Chapter 5
Exponential and Logarithmic Functions

Toolbox Exercises

1. a. $x^4 \bullet x^3 = x^{4+3} = x^7$

b. $\frac{x^{12}}{x^7} = x^{12-7} = x^5$

c. $(4ay)^4 = (4)^4 a^4 y^4 = 256a^4 y^4$

d. $\left(\frac{3}{z}\right)^4 = \frac{3^4}{z^4} = \frac{81}{z^4}$

e. $2^3 \bullet 2^2 = 2^{3+2} = 2^5 = 32$

f. $(x^4)^2 = x^{4 \bullet 2} = x^8$

2. a. $y^5 \bullet y = y^5 \bullet y^1 = y^{5+1} = y^6$

b. $\frac{w^{10}}{w^4} = w^{10-4} = w^6$

c. $(6bx)^3 = (6)^3 b^3 x^3 = 216b^3 x^3$

d. $\left(\frac{5z}{2}\right)^3 = \frac{5^3 z^3}{2^3} = \frac{125z^3}{8}$

e. $3^2 \bullet 3^3 = 3^{2+3} = 3^5 = 243$

f. $(2y^3)^4 = 2^4 y^{3 \bullet 4} = 16y^{12}$

3. $10^{5^0} = 10^{(1)} = 10$

4. $4^{2^2} = 4^4 = 256$

5. $x^{-4} \bullet x^{-3} = x^{-4+-3} = x^{-7} = \frac{1}{x^7}$

6. $y^{-5} \bullet y^{-3} = y^{-5+-3} = y^{-8} = \frac{1}{y^8}$

7. $(c^{-6})^3 = c^{-6 \bullet 3} = c^{-18} = \frac{1}{c^{18}}$

8. $(x^{-2})^4 = x^{-2 \bullet 4} = x^{-8} = \frac{1}{x^8}$

9. $\frac{a^{-4}}{a^{-5}} = a^{-4-(-5)} = a^1 = a$

10. $\frac{b^{-6}}{b^{-8}} = b^{-6-(-8)} = b^2$

11. $\left(x^{-\frac{1}{2}}\right)\left(x^{\frac{2}{3}}\right) = x^{-\frac{1}{2}+\frac{2}{3}} = x^{-\frac{3}{6}+\frac{4}{6}} = x^{\frac{1}{6}}$

12. $\left(y^{-\frac{1}{3}}\right)\left(y^{\frac{2}{5}}\right) = y^{-\frac{1}{3}+\frac{2}{5}} = y^{-\frac{5}{15}+\frac{6}{15}} = y^{\frac{1}{15}}$

13. $(3a^{-3}b^2)(2a^2b^{-4})$

$= 6a^{-3+2}b^{2+-4}$

$= 6a^{-1}b^{-2}$

$= \frac{6}{ab^2}$

14. $(4a^{-2}b^3)(-2a^4b^{-5})$

$= -8a^{-2+4}b^{3+-5}$

$= -8a^2b^{-2}$

$= \frac{-8a^2}{b^2}$

15. $\left(\frac{2x^{-3}}{x^2}\right)^{-2} = \left(2x^{-3-2}\right)^{-2}$

$= \left(2x^{-5}\right)^{-2}$

$= (2)^{-2}\left(x^{-5}\right)^{-2}$

$= \frac{1}{2^2}x^{-5\bullet-2}$

$= \frac{1}{4}x^{10}$

$= \frac{x^{10}}{4}$

16. $\left(\frac{3y^{-4}}{2y^2}\right)^{-3} = \left(\frac{2y^2}{3y^{-4}}\right)^3$

$= \frac{\left(2y^2\right)^3}{\left(3y^{-4}\right)^3}$

$= \frac{8y^6}{27y^{-12}}$

$= \frac{8y^{6-(-12)}}{27}$

$= \frac{8y^{18}}{27}$

17. $\frac{28a^4b^{-3}}{-4a^6b^{-2}} = -7a^{4-6}b^{-3-(-2)}$

$= -7a^{-2}b^{-1}$

$= \frac{-7}{a^2b}$

18. $\frac{36x^5y^{-2}}{-6x^6y^{-4}} = -6x^{5-6}y^{-2-(-4)}$

$= -6x^{-1}y^2$

$= \frac{-6y^2}{x}$

19. 4.6×10^7

20. 8.62×10^{11}

21. 9.4×10^{-5}

22. 2.78×10^{-6}

23. $437,200$

24. $7,910,000$

25. 0.00056294

26. 0.0063478

27. $\left(6.25\times10^7\right)\left(5.933\times10^{-2}\right)$

$(6.25\times5.933)\times10^{7+-2}$

37.08125×10^5

Rewriting in scientific notation

3.708125×10^6

28. $\frac{2.961\times10^{-2}}{4.583\times10^{-4}}$

$\frac{2.961}{4.583}\times10^{-2-(-4)}$

0.6460833515×10^2

Rewriting in scientific notation

6.460833515×10^1

29. $x^{1/2}\bullet x^{5/6} = x^{1/2+5/6} = x^{3/6+5/6} = x^{8/6} = x^{4/3}$

30. $y^{2/5}\bullet y^{1/4} = y^{2/5+1/4} = y^{8/20+5/20} = y^{13/20}$

31. $\left(c^{2/3}\right)^{5/2} = c^{2/3 \cdot 5/2} = c^{5/3}$

32. $\left(x^{3/2}\right)^{3/4} = x^{3/2 \cdot 3/4} = x^{9/8}$

33. $\dfrac{x^{3/4}}{x^{1/2}} = x^{3/4-1/2} = x^{3/4-2/4} = x^{1/4}$

34. $\dfrac{y^{3/8}}{y^{1/4}} = y^{3/8-1/4} = y^{3/8-2/8} = y^{1/8}$

Section 5.1 Skills Check

1. Functions c), d), and e) represent exponential functions. They both fit the form $y = a^x$, where a is a constant, $a > 0$ and $a \neq 1$.

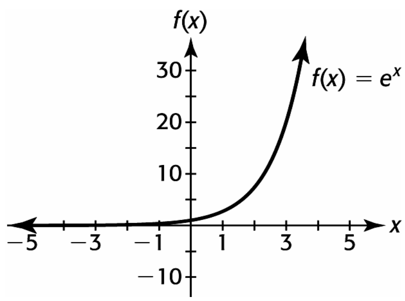
2. **a.** Growth. $k = 0.1 > 0$, and base is > 1.

b. Decay. $k = -1.4 < 0$, and base is > 1.

c. Decay. $k = -5 < 0$, and base is > 1.

d. Decay. $k = 3 > 0$, but base is < 1.

3. **a.**



b. $f(1) = e^1 = e \approx 2.718$

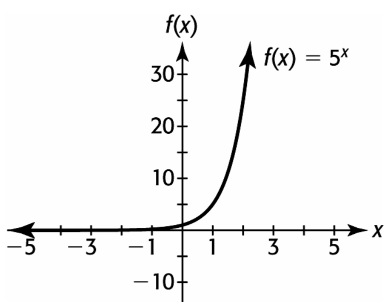
$f(-1) = e^{-1} = \frac{1}{e} \approx 0.368$

$f(4) = e^4 \approx 54.598$

c. $y = 0$, the x-axis.

d. $(0,1)$ since $f(0) = 1$.

4. **a.**



b. $f(1) = 5^1 = 5$

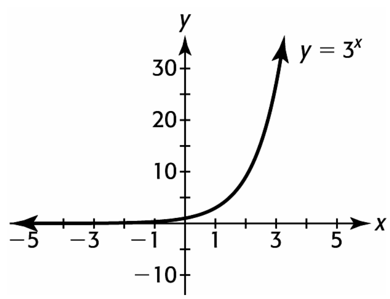
$f(3) = 5^3 = 125$

$f(-2) = 5^{-2} = \frac{1}{5^2} = \frac{1}{25} = 0.04$

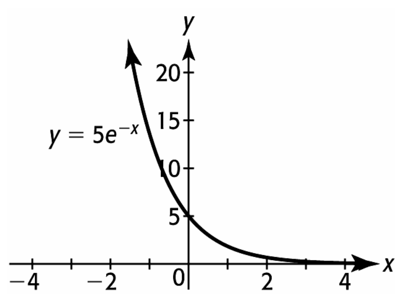
c. $y = 0$, the x-axis.

d. $(0,1)$ since $f(0) = 1$.

5.

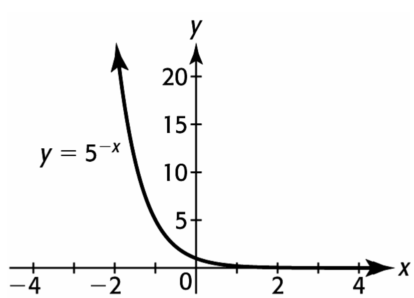


6.

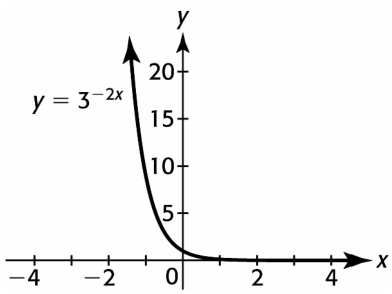


Notice that the y-intercept is (0, 5).

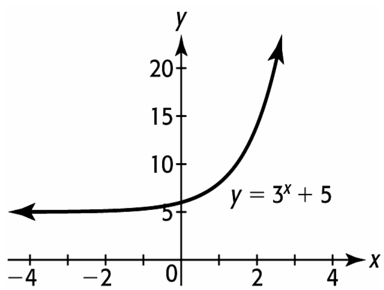
7.



8.

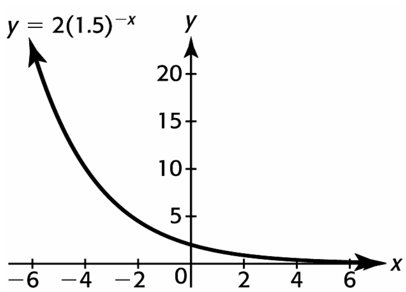


9.



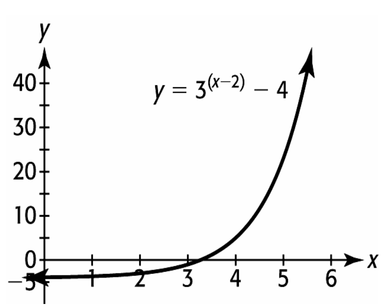
Notice that the y-intercept is $(0,6)$.

10.



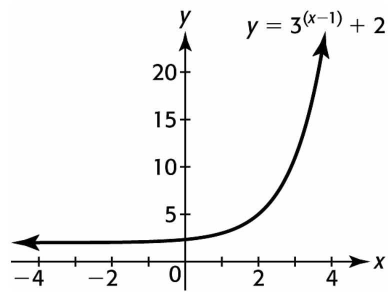
Notice that the y-intercept is $(0,2)$.

11.



Notice that the y-intercept is $(0,-3.\overline{8})$.

12.



Notice that the y-intercept is $(0,2.\overline{3})$.

13. The equation matches graph B.

14. The equation matches graph C.

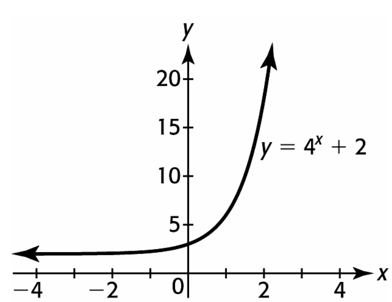
15. The equation matches graph A.

16. The equation matches graph F.

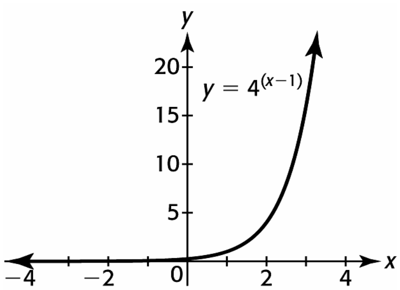
17. The equation matches graph E.

18. The equation matches graph D.

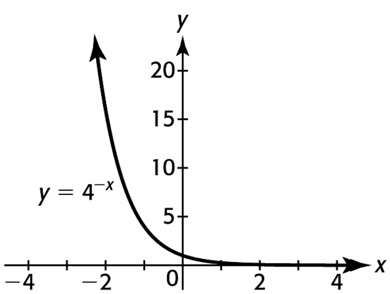
19. In comparison to 4^x, the graph has the same shape but shifted 2 units up.



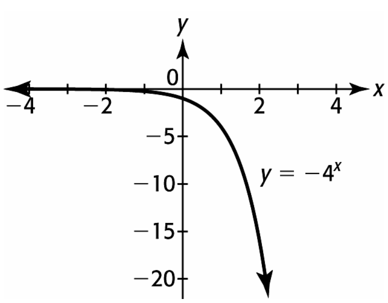
20. In comparison to 4^x, the graph has the same shape but has a shift 1 unit right.



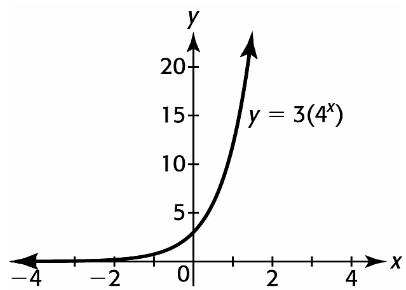
21. In comparison to 4^x, the graph has the same shape but is reflected across the *y*-axis.



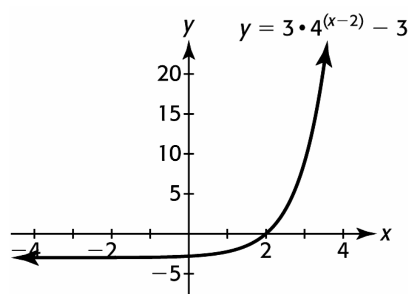
22. In comparison to 4^x, the graph has the same shape but is reflected across the *x*-axis.



23. In comparison to 4^x, the graph has a vertical stretch by a factor of 3.



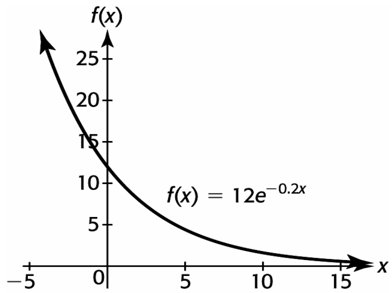
24. In comparison to 4^x, the graph has a vertical stretch by a factor of 3, a shift 2 units right, and a shift 3 units down.



25. Both graphs have a vertical stretch by a factor of 3 in comparison with 4^x. Therefore, the graph in Exercise 24 has the same shape as the graph in Exercise 23, but it has a shift 2 units right and 3 units down.

26. All are increasing except for Exercises 21 and 22, which are decreasing.

27. a.



b. $f(10) = 12e^{-0.2(10)} = 12e^{-2} = \frac{12}{e^2} \approx 1.624$

$f(-10) = 12e^{-0.2(-10)} = 12e^{2} \approx 88.669$

c. Since the function is decreasing, it represents decay. Notice that the y-intercept is $(0,12)$.

28. a. $y = 200\left(2^{-0.01(20)}\right)$

$= 200\left(2^{-0.2}\right)$

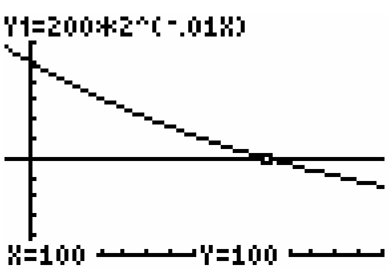
≈ 174.11

b.

X	Y1	Y2
97	102.1	100
98	101.4	100
99	100.7	100
100	100	100
101	99.309	100
102	98.623	100
103	97.942	100

X=100

The value of x is 100.



[–10, 150] by [–10, 250]

Section 5.1 Exercises

29. a. Let $x = 0$ and solve for y.

$y = 12{,}000\left(2^{-0.08 \cdot 0}\right)$

$= 12{,}000\left(2^{0}\right)$

$= 12{,}000(1)$

$= 12{,}000$

At the end of the ad campaign, sales were $12,000 per week.

b. Let $x = 6$ and solve for y.

$y = 12{,}000\left(2^{-0.08 \cdot 6}\right)$

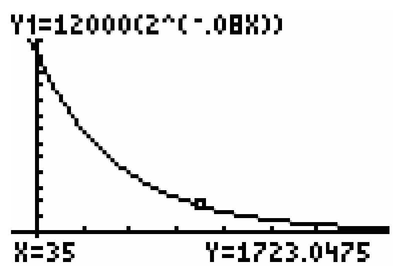
$= 12{,}000\left(2^{-0.48}\right)$

$= 12{,}000(0.716977624)$

≈ 8603.73

Six weeks after the end of the ad campaign, sales were $8603.73 per week.

c. No. Sales approach a level of zero but never actually reach that level. Consider the graph of the model below.



[–5, 75] by [–2000, 15,000]

30. a. Let $x = 0$ and solve for y.

$y = 10{,}000\left(3^{-0.05 \cdot 0}\right)$

$= 10{,}000\left(3^{0}\right)$

$= 10{,}000(1)$

$= 10{,}000$

At the end of the ad campaign, sales were $10,000 per week.

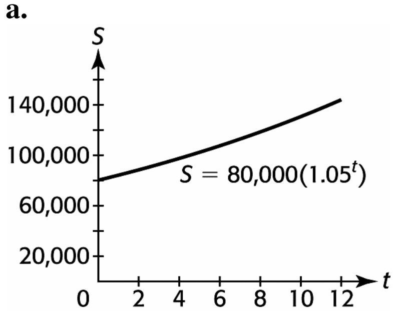
b. Let $x = 8$ and solve for y.

$$\begin{aligned} y &= 10{,}000\left(3^{-0.05 \cdot 8}\right) \\ &= 10{,}000\left(3^{-0.40}\right) \\ &= 10{,}000(0.644394015) \\ &\approx 6443.94 \end{aligned}$$

Eight weeks after the end of the ad campaign, sales were \$6,443.94 per week.

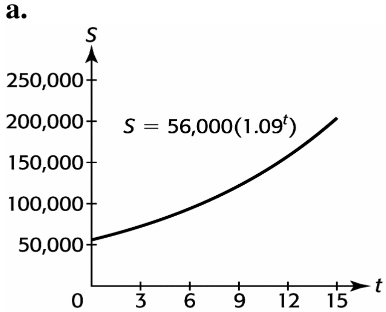
c. The equation is of the form $y = b^{kx}$, with $b = 3 > 1$ and $k = -0.05 < 0$. Since b is positive and $b > 1$, while k is negative, the function is decreasing.

31. a.



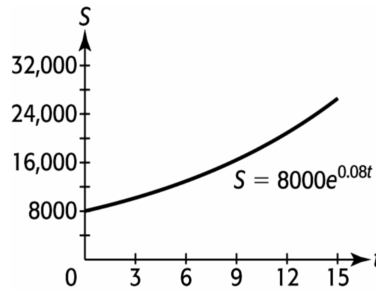
b. $$\begin{aligned} S &= 80{,}000\left(1.05^{10}\right) \\ &= 80{,}000(1.628894627) \\ &\approx 130{,}311.57 \text{ after 10 years} \end{aligned}$$

32. a.



b. $$\begin{aligned} S &= 56{,}000\left(1.09^{13}\right) \\ &= 56{,}000(3.065804612) \\ &\approx 171{,}685.06 \text{ after 13 years} \end{aligned}$$

33. a.

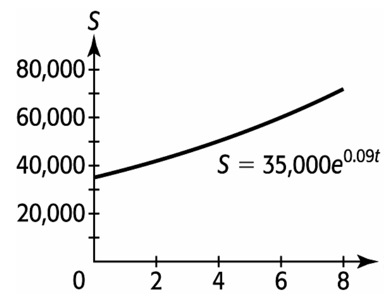


b. The future value will be \$20,000 in approximately 11.45 years.

c.

t (Year)	S (\$)
10	17,804.33
20	39,624.26
22	46,449.50

34. a.



b. The future value will be \$60,000 in about 6 years.

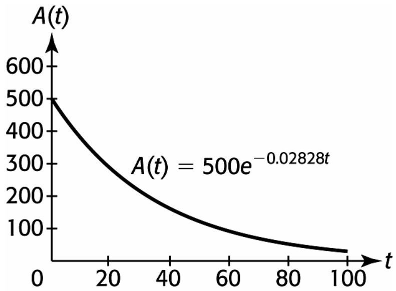
c.

t (Year)	S (\$)
10	86,086.11
15	135,009.89
22	253,496

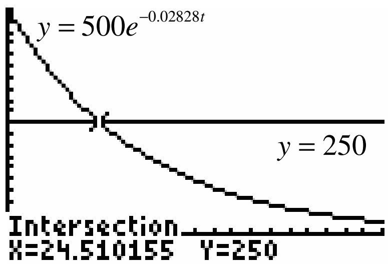
35. a. $$\begin{aligned} A(10) &= 500e^{-0.02828(10)} \\ &= 500e^{-0.2828} \\ &= 500(0.7536705069) \\ &\approx 376.84 \end{aligned}$$

Approximately 376.84 grams remain after 10 years.

b.



c.



[0, 100] by [–50, 500]

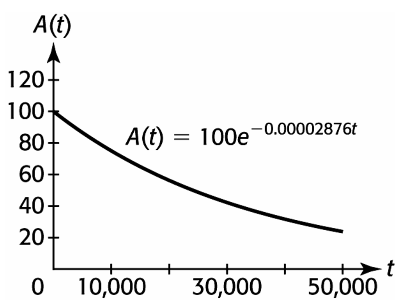
The half-life is approximately 24.5 years.

36. a.

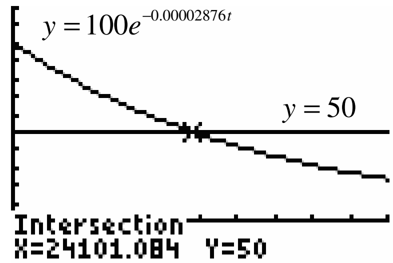
$$\begin{aligned} A(100) &= 100e^{-0.00002876(100)} \\ &= 100e^{-0.002876} \\ &= 100(0.9971281317) \\ &\approx 99.71 \end{aligned}$$

Approximately 99.71 grams remain after 100 years.

b.



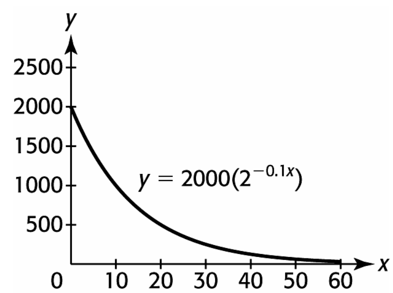
c.



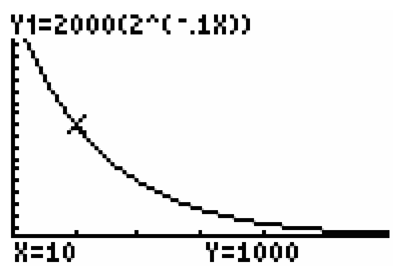
[0, 50,000] by [–20, 120]

The half-life is approximately 24,101 years.

37. a.



b.

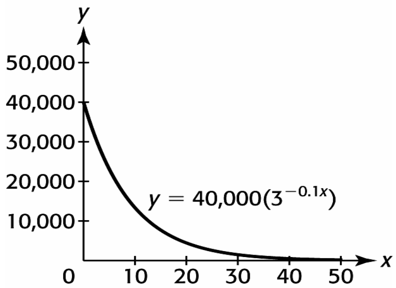


[0, 60] by [–200, 2000]

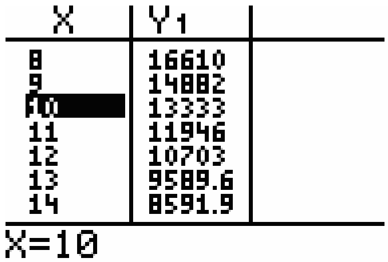
Ten weeks after the campaign ended, the weekly sales were $1000.

c. Weekly sales dropped by half, from $2000 to $1000, ten weeks after the end of the ad campaign. It is important for this company to advertise.

38. a.



b.



Ten weeks after the campaign ended, weekly sales were $13,333.

c. Yes. Spending $5000 to boost sales to $40,000, especially considering the rapid drop in sales over just a few weeks, is a good idea.

39. a. $P = 40{,}000\left(0.95^{20}\right)$

$= 40{,}000(0.3584859224)$

$= 14{,}339.4369$

$\approx 14{,}339.44$

The purchasing power will be $14,339.44.

b. Since the purchasing power of $40,000 will decrease to $14,339 over the next twenty years, people who retire at age 50 should continue to save money to offset the decrease due to inflation. Answers to part b) could vary.

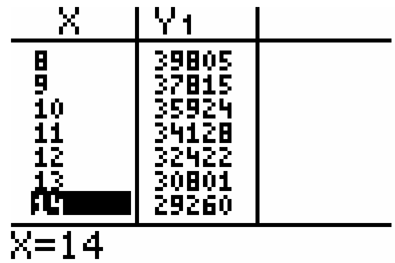
40. a. $P = 60{,}000\left(0.95^{4}\right)$

$= 60{,}000(0.81450625)$

$= 48{,}870.375$

After four years, the purchasing power drops to $48,870.38.

b.



After 14 or more years, the purchasing power is below $30,000.

41. a. $y = 100{,}000e^{0.05(4)}$

$= 100{,}000e^{0.2}$

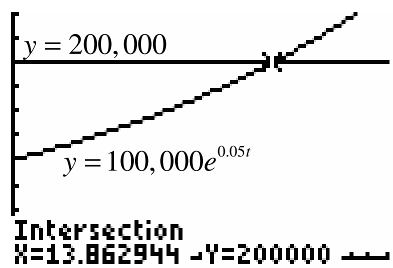
$= 100{,}000(1.221402758)$

$= 122{,}140.2758$

$\approx 122{,}140.28$

The value of this property after 4 years will be $122,140.28.

b.



[0, 20] by [–5000, 250,000]

The value of this property doubles in 13.86 years or approximately 14 years.

42. a. $v(0) = 850\left(1.04^0\right) = 850(1) = 850$

The table was worth $850 in 1990.

b. $v(15) = 850\left(1.04^{15}\right)$

$= 850(1.800943506)$

≈ 1530.80

In 2005, the value of the antique table is $1530.80.

c.

X	Y1	
13	1415.3	
14	1471.9	
15	1530.8	
16	1592	
17	1655.7	
18	1721.9	
19	1790.8	

X=18

The antique table doubles in value in approximately 2008.

43. a. Increasing. The exponent is positive for all values of $t \geq 0$.

b. $P(5) = 53{,}000e^{0.015(5)}$

$= 53{,}000e^{0.075}$

$= 53{,}000(1.077884151)$

$\approx 57{,}128$

The population was 57,128 in 2005.

c. $P(10) = 53{,}000e^{0.015(10)}$

$= 53{,}000e^{0.15}$

$= 53{,}000(1.161834243)$

$\approx 61{,}577$

The population was 61,577 in 2010.

d. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{61{,}577 - 53{,}000}{10 - 0}$

$= \dfrac{8577}{10}$

$= 857.7$

The average rate of growth in population between 2000 and 2010 is approximately 858 people per year.

44. a. Since the coefficient of the variable exponent is negative, the model indicates that the population is decreasing.

b. $P(7) = 800{,}000e^{-0.020(7)}$

$= 800{,}000e^{-0.14}$

$\approx 695{,}486.59$

The population in 2010 was 695,487.

c. $P(17) = 800{,}000e^{-0.020(17)}$

$= 800{,}000e^{-0.34}$

$\approx 569{,}416.26$

The population in 2020 is estimated to be $569{,}416$.

d. $\dfrac{P(17) - P(7)}{17 - 7} = \dfrac{569{,}416 - 695{,}487}{10}$

$= \dfrac{-126{,}071}{10}$

$= -12{,}607.1$

The average rate of change is –12,607 people per year. The population decreases on average by 12,607 people per year.

45. a. $y = 100e^{-0.00012097(1000)}$

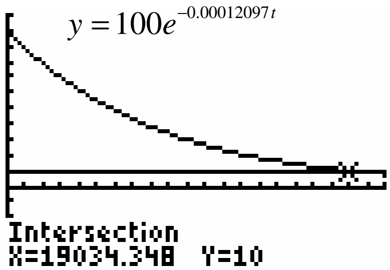
$= 100e^{-0.12097}$

$= 100(0.886060541)$

≈ 88.61

Appriximately 88.61 grams remain after 1000 years.

b.



[0, 21,000] by [–50, 110]

After approximately 19,034 years, 10 grams of Carbon-14 will remain.

46. a. $y = 100\left(1 - e^{-0.35(10-2)}\right)$

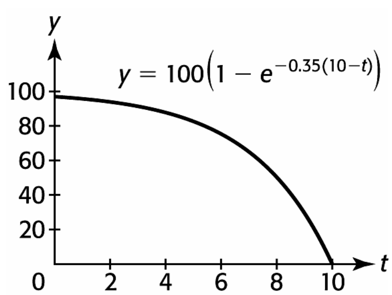
$= 100\left(1 - e^{-2.8}\right)$

$= 100(0.9391899374)$

≈ 93.92

After two hours, 93.92% of the drug remains in the bloodstream.

b.



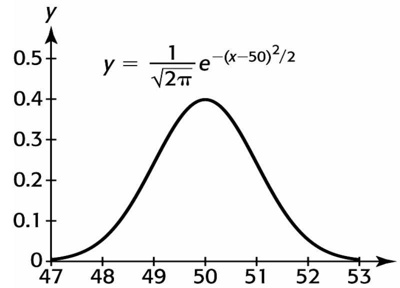
c.

X	Y1
8	50.341
9	29.531
10	0
11	-41.91
12	-101.4
13	-185.8
14	-305.5

X=10

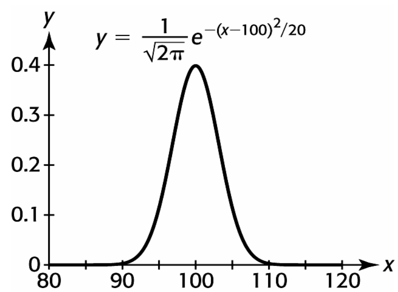
After 10 hours, the drug is totally gone from the bloodstream.

47. a.



b. The average score is 50.

48.



Section 5.2 Skills Check

1. $y = \log_3 x \Leftrightarrow 3^y = x$

2. $2y = \log_5 x \Leftrightarrow 5^{2y} = x$

3. $y = \ln(2x) = \log_e(2x) \Leftrightarrow e^y = 2x$

4. $y = \log(-x) = \log_{10}(-x) \Leftrightarrow 10^y = -x$

5. $x = 4^y \Leftrightarrow \log_4 x = y$

6. $m = 3^p \Leftrightarrow \log_3 m = p$

7. $32 = 2^5 \Leftrightarrow \log_2 32 = 5$

8. $9^{2x} = y \Leftrightarrow \log_9 y = 2x$

9. a. 0.845

b. 4.454

c. 4.806

10. a. 2.659

b. Undefined.

c. 2.303

11. a. $y = \log_2 32 \Leftrightarrow 2^y = 32 = 2^5$
Therefore, $y = 5$.

b. $y = \log_9 81 \Leftrightarrow 9^y = 81 = 9^2$
Therefore, $y = 2$.

c. $y = \log_3 27 \Leftrightarrow 3^y = 27 = 3^3$
Therefore, $y = 3$.

d. $y = \log_4 64 \Leftrightarrow 4^y = 64 = 4^3$
Therefore, $y = 3$.

e. $y = \log_5 625 \Leftrightarrow 5^y = 625 = 5^4$
Therefore, $y = 4$.

12. a. $y = \log_2 64 \Leftrightarrow 2^y = 64 = 2^6$
Therefore, $y = 6$.

b. $y = \log_9 27 \Leftrightarrow 9^y = 27 = 3^3$

$9^y = (3^2)^y = 3^{2y} = 3^3$

$2y = 3$

$y = \frac{3}{2}$

c. $y = \log_4 2 \Leftrightarrow 4^y = 2$

$4^y = (2^2)^y = 2^{2y} = 2^1$

$2y = 1$

$y = \frac{1}{2}$

d. $y = \ln(e^3) = \log_e(e^3) \Leftrightarrow e^y = e^3$
Therefore, $y = 3$.

e. $y = \log(100) = \log_{10}(100)$

$\log_{10}(100) \Leftrightarrow 10^y = 100 = 10^2$

Therefore, $y = 2$.

13. a. $y = \log_3\left(\frac{1}{27}\right) \Leftrightarrow 3^y = \frac{1}{27} = \frac{1}{3^3}$

$3^y = \frac{1}{3^3} = 3^{-3}$

Therefore, $y = -3$.

b. $y = \ln(1) = \log_e(1) \Leftrightarrow e^y = 1 = e^0$

Therefore, $y = 0$.

c. $y = \ln(e) = \log_e(e) \Leftrightarrow e^y = e = e^1$

Therefore, $y = 1$.

d. $y = \log(0.0001) = \log_{10}(0.0001)$

$y = \log_{10}(0.0001) \Leftrightarrow 10^y = 0.0001$

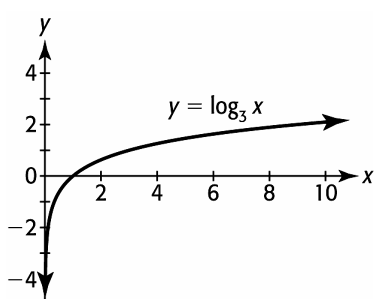
$10^y = \dfrac{1}{10,000}$

$10^y = \dfrac{1}{10^4} = 10^{-4}$

Therefore, $y = -4$.

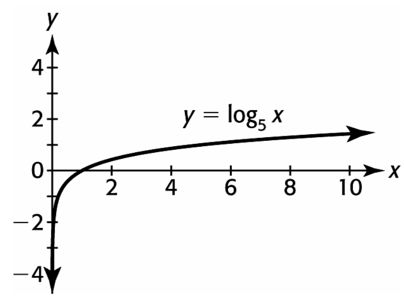
14. a. $y = \log_3 x \Leftrightarrow x = 3^y$

$x = 3^y$	y
1/27	−3
1/9	−2
1/3	−1
1	0
3	1
9	2

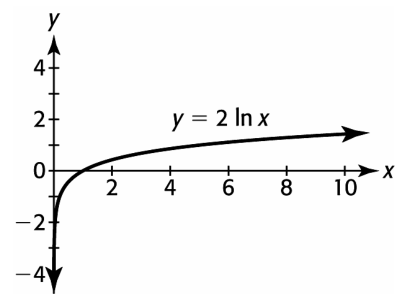


b. $y = \log_5 x \Leftrightarrow x = 5^y$

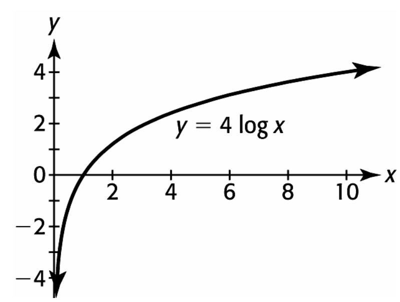
$x = 5^y$	y
1/125	−3
1/25	−2
1/5	−1
1	0
5	1
25	2



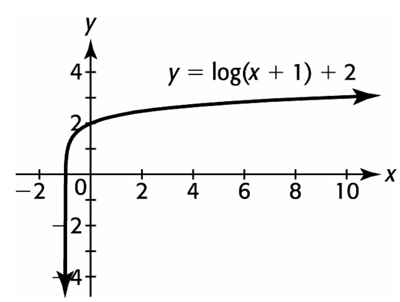
15.



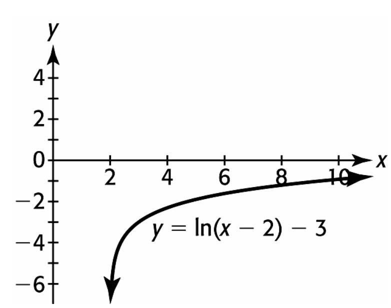
16.



17.



18.



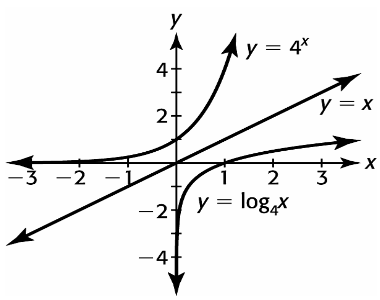
19. a. $y = 4^x$

$x = 4^y \Leftrightarrow \log_4 x = y$

Therefore, the inverse function is

$y = \log_4 x.$

b.



The graphs are symmetric about the line $y = x$.

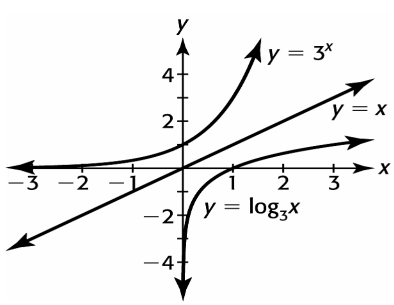
20. a. $y = 3^x$

$x = 3^y \Leftrightarrow \log_3 x = y$

Therefore, the inverse function is

$y = \log_3 x.$

b.



The graphs are symmetric about the line $y = x$.

21. $\log_a a = x \Leftrightarrow a^x = a = a^1$

If $a > 0$ and $a \neq 1$, then $x = 1$,

and therefore, $\log_a a = 1$.

22. $\log_a 1 = x \Leftrightarrow a^x = 1 = a^0$

If $a > 0$ and $a \neq 1$, then $x = 0$,

and therefore, $\log_a 1 = 0$.

23. $$\begin{aligned}\log 10^{14} &= \log_{10} 10^{14}\\ &= 14\log_{10} 10\\ &= 14(1) = 14\end{aligned}$$

24. $\ln\left(e^5\right) = 5\ln(e) = 5(1) = 5$

25. $10^{\log_{10} 12} = 12$

26. $6^{\log_6 25} = 25$

27. $$\begin{aligned}\log_a(100) &= \log_a(20 \bullet 5)\\ &= \log_a(20) + \log_a(5)\\ &= 1.4406 + 0.7740\\ &= 2.2146\end{aligned}$$

28. $$\begin{aligned}\log_a(4) &= \log_a\left(\frac{20}{5}\right)\\ &= \log_a(20) - \log_a(5)\\ &= 1.4406 - 0.7740\\ &= 0.6666\end{aligned}$$

29. $$\begin{aligned}\log_a 5^3 &= 3\log_a 5\\ &= 3(0.7740)\\ &= 2.322\end{aligned}$$

30. $$\begin{aligned}\log_a \sqrt{20} &= \log_a (20)^{\frac{1}{2}}\\ &= \frac{1}{2}\log_a(20)\\ &= \frac{1}{2}(1.4406)\\ &= 0.7203\end{aligned}$$

31. $\ln\left(\frac{3x-2}{x+1}\right) = \ln(3x-2) - \ln(x+1)$

32. $\log\left[x^3(3x-4)^5\right]$
$=\log\left(x^3\right)+\log(3x-4)^5$
$=3\log x+5\log(3x-4)$

33. $\log_3 \dfrac{\sqrt[4]{4x+1}}{4x^2}$
$=\log_3\left(\sqrt[4]{4x+1}\right)-\log_3\left(4x^2\right)$
$=\log_3\left[(4x+1)^{\frac{1}{4}}\right]-\left[\log_3(4)+\log_3\left(x^2\right)\right]$
$=\dfrac{1}{4}\log_3(4x+1)-\left[\log_3(4)+2\log_3(x)\right]$
$=\dfrac{1}{4}\log_3(4x+1)-\log_3(4)-2\log_3(x)$

34. $\log_3 \dfrac{\sqrt[3]{3x-1}}{5x^2}$
$=\log_3\left(\sqrt[3]{3x-1}\right)-\log_3\left(5x^2\right)$
$=\log_3\left[(3x-1)^{\frac{1}{3}}\right]-\left[\log_3(5)+\log_3\left(x^2\right)\right]$
$=\dfrac{1}{3}\log_3(3x-1)-\left[\log_3(5)+2\log_3(x)\right]$
$=\dfrac{1}{3}\log_3(3x-1)-\log_3(5)-2\log_3(x)$

35. $3\log_2 x+\log_2 y$
$=\log_2 x^3+\log_2 y$
$=\log_2\left(x^3y\right)$

36. $\log x-\dfrac{1}{3}\log y$
$=\log x-\log(y)^{\frac{1}{3}}$
$=\log x-\log\left(\sqrt[3]{y}\right)$
$=\log\left(\dfrac{x}{\sqrt[3]{y}}\right)$

37. $4\ln(2a)-\ln(b)$
$=\ln(2a)^4-\ln(b)$
$=\ln\left(\dfrac{(2a)^4}{b}\right)=\ln\left(\dfrac{16a^4}{b}\right)$

38. $6\ln(5y)+2\ln x$
$=\ln(5y)^6+\ln x^2$
$=\ln\left[(5y)^6x^2\right]$
$=\ln\left(15{,}625x^2y^6\right)$

Section 5.2 Exercises

39. a. In 1925, $x=1925-1900=25$.
$f(25)=11.027+14.304\ln(25)$
$f(25)=57.0698$
In 1925, the expected life span is approximately 57 years.

In 2007, $x=2007-1900=107$.
$f(107)=11.027+14.304\ln(107)$
$f(107)=77.8671$
In 2007, the expected life span is approximately 78 years.

b. Based on the model, life span increased tremendously between 1925 and 2007. The increase could be due to multiple factors, including improved healthcare and nutrition/better diet.

40. In 2000, $x = 2000 - 1980 = 20$.

$y = 114.016 + 4.267\ln(20)$

$y \approx 126.799$

In 2000, the population of Japan was 126.8 million.

In 2020, $x = 2020 - 1980 = 40$.

$y = 114.016 + 4.267\ln(40)$

$y \approx 129.756$

In 2020, the population of Japan was 129.8 million.

41. a. In 2015, $x = 2015 - 1980 = 35$.

$f(35) = -3{,}130.3 + 4{,}056.8\ln(35)$

$f(35) = 11{,}293.04$

In 2015, the official single poverty level is approximately \$11,293.

In 2020, $x = 2020 - 1980 = 40$.

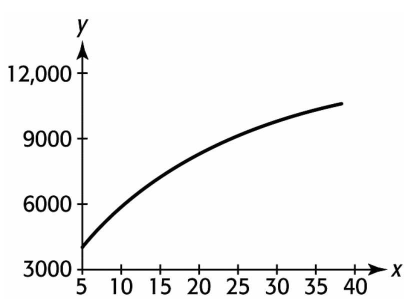
$f(40) = -3{,}130.3 + 4{,}056.8\ln(40)$

$f(40) = 11{,}834.75$

In 2020, the official single poverty level is approximately \$11,835.

b. Based on the solutions to part a), the function seems to be increasing.

c.



42. $t = \dfrac{\ln(16274.54) - \ln(6274.54)}{\ln(1.1)}$

$= \dfrac{9.697 - 8.744}{.0953} = 10$ years

43. $p = 20 + 6\ln(2 \times 5200 + 1)$

$= \$75.50$

44. $p = \dfrac{500}{\ln(6400 + 1)}$

$= \$57.05$

45. a. In 2011, $x = 2011 - 1960 = 51$.

$f(51) = 27.4 + 5.02\ln(51)$

$f(51) = 47.138$

In 2011, the % of female workers in the work force will be 47.1%.

In 2015, $x = 2015 - 1960 = 55$.

$f(55) = 27.4 + 5.02\ln(55)$

$f(55) = 47.517$

In 2015, the % of female workers in the work force will be 47.5%.

b. Based on part a), it appears the % is increasing.

46. a. In 2015, $x = 2015 - 1960 = 55$.

$f(55) = -37.016 + 19.278\ln(55)$

$f(55) = 40.237$

In 2015, the % of live births to unwed mothers will be 40.2%.

In 2022, $x = 2022 - 1960 = 62$.

$f(62) = -37.016 + 19.278\ln(62)$

$f(62) = 42.547$

In 2022, the % of live births to unwed mothers will be 42.5%.

b. Based on part a), it appears the % is increasing.

47. $\frac{\ln 2}{0.10} \approx 6.9$ years

48. $\frac{\ln 2}{0.07} \approx 9.9$ years

49. $n = \frac{\log 2}{0.0086}$

$n = 35.0035 \approx 35$

Since it takes approximately 35 quarters for an investment to double under this scenario, then in terms of years the time to double is approximately $\frac{35}{4} = 8.75$ years.

50. $n = \frac{\log 2}{0.0253} = 11.898 \approx 12$

Since the compounding is semi-annual, 12 compounding periods corresponds to approximately 6 years.

51. $t = \frac{\ln 2}{\ln(1+.08)} = 9.0$ years

52. $t = \frac{\ln 2}{\ln(1+.123)} = 6$ years

53. $R = \log\left(\frac{I}{I_0}\right)$

$R = \log\left(\frac{25,000 I_0}{I_0}\right)$

$R = \log(25,000) = 4.3979 \approx 4.4$

The earthquake measures 4.4 on the Richter scale.

54. a. $R = \log\left(\frac{I}{I_0}\right)$

$R = \log\left(\frac{250,000 I_0}{I_0}\right)$

$R = \log(250,000)$

$= 5.397940009 \approx 5.4$

The earthquake measures 5.4 on the Richter scale.

b. Suppose one earthquake has a magnitude of AI_0, while another earthquake has a magnitude of $10AI_0$.

$R_1 = \log\left(\frac{AI_0}{I_0}\right) = \log(A)$

$R_2 = \log\left(\frac{10AI_0}{I_0}\right)$

$= \log(10A)$

$= \log 10 + \log A$

$= 1 + \log A$

$= 1 + R_1$

The stronger earthquake measures one more unit on the Richter scale than the weaker earthquake.

55. $R = \log\left(\frac{I}{I_0}\right)$

$6.4 = \log\left(\frac{I}{I_0}\right)$

$10^{6.4} = \left(\frac{I}{I_0}\right) = 2,511,886.4$

Thus $I = 10^{6.4} I_0 = 2,511,886.4 I_0$.

56. $R = \log\left(\frac{I}{I_0}\right)$

$8.25 = \log\left(\frac{I}{I_0}\right)$

$10^{8.25} = \left(\frac{I}{I_0}\right)$

Thus $I = 10^{8.25} I_0 = 177{,}827{,}941 I_0$.

57. $R = \log\left(\frac{I}{I_0}\right)$

$7.1 = \log\left(\frac{I}{I_0}\right)$

$10^{7.1} = \left(\frac{I}{I_0}\right)$

Thus $I = 10^{7.1} I_0 = 12{,}589{,}254 I_0$.

58.

$10^R = \frac{I}{I_0}$

$I = 10^R I_0$

In China, $I = 10^{7.9} I_0$

In Algeria, $I = 10^{4.81} I_0$

$\frac{10^{7.9} I_0}{10^{4.81} I_0} = 10^{3.09} = 1230.27$

Thus the China earthquake was 1230 times more intense than the Algerian earthquake.

59. The difference in the Richter scale measurements is $8.25 - 7.1 = 1.15$.

Therefore, the intensity of the 1906 earthquake was $10^{1.15} \approx 14.13$ times stronger than the intensity of the 1989 earthquake.

60. The difference in the Richter scale measurements is $9.0 - 8.25 = 0.75$.

Therefore, the intensity of the 2011 earthquake was $10^{0.75} \approx 5.62$ times stronger than the intensity of the 1983 earthquake.

61. The difference in the Richter scale measurements is $9.0 - 6.8 = 2.2$.

Therefore, the intensity of the 2011 earthquake was $10^{2.2} \approx 158.5$ times stronger than the intensity of the 2008 earthquake.

62. $L = 10\log\left(\frac{I}{I_0}\right)$

$L = 10\log\left(\frac{20{,}000 I_0}{I_0}\right)$

$L = 10\log(20{,}000) \approx 43$

The decibel level is approximately 43.

63. Suppose the intensity of one sound is AI_0, while the intensity of a second sound is $100AI_0$. Then,

$$L_1 = 10\log\left(\frac{AI_0}{I_0}\right) = 10\log(A)$$

$$\begin{aligned} L_2 &= 10\log\left(\frac{100AI_0}{I_0}\right) \\ &= 10\log(100A) \\ &= 10(\log 100 + \log A) \\ &= 20 + 10\log A \\ &= 20 + L_1 \end{aligned}$$

As a decibel level, the higher intensity sound measures 20 more than the lower intensity sound.

64. $L = 10\log\left(\frac{I}{I_0}\right)$

$40 = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\log_{10}\left(\frac{I}{I_0}\right) = 4 \Leftrightarrow 10^4 = \frac{I}{I_0}$

$\frac{I}{I_0} = 10^4$

$I = 10^4 I_0 = 10{,}000 I_0$

65. $L = 10\log\left(\frac{I}{I_0}\right)$

$140 = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\log_{10}\left(\frac{I}{I_0}\right) = 14 \Leftrightarrow 10^{14} = \frac{I}{I_0}$

$\frac{I}{I_0} = 10^{14}$

$I = 10^{14} I_0 = 100{,}000{,}000{,}000{,}000 I_0$

66. $L_1 = 10\log\left(\frac{115 I_0}{I_0}\right)$

$= 10\log(115)$

≈ 20.6

$L_2 = 10\log\left(\frac{9{,}500{,}000 I_0}{I_0}\right)$

$= 10\log(9{,}500{,}000)$

≈ 69.8

The decibel level on a busy street is approximately 49 more than the decibel level of a whisper.

67. $L = 10\log\left(\frac{I}{I_0}\right)$

Let $L = 140$.

$140 = 10\log\left(\frac{I}{I_0}\right)$

$14 = \log\left(\frac{I}{I_0}\right) \Leftrightarrow 10^{14} = \frac{I}{I_0}$

$I = 10^{14} I_0$

Let $L = 120$.

$120 = 10\log\left(\frac{I}{I_0}\right)$

$12 = \log\left(\frac{I}{I_0}\right) \Leftrightarrow 10^{12} = \frac{I}{I_0}$

$I = 10^{12} I_0$

Comparing the intensity levels:

$\frac{10^{14}}{10^{12}} = 10^2 = 100$

The decibel level of 140 is one hundred times as intense as a decibel level of 120.

68. $\text{pH} = -\log\left[\text{H}^+\right]$

$= -\log(0.0000631)$

$= 4.2$

69. $7.79 = -\log\left[\text{H}^+\right]$

multiply both sides by -1

$-7.79 = \log_{10}\left[\text{H}^+\right] \Leftrightarrow 10^{-7.79} = \left[\text{H}^+\right]$

$\left[\text{H}^+\right] = 10^{-7.79} \approx 0.0000000162$

70. $pH = -\log\left[H^+\right]$

multiply both sides by -1

$-pH = \log_{10}\left[H^+\right] \Leftrightarrow 10^{-pH} = H^+$

$H^+ = 10^{-pH}$

If $pH = 1,$ then $H^+ = 10^{-1}$

If $pH = 14,$ then $H^+ = 10^{-14}$

Thus, $10^{-14} \le \left[H^+\right] \le 10^{-1}$

71. If the pH of ketchup is 3.9,

then H^+ for ketchup $= 10^{-3.9}$, and

if the pH for peanut butter is 6.3,

then H^+ for peanut butter $= 10^{-6.3}$.

Thus, $\dfrac{10^{-3.9}}{10^{-6.3}} = 10^{2.4} = 251.2$

Thus ketchup is 251.2 times as acidic as peanut butter.

72. If the pH of aquarium water is 8,

then H^+ for aquarium water = $10^{-8.0}$, and

if the pH for pure sea water is 8.3,

then H^+ for pure sea water $= 10^{-8.3}$.

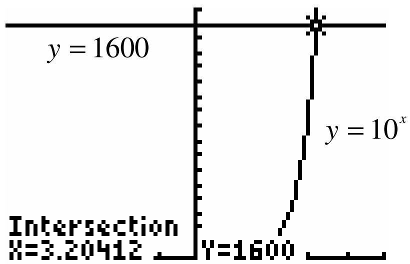
Thus, $\dfrac{10^{-8.0}}{10^{-8.3}} = 10^{0.3} = 1.995 = 2$

Thus, water in an aquarium is 2 times as acidic as pure sea water.

Section 5.3 Skills Check

1. $1600 = 10^x$

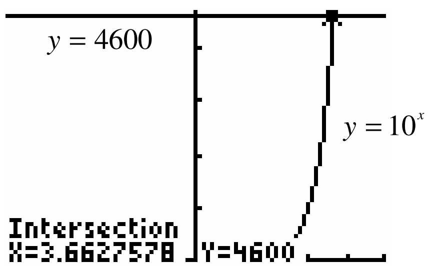
$x = \log(1600) \approx 3.204$



[–5, 5] by [–10, 1700]

2. $4600 = 10^x$

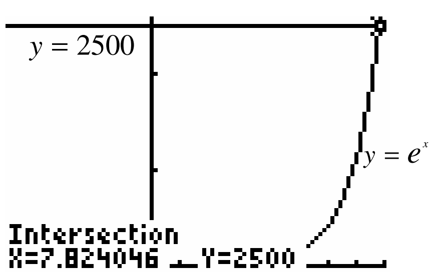
$x = \log(4600) \approx 3.663$



[–5, 5] by [–10, 4700]

3. $2500 = e^x$

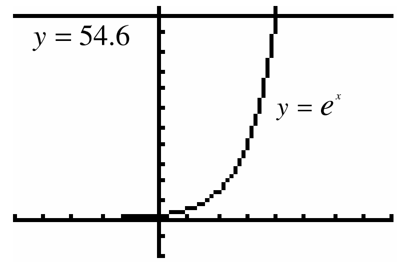
$x = \ln(2500) \approx 7.824$



[–5, 8] by [–10, 2600]

4. $54.6 = e^x$

$x = \ln(54.6) \approx 4.000$

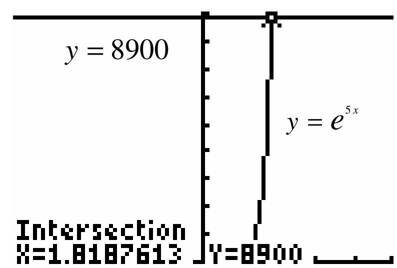


[–5, 8] by [–10, 57]

5. $8900 = e^{5x}$

$5x = \ln(8900)$

$x = \dfrac{\ln(8900)}{5} \approx 1.819$

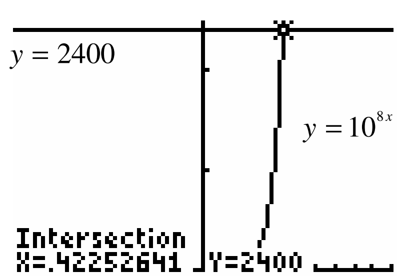


[–5, 5] by [–10, 9000]

6. $2400 = 10^{8x}$

$8x = \log(2400)$

$x = \dfrac{\log(2400)}{8} \approx 0.423$



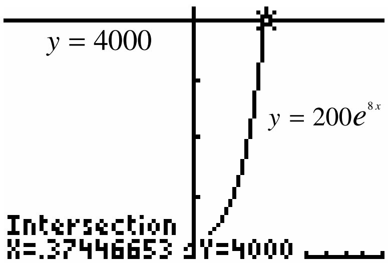
[–1, 1] by [–10, 2500]

7. $4000 = 200e^{8x}$

$20 = e^{8x}$

$8x = \ln(20)$

$x = \dfrac{\ln(20)}{8} \approx 0.374$



[–1, 1] by [–10, 4200]

8. $5200 = 13e^{12x}$

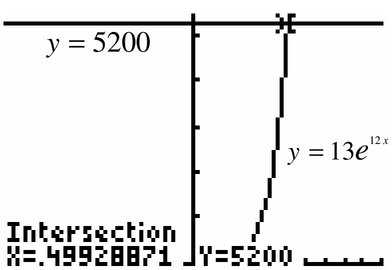
$\dfrac{5200}{13} = e^{12x}$

$400 = e^{12x}$

$12x = \ln(400)$

$x = \dfrac{\ln(400)}{12}$

$x \approx 0.499$

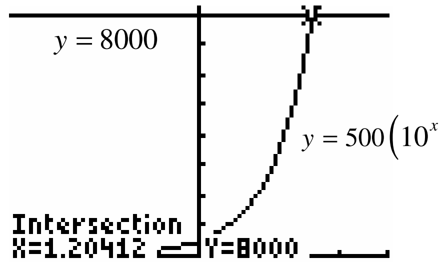


[–1, 1] by [–10, 5400]

9. $8000 = 500(10^x)$

$16 = 10^x$

$x = \log(16) \approx 1.204$

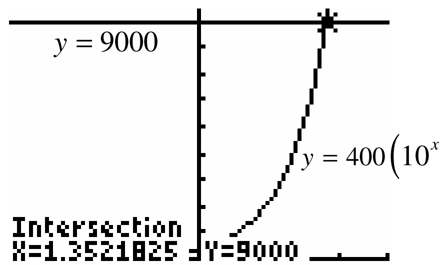


[–2, 2] by [–10, 8200]

10. $9000 = 400(10^x)$

$22.5 = 10^x$

$x = \log(22.5) \approx 1.352$



[–2, 2] by [–10, 9400]

11. $\log_6(18) = \dfrac{\ln(18)}{\ln(6)} = 1.6131$

or

$\log_6(18) = \dfrac{\log(18)}{\log(6)} = 1.6131$

12. $\log_7(215) = \dfrac{\ln(215)}{\ln(7)} = 2.7600$

or

$\log_7(215) = \dfrac{\log(215)}{\log(7)} = 2.7600$

13. $\log_8\left(\sqrt{2}\right)=\left(\dfrac{\ln\left(\sqrt{2}\right)}{\ln(8)}\right)=0.1667$

or

$\log_8\left(\sqrt{2}\right)=\left(\dfrac{\log\left(\sqrt{2}\right)}{\log(8)}\right)=0.1667$

14. $\log_4\left(\sqrt[3]{10}\right)=\left(\dfrac{\ln\left(\sqrt[3]{10}\right)}{\ln(4)}\right)=0.5537$

or

$\log_4\left(\sqrt[3]{10}\right)=\left(\dfrac{\log\left(\sqrt[3]{10}\right)}{\log(4)}\right)=0.5537$

15.
$$\begin{aligned}8^x&=1024\\ \left(2^3\right)^x&=2^{10}\\ 2^{3x}&=2^{10}\\ 3x&=10\\ x&=3.\overline{3}=\frac{10}{3}\end{aligned}$$

16.
$$\begin{aligned}9^x&=2187\\ \left(3^2\right)^x&=3^7\\ 3^{2x}&=3^7\\ 2x&=7\\ x&=3.5\end{aligned}$$

17.
$$\begin{aligned}2\left(5^{3x}\right)&=31,250\\ 5^{3x}&=15,625=5^6\\ 3x&=6\\ x&=2\end{aligned}$$

18.
$$\begin{aligned}2\left(6^{2x}\right)&=2592\\ 6^{2x}&=1296=6^4\\ 2x&=4\\ x&=2\end{aligned}$$

19.
$$\begin{aligned}5^{x-2}&=11.18\\ \ln\left(5^{x-2}\right)&=\ln(11.18)\\ (x-2)\ln(5)&=\ln(11.18)\\ x-2&=\frac{\ln(11.18)}{\ln(5)}\\ x&=\frac{\ln(11.18)}{\ln(5)}+2\\ x&\approx 3.5\end{aligned}$$

20.
$$\begin{aligned}3^{x-4}&=140.3\\ \ln\left(3^{x-4}\right)&=\ln(140.3)\\ (x-4)\ln(3)&=\ln(140.3)\\ x-4&=\frac{\ln(140.3)}{\ln(3)}\\ x&=\frac{\ln(140.3)}{\ln(3)}+4\\ x&\approx 8.5\end{aligned}$$

21.
$$\begin{aligned}18,000&=30\left(2^{12x}\right)\\ 600&=2^{12x}\\ \log(600)&=\log\left(2^{12x}\right)\\ 12x\log(2)&=\log(600)\\ x&=\frac{\log(600)}{12\log(2)}\\ x&\approx 0.769\end{aligned}$$

22.
$$5880 = 21\left(2^{3x}\right)$$
$$\frac{5880}{21} = 2^{3x}$$
$$280 = 2^{3x}$$
$$\log(280) = \log\left(2^{3x}\right)$$
$$3x\log(2) = \log(280)$$
$$x = \frac{\log(280)}{3\log(2)}$$
$$x \approx 2.710$$

23. $\log_2 x = 3 \Leftrightarrow 2^3 = x$
$x = 8$

24. $\log_4 x = -2 \Leftrightarrow 4^{-2} = x$
$$x = \frac{1}{4^2}$$
$$x = \frac{1}{16}$$

25. $5 + 2\ln x = 8$
$2\ln x = 3$
$$\ln x = \frac{3}{2}$$
$$\log_e x = \frac{3}{2} \Leftrightarrow e^{\frac{3}{2}} = x$$
$$x = e^{\frac{3}{2}} \approx 4.482$$

26. $4 + 3\log x = 10$
$3\log x = 6$
$\log x = 2$
$\log_{10} x = 2 \Leftrightarrow 10^2 = x$
$x = 100$

27.
$$5 + \ln(8x) = 23 - 2\ln(x)$$
$$\ln(8x) + 2\ln(x) = 23 - 5$$
$$\ln(8x) + \ln(x)^2 = 18$$
$$\ln\left(8x \bullet x^2\right) = 18$$
$$\ln\left(8x^3\right) = 18$$
$$8x^3 = e^{18}$$
$$x = \sqrt[3]{\frac{e^{18}}{8}}$$
$$x = \frac{e^6}{2} \approx 201.7$$

28.
$$3\ln x + 8 = \ln(3x) + 12.18$$
$$3\ln(x) - \ln(3x) = 12.18 - 8$$
$$\ln\left(x^3\right) - \ln(3x) = 4.18$$
$$\ln\left(\frac{x^3}{3x}\right) = 4.18$$
$$\ln\left(\frac{x^2}{3}\right) = 4.18$$
$$\frac{x^2}{3} = e^{4.18}$$
$$x = \sqrt{3e^{4.18}} \approx 14$$

29.
$$2\log(x) - 2 = \log(x - 25)$$
$$\log\left(x^2\right) - \log(x - 25) = 2$$
$$\log\left(\frac{x^2}{x - 25}\right) = 2$$
$$\frac{x^2}{x - 25} = 10^2 = 100$$
$$x^2 = 100(x - 25)$$
$$x^2 = 100x - 2500$$
$$x^2 - 100x + 2500 = 0$$
$$(x - 50)(x - 50) = 0$$
$$x = 50$$

30.

$$\ln(x-6)+4=\ln x+3$$
$$\ln(x-6)-\ln x=3-4$$
$$\ln\left(\frac{x-6}{x}\right)=-1$$
$$\frac{x-6}{x}=e^{-1}$$
$$x-6=e^{-1}x$$
$$x-e^{-1}x=6$$
$$x(1-e^{-1})=6$$
$$x=\frac{6}{1-e^{-1}}$$
$$x=\frac{6}{.632}=9.49$$

31. $\log_3 x+\log_3 9=1$

$\log_3(9\bullet x)=1$

$9\bullet x=3^1$

$x=3/9=1/3$

32. $\log_2 x+\log_2(x-6)=4$

$\log_2 x(x-6)=4$

$x^2-6x=2^4$

$x^2-6x-16=0$

$(x-8)(x+2)=0$

$x=8,$ or $x=-2,$ but $x=-2$

does not check in the original

equation since $\log_2(-2)$ is undefined.

33. $\log_2 x=\log_2 5+3$

$\log_2 x-\log_2 5=3$

$\log_2\frac{x}{5}=3$

$\frac{x}{5}=2^3=8$

$x=40$

34. $\log_2 x=3-\log_2 2x$

$\log_2 x+\log_2 2x=3$

$\log_2(x\bullet 2x)=3$

$2x^2=2^3=8$

$x^2=4$

$x=\pm 2,$ but $x=-2$

does not check in the original

equation since $\log_2(-2)$ is undefined.

35.

$\log 3x+\log 2x=\log 150$

$\log(2x\bullet 3x)=\log 150$

$\log(6x^2)=\log 150$

$6x^2=150$

$x^2=25$

$x=\pm 5,$ but $x=-5$

does not check in the original

equation since $\log(-10)$ nor

$\log(-15)$ is undefined.

36.

$\ln(x+2)+\ln x=\ln(x+12)$

$\ln x(x+2)=\ln(x+12)$

$\ln(x^2+2x)=\ln(x+12)$

$x^2+2x=x+12$

$x^2+2x-x-12=0$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4, x=3,$ but $x=-4$

does not check in the original

equation since $\ln(-4)$ is undefined.

37.
$$
\begin{aligned}
3^x &< 243\\
\ln\left(3^x\right) &< \ln(243)\\
x\ln(3) &< \ln(243)\\
x &< \frac{\ln(243)}{\ln(3)}\\
x &< 5
\end{aligned}
$$

38.
$$
\begin{aligned}
7^x &\ge 2401\\
\ln\left(7^x\right) &\ge \ln(2401)\\
x\ln(7) &\ge \ln(2401)\\
x &\ge \frac{\ln(2401)}{\ln(7)}\\
x &\ge 4
\end{aligned}
$$

39.
$$
\begin{aligned}
5\left(2^x\right) &\ge 2560\\
\ln\left(2^x\right) &\ge \ln\left(\frac{2560}{5}\right)\\
x\ln(2) &\ge \ln(512)\\
x &\ge \frac{\ln(512)}{\ln(2)}\\
x &\ge 9
\end{aligned}
$$

40.
$$
\begin{aligned}
15\left(4^x\right) &\le 15{,}360\\
\ln\left(4^x\right) &\le \ln\left(\frac{15{,}360}{15}\right)\\
x\ln(4) &\le \ln(1024)\\
x &\le \frac{\ln(1024)}{\ln(4)}\\
x &\le 5
\end{aligned}
$$

Section 5.3 Exercises

41.
$$
\begin{aligned}
10{,}880 &= 340\left(2^q\right)\\
2^q &= \frac{10{,}880}{340}\\
2^q &= 32\\
\ln\left(2^q\right) &= \ln(32)\\
q\ln(2) &= \ln(32)\\
q &= \frac{\ln(32)}{\ln(2)}\\
q &= 5
\end{aligned}
$$

When the price is \$10,880, the quantity supplied is 5.

42.
$$
\begin{aligned}
256.60 &= 4000\left(3^{-q}\right)\\
3^{-q} &= \frac{256.60}{4000}\\
3^{-q} &= 0.06415\\
\ln\left(3^{-q}\right) &= \ln(0.06415)\\
-q\ln(3) &= \ln(0.06415)\\
q &= \frac{\ln(0.06415)}{-\ln(3)}\\
q &= 2.5
\end{aligned}
$$

When the price is \$256.60, the quantity supplied is 2.5 thousand.

43. a.
$$
\begin{aligned}
S &= 25{,}000e^{-0.072x}\\
\frac{S}{25{,}000} &= e^{-0.072x}\\
\Leftrightarrow \ln\left(\frac{S}{25{,}000}\right) &= -0.072x
\end{aligned}
$$

b. $\ln\left(\dfrac{16{,}230}{25{,}000}\right) = -0.072x$

$$x = \frac{\ln\left(\dfrac{16{,}230}{25{,}000}\right)}{-0.072}$$

$$x = 6$$

Six weeks after the completion of the campaign, the weekly sales fell to $16,230.

44. a. $S = 3200e^{-0.08x}$

$$\frac{S}{3200} = e^{-0.08x}$$

$$\Leftrightarrow \ln\left(\frac{S}{3200}\right) = -0.08x$$

b. $\ln\left(\dfrac{2145}{3200}\right) = -0.08x$

$$x = \frac{\ln\left(\dfrac{2145}{3200}\right)}{-0.08}$$

$$x = 5$$

After five days, the daily sales fell to $2145.

45. a. $S = 3200e^{-0.08(0)} = 3200e^{0} = 3200$

At the end of the ad campaign, daily sales were $3200.

b.

$$S = 3200e^{-0.08x}$$

$$1600 = 3200e^{-0.08x}$$

$$\frac{1}{2} = e^{-0.08x}$$

$$-0.08x = \ln\left(\frac{1}{2}\right)$$

$$x = \frac{\ln\left(\dfrac{1}{2}\right)}{-0.08} = 8.664$$

Approximately 9 days after the completion of the ad campaign, daily sales dropped below half of what they were at the end of the campaign.

46. a. $S = 25{,}000e^{-0.072(0)}$

$$= 25{,}000e^{0}$$

$$= 25{,}000$$

At the end of the campaign, weekly sales were $25,000.

b.

X	Y1
8	14054
9	13077
10	12169
11	11323
12	10537
13	9804.8
14	9123.7

X=10

In the tenth week, weekly sales dropped below half the initial amount of $25,000.

47. a. $y = 0.0000966\left(1.101^{x}\right)$

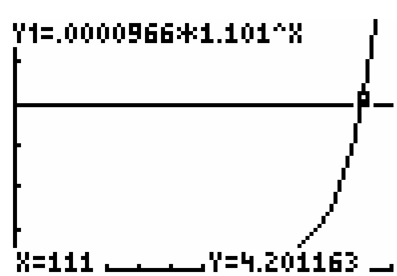
When x = 100, $(2000-1900)$

$$y = 0.0000966\left(1.101^{100}\right)$$

$$y = 1{,}457{,}837$$

Based on the model, in 2000, the cost of a 30-second Super Bowl ad was $1,457,837.

b. Applying the intersection of graphs method

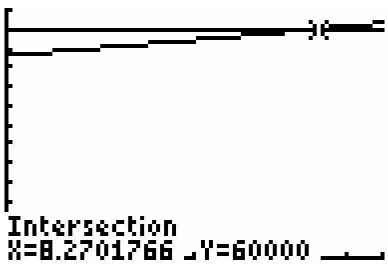


[0, 120] by [0, 6000]

Revenues would reach $4,000,000 in 2011.

48.
$$60{,}000 = 53{,}000e^{0.015t}$$
$$e^{0.015t} = 1.1321$$
$$\ln\left(e^{0.015t}\right) = \ln(1.1321)$$
$$0.015t = \ln(1.1321)$$
$$t = \frac{\ln(1.1321)}{0.015}$$
$$t = 8.27$$

The population was predicted to reach 60,000 people between 8 and 9 years, which corresponds to 2009.



[0, 10] by [0, 65000]

49.
$$20{,}000 = 40{,}000e^{-0.05t}$$
$$e^{-0.05t} = 0.5$$
$$\ln\left(e^{-0.05t}\right) = \ln(0.5)$$
$$-0.05t = \ln(0.5)$$
$$t = \frac{\ln(0.5)}{-0.05}$$
$$t = 13.86294361$$

It will take approximately 13.86 years for the $40,000 pension to decrease to $20,000 in purchasing power.

50.
$$30{,}000 = 60{,}000e^{-0.05t}$$
$$60{,}000e^{-0.05t} = 30{,}000$$
$$e^{-0.05t} = 0.5$$
$$\ln\left(e^{-0.05t}\right) = \ln(0.5)$$
$$-0.05t = \ln(0.5)$$
$$t = \frac{\ln(0.5)}{-0.05}$$
$$t = 13.86294361$$

It will take approximately 13.86 years for the $60,000 in purchasing power to decrease to $30,000 in purchasing power.

51.
$$200{,}000 = 100{,}000e^{0.03t}$$
$$2 = e^{0.03t}$$
$$\ln(2) = \ln\left(e^{0.03t}\right)$$
$$\ln(2) = 0.03t$$
$$t = \frac{\ln(2)}{0.03}$$
$$t = 23.1049$$

It will take approximately 23.1 years for the value of the property to double.

52.
$$254{,}250 = 200{,}000e^{0.05t}$$
$$1.27125 = e^{0.05t}$$
$$\ln(1.27125) = \ln\left(e^{0.05t}\right)$$
$$\ln(1.27125) = 0.05t$$
$$t = \frac{\ln(1.27125)}{0.05}$$
$$t = 4.8$$

It will take approximately 4.8 years for the value of the property to reach $254,250.

53. a. $A(0)=500e^{-0.02828(0)}=500e^{0}=500$

The initial quantity is 500 grams.

b.
$$250=500e^{-0.02828t}$$
$$0.5=e^{-0.02828t}$$
$$\ln(0.5)=\ln\left(e^{-0.02828t}\right)$$
$$-0.02828t=\ln(0.5)$$
$$t=\frac{\ln(0.5)}{-0.02828}=24.51$$

The half-life, the time it takes the initial quantity to become half, is approximately 24.51 years.

54.
$$318=500e^{-0.02828t}$$
$$e^{-0.02828t}=\frac{318}{500}$$
$$\ln\left(e^{-0.02828t}\right)=\ln\left(\frac{318}{500}\right)$$
$$-0.02828t=\ln\left(\frac{318}{500}\right)$$
$$t=\frac{\ln\left(\frac{318}{500}\right)}{-0.02828}$$
$$t\approx 16$$

The amount of thorium reaches 318 grams in about 16 years.

55.

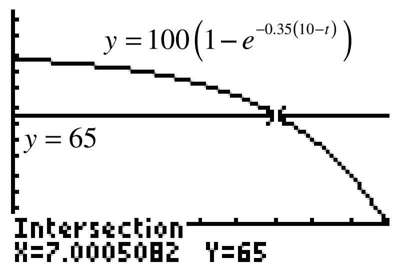
X	Y1
-1	93.967
0	91.759
1	88.741
2	84.618
3	78.986
4	71.292
5	60.781

X=3

The concentration reaches 79% in about 3 hours.

Solving algebraically:
$$79=100(1-e^{-0.312(8-t)})$$
$$\frac{79}{100}=1-e^{-0.312(8-t)}$$
$$e^{-0.312(8-t)}=1-.79=.21$$
$$\ln\left(e^{-0.312(8-t)}\right)=\ln(.21)=-1.5606$$
$$-0.312(8-t)=-1.5606$$
$$8-t=\frac{-1.5606}{-0.312}=5$$
$$t=3$$

56. Applying the intersection of graphs method:

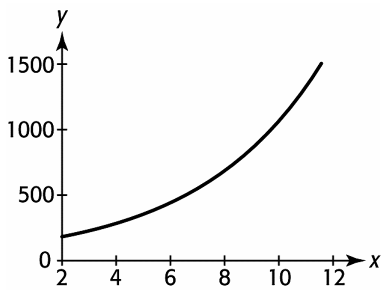


[0, 10] by [–20, 125]

After approximately 7 hours, the percent of the maximum dosage present is 65%.

Solving algebraically:
$$65=100(1-e^{-0.35(10-t)})$$
$$\frac{65}{100}=1-e^{-0.35(10-t)}$$
$$e^{-0.35(10-t)}=1-.65=.35$$
$$\ln\left(e^{-0.35(10-t)}\right)=\ln(.35)=-1.0498$$
$$-0.35(10-t)=-1.0498$$
$$10-t=\frac{-1.0498}{-0.35}=3$$
$$t=7$$

57. a. $y = 117.911(1.247^x)$



b. Based on the model in part a), in the year 2010, the price of an ounce of gold will be $1000.

c. No, the model increases and reaches approximately $1500 an ounce when x = 11. This is not reasonable since the price of gold has historically fluctuated and is currently on the rise

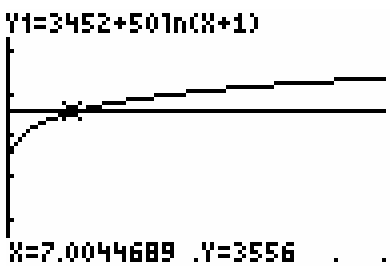
.

58.

$$50 = 100e^{-0.00002876t}$$
$$0.5 = e^{-0.00002876t}$$
$$\ln(0.5) = \ln\left(e^{-0.00002876t}\right)$$
$$-0.00002876t = \ln(0.5)$$
$$t = \frac{\ln(0.5)}{-0.00002876}$$
$$t \approx 24{,}101$$

The half-life is approximately 24,101 years.

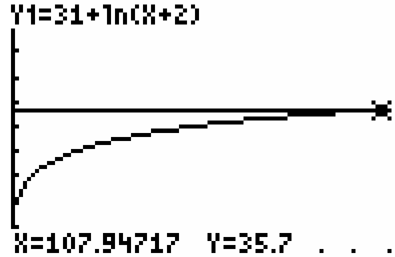
59.



[0, 40] by [3200, 3800]

When the cost is $3556, approximately 7 units are produced.

60.



[0, 110] by [30, 40]

When the cost is $35.70, approximately 108 units are supplied.

61. $S = P(1.07)^t$

Note that the initial investment is P and that double the initial investment is $2P$.

$$2P = P(1.07)^t$$
$$2 = 1.07^t$$
$$\ln(2) = \ln\left(1.07^t\right)$$
$$t = \frac{\ln(2)}{\ln(1.07)}$$

The time to double is $\ln(2)$ divided by $\ln(1.07)$.

62.

$$S = P(1.10)^n$$
$$P(1.10)^n = S$$
$$1.10^n = \frac{S}{P}$$
$$\log\left(1.10^n\right) = \log\left(\frac{S}{P}\right)$$
$$n\log(1.10) = \log\left(\frac{S}{P}\right)$$

$$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1.10)}$$

Let $S = 2P$, since the investment doubles.

$$n = \frac{\log\left(\frac{2P}{P}\right)}{\log(1.10)}$$

$$n = \frac{\log(2)}{\log(1.10)}$$

$$n = \log_{1.10} 2$$

63. $S = 20,000(1+.07)^t$

$48,196.90 = 20,000(1+.07)^t$

$\frac{48,196.90}{20,000} = (1.07)^t$

$2.409845 = (1.07)^t$

$t = \log_{1.07}(2.409845)$

$t = 13$

64. $S = 30,000(1+.09)^t$

$129,829 = 30,000(1+.09)^t$

$\frac{129,829}{30,000} = (1.09)^t$

$4.32763 = (1.09)^t$

$t = \log_{1.09}(4.32763)$

$t = 16.9 \approx 17$ years

65. $S = 40,000(1+.10)^t$

$64,420.40 = 40,000(1+.10)^t$

$\frac{64,420.40}{40,000} = (1.10)^t$

$1.61051 = (1.10)^t$

$t = \log_{1.10}(1.61051)$

$t = 5$ years

66. $S = 40,000(1+.08)^t$

$86,357 = 40,000(1+.08)^t$

$\frac{86,357}{40,000} = (1.08)^t$

$2.158925 = (1.08)^t$

$t = \log_{1.08}(2.158925)$

$t = 10$ years

67. a. $f(x) = 11.027 + 14.304 \ln x$

For $f(x) = 78$,

$78 = 11.027 + 14.304 \ln x$

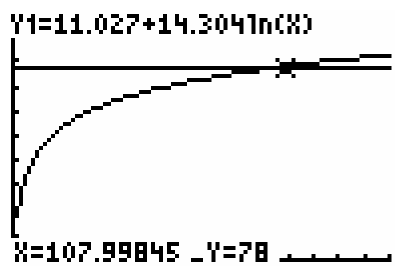
$\frac{78 - 11.027}{14.304} = \ln x$

$4.682 = \ln x$

$x = e^{4.682} = 107.99 \approx 108$ years

Thus, for an expected life span of 78 years, the birth year is 1900 + 108 = 2008.

b.



[0, 150] by [0, 100]

Yes, it agrees with part a).

68.

$p = 20 + 6\ln(2q+1)$

For $p = 68.04$,

$68.04 = 20 + 6\ln(2q+1)$

$\frac{68.04-20}{6} = \ln(2q+1)$

$8.007 = \ln(2q+1)$

$e^{8.007} = (2q+1)$,

$3000.90 = 2q+1$

$q \approx 1500$ units

69. a. $G(x) = 174.075(1.378^x)$

For x = 0 months after Dec. 2009,

$G(0) = 174.075(1.378^0)$

$= \$174.08$ million

b. For x = 12 months after Dec. 2009,

$G(12) = 174.075(1.378^{12})$

$= \$8160.69$ million

c. $\frac{8160.69-174.08}{174.08} = 4588\%$

The percent increase from December 2009 to December 2010 was 4588%.

70. $p = \frac{500}{\ln(q+1)}$

For $p = 61.71$,

$61.71 = \frac{500}{\ln(q+1)}$

$\ln(q+1) = \frac{500}{61.71} = 8.102$

$q+1 = e^{8.102} = 3302$

$q = 3301$

71. $y = 4899.7601(1.0468^x)$

$6447 = 4899.7601(1.0468^x)$

$1.316 = 1.0468^x$

$x = \log_{1.0468}(1.316) = 6.0$

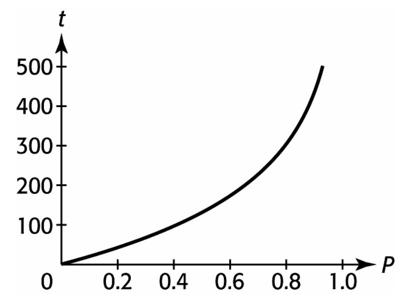
Thus $1990 + 6 = 1996$

72. a. $\ln(1-P) = -0.0034 - 0.0053t$

$\ln(1-P) + 0.0034 = -0.0053t$

$\frac{\ln(1-P)+0.0034}{-0.0053} = t$

$\frac{-(\ln(1-P)+0.0034)}{0.0053} = t$



b. For $P = 30\%$,

$t = \frac{\ln(1-.30)+0.0034}{-0.0053}$

$t = \$66.66$ per ton of carbon

73. a.

Annual Interest Rate	Rule of 72 Years	Exact Years
2%	36	34.66
3%	24	23.10
4%	18	17.33
5%	14.4	13.86
6%	12	11.55
7%	10.29	9.90
8%	9	8.66
9%	8	7.70
10%	7.2	6.93
11%	6.55	6.30

b. The differences between the two sets of outputs are: 1.34, 0.90, 0.67, 0.54, 0.45, 0.39, 0.34, 0.30, 0.27, and 0.25.

c. As interest rate increases, the estimate gets closer to actual value.

74. a. $n = \log_{1.02} 2$

$n = \dfrac{\log 2}{\log 1.02}$

$n = 35.0027 \approx 35$

b. Since it takes approximately 35 quarters for an investment to double under this scenario, then in terms of years, the time to double is $\dfrac{35}{4} = 8.75$ years.

75. a. $n = \log_{1.06} 2 = \dfrac{\ln 2}{\ln 1.06} \approx 11.9$

b. Since the compounding is semi-annual, 11.9 compounding periods correspond to approximately 6 years.

76. $t = \log_{1.05} 2$

$t = \dfrac{\log 2}{\log 1.05}$

$t = 14.20669908$

$t \approx 14.2$

The future value will be $40,000 in approximately 15 years.

77. $t = \log_{1.08} 3.4$

$t = \dfrac{\log 3.4}{\log 1.08}$

$t = 15.9012328$

$t \approx 15.9$

The future value will be $30,000 in approximately 16 years.

78. $y = -3.91435 + 2.62196 \ln t$

$7 < -3.91435 + 2.62196 \ln t$

$\dfrac{7 + 3.91435}{2.62196} < \ln t$

$4.1627 < \ln t$

$e^{4.1627} < t$, so $t > 64.24 \approx 65$

Thus, there will be more than 7 hectares destroyed per year in the year 1950 + 65 = 2015, and after.

79. $m = 20 \ln \dfrac{50}{50 - x}$, for $x < 50$

$m = 20 \ln \dfrac{50}{50 - 45}$

$m = 20 \ln \dfrac{50}{5} = 20 \ln(10)$

$m = 20(2.303) = 46.052$ months

Thus, the market share is more than 45% for approximately 46 months.

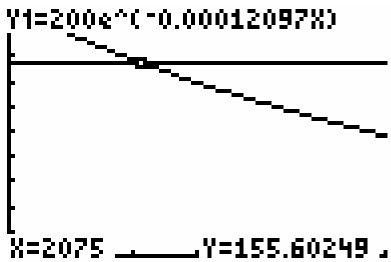
80.

X	Y1	
2	84.618	
3	78.986	
4	71.292	
5	60.781	
6	46.42	
7	26.802	
8	0	

X=5

For the first five hours, the drug dosage remains below 60%.

81. Applying the intersection of graphs method:



[0, 6000] by [0, 400]

After approximately 2075 years, 155.6 grams of carbon-14 remain.

82. a.

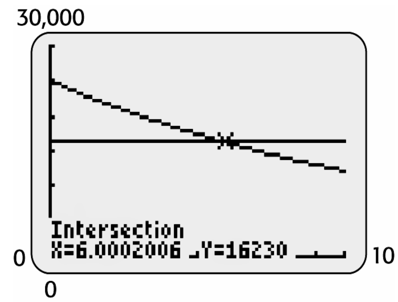
$$S = 25{,}000e^{-0.072x}$$
$$16{,}230 = 25{,}000e^{-0.072x}$$
$$e^{-0.072x} = \frac{16{,}230}{25{,}000}$$
$$-0.072x = \ln\left(\frac{16{,}230}{25{,}000}\right)$$
$$x = \frac{\ln\left(\frac{16{,}230}{25{,}000}\right)}{-0.072}$$
$$x \approx 6$$

Sales fell below $16,230 approximately 6 weeks after the end of the campaign.

b.

X	Y1
5	17442
6	16230
7	15103
8	14054
9	13077
10	12169
11	11323

X=6



83. a.

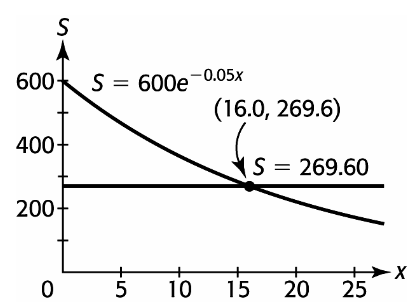
$$S = 600e^{-0.05x}$$
$$269.60 = 600e^{-0.05x}$$
$$e^{-0.05x} = \frac{269.60}{600}$$
$$-0.05x = \ln\left(\frac{269.60}{600}\right)$$
$$x = \frac{\ln\left(\frac{269.60}{600}\right)}{-0.05}$$
$$x \approx 16$$

Sixteen weeks after the end of the campaign, sales dropped below $269.60 thousand.

b.

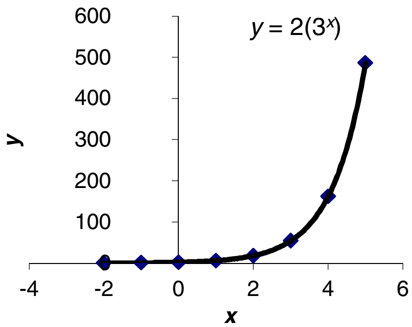
X	Y1
13	313.23
14	297.95
15	283.42
16	269.6
17	256.45
18	243.94
19	232.04

X=16



Section 5.4 Skills Check

1.



2.

x	$f(x)$	First Differences	Percent Change
1	4		
2	16	12	300.00%
3	64	48	300.00%
4	256	192	300.00%
5	1024	768	300.00%
6	4096	3072	300.00%

Since the percent change in the table is constant, $f(x)$ is exactly exponential.

3.

x	$g(x)$	First Differences	Percent Change
1	2.5		
2	6	3.5	140.00%
3	8.5	2.5	41.67%
4	10	1.5	17.65%
5	8	–2	–20.00%
6	6	–2	–25.00%

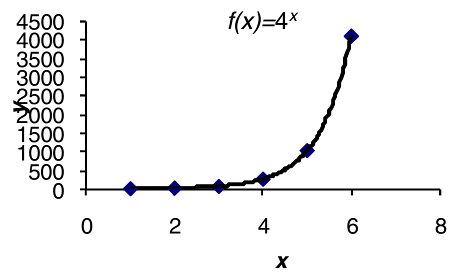
Since the percent change in the table is both positive and negative, $g(x)$ is not exponential.

4.

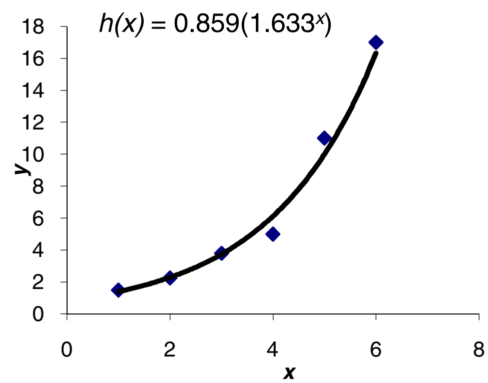
x	$h(x)$	First Differences	Percent Change
1	1.5		
2	2.25	0.75	50.00%
3	3.8	1.55	68.89%
4	5	1.2	31.58%
5	11	6	120.00%
6	17	6	54.55%

Since the percent change in the table is approximately 50%, except for the 120% increase from 5 to 11, $h(x)$ is approximately exponential.

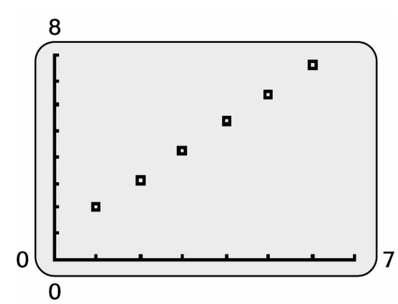
5.



6.

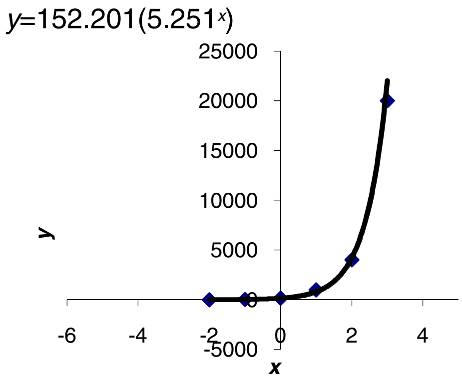


7. a.



b. Considering the scatter plot from part a), a linear model fits the data very well.

8.

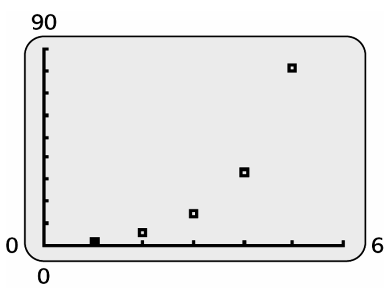


9.

x	y	First Differences	Percent Change
1	2		
2	6	4	200.00%
3	14	8	133.33%
4	34	20	142.86%
5	81	47	138.24%

Since the percent change is approximately constant and the first differences vary, an exponential function will fit the data best. Also, since the first differences are not constant, it cannot be linear.

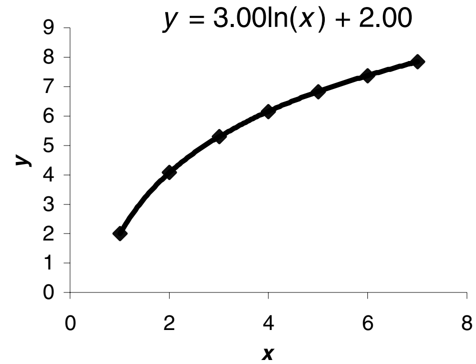
10.



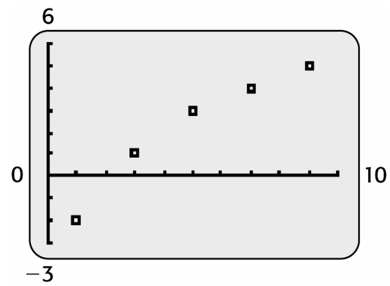
An exponential model is the better fit based on the scatter plot.

11. Using technology yields, $y = 0.876\left(2.494^x\right)$.

12.

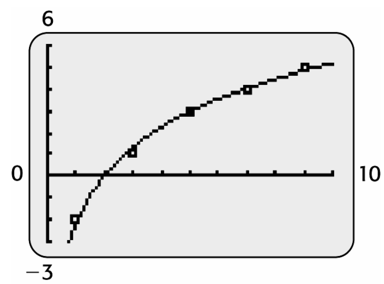


13. a.

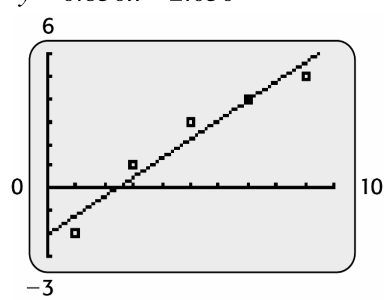


b. Based on the scatter plot, it appears that a logarithmic model fits the data better.

14. a. $y = 3.183\ \ln x - 2.161$

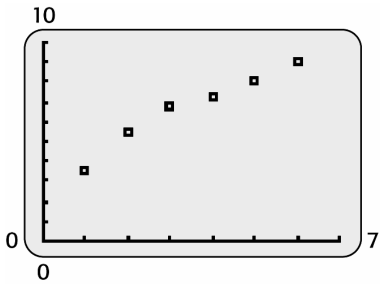


b. $y = 0.850x - 2.050$



c. The logarithmic model is a much better fit based on the two scatter plots.

15. a.

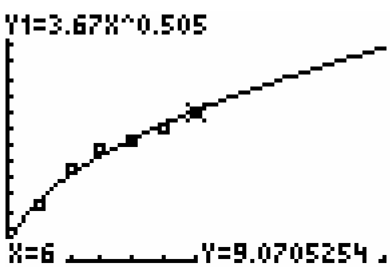


b. Using technology, $y = 3.671x^{0.505}$ is a power function that models the data.

c. Using technology, $y = -0.125x^2 + 1.886x + 1.960$ is a quadratic function that models the data.

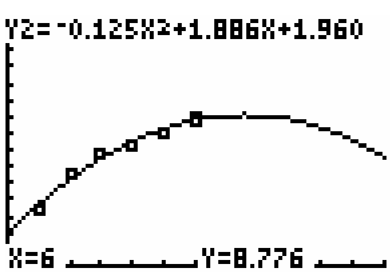
d. Using technology, $y = 3.468 + 2.917 \ln x$ is a logarithmic function that models the data.

16. From problem #15:
$f(x) = 3.671x^{0.505}$



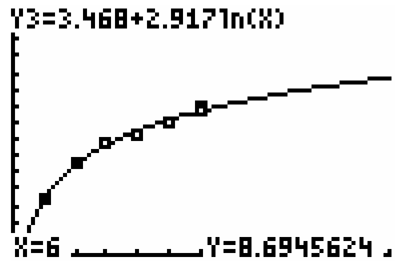
[0, 12] by [0, 15]

$g(x) = -0.125x^2 + 1.886x + 1.960$



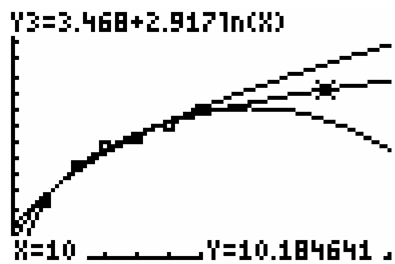
[0, 12] by [0, 15]

$h(x) = 3.468 + 2.917 \ln x \quad (y_3)$



[0, 12] by [0, 15]

With all on the same axis:



The logarithmic model *h(x)* appears to be a slightly better fit.

Section 5.4 Exercises

17. a. $y = a(1+r)^x$

$y = 30{,}000(1+0.04)^t$

$y = 30{,}000(1.04^t)$

b. $y = 30{,}000(1.04^t)$

$y = 30{,}000(1.04^{15}) \approx 54{,}028.31$

In 2015, the retail price of the automobile is predicted to be $54,028.31.

18. a. $y = a(1+r)^x$

$y = 190{,}000(1+0.03)^t$

$y = 190{,}000(1.03^t)$

b. $y = 190{,}000\left(1.03^t\right)$

$= 190{,}000\left(1.03^{10}\right)$

$\approx 255{,}344.11$

In 2010, the population was predicted to be 255,344.

19. a. $y = a(1+r)^x$

$y = 20{,}000(1-0.02)^x$

$y = 20{,}000\left(0.98^x\right)$

b. $y = 20{,}000\left(0.98^t\right)$

$= 20{,}000\left(0.98^5\right)$

$= 18{,}078.42$

In five weeks, the sales are predicted to decline to $18,078.42.

20. a. $y = a(1+r)^x$

$y = 220{,}000(1+0.03)^t$

$y = 220{,}000\left(1.03^t\right)$

b. $y = 220{,}000\left(1.03^5\right) \approx 255{,}040.30$

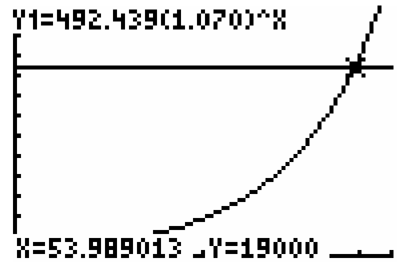
In 2013, the value of the home is predicted to increase to $255,040.30.

21. a. Using technology, $y = 492.439\left(1.070^x\right)$, correct to three decimal places.

b. Using the unrounded model for the year 2015, $2015 - 1960 = 55$

$y = \$20{,}100.80$ billion

c.



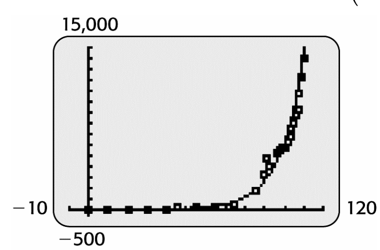
[0, 60] by [0, 25,000]

To reach 19 trillion, $x = 53.989$, approximately 54 years. Thus 1960 + 54 = the year 2014.

22. a. Using technology, $y = 408.705\left(1.064^x\right)$

b. For the year 2014, $2014 - 1960 = 54$ the unrounded model indicates the number of cohabiting households will be 11,835 thousand.

23. a. Using technology, $y = 1.756\left(1.085^x\right)$

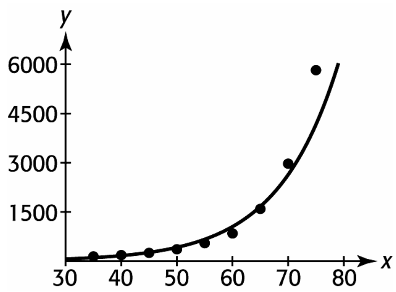


b. For the year 2013, $x = 113$, and using the unrounded model, $y = \$17{,}749$ billion.

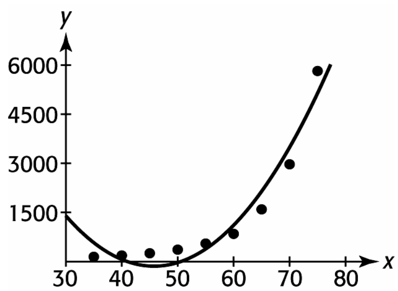
c. For $y = \$25$ trillion, x will equal 117.2 years, equivalent to the year 2018.

d. Events that may affect the accuracy of the model and predictions of future debt include involvement in war activities, catastrophic natural events, world assistance programs, etc.

24. a. $y = 4.304(1.096)^x$



b. $y = 6.182x^2 - 565.948x + 12{,}810.482$



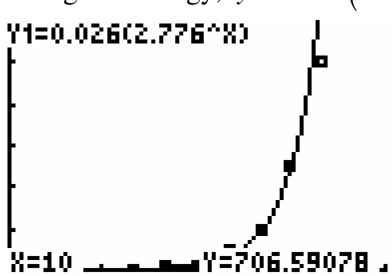
c. Considering parts a) and b), the exponential model is the better fit.

25. a. Using technology, $y = 2.919(1.041^x)$

b. In the year 2013, $x = 113$, and using the unrounded model, y = 269.6.

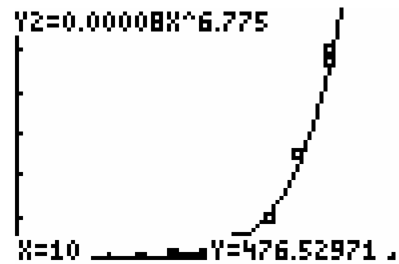
c. For $y = 300$, x will equal 115.29 years, equivalent to the year 2016.

26. a. Using technology, $y = 0.026(2.776^x)$



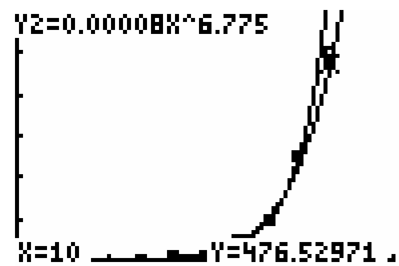
[0, 12] by [0, 600]

b. Using technology, $y = 0.00008x^{6.775}$



[0, 12] by [0, 600]

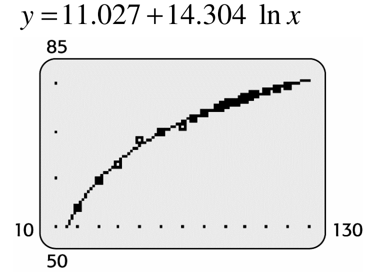
c.



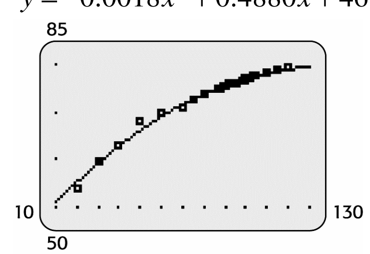
[0, 12] by [0, 600]

The power model seems to fit better.

27. a. $y = 11.027 + 14.304 \ln x$



b. $y = -0.0018x^2 + 0.4880x + 46.2495$

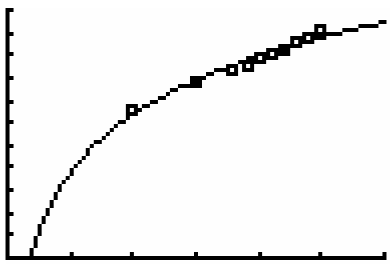


c. Based on the graphs in parts a) and b), it appears that the logarithmic model is the better fit.

d. In 2016, $x = 116$.
Using the unrounded model of the logarithmic function, $y = 79.0$.

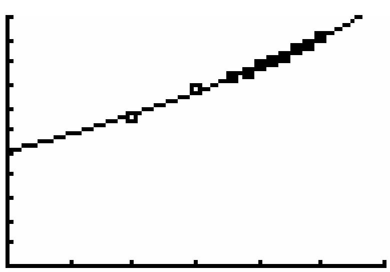
Using the unrounded model of the quadratic function, $y = 78.6$.

28. a. Using technology,
$y = -2179.067 + 3714.021 \ln x$
with x as the number of years after 1980.



[0, 30] by [0, 11000]

b. Using technology,
$y = 5069.388(1.028^x)$
with $x = 0$ in 1980.



[0, 30] by [0, 11000]

c. The exponential model appears to be a better fit for the data.

29. a. Using technology,
$y = 27.496 + 4.929 \ln x$
with x as the number of years since 1960.

b. For $y = 48\%$, $x = 64$, so the year is
$1960 + 64 = 2024$

30. a. Using technology,
$y = 33.266 + 7.297 \ln x$
with x as the number of years since 2000.

b. For the year 2007, $2007 - 2000 = 7$
$y = 33.266 + 7.297 \ln(7) = 47.5\%$

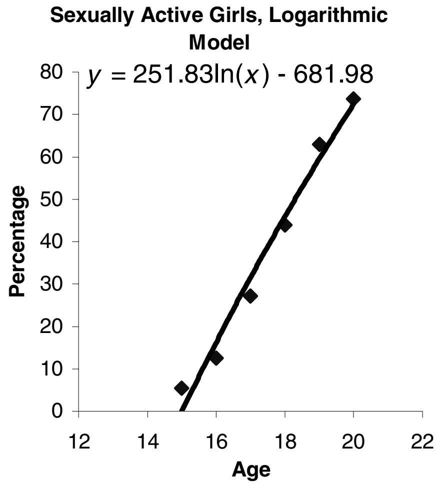
31. a. Using technology,
$y = 2400.49(1.062^x)$
with x = the number of years from 1980 to the end of the academic year.

b. For the year 2016, $2016 - 1980 = 36$
Using the unrounded model, the tuition for 2015-16 will be $21,244. This is an extrapolation since it is outside the table's data.

32. a. Using technology,
$y = 1318.744(1.064^x)$
with x = the number of years after 2000.

b. For the year 2020, $2020 - 2000 = 20$
Using the unrounded model, the estimated U.S. expenditures for health services and supplies in 2020 is $4600 billion.

33. a.



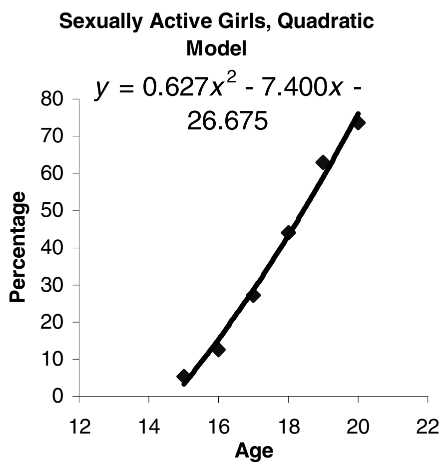
b. $y = 251.83\ln(x) - 681.98$

$y = 251.83\ln(17) - 681.98$

Substituting into the unrounded model yields $y \approx 31.5$.

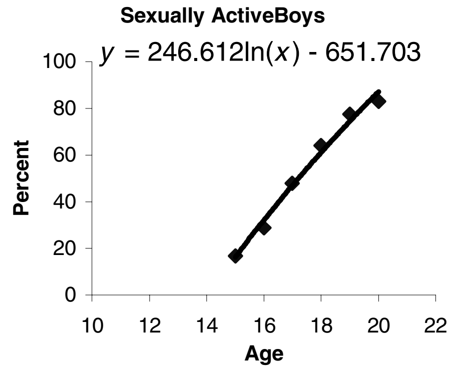
The percentage of girls 17 or younger who have been sexually active is 31.5%.

c.



d. Based on the graphs in parts a) and c), the quadratic function seems to be the better fit.

34. a.



b. $y = -651.703 + 246.612\ln(17)$

Substituting into the unrounded model yields $y \approx 47.021$.

The percentage of boys 17 or younger who have been sexually active is 47.0%.

c. Based on the answers to problems 33 and 34, it seems that more males than females are sexually active at given ages.

35. a. Using technology,

$y = 0.028(1.381^x)$

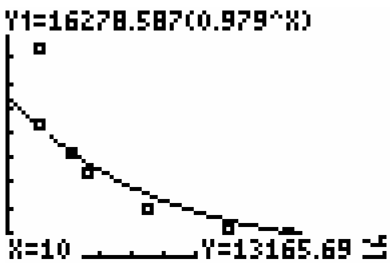
with x = the number of years after 1990.

b. For the year 2015, $2015 - 1990 = 25$

Using the unrounded model, the estimate of the number of FFVs in use in 2015 is 89.5 million.

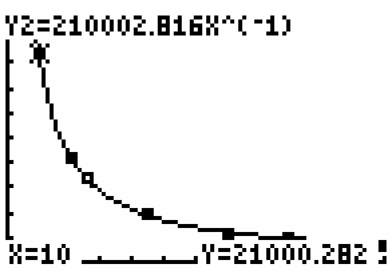
36. a. Based on the figure shown, the data should be modeled by an exponential decay function.

b. Using technology,
$y = 16,278.587(0.979^x)$



[0, 120] by [0, 25,000]

c. Using technology,
$y = 210,002.816x^{-1}$



[0, 120] by [0, 25,000]

d. The power function appears to be the better fit to the data.

e. For x = a fuel economy of 100 mpg, according the power model, its lifetime gasoline use would be 2100 gallons.

X	Y2	
97	2165	
98	2142.9	
99	2121.2	
100	2100	
101	2079.2	
102	2058.9	
103	2038.9	

X=100

Section 5.5 Skills Check

1. $15{,}000e^{0.06(20)}$
$=15{,}000e^{1.2}$
$=15{,}000(3.320116923)$
$=49{,}801.75$

2. $8000e^{0.05(10)}$
$=8000e^{0.5}$
$=8000(1.648721271)$
$=13{,}189.77$

3. $3000(1.06)^{30}$
$=3000(5.743491173)$
$=17{,}230.47$

4. $20{,}000(1.07)^{20}$
$=20{,}000(3.869684462)$
$=77{,}393.69$

5. $12{,}000\left(1+\frac{0.10}{4}\right)^{(4)(8)}$
$=12{,}000(1+.025)^{32}$
$=12{,}000(1.025)^{32}$
$=12{,}000(2.203756938)$
$=26{,}445.08$

6. $23{,}000\left(1+\frac{0.08}{12}\right)^{(12)(20)}$
$=23{,}000(1.00\overline{6})^{240}$
$=23{,}000(4.926802771)$
$=113{,}316.46$

7. $P\left(1+\frac{r}{k}\right)^{kn}$
$=3000\left(1+\frac{0.08}{2}\right)^{(2)(18)}$
$=3000(1.04)^{36}$
$=12{,}311.80$

8. $P\left(1+\frac{r}{k}\right)^{kn}$
$=8000\left(1+\frac{0.12}{12}\right)^{(12)(8)}$
$=8000(1.01)^{96}$
$=20{,}794.18$

9. $300\left[\frac{1.02^{240}-1}{0.02}\right]$
$=300\left[\frac{115.8887352-1}{0.02}\right]$
$=300\left[\frac{114.8887352}{0.02}\right]$
$=300[5744.436758]$
$=1{,}723{,}331.03$

10. $2000\left[\frac{1.10^{12}-1}{0.10}\right]$
$=2000\left[\frac{3.138428377-1}{0.10}\right]$
$=2000\left[\frac{2.138428377}{0.10}\right]$
$=2000[21.38428377]$
$=42{,}768.57$

11. $g(2.5)=1123.60$
$g(3)=1191.00$
$g(3.5)=1191.00$

12. $f(2) = 300$

$f(1.99) = 200$

$f(2.1) = 300$

13.

$$S = P\left(1+\frac{r}{k}\right)^{kn}$$

$$P\left(1+\frac{r}{k}\right)^{kn} = S$$

$$P = \frac{S}{\left(1+\frac{r}{k}\right)^{kn}}$$

$$P = S\left(1+\frac{r}{k}\right)^{-kn}$$

14. $S = P(1+i)^n$

$$P = \frac{S}{(1+i)^n}$$

$$P = S(1+i)^{-n}$$

Section 5.5 Exercises

15. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.08,\ k = 1,\ t = 8$

$$S = 8800\left(1+\frac{0.08}{1}\right)^{(1)(8)}$$

$S = 8800(1.08)^8$

$S = 16,288.19$

The future value is \$16,288.19.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.08,\ k = 1,\ t = 30$

$$S = 8800\left(1+\frac{0.08}{1}\right)^{(1)(30)}$$

$S = 8800(1.08)^{30}$

$S = 88,551.38$

The future value is \$88,551.38.

16. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 6400,\ r = 0.07,\ k = 1,\ t = 10$

$$S = 6400\left(1+\frac{0.07}{1}\right)^{(1)(10)}$$

$S = 6400(1.07)^{10}$

$S = 12,589.77$

The future value is \$12,589.77.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 6400,\ r = 0.07,\ k = 1,\ t = 30$

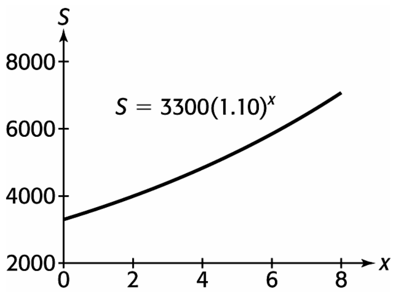
$S = 6400\left(1+\frac{0.07}{1}\right)^{(1)(30)}$

$S = 6400(1.07)^{30}$

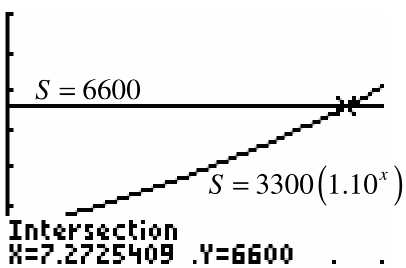
$S = 48{,}718.43$

The future value is \$48,718.43.

17. a.



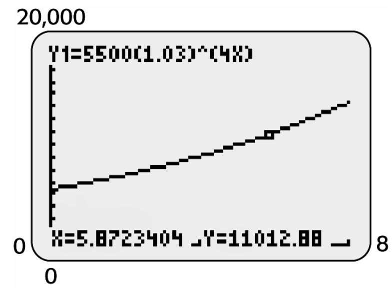
b.



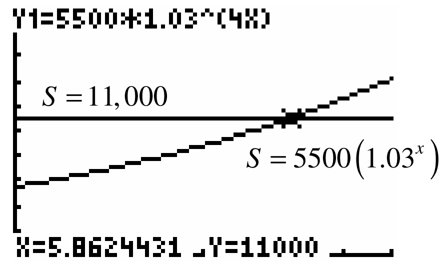
[0, 8] by [2500, 9000]

The initial investment doubles in approximately 7.3 years. After 8 years compounded annually, the initial investment will be more than doubled.

18. a.



b.



[0, 8] by [0, 20,000]

The initial investment doubles in approximately 5.86 years. After 6 years compounded annually, the initial investment will be more than doubled.

19. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10{,}000,\ r = 0.12,\ k = 4,\ t = 10$

$S = 10{,}000\left(1+\frac{0.12}{4}\right)^{(4)(10)}$

$S = 10{,}000(1.03)^{40}$

$S = 32{,}620.38$

The future value is \$32,620.38.

20. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.06,\ k = 2,\ t = 10$

$S = 8800\left(1+\frac{0.06}{2}\right)^{(2)(10)}$

$S = 8800(1.03)^{20}$

$S = 15,893.78$

The future value is $15,893.78.

21. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10,000,\ r = 0.12,\ k = 365,\ t = 10$

$S = 10,000\left(1+\frac{0.12}{365}\right)^{(365)(10)}$

$S = 10,000(1.0003287671233)^{3650}$

$S = 33,194.62$

The future value is $33,194.62.

b. Since the compounding occurs more frequently in Exercise 21 than in Exercise 19, the future value in Exercise 21 is greater.

22. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.06,\ k = 365,\ t = 10$

$S = 8800\left(1+\frac{0.06}{365}\right)^{(365)(10)}$

$S = 8800(1.000164384)^{3650}$

$S = 16,033.85$

The future value is $16,033.85.

b. Since the compounding occurs more frequently in Exercise 22 than in Exercise 20, the future value in Exercise 22 is greater.

23. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10,000,\ r = 0.12,\ k = 12,\ t = 15$

$S = 10,000\left(1+\frac{0.12}{12}\right)^{(12)(15)}$

$S = 10,000(1.01)^{180}$

$S = 59,958.02$

The future value is $59,958.02. The interest earned is the future value minus the original investment. In this case, $59,958.02 – $10,000 = $49,958.02.

24. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 20,000,\ r = 0.08,\ k = 4,\ t = 25$

$S = 20,000\left(1+\frac{0.08}{4}\right)^{(4)(25)}$

$S = 20,000(1.02)^{100}$

$S = 144,892.92$

The future value is $144,892.92. The interest earned is the future value minus the original investment. In this case, $144,892.92 – $20,000 = $124,892.92.

25. a. $S = Pe^{rt}$

$P = 10,000,\ r = 0.06,\ t = 12$

$S = 10,000e^{(0.06)(12)}$

$S = 10,000e^{0.72}$

$S = 20,544.33$

The future value is $20,544.33.

b. $S = Pe^{rt}$

$P = 10,000,\ r = 0.06,\ t = 18$

$S = 10,000e^{(0.06)(18)}$

$S = 10,000e^{1.08}$

$S = 29,446.80$

The future value is $29,446.80.

26. a. $S = Pe^{rt}$

$P = 42,000,\ r = 0.07,\ t = 10$

$S = 42,000e^{(0.07)(10)}$

$S = 42,000e^{0.7}$

$S = 84,577.61$

The future value is $84,577.61.

b. $S = Pe^{rt}$

$P = 42,000,\ r = 0.07,\ t = 20$

$S = 42,000e^{(0.07)(20)}$

$S = 42,000e^{1.4}$

$S = 170,318.40$

The future value is $170,318.40.

27. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10,000,\ r = 0.06,\ k = 1,\ t = 18$

$S = 10,000\left(1+\frac{0.06}{1}\right)^{(1)(18)}$

$S = 10,000(1.06)^{18}$

$S \approx 28,543.39$

The future value is $28,543.39.

b. Continuous compounding yields a higher future value, $29,446.80 – $28,543.39 = $903.41 additional dollars.

28. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 42,000,\ r = 0.07,\ k = 1,\ t = 20$

$S = 42,000\left(1+\frac{0.07}{1}\right)^{(1)(20)}$

$S = 42,000(1.07)^{20}$

$S \approx 162,526.75$

The future value is $162,526.75.

b. Continuous compounding yields a higher future value, $170,318.40 – $162,526.75 = $7791.65 additional dollars.

29. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

Doubling the investment implies $S = 2P$.

$2P = P\left(1+\frac{r}{k}\right)^{kt}$

$\frac{2P}{P} = \frac{P\left(1+\frac{r}{k}\right)^{kt}}{P}$

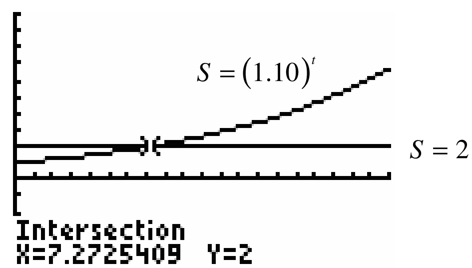
$2 = \left(1+\frac{r}{k}\right)^{kt}$

$k = 1,\ r = 0.10$

$2 = \left(1+\frac{0.10}{1}\right)^{(1)t}$

$2 = (1.10)^t$

Applying the intersection of graphs method:



[0, 20] by [–5, 10]

The time to double is approximately 7.27 years.

b. $S = Pe^{rt}$

Doubling the investment implies

$S = 2P.$

$2P = Pe^{rt}$

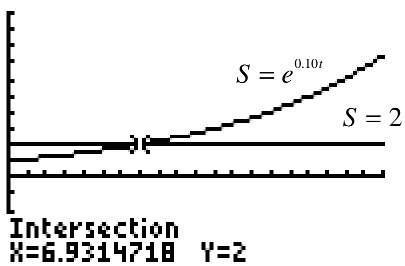
$\frac{2P}{P} = \frac{Pe^{rt}}{P}$

$2 = e^{rt}$

$r = 0.10$

$2 = e^{0.10t}$

Applying the intersection of graphs method:



[0, 20] by [–5, 10]

The time to double is approximately 6.93 years.

30. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

Doubling the investment implies

$S = 2P.$

$2P = P\left(1+\frac{r}{k}\right)^{kt}$

$\frac{2P}{P} = \frac{P\left(1+\frac{r}{k}\right)^{kt}}{P}$

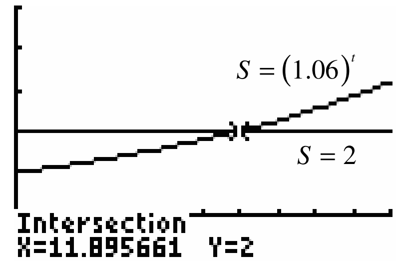
$2 = \left(1+\frac{r}{k}\right)^{kt}$

$k = 1, r = 0.06$

$2 = \left(1+\frac{0.06}{1}\right)^{(1)t}$

$2 = (1.06)^t$

Applying the intersection of graphs method:



[0, 20] by [–1, 5]

The time to double is approximately 11.9 years. In terms of discrete units, the time to double is 12 years.

b. $S = Pe^{rt}$

Doubling the investment implies

$S = 2P.$

$2P = Pe^{rt}$

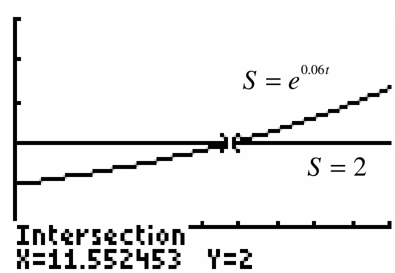
$\frac{2P}{P} = \frac{Pe^{rt}}{P}$

$2 = e^{rt}$

$r = 0.06$

$2 = e^{0.06t}$

Applying the intersection of graphs method:



[0, 20] by [–1, 5]

The time to double is approximately 11.55 years.

31. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 2000,\ r = 0.05,\ k = 1,\ t = 8$

$S = 2000\left(1+\frac{0.05}{1}\right)^{(1)(8)}$

$S = 2000(1.05)^8$

$S = 2954.91$

The future value is \$2954.91.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 2000,\ r = 0.05,\ k = 1,\ t = 18$

$S = 2000\left(1+\frac{0.05}{1}\right)^{(1)(18)}$

$S = 2000(1.05)^{18}$

$S = 4813.24$

The future value is \$4813.24.

32. a.

$S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 12{,}000,\ r = 0.08,\ k = 4,$

$t = \frac{1}{2}$ (2 quarters of a year)

$S = 12{,}000\left(1+\frac{0.08}{4}\right)^{(4)\left(\frac{1}{2}\right)}$

$S = 12{,}000(1.02)^2$

$S = 12{,}484.80$

The future value is \$12,484.80.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 12{,}000,\ r = 0.08,\ k = 4,\ t = 10$

$S = 12{,}000\left(1+\frac{0.08}{4}\right)^{(4)(10)}$

$S = 12{,}000(1.02)^{40}$

$S = 26{,}496.48$

The future value is \$26,496.48.

33. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 3000,\ r = 0.06,\ k = 12,\ t = 12$

$S = 3000\left(1+\frac{0.06}{12}\right)^{(12)(12)}$

$S = 3000(1.005)^{144}$

$S = 6152.25$

The future value is \$6152.25.

34. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 9000,\ r = 0.08,\ k = 4,\ t = 0.5$

$S = 9000\left(1+\frac{0.08}{4}\right)^{(4)(0.5)}$

$S = 9000(1.02)^2$

$S = 9363.60$

The future value is \$9363.60.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 9000,\ r = 0.08,\ k = 4,\ t = 15$

$S = 9000\left(1+\frac{0.08}{4}\right)^{(4)(15)}$

$S = 9000(1.02)^{60}$

$S = 29{,}529.28$

The future value is \$29,529.28.

35. a.

Years	Future Value
0	1000
7	2000
14	4000
21	8000
28	16,000

b. $S = 1000\left(1 + \frac{0.10}{4}\right)^{4t}$

$S = 1000(1.025)^{4t}$

$S = 1000\left((1.025)^4\right)^t$

$S = 1000(1.104)^t$

c. After five years, the investment is worth $S = 1000(1.104)^5 = \$1640.01$.
After 10.5 years, the investment is worth $S = 1000(1.104)^{10.5} = \$2826.02$.

36. a.

Years	Future Value
0	1000
6	2000
12	4000
18	8000
24	16,000

b. $S = 1000\left(1 + \frac{0.116}{12}\right)^{12t}$

$S = 1000(1.009\overline{6})^{12t}$

$S = 1000\left((1.009\overline{6})^{12}\right)^t$

$S = 1000(1.122)^t$

c. After two months, the value of the investment is
$S = 1000(1.122)^{\left(\frac{2}{12}\right)} = \$1019.37$.

After four years, the investment is worth $S = 1000(1.122)^4 = \$1584.79$.

After 12.5 years, the investment is worth $S = 1000(1.122)^{12.5} = \$4216.10$.

37.

$$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$65{,}000 = P\left(1 + \frac{0.10}{12}\right)^{(12)(8)}$$

$$65{,}000 = P(1.008\overline{3})^{96}$$

$$P = \frac{65{,}000}{(1.008\overline{3})^{96}}$$

$$P = \$29{,}303.36$$

38.

$$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$30{,}000 = P\left(1 + \frac{0.08}{4}\right)^{(4)(12)}$$

$$30{,}000 = P(1.02)^{48}$$

$$P = \frac{30{,}000}{(1.02)^{48}}$$

$$P = \$11{,}596.13$$

39.

$$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$10{,}000 = P\left(1 + \frac{0.06}{1}\right)^{(1)(10)}$$

$$10{,}000 = P(1.06)^{10}$$

$$P = \frac{10{,}000}{(1.06)^{10}}$$

$$P = \$5{,}583.95$$

An initial amount of \$5583.95 will grow to \$10,000 in 10 years if invested at 6% compounded annually.

40.

$$S = P\left(1+\frac{r}{k}\right)^{kt}$$

$$30,000 = P\left(1+\frac{0.07}{1}\right)^{(1)(15)}$$

$$30,000 = P(1.07)^{15}$$

$$P = \frac{30,000}{(1.07)^{15}}$$

$$P = \$10,873.38$$

An initial amount of $10,873.38 will grow to $30,000 in 15 years if invested at 7% compounded annually.

41.

$$S = P\left(1+\frac{r}{k}\right)^{kt}$$

$$30,000 = P\left(1+\frac{0.10}{12}\right)^{(12)(18)}$$

$$30,000 = P\left(1.008\overline{3}\right)^{216}$$

$$P = \frac{30,000}{\left(1.008\overline{3}\right)^{216}}$$

$$P = \$4,996.09$$

An initial amount of $4996.09 will grow to $30,000 in 18 years if invested at 10% compounded monthly.

42.

$$S = P\left(1+\frac{r}{k}\right)^{kt}$$

$$1,000,000 = P\left(1+\frac{0.11}{12}\right)^{(12)(50)}$$

$$1,000,000 = P\left(1.0091\overline{6}\right)^{600}$$

$$P = \frac{1,000,000}{\left(1.0091\overline{6}\right)^{600}}$$

$$P = \$4,190.46$$

An initial amount of $4190.46 will grow to $1,000,000 in 50 years if invested at 11% compounded monthly.

43.

$$S = P\left(1+\frac{r}{k}\right)^{kt}$$

$$80,000 = P\left(1+\frac{0.10}{12}\right)^{(12)(12)}$$

$$80,000 = P\left(1.008\overline{3}\right)^{144}$$

$$P = \frac{80,000}{\left(1.008\overline{3}\right)^{144}}$$

$$P = \$24,215.65$$

An initial amount of $24,215.65 will grow to $80,000 in 12 years if invested at 10% compounded monthly.

44.

$$S = P\left(1+\frac{r}{k}\right)^{kt}$$

$$1,000,000 = P\left(1+\frac{0.10}{12}\right)^{(12)(25)}$$

$$1,000,000 = P\left(1.008\overline{3}\right)^{300}$$

$$P = \frac{1,000,000}{\left(1.008\overline{3}\right)^{300}}$$

$$= \frac{1,000,000}{12.056945}$$

$$P = \$82,939.75$$

They should invest an initial amount of $82,939.75 which will grow to $1,000,000 in 25 years if invested at 10% compounded monthly.

45.

$$40,000 = 10,000\left(1+\frac{0.08}{12}\right)^{12t}$$

$$4 = \left(1+\frac{0.08}{12}\right)^{12t}$$

$$\ln(4) = \ln\left[\left(1+\frac{0.08}{12}\right)^{12t}\right]$$

$$12t\ln\left(1.00\overline{6}\right) = \ln(4)$$

$$t = \frac{\ln(4)}{12\ln\left(1.00\overline{6}\right)}$$

$$t \approx 17.3864$$

It will take approximately 17.39 years, or 17 years and 5 months, for the initial investment of \$10,000 to grow to \$40,000.

46.

$$60,000 = 25,000\left(1+\frac{0.12}{4}\right)^{4t}$$

$$2.4 = (1.03)^{4t}$$

$$\ln(2.4) = \ln\left[(1.03)^{4t}\right]$$

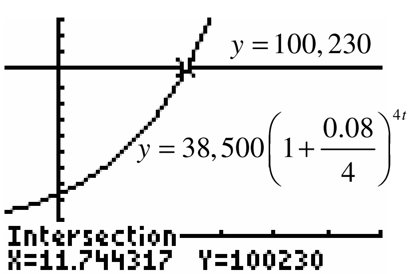
$$4t\ln(1.03) = \ln(2.4)$$

$$t = \frac{\ln(2.4)}{4\ln(1.03)}$$

$$t \approx 7.4046729$$

It will take approximately 7.4 years, or 7 years and 5 months, for the initial investment of \$25,000 to grow to \$60,000.

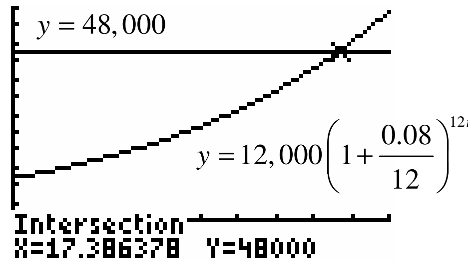
47. Applying the intersection of graphs method:



[–5, 30] by [–20,000, 130,000]

After 12 years, the future value of the investment will be greater than \$100,230.

48. Applying the intersection of graphs method:



[0, 20] by [–10,000, 60,000]

For the first 17 years and 4 months, the future value of the investment will be below \$48,000.

49.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$2P = P\left(1+\frac{r}{m}\right)^{mt}$$

$$2 = \left(1+\frac{r}{m}\right)^{mt}$$

$$\ln(2) = \ln\left[\left(1+\frac{r}{m}\right)^{mt}\right]$$

$$\ln(2) = mt\left[\ln\left(1+\frac{r}{m}\right)\right]$$

$$t = \frac{\ln(2)}{m\ln\left(1+\frac{r}{m}\right)}$$

Section 5.6 Skills Check

1.
$$S = P(1+i)^n$$
$$\frac{S}{(1+i)^n} = \frac{P(1+i)^n}{(1+i)^n}$$
$$P = \frac{S}{(1+i)^n}$$

2. Considering the answer to Exercise 1 and continuing the algebra yields
$$P = \frac{S}{(1+i)^n} = S(1+i)^{-n}.$$

3.
$$A\,i = R\left[1-(1+i)^{-n}\right]$$
$$\frac{A\,i}{i} = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$
$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

4.
$$2000\left[\frac{1-(1+0.01)^{-240}}{0.01}\right]$$
$$= 2000\left[\frac{1-(1.01)^{-240}}{0.01}\right]$$
$$= 2000\left[\frac{1-0.0918058365}{0.01}\right]$$
$$= 2000\left[\frac{0.9081941635}{0.01}\right]$$
$$= 2000[90.81941635]$$
$$= \$181,638.83$$

5.
$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$i\ A = i\left(R\left[\frac{1-(1+i)^{-n}}{i}\right]\right)$$
$$iA = R\left[1-(1+i)^{-n}\right]$$
$$\frac{iA}{\left[1-(1+i)^{-n}\right]} = \frac{R\left[1-(1+i)^{-n}\right]}{\left[1-(1+i)^{-n}\right]}$$
$$R = \frac{iA}{\left[1-(1+i)^{-n}\right]} = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$

6.
$$240,000\left[\frac{0.01}{1-(1+0.01)^{-120}}\right]$$
$$= 240,000\left[\frac{0.01}{1-(1.01)^{-120}}\right]$$
$$= 240,000\left[\frac{0.01}{1-(0.3029947797)}\right]$$
$$= 240,000\left[\frac{0.01}{0.6970052203}\right]$$
$$= 240,000[0.0143470948]$$
$$= 3443.30$$

Section 5.6 Exercises

7. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 4000\left[\frac{(1+.06)^{10} - 1}{.06}\right] = 52{,}723.18$

8. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 5000\left[\frac{(1+.09)^{20} - 1}{.09}\right] = 255{,}800.60$

9. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 1000\left[\frac{(1+.08/2)^{2(8)} - 1}{.04}\right] = 21{,}824.53$

10. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 2600\left[\frac{(1+.06/4)^{4(5)} - 1}{.015}\right] = 60{,}121.53$

11. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 600\left[\frac{(1+.07/12)^{12(25)} - 1}{.00583̄}\right] = 486{,}043.02$

12. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 800\left[\frac{(1+.07/12)^{12(5)} - 1}{.0058\bar{3}}\right] = 57{,}274.32$

13. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 1000\left[\frac{(1+.10/2)^{2(4)} - 1}{.05}\right] = 9549.11$

14. $A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$A = 1000\left[\frac{(1+.06)^{19} - 1}{.06}\right] = 33{,}759.99$

15. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$A = 1000\left[\frac{1-(1+0.07)^{-10}}{0.07}\right]$

$A = 1000\left[\frac{1-(1.07)^{-10}}{0.07}\right]$

$A = 1000\left[\frac{1-(0.5083492921)}{0.07}\right]$

$A = 1000[7.023581541]$

$A = 7023.58$

Investing \$7023.58 initially will produce an income of \$1000 per year for 10 years if the interest rate is 7% compounded annually.

16. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$A = 500\left[\frac{1-(1+0.09)^{-20}}{0.09}\right]$

$A = 500\left[\frac{1-(1.09)^{-20}}{0.09}\right]$

$A = 500\left[\frac{1-(0.1784308898)}{0.09}\right]$

$A = 500[9.128545669]$

$A \approx 4564.27$

Investing \$4564.27 initially will produce an income of \$500 per year for 20 years if the interest rate is 9% compounded annually.

17. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$A = 50,000\left[\frac{1-(1+0.08)^{-19}}{0.08}\right]$

$A = 50,000\left[\frac{1-(1.08)^{-19}}{0.08}\right]$

$A = 50,000\left[\frac{1-(0.231712064)}{0.08}\right]$

$A = 50,000[9.6035992]$

$A = 480,179.96$

The formula above calculates the present value of the annuity given the payment made at the end of each period. Twenty total payments were made, but only nineteen occurred at the end of a compounding period. The first payment of \$50,000 was made up front. Therefore, the total value of the lottery winnings is $\$50,000 + \$480,179.96 = \$530,179.96$.

18.

$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$i = \frac{0.08}{2} = 0.04,\ n = 4\times 2 = 8$

$A = 3000\left[\frac{1-(1+0.04)^{-8}}{0.04}\right]$

$A = 3000\left[\frac{1-(1.04)^{-8}}{0.04}\right]$

$A = 3000[6.732744875]$

$A \approx 20,198.23$

A lump sum of \$20,198.23 is required to generate the annuity.

19. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$A = 3000\left[\frac{1-\left(1+\frac{0.09}{12}\right)^{-(30)(12)}}{\frac{0.09}{12}}\right]$

$A = 3000\left[\frac{1-(1.0075)^{-360}}{0.0075}\right]$

$A = 3000\left[\frac{1-0.0678860074}{0.0075}\right]$

$A = 3000[124.2818657]$

$A = 372,845.60$

The disabled man should seek a lump sum payment of \$372,845.60.

20. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$A = 400\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-(12)(4)}}{\frac{0.08}{12}}\right]$$

$$A = 400\left[\frac{1-(1.00\overline{6})^{-48}}{0.00\overline{6}}\right]$$

$$A = 400[40.96191296]$$

$$A = 16,384.77$$

A fair offer for the car would be \$16,384.77.

21. a.

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$A = 122,000\left[\frac{1-(1+0.10)^{-9}}{0.10}\right]$$

$$A = 122,000\left[\frac{1-(1.10)^{-9}}{0.10}\right]$$

$$A = 122,000\left[\frac{1-0.4240976184}{0.10}\right]$$

$$A = 122,000[5.759023816]$$

$$A = \$702,600.91$$

b.

$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$

$$R = 700,000\left[\frac{0.10}{1-(1+0.10)^{-9}}\right]$$

$$R = 700,000\left[\frac{0.10}{1-(1.10)^{-9}}\right]$$

$$R = 700,000\left[\frac{0.10}{1-0.4240976184}\right]$$

$$R = 700,000[0.1736405391]$$

$$R = \$121,548.38$$

The annuity payment is \$121,548.38.

c. The \$100,000 plus the annuity yields a higher present value and therefore would be the better choice. Over the nine year annuity period, the \$100,000 cash plus \$122,000 annuity yields \$452 more per year than investing \$700,000 in cash.

22. a. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$A = 250,000\left[\frac{1-(1+0.07)^{-5}}{0.07}\right]$$

$$A = 250,000\left[\frac{1-(1.07)^{-5}}{0.07}\right]$$

$$A = 250,000[4.100197436]$$

$$A = 1,025,049.36$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. Six total payments were made, but only five occurred at the end of a compounding period. The first payment of \$200,000 was made up front. Therefore, the total value of the sale is $\$200,000 + \$1,025,049.36 = \$1,225,049.36$.

b.

$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$

$$R = 1{,}000{,}000\left[\frac{0.07}{1-(1+0.07)^{-5}}\right]$$

$$R = 1{,}000{,}000\left[\frac{0.07}{1-(1.07)^{-5}}\right]$$

$$R = 1{,}000{,}000[0.2438906944]$$

$$R = \$243{,}890.69$$

The annuity payment is $243,890.69.

c. The present value of the all cash transaction is $1,200,000, while the present value of the cash plus annuity transaction is $1,225,049. The cash plus annuity is better. Over the 5-year annuity period, the $200,000 cash plus $250,000 annuity yields approximately $6109 per month more than investing $1,000,000 in cash.

23. a.

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$A = 1600\left[\frac{1-\left(1+\frac{0.09}{12}\right)^{-(30)(12)}}{\frac{0.09}{12}}\right]$$

$$A = 1600\left[\frac{1-(1.0075)^{-360}}{0.0075}\right]$$

$$A = 1600\left[\frac{1-0.0678860074}{0.0075}\right]$$

$$A = 1600[124.2818657]$$

$$A = \$198{,}850.99$$

The couple can afford to pay $198,850.99 for a house.

b. $(\$1600 \text{ per month})\times(12 \text{ months})$
$\times(30 \text{ years}) = \$576{,}000$

c. $\$576{,}000 - \$198{,}850.99 = \$377{,}149.01$

24. a.

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$A = 400\left[\frac{1-\left(1+\frac{0.12}{12}\right)^{-(4)(12)}}{\frac{0.12}{12}}\right]$$

$$A = 400\left[\frac{1-(1.01)^{-48}}{0.01}\right]$$

$$A = 400[37.97395949]$$

$$A = \$15{,}189.58$$

A total of $15,189.58 can be paid for the car in order for the payment to remain $400 per month.

b. $(\$400 \text{ per month})\times(48 \text{ months})$
$= \$19{,}200$

c. $19{,}200 - 15{,}189.58 = \$4010.42$
The interest is $4010.42.

25. a. $\frac{8}{4} = 2\%$

b. $(4 \text{ years})\times(4 \text{ payments per year})$
$= 16 \text{ payments}$

c.

$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$
$$R = 10,000\left[\frac{0.02}{1-(1+0.02)^{-16}}\right]$$
$$R = 10,000\left[\frac{0.02}{1-(1.02)^{-16}}\right]$$
$$R = 10,000\left[\frac{0.02}{1-0.7284458137}\right]$$
$$R = 10,000[0.0736501259]$$
$$R = \$736.50$$

The quarterly payment is $736.50.

26. a. $\frac{6}{12} = 0.5\%$

b. $(6 \text{ years}) \times (12 \text{ payments per year})$
$= 72$ payments

c.

$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$
$$R = 36,000\left[\frac{0.005}{1-(1+0.005)^{-72}}\right]$$
$$R = 36,000\left[\frac{0.005}{1-(1.005)^{-72}}\right]$$
$$R = 36,000[0.0165728879]$$
$$R = \$596.62$$

The monthly car payment is $596.62.

27. a.

$$i = \frac{0.06}{12} = 0.005,\ n = 360$$
$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$
$$R = 250,000\left[\frac{0.005}{1-(1+0.005)^{-360}}\right]$$
$$R = 250,000\left[\frac{0.005}{1-0.166041928}\right]$$
$$R = 250,000\left[\frac{0.005}{0.833958072}\right]$$
$$R = 250,000[0.0059955053]$$
$$R = \$1498.88$$

The monthly mortgage payment is $1498.88.

b. $(30 \text{ years}) \times (12 \text{ payments per year})$
$\times(\$1498.88) = \$539,596.80$
Including the down payment, the total cost of the house is $639,596.80.

c. $\$639,596.80 - \$350,000 = \$289,596.80$

28. a.

$$i = \frac{0.08}{4} = 0.02,\ n = 100$$
$$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$
$$R = 450,000\left[\frac{0.02}{1-(1+0.02)^{-100}}\right]$$
$$R = 450,000\left[\frac{0.02}{1-(1.02)^{-100}}\right]$$
$$R = 450,000[0.0232027435]$$
$$R = \$10,441.23$$

The monthly payment is $10,441.23.

b. $(25 \text{ years}) \times (4 \text{ payments per year})$
$\times(\$10,441.23) = \$1,044,123$
Including the down payment, the total cost of the restaurant is $1,344,123.

c. $1,344,123 - 750,000 = \$594,123$

Section 5.7 Skills Check

1. $\dfrac{79.514}{1+0.835e^{-0.0298(80)}}$

$=\dfrac{79.514}{1+0.835e^{-2.384}}$

$=\dfrac{79.514}{1+0.835(0.0921811146)}$

$=\dfrac{79.514}{1.076971231}$

$=73.83112727$

≈ 73.83

2. a. $y=\dfrac{79.514}{1+0.835e^{-0.0298x}}$

$y=\dfrac{79.514}{1+0.835e^{-0.0298(10)}}$

$y=\dfrac{79.514}{1+0.835(0.7423013397)}$

$y=\dfrac{79.514}{1.619821619}$

$y=49.08812124$

$y\approx 49.09$

b. $y=\dfrac{79.514}{1+0.835e^{-0.0298x}}$

$y=\dfrac{79.514}{1+0.835e^{-0.0298(50)}}$

$y=\dfrac{79.514}{1+0.835(0.2253726555)}$

$y=\dfrac{79.514}{1.188186167}$

$y=66.92048955$

$y\approx 66.92$

3. $1000(0.06)^{0.2^t}$

Let $t=4$.

$1000(0.06)^{0.2^4}=1000(0.06)^{0.0016}$

$=1000(0.9955086592)$

≈ 995.51

Let $t=6$.

$1000(0.06)^{0.2^6}=1000(0.06)^{0.000064}$

$=1000(0.9998199579)$

≈ 999.82

4. $2000(0.004)^{0.5^t}$

Let $t=5$.

$2000(0.004)^{0.5^5}=2000(0.004)^{0.03125}$

$=2000(0.8415198695)$

≈ 1683.04

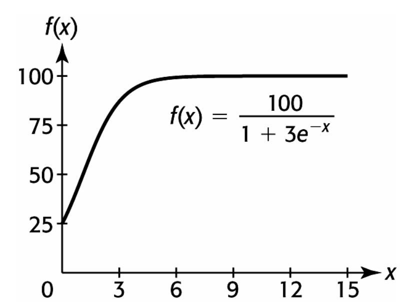
Let $t=10$.

$2000(0.004)^{0.5^{10}}=2000(0.004)^{0.0009765625}$

$=2000(0.9946224593)$

≈ 1989.24

5. a.



b.

X	Y1
0	25
5	98.019
10	99.986
15	100
20	100
25	100
30	100

X=0

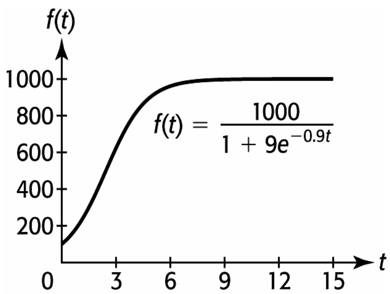
$f(0) = 25$

$f(10) = 99.986 = 99.99$

c. The graph is increasing.

d. Based on the graph, the y-values of the function approach 100. Therefore the limiting value of the function is 100. $y = c = 100$ is a horizontal asymptote of the function.

6. a.



b.

X	Y1
0	100
1	214.63
2	401.98
3	623.11
4	802.62
5	909.11
6	960.94

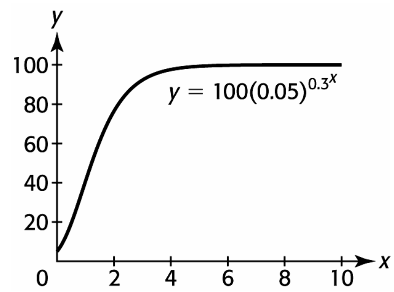
X=2

$f(2) = 401.98$

$f(5) = 909.11$

c. Based on the graph, the y-values of the function approach 1000. Therefore the limiting value of the function is 1000. $y = c = 1000$ is a horizontal asymptote.

7. a.



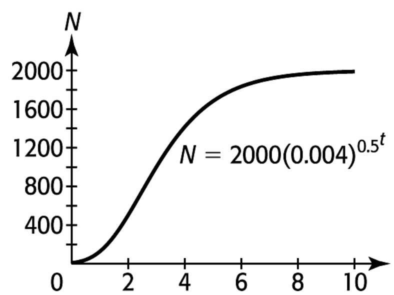
b. Let $x = 0$, and solve for y.

$y = 100(0.05)^{0.3^0} = 100(0.05)^1 = 5$

The initial value is 5.

c. The limiting value is c. In this case, $c = 100$.

8. a.



b. Let $t = 0$, and solve for N.

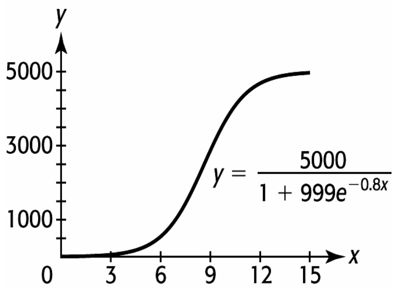
$N = 2000(0.004)^{0.5^0} = 2000(0.004)^1 = 8$

The initial value is 8.

c. The limiting value is c. In this case, $c = 2000$.

Section 5.7 Exercises

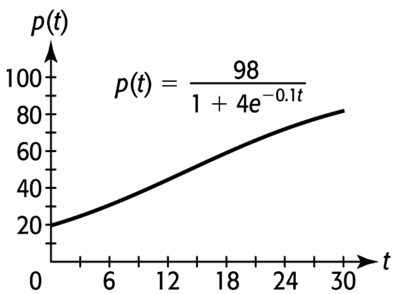
9. a.



b. At $x = 0$, the number of infected students is the value of the y-intercept of the function. The y-intercept is $\dfrac{5000}{1+999e^{-0.8(0)}} = \dfrac{5000}{1+999} = 5$.

c. The upper limit is $c = 5000$ students.

10. a.



b. $p(10) = \dfrac{98}{1+4e^{-0.1(10)}}$

$= \dfrac{98}{1+4e^{-1}} \approx 39.652$

The population in 1998 is approximately 39,652 people.

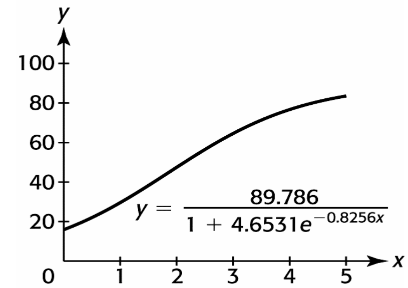
c. $p(100) = \dfrac{98}{1+4e^{-0.1(100)}}$

$= \dfrac{98}{1+4e^{-10}} \approx 97.982$

The population in 2088 is approximately 97,982 people.

d. The upper limit is $c = 98$ or 98,000 people.

11. a.



b.

X	Y1
0	15.883
1	29.555
2	47.442
3	64.552
4	76.661
5	83.523
6	86.931

X=1

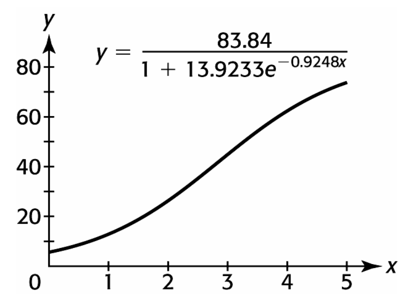
The model indicates that 29.56% of 16-year old boys have been sexually active

c. Consider the table in part b) above.

The model indicates that 86.93% of 21-year old boys have been sexually active

d. The upper limit is $c = 89.786\%$.

12. a.



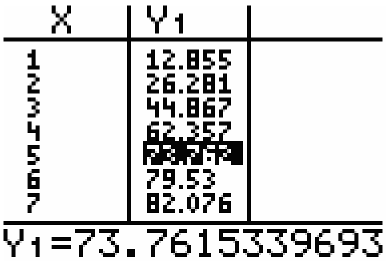
b.

X	Y1
-1	2.3221
0	5.6181
1	12.855
2	26.281
3	44.867
4	62.357
5	73.762

Y1=12.8546958027

The model indicates that approximately 12.85% of 16-year old girls have been sexually active.

c.



The model indicates that 73.76% of 20-year old girls have been sexually active.

d. The upper limit is c = 83.84%.

13. a. Let $t=1$ and solve for N.

$$\begin{aligned} N &= \frac{10,000}{1+100e^{-0.8(1)}} \\ &= \frac{10,000}{1+100e^{-0.8}} \\ &= \frac{10,000}{45.393289641} \\ &\approx 218 \end{aligned}$$

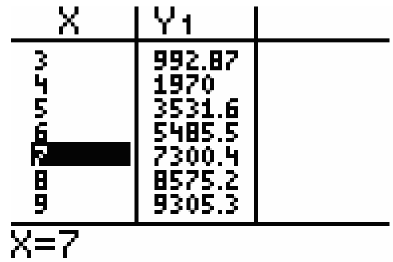
Approximately 218 people have heard the rumor by the end of the first day.

b. Let $t=4$ and solve for N.

$$\begin{aligned} N &= \frac{10,000}{1+100e^{-0.8(4)}} \\ &= \frac{10,000}{1+100e^{-3.2}} \\ &= \frac{10,000}{5.076220398} \\ &\approx 1970 \end{aligned}$$

Approximately 1970 people have heard the rumor by the end of the fourth day.

c.



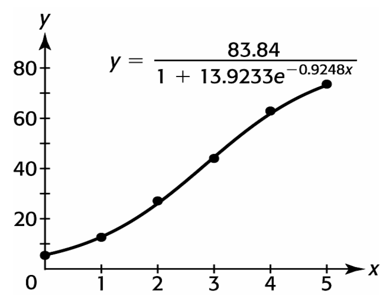
By the end of the seventh day, 7300 people have heard the rumor.

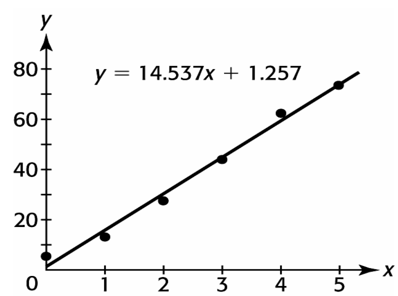
14. a. $y=\dfrac{83.84}{1+13.9233e^{-0.9248x}}$

b. Yes, the models are the same.

c. $y=14.537x+1.257$

d.





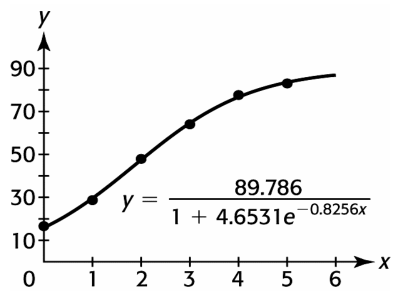
The logistic model is a better fit for the data.

15. a. $y=\dfrac{89.786}{1+4.6531e^{-0.8256x}}$

b. Yes, the models are the same.

c. $y=14.137x+17.624$

d.



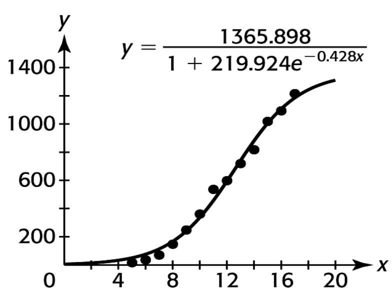
The logistic model is a better fit.

16. a. $y=\dfrac{1365.898}{1+219.924e^{-0.428x}}$

b. Using the unrounded model for $x = 25$, the number of users in 2015 is 1359.100 million.

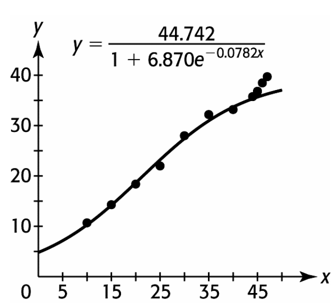
c. Approximately, 1366 million

d. Yes, the model is a good fit.



17. a. $y=\dfrac{44.742}{1+6.870e^{-0.0782x}}$

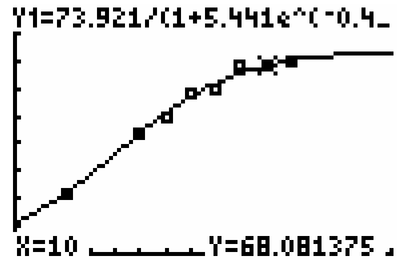
b.



c. 40.9% in 2015.

18. a. $y=\dfrac{73.921}{1+5.441e^{-0.415x}}$

b. It appears to be a good fit.



[0, 15] by [0, 90]

c. 73.1% in 2010.

19. a. $y=\dfrac{82.488}{1+0.816e^{-0.024x}}$

b. The expected life span for a person born in 1955 was 67.9 years, and in 2006, it was 77.7 years.

c. The upper limit for a person's life span is 82.5 years.

20. a. $y=\dfrac{1241}{1+0.489e^{-0.08x}}$

b. In 2015, the SAT composite score for this school is estimated to be1164.

21. a. Let $t = 0$ and solve for N.

$$\begin{aligned} N &= 10{,}000(0.4)^{0.2^0} \\ &= 10{,}000(0.4)^1 \\ &= 10{,}000(0.4) \\ &= 4000 \end{aligned}$$

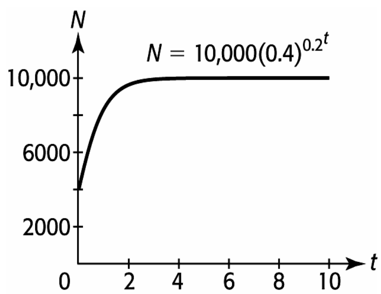
The initial population size is 4000 students.

b. Let $t = 4$ and solve for N.

$$\begin{aligned} N &= 10{,}000(0.4)^{0.2^4} \\ &= 10{,}000(0.4)^{0.0016} \\ &= 10{,}000(0.998535009) \\ &= 9985.35009 \\ &\approx 9985 \end{aligned}$$

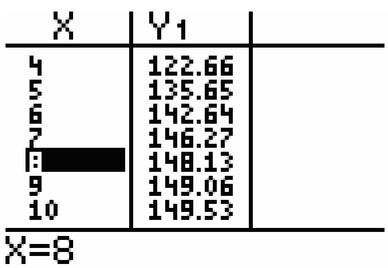
After four years, the population is approximately 9985 students.

c.



The upper limit appears to be 10,000.

22. a.

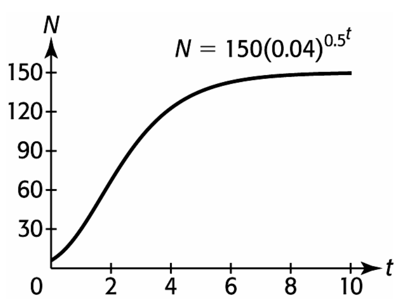


X	Y1
4	122.66
5	135.65
6	142.64
7	146.27
8	148.13
9	149.06
10	149.53

X=8

In eight years the number of employees is approximately 148.

b.



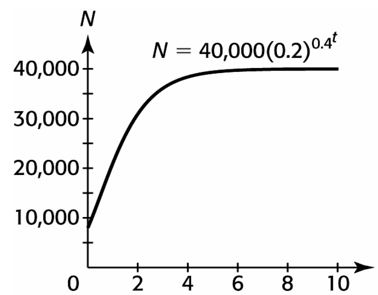
As the time increases, the number of employees approaches 150.

23. a. Let $t = 1$ and solve for N.

$$\begin{aligned} N &= 40{,}000(0.2)^{0.4^1} \\ &= 40{,}000(0.2)^{0.4} \\ &= 40{,}000(0.5253055609) \\ &= 21{,}012.22244 \\ &\approx 21{,}012 \end{aligned}$$

After one month, the approximately 21,012 units will be sold.

b.



c. The upper limit appears to be 40,000.

24. a. Let $t = 0$ and solve for N.

$$\begin{aligned} N &= 1600(0.6)^{0.2^0} \\ &= 1600(0.6)^1 \\ &= 1600(0.6) \\ &= 960 \end{aligned}$$

Initially the company has 960 employees.

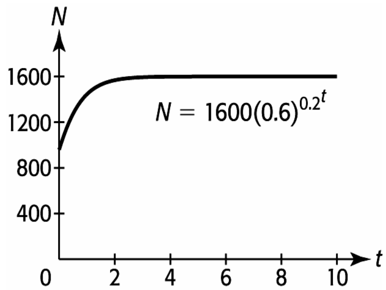
b. Let $t = 3$ and solve for N.

$$\begin{aligned} N &= 1600(0.6)^{0.2^3} \\ &= 1600(0.6)^{0.008} \\ &= 1600(0.9959217338) \\ &\approx 1593.47 \end{aligned}$$

After three years, the company had approximately 1593 employees.

c. The upper limit is 1600 employees.

d.



25. a. Let $t = 0$ and solve for N.

$$N = 1000(0.01)^{0.5^0}$$
$$= 1000(0.01)^1$$
$$= 1000(0.01)$$
$$= 10$$

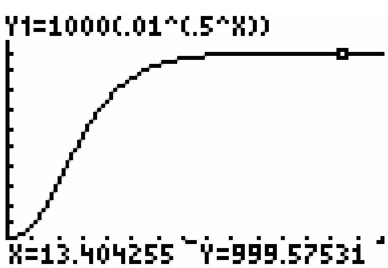
Initially the company had 10 employees.

b. Let $t = 1$ and solve for N.

$$N = 1000(0.01)^{0.5^1}$$
$$= 1000(0.01)^{0.5}$$
$$= 1000(0.1)$$
$$= 100$$

After one year, the company had 100 employees.

c. The upper limit is 1000 employees.



[0, 15] by [0, 1200]

d.

X	Y1
3	562.34
4	749.89
5	865.96
6	930.57
7	964.66
8	982.17
9	991.05

X=6

In the sixth year 930 people were employed by the company.

26. a. Let $t = 0$ and solve for N.

$$N = 8000(0.1)^{0.3^0}$$
$$= 8000(0.1)^1$$
$$= 8000(0.1)$$
$$= 800$$

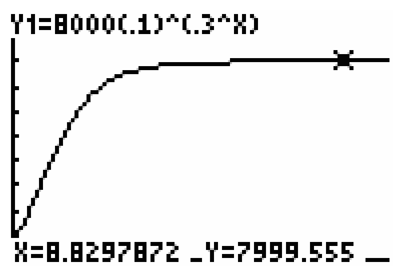
Initially the company sold 800 units.

b. Let $t = 1$ and solve for N

$$N = 8{,}000(0.1)^{0.3^1}$$
$$= 8{,}000(0.1)^{0.3}$$
$$= 8{,}000(0.5011872336) \approx 4009.50$$

After three weeks, the company sold approximately 4009 units.

c. The upper limit is 8000 units.



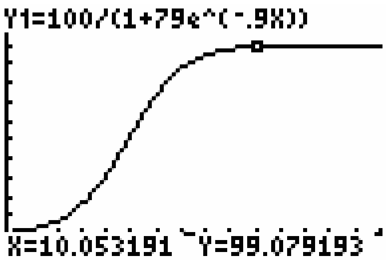
[0, 10] by [0, 10,000]

d.

X	Y1
0	800
1	4009.5
2	6502.6
3	7517.8
4	7852.2
5	7955.4
6	7986.6

X=2

In the second week, approximately 6500 units were sold by the company.

27. 

[0, 15] by [0, 120]

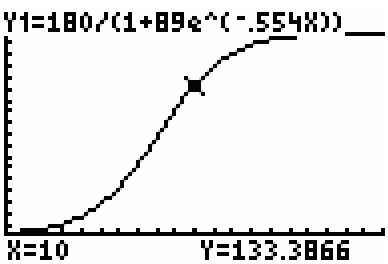
After 10 days, 99 people are infected.

28.

X	Y1
6.75	5105.3
7	5601
7.25	6111.7
7.5	6632.2
[illegible]	7156.9
8	7679.8
8.25	8195.1

X=7.75

In about eight weeks, half the community has been reached by the advertisement.

29. 

[0, 20] by [–20, 200]

X	Y1
7	63.345
8	87.452
9	111.93
10	133.39
[illegible]	149.9
12	161.38
13	168.81

X=11

In approximately 11 years, the deer population reaches a level of 150.

30.

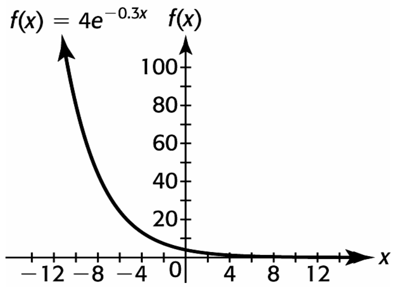
X	Y1
6.75	281.95
7	324.25
7.25	368.39
7.5	413.32
7.75	457.92
[illegible]	501.09
8.25	541.88

X=8

Five hundred students in the elementary school will be infected in approximately 8 days.

Chapter 5 Skills Check

1. a.

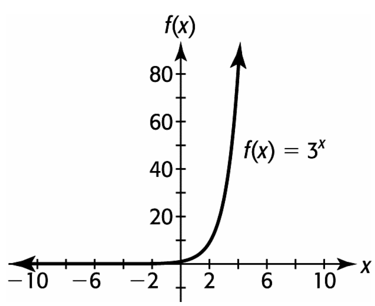


b. $f(-10) = 4e^{-0.3(-10)}$
$= 4e^{3}$
≈ 80.342

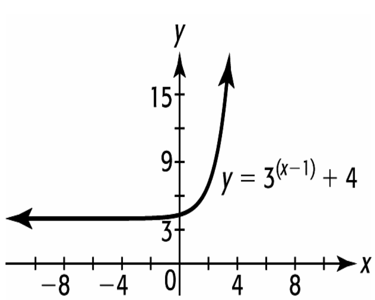
$f(10) = 4e^{-0.3(10)}$
$= 4e^{-3}$
≈ 0.19915

2. The function in Exercise 1, $f(x) = 4e^{-0.3x}$, is decreasing.

3.



4.



5. The graph in Exercise 4 is shifted right one unit and up four units in comparison with the graph in Exercise 3.

6. The function in Exercise 4 is increasing.

7. a. $y = 1000(2)^{-0.1x}$
$= 1000(2)^{-0.1(10)}$
$= 1000(2)^{-1}$
$= 500$

b.

X	Y1
15	353.55
16	329.88
17	307.79
18	287.17
19	267.94
20	250
21	233.26

X=20

When $y = 250$, $x = 20$.

8. $x = 6^{y} \Leftrightarrow \log_6 x = y$

9. $y = 7^{3x} \Leftrightarrow \log_7 y = 3x$

10. $y = \log_4 x \Leftrightarrow x = 4^{y}$

11. $y = \log(x) = \log_{10} x$
$y = \log_{10} x \Leftrightarrow x = 10^{y}$

12. $y = \ln x = \log_e x$
$y = \log_e x \Leftrightarrow x = e^{y}$

13. $y = 4^x$

$x = 4^y$

$x = 4^y \Leftrightarrow \log_4 x = y$

Therefore, the inverse function is $y = \log_4 x$.

14. $\log 22 = \log_{10} 22 = 1.3424$

15. $\ln 56 = \log_e 56 = 4.0254$

16. $\log 10 = \log_{10} 10 = 1$

17. $\log_2 16$

$y = \log_2 16 \Leftrightarrow 2^y = 16$

$y = 4$

18. $\ln\left(e^4\right) = \log_e\left(e^4\right)$

$y = \log_e\left(e^4\right) \Leftrightarrow e^y = e^4$

$y = 4$

19. $\log(0.001) = \log_{10}\left(\frac{1}{1000}\right)$

$y = \log_{10}\left(\frac{1}{1000}\right) \Leftrightarrow 10^y = \frac{1}{1000}$

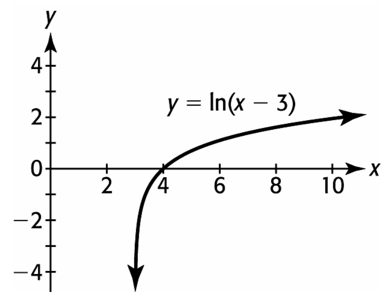
$10^y = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

$y = -3$

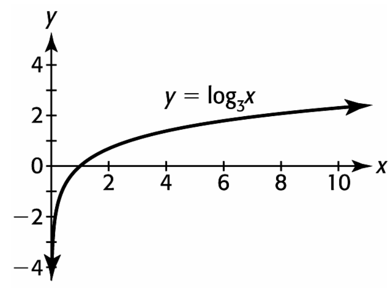
20. $\log_3(54) = \frac{\ln(54)}{\ln(3)} = 3.6039$

21. $\log_8(56) = \frac{\ln(56)}{\ln(8)} = 1.9358$

22.



23.



24. $340 = e^x$

$x = \ln(340)$

$x \approx 5.8289$

25. $1500 = 300e^{8x}$

$\frac{1500}{300} = \frac{300e^{8x}}{300}$

$5 = e^{8x}$

$8x = \ln(5)$

$x = \frac{\ln(5)}{8}$

$x \approx 0.2012$

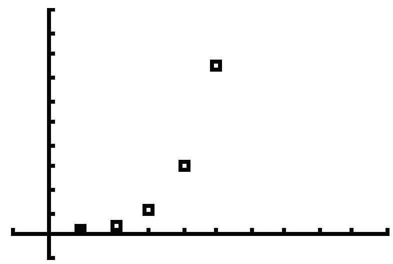
26.
$$9200 = 23\left(2^{3x}\right)$$
$$\frac{9200}{23} = \frac{23\left(2^{3x}\right)}{23}$$
$$2^{3x} = 400$$
$$\ln\left(2^{3x}\right) = \ln(400)$$
$$3x\ln(2) = \ln(400)$$
$$x = \frac{\ln(400)}{3\ln(2)}$$
$$x \approx 2.8813$$

27.
$$4\left(3^{x}\right) = 36$$
$$3^{x} = 9$$
$$\log\left(3^{x}\right) = \log(9)$$
$$x\log(3) = \log(9)$$
$$x = \frac{\log(9)}{\log(3)}$$
$x = 2$, or since
$$3^{x} = 9 = 3^{2}$$
$$x = 2$$

28. $\ln\left[\frac{(2x-5)^{3}}{x-3}\right]$
$$= \ln(2x-5)^{3} - \ln(x-3)$$
$$= 3\ln(2x-5) - \ln(x-3)$$

29. $6\log_4 x - 2\log_4 y$
$$= \log_4 x^{6} - \log_4 y^{2}$$
$$= \log_4\left(\frac{x^{6}}{y^{2}}\right)$$

30.



[–1, 10] by [–10, 100]

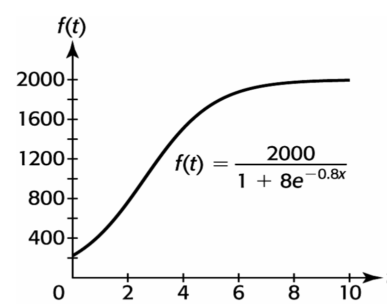
The data is best modeled by an exponential function.

$$y = 0.810(2.470)^{x}$$

31. $P\left(1+\frac{r}{k}\right)^{kn}$
$$= 1000\left(1+\frac{0.08}{12}\right)^{(12)(20)}$$
$$= 1000\left(1.00\overline{6}\right)^{240}$$
$$\approx 4926.80$$

32. $1000\left[\frac{1-1.03^{-240+120}}{0.03}\right]$
$$= 1000\left[\frac{1-1.03^{-120}}{0.03}\right]$$
$$= 1000\left[\frac{0.9711906782}{0.03}\right]$$
$$= 1000[32.37302261]$$
$$= 32,373.02$$

33. a.

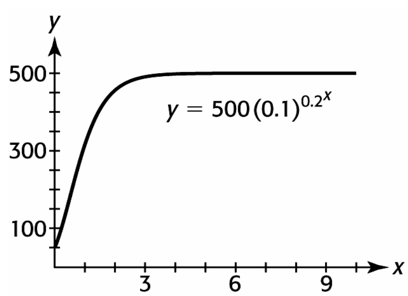


b. $$\begin{aligned} f(0) &= \frac{2000}{1+8e^{-0.8(0)}} \\ &= \frac{2000}{1+8e^{0}} \\ &= \frac{2000}{9} \\ &\approx 222.22 \end{aligned}$$

$$\begin{aligned} f(8) &= \frac{2000}{1+8e^{-0.8(8)}} \\ &= \frac{2000}{1+8e^{-6.4}} \\ &= \frac{2000}{1.013292458} \\ &\approx 1973.76 \end{aligned}$$

c. The limiting value of the function is 2000.

34. a.



b. $$\begin{aligned} y &= 500(0.1)^{0.2^{0}} \\ &= 500(0.1)^{1} \\ &= 500(0.1) \\ &= 50 \end{aligned}$$

c. The limiting value is 500.

Chapter 5 Review Exercises

35. Let $x = 17$ (months after Apr 1, 2010)

$$y = 0.554\left(1.455^x\right)$$
$$y = 0.554\left(1.455^{17}\right)$$
$$= 325.223 \approx 325$$

Seventeen months after April 1, 2010, the total number of iPads sold was 325 million.

36. Let $x = 4$.

$$y = 2000(2)^{-0.1(4)}$$
$$= 2000(2)^{-0.4}$$
$$= 2000(0.7578582833)$$
$$\approx 1515.72$$

Four weeks after the end of the advertising campaign, the daily sales in dollars will be $1515.72.

37.

$B(t) = 1.337e^{0.718t}$ where t is the number of years after 1985.

$$1.337e^{0.718t} > 10$$
$$e^{0.718t} > \frac{10}{1.337}$$
$$\ln e^{0.718t} > \ln\frac{10}{1.337} = 2.012$$
$$0.718t > 2.012$$
$$t > 2.802$$

Annual revenue exceeded $10 million during 1988 (1985 + 2.8).

38. a. $R = \log\left(\frac{I}{I_0}\right)$

$$R = \log\left(\frac{1000I_0}{I_0}\right)$$
$$R = \log(1000) = 3$$

The earthquake measures 3 on the Richter scale.

b. $10^R = \frac{I}{I_0}$

$$I = 10^R I_0$$
$$I = 10^{6.5} I_0$$
$$I = 3{,}162{,}277.66I_0$$

39. The difference in the Richter scale measurements is $7.9 - 4.8 = 3.1$. Therefore the intensity of the Indian earthquake was $10^{3.1} \approx 1259$ times as intense as the American earthquake.

40. $t = \log_{1.12} 3 = \frac{\ln 3}{\ln 1.12} \approx 9.69$

The investment will triple in approximately 10 years.

41. a. $S = 1000(2)^{\left(\frac{x}{7}\right)}$

$$\frac{S}{1000} = (2)^{\left(\frac{x}{7}\right)} \Leftrightarrow \log_2\left(\frac{S}{1000}\right) = \frac{x}{7}$$
$$x = 7\log_2\left(\frac{S}{1000}\right)$$

b. $x = 7\log_2\left(\frac{19{,}504}{1000}\right)$

$$= 7\log_2(19.504)$$
$$= 7\left(\frac{\ln 19.504}{\ln 2}\right)$$
$$\approx 29.99989$$

In about 30 years, the future value will be $19,504.

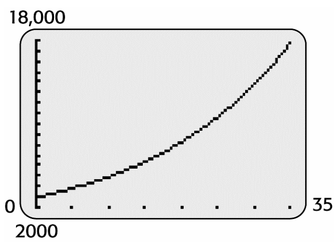
42.
$$1000 = 2000(2)^{-0.1x}$$
$$0.5 = 2^{-0.1x}$$
$$\ln(0.5) = \ln\left(2^{-0.1x}\right)$$
$$\ln(0.5) = -0.1x\ln 2$$
$$x = \frac{\ln 0.5}{-0.1\ln 2}$$
$$x = 10$$

In 10 weeks, sales will decay by half.

43. a. $P = 2969e^{0.051t}$

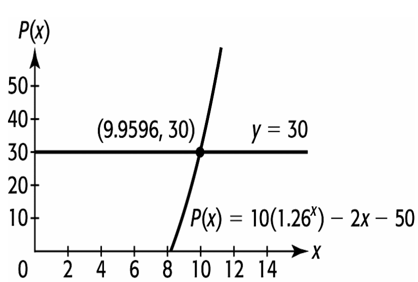
This model is exponential growth since the base (e) is > 1, and the exponent has a positive coefficient.

b.



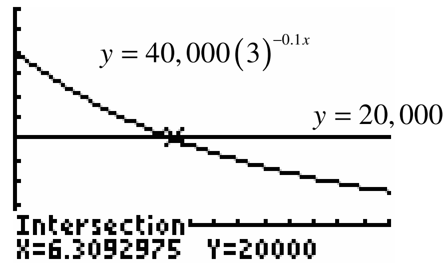
44. a.
$$P(x) = R(x) - C(x)$$
$$P(x) = 10\left(1.26^x\right) - (2x + 50)$$
$$= 10\left(1.26^x\right) - 2x - 50$$

b. Applying the intersection of graphs method *P(x)* is in thousands:



Selling at least 10 mobile homes produces a profit of at least $30,000.

45. Applying the intersection of graphs method:



[0, 15] by [–7500, 50,000]

After seven weeks, sales will be less than half.

46. a.
$$y = 100e^{-0.00012097(5000)}$$
$$= 100e^{-0.60485}$$
$$= 100(0.5461563439)$$
$$\approx 54.62$$

After 5000 years, approximately 54.62 grams of carbon-14 remains.

b.
$$36\% y_0 = y_0e^{-0.00012097t}$$
$$0.36 = e^{-0.00012097t}$$
$$\ln(0.36) = \ln\left(e^{-0.00012097t}\right)$$
$$\ln(0.36) = -0.00012097t$$
$$-0.00012097t = \ln(0.36)$$
$$t = \frac{\ln(0.36)}{-0.00012097}$$
$$t \approx 8445.49$$

The wood was cut approximately 8445 years ago.

47. $P = 60000e^{-0.05t}$

X	Y_1
11	34617
12	32929
13	31323
14	29795
15	28342
16	26960
17	25645

X=14

After approximately 14 years, the purchasing power will be less than half of the original $60,000 income.

48. $S = 2000e^{0.08(10)} = 2000e^{0.8} \approx 4451.08$

The future value is approximately $4451.08 after 10 years.

49.

$$13{,}784.92 = 3300(1.10)^x$$
$$(1.10)^x = 4.177248485$$
$$\ln\left[(1.10)^x\right] = \ln[4.177248485]$$
$$x\ln(1.10) = \ln(4.177248485)$$
$$x = \frac{\ln(4.177248485)}{\ln(1.10)}$$
$$x \approx 15$$

The investment reaches the indicated value in 15 years.

50. Using technology, $y = 108.319(1.315^x)$

51. **a.** Using technology, $y = 98.221(0.870^x)$

b. This model is exponential decay since the base is <1 and the exponent has a positive coefficient.

c. For the year 2010, $x = 2010 - 1980 = 30$

$y = 98.221(0.870^{30}) = 1.5$

Thus this model predicts there will be 1.5 students per computer in the year 2010.

52. **a.** Using technology, $y = 220.936(1.347^x)$

b. For the year 2007, $x = 2007 - 1980 = 27$

$y = 220.936(1.347^{27}) = 687{,}371.58$

Thus this model predicts there were 687,372 thousand subscribers in the year 2007. Using the unrounded model gives 686,377 thousand subscribers.

Using either result is a number that is twice the current population of the Unites States. Therefore, it is not a good model.

53. Using technology, $y = 165.893(1.055^x)$

For the year 2015, $x = 2015 - 1980 = 35$

$y = 165.893(1.055^{35}) = 1080.60$

Thus this model predicts the receipts will be $1081 billion in the year 2015. Using the unrounded model gives $1065 billion.

54. **a.** Using technology, $y = 0.940 + 28.672\ln x$

b. For the year 2015, $x = 2015 - 1995 = 20$

$y = 0.940 + 28.672\ln 20 = 86.83$

Thus this model predicts the percent of users will be 86.8% in the year 2015.

c. Since the model continues to increase, it will become invalid in the year 2027 when the predicted percent of users becomes > 100.

55. Using technology, $y = 4.337 + 40.890 \ln x$

56. a. $y = \dfrac{129.619}{1 + 0.106e^{-0.0786x}}$ for x equal to the number of years from 1980.

b. The model is an excellent fit.

57. $S = Pe^{rt}$

$S = 12{,}500e^{(0.05)(10)}$

$= 12{,}500e^{0.5} \approx 20{,}609.02$

The future value is $20,609.02.

58. $S = P\left(1 + \dfrac{r}{k}\right)^{kt}$

$S = 20{,}000\left(1 + \dfrac{0.06}{1}\right)^{(1)(7)}$

$S = 20{,}000(1.06)^7 \approx 30{,}072.61$

The future value is $30,072.61.

59.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$S = 1000\left[\frac{\left(1 + \frac{0.12}{4}\right)^{(4)(6)} - 1}{\frac{0.12}{4}}\right]$$

$$S = 1000\left[\frac{(1.03)^{24} - 1}{0.03}\right]$$

$$S = 1000(34.42647022) \approx 34{,}426.47$$

The future value is $34,426.47.

60. $S = R\left[\dfrac{(1+i)^n - 1}{i}\right]$

$$S = 1500\left[\frac{\left(1 + \frac{0.08}{12}\right)^{(12)(10)} - 1}{\frac{0.08}{12}}\right]$$

$$S = 1500\left[\frac{(1.00\overline{6})^{120} - 1}{0.00\overline{6}}\right]$$

$$S = 1500(182.9460352)$$

$$S \approx 274{,}419.05$$

The future value is $274,419.05.

61. $A = R\left[\dfrac{1 - (1+i)^{-n}}{i}\right]$

$$A = 2000\left[\frac{1 - \left(1 + \frac{0.08}{12}\right)^{-(12)(15)}}{\frac{0.08}{12}}\right]$$

$$A = 2000\left[\frac{1 - (1.00\overline{6})^{-180}}{0.00\overline{6}}\right]$$

$$A = 2000[104.6405922] \approx 209{,}281.18$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. The present value is $209,218.18.

62. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$A = 500\left[\dfrac{1-\left(1+\dfrac{0.10}{2}\right)^{-(2)(12)}}{\dfrac{0.10}{2}}\right]$$

$$A = 500\left[\dfrac{1-(1.05)^{-24}}{0.05}\right]$$

$$A = 500[13.79864179]$$

$$A \approx 6899.32$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. The present value is $6899.32.

63. $R = A\left[\dfrac{i}{1-(1+i)^{-n}}\right]$

$$R = 2000\left[\dfrac{\dfrac{0.12}{12}}{1-\left(1+\dfrac{0.12}{12}\right)^{-36}}\right]$$

$$R = 2000\left[\dfrac{0.01}{1-(1.01)^{-36}}\right]$$

$$R = 2000[0.0332143098]$$

$$R \approx 66.43$$

The monthly payment is $66.43.

64. $R = A\left[\dfrac{i}{1-(1+i)^{-n}}\right]$

$$R = 120{,}000\left[\dfrac{\dfrac{0.06}{12}}{1-\left(1+\dfrac{0.06}{12}\right)^{-(12)(25)}}\right]$$

$$R = 120{,}000\left[\dfrac{0.005}{1-(1.005)^{-300}}\right]$$

$$R = 120{,}000[0.006443014]$$

$$R \approx 773.16$$

The monthly payment is $773.16.

65. a. In 1990, $x = 1990 - 1960 = 30$.

$$\begin{aligned} y &= \frac{44.472}{1+6.870e^{-0.0782(30)}} \\ &= \frac{44.472}{1+6.870e^{-2.346}} \\ &= \frac{44.472}{1+0.6578121375} \\ &\approx 26.989 \end{aligned}$$

Based on the model, the percentage of live births to unmarried mothers in 1990 was 26.989%.

In 1996, $x = 1996 - 1960 = 36$.

$$\begin{aligned} y &= \frac{44.742}{1+6.870e^{-0.0782(36)}} \\ &= \frac{44.742}{1+6.870e^{-2.8152}} \\ &= \frac{44.742}{1+0.4114631169} \\ &\approx 31.699 \end{aligned}$$

Based on the model, the percentage of live births to unmarried mothers in 1996 was 31.699%.

b. The upper limit on the percentage of live births to unmarried mothers is 44.742%.

66. a. Let $x = 14.$

$$y = \frac{1400}{1 + 200e^{-0.5(14)}}$$
$$= \frac{1400}{1 + 200e^{-7}}$$
$$= \frac{1400}{1 + 0.1823763931}$$
$$\approx 1184.06$$

After 14 days, approximately 1184 students are infected.

b.

X	Y_1	
13	1076.4	
14	1184.1	
15	1260.6	
16	1312	
17	1345.3	
18	1366.3	
19	1379.4	

X=16

After 16 days, 1312 students are infected.

67. a. $N = 4000(0.06)^{0.4^{(2-1)}}$

$$= 4000(0.06)^{0.4^1}$$
$$= 4000(0.06)^{0.4}$$
$$\approx 1298.13$$

At the beginning of the second year, the enrollment will be approximately 1298 students.

b. $N = 4000(0.06)^{0.4^{(10-1)}}$

$$= 4000(0.06)^{0.4^9}$$
$$= 4000(0.06)^{0.000262144}$$
$$\approx 3997.05$$

At the beginning of the tenth year, the enrollment will be approximately 3997 students.

c. The upper limit on the number of students is 4000.

68. a. $N = 18{,}000(0.03)^{0.4^{10}}$

$$= 18{,}000(0.03)^{0.0001048576}$$
$$\approx 17{,}993.38$$

After ten months, the number of units sold in a month will be approximately 17,993.

b. The upper limit on the number of units sold per month is 18,000.

69. a. Using technology,

$$y = \frac{627.044}{1 + 268.609e^{-0.324x}}$$

for x as the number of years past 1975.

b. For the year 2012, $x = 2012 - 1975 = 37$

$$y = \frac{627.044}{1 + 268.609e^{-0.324(37)}} = 625.998$$

Thus this model predicts the number of species of endangered plants will be 626 in the year 2012.

c. To predict when 627 plant species are endangered, let $y = 627$ and solve for x:

$$627 = \frac{627.044}{1 + 268.609e^{-0.324(x)}}$$

$x = 47.08$

Thus, 1975 + 47 = 2022, the year when the number of species of endangered plants will be 627.

Group Activity/Extended Applications

1. The first person on the list receives \$36. Each of the original six people on the list sends their letter to six people. Therefore, 36 people receive letters with the original six names, and each of the 36 forwards a dollar to the first person on the original list.

2. The 36 people receiving the first letter place their name on the bottom of the list, shift up the second person to first place. The 36 people send out six letters each, for a total of $36 \times 6 = 216$ letters. Therefore the second person on the original list receives \$216.

3.

Cycle Number	Money Sent to the Person on Top of the List
1	$6^2 = 36$
2	$6^3 = 216$
3	$6^4 = 1296$
4	$6^5 = 7776$
5	$6^6 = 46,656$

4. Position 5 generates the most money!

5.

```
QuadReg
 y=ax²+bx+c
 a=5914.285714
 b=-25405.71429
 c=22356
```

```
PwrReg
 y=a*x^b
 a=20.33965715
 b=4.338874682
```

```
ExpReg
 y=a*b^x
 a=6
 b=6
```

The exponential model, $y = 6(6)^x = 6^{x+1}$, fits the data exactly.

6. $y = 6^{6+1} = 6^7 = 279,936$
The sixth person on the original list receives \$279,936.

7. The total number of responses on the sixth cycle would be
$6 + 36 + 216 + 1296 + 7776 + 46,656 + 279,936 = 335,922$

8. $y = 6^{10+1} = 6^{11} = 362,797,056$
On the tenth cycle 362,797,056 people receive the chain letter and are supposed to respond with \$1.00 to the first name on the list.

9. The answer to problem 8 is larger than the U.S. population. There is no unsolicited person in the U.S. to whom to send the letter.

10. Chain letters are illegal since people entering lower on the chain have a very small chance of earning money from the scheme.