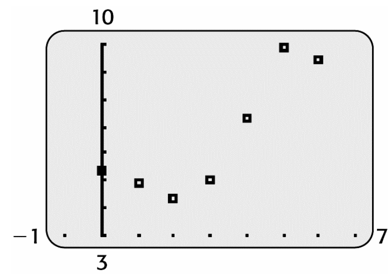
[0, 75] by [0, 75,000]

b. The model appears to fit the data points very well.

c. $x = -\dfrac{b}{2a} = -\dfrac{4910.104}{2(-54.966)} = 44.665$

Substituting 44.665 into the function, results in 65,696.78. Thus the vertex of the parabola is (44.665, 65696.78) and means a 45-year old person produces the maximum median income of $65,696.78.

26. a.



b. $y = 0.225x^2 + 0.461x + 5.071$

c. Yes, in 2004, the unemployment was approximately 5%.

27. a. $y = 0.0052x^2 - 0.62x + 15.0$

b.

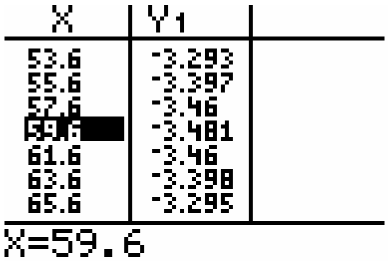
X	Y1
47	⁻2.653
48	⁻2.779
49	⁻2.895
50	⁻3
51	⁻3.095
52	⁻3.179
53	⁻3.253

X=50

Using the Table feature of the calculator, at 50 mph, the wind chill will be minus 3 degrees.

Copyright ©2013 Pearson Education. Inc.

c.

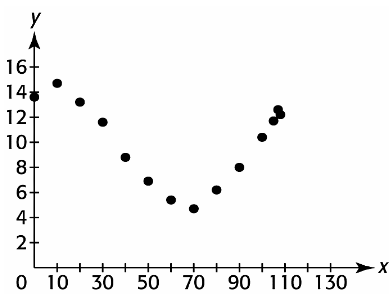


X	Y_1
53.6	-3.293
55.6	-3.397
57.6	-3.46
59.6	-3.481
61.6	-3.46
63.6	-3.398
65.6	-3.295

X=59.6

No, it appears the model predicts the wind chill temperature will be a minimum near 59.6 mph, and it is illogical that it would be warmer for winds greater than 60 mph.

28. a.



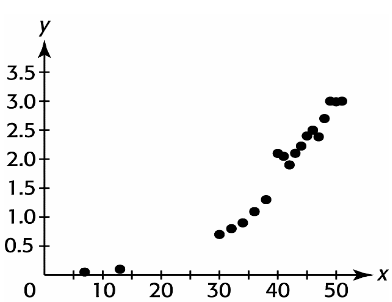
b. Yes.

c. $y = 0.0026x^2 - 0.3217x + 16.6572$

d. In 2015, $x = 2015 - 1900 = 115$

$$y = 0.0026(115)^2 - 0.3217(115) + 16.6572 = 14.0\%$$

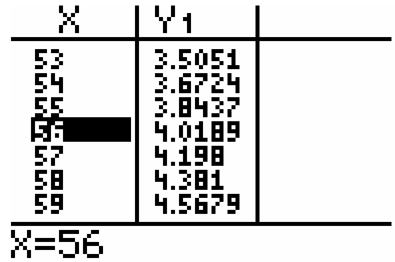
29. a.



b. Yes, a quadratic function could be a good model.

c. The best function to model this data is $y = 0.002x^2 - 0.043x + 0.253$

d.

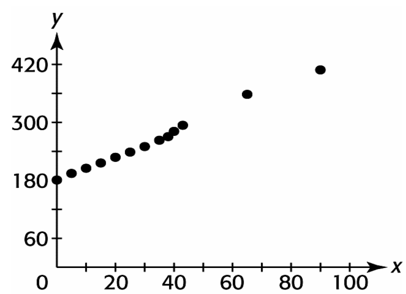


X	Y_1
53	3.5051
54	3.6724
55	3.8437
56	4.0189
57	4.198
58	4.381
59	4.5679

X=56

In the year 2016, 56 years after 1960, the cost for a 30-second ad will be \$4,000,000.

30. a.



b. The best function to model this data is $y = 0.00138x^2 + 2.488x + 178.792$

31. a. $y = 511.873x^{-0.046}$

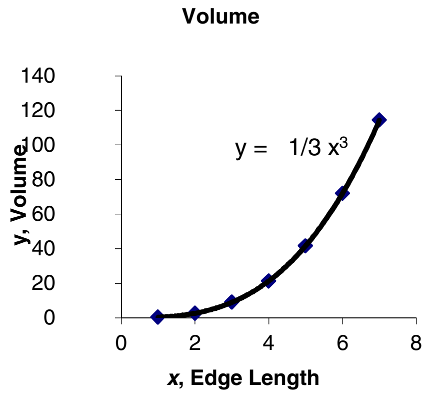
b. 454.7 crimes per 100,000 residents

32. a. The second differences are not constant. The situation is not modeled by a quadratic function.

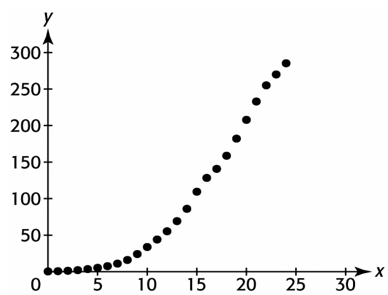
x	y	first differences	second differences
1	1/3		
2	2 2/3	2 1/3	
3	9	6 1/3	4
4	21 1/3	12 1/3	6
5	41 2/3	20 1/3	8
6	72	30 1/3	10

Copyright ©2013 Pearson Education. Inc.

b. Modeling with a power function yields:



33. a.



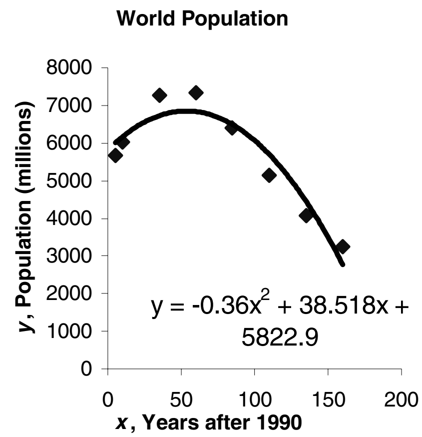
Yes. It appears a quadratic function will fit the data well.

b. The best function to model this data is
$y = 0.632x^2 - 2.651x + 1.209$

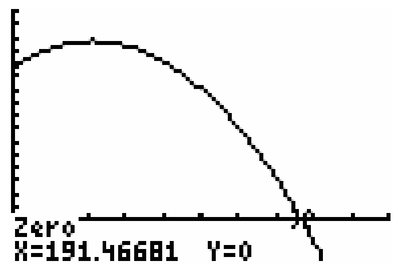
c. In 2011, $x = 2011 - 1985 = 26$
$y = 0.632(26)^2 - 2.651(26) + 1.209$
$y = 359.52$ million

d. There are about 310 million people in the U.S. Therefore, approximately 100% have cell phones.

34. a.



b. The x-intercept is approximately 191. The corresponding year is 2181 (1990 + 191).



[0, 250] by [–1500, 8000]

X	Y1
188	340.07
189	242.87
190	144.94
191	46.296
192	-53.07
193	-153.2
194	-254

X=191

c. The model is not valid beyond the year 2181. The model predicts that the population will become negative beginning in 2182!

Copyright ©2013 Pearson Education. Inc.

35. a. The best function to model this data is $y = 0.011x^2 - 0.957x + 24.346$

b.

$$x = -\frac{b}{2a} = -\frac{-0.957}{2(0.011)} = 43.5$$

Substituting 43.5 into the function, results in 3.531. Thus the vertex of the parabola is (43.5, 3.531) and means that between the years 1980 and 2009, the lowest savings rate was 3.5% and occurred in 2004 (1960 + 44).

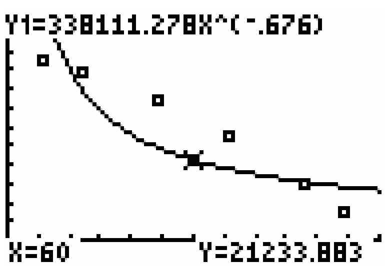
c.

X	Y1	
56	5.25	
57	5.536	
58	5.844	
59	6.174	
60	6.526	
61	6.9	
62	7.296	

X=59

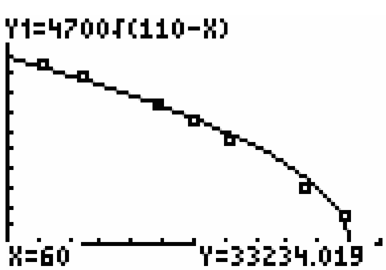
After 1990, the personal savings rate will again reach 6% in the year 2019 (1960 + 59).

36. a.



[0, 120] by [–5000, 60,000]

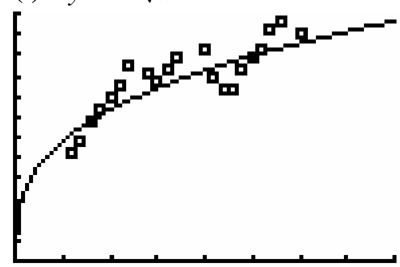
b.



[0, 120] by [–5000, 60,000]

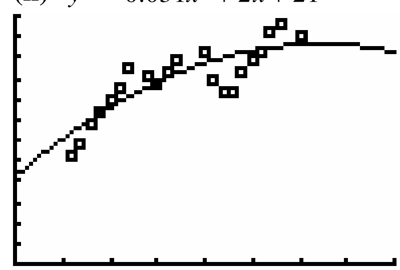
Equation b) fits the data much better.

37. a. **(i)** $y = 17\sqrt[3]{x}$



[0, 40] by [0, 60]

(ii) $y = -0.031x^2 + 2x + 21$



[0, 40] by [0, 60]

The quadratic model appears to be slightly better.

b. Since beyond 2005 the cube root model appears to be a better fit, to make a prediction about 2020, apply the cube root model.

38. a. $y = 0.004x^2 - 0.338x + 7.687$

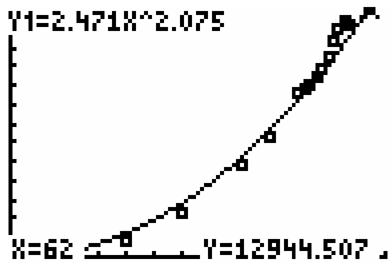
b. The model gives the minimum index in the year 1943 (1900 + 43).

$$-\frac{b}{2a} = -\frac{-(-0.338)}{2(0.004)} = 42.25 \approx 43$$

c. The Great Depression occurred in this time period.

Copyright ©2013 Pearson Education. Inc.

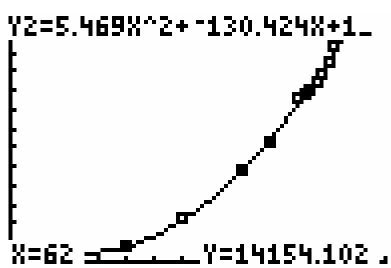
39. a. The power function is $y = 2.471x^{2.075}$



[0, 65] by [0, 13,000]

b. The total U. S. personal income in 2012, when $x = 62$, will be $12.945 trillion.

c. The quadratic function is
$y = 5.469x^2 - 130.424x + 1217.554$



[0, 65] by [0, 13,000]

d. It appears the quadratic model is better.

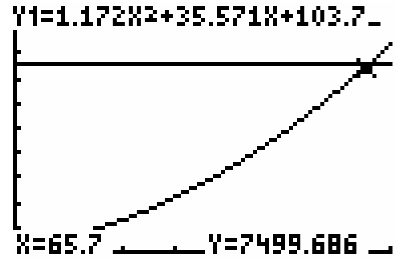
40. a. $y = 0.114x^2 - 2.322x + 45.445$

b. In 2013, $x = 2013 - 1990 = 23$
$y = 0.114(23)^2 - 2.322(23) + 45.445$
$y = 52.3$ million

The number of persons living below poverty level in the year 2013 is predicted to be 52.3 million.

41. a. $y = 1.172x^2 + 35.571x + 103.745$

b.

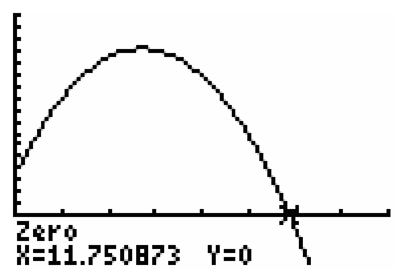


[0, 70] by [0, 10000]

When x is approximately 65.7 years after 1960, during the year 2026, the tax per capita will reach $7500.

42. a. Based on the table, the Medicare trust fund is broke in 2002.

b. The best fitting quadratic model is
$y = -3.556x^2 + 38.704x + 36.25$



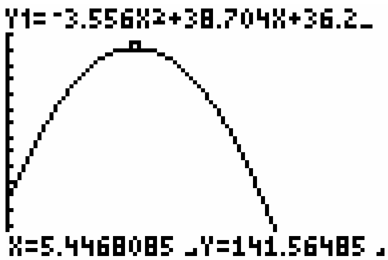
[0, 16] by [0, 170]

The model predicts that the Medicare Trust Fund balance would be 0, when x is 12. The year is 2002.

c. Assuming the pamphlet was issued in 1994, the data confirm the statement.

Copyright ©2013 Pearson Education. Inc.

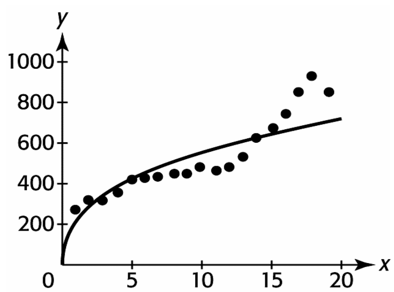
d.



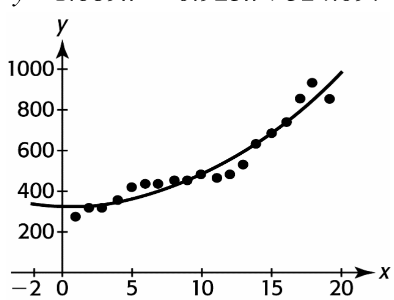
[0, 16] by [0, 170]

The vertex is approximately $(5.44, 141.56)$. The highest balance of the trust fund is 141.56 billion in 1996.

43. a. Power function is $y = 222.434x^{0.3943}$



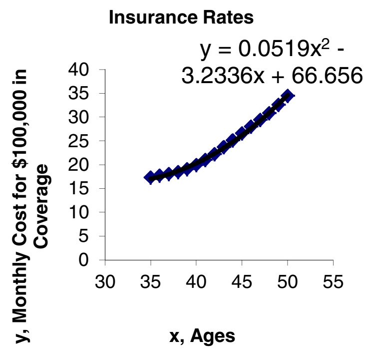
b. The best fitting quadratic model is $y = 1.689x^2 - 0.923x + 324.097$



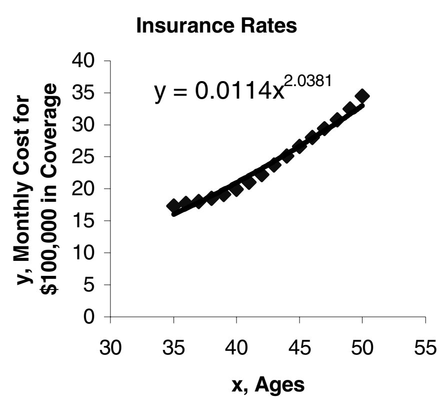
c. Based on the models shown in part a) and part b), the quadratic model appears to be the better fit.

44. a.

b. Yes. It appears that a quadratic function will fit the data.



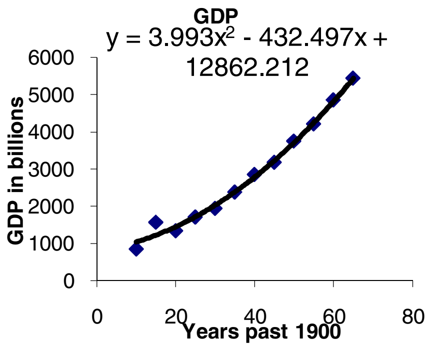
c.



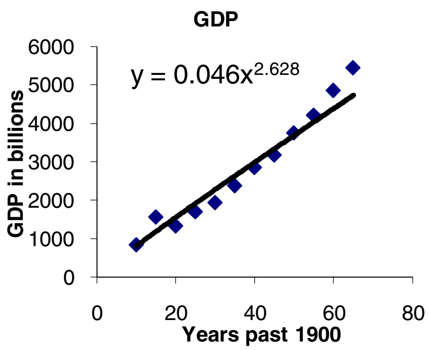
d. It appears that the quadratic model, based on the scatter plots, is the best fit.

Copyright ©2013 Pearson Education. Inc.

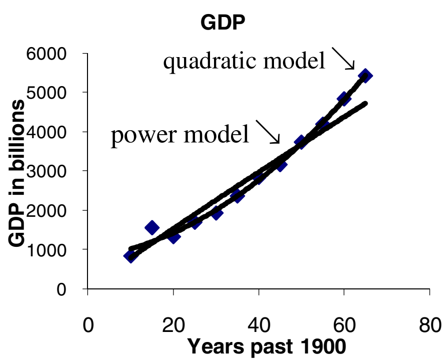
45. a.



b.



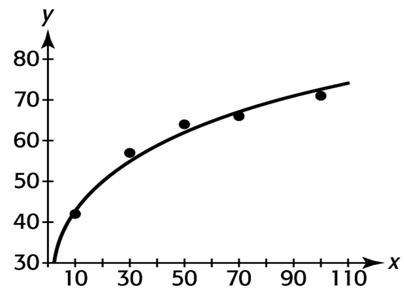
c.



The quadratic model fits the data better.

46. a. Power function is $y = 25.425x^{0.228}$

b.

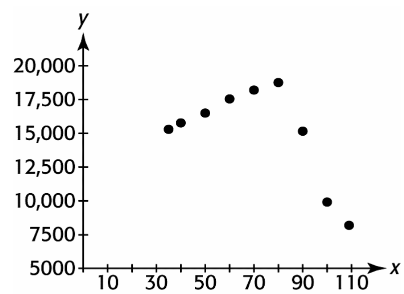


c. $y = 25.425x^{0.228}$

$y = 25.425(80)^{0.228} = 69.05$

Thus, the noise level at 80 mph will be approximately, 69 db.

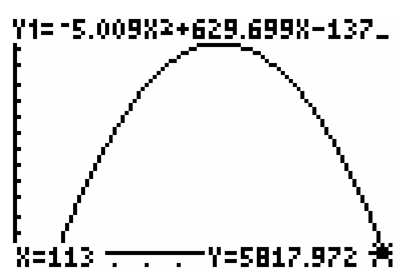
47.



b. $y = -5.009x^2 + 629.699x - 1378.094$

c. In 2007, when $x = 107$, the number of banks is estimated to be 8,649.

d.



[0, 115] by [5000, 20000]

The model estimates the number of banks in the U.S. will be 5818 when x = 113, in the year 2013.

48. a. The model that best fits the revenues as a function of the number of years after 1970 is a power function,
$y = 0.2794(x)^{0.9649}$

b. In 2015, $x = 45$, and the unrounded model estimate as the revenue is \$11 billion.

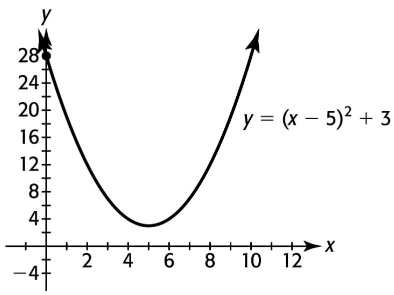
c. Since 2015 is outside the range of the original data, the answer in part b) is an extrapolation. If conditions change in future years, the model may no longer be valid.

Copyright ©2013 Pearson Education. Inc.

Chapter 3 Skills Check

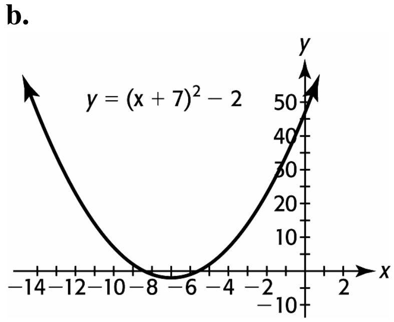
1. **a.** The vertex is at (5, 3).

b.



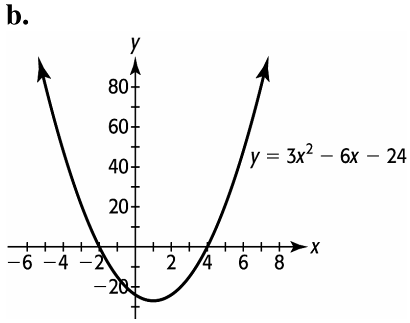
2. **a.** The vertex is at $(-7,-2)$.

b.



3. **a.** The vertex is at $(1,-27)$.

b.



$f(x)=3x^2-6x-24$

$x=\dfrac{-b}{2a}=\dfrac{-(-6)}{2(3)}=\dfrac{6}{6}=1$

$y=f(x)=f(1)$

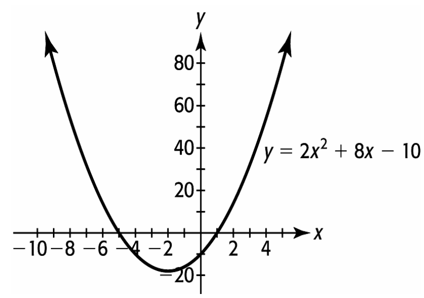
$f(1)=3(1)^2-6(1)-24$

$f(1)=3-6-24=-27$

The vertex is $(1,-27)$.

4. **a.** The vertex is at $(-2,-18)$.

b.



$f(x)=2x^2+8x-10$

$x=\dfrac{-b}{2a}=\dfrac{-(8)}{2(2)}=\dfrac{-8}{4}=-2$

$y=f(x)=f(-2)$

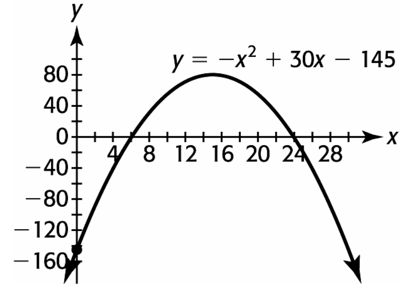
$f(-2)=2(-2)^2+8(-2)-10$

$f(-2)=2(4)+8(-2)-10=-18$

The vertex is $(-2,-18)$.

5. **a.** The vertex is at (15, 80).

b.



Copyright ©2013 Pearson Education. Inc.

$f(x) = -x^2 + 30x - 145$

$x = \dfrac{-b}{2a} = \dfrac{-(30)}{2(-1)} = \dfrac{-30}{-2} = 15$

$y = f(x) = f(15)$

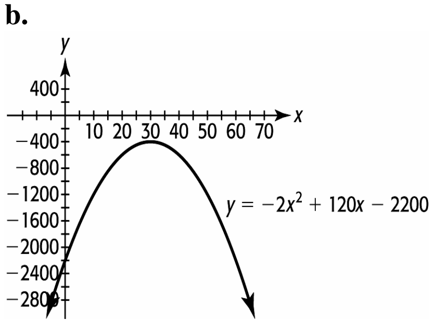
$f(15) = -(15)^2 + 30(15) - 145$

$f(15) = -225 + 450 - 145$

The vertex is $(15, 80)$.

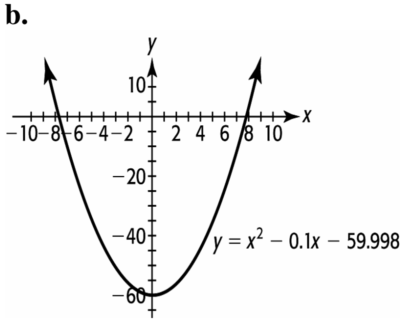
6. **a.** The vertex is at $(30, -400)$. Use the procedure in #3 to find the vertex point.

b.



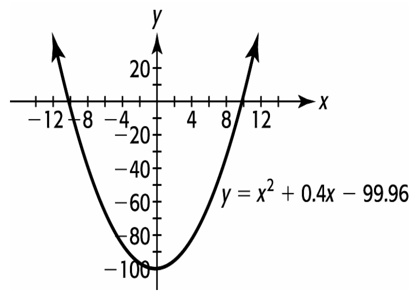
7. **a.** The vertex is at $(0.05, -60.0005)$. Use the procedure in #3 to find the vertex point.

b.



8. **a.** The vertex is at $(-0.2, -100)$. Use the procedure in #3 to find the vertex point.

b.



9. $x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, x = 1$

10.

$6x^2 + x - 2 = 0$

Note that $(6x^2)(-2) = -12x^2$. Look for two terms whose product is $-12x^2$ and whose sum is the middle term, x $(+4x)$ and $(-3x)$.

$6x^2 + 4x - 3x - 2 = 0$

$(6x^2 + 4x) + (-3x - 2) = 0$

$2x(3x+2) - 1(3x+2) = 0$

$(2x-1)(3x+2) = 0$

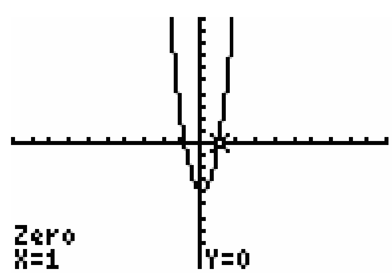
$2x - 1 = 0, 3x + 2 = 0$

$x = \dfrac{1}{2}, x = \dfrac{-2}{3}$

11. $5x^2 - x - 4 = 0$

$(x-1)(5x+4) = 0$

$x = 1, x = -\dfrac{4}{5}$



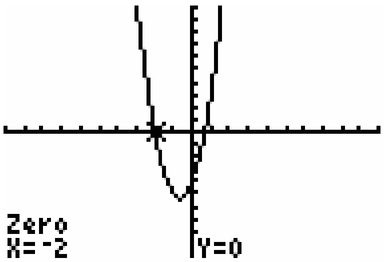
[–10, 10] by [–10, 10]

Copyright ©2013 Pearson Education. Inc.

12. $3x^2+4x-4=0$

$(x+2)(3x-2)=0$

$x=-2, x=\frac{2}{3}$



[–10, 10] by [–10, 10]

13. $x^2-4x+3=0$

$a=1, b=-4, c=3$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(3)}}{2(1)}$

$x=\frac{4\pm\sqrt{16-12}}{2}$

$x=\frac{4\pm\sqrt{4}}{2}$

$x=\frac{4\pm 2}{2}$

$x=3, x=1$

14. $4x^2+4x-3=0$

$a=4, b=4, c=-3$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(4)\pm\sqrt{(4)^2-4(4)(-3)}}{2(4)}$

$x=\frac{-4\pm\sqrt{16+48}}{8}$

$x=\frac{-4\pm\sqrt{64}}{8}$

$x=\frac{-4\pm 8}{8}$

$x=\frac{-4+8}{8}, x=\frac{-4-8}{8}$

$x=\frac{4}{8}=\frac{1}{2}, x=-\frac{12}{8}=-\frac{3}{2}$

15. a. Algebraically:

Let $f(x)=0$

$3x^2-6x-24=0$

$3(x^2-2x-8)=0$

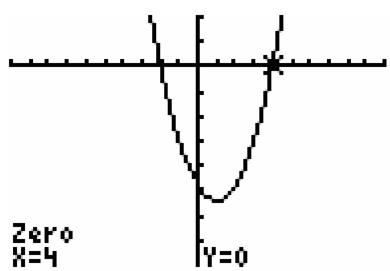
$3(x-4)(x+2)=0$

$x-4=0, x+2=0$

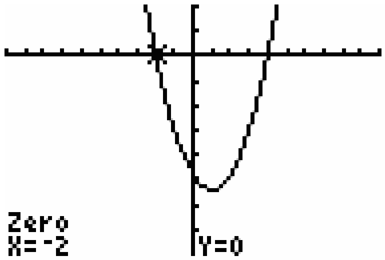
$x=4, x=-2$

The x-intercepts are $(4,0)$ and $(-2,0)$.

Graphically:



[–10, 10] by [–40, 10]



[–10, 10] by [–40, 10]

Again, the x-intercepts are $(4,0)$ and $(-2,0)$.

b. Solving $f(x)=0$ produces the x-intercepts, as shown in part a). The x-intercepts are $(4,0)$ and $(-2,0)$.

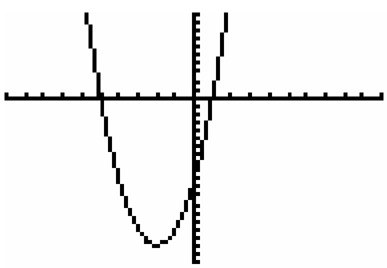
16. a. Algebraically:

Let $f(x)=0$

$$2x^2+8x-10=0$$
$$2\left(x^2+4x-5\right)=0$$
$$2(x-1)(x+5)=0$$
$$x-1=0, x+5=0$$
$$x=1, x=-5$$

The x-intercepts are $(1,0)$ and $(-5,0)$.

Graphically:



[–10, 10] by [–20, 10]

Again, the x-intercepts are $(1,0)$ and $(-5,0)$.

b. Solving $f(x)=0$ produces the x-intercepts, as shown in part a) above.

17. $5x^2-20=0$

$$5x^2=20$$
$$x^2=4$$
$$x=\pm 2$$

18. $(x-4)^2=25$

$$x-4=\pm 5$$
$$x=4\pm 5=4+5, \text{ or } 4-5$$
$$x=9 \text{ or } -1$$

19. $z^2-4z+6=0$

$$a=1, b=-4, c=6$$
$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$z=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(6)}}{2(1)}$$
$$z=\frac{4\pm\sqrt{16-24}}{2}$$
$$z=\frac{4\pm\sqrt{-8}}{2}$$
$$z=\frac{4\pm 2i\sqrt{2}}{2}$$
$$z=2\pm i\sqrt{2}$$

20. $w^2-4w+5=0$

$$a=1, b=-4, c=5$$
$$w=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$w=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$$
$$w=\frac{4\pm\sqrt{16-20}}{2}$$
$$w=\frac{4\pm\sqrt{-4}}{2}$$
$$w=\frac{4\pm 2i}{2}$$
$$w=2\pm i$$

Copyright ©2013 Pearson Education. Inc.

21. $4x^2 - 5x + 3 = 0$

$a = 4, b = -5, c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{5 \pm \sqrt{25 - 48}}{8}$$

$$x = \frac{5 \pm \sqrt{-23}}{8}$$

$$x = \frac{5 \pm i\sqrt{23}}{8}$$

22.

$4x^2 + 2x + 1 = 0$

$a = 4, b = 2, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{4 - 16}}{8}$$

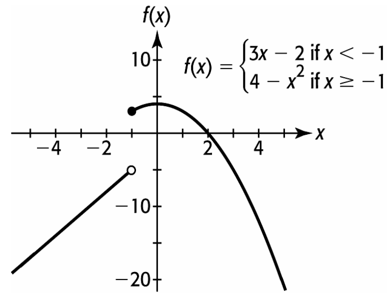
$$x = \frac{-2 \pm \sqrt{-12}}{8}$$

$$x = \frac{-2 \pm \sqrt{-4 \times 3}}{8}$$

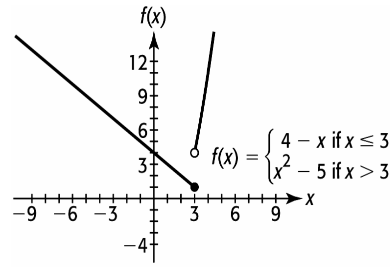
$$x = \frac{-2 \pm 2i\sqrt{3}}{8}$$

$$x = \frac{-1 \pm i\sqrt{3}}{4}$$

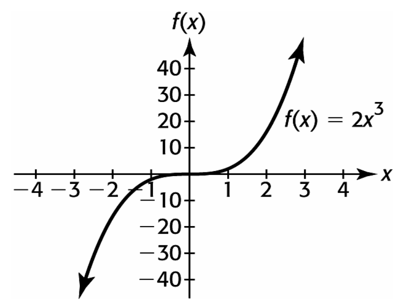
23.



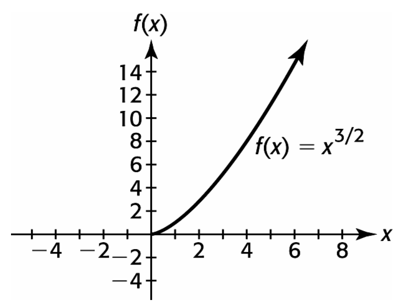
24.



25.

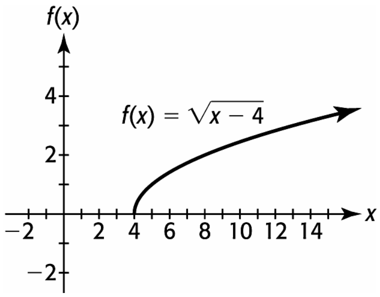


26.

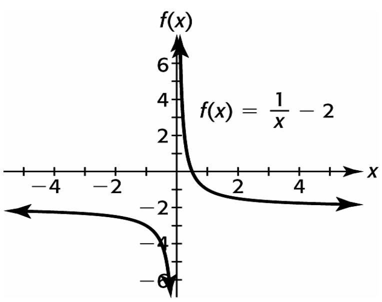


Copyright ©2013 Pearson Education. Inc.

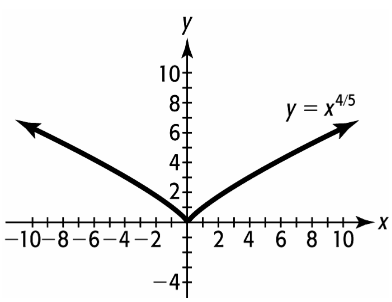
27.



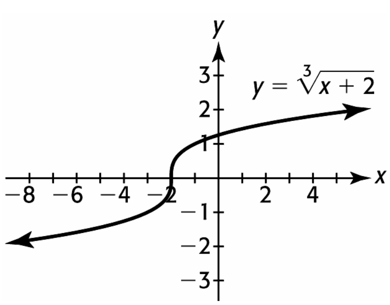
28.



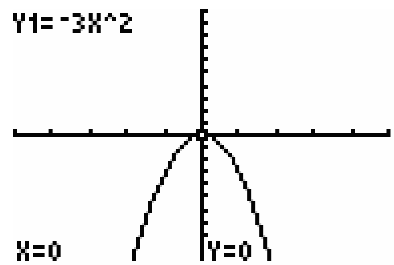
29.



30.



31.

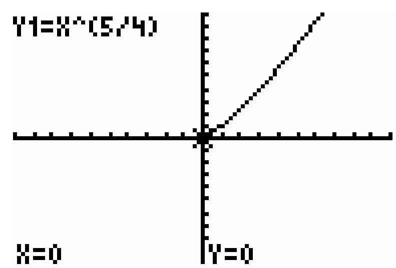


[–5, 5] by [–10, 10]

a. The above function is increasing when $x < 0$. In interval notation, the function is increasing on the interval $(-\infty, 0)$.

b. The above function is decreasing when $x > 0$. In interval notation, the function is decreasing on the interval $(0, \infty)$.

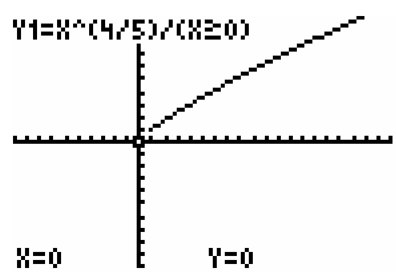
32. a.



[–10, 10] by [–10, 10]

The graph is concave up.

b.



[–10, 10] by [–10, 10]

The graph is concave down.

Copyright ©2013 Pearson Education. Inc.

33. $|3x-6|=24$

$$\begin{array}{l|l} 3x-6=24 & 3x-6=-24 \\ 3x=30 & 3x=-18 \\ x=10 & x=-6 \end{array}$$

$x=10, x=-6$

34. $|2x+3|=13$

$$\begin{array}{l|l} 2x+3=13 & 2x+3=-13 \\ 2x=10 & 2x=-16 \\ x=5 & x=-8 \end{array}$$

$x=5, x=-8$

35.

Given the quadratic model: $y=ax^2+bx+c$, the following system of equations can be determined from the given points and then solved for a, b, and c:

$-2=a(0)^2+b(0)+c=0a+0b+c$

$12=a(-2)^2+b(-2)+c=4a-2b+c$

$7=a(3)^2+b(3)+c=9a+3b+c$

so that $a=2, b=-3, c=-2$ and the quadratic equation is $y=2x^2-3x-2$.

36.

Given the quadratic model: $y=ax^2+bx+c$, the following system of equations can be determined from the given points and then solved for a, b, and c:

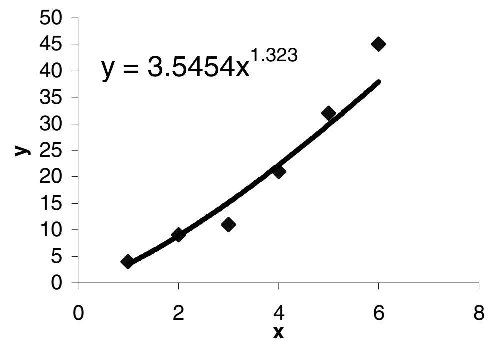
$-9=a(-2)^2+b(-2)+c=4a-2b+c$

$7=a(2)^2+b(2)+c=4a+2b+c$

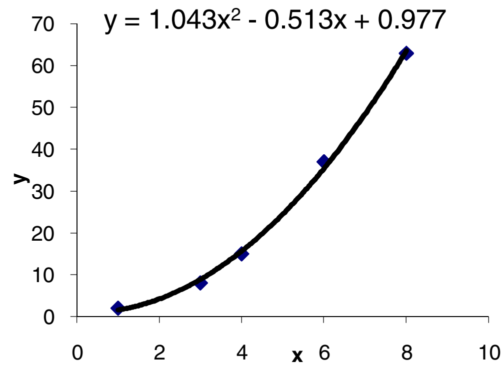
$-9=a(4)^2+b(4)+c=16a+4b+c$

so that $a=-2, b=4, c=7$ and the quadratic equation is $y=-2x^2+4x+7$.

37.



38.



39. $q=kp^{\frac{3}{2}}$

$16=k(4)^{\frac{3}{2}}=k(8)$

$\frac{16}{8}=k=2$

$q=2p^{\frac{3}{2}}$

$q=2(16)^{\frac{3}{2}}=2(64)=128$

40. $f(-8)=3(-8)-2=-26$

$f(0)=0^2-4=-4$

$f(4)=-5$

Copyright ©2013 Pearson Education. Inc.

Chapter 3 Review Exercises

41. a. The maximum profit occurs at the vertex.

$P(x) = -0.01x^2 + 62x - 12,000$

$x = \frac{-b}{2a} = \frac{-62}{2(-0.01)} = \frac{-62}{-0.02} = 3100$

$P(3100) = -0.01(3100)^2 + 62(3100)$
$\quad -12,000$

$P(3100) = -96,100 + 192,200 - 12,000$

$p(3100) = 84,100$

The vertex is $(3100,\ 84,100)$.

Producing and selling 3100 units produces the maximum profit of $84,100.

b. The maximum possible profit is $84,100.

42. a.

$P(x) = R(x) - C(x)$

$= (200x - 0.01x^2) - (38x + 0.01x^2 + 16,000)$

$= 200x - 0.01x^2 - 38x - 0.01x^2 - 16,000$

$= -0.02x^2 + 162x - 16,000$

$x = \frac{-b}{2a} = \frac{-162}{2(-0.02)} = \frac{-162}{-0.04} = 4050$

$P(4050) = -0.02(4050)^2 + 162(4050)$
$\quad -16,000$

$P(4050) = -328,050 + 656,100 - 16,000$

$p(4050) = 312,050$

The vertex is $(4050,\ 312,050)$.

Producing and selling 4050 units gives the maximum profit of $312,050.

b. The maximum possible profit is $312,050.

43. a. $h = \frac{-b}{2a} = \frac{-64}{2(-16)} = 2$. The ball reaches its maximum height in 2 seconds.

b. $S(2) = 192 + 64(2) - 16(2)^2$
$= 256$

The ball reaches its maximum height at 256 feet.

44. a. $h = \frac{-b}{2a} = \frac{-29.4}{2(-9.8)} = 1.5$.

The ball reaches its maximum height in 1.5 seconds.

b. $S(1.5) = 60 + 29.4(1.5) - 9.8(1.5)^2$
$= 82.05$

The ball reaches its maximum height at 82.05meters.

Copyright ©2013 Pearson Education. Inc.

45. a. $y = -1.48x^2 + 38.901x - 118.429$

$$x = \frac{-b}{2a} = \frac{-38.901}{2(-1.48)} = \frac{-38.901}{-2.96} = 13.1$$

The year is 13.1 after 1990 = 2004.

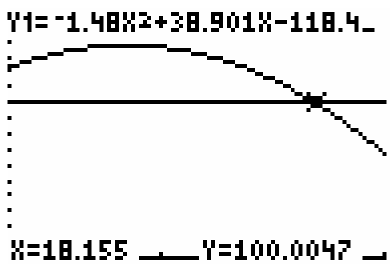
b. $y = -1.48x^2 + 38.901x - 118.429$

$$y = -1.48(13.1)^2 + 38.901(13.1) - 118.429$$

$$y = -253.9828 + 509.6031 - 118.429$$

$$y = 137.1913 = 137 \text{ thousand visas.}$$

c. In 2008 (1990 + 18), the number of visas will be 100 thousand.



[10, 20] by [0, 160]

46. $3600 - 150x + x^2 = 0$

$$x^2 - 150x + 3600 = 0$$

$$(x-30)(x-120) = 0$$

$$x = 30, x = 120$$

Break-even occurs when 30 or 120 units are produced and sold.

47. The ball is on the ground when $S = 0$.

$$400 - 16t^2 = 0$$

$$-16t^2 = -400$$

$$t^2 = 25$$

$$t = \pm 5$$

In the physical context of the question, $t \geq 0$. The ball reaches the ground in 5 seconds.

48.

$$-0.3x^2 + 1230x - 120,000 = 324,000$$

$$-0.3x^2 + 1230x - 444,000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(1230) \pm \sqrt{(1230)^2 - 4(-0.3)(-444,000)}}{2(-0.3)}$$

$$= \frac{-1230 \pm \sqrt{1,512,900 - 532,800}}{-0.6}$$

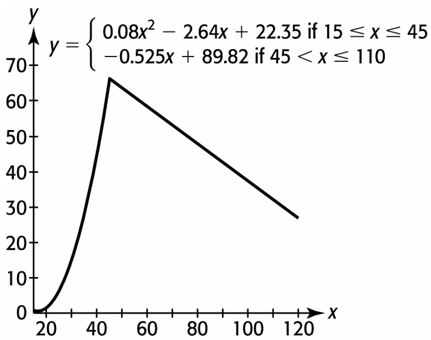
$$= \frac{-1230 \pm \sqrt{980,100}}{-0.6}$$

$$= \frac{-1230 \pm 990}{-0.6}$$

$$x = 400, x = 3700$$

A profit of \$324,000 occurs when 400 or 3700 units are produced and sold.

49. a.



b.

Since 1990 is 90 years after 1900, $x = 90$. Since 90 fits the second piece of the function, substitute 90 into:

$T(x) = -0.525x + 89.82$

$T(90) = -0.525(90) + 89.82 = 42.57$

Thus, the tax rate for a millionaire head of household in 1990 was 42.57%.

c.

Since 2010 is 110 years after 1900, $x = 110$. Since 110 fits the second piece of the function, substitute 110 into:

$T(x) = -0.525x + 89.82$

$T(110) = -0.525(110) + 89.82 = 32.07$

Thus, the tax rate for a millionaire head of household in 2010 was 32.07%, so the model is fairly good.

50. a. $y = -0.00482x^2 + 0.754x + 8.512$

b. In 2022, $x = 2022 - 2000 = 22$

$y = -0.00482(22)^2 + 0.754(22)$
$+8.512 = 22.767 \approx 22.8\%$

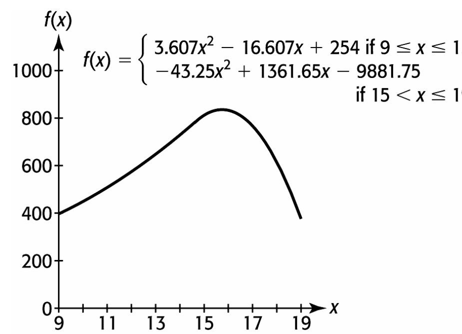
c.

X	Y1
35	28.998
36	29.409
37	29.811
38	30.204
39	30.587
40	30.96
41	31.324

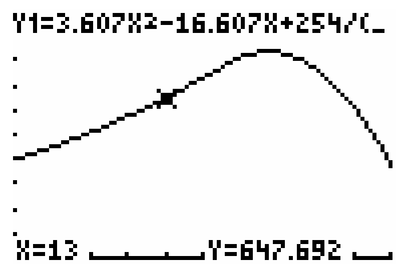
X=38

Using the table feature of the calculator, the model predicts the percent to 30.2% in the year 2038.

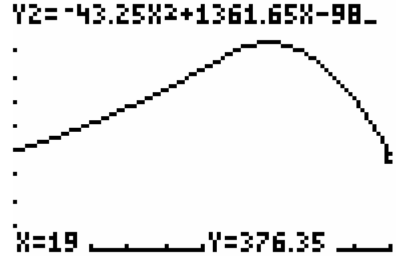
51. a.



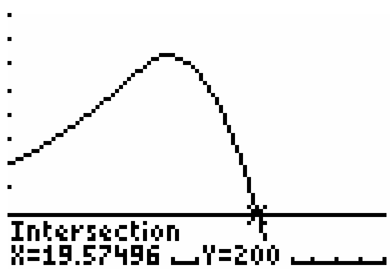
b. Using $x = 13$ for the year 2003, the number of deaths in that year were approximately 648.



c. Using $x = 19$ for the year 2009, the number of deaths in that year were approximately 376.



d. Extending the graphing window to x = 25, and using the Intersect feature of the calculator, the model predicts the number of deaths to be 200, when x = 20, in the year 2010.



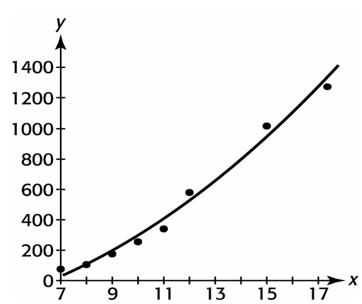
[9, 25] by [0, 1000]

52. Since x is the mass of the animal measured in grams, and the given bobcat weight is 1.6 kg, the value of x is 1600. Thus, the home range of this meat-eating mammal would be:

$$H(x) = 0.11x^{1.36} = 0.11(1600)^{1.36}$$
$$= 2506 \text{ hectares}$$

53. a. $y = 5.126x^2 + 0.370x - 221.289$

b.



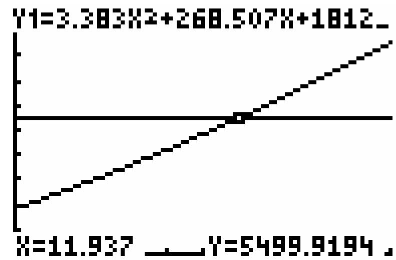
c. Yes. The function seems to fit the data very well.

d. Since the table is shown in millions, and 2 billion = 2000 million, y = 2000.

$$y = 5.126x^2 + 0.370x - 221.289$$
$$2000 = 5.126x^2 + 0.370x - 221.289$$
$x = 20.78,$ and the year will be 2011.

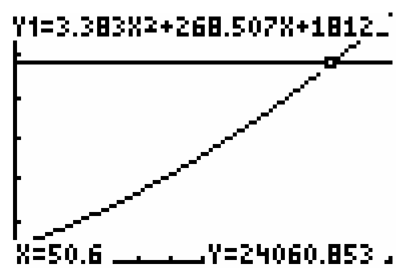
54. a. $y = 3.383x^2 + 268.507x + 1812.701$

b. Using the model:
Personal income reaches $5500 billion in 1992 (1980 + 11.9).



[0, 20] by [0, 10,000]

c. The personal income level in 2009 was $12,026 billion. Twice that amount is $24,052 billion. Using the model, the personal income level doubles during 2031 (1980 + 51).

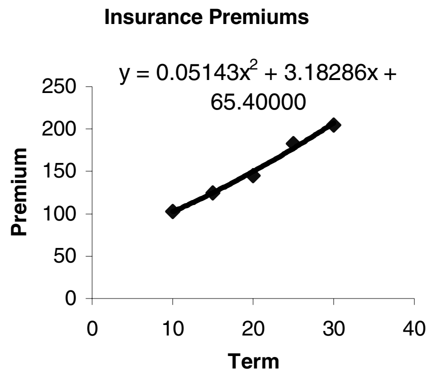


[0, 60] by [0, 30,000]

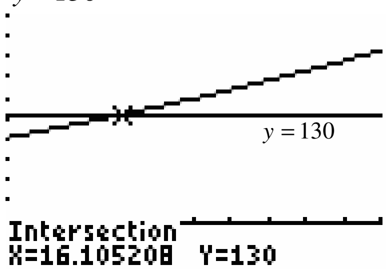
55. $y = 18.624x^2 - 440.198x + 20823.439$
thousand people in the U. S.
in year x, where x is the number
of years after 1980.

Copyright ©2013 Pearson Education. Inc.

56. a.



b. Applying the intersection of graphs method using unrounded model and $y = 130$



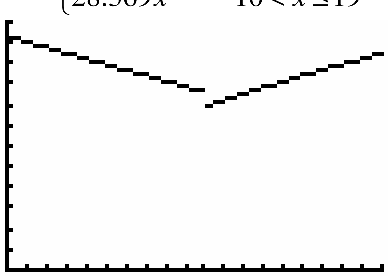
[10, 30] by [–50, 250]

A premium of \$130 would purchase a term of 16 years.

57. a. $y = -2.7x + 113$, for years 1990 to 2000, where x is the years after 1990.

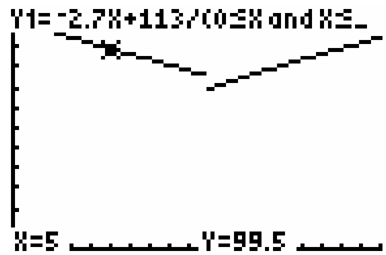
b. $y = 28.369x^{0.443}$, for years 2000 to 2009, where x is the years after 1990.

c. $y = \begin{cases} -2.7x + 113 & 0 \le x \le 10 \\ 28.369x^{0.443} & 10 < x \le 19 \end{cases}$

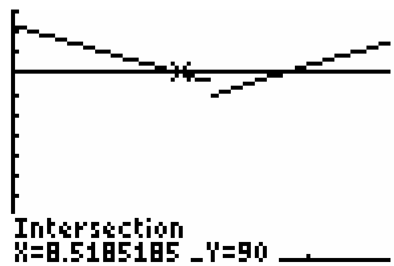


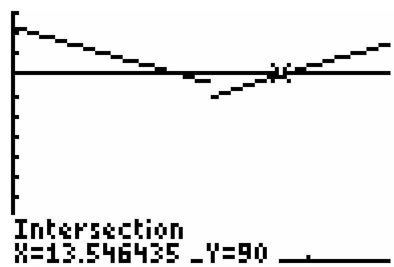
[0, 19] by [0, 120]

d. i. For the year 1995, $x = 5$, and as shown in the model, the number of thousands employed in 1995 is approximately 99.5 thousand.

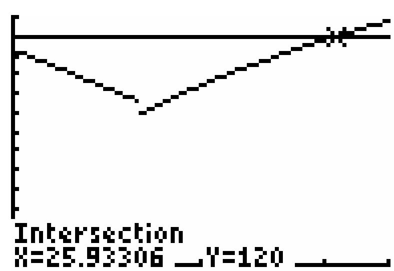


ii. When $y = 90$ in thousands, the model below indicates that x, the years after 1990, is approximately 9 and 14, that is, 1999 and 2004.





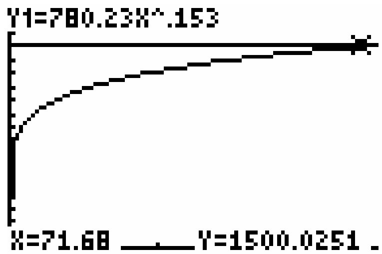
iii. By extending the calculator window to accommodate until the year 2020, assuming the model remains accurate that long, x is approximately 26, which represents the year 2016.



Copyright ©2013 Pearson Education. Inc.

58. a. $y = 780.23x^{0.153}$

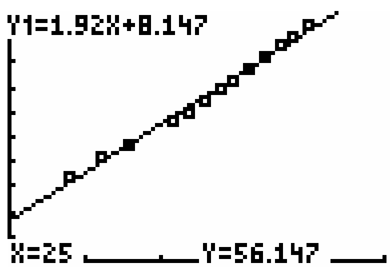
b.



[0, 75] by [0, 1800]

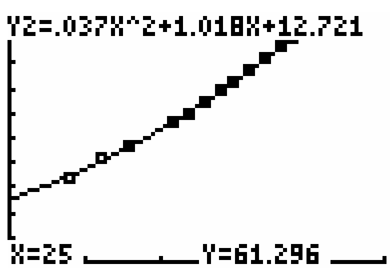
In 2052, 72 years after 1980, the Hawaiian population will rise to 1.5 million people.

59. a. $y = 1.92x + 8.147$, good fit to the data



[0, 25] by [0, 50]

b. $y = 0.037x^2 + 1.018x + 12.721$, better fit than the linear function to the data

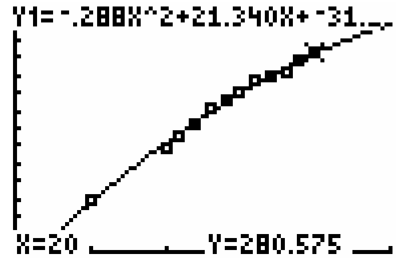


[0, 25] by [0, 50]

c. In the year 2015, $x = 25$. Thus,
$y = 1.92(25) + 8.147 = 56.147 = 56.1$
$y = 0.037(25)^2 + 1.018(25) + 12.721$
$= 61.296 = 61.3$

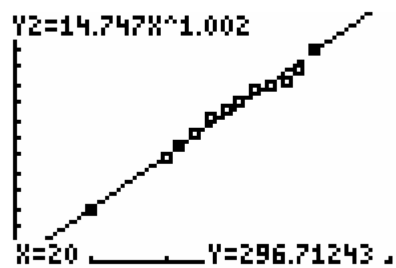
Each model predicts the amount of money Americans will spend on their pets in 2015 to be, 56.1 billion dollars and 61.3 billion dollars, respectively.

60. a. $y = -0.288x^2 + 21.340x - 31.025$



[0, 25] by [0, 350]

b. $y = 14.747x^{1.002}$



[0, 25] by [0, 350]

c. The power model is better, as the quadratic model will turn and begin decreasing.

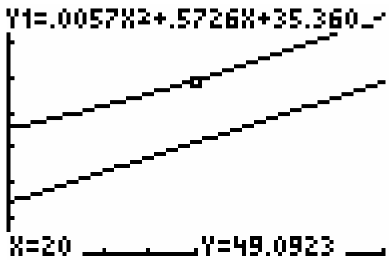
Copyright ©2013 Pearson Education. Inc.

Group Activity/Extended Application II

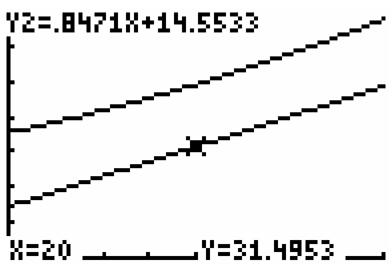
1. $f(x) = 0.0057x^2 + 0.5726x + 35.3603$

2. $g(x) = 0.8471x + 14.5533$

3.



[0, 40] by [0, 70]



[0, 40] by [0, 70]

From the graphs of these functions, it does not appear that they will intersect.

4.

Year	Female-to-Male Earnings Ratio
1970	0.594
1975	0.588
1980	0.602
1985	0.646
1990	0.716
1995	0.714
2000	0.737
2005	0.770
2006	0.769

5. $y = -0.00028x^2 + 0.0123x + 0.6503$

6.

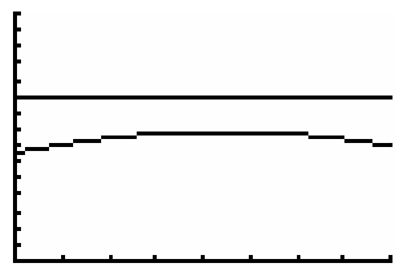
$y = -0.00028x^2 + 0.0123x + 0.6503$

$1.0000 = -0.00028x^2 + 0.0123x + 0.6503$

$0 = -0.00028x^2 + 0.0123x - 0.3497$

Using the quadratic formula, a negative result is found under the radical. Therefore there is no real solution to the equation, and one should conclude that median female earnings does not ever equal the median male earnings.

7.



[0, 40] by [0, 1.5]

8. Answers may vary.

Copyright ©2013 Pearson Education. Inc.