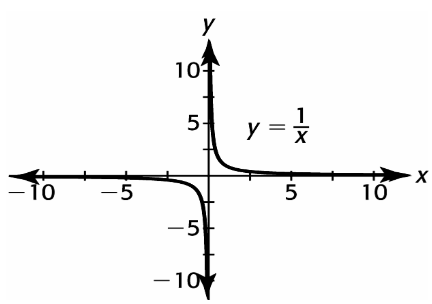
Chapter 4
Additional Topics with Functions

Toolbox Exercises

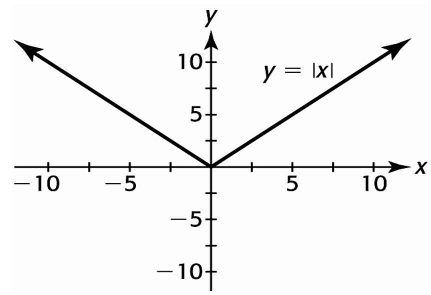
1. Since the reciprocal function has the form, $f(x)=\frac{1}{x}$, and since *x* cannot = 0, the domain of the reciprocal function is $(-\infty,0)\cup(0,\infty)$. In this function, *f(x)* also cannot = 0, so the range of the reciprocal function is also $(-\infty,0)\cup(0,\infty)$.

2. Since the constant function has the form, $g(x)=k$, there is no restriction on the value of *x* while the value of *g(x)* can only equal whatever the value of *k* is. Thus the domain of the constant function is $(-\infty,\infty)$ and the range is $\{k\}$.

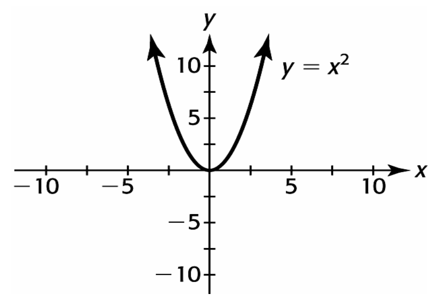
3. As shown in the model, there is no interval where the reciprocal function increases, so the function decreases on its entire domain, $(-\infty,0)\cup(0,\infty)$.



4. As shown in the model, the absolute value function increases on the interval $(0,\infty)$, and it decreases on the interval $(-\infty,0)$.

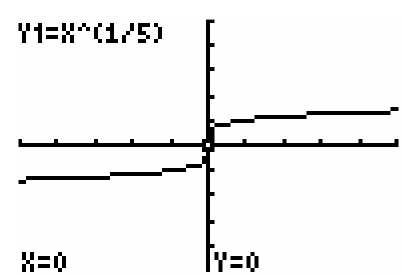


5. As shown in the model, the range of the squaring function is $[0,\infty)$.



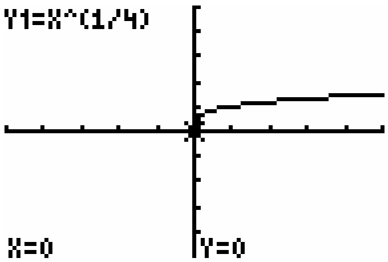
6. As shown in the model above, the domain of the squaring function is $(-\infty,\infty)$.

7. As shown in the model, the root function, $g(x)=\sqrt[5]{x}$, is increasing on the interval $(-\infty,\infty)$.



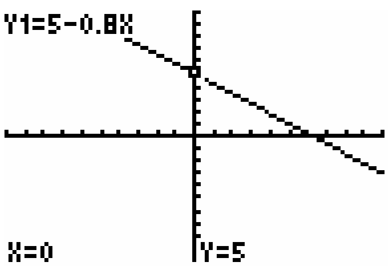
[–5, 5] by [–5, 5]

8. As shown in the model, the root function, $h(x) = \sqrt[4]{x}$, is increasing on the interval $[0, \infty)$.



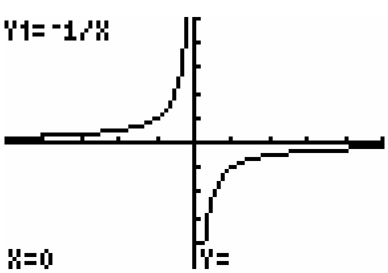
[–5, 5] by [–5, 5]

9. As shown in the model, the linear function, $f(x) = 5 - 0.8x$, is decreasing on the interval $(-\infty, \infty)$.



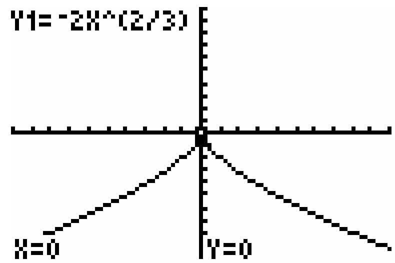
[–10, 10] by [–10, 10]

10. As shown in the model, the reciprocal function, $f(x) = \frac{-1}{x}$, is increasing on the interval $(-\infty, 0)$.



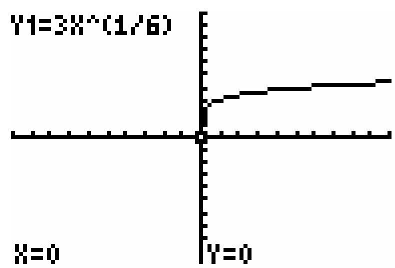
[–5, 5] by [–5, 5]

11. As shown in the model, the power function, $g(x) = -2x^{\frac{2}{3}}$, is decreasing on the interval $(0, \infty)$.



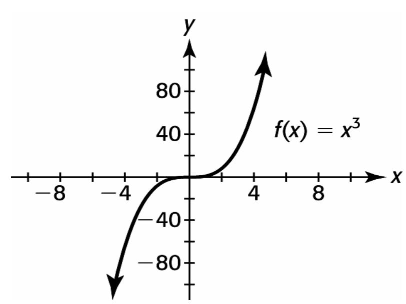
[–10, 10] by [–10, 10]

12. As shown in the model, the power function, $h(x) = 3x^{\frac{1}{6}}$, is increasing on the interval $[0, \infty)$.

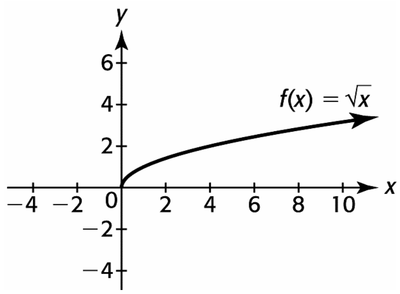


[–10, 10] by [–10, 10]

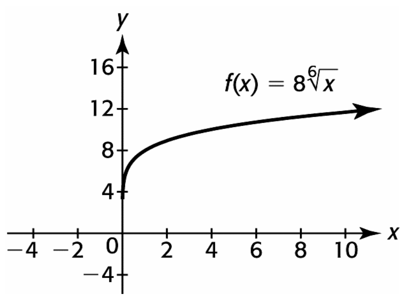
13. This is a cubing function.



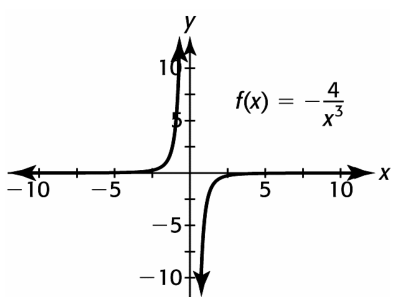
14. This is a square root function.



15. This is a power function, with power 1/6.



16. This is a power function, with power -3.



17. No, the graph would fail the vertical line test.

18. No, the graph would fail the vertical line test.

19. Yes, this is a function since it passes the vertical line test.

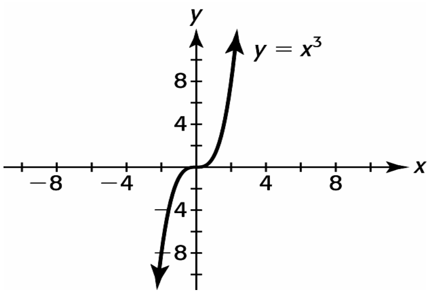
20. Yes, this is a function since it passes the vertical line test.

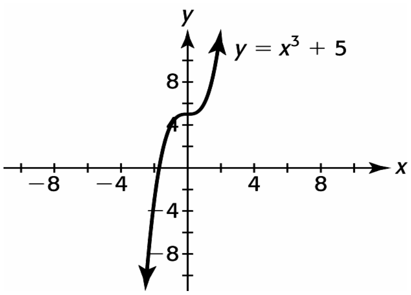
21. Yes, this is a function since it passes the vertical line test.

22. Yes, this is a function since it passes the vertical line test.

Section 4.1 Skills Check

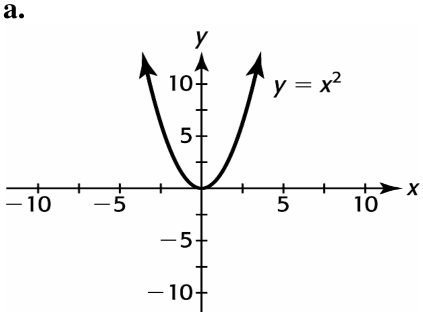
1. a.

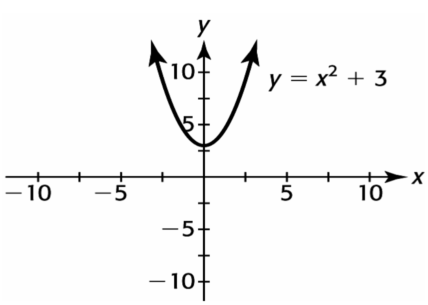




b. The graph of the function has a vertical shift 5 units up.

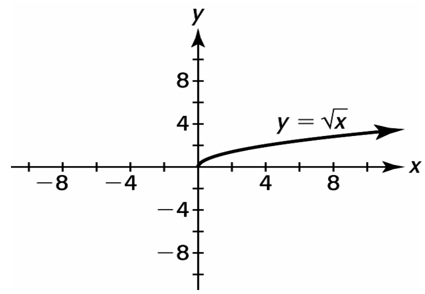
2. a.

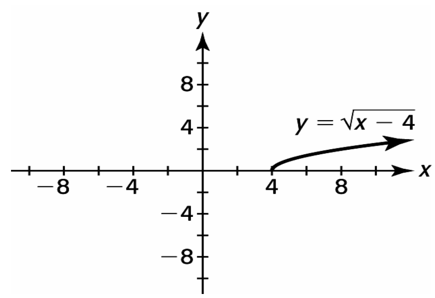




b. The graph of the function has a vertical shift 3 units up.

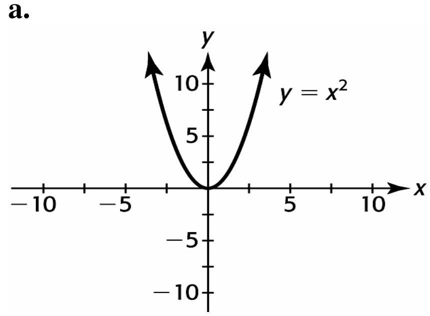
3. a.

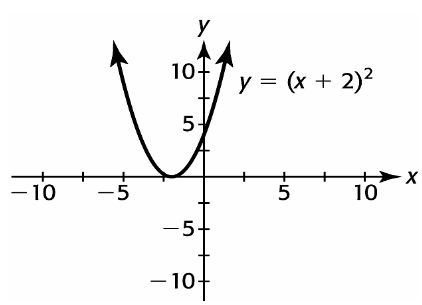




b. The graph of the function has a horizontal shift 4 units right.

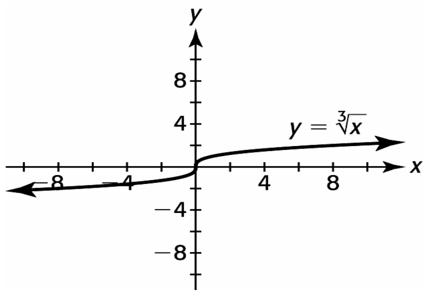
4. a.

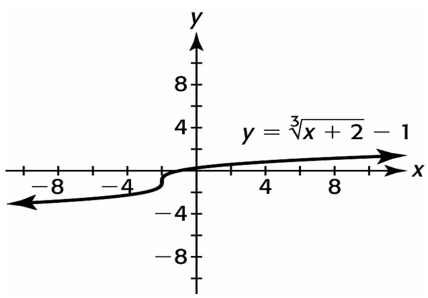




b. The graph of the function has a horizontal shift 2 units left.

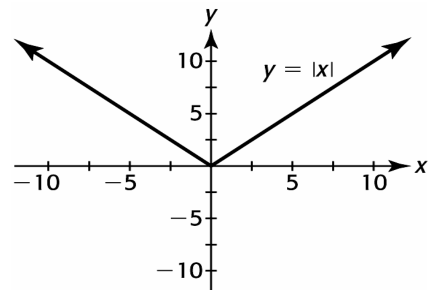
5. a.

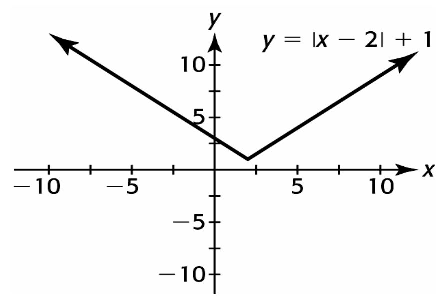




b. The graph of the function has a horizontal shift 2 units left and a vertical shift 1 unit down.

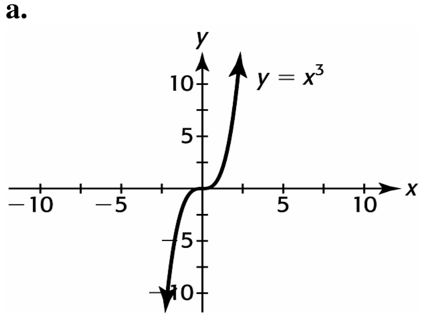
7. a.

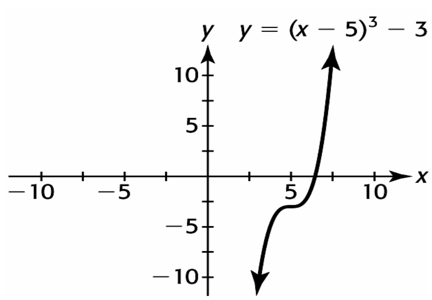




b. The graph of the function has a horizontal shift 2 units right and a vertical shift 1 unit up.

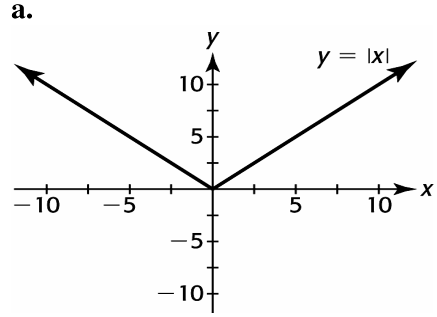
6. a.

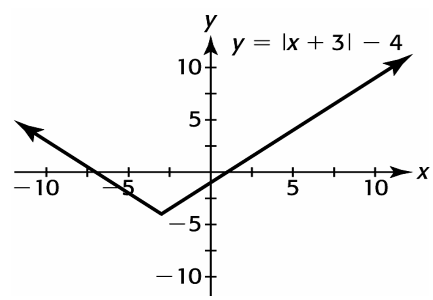




b. The graph of the function has a horizontal shift 5 units right and a vertical shift 3 units down.

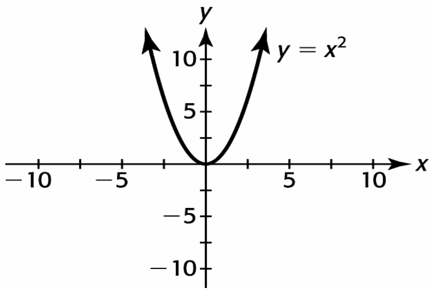
8. a.

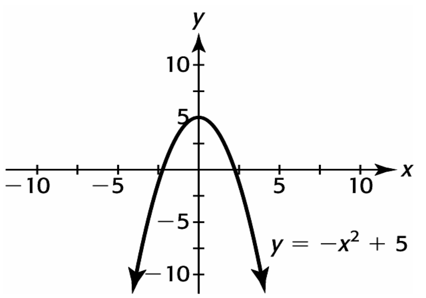




b. The graph of the function has a horizontal shift 3 units left and a vertical shift 4 units down.

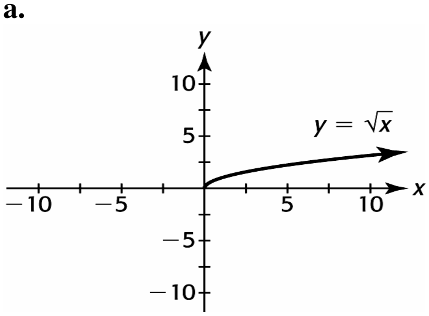
9. a.

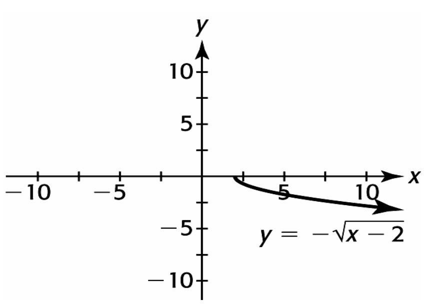




b. The graph of the function has a vertical reflection across the *x*-axis and then a vertical shift 5 units up.

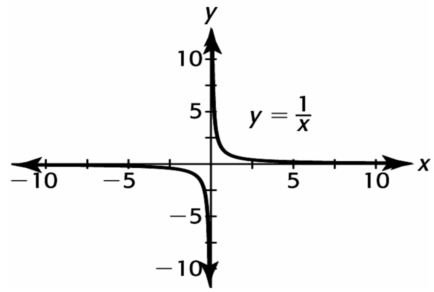
10. a.

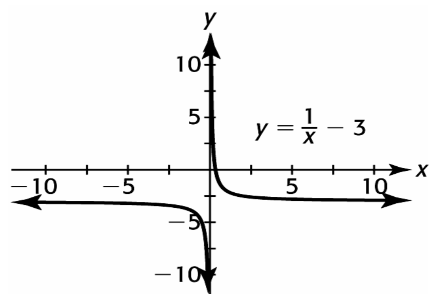




b. The graph of the function has a vertical reflection across the *x*-axis after a horizontal shift 2 units right.

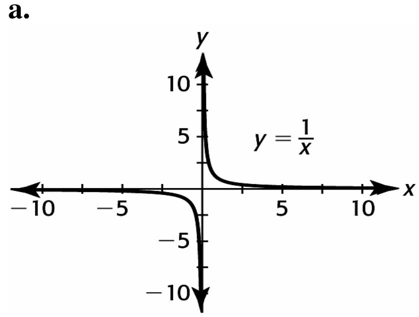
11. a.

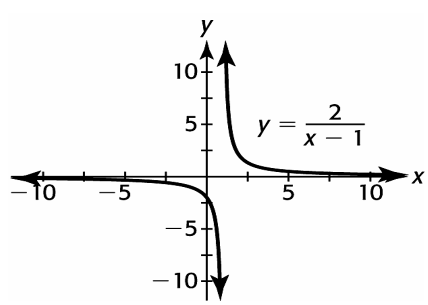




b. The graph of the function has a vertical shift 3 units down.

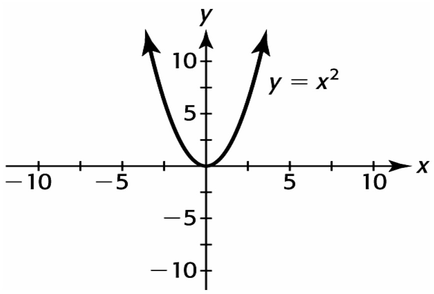
12. a.

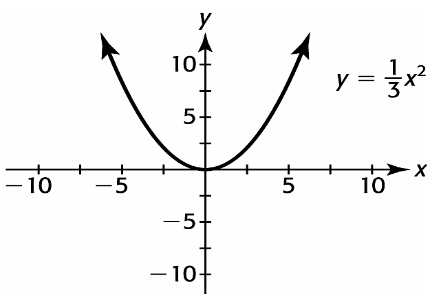




b. The graph of the function has a vertical stretch by a factor of 2 and a horizontal shift 1 unit right.

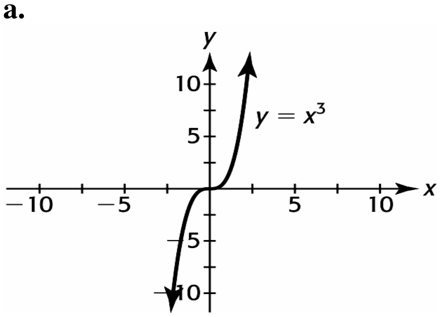
13. a.

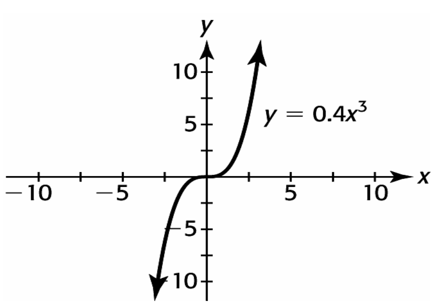




b. The graph of the function has a vertical compression by a factor of 1/3.

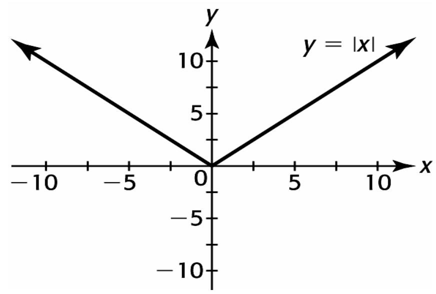
14. a.

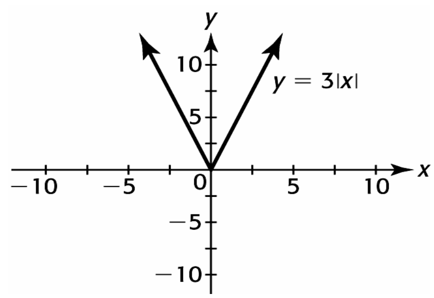




b. The graph of the function has a vertical compression by a factor of 0.4.

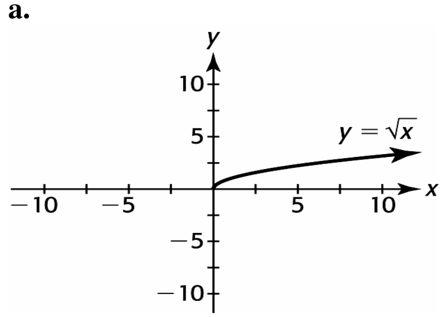
15. a.

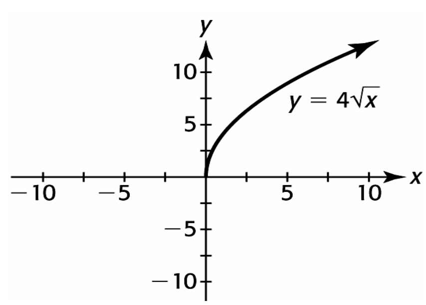




b. The graph of the function has a vertical stretch by a factor of 3.

16. a.





b. The graph of the function has a vertical stretch by a factor of 4.

17. The graph of the function is shifted 2 units right and 3 units up.

18. The graph of the function is shifted 4 units left and 2 units down.

19. $y = (x+4)^{\frac{3}{2}}$

20. $y = (x-4)^{\frac{3}{2}} - 5$

21. $y = 3x^{\frac{3}{2}} + 5$

22. $y = \frac{1}{5}(x-6)^{\frac{2}{3}}$

23. $g(x) = -x^2 + 2$

24. $g(x) = -x^2 - 1$

25. $g(x) = |x+3| - 2$

26. $g(x) = (x-4)^2 + 2$

27. The graph has y-axis symmetry.

28. The graph has x-axis and y-axis symmetry.

29. The graph has x-axis symmetry.

30. The graph has neither x-axis nor y-axis symmetry.

31. Yes, the graph would have y-axis symmetry.

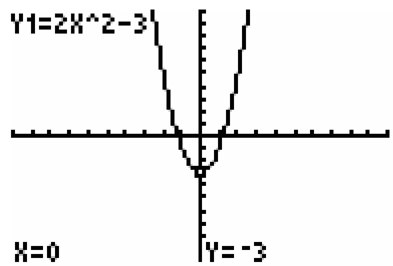
32. Yes, the graph would have y-axis symmetry.

33. ***y*-axis symmetry.**

Let $x = -x$

$y = 2(-x)^2 - 3 = 2x^2 - 3$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the y-axis.



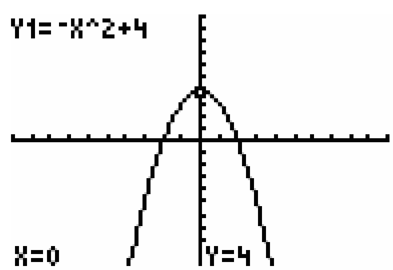
[–10, 10] by [–10, 10]

34. ***y*-axis symmetry.**

Let $x = -x$

$y = -(-x)^2 + 4 = -x^2 + 4$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the y-axis.



[–10, 10] by [–10, 10]

35. Origin symmetry.

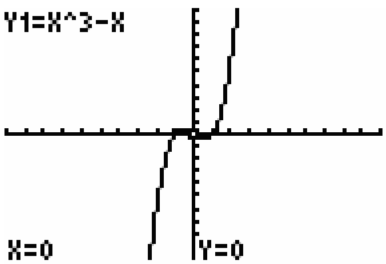
Let $x = -x$, $y = -y$

$-y = (-x)^3 - (-x)$

$-y = -x^3 + x$

$y = x^3 - x$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.



[–10, 10] by [–10, 10]

36. Origin symmetry.

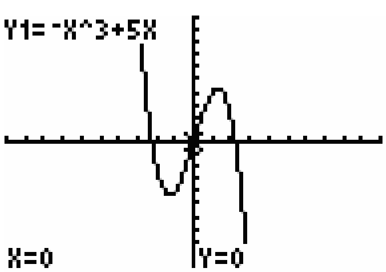
Let $x = -x$, $y = -y$

$-y = -(-x)^3 + 5(-x)$

$-y = x^3 - 5x$

$y = -x^3 + 5x$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.



[–10, 10] by [–10, 10]

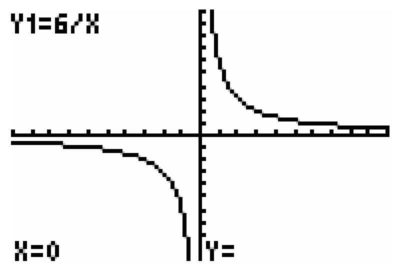
37. Origin symmetry.

Let $x = -x$, $y = -y$

$-y = \dfrac{6}{-x}$

$y = \dfrac{6}{x}$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.



[–10, 10] by [–10, 10]

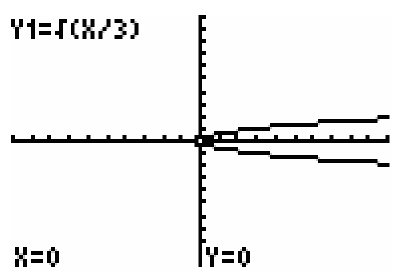
38. *x*-axis symmetry.

Let $y = -y$

$x = 3(-y)^2$

$x = 3y^2$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the *x*-axis. Also since the given equation is not a function, it can not be easily graphed using the graphing calculator. The equation must be rewritten as $y = \pm\sqrt{\dfrac{x}{3}}$



[–10, 10] by [–10, 10]

39. x-axis symmetry.

Let $y = -y$

$x^2 + (-y)^2 = 25$

$x^2 + y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the x-axis.

y-axis symmetry.

Let $x = -x$

$(-x)^2 + y^2 = 25$

$x^2 + y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the y-axis.

origin symmetry.

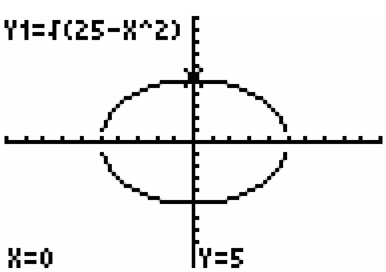
Let $x = -x, y = -y$

$(-x)^2 + (-y)^2 = 25$

$x^2 + y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.

Since the given equation is not a function, it can not be easily graphed using the graphing calculator. The equation must be rewritten as $y = \pm\sqrt{25 - x^2}$



[–10, 10] by [–10, 10]

40. x-axis symmetry.

Let $y = -y$

$x^2 - (-y)^2 = 25$

$x^2 - y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the x-axis.

y-axis symmetry.

Let $x = -x$

$(-x)^2 - y^2 = 25$

$x^2 - y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the y-axis.

origin symmetry.

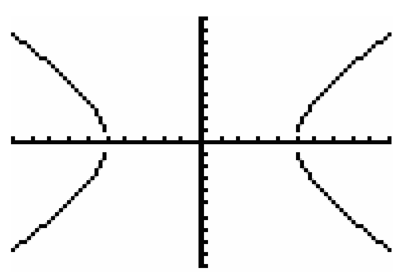
Let $x = -x, y = -y$

$(-x)^2 - (-y)^2 = 25$

$x^2 - y^2 = 25$

Since the result matches the original equation, the graph of the equation is symmetric with respect to the origin.

Since the given equation is not a function, it can not be easily graphed using the graphing calculator. The equation must be rewritten as $y = \pm\sqrt{x^2 - 25}$



[–10, 10] by [–10, 10]

41. $f(-x) = |-x| - 5$
$= |-1(x)| - 5$
$= |-1||x| - 5$
$= |x| - 5$
$= f(x)$

Since $f(-x) = f(x)$, the function has y-axis symmetry and is even.

42. $f(-x) = |(-x) - 2|$
$= |-x - 2|$
$= |-1(x + 2)|$
$= |x + 2|$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is neither even nor odd.

43. $g(-x) = \sqrt{(-x)^2 + 3}$
$= \sqrt{x^2 + 3}$

Since $g(-x) = g(x)$, the function has y-axis symmetry and is even.

44. $f(-x) = \frac{1}{2}(-x)^3 - (-x)$
$= -\frac{1}{2}x^3 + x$
$= -\left(\frac{1}{2}x^3 - x\right)$

Since $f(-x) = -f(x)$, the function has origin symmetry and is odd.

45. $g(-x) = \frac{5}{-x}$
$= -\frac{5}{x}$
$= -\left(\frac{5}{x}\right)$

Since $g(-x) = -g(x)$, the function has origin symmetry and is odd.

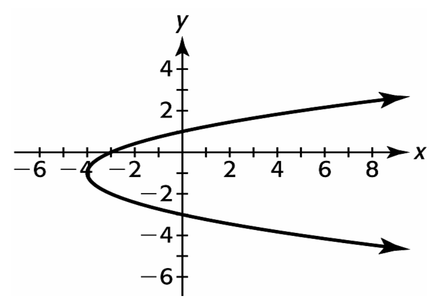
46. $g(-x) = 4(-x) + (-x)^2$
$= -4x + x^2$

Since $g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$, the function is neither even nor odd.

47. The graph has y-axis symmetry and is therefore even.

48. The graph has origin symmetry and is therefore odd.

49. Since this equation's graph would be a parabola opening to the side, it would not be a function at all.



50. $y = 3$ would be the line of symmetry.

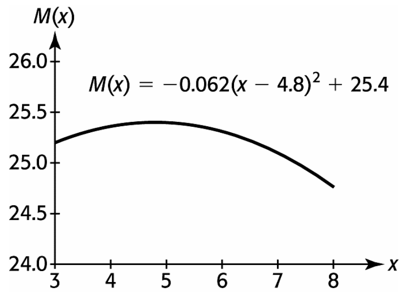
Section 4.1 Exercises

51. a. This function is a shifted graph of $M(x) = x^2$ $\quad (y = x^2)$

b. $M(3) = -0.062(3-4.8)^2 + 25.4$
$= -0.062(-1.8)^2 + 25.4$
$= 25.199$

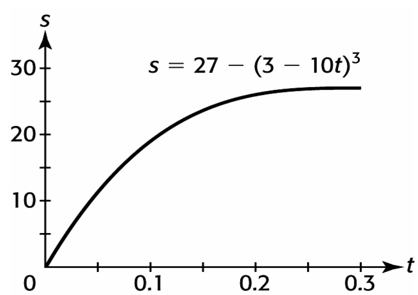
In 2003, 25.199 million (25,199,000) people 12 and older used marijuana.

c.



52. a. This function is a shifted graph of $s = t^3 \left(y = x^3\right)$

b.



c. $f(0.3) = 27 - (3 - 10(0.3))^3$
$= 27 - (3-3)^3$
$= 27 - 0$
$= 27$

After 0.3 seconds the bullet has traveled 27 inches.

53. a. Since q is in the numerator with an exponent of one, $p = \frac{180+q}{6}$ is a linear function.

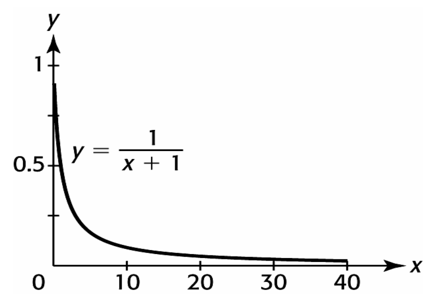
b. Since there is a variable in the denominator, $p = \frac{30{,}000}{q} - 20$ is a shift of the reciprocal function. It can be obtained from the basic reciprocal function by a vertical stretch by a factor of 30,000 and a shift 20 units down.

54. a. Since q is in the numerator and with an exponent of one, $p = 58 + \frac{q}{2}$ is a linear function.

b. Since there is a variable in the denominator, $p = \frac{2555}{q+5}$ is a shift of the reciprocal function. It can be obtained from the basic reciprocal function by a vertical stretch by a factor of 2555 and a shift five units to the left.

55. a. $y = \frac{1}{x}$. Shift one unit left.

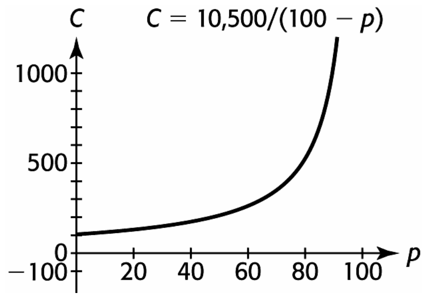
b.



c. The function is decreasing. Therefore, the amount of self-attentiveness of a person decreases as the size of the crowd increases.

56. a. $C = -10{,}500\left(\frac{1}{p-100}\right)$.
Shift 100 units right, reflect about the p-axis, and a vertical stretch by a factor of 10,500.

b.



c. $C = \dfrac{10{,}500}{100 - p}$

$= \dfrac{10{,}500}{100 - 80}$

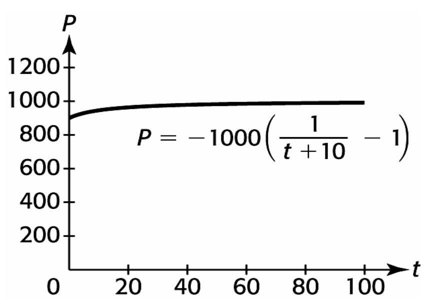
$= \dfrac{10{,}500}{20}$

$= \$525$

The daily cost of removing 80% of the pollution is \$525.

57. a. Shift 10 units left and one unit down. Reflect about the *x*-axis and a vertical stretch by a factor of 1000.

b.



58. a. Yes, this is a shifted root function.

b. $(0, 110]$

c. Shift 110 units right, reflect about the *y*-axis, and stretch vertically by a factor of 4700.

59. $C(x) = 306.472(x+5)^{0.464}$

60. $V(x) = 0.084(x-10)^{0.675}$

61. Since in the given function *x* represents the years after 1980, $x+10$ represents the years after 1990. Therefore, the new function would be

$$T(x) = -13.898(x+10)^2 + 255.467(x+10) + 5425.618$$

62. Since in the given function *x* represents the years after 2000, $x-10$ represents the years after 1990. Therefore, the new function would be

$$A(x) = -17.560(x-10)^2 + 126.488(x-10) + 7986.786$$

63. a. $t = 2005 - 1980 = 25$

$S(25) = 0.00056(25)^4$

≈ 218.75

The number of subscribers in 2005 was approximately 218.75 million.

b. Since in the given function t represents the years after 1980, $t+5$ represents the years after 1985. Therefore, the new function would be

$C(t) = 0.00056(t+5)^4$

c. $t = 2005 - 1985 = 20$

$C(20) = 0.00056(20+5)^4$

$= 0.00056(25)^4 = 218.75$

Using the shifted model yields the same result as part a) above.

64. a. $C(p)=\dfrac{120,000}{p}-1200$

$C(100)=\dfrac{120,000}{100}-1200$
$=1200-1200$
$=0$

Completely impure water is free!

b. $C(p)=\dfrac{120,000}{p}-1200$

$C(50)=\dfrac{120,000}{50}-1200$
$=2400-1200$
$=1200$

The cost of drinking water that is 50% impure is $1200.

c. It has a vertical stretch by a factor of 120,000 and a shift down 1200 units.

65. a. $t=2013-1970=43$

$P(43)=638.57(43)^{0.775}$
$\approx 11,780$

The poverty threshold in 2013 is approximately $11,780.

b. Since in the given function t represents the years since 1970, $t+20$ represents the years since 1990. Therefore, the new function would be
$S(t)=638.57(t+20)^{0.775}$

c. $t=2013-1990=23$

$S(43)\ =638.57(23+20)^{0.775}$
$=638.57(43)^{0.775}\approx 11,780$

Using the shifted model yields the same result.

66. a. $100C-Cp=10,500$

$C(100-p)=10,500$

$C=\dfrac{10,500}{100-p}$

$C(p)=\dfrac{10,500}{100-p}$

b. $C(50)=\dfrac{10,500}{100-50}=\dfrac{10,500}{50}=210$

The daily cost of removing 50% of the pollution is $210.

c. $C(99)=\dfrac{10,500}{100-99}=10,500$

The daily cost of removing 99% of the pollution is $10,500. The company would likely resist such a high daily cost.

Section 4.2 Skills Check

1. a. $(f+g)(x)$
$= f(x)+g(x)$
$=(3x-5)+(4-x)$
$=2x-1$

b. $(f-g)(x)$
$= f(x)-g(x)$
$=(3x-5)-(4-x)$
$=3x-5-4+x$
$=4x-9$

c. $(f \bullet g)(x)$
$= f(x)\bullet g(x)$
$=(3x-5)(4-x)$
$=-3x^2+17x-20$

d. $\left(\dfrac{f}{g}\right)(x)$
$=\dfrac{f(x)}{g(x)}$
$=\dfrac{3x-5}{4-x}$

e. $g(x)\neq 0$
$4-x=0$
$x=4$
Domain: $(-\infty,4)\cup(4,\infty)$

2. a. $(f+g)(x)$
$= f(x)+g(x)$
$=(2x-3)+(5-x)$
$=x+2$

b. $(f-g)(x)$
$= f(x)-g(x)$
$=(2x-3)-(5-x)$
$=2x-3-5+x$
$=3x-8$

c. $(f \bullet g)(x)$
$= f(x)\bullet g(x)$
$=(2x-3)(5-x)$
$=-2x^2+13x-15$

d. $\left(\dfrac{f}{g}\right)(x)$
$=\dfrac{f(x)}{g(x)}$
$=\dfrac{2x-3}{5-x}$

e. $g(x)\neq 0$
$5-x=0$
$x=5$
Domain: $(-\infty,5)\cup(5,\infty)$

3. a. $(f+g)(x)$
$= f(x)+g(x)$
$=\left(x^2-2x\right)+(1+x)$
$=x^2-x+1$

b. $(f-g)(x)$
$= f(x)-g(x)$
$=\left(x^2-2x\right)-(1+x)$
$=x^2-2x-1-x$
$=x^2-3x-1$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= (x^2 - 2x)(1 + x)$
$= x^3 - x^2 - 2x$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{x^2 - 2x}{1 + x}$

e. $g(x) \neq 0$
$1 + x = 0$
$x = -1$
Domain: $(-\infty, -1) \cup (-1, \infty)$

4. a. $(f + g)(x)$
$= f(x) + g(x)$
$= (2x^2 - x) + (2x + 1)$
$= 2x^2 + x + 1$

b. $(f - g)(x)$
$= f(x) - g(x)$
$= (2x^2 - x) - (2x + 1)$
$= 2x^2 - x - 2x - 1$
$= 2x^2 - 3x - 1$

c. $(f \bullet g)(x)$
$= f(x) \bullet g(x)$
$= (2x^2 - x)(2x + 1)$
$= 4x^3 - x$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{2x^2 - x}{2x + 1} = \frac{x(2x - 1)}{2x + 1}$

e. $g(x) \neq 0$
$2x + 1 = 0$
$x = -\frac{1}{2}$
Domain: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

5. a.
$(f + g)(x)$
$= f(x) + g(x)$
$= \left(\frac{1}{x}\right) + \left(\frac{x + 1}{5}\right)$
$LCD: 5x$
$= \frac{5}{5}\left(\frac{1}{x}\right) + \frac{x}{x}\left(\frac{x + 1}{5}\right)$
$= \left(\frac{5}{5x}\right) + \left(\frac{x^2 + x}{5x}\right)$
$= \frac{x^2 + x + 5}{5x}$

b.
$(f - g)(x)$
$= f(x) - g(x)$
$= \left(\frac{1}{x}\right) - \left(\frac{x + 1}{5}\right)$
$LCD: 5x$
$= \frac{5}{5}\left(\frac{1}{x}\right) - \frac{x}{x}\left(\frac{x + 1}{5}\right)$
$= \left(\frac{5}{5x}\right) - \left(\frac{x^2 + x}{5x}\right)$
$= \frac{-x^2 - x + 5}{5x}$

c. $(f \bullet g)(x)$

$= f(x) \bullet g(x)$

$= \left(\frac{1}{x}\right)\left(\frac{x+1}{5}\right)$

$= \frac{x+1}{5x}$

d. $\left(\frac{f}{g}\right)(x)$

$= \frac{f(x)}{g(x)}$

$= \frac{\frac{1}{x}}{\frac{x+1}{5}}$

$= \frac{1}{x} \bullet \frac{5}{x+1}$

$= \frac{5}{x(x+1)}$

e. $g(x) \neq 0$

$x(x+1) = 0$

$x = 0, -1$

Domain: $(-\infty,-1) \cup (-1,0) \cup (0,\infty)$

6. a.

$(f+g)(x)$

$= f(x) + g(x)$

$= \left(\frac{x-2}{3}\right) + \left(\frac{1}{x}\right)$

$LCD: 3x$

$= \frac{x}{x}\left(\frac{x-2}{3}\right) + \frac{3}{3}\left(\frac{1}{x}\right)$

$= \left(\frac{x^2-2x}{3x}\right) + \left(\frac{3}{3x}\right)$

$= \frac{x^2-2x+3}{3x}$

b.

$(f-g)(x)$

$= f(x) - g(x)$

$= \left(\frac{x-2}{3}\right) - \left(\frac{1}{x}\right)$

$LCD: 3x$

$= \frac{x}{x}\left(\frac{x-2}{3}\right) - \frac{3}{3}\left(\frac{1}{x}\right)$

$= \left(\frac{x^2-2x}{3x}\right) - \left(\frac{3}{3x}\right)$

$= \frac{x^2-2x-3}{3x}$

c. $(f \bullet g)(x)$

$= f(x) \bullet g(x)$

$= \left(\frac{x-2}{3}\right)\left(\frac{1}{x}\right)$

$= \frac{x-2}{3x}$

d. $\left(\frac{f}{g}\right)(x)$

$= \frac{f(x)}{g(x)}$

$= \frac{\frac{x-2}{3}}{\frac{1}{x}}$

$= \frac{x-2}{3} \bullet \frac{x}{1}$

$= \frac{x^2-2x}{3}$

e. $g(x) \neq 0$

$3 = 0$ is impossible.

Note that $g(x)$ is undefined when $x = 0$. Therefore the domain of the composition is not all real numbers.

Domain: $(-\infty,0) \cup (0,\infty)$

7. a. $(f+g)(x)$
$= f(x)+g(x)$
$= (\sqrt{x})+(1-x^2)$
$= \sqrt{x}+1-x^2$

b. $(f-g)(x)$
$= f(x)-g(x)$
$= (\sqrt{x})-(1-x^2)$
$= \sqrt{x}-1+x^2$

c. $(f \cdot g)(x)$
$= f(x) \cdot g(x)$
$= (\sqrt{x})(1-x^2)$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{\sqrt{x}}{1-x^2}$

e. $g(x) \neq 0$
$1-x^2 \neq 0$
$x \neq -1, 1$
And in $f(x)=\sqrt{x},\ x \geq 0$
Domain: $[0,1) \cup (1,\infty)$

8. a. $(f+g)(x)$
$= f(x)+g(x)$
$= (x^3)+(\sqrt{x+3})$
$= x^3+\sqrt{x+3}$

b. $(f-g)(x)$
$f(x)-g(x)$
$(x^3)-(\sqrt{x+3})$
$x^3-\sqrt{x+3}$

c. $(f \cdot g)(x)$
$= f(x) \cdot g(x)$
$= (x^3)(\sqrt{x+3})$

d. $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{x^3}{\sqrt{x+3}}$

e. $g(x) \neq 0$
$x+3=0$
$x=-3$
Additionally, because of the square root in the denominator
$x+3>0$
$x>-3$
Domain: $(-3,\infty)$

9. a. $(f+g)(2)$
$= f(2)+g(2)$
$= (2^2-5(2))+(6-(2)^3)$
$= -6-2$
$= -8$

b. $(g-f)(-1)$
$= g(-1)-f(-1)$
$= (6-(-1)^3)-((-1)^2-5(-1))$
$= 7-6$
$= 1$

c. $(f \cdot g)(-2)$
$= f(-2) \cdot g(-2)$
$= \left((-2)^2 - 5(-2)\right) \cdot \left(6 - (-2)^3\right)$
$= (14)(14)$
$= 196$

d. $\left(\frac{g}{f}\right)(3)$
$= \frac{g(3)}{f(3)}$
$= \frac{\left(6-(3)^3\right)}{\left(3^2 - 5(3)\right)}$
$= \frac{-21}{-6}$
$= 3.5$

10. a. $(f+g)(1)$
$= f(1) + g(1)$
$= \left(4-(1)^2\right) + \left((1)^3 + (1)\right)$
$= 3 + 2$
$= 5$

b. $(f-g)(-2)$
$= f(-2) - g(-2)$
$= \left(4-(-2)^2\right) - \left((-2)^3 + (-2)\right)$
$= 0 - (-10)$
$= 10$

c. $(f \cdot g)(-3)$
$= f(-3) \cdot g(-3)$
$= \left(4-(-3)^2\right)\left((-3)^3 + (-3)\right)$
$= (-5)(-30)$
$= 150$

d. $\left(\frac{g}{f}\right)(2)$
$= \frac{g(2)}{f(2)}$
$= \frac{\left((2)^3 + (2)\right)}{\left(4-(2)^2\right)}$
$= \frac{10}{0}$
Undefined expression

11. a. $(f \circ g)(x)$
$= f(g(x))$
$= 2(3x-1) - 6$
$= 6x - 8$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 3(2x-6) - 1$
$= 6x - 19$

12. a. $(f \circ g)(x)$
$= f(g(x))$
$= 3(2x-2) - 2$
$= 6x - 8$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 2(3x-2) - 2$
$= 6x - 6$

13. a. $(f \circ g)(x)$
$= f(g(x))$
$= \left(\frac{1}{x}\right)^2$
$= \frac{1}{x^2}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \frac{1}{x^2}$

14. a. $(f \circ g)(x)$
$= f(g(x))$
$= \left(\frac{2}{x}\right)^3$
$= \frac{8}{x^3}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \frac{2}{x^3}$

15. a. $(f \circ g)(x)$
$= f(g(x))$
$= \sqrt{(2x-7)-1}$
$= \sqrt{2x-8}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 2\left(\sqrt{x-1}\right)-7$
$= 2\sqrt{x-1}-7$

16. a. $(f \circ g)(x)$
$= f(g(x))$
$= \sqrt{3-(x-5)}$
$= \sqrt{8-x}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \left(\sqrt{3-x}\right)-5$
$= \sqrt{3-x}-5$

17. a. $(f \circ g)(x)$
$= f(g(x))$
$= |(4x)-3|$
$= |4x-3|$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 4(|x-3|)$
$= 4|x-3|$

18. a. $(f \circ g)(x)$
$= f(g(x))$
$= |4-(2x+1)|$
$= |3-2x|$

b. $(g \circ f)(x)$
$= g(f(x))$
$= 2(|4-x|)+1$
$= 2|4-x|+1$

19. a. $(f \circ g)(x)$
$= f(g(x))$
$= \dfrac{3\left(\dfrac{2x-1}{3}\right)+1}{2}$
$= \dfrac{2x-1+1}{2}$
$= \dfrac{2x}{2}$
$= x$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \dfrac{2\left(\dfrac{3x+1}{2}\right)-1}{3}$
$= \dfrac{3x+1-1}{3}$
$= \dfrac{3x}{3}$
$= x$

20. a. $(f \circ g)(x)$
$= f(g(x))$
$= \sqrt[3]{(x^3+1)+1}$
$= \sqrt[3]{x^3+2}$

b. $(g \circ f)(x)$
$= g(f(x))$
$= \left(\sqrt[3]{x+1}\right)^3+1$
$= x+1+1$
$= x+2$

21. a. $f(g(2)) = 2\left(\dfrac{2-5}{3}\right)^2 = 2(-1)^2 = 2$

b. $g(f(-2)) = \dfrac{\left[2(-2)^2\right]-5}{3}$
$= \dfrac{8-5}{3}$
$= \dfrac{3}{3}$
$= 1$

22. a. $f(g(2)) = ((3(2)-1)-1)^2$
$= (5-1)^2$
$= 4^2$
$= 16$

b. $g(f(-2)) = 3\left([-2-1]^2\right)-1$
$= 3\left([-3]^2\right)-1$
$= 3(9)-1$
$= 26$

23. a. $(f+g)(2) = f(2)+g(2) = 1+(-3) = -2$

b. $(f \circ g)(-1) = f(g(-1)) = f(0) = -1$

c. $\left(\dfrac{f}{g}\right)(4) = \dfrac{f(4)}{g(4)} = \dfrac{3}{-1} = -3$

d. $(f \circ g)(1) = f(g(1)) = f(-2) = -3$

e. $(g \circ f)(-2) = g(f(-2)) = g(-3) = 2$

24. a. $(g-f)(-2) = g(-2)-f(-2)$
$= 1-(-3)$
$= 4$

b. $(f \circ g)(3) = f(g(3)) = f(-2) = -3$

c. $\left(\frac{f}{g}\right)(0)=\frac{f(0)}{g(0)}=\frac{-1}{-1}=1$

d. $(f\circ g)(-2)=f(g(-2))=f(1)=0$

e. $(g\circ f)(2)=g(f(2))=g(1)=-2$

Section 4.2 Exercises

25. a. $$\begin{aligned}P(x)&=R(x)-C(x)\\&=(89x)-(23x+3420)\\&=89x-23x-3420\\&=66x-3420\end{aligned}$$

b. $P(150)=66(150)-3420=6480$
The profit on the production and sale of 150 bicycles is $6480.

26. a. $$\begin{aligned}P(x)&=R(x)-C(x)\\&=(988x)-(189x+5460)\\&=988x-189x-5460\\&=799x-5460\end{aligned}$$

b. $P(80)=799(80)-5460=58,460$
The profit on the production and sale of 80 televisions is $58,460.

27. a. The revenue function is linear, while the cost function is quadratic. Note that *C(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

b. $P(x)=R(x)-C(x)$
$P(x)=1050x-(10,000+30x+x^2)$
$P(x)=1050x-10,000-30x-x^2$
$P(x)=-x^2+1020x-10,000$

c. Quadratic. Note that *P(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

28. a. The revenue function is linear, while the cost function is quadratic. Note that *C(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

b. $$\begin{aligned}&P(x)\\&=R(x)-C(x)\\&=26,600x-(200,000+4600x+2x^2)\\&=26,600x-200,000-4600x-2x^2\\&=-2x^2+22,000x-200,000\end{aligned}$$

c. Quadratic. Note that *P(x)* fits the form
$f(x)=ax^2+bx+c, a\neq 0.$

29. a. $P(x)=R(x)-C(x)$
$P(x)=550x-(10,000+30x+x^2)$
$P(x)=550x-10,000-30x-x^2$
$P(x)=-x^2+520x-10,000$

b. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.
$x=\frac{-b}{2a}=\frac{-520}{2(-1)}=\frac{-520}{-2}=260$

c. $$\begin{aligned}y&=P(x)\\&=P(260)\\&=-(260)^2+520(260)-10,000\\&=-67,600+135,200-10,000\\&=57,600\end{aligned}$$

Producing and selling 260 cameras yields a maximum profit of $57,600.

30. a. $P(x)=R(x)-C(x)$
$P(x)=6600x-(2000+4800x+2x^2)$
$P(x)=6600x-2000-4800x-2x^2$
$P(x)=-2x^2+1800x-2000$

b. Note that the maximum profit occurs at the vertex of the quadratic function, since the function is concave down.

$$x = \frac{-b}{2a} = \frac{-1800}{2(-2)} = \frac{-1800}{-4} = 450$$

c.
$$\begin{aligned} y &= P(x) \\ &= P(450) \\ &= -2(450)^2 + 1800(450) - 2000 \\ &= -405{,}000 + 810{,}000 - 2000 \\ &= 403{,}000 \end{aligned}$$

Producing and selling 450 camcorders yields a maximum profit of \$403,000.

31. a. $\overline{C}(x)$ fits the form $\left(\frac{f}{g}\right)(x)$ where $f(x) = C(x)$ and $g(x) = x$. Note that $\overline{C}(x) = \frac{C(x)}{x}$.

b. Let $x = 3000$ and calculate $\overline{C}(x)$.

$$\begin{aligned} \overline{C}(3000) &= \frac{50{,}000 + 105(3000)}{3000} \\ &= \frac{365{,}000}{3000} \\ &= 121.\overline{6} \approx \$121.67 \text{ per set} \end{aligned}$$

32. a. $C(p)$ fits the form $(f - g)(p)$ where $f(p) = \frac{120{,}000}{p}$ and $g(p) = 1200$.

b.
$$C(p) = \frac{120{,}000}{p} - 1200$$
$$\begin{aligned} C(80) &= \frac{120{,}000}{80} - 1200 \\ &= 1500 - 1200 = 300 \end{aligned}$$

The cost of water that is 80% impure is \$300.

33. a. $\overline{C}(x) = \frac{C(x)}{x} = \frac{3000 + 72x}{x}$

b.
$$\begin{aligned} \overline{C}(100) &= \frac{C(100)}{100} \\ &= \frac{3000 + 72(100)}{100} \\ &= \frac{3000 + 7200}{100} \\ &= \frac{10{,}200}{100} \\ &= 102 \text{ or } \$102 \text{ per printer} \end{aligned}$$

34. a. $\overline{C}(x) = \frac{C(x)}{x} = \frac{2.15x + 2350}{x}$

b.
$$\begin{aligned} \overline{C}(100) &= \frac{2.15(100) + 2350}{100} \\ &= \frac{215 + 2350}{100} \\ &= \frac{2565}{100} = 25.65 \end{aligned}$$

The average cost for the production of 100 components is \$25.65 per component.

35. a. Let $T(p)$ represent the total number of tickets for a home football game.

$$\begin{aligned} T(p) &= S(p) + N(p) \\ &= (62p + 8500) + (0.5p^2 + 16p + 4400) \\ &= 0.5p^2 + 78p + 12{,}900 \end{aligned}$$

b. Since p represents the winning percentage for the football team, $0 \le p \le 100$. Therefore the domain of the function is $[0, 100]$.

c.
$$\begin{aligned} T(90) &= 0.5(90)^2 + 78(90) + 12{,}900 \\ &= 4050 + 7020 + 12{,}900 \\ &= 23{,}970 \end{aligned}$$

The stadium holds 23,970 people.

36. $(T \cdot P)(c) = T(c) \cdot P(c)$
The function represents the number of T-shirts sold multiplied by the price per shirt. The result is the revenue for selling shirts with *c* colors.

37. a. $B(8) = 6(8+1)^{\frac{3}{2}} = 6(27) = 162$
On May 8th the number of bushels of tomatoes harvested was 162.

b. $P(8) = 8.5 - 0.12(8) = 7.54$
On May 8th the price per bushel of tomatoes was \$7.54.

c. $(B \cdot P)(x)$ represents the worth of the tomatoes on the x^{th} day of May.

$$\begin{aligned}(B \cdot P)(8) &= B(8) \cdot P(8)\\ &= 162 \cdot 7.54\\ &= 1221.48\end{aligned}$$

On May 8th the worth was \$1221.48.

d.
$$\begin{aligned}W(x) &= (B \cdot P)(x)\\ &= B(x) \cdot P(x)\\ &= \left[6(x+1)^{\frac{3}{2}}\right] \cdot (8.5 - 0.12x)\\ &= 6(x+1)^{\frac{3}{2}}(8.5 - 0.12x)\end{aligned}$$

38. $C(x) = 3000 + 3.30x^2$

39. a.
$$\begin{aligned}P(x) &= R(x) - C(x)\\ &= (592x) - (32{,}000 + 432x)\\ &= 592x - 32{,}000 - 432x\\ &= 160x - 32{,}000\end{aligned}$$

b. $P(600) = 160(600) - 32{,}000 = 64{,}000$
The profit for producing and selling 600 satellite systems in one month is \$64,000.

c. Since the function is linear, the rate of change is constant. For every one unit increase in production, the profit increases by \$160 per unit.

40. a.
$$\begin{aligned}P(x) &= R(x) - C(x)\\ &= (295x) - (87{,}500 + 87x)\\ &= 295x - 87{,}500 - 87x\\ &= 208x - 87{,}500\end{aligned}$$

b. $P(700) = 208(700) - 87{,}500 = 58{,}100$
The profit for producing and selling 700 computers in one month is \$58,100.

c. $(0, -87{,}500)$. It represents the fixed costs for manufacturing and selling the computers. If no computers are manufactured and sold, the company will lose \$87,500 per month.

41. a. No. Adding the percentages is not valid since the percentages are based on different populations of people. Adding the number of males and the number of females completing college and then dividing by the total is a legitimate approach.

b. Based on results using the Table feature of the TI-83 calculator, the percentage in 2002 is 64.78%, and the percentage in 2008 is 67.76 %.

X	Y1
3	64.778
4	65.547
5	66.149
6	66.645
7	67.068
8	67.436
9	67.763

X=3

c.
$$2002 \Rightarrow \frac{62.1 + 68.4}{2} = 65.25$$
$$2008 \Rightarrow \frac{65.9 + 71.6}{2} = 68.75$$

The percentages are relatively close but not the same.

42. a. Let $P(t)$ represent the total U.S. population age 5 and under in millions.
Then, $P(t) = B(t) + G(t)$

$$P(t) = \left(0.0076t^2 - 0.1752t + 10.705\right) + \left(0.0064t^2 - 0.1448t + 10.12\right)$$
$$P(t) = 0.014t^2 - 0.32t + 20.825$$

b. $P(2003-1990)$
$= P(13)$
$= 0.014(13)^2 - 0.32(13) + 20.825$
$= 2.366 - 4.16 + 20.825$
$= 19.031$

The model predicts that in 2003 there are 19.03 million children age 5 and under.

43. For a)–d), consider the output from the function.

a. Meat put in a container.

b. Meat ground.

c. Meat ground and then ground again.

d. Meat ground and put in a container.

e. Meat put in a container, and then both ground.

f. Only part d), unless the reader enjoys ground styrofoam!

44. a. A sock is placed on the left foot.

b. A sock is placed on the left foot, followed by a second sock on the left foot. The result is two socks on the left foot.

c. $(g \circ f)(\text{right foot})$
$= g\left(f(\text{right foot})\right)$
$= g(\text{sock on right foot})$
$= \text{sock on right foot removed}$

A sock is placed on the right foot and then removed. The net result is that there is no sock on the right foot.

45. Let $B(x)$ convert a Japanese shoe size, x, into a British shoe size, $B(x)$.
Japanese $\to$ U.S. $\to$ British

$B(x) = (p \circ s)(x)$
$= p\left(s(x)\right)$
$= (x - 17) - 1.5$
$= x - 18.5$

46. Let $C(x)$ convert a British shoe size, x, into a Continental shoe size, $C(x)$.
British $\to$ U.S. $\to$ Continental

$C(x) = (t \circ d)(x)$
$= t\left(d(x)\right)$
$= (x + 0.5) + 34.5$
$= x + 35$

47.
Chilean pesos $\to$ Japanese yen $\to$ Russian rubles

Let $R(x) = 0.34954x$, where x is yen and $R(x)$ is rubles.
Let $S(x) = 0.171718x$, where x is pesos and $S(x)$ is yen.
Let $V(x) = (R \circ S)(x)$, where x is pesos and $V(x)$ is rubles.

$V(x) = R\left(S(x)\right)$
$= 0.34954(0.171718x)$
$= 0.060022x$

$V(100) = 0.060022(100)$
$= 6 \text{ rubles}$

48.
Mexican pesos $\rightarrow$
Euros $\rightarrow$ U.S. dollars

Let $E(x)=1.3773x,$ where x is Mexican pesos and $E(x)$ is euros.
Let $U(x)=0.06047x,$ where x is euros and $U(x)$ is U.S. dollars.
Let $V(x)=(U\circ E)(x),$ where x is Mexican pesos and $V(x)$ is U.S. dollars.

$$V(x)=U\left(E(x)\right)=0.06047(1.3773x)=0.083285x$$

$$V(100)=0.083285(100)=\$8.33\text{ U.S.}$$

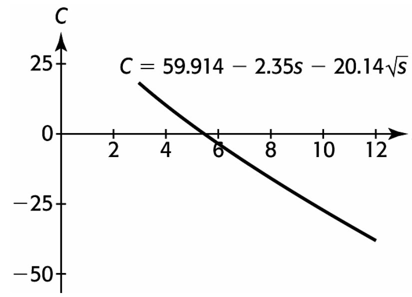
49. The function is $100\bullet\left(\frac{g}{f}\right)(x)$. Note that $\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}=$ the percent of Facebook users "who are" MySpace users. Multiplying by 100 creates a percentage.

50. number of homes with computers
$=$ (percentage of homes with computers) $\times$ (number of homes)
In symbols, $(f\bullet g)(x)$ or $(g\bullet f)(x)$

51. The function is $(f+g)(x)=f(x)+g(x)$.

52. a. If $f_1=59.914-2.35s$ and $f_2=20.14\sqrt{s}$, then $C=f_1-f_2$. Answers may vary.

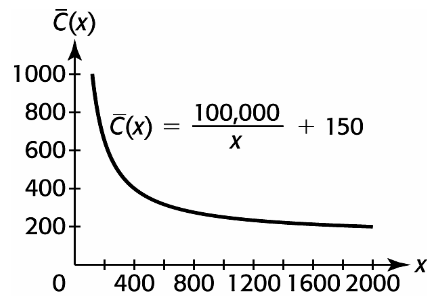
b.



c. Based on the graph in part b), the function is decreasing. As s increases, C decreases.

53. The normal price is $0.50x$ where x represents retail price. Since the sale price is 20% off the normal price, the sale price is $0.50x-(0.20)(0.50x)=0.50x-0.10x=0.40x.$ Therefore the books are on sale for 40% of retail price.

54. a.

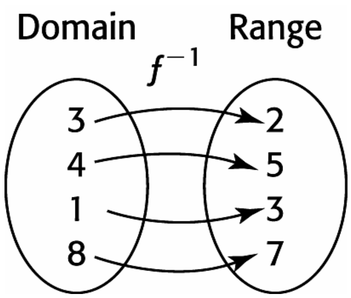


b. The function, and therefore the average cost, is decreasing as the number of sets produced increases.

c. It is a stretch vertically by a factor of 100,000 and a shift up 150 units.

Section 4.3 Skills Check

1. Yes, the function is one-to-one and has an inverse.



2. No, the function is not one-to-one. Inputs of 4 and 18 correspond with an output of 7.

3. No, the function is not one-to-one, and therefore has no inverse. Inputs of 5 and 6 correspond with an output of 2.

4. Yes, the function is one-to-one and has an inverse with the following ordered pairs.
$\{(8,2),(9,3),(10,4),(11,5)\}$

5. **a.** $f(g(x))=(f\circ g)(x)=3\left(\dfrac{x}{3}\right)=x$

$g(f(x))=(g\circ f)(x)=\dfrac{(3x)}{3}=x$

b. Yes, since $(f\circ g)(x)=(g\circ f)(x)=x$.

6. **a.**
$$\begin{aligned} f(g(x)) &= (f\circ g)(x)\\ &= 4\left(\frac{x+1}{4}\right)-1\\ &= x+1-1\\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= (g\circ f)(x)\\ &= \frac{(4x-1)+1}{4}\\ &= \frac{4x}{4}=x \end{aligned}$$

b. Yes, since $(f\circ g)(x)=(g\circ f)(x)=x$.

7. Is $(f\circ g)(x)=(g\circ f)(x)=x$?

$$\begin{aligned} (f\circ g)(x) &= f(g(x))\\ &= \left(\sqrt[3]{x-1}\right)^3+1\\ &= x-1+1\\ &= x \end{aligned}$$

$$\begin{aligned} (g\circ f)(x) &= g(f(x))\\ &= \sqrt[3]{x^3+1-1}\\ &= \sqrt[3]{x^3}\\ &= x \end{aligned}$$

Yes, f and g are inverse functions.

8. Is $(f\circ g)(x)=(g\circ f)(x)=x$?

$$\begin{aligned} (f\circ g)(x) &= f(g(x))\\ &= \left(\sqrt[3]{x+2}-2\right)^3 \neq x \end{aligned}$$

$$\begin{aligned} (g\circ f)(x) &= g(f(x))\\ &= \sqrt[3]{(x-2)^3+2} \neq x \end{aligned}$$

No, f and g are not inverse functions.

9. See the completed tables below.

x	$f(x)$
–1	–7
0	**–4**
1	**–1**
2	**2**
3	**5**

x	$f^{-1}(x)$
–7	–1
–4	**0**
–1	**1**
2	**2**
5	**3**

Note that values for x in the second table are the values for $f(x)$ in the first table.

10. See the completed tables below.

x	$g(x)$
–2	–17
–1	**–3**
0	**–1**
1	**1**
2	**15**

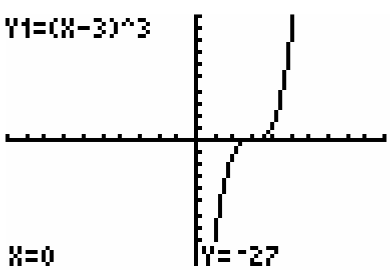
x	$g^{-1}(x)$
–17	–2
–3	**–1**
–1	**0**
1	**1**
15	**2**

Note that values for x in the second table are the values for $g(x)$ in the first table.

11. Not one-to-one since (1, 5) and (4, 5) make the output 5 have more than one input.

12. Yes, this function is one-to-one since each input is matched with exactly one output, and each output with exactly one input.

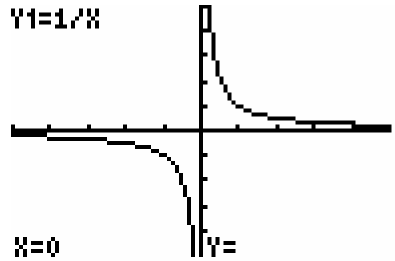
13.



[–10, 10] by [–10, 10]

Since the graph of $f(x)=(x-3)^3$ passes the horizontal line test, this is a one-to-one function.

14.



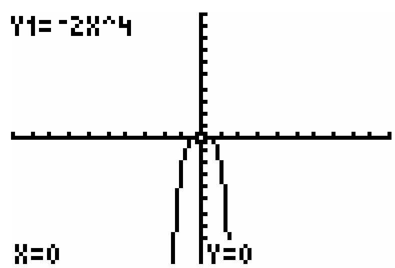
[–5, 5] by [–5, 5]

Since the graph of $f(x)=\frac{1}{x}$ passes the horizontal line test, this is a one-to-one function.

15. Since the given graph passes both the vertical line test and the horizontal line test, this is a one-to-one function.

16. Since the given graph passes both the vertical line test and the horizontal line test, this is a one-to-one function.

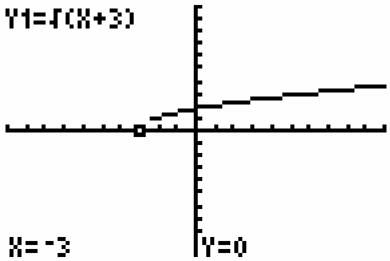
17.



[–10, 10] by [–10, 10]

Since the graph of $f(x)=-2x^4$ fails the horizontal line test, this is not a one-to-one function.

18.



[–10, 10] by [–10, 10]

Since the graph of $f(x)=\sqrt{x+3}$ passes the horizontal line test, this is a one-to-one function.

19. a.

$$f(x)=3x-4$$
$$y=3x-4$$
$$x=3y-4$$
$$3y=x+4$$
$$y=\frac{x+4}{3}$$
$$f^{-1}(x)=\frac{x+4}{3}$$

b. Yes. Substituting the *x*-values from the table into $f^{-1}(x)$ generates the $f^{-1}(x)$ outputs found in the table.

20. a.

$$g(x)=2x^3-1$$
$$y=2x^3-1$$
$$x=2y^3-1$$
$$x+1=2y^3$$
$$\frac{x+1}{2}=y^3$$
$$y=\sqrt[3]{\frac{x+1}{2}}$$
$$g^{-1}(x)=\sqrt[3]{\frac{x+1}{2}}$$

b. Yes. Substituting the *x*-values from the table into $g^{-1}(x)$ generates the $g^{-1}(x)$ outputs found in the table.

21. $h^{-1}(-2)=3 \Leftrightarrow h(3)=-2$, if h and h^{-1} are inverse functions.

22.

$$f(x)=\frac{1}{x}$$
$$y=\frac{1}{x}$$
$$x=\frac{1}{y}$$
$$xy=1$$
$$y=\frac{1}{x}$$
$$f^{-1}(x)=\frac{1}{x}$$

Note that in this example the function and its inverse are the same function!

23.

$$g(x)=4x+1$$
$$y=4x+1$$
$$x=4y+1$$
$$4y=x-1$$
$$y=\frac{x-1}{4}$$
$$g^{-1}(x)=\frac{x-1}{4}$$

24.

$$f(x)=4x^2$$
$$y=4x^2$$
$$x=4y^2$$
$$y^2=\frac{x}{4}$$
$$y=\pm\sqrt{\frac{x}{4}}$$

Since $x \geq 0$ is given for $f(x)$,
then $y \geq 0$ for $f^{-1}(x)$.
Thus only the positive solution represents the inverse function.

$$f^{-1}(x) = \sqrt{\frac{x}{4}} = \frac{\sqrt{x}}{2}$$

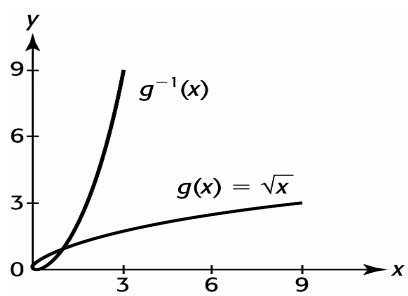
25. $g(x) = x^2 - 3$

$$y = x^2 - 3$$
$$x = y^2 - 3$$
$$y^2 = x + 3$$
$$y = \pm\sqrt{x+3}$$

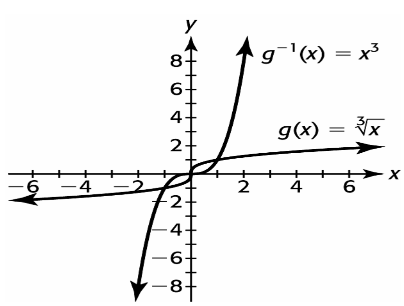
Since $x \geq 0$ is given for $g(x)$,
then $y \geq 0$ for $g^{-1}(x)$.
Thus only the positive solution represents the inverse function.

$$g^{-1}(x) = \sqrt{x+3}$$

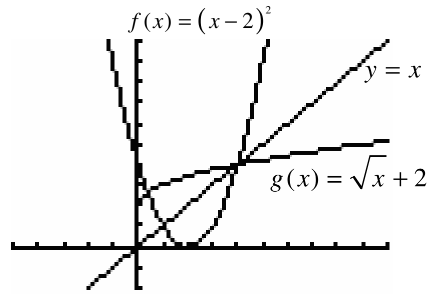
26.



27.

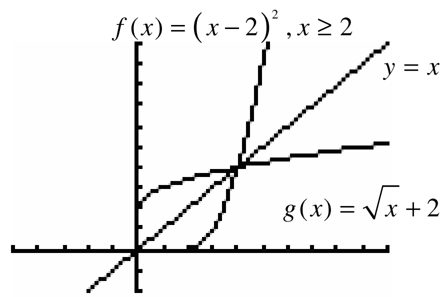


28.



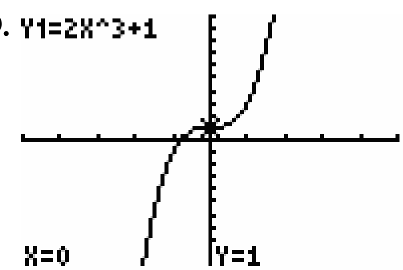
[–5, 10] by [–2, 10]

Restricting the domain of *f(x)* to $[2, \infty)$ and the domain of *g(x)* to $[0, \infty)$ forces *f* and *g* to become inverse functions.



[–5, 10] by [–2, 10]

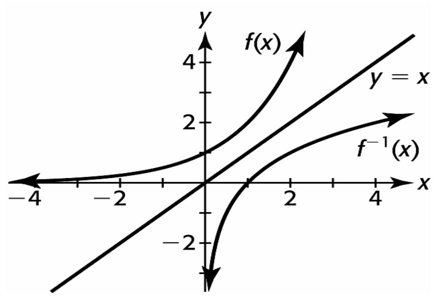
29.



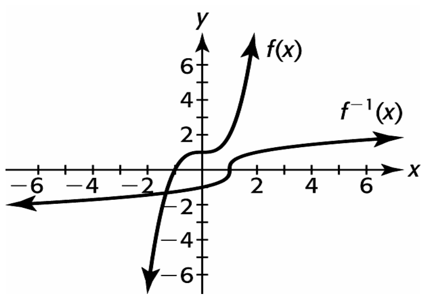
[–5, 5] by [–10, 10]

The function passes the horizontal line test, is one-to-one, and has an inverse function.

30.



31.



Section 4.3 Exercises

32. a.
$$\begin{aligned} d(x) &= x + 0.5 \\ y &= x + 0.5 \\ x &= y + 0.5 \\ y &= x - 0.5 \\ d^{-1}(x) &= x - 0.5 \end{aligned}$$

b. $d^{-1}(8.5) = 8.5 - 0.5 = 8$
The British shoe size is 8.

33. a.
$$\begin{aligned} t(x) &= x + 34.5 \\ y &= x + 34.5 \\ x &= y + 34.5 \\ y &= x - 34.5 \\ t^{-1}(x) &= x - 34.5 \end{aligned}$$

b. $t^{-1}(43) = 43 - 34.5 = 8.5$
The U.S. shoe size is 8.5 or $8\frac{1}{2}$.

34. a.
$$\begin{aligned} S(x) &= x + 0.6x \\ y &= x + 0.6x \\ x &= y + 0.6y \\ x &= 1.6y \\ y &= \frac{x}{1.6} \\ S^{-1}(x) &= \frac{x}{1.6} \end{aligned}$$

b. Given the future value of an investment, the function $S^{-1}(x)$ calculates the amount originally invested.

c. $S^{-1}(24{,}000) = \dfrac{24{,}000}{1.6} = \$15{,}000$

35.
$f(t) = 82.074 - 2.087t$
$y = 82.074 - 2.087t$
Switch y and t to form inverse:
$t = 82.074 - 2.087y$
$t - 82.074 = -2.087y$
$y = \dfrac{t - 82.074}{-2.087}$
$y = \dfrac{82.074 - t}{2.087}$
$f^{-1}(t) = \dfrac{82.074 - t}{2.087}$

$$\begin{aligned} f^{-1}(41) &= \frac{82.074 - 41}{2.087} \\ &= \frac{41.074}{2.087} \\ &= 19.681 \approx 20 \end{aligned}$$

The percentage dropped below 41% in the year 2010 (1990 + 20).

36. $$A(x) = 82.35 + 29.3x$$
$$y = 82.35 + 29.3x$$
$$x = 82.35 + 29.3y$$
$$29.3y = x - 82.35$$
$$y = \frac{x - 82.35}{29.3}$$
$$A^{-1}(x) = \frac{x - 82.35}{29.3}$$

$$A^{-1}(97) = \frac{97 - 82.35}{29.3} = 0.5 = 50\%$$

A temperature of 97° corresponds to 50% relative humidity.

37. a. $$f(x) = -0.085x + 2.97$$
$$y = -0.085x + 2.97$$
$$x = -0.085y + 2.97$$
$$x - 2.97 = -0.085y$$
$$y = \frac{x - 2.97}{-0.085}$$
$$f^{-1}(x) = \frac{x - 2.97}{-0.085} = \frac{2.97 - x}{0.085}$$

The inverse function will calculate the number of years after 2000 in which the percent of children ages 0 to 19 taking antidepressants from 2004 to 2009 reaches a given level.

b. $f^{-1}(2.3) = \frac{2.97 - x}{0.085} = 7.88 \approx 8$

The percent will reach 2.3% in 2008 (2000 + 8).

38. a. $f(x) = 4\sqrt{4x+1}$

To determine the domain, solve $4x + 1 \geq 0$.

$4x \geq -1$

$x \geq -\frac{1}{4}$

Domain: $\left[-\frac{1}{4}, \infty\right)$

Consequently, the range is $[0, \infty)$.

b. $$f(x) = 4\sqrt{4x+1}$$
$$y = 4\sqrt{4x+1}$$
$$x = 4\sqrt{4y+1}$$
$$\frac{x}{4} = \sqrt{4y+1}$$
$$\left(\frac{x}{4}\right)^2 = \left(\sqrt{4y+1}\right)^2$$
$$\frac{x^2}{16} = 4y + 1$$
$$4y = \frac{x^2}{16} - 1$$
$$y = \frac{\frac{x^2 - 16}{16}}{4}$$
$$y = \frac{x^2 - 16}{64}$$
$$f^{-1}(x) = \frac{x^2 - 16}{64}$$

c. The domain of the inverse function is the range of the original function, and the range of the inverse function is the domain of the original function. Therefore,

Domain: $[0, \infty)$

Range: $\left[-\frac{1}{4}, \infty\right)$

d. For the inverse function,

Domain: $[4, \infty)$

Range: $[0, \infty)$

In the context of the application, the range of the inverse function represents the wind speed and therefore must be greater than or equal to 0, while the domain of the inverse function must be greater than or equal to 4: $x^2 - 16 \geq 0$, to ensure the range is not negative.

39. a.

$$W(x) = 0.002x^3$$
$$y = 0.002x^3$$
$$x = 0.002y^3$$
$$y^3 = \frac{x}{0.002}$$
$$y = \sqrt[3]{\frac{x}{0.002}} = \sqrt[3]{500x}$$
$$W^{-1}(x) = \sqrt[3]{500x}$$

b. Given the weight, the inverse function calculates the length.

c. $W^{-1}(2) = \sqrt[3]{500(2)} = \sqrt[3]{1000} = 10$

The length of the fish is 10 inches.

d. Both the domain and the range are $(0, \infty)$. The weight and the length must be greater than zero or there is no fish to measure.

40.

$$C(x) = x + 3$$
$$y = x + 3$$
$$x = y + 3$$
$$x - 3 = y$$
$$y = x - 3$$
$$C^{-1}(x) = x - 3$$

The decoded numerical sequence is
{20 8 5 27 18 5 1 12 27 20 8 9 14 7},
which translates into "THE REAL THING".

41.

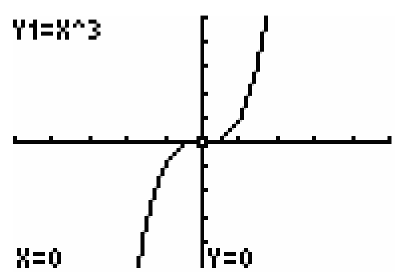
$$C(x) = 3x + 2$$
$$y = 3x + 2$$
$$x = 3y + 2$$
$$3y = x - 2$$
$$y = \frac{x-2}{3}$$
$$C^{-1}(x) = \frac{x-2}{3}$$

The decoded numerical sequence is
{13 1 11 5 27 13 25 27 4 1 25},
which translates into "MAKE MY DAY".

42. Yes. Each person has a unique social security number. Since no two people have the same number, the function is one-to-one.

43. No. The given function is not one-to-one. More than one check could correspond to the same dollar amount.

44. a. Yes. The function is one-to-one. The graph passes the horizontal line test.



[–5, 5] by [–5, 5]

b.

$$f(x) = x^3$$
$$y = x^3$$
$$x = y^3$$
$$\sqrt[3]{x} = \sqrt[3]{y^3}$$
$$y = \sqrt[3]{x}$$
$$f^{-1}(x) = \sqrt[3]{x}$$

c. Both domain and the range are $(0,\infty)$ based on the physical context of the question. The side of the cube and the volume cannot = 0.

d. The inverse function is used to convert from the volume of a cube to the length of its side.

45. a. Yes. The graph of the equation would pass the horizontal line test.

b.
$$f(x)=\frac{4}{3}\pi x^3$$
$$y=\frac{4}{3}\pi x^3$$
$$x=\frac{4}{3}\pi y^3$$
$$3(x)=3\left(\frac{4}{3}\pi y^3\right)$$
$$3x=4\pi y^3$$
$$y^3=\frac{3x}{4\pi}$$
$$y=\sqrt[3]{\frac{3x}{4\pi}}$$
$$f^{-1}(x)=\sqrt[3]{\frac{3x}{4\pi}}$$

c. Both the domain and range are $(0,\infty)$. If the domain is less than or equal to zero, then there is no sphere.

d. Given its volume, the inverse function can be used to calculate the radius of a sphere.

e.
$$f^{-1}(65,450)=\sqrt[3]{\frac{3(65,450)}{4\pi}}$$
$$\approx 25$$

If the volume is 65,450 cubic inches, the radius is approximately 25 inches.

46. a. The model makes sense as long as the length of the side of the cube is greater than zero. Therefore the domain is $(0,\infty)$. The function is one-to-one over this restricted domain.

b.
$$y=6x^2 \quad x>0$$
$$x=6y^2$$
$$y^2=\frac{x}{6}$$
$$y=\pm\sqrt{\frac{x}{6}}$$
Based on the original restricted domain, the inverse function is
$$y=\sqrt{\frac{x}{6}}.$$

c. Given its surface area, the inverse function can be used to calculate the length of the edge of a cube.

47. a.
$$f(x)=1.0136x$$
$$y=1.0136x$$
$$x=1.0136y$$
$$y=\frac{x}{1.0136}$$
$$f^{-1}(x)=\frac{x}{1.0136}=0.9866x$$

The inverse function converts U.S. dollars into Canadian dollars.

b. $f^{-1}(500 \text{ U.S. dollars})=500/1.0136$
$y=493.30$ Canadian dollars

$f(493.30)=1.0136(493.30)$
$=500$ U.S. dollars

If you convert \$500 from U.S. to Canadian dollars and then convert the money back to U.S. dollars, you will still have \$500 U.S currency. (Note:

This assumes there are no transaction fees for the conversion and that the exchange rate has not changed.)

48. a. Considering the graph of the function, it is not one-to-one.

b. Since the number of units cannot be negative, the domain is $[0,\infty)$.

c. Based on the restricted domain arising from the physical context of the problem, the function is one-to-one.

d. $p(x)=\frac{1}{4}x^2+20,\ x\ge 0$

$$y=\frac{1}{4}x^2+20$$
$$x=\frac{1}{4}y^2+20$$
$$4x=y^2+80$$
$$y^2=4x-80$$
$$y=\pm\sqrt{4x-80}$$

Based on the restricted domain,

$$p^{-1}(x)=\sqrt{4x-80}=2\sqrt{x-20}$$
$$p^{-1}(101)=2\sqrt{101-20}$$
$$=2\sqrt{81}$$
$$=2(9)$$
$$=18$$

The manufacturer will supply 18,000 units if the price is $101 per unit.

49. a. No. The function is not one-to-one. Note that $I(-1)=I(1)=300{,}000$.

b. In the given physical context, since x represents distance, the domain is $(0,\infty)$.

$x\ne 0,$ since $\frac{300{,}000}{x^2}$ is undefined.

c. Yes. Based on the restricted domain $(0,\infty)$, the function is one-to-one.

d. $I(x)=\frac{300{,}000}{x^2}, x>0$

$$y=\frac{300{,}000}{x^2}$$
$$x=\frac{300{,}000}{y^2}$$
$$xy^2=300{,}000$$
$$y^2=\frac{300{,}000}{x}$$
$$y=\pm\sqrt{\frac{300{,}000}{x}}$$

Based on the physical context,

$$I^{-1}(x)=\sqrt{\frac{300{,}000}{x}}$$
$$I^{-1}(75{,}000)=\sqrt{\frac{300{,}000}{75{,}000}}$$
$$=\sqrt{4}$$
$$=2$$

When the distance is 2 feet, the intensity of light is 75,000 candlepower.

50. a. $P(w)=\begin{cases} 44 & 0<w\le 1 \\ 64 & 1<w\le 2 \\ 84 & 2<w\le 3 \end{cases}$

b. No. The function is not one-to-one. The postage is the same for a variety of weights.

51. a. $f(x)=1.6249x$

$$y=1.6249x$$
$$x=1.6249y$$
$$y=\frac{x}{1.6249}=\frac{1}{1.6249}x$$
$$f^{-1}(x)=0.6154x$$

The inverse function converts U.S. dollars into British pounds.

b. $f^{-1}(1000 \text{ U.S. dollars}) = 0.6154(1000)$ If $y = 615.40$ U.K. pounds

$f(615.40) = 1.6249(615.40)$
$= 1000$ U.S. dollars

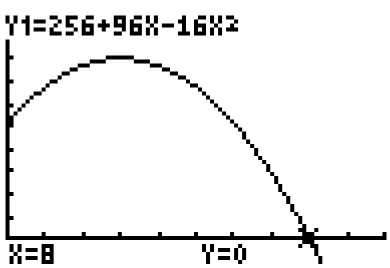
If you convert $1000 from U.S. to British currency and then convert the money back to U.S. dollars, you will still have $1000 U.S currency. (Note: This assumes there are no transaction fees for the conversion and that the exchange rate has not changed.)

52. a. The ball reaches the ground when its height equals zero. Solve $f(x) = 0$.

$256 + 96x - 16x^2 = 0$
$-16(x^2 - 6x - 16) = 0$
$-16(x-8)(x+2) = 0$
$x - 8 = 0, x + 2 = 0$
$x = 8, x = -2$
Since x represents time, $x \geq 0$.
$x = 8$

The ball remains in the air for 8 seconds.

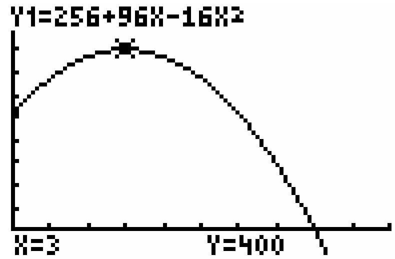
b.



[0, 10] by [–50, 500]

The function is not one-to-one on the interval $[0,8]$. The graph does not pass the horizontal line test.

c. The function is one-to-one on the interval $[0,3]$ or on the interval $[3,8]$.



[0, 10] by [–50, 500]

d.

$f(x) = 256 + 96x - 16x^2$
$y = -16(x^2 - 6x - 16)$
$x = -16(y^2 - 6y - 16)$
Completing the square yields,
$x = -16[(y^2 - 6y + 9) + (-9 - 16)]$
$x = -16[(y-3)^2 - 25]$
$x = -16(y-3)^2 + 400$
$x - 400 = -16(y-3)^2$
$(y-3)^2 = \frac{x-400}{-16} = \frac{400-x}{16}$
$\sqrt{(y-3)^2} = \pm\sqrt{\frac{400-x}{16}}$
$y - 3 = \pm\sqrt{\frac{400-x}{16}}$
$y = 3 \pm \sqrt{\frac{400-x}{16}}$
For the interval $[0,3]$,
$f^{-1}(x) = 3 - \sqrt{\frac{400-x}{16}} = 3 - \frac{\sqrt{400-x}}{4}$

The inverse function calculates the time the ball is in air between 0 and 3 seconds, given the height of the ball.

Section 4.4 Skills Check

1. $\sqrt{2x^2-1}-x=0$

$\sqrt{2x^2-1}=x$

$\left(\sqrt{2x^2-1}\right)^2=(x)^2$

$2x^2-1=x^2$

$x^2-1=0$

$(x+1)(x-1)=0$

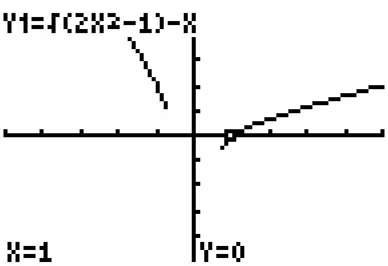
$x=1, x=-1$

-1 does not check

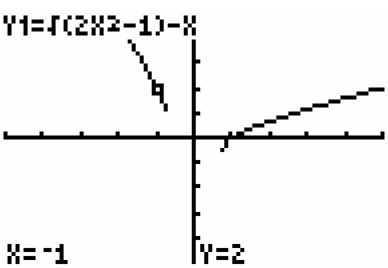
$\sqrt{2(-1)^2-1}-(-1)=$

$\sqrt{2(1)-1}+1=\sqrt{1}+1=2\neq 0$

Applying the intersection of graphs method to check graphically:



[–5, 5] by [–5, 5]



[–5, 5] by [–5, 5]

The only solution that checks is $x = 1$.

2. $\sqrt{3x^2+4}-2x=0$

$\sqrt{3x^2+4}=2x$

$\left(\sqrt{3x^2+4}\right)^2=(2x)^2$

$3x^2+4=4x^2$

$x^2-4=0$

$x=2, x=-2$

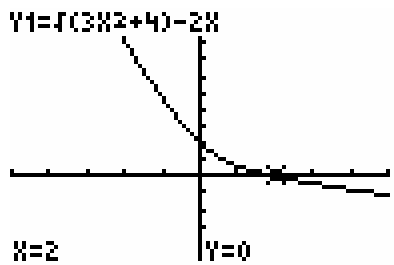
-2 does not check

$\sqrt{3(-2)^2+4}-2(-2)$

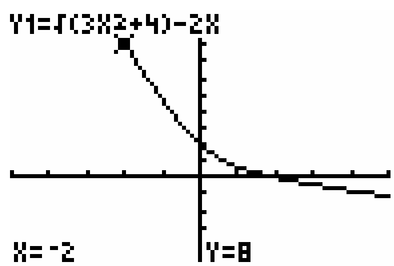
$\sqrt{16}+4$

$8\neq 0$

Applying the intersection of graphs method to check graphically:



[–5, 5] by [–5, 10]



[–5, 5] by [–5, 10]

The only solution that checks is $x = 2$.

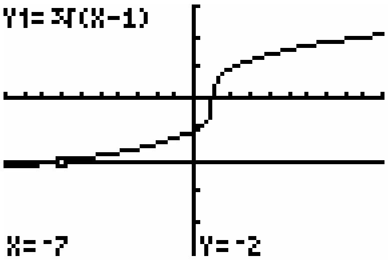
3. $\sqrt[3]{x-1}=-2$

$\left(\sqrt[3]{x-1}\right)^3=(-2)^3$

$x-1=-8$

$x=-7$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–5, 3]

The solution is $x = -7$.

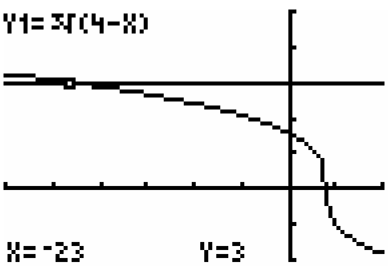
4. $\sqrt[3]{4-x} = 3$

$\left(\sqrt[3]{4-x}\right)^3 = (3)^3$

$4 - x = 27$

$x = -23$

Applying the intersection of graphs method to check graphically:



[–30, 10] by [–2, 5]

The solution is $x = -23$.

5. $\sqrt{3x-2} + 2 = x$

$\sqrt{3x-2} = x - 2$

$\left(\sqrt{3x-2}\right)^2 = (x-2)^2$

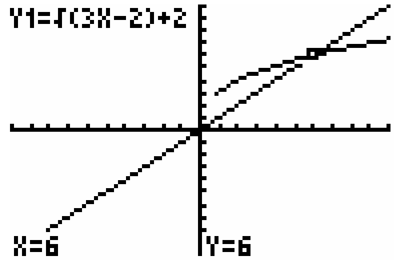
$3x - 2 = x^2 - 4x + 4$

$x^2 - 7x + 6 = 0$

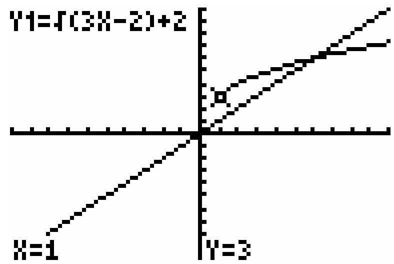
$(x-6)(x-1) = 0$

$x = 6, x = 1$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

There is only one solution. The x-value of 1 does not check. The solution is $x = 6$.

6. $\sqrt{x-2} + 2 = x$

$\sqrt{x-2} = x - 2$

$\left(\sqrt{x-2}\right)^2 = (x-2)^2$

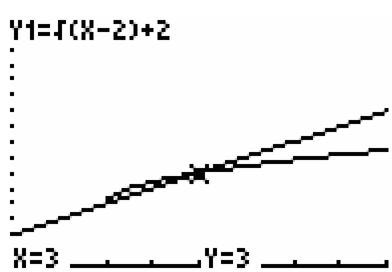
$x - 2 = x^2 - 4x + 4$

$x^2 - 5x + 6 = 0$

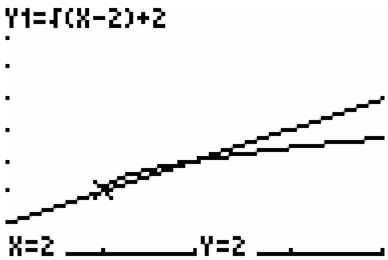
$(x-3)(x-2) = 0$

$x = 3, x = 2$

Applying the intersection of graphs method to check graphically:



[1, 5] by [0, 8]



[1, 5] by [0, 8]

Both solutions check.

7. $\sqrt[3]{4x+5}=\sqrt[3]{x^2-7}$

$\left(\sqrt[3]{4x+5}\right)^3=\left(\sqrt[3]{x^2-7}\right)^3$

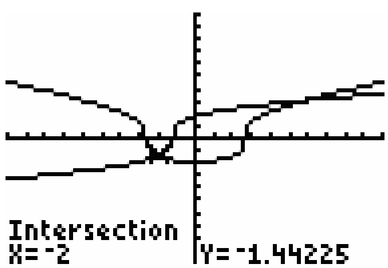
$4x+5=x^2-7$

$x^2-4x-12=0$

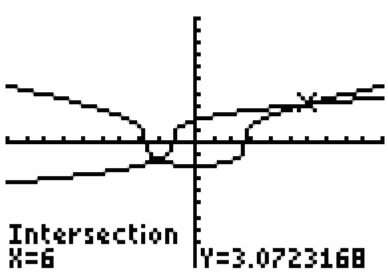
$(x-6)(x+2)=0$

$x=6, x=-2$

Applying the intersection of graphs method to check graphically:



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]
Both solutions check.

8. $\sqrt{5x-6}=\sqrt{x^2-2x}$

$\left(\sqrt{5x-6}\right)^2=\left(\sqrt{x^2-2x}\right)^2$

$5x-6=x^2-2x$

$x^2-7x+6=0$

$(x-6)(x-1)=0$

$x=6, x=1$

Checking by substitution

$x=6$

$\sqrt{5x-6}\overset{?}{=}\sqrt{x^2-2x}$

$\sqrt{5(6)-6}\overset{?}{=}\sqrt{(6)^2-2(6)}$

$\sqrt{24}=\sqrt{24}$

$x=1$

$\sqrt{5x-6}\overset{?}{=}\sqrt{x^2-2x}$

$\sqrt{5(1)-6}\overset{?}{=}\sqrt{(1)^2-2(1)}$

$\sqrt{-1}=\sqrt{-1}\leftarrow$ Undefined expressions

Only $x = 6$ checks.

9. $\sqrt{x}-1=\sqrt{x-5}$

$\left(\sqrt{x}-1\right)^2=\left(\sqrt{x-5}\right)^2$

$x-2\sqrt{x}+1=x-5$

$-2\sqrt{x}=-6$

$\sqrt{x}=3$

$\left(\sqrt{x}\right)^2=(3)^2$

$x=9$

Checking by substitution

$x=9$

$\sqrt{x}-1\overset{?}{=}\sqrt{x-5}$

$\sqrt{9}-1\overset{?}{=}\sqrt{9-5}$

$2=2$

The solution is $x=9$.

10. $\sqrt{x}-10=-\sqrt{x-20}$

$\left(\sqrt{x}-10\right)^2=\left(-\sqrt{x-20}\right)^2$

$x-20\sqrt{x}+100=x-20$

$-20\sqrt{x}=-120$

$\sqrt{x}=6$

$\left(\sqrt{x}\right)^2=(6)^2$

$x=36$

Checking by substitution

$x=36$

$\sqrt{x}-10\stackrel{?}{=}-\sqrt{x-20}$

$\sqrt{36}-10\stackrel{?}{=}-\sqrt{36-20}$

$-4=-4$

The solution is $x=36$.

11. $(x+4)^{\frac{2}{3}}=9$

$\sqrt[3]{(x+4)^2}=9$

$\left[\sqrt[3]{(x+4)^2}\right]^3=[9]^3$

$(x+4)^2=729$

$\sqrt{(x+4)^2}=\pm\sqrt{729}$

$x+4=\pm 27$

$x=-4\pm 27$

$x=23, x=-31$

Checking by substitution

$x=23$

$(23+4)^{\frac{2}{3}}=9$

$(27)^{\frac{2}{3}}=9$

$9=9$

$x=-31$

$(-31+4)^{\frac{2}{3}}=9$

$(-27)^{\frac{2}{3}}=9$

$(-3)^2=9$

$9=9$

The solutions are $x=23, x=-31$.

12. $(x-5)^{\frac{3}{2}}=64$

$\sqrt[2]{(x-5)^3}=64$

$\left[\sqrt[2]{(x-5)^3}\right]^2=[64]^2$

$(x-5)^3=4096$

$\sqrt[3]{(x-5)^3}=\sqrt[3]{4096}$

$x-5=16$

$x=21$

Checking by substitution

$x=21$

$(21-5)^{\frac{3}{2}}=64$

$(16)^{\frac{3}{2}}=64$

$(4)^3=64$

$64=64$

The solution is $x=21$.

13. $x^2 + 4x < 0$

$x(x+4) < 0$

$x(x+4) = 0$

$x = 0, x = -4$

sign of x	$\leftarrow - - -_{-4} - - -_{0} + + + \rightarrow$
sign of $(x+4)$	$\leftarrow - - -_{-4} + + +_{0} + + + \rightarrow$
sign of $x(x+4)$	$\leftarrow + + +_{-4} - - -_{0} + + + \rightarrow$

Considering the inequality symbol in the original question, the solution is $(-4,0)$.

14. $x^2 - 25x < 0$

$x(x-25) < 0$

$x(x-25) = 0$

$x = 0, x = 25$

sign of x	$\leftarrow - - -_{0} + + +_{25} + + + \rightarrow$
sign of $(x-25)$	$\leftarrow - - -_{0} - - -_{25} + + + \rightarrow$
sign of $x(x-25)$	$\leftarrow + + +_{0} - - -_{25} + + + \rightarrow$

Considering the inequality symbol in the original question, the solution is $(0,25)$.

15. $9 - x^2 \geq 0$

$-1(x^2 - 9) \geq 0$

$$\frac{-1(x^2-9)}{-1} \leq \frac{0}{-1}$$

$\boxed{x^2 - 9 \leq 0}$ ← Solve the simplified question

$(x+3)(x-3) \leq 0$

$(x+3)(x-3) = 0$

$x = -3, x = 3$

sign of $(x+3)$	$\leftarrow - - -_{-3} + + +_{3} + + + \rightarrow$
sign of $(x-3)$	$\leftarrow - - -_{-3} - - -_{3} + + + \rightarrow$
sign of $(x+3)(x-3)$	$\leftarrow + + +_{-3} - - -_{3} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $[-3,3]$.

16. $x > x^2$

$0 > x^2 - x$

$\boxed{x^2 - x < 0}$ ← Solve the simplified question

$x(x-1) < 0$

$x(x-1) = 0$

$x = 0, x = 1$

sign of x	$\leftarrow - - -_{0} + + +_{1} + + + \rightarrow$
sign of $(x-1)$	$\leftarrow - - -_{0} - - -_{1} + + + \rightarrow$
sign of $x(x-1)$	$\leftarrow + + +_{0} - - -_{1} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $(0,1)$.

17. $-x^2 + 9x - 20 > 0$

$-1(x^2 - 9x + 20) > 0$

$$\frac{-1(x^2-9x+20)}{-1} < \frac{0}{-1}$$

$\boxed{x^2 - 9x + 20 < 0}$ ← Solve the simplified question

$(x-5)(x-4) < 0$

$(x-5)(x-4) = 0$

$x = 5, x = 4$

sign of $(x-5)$	$\leftarrow - - -_{4} - - -_{5} + + + \rightarrow$
sign of $(x-4)$	$\leftarrow - - -_{4} + + +_{5} + + + \rightarrow$
sign of $(x-5)(x-4)$	$\leftarrow + + +_{4} - - -_{5} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $(4,5)$.

18. $2x^2 - 8x < 0$

$2x(x-4) < 0$

$2x(x-4) = 0$

$x = 0, x = 4$

sign of x	$\leftarrow - - -_{0} + + +_{4} + + + \rightarrow$
sign of $(x-4)$	$\leftarrow - - -_{0} - - -_{4} + + + \rightarrow$
sign of $x(x-4)$	$\leftarrow + + +_{0} - - -_{4} + + + \rightarrow$

Considering the inequality symbol in the original question, the solution is $(0,4)$.

19. $2x^2 - 8x \geq 24$

$\boxed{2x^2 - 8x - 24 \geq 0}$ ← Solve the simplified question

$2(x^2 - 4x - 12) \geq 0$

$2(x-6)(x+2) \geq 0$

$2(x-6)(x+2) = 0$

$2 \neq 0, x = 6, x = -2$

sign of $(x-6)$ $\leftarrow ---_{-2} ---_{6} +++ \rightarrow$

sign of $(x+2)$ $\leftarrow ---_{-2} +++_{6} +++ \rightarrow$

sign of $(x-6)(x+2)$ $\leftarrow +++_{-2} ---_{6} +++ \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $(-\infty, -2] \cup [6, \infty)$.

20. $t^2 + 17t \leq 8t - 14$

$\boxed{t^2 + 9t + 14 \leq 0}$ ← Solve the simplified question

$(t+7)(t+2) \leq 0$

$(t+7)(t+2) = 0$

$t = -7, t = -2$

sign of $(t+7)$ $\leftarrow ---_{-7} +++_{-2} +++ \rightarrow$

sign of $(t+2)$ $\leftarrow ---_{-7} ---_{-2} +++ \rightarrow$

sign of $(t+7)(t+2)$ $\leftarrow +++_{-7} ---_{-2} +++ \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $[-7, -2]$.

21. $x^2 - 6x < 7$

$\boxed{x^2 - 6x - 7 < 0}$ ← Solve the simplified question

$(x-7)(x+1) < 0$

$(x-7)(x+1) = 0$

$x = 7, x = -1$

sign of $(x-7)$ $\leftarrow ---_{-1} ---_{7} +++ \rightarrow$

sign of $(x+1)$ $\leftarrow ---_{-1} +++_{7} +++ \rightarrow$

sign of $(x-7)(x+1)$ $\leftarrow +++_{-1} ---_{7} +++ \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $(-1, 7)$.

22. $4x^2 - 4x + 1 > 0$

$(2x-1)(2x-1) > 0$

$(2x-1)(2x-1) = 0$

$2x - 1 = 0, 2x - 1 = 0$

$x = \frac{1}{2}, x = \frac{1}{2}$

sign of $(2x-1)$ $\leftarrow ----_{\frac{1}{2}} ++++ \rightarrow$

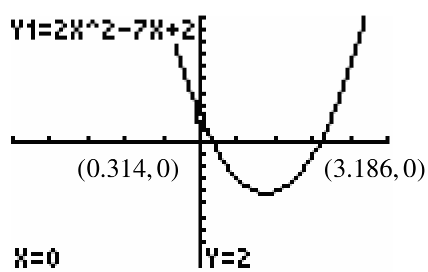
sign of $(2x-1)$ $\leftarrow ----_{\frac{1}{2}} ++++ \rightarrow$

sign of $(2x-1)(2x-1)$ $\leftarrow ++++_{\frac{1}{2}} ++++ \rightarrow$

Based on the inequality symbol and the sign chart, the equation is greater than zero for all real numbers except $\frac{1}{2}$. When $x = \frac{1}{2}$, the equation equals zero. Therefore, the solution is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

23. $2x^2 - 7x + 2 \geq 0$

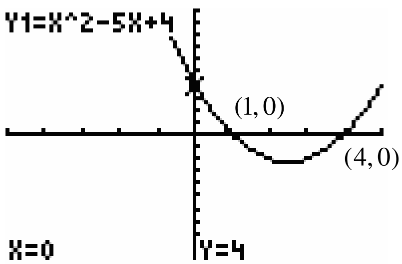
Solving graphically means to find the x-intercepts, in this case, 0.314 and 3.186. Then the solution to $2x^2 - 7x + 2 \geq 0$ is the interval where the graph is above the x-axis, in this case to the left of 0.314 and to the right of 3.186, including both values. Thus the solution to the inequality is $(-\infty, 0.314] \cup [3.186, \infty)$



[–5, 5] by [–10, 10]

24. $w^2-5w+4>0$

Solving graphically means to find the *x*-intercepts, in this case, $w = 1$ and $w = 4$. Then the solution to $w^2-5w+4>0$ is the interval where the graph is above the *x*-axis, in this case to the left of 1 and to the right of 4. Thus the solution to the inequality is $(-\infty,1)\cup(4,\infty)$

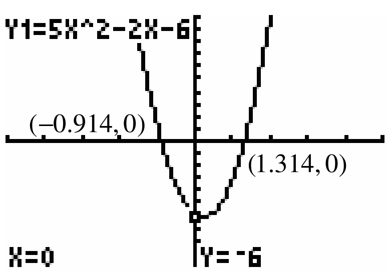


[–5, 5] by [–10, 10]

25. $5x^2 \ge 2x+6$

$\boxed{5x^2-2x-6\ge 0}$ ← Solve the simplified question

Solving graphically means to find the *x*-intercepts, in this case, $x=-0.914$ and $x =$ 1.314. Then the solution to $5x^2-2x-6\ge 0$ is the interval where the graph is above the *x*-axis, in this case to the left of -0.914 and to the right of 1.314, including both values. Thus the solution to the inequality is $(-\infty,-0.914]\cup[1.314,\infty)$

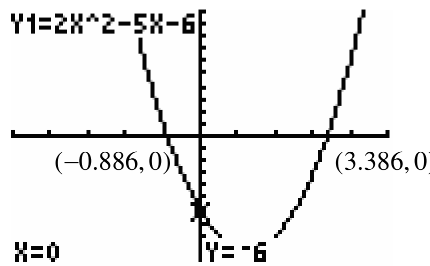


[–5, 5] by [–10, 10]

26. $2x^2 \le 5x+6$

$\boxed{2x^2-5x-6\le 0}$ ← Solve the simplified question

Solving graphically means to find the *x*-intercepts, in this case, $x=-0.886$ and $x =$ 3.386. Then the solution to $2x^2-5x-6\le 0$ is the interval where the graph is below the *x*-axis, in this case between and including 0.886 and 3.386. Thus the solution to the inequality is $[-0.886, 3.386]$



[–5, 5] by [–10, 10]

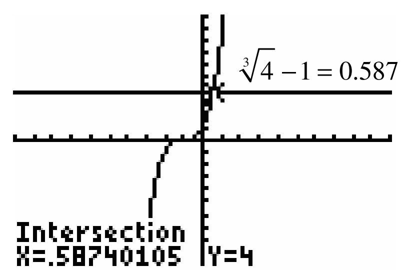
27. $(x+1)^3<4$

$\sqrt[3]{(x+1)^3}<\sqrt[3]{4}$

$x+1<\sqrt[3]{4}$

$x<\sqrt[3]{4}-1$

Applying the intersections of graphs method



[–10, 10] by [–10, 10]

Considering the graph, the solution is $\left(-\infty,\sqrt[3]{4}-1\right)$ or approximately $(-\infty, 0.587)$.

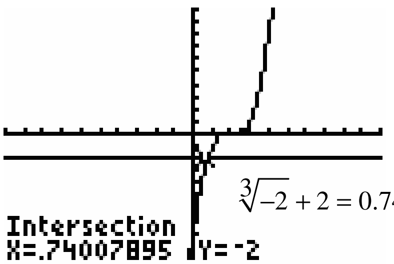
28. $(x-2)^3 \ge -2$

$\sqrt[3]{(x-2)^3} \ge \sqrt[3]{-2}$

$x-2 \ge \sqrt[3]{-2}$

$x \ge \sqrt[3]{-2}+2$

Applying the intersections of graphs method



[–10, 10] by [–10, 10]

Considering the graph, the solution is $\left[\sqrt[3]{-2}+2,\infty\right)$ or approximately $[0.74,\infty)$.

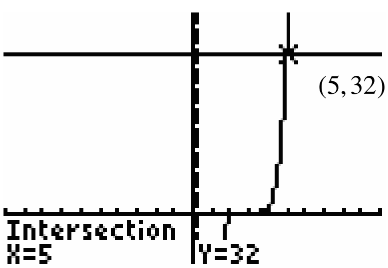
29. $(x-3)^5 < 32$

$\sqrt[5]{(x-3)^5} < \sqrt[5]{32}$

$x-3<2$

$x<5$

Applying the intersections of graphs method



[–10, 10] by [–10, 40]

Considering the graph, the solution is $(-\infty,5)$.

30. $(x+5)^4 > 16$

$(x+5)^4 = 16$

$\sqrt[4]{(x+5)^4} = \pm\sqrt[4]{16}$

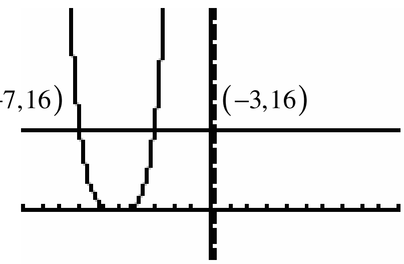
$x+5=\pm 2$

$x=-5\pm 2$

$x=-3, x=-7$

sign of $(x+7)$	$\leftarrow - - -_{-7} - - -_{-3} + + + \rightarrow$
sign of $(x+3)$	$\leftarrow - - -_{-7} + + +_{-3} + + + \rightarrow$
sign of $(x+7)(x+3)$	$\leftarrow + + +_{-7} - - -_{-3} + + + \rightarrow$

Applying the intersections of graphs method



[–10, 10] by [–10, 40]

Considering the graph, the solution is $(-\infty,-7)\cup(-3,\infty)$.

31. $|2x-1|<3$

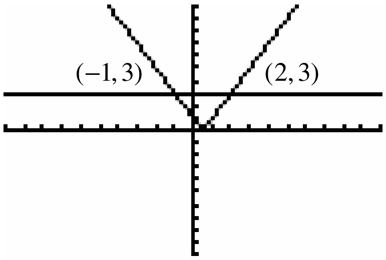
AND

$2x-1<3$	$2x-1>-3$
$2x<4$	$2x>-2$
$x<2$	$x>-1$

$x<2$ and $x>-1$

$(-1,2)$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]

32. $|3x+1| \le 5$

AND

$3x+1 \le 5 \quad | \quad 3x+1 \ge -5$

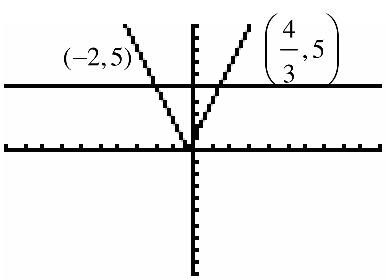
$3x \le 4 \quad | \quad 3x \ge -6$

$x \le \frac{4}{3} \quad | \quad x \ge -2$

$x \le \frac{4}{3}$ and $x \ge -2$

$\left[-2, \frac{4}{3}\right]$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 10] by [–10, 10]

33. $|x-6| \ge 2$

OR

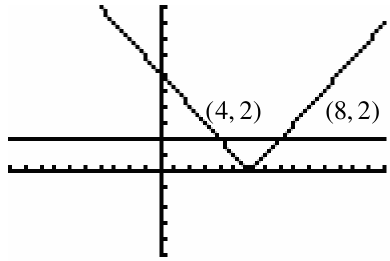
$x-6 \ge 2 \quad | \quad x-6 \le -2$

$x \ge 8 \quad | \quad x \le 4$

$x \le 4$ or $x \ge 8$

$(-\infty, 4] \cup [8, \infty)$

Using the intersections of graphs method to check the solution graphically yields,



[–10, 15] by [–5, 10]

34. $|x+8| > 7$

OR

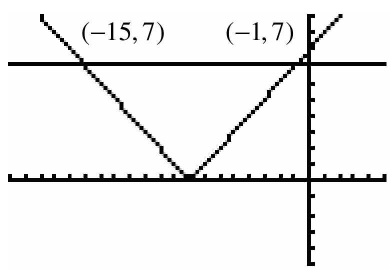
$x+8 > 7 \quad | \quad x+8 < -7$

$x > -1 \quad | \quad x < -15$

$x < -15$ or $x > -1$

$(-\infty, -15) \cup (-1, \infty)$

Using the intersections of graphs method to check the solution graphically yields,



[–20, 5] by [–5, 10]

35. a. $x \le -2$ or $x \ge 3$, or $(-\infty, -2] \cup [3, \infty)$

b. $-2 < x < 3$, or $(-2, 3)$

36. a. $-4 \le x \le 2$, or $[-4, 2]$

b. $x < -4$ or $x > 2$, or $(-\infty, -4) \cup (2, \infty)$

37. a. no solution, since no part of the graph is above the x- axis

b. all real numbers $(-\infty, \infty)$, since the entire graph is below the x- axis

38. a. all real numbers $(-\infty, \infty)$, since the entire graph is above the x- axis

b. no solution, since no part of the graph is below the x- axis

39. $f(x) \le g(x)$, that is, the graph of $f(x)$ is below the graph of $g(x)$, where x is in the interval $[-3, 2.5]$.

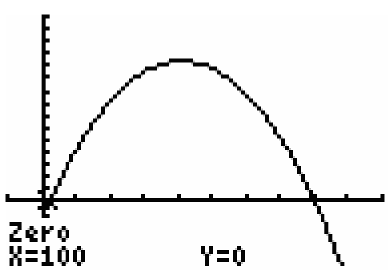
40. $f(x) \le g(x)$, that is, the graph of $f(x)$ is below the graph of $g(x)$, where x is in the interval $(-\infty, -3.5] \cup [1, 4.5]$.

Section 4.4 Exercises

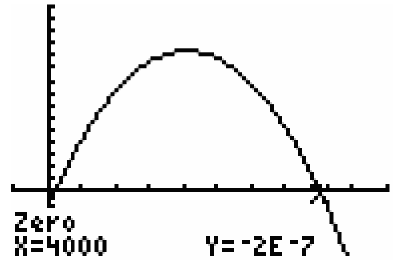
41. $P(x) > 0$

$-0.3x^2 + 1230x - 120{,}000 > 0$

Applying the x-intercept method



[–500, 5000] by [–500,000, 1,500,000]



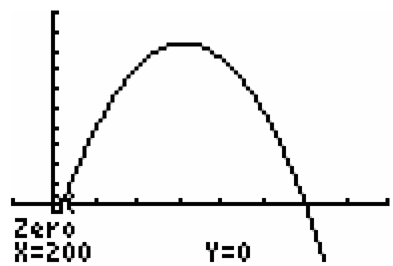
[–500, 5000] by [–500,000, 1,500,000]

Considering the graphs, the function is greater than zero over the interval $(100, 4000)$. Producing and selling between 100 and 4000 units, not inclusive, will result in a profit.

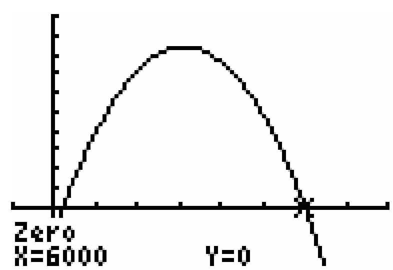
42. $P(x) > 0$

$-0.01x^2 + 62x - 12{,}000 > 0$

Applying the x-intercept method



[–1000, 8000] by [–30,000, 100,000]



[–1000, 8000] by [–30,000, 100,000]

Considering the graphs, the function is greater than zero over the interval $(200, 6000)$. Producing and selling between 200 and 6000 units, not inclusive, will result in a profit.

43. $P(x) = R(x) - C(x)$

$= \left(200x - 0.01x^2\right) - \left(38x + 0.01x^2 + 16{,}000\right)$

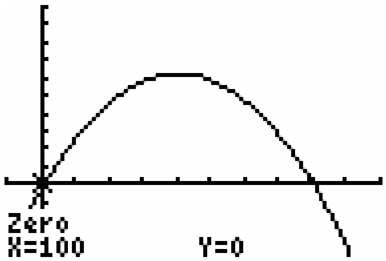
$= 200x - 0.01x^2 - 38x - 0.01x^2 - 16{,}000$

$= -0.02x^2 + 162x - 16{,}000$

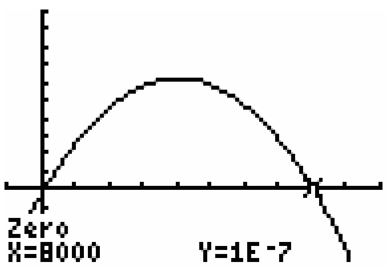
$P(x) > 0$

$-0.02x^2 + 162x - 16{,}000 > 0$

Applying the x-intercept method



[–1000, 10,000] by [–200,000, 500,000]



[–1000, 10,000] by [–200,000, 500,000]

Considering the graphs, the function is greater than zero over the interval $(100, 8000)$. Producing and selling between 100 and 8000 units, not inclusive, will result in a profit.

44. $P(x) = R(x) - C(x)$

$= \left(600x - 0.01x^2\right) - \left(77x + 0.02x^2 + 52{,}000\right)$

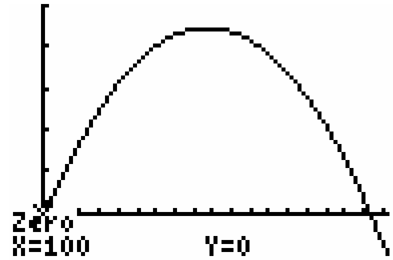
$= 600x - 0.01x^2 - 77x - 0.02x^2 - 52{,}000$

$= -0.03x^2 + 523x - 52{,}000$

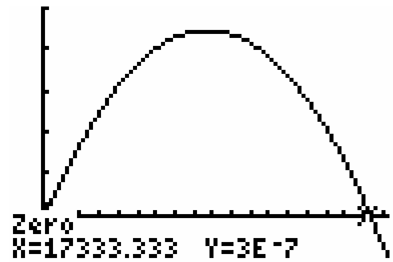
$P(x) > 0$

$-0.03x^2 + 523x - 52{,}000 > 0$

Applying the x-intercept method



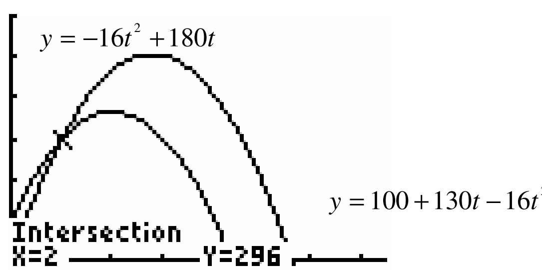
[–1000, 20,000] by [–500,000, 2,500,000]



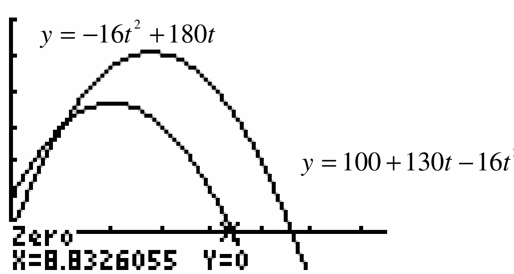
[–1000, 20,000] by [–500,000, 2,500,000]

Considering the graphs, the function is greater than zero over the interval $(100,\ 17{,}333.33)$. Producing and selling between 100 and 17,333 units will result in a profit.

45. Applying the intersection of graphs method



[0, 15] by [–10, 600]



[0, 15] by [–10, 600]

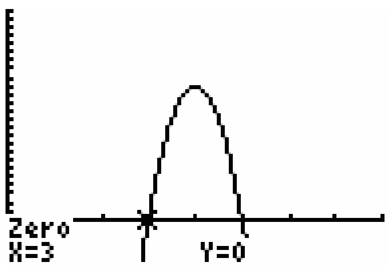
Considering the graphs, the second projectile is above the first projectile over the interval $(2, 8.83)$. Between 2 seconds and 8.83 seconds the height of the second projectile exceeds the height of the first projectile.

46. $s \geq 240$

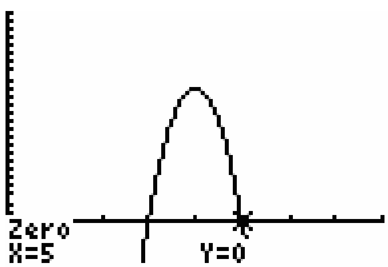
$128t - 16t^2 \geq 240$

$-16t^2 + 128t - 240 \geq 0$

Applying the *x*-intercept method



[0, 8] by [–5, 25]



[0, 8] by [–5, 25]

Considering the graphs, the function is greater than or equal to zero over the interval $[3, 5]$. Therefore the height of the rocket is greater than or equal to 240 feet when the time is between 3 and 5 seconds inclusive.

47.

If domestic sales are at least $6,000,000,000,

$y \geq 6{,}000$ (6000 million). Therefore,

$$-13.898x^2 + 255.467x + 5425.618 \geq 6000$$

$$\boxed{-13.898x^2 + 255.467x - 574.382 \geq 0}$$

Solve the simplified problem above.

$$-13.898x^2 + 255.467x - 574.382 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

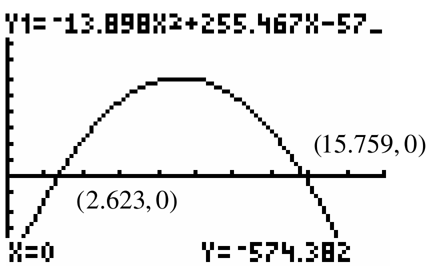
$$x = \frac{-(255.467) \pm \sqrt{(255.467)^2 - 4(-13.898)(-574.382)}}{2(-13.898)}$$

$$x = \frac{-255.467 \pm \sqrt{65263.38809 - 31931.04414}}{-27.796}$$

$$x = \frac{-255.467 \pm \sqrt{33332.34395}}{-27.796}$$

$x = 2.623, x = 15.759$

Applying the x-intercept method



[0, 20] by [–500, 1000]

Considering the graph, the equation is greater than or equal to zero over the interval $[2.623, 15.759]$. Therefore, the sales (in millions) of fine-cut cigarettes in Canada after the year 1980 total at least $6 billion between 1983 and 1995, inclusive.

48.

If the population is above 6,000,000, then $y > 6$. Therefore,

$-0.00036x^2 + 0.0385x + 5.823 > 6.$

$\boxed{-0.00036x^2 + 0.0385x - 0.177 > 0}$

Solve the simplified problem above.

$-0.00036x^2 + 0.0385x - 0.177 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.0385) \pm \sqrt{(0.0385)^2 - 4(-0.00036)(-0.177)}}{2(-0.00036)}$$

$$x = \frac{-0.0385 \pm \sqrt{0.00148225 - 0.00025488}}{-0.00072}$$

$$x = \frac{-0.0385 \pm \sqrt{0.00122737}}{-0.00072}$$

$$x = \frac{-0.0385 \pm 0.0350338408}{-0.00072}$$

$x = 4.814, x = 102.130$

Therefore, world population is above 6 billion between 1995 and 2092 inclusive.

49.

If the percent is at most 12.62, then $y \leq 12.62$. Therefore,

$0.003x^2 - 0.438x + 20.18 \leq 12.62$

$\boxed{0.003x^2 - 0.438x + 7.56 \leq 0}$

Solve the simplified problem above.

$0.003x^2 - 0.438x + 7.56 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.438) \pm \sqrt{(-0.438)^2 - 4(0.003)(7.56)}}{2(0.003)}$$

$$x = \frac{0.438 \pm \sqrt{0.191844 - 0.09072}}{0.006}$$

$$x = \frac{0.438 \pm \sqrt{0.101124}}{0.006}$$

$$x = \frac{0.438 \pm 0.318}{0.006}$$

$x = 126, x = 20$

Thus between 1920 and 2026, inclusive, the percentage of the U.S. population that is foreign born is at most 12.62%.

50.

If the GDP is less than 4 trillion, then $y < 4000$. Therefore,

$3.99x^2 - 432.50x + 12,862.21 < 4000.$

$\boxed{3.99x^2 - 432.50x + 8,862.21 < 0}$

Solve the simplified problem above.

$$3.99x^2 - 432.50x + 8862.21 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-432.50) \pm \sqrt{(-432.50)^2 - 4(3.99)(8862.21)}}{2(3.99)}$$

$$x = \frac{432.50 \pm \sqrt{187056.25 - 141440.8716}}{7.98}$$

$$x = \frac{432.50 \pm \sqrt{45615.3784}}{7.98}$$

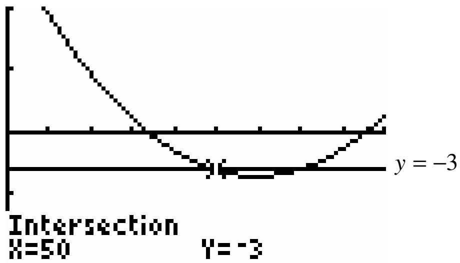
$$x = \frac{432.50 \pm 213.57757}{7.98}$$

$x = 80.962,\ x = 27.434$

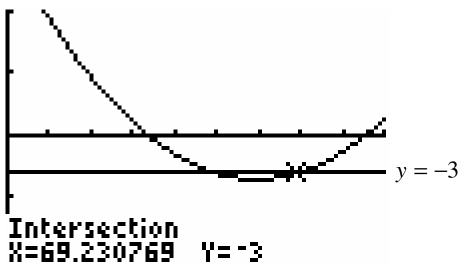
Thus adding each result to the year 1900, gives 1981 and 1928. The gross domestic product was less than \$4 trillion between 1928 and 1981.

51. $y = 0.0052x^2 - 0.62x + 15.0$

Applying the intersection of graphs method



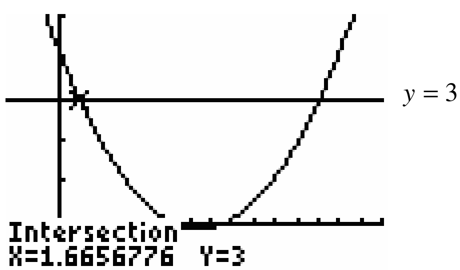
[0, 90] by [–10, 10]



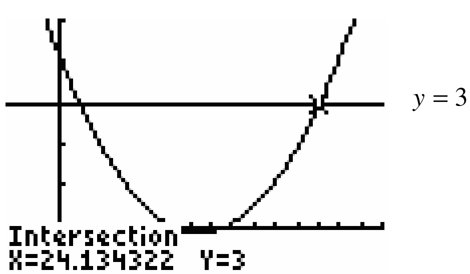
Considering the graphs, the wind chill temperature is $-3°F$ or below when the wind speed is between 50 mph and 69.2 mph.

52. $p = 0.025x^2 - 0.645x + 4.005$

Applying the intersection of graphs method



[–5, 30] by [–1, 5]

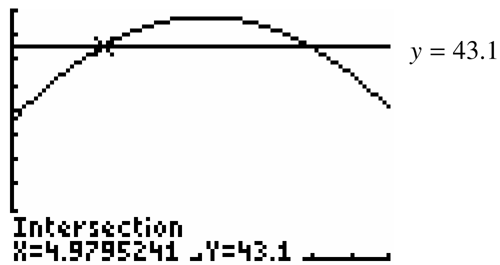


[–5, 30] by [–1, 5]

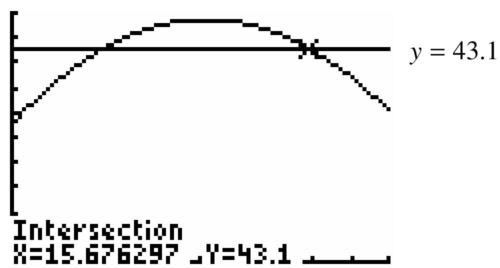
Considering the graphs, the percentage change in hotel room supply is less than 3% over the interval $(1.67, 24.13)$. Between 2000 and 2022 inclusive percentage change in hotel room supply is less than 3%.

53. $y = -0.1967x^2 + 4.063x + 27.7455$

Applying the intersection of graphs method



[0, 20] by [0, 50]



Considering the graphs, the % of marijuana use is above 43.1% over the interval $(4.98, 15.68)$. Thus between 1995 and 2005, the % of marijuana use is above 43.1%.

54.

If the number of airplane crashes is below 3146, then $y < 3.146$. Therefore,

$0.0057x^2 - 0.197x + 3.613 < 3.146$

$\boxed{0.0057x^2 - 0.197x + 0.467 < 0}$

Solve the simplified question above.

$$0.0057x^2 - 0.197x + 0.467 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.197) \pm \sqrt{(-0.197)^2 - 4(0.0057)(0.467)}}{2(0.0057)}$$

$$x = \frac{0.197 \pm \sqrt{0.038809 - 0.0106476}}{0.0114}$$

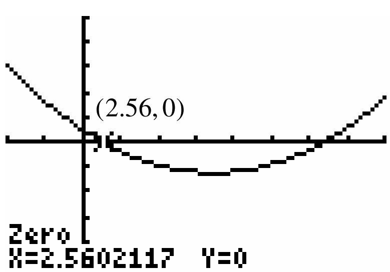
$$x = \frac{0.197 \pm \sqrt{0.0281614}}{0.0114}$$

$$x = \frac{0.197 \pm 0.1678135871}{0.0114}$$

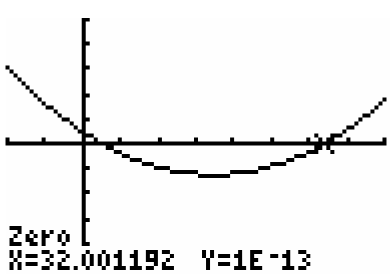
$x = 32.001,\ x = 2.560$

Find the year: 1980 + 32 = 2012 and 1980 + 2.56 = 1983. Thus, the number of airplane crashes is below 3146 in the years between 1983 and 2012.

Applying the x-intercept method



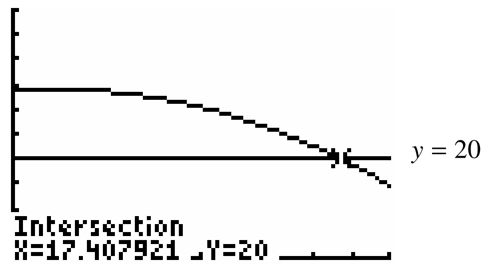
[–10, 40] by [–5, 5]



Considering the graph, the equation is less than 3146 over the interval $(2.56, 32)$. Therefore, the number of plane crashes is less than 3146 between 1983 and 2012, inclusive.

55. $y = -0.061x^2 + 0.275x + 33.698$

Applying the intersection of graphs method



[0, 20] by [0, 50]

Therefore, the percent of high school students who smoked cigarettes on 1 or more of the 30 days preceding the survey, is greater than 20% from 1990 to 2007.

56. a. Let x equal a person's height in inches. Then, $|x - 68| > 8$ represents heights that will be uncomfortable.

b.

$$|x - 68| > 8$$

OR

$x - 68 > 8$	$x - 68 < -8$
$x > 76$	$x < 60$

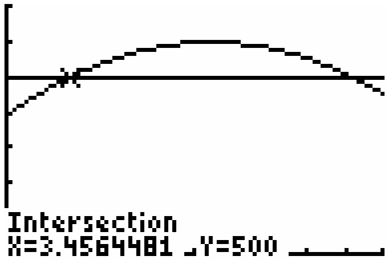
$x < 60$ or $x > 76$

$(-\infty, 60) \cup (76, \infty)$

In the context of the problem, people with heights below 60 inches, or above 76 inches, will be uncomfortable

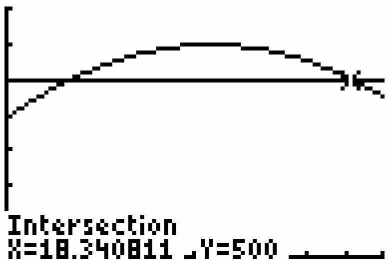
57. $S(x) = -1.751x^2 + 38.167x + 388.997$

Applying the intersection of graphs method



$y = 500$

[0, 20] by [0, 50]



$y = 500$

Therefore, retail sales are above 500 billion dollars between the years 2004 and 2018.

58. a. Let x equal voltage. Then, $|x - 220| \le 10$ represents voltages that allow the oven to work normally.

b.

$$|x - 220| \le 10$$

AND

$x - 220 \le 10$	$x - 220 \ge -10$
$x \le 230$	$x \ge 210$

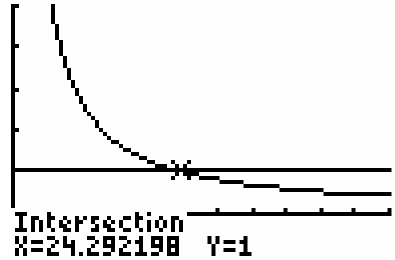
$$x \le 230 \text{ and } x \ge 210$$

$$[210, 230]$$

In the context of the problem, voltages between 210 volts and 230 volts, inclusive, will allow the oven to work normally

59. $y = 34.394x^{-1.109}$

Applying the intersection of graphs method



[0, 20] by [0, 50]

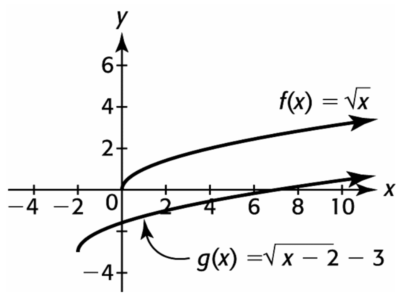
Therefore, based on the current model, the purchasing power of a 1983 dollar, is less than $1.00, from 1985 through 2012.

Chapter 4 Skills Check

1. The graph of the function is shifted 8 units right and 7 units up.

2. The graph of the function is shifted 1 unit left, vertically stretched by a factor of 2, and reflected over the x-axis.

3. a.



b. The graph of *g(x)* is shifted two units left and three units down in comparison to *f(x)*.

4. $g(x)=\sqrt{x+2}-3$

$x+2\geq 0$

$x\geq -2$

Domain: $[-2,\infty)$.

5. $y=(x-6)^{1/3}+4$

6. $y=3x^{1/3}-5$

7. The graph matches equation f.

8. The graph matches equation c.

9. The graph matches equation e.

10. To test for origin symmetry:

Let $x=-x$ and check if $f(-x)=-f(x)$

since $f(x)=x^3-4x$,

$f(-x)=(-x)^3-4(-x)$

$f(-x)=-x^3+4x$

Thus, $f(-x)=-f(x)$

Since $f(-x)=-f(x)$, the graph of the equation is symmetric with respect to the origin.

11. To test for y-axis symmetry:

Let $x=-x$ and check if $f(-x)=f(x)$

since $f(x)=-x^2+5$,

$f(-x)=-(-x)^2+5$

$f(-x)=-x^2+5$

Thus, $f(-x)=f(x)$

Since $f(-x)=f(x)$, the graph of the equation is symmetric with respect to the y-axis.

12. Since an odd function has origin symmetry, one should test for origin symmetry.

Let $x=-x$ and check if $f(-x)=-f(x)$

since $f(x)=-\dfrac{2}{x}$,

$f(-x)=-\dfrac{2}{-x}$

$f(-x)=\dfrac{2}{x}$

Thus, $f(-x)=-f(x)$

Since $f(-x)=-f(x)$, the graph of the equation is symmetric with respect to the origin and the function can be called an odd function.

13. $(f+g)(x) =$
$(3x^2 - 5x) + (6x - 4) =$
$3x^2 + x - 4$

14. $(h-g)(x) =$
$(5 - x^3) - (6x - 4) =$
$-x^3 - 6x + 9$

15. $(g \cdot f)(x) =$
$(6x-4)\cdot(3x^2 - 5x) =$
$18x^3 - 42x^2 + 20x$

16. $(h/g)(x) =$
$(5 - x^3)/(6x - 4)$

17. $(f-g)(x) =$
$(3x^2 - 5x) - (6x - 4) =$
$3x^2 - 11x + 4$, then
$(f-g)(-2) =$
$3(-2)^2 - 11(-2) + 4 = 38$

18. $(f \circ g)(x) = f(g(x))$
$3(6x-4)^2 - 5(6x-4) =$
$3(36x^2 - 48x + 16) - 30x + 20 =$
$108x^2 - 144x + 48 - 30x + 20 =$
$108x^2 - 174x + 68$

19. $(g \circ f)(x) = g(f(x))$
$6(3x^2 - 5x) - 4 =$
$18x^2 - 30x - 4$

20. $(g \circ h)(x) = g(h(x)) =$
$6(5 - x^3) - 4 =$
$30 - 6x^3 - 4 = 26 - 6x^3$, then
$(g \circ h)(-3) = 26 - 6(-3)^3 = 188$

21. a. $f\left(g\left(x\right)\right) = 2\left(\frac{x+5}{2}\right) - 5 = x$

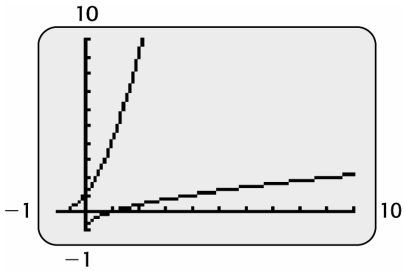
$g\left(f\left(x\right)\right) = \frac{(2x-5)+5}{2} = x$

b. Since $f(g(x)) = g(f(x))$, *f(x)* and *g(x)* are inverse functions.

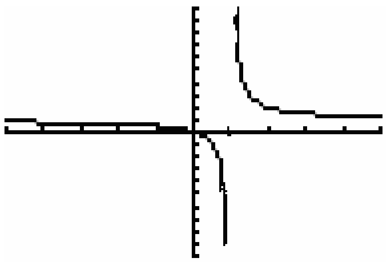
22.
$$\begin{aligned} f(x) &= 3x - 2 \\ y &= 3x - 2 \\ x &= 3y - 2 \\ 3y &= x + 2 \\ y &= \frac{x+2}{3} \\ f^{-1}(x) &= \frac{x+2}{3} \end{aligned}$$

23.
$$\begin{aligned} g(x) &= \sqrt[3]{x-1} \\ y &= \sqrt[3]{x-1} \\ x &= \sqrt[3]{y-1} \\ x^3 &= \left(\sqrt[3]{y-1}\right)^3 \\ x^3 &= y - 1 \\ y &= x^3 + 1 \\ g^{-1}(x) &= x^3 + 1 \end{aligned}$$

24. Note that $f\left(x\right)$ is one-to-one on the given interval [–1, 10]. Therefore,
$$\begin{aligned} f\left(x\right) &= \left(x+1\right)^2 \\ y &= \left(x+1\right)^2 \\ x &= \left(y+1\right)^2 \\ y + 1 &= \sqrt{x}, \quad x \ge 0, y \ge -1 \\ y &= -1 + \sqrt{x} \\ f^{-1}\left(x\right) &= -1 + \sqrt{x} \end{aligned}$$



25.



[–5, 5] by [–10, 10]

Since the function passes the horizontal line test. It is one-to-one.

26. Since $f(x) = x^2 - 3x$ is a parabola whose graph opens upward, it would fail the horizontal line test, and would therefore not have an inverse.

27. $\sqrt{4x^2+1} = 2x+2$

$\left(\sqrt{4x^2+1}\right)^2 = (2x+2)^2$

$4x^2+1 = 4x^2+8x+4$

$8x = -3$

$x = -\frac{3}{8}$

28.

$\sqrt{3x^2-8} + x = 0$

$\left(\sqrt{3x^2-8}\right)^2 = (-x)^2$

$3x^2 - 8 = x^2$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

Substituting to check $x = 2$:

$\sqrt{3(2)^2-8} + 2 \stackrel{?}{=} 0$

$4 \neq 0$

Substituting to check $x = -2$:

$\sqrt{3(-2)^2-8} + (-2) \stackrel{?}{=} 0$

$0 = 0$

The only solution is $x = -2$.

29. $x^2 - 7x \leq 18$

$x^2 - 7x - 18 \leq 0$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$x = 9, x = -2$

sign of $(x-9)$ $\leftarrow - - -_{-2} - - -_{9} + + + \rightarrow$

sign of $(x+2)$ $\leftarrow - - -_{-2} + + +_{9} + + + \rightarrow$

sign of $(x-9)(x+2)$ $\leftarrow + + +_{-2} - - -_{9} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is $[-2, 9]$.

30. $2x^2+5x\ge 3$

$2x^2+5x-3\ge 0$

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$x=\dfrac{1}{2}, x=-3$

sign of $(2x-1)$	$\leftarrow - - -_{-3} - - -_{\frac{1}{2}} + + + \rightarrow$
sign of $(x+3)$	$\leftarrow - - -_{-3} + + +_{\frac{1}{2}} + + + \rightarrow$
sign of $(2x-1)(x+3)$	$\leftarrow + + +_{-3} - - -_{\frac{1}{2}} + + + \rightarrow$

Considering the inequality symbol in the simplified question, the solution is

$(-\infty,-3]\cup\left[\dfrac{1}{2},\infty\right)$.

31. $|2x-4|\le 8$

AND

$2x-4\le 8$	$2x-4\ge -8$
$2x\le 12$	$2x\ge -4$
$x\le 6$	$x\ge -2$

$x\ge -2$ and $x\le 6$

$[-2,6]$

32. $|4x-3|\ge 15$

OR

$4x-3\ge 15$	$4x-3\le -15$
$4x\ge 18$	$4x\le -12$
$x\ge \dfrac{9}{2}$	$x\le -3$

$x\ge \dfrac{9}{2}$ or $x\le -3$

$(-\infty,-3]\cup\left[\dfrac{9}{2},\infty\right)$

33. $(x-4)^3<4096$

$\sqrt[3]{(x-4)^3}<\sqrt[3]{4096}$

$x-4<16$

$x<20$

The solution is $(-\infty,20)$.

34. $(x+2)^2\ge 512$

$\sqrt{(x+2)^2}=\sqrt{512}$

$x+2=\pm\sqrt{256\bullet 2}$

$x+2=\pm 16\sqrt{2}$

$x=-2\pm 16\sqrt{2}$

Considering $(x+2)^2-512\ge 0$

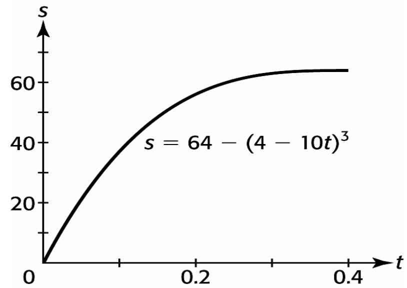
$\leftarrow + + + +_{-2-16\sqrt{2}} - - - -_{-2+16\sqrt{2}} + + + + \rightarrow$

The solution is

$\left(-\infty,-2-16\sqrt{2}\right]\cup\left[-2+16\sqrt{2},\infty\right)$.

Chapter 4 Review Exercises

35. a.



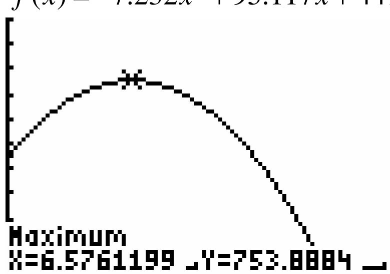
b. The bullet travels 64 inches in 0.4 seconds.

36. a. Note that the function is quadratic written in vertex form, $y = a(x-h)^2 + k$. The vertex is $(h,k) = (4,380)$. Therefore the maximum height occurs 4 seconds into the flight of the rocket.

b. Referring to part *a*, the maximum height of the rocket is 380 feet.

c. In comparison to $y = t^2$ the graph is shifted 4 units right, 380 up, stretched by a factor of 16, and reflected across the *x*-axis.

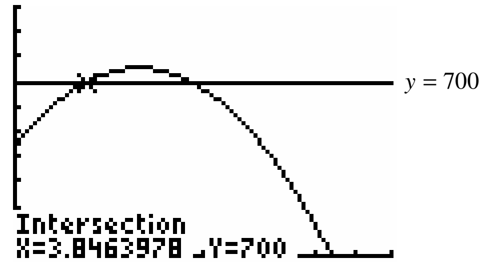
37. a. $f(x) = -7.232x^2 + 95.117x + 441.138$



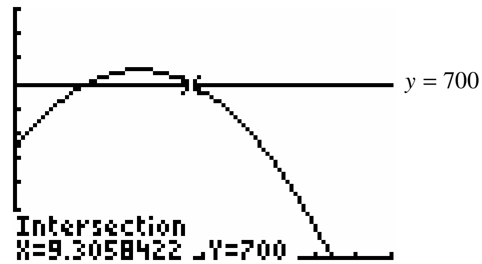
[0, 20] by [0, 1000]

Thus, the year in which the number of passengers was at a maximum was 2007.

b. Applying the intersection of graphs method



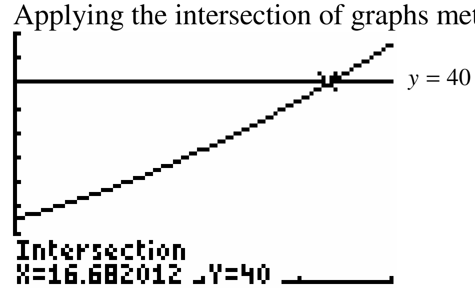
[0, 20] by [0, 1000]



From the graphs, the number of passengers was above 700 million between the years 2004 and 2009.

38. $y = 0.037x^2 + 1.018x + 12.721$

Applying the intersection of graphs method

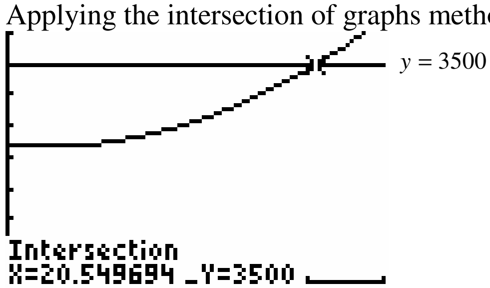


[0, 20] by [0, 50]

From 2007 through 2010, Americans spent more than 40 million dollars on their pets.

39. $y = 3.980x^2 - 17.597x + 2180.899$

Applying the intersection of graphs method

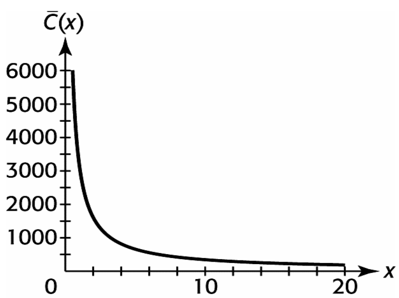


[0, 25] by [0, 4000]

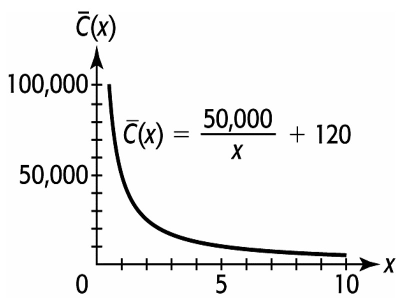
From 1990 to 2010, the number of postsecondary degrees earned was less than 3500.

40. **a.** $\bar{C}(x) = \dfrac{30x + 3150}{x}$

b.



41. **a.**

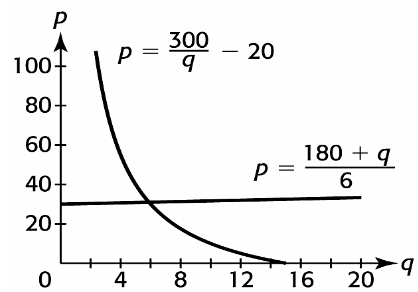


b. This is a decreasing function.

c. The graph of this function has a vertical stretch by a factor of 50,000 and a vertical shift 120 units up.

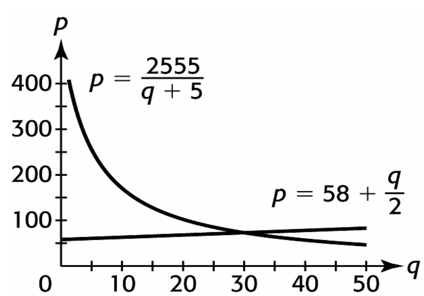
42. **a.** Since there is a variable in the denominator of the demand function, $p = \dfrac{300}{q} - 20$ is a shifted reciprocal function.

b.



43. **a.** Since there is a variable in the denominator of the demand function, $p = \dfrac{2555}{q+5}$ is a shifted reciprocal function.

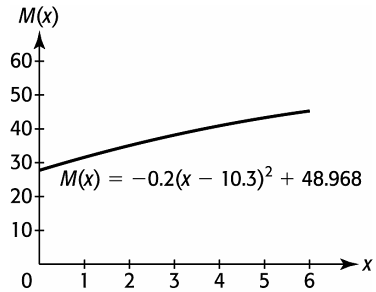
b.



44. **a.** The transformation of $f(x) = x^2$ includes a horizontal shift to the right of 10.3 units, a vertical shift up of 48.968 units, a vertical reflection across the x-axis, and a vertical compression by a factor of 0.2.

b. M(1) = % in 1991 = 31.67
M(2) = % in 1992 = 35.19
M(4) = % in 1994 = 41.03
M(6) = % in 1996 = 45.27

c.



d. The domain of the discrete function M is: {1, 2, 3, 4, 5, …, 16}.

45. a. $R(E(t)) = 0.165(0.017t^2 + 2.164t + 8.061) - 0.226$

$$= 0.002805t^2 + 0.35706t + 1.330065 - 0.226$$

$$= 0.002805t^2 + 0.35706t + 1.104065$$

The function calculates the revenue for Southwest Airlines given the number of years past 1990.

b. $R(E(t)) = 0.002805t^2 + 0.35706t + 1.104065$

$R(E(3)) = 0.002805(3)^2 + 0.35706(3) + 1.104065$

$R(E(3)) = 2.2008995 \approx 2.2$

In 1993 Southwest Airlines had revenue of $2.2 billion.

c. $E(7) = 0.017(7)^2 + 2.164(7) + 8.061$

$= 24.042$

In 1997 Southwest Airlines has 24,042 employees.

d. $R(E(t)) = 0.002805t^2 + 0.35706t + 1.104065$

$R(E(7)) = 0.002805(7)^2 + 0.35706(7) + 1.104065$

$R(E(7)) = 3.74093 \approx 3.7$

In 1997 Southwest Airlines had revenue of $3.7 billion.

46. a.

$$f(x) = 0.554x - 2.886$$
$$y = 0.554x - 2.886$$
$$x = 0.554y - 2.886$$
$$0.554y = x + 2.886$$
$$y = \frac{x + 2.886}{0.554}$$
$$f^{-1}(x) = \frac{x + 2.886}{0.554}$$

b. The inverse function calculates the mean length of the original prison sentence given the mean time spent in prison.

47. a.

$$f(x) = 5.582x + 28.093$$
$$y = 5.582x + 28.093$$
$$x = 5.582y + 28.093$$
$$5.582y = x - 28.093$$
$$y = \frac{x - 28.093}{5.582}$$
$$f^{-1}(x) = \frac{x - 28.093}{5.582}$$

b. The inverse function calculates the year in which the number of billion dollars was spent for higher education

c.

$$f(10) = 5.582(10) + 28.093$$
$$= 83.913$$
$$f^{-1}(83.913) = \frac{83.913 - 28.093}{5.582}$$
$$= \frac{55.82}{5.582} = 10$$

48. $P(x) = 34.394x^{-1.109}$

a. $f(x) = \dfrac{50}{1.6x} - 0.2$

f(x) is a transformed reciprocal function.

b. For $x = 50,\ (2010 - 1960)$

$$P(50) = 34.394(50)^{-1.109} = \$0.449$$
$$f(50) = \frac{50}{1.6(50)} - 0.2 = \$0.425$$

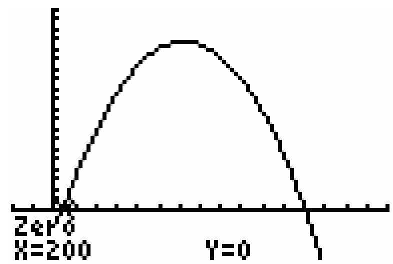
The function P(x) gives the better estimate of the purchasing power of a 1983 dollar in 2010.

49. $P(x) = -0.01x^2 + 62x - 12{,}000$

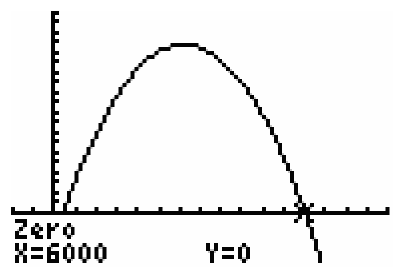
$P(x) > 0$

$-0.01x^2 + 62x - 12{,}000 > 0$

Applying the *x*-intercept method



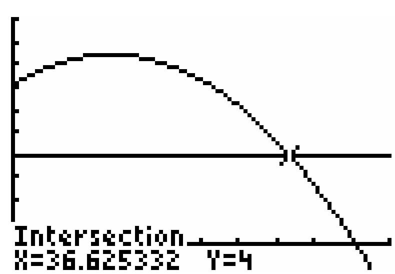
[–1000, 8000] by [–25,000, 100,000]



[–1000, 8000] by [–25,000, 100,000]

Manufacturing and producing between 200 and 6000 units will result in a profit.

50.

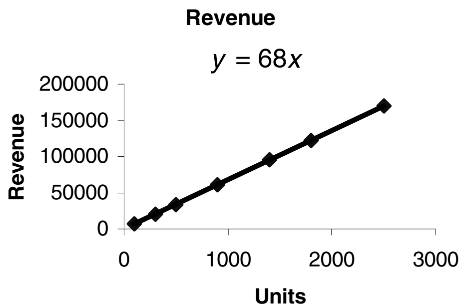


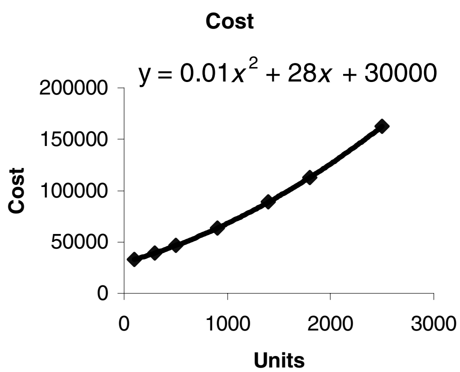
[0, 50] by [–1, 10]

The model indicates that the personal savings rate is above 4% between 1960 and 1996.

Group Activity/Extended Application I

1.





2. **a.** $P(x) = R(x) - C(x)$

$$= 68x - \left(0.01x^2 + 28x + 30{,}000\right)$$

$$= -0.01x^2 + 40x - 30{,}000$$

b.

0	–30,000
100	–26,100
600	–9600
1600	8400
2000	10,000
2500	7500

3. $R(x) = C(x)$

$$68x = 0.01x^2 + 28x + 30{,}000$$

$$0.01x^2 - 40x + 30{,}000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(0.01)(30{,}000)}}{2(0.01)}$$

$$x = \frac{40 \pm \sqrt{1600 - 1200}}{0.02}$$

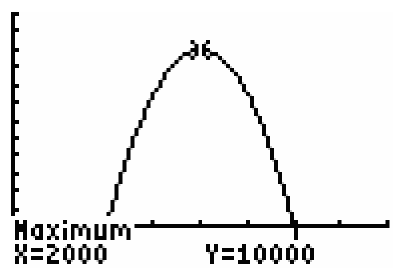
$$x = \frac{40 \pm \sqrt{400}}{0.02}$$

$$x = \frac{40 \pm 20}{0.02}$$

$$x = 3000, x = 1000$$

Producing and selling either 3000 or 1000 units forces the profit to equal the cost.

4.



[0, 4000] by [–2000, 12,000]

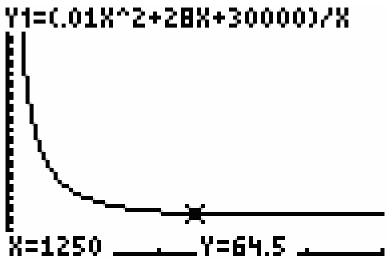
Producing and selling 2000 units results in a maximum profit of $10,000.

5. **a.** $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.01x^2 + 28x + 30{,}000}{x}$

b.

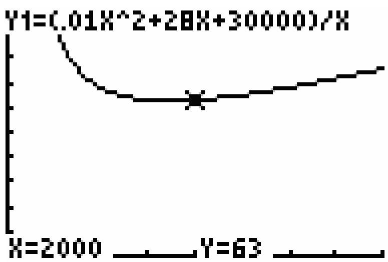
1	30,028.01
100	329.00
300	131.00
1400	63.43
2000	63.00
2500	65.00

c.

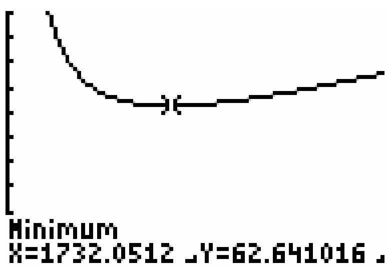


[0, 2500] by [0, 400]

6.



[0, 4000] by [0, 100]



[0, 4000] by [0, 100]

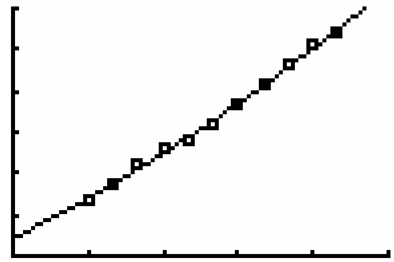
The minimum average cost is \$62.64 which occurs when 1732 units are produced and sold.

7. The values are different. However, they are relatively close together. The number of units that maximizes profit is most important. While keeping costs low is important, the key to a successful business is generating profit.

Group Activity/Extended Application II

1. $f(t) = 0.336t^2 + 15.116t + 21.608$

2.

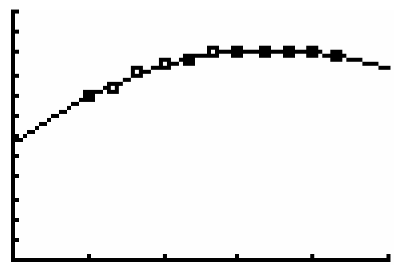


[0, 15] by [0, 300]

The model is a fairly good fit.

3. $g(t) = -0.216t^2 + 4.449t + 27.864$

4.

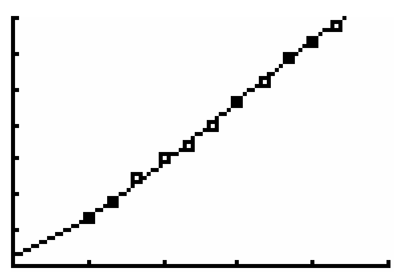


[0, 15] by [0, 60]

The model appears to be a good fit.

5. $(f \cdot g)(t) = -0.090t^4 - 2.850t^3 + 77.190x^2$
$+500.420x + 601.440$

6.



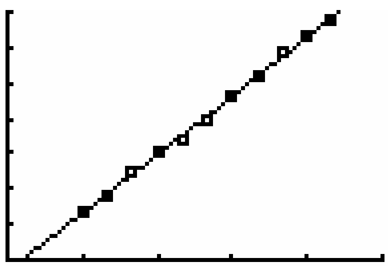
[0, 15] by [0, 14,000]

The model is a fairly good fit.

7.

Year	Average Revenue
1998	2728.91
1999	3548.58
2000	4956.07
2001	6081.12
2002	6813.07
2003	7921.82
2004	9223.57
2005	10390.64
2006	11782.55
2007	12754.48
2008	13459.93

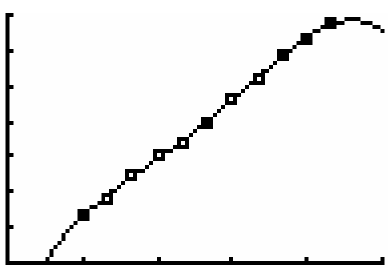
8. $r(t) = 1.788t^2 + 1080.376t - 624.321$



[0, 15] by [0, 14,000]

The model is a very good fit.

9. $s(t) = -1.722t^4 + 51.941t^3 - 540.319x^2$
$+3365.542x - 3873.392$



[0, 15] by [0, 14,000]

Both models fit about the same. Neither is an exact match.