Chapter 6
Higher-Degree Polynomial and Rational Functions

Toolbox Exercises

1. **a.** The polynomial is 4^{th} degree.

b. The leading coefficient is 3.

2. **a.** The polynomial is 3^{rd} degree.

b. The leading coefficient is 5.

3. **a.** The polynomial is 5^{th} degree.

b. The leading coefficient is –14.

4. **a.** The polynomial is 6^{th} degree.

b. The leading coefficient is –8.

5. $4x^3 - 8x^2 - 140x$
$= 4x(x^2 - 2x - 35)$
$= 4x(x-7)(x+5)$

6. $4x^2 + 7x^3 - 2x^4$
$= -2x^4 + 7x^3 + 4x^2$
$= -1x^2(2x^2 - 7x - 4)$
$= -x^2(2x+1)(x-4)$

7. $x^4 - 13x^2 + 36$
$= (x^2 - 9)(x^2 - 4)$
$= (x+3)(x-3)(x+2)(x-2)$

8. $x^4 - 21x^2 + 80$
$= (x^2 - 16)(x^2 - 5)$
$= (x+4)(x-4)(x^2 - 5)$

9. $2x^4 - 8x^2 + 8$
$= 2(x^4 - 4x^2 + 4)$
$= 2(x^2 - 2)(x^2 - 2)$
$= 2(x^2 - 2)^2$

10. $3x^5 - 24x^3 + 48x$
$= 3x(x^4 - 8x^2 + 16)$
$= 3x(x^2 - 4)(x^2 - 4)$
$= 3x(x+2)(x-2)(x+2)(x-2)$
$= 3x(x+2)^2(x-2)^2$

11. $\dfrac{x-3y}{3x-9y} = \dfrac{x-3y}{3(x-3y)} = \dfrac{1}{3}$

12. $\dfrac{x^2-9}{4x+12} = \dfrac{(x+3)(x-3)}{4(x+3)} = \dfrac{x-3}{4}$

13. $\dfrac{2y^3 - 2y}{y^2 - y}$
$= \dfrac{2y(y^2-1)}{y(y-1)}$
$= \dfrac{2y(y+1)(y-1)}{y(y-1)}$
$= 2(y+1) = 2y + 2$

14. $\dfrac{4x^3-3x}{x^2-x}$

$=\dfrac{x\left(4x^2-3\right)}{x(x-1)}$

$=\dfrac{4x^2-3}{x-1}$

15. $\dfrac{x^2-6x+8}{x^2-16}$

$=\dfrac{(x-4)(x-2)}{(x+4)(x-4)}$

$=\dfrac{x-2}{x+4}$

16. $\dfrac{3x^2-7x-6}{x^2-4x+3}$

$=\dfrac{(3x+2)(x-3)}{(x-3)(x-1)}$

$=\dfrac{3x+2}{x-1}$

17. $\dfrac{x-3}{x^3}\bullet\dfrac{x(x-4)}{(x-4)(x-3)}$

$=\dfrac{x}{x^3}$

$=\dfrac{1}{x^2}$

18. $(x+2)(x-2)\left(\dfrac{2x-3}{x+2}\right)$

$=(x-2)(2x-3)$

$=2x^2-7x+6$

19. $\dfrac{4x+4}{x-4}\div\dfrac{8x^2+8x}{x^2-6x+8}$

$=\dfrac{4x+4}{x-4}\bullet\dfrac{x^2-6x+8}{8x^2+8x}$

$=\dfrac{4(x+1)}{x-4}\bullet\dfrac{(x-2)(x-4)}{8x(x+1)}$

$=\dfrac{x-2}{2x}$

20. $\dfrac{6x^2}{4x^2y-12xy}\div\dfrac{3x^2+12x}{x^2+x-12}$

$=\dfrac{6x^2}{4x^2y-12xy}\bullet\dfrac{x^2+x-12}{3x^2+12x}$

$=\dfrac{6x^2}{4xy(x-3)}\bullet\dfrac{(x+4)(x-3)}{3x(x+4)}$

$=\dfrac{1}{2y}$

21. $3+\dfrac{1}{x^2}-\dfrac{2}{x^3}$ LCD: x^3

$=\dfrac{3x^3}{x^3}+\dfrac{x}{x^3}-\dfrac{2}{x^3}$

$=\dfrac{3x^3+x-2}{x^3}$

22.

$\dfrac{5}{x}-\dfrac{x-2}{x^2}+\dfrac{4}{x^3}$ LCD: x^3

$=\dfrac{5x^2}{x^3}-\dfrac{x(x-2)}{x^3}+\dfrac{4}{x^3}$

$=\dfrac{5x^2-\left(x^2-2x\right)+4}{x^3}$

$=\dfrac{5x^2-x^2+2x+4}{x^3}$

$=\dfrac{4x^2+2x+4}{x^3}$

23.

$$\frac{a}{a^2-2a}-\frac{a-2}{a^2}=\frac{a}{a(a-2)}-\frac{a-2}{a^2} \qquad \text{LCD: } a^2(a-2)$$

$$=\frac{a(a)}{a(a)(a-2)}-\frac{(a-2)(a-2)}{a^2(a-2)}$$

$$=\frac{a^2}{a^2(a-2)}-\frac{a^2-4a+4}{a^2(a-2)}$$

$$=\frac{a^2-(a^2-4a+4)}{a^2(a-2)}$$

$$=\frac{a^2-a^2+4a-4}{a^2(a-2)}$$

$$=\frac{4a-4}{a^2(a-2)}$$

$$=\frac{4(a-1)}{a^2(a-2)}$$

$$=\frac{4a-4}{a^3-2a^2}$$

24.

$$\frac{5x}{x^4-16}+\frac{8x}{x+2}=\frac{5x}{(x^2+4)(x^2-4)}+\frac{8x}{x+2}$$

$$=\frac{5x}{(x^2+4)(x+2)(x-2)}+\frac{8x}{x+2} \qquad \text{LCD: } (x^2+4)(x+2)(x-2)$$

$$=\frac{5x}{(x^2+4)(x+2)(x-2)}+\frac{8x(x^2+4)(x-2)}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{5x+8x(x^3-2x^2+4x-8)}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{5x+8x^4-16x^3+32x^2-64x}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{8x^4-16x^3+32x^2-59x}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{8x^4-16x^3+32x^2-59x}{x^4-16}$$

25.

$$\frac{x-1}{x+1}-\frac{2}{x(x+1)}$$

$$\{\text{LCD: } x(x+1)\}$$

$$=\frac{x(x-1)}{x(x+1)}-\frac{2}{x(x+1)}$$

$$=\frac{x^2-x-2}{x(x+1)}$$

$$=\frac{(x-2)(x+1)}{x(x+1)}$$

$$=\frac{x-2}{x}$$

26.

$$\frac{2x+1}{2(2x-1)}+\frac{5}{2x}-\frac{x+1}{x(2x-1)}$$

$$\{\text{LCD: } 2x(2x-1)\}$$

$$=\frac{x(2x+1)}{2x(2x-1)}+\frac{5(2x-1)}{2x(2x-1)}-\frac{2(x+1)}{2x(2x-1)}$$

$$=\frac{2x^2+x+(10x-5)-(2x+2)}{2x(2x-1)}$$

$$=\frac{2x^2+x+10x-5-2x-2}{2x(2x-1)}$$

$$=\frac{2x^2+9x-7}{2x(2x-1)}=\frac{2x^2+9x-7}{4x^2-2x}$$

27.

$$\begin{array}{r}
x^4-x^3+2x^2-2x+2 \\
x+1\overline{)x^5+0x^4+x^3+0x^2+0x-1} \\
\underline{x^5+x^4} \qquad\qquad\qquad\qquad \\
-x^4+x^3 \qquad\qquad\qquad \\
\underline{-x^4-x^3} \qquad\qquad\qquad \\
2x^3+0x^2 \qquad\qquad \\
\underline{2x^3+2x^2} \qquad\qquad \\
-2x^2+0x \qquad \\
\underline{-2x^2-2x} \qquad \\
2x-1 \\
\underline{2x+2} \\
-3
\end{array}$$

Thus, the quotient is:

$x^4-x^3+2x^2-2x+2$ with remainder -3.

or

$$x^4-x^3+2x^2-2x+2-\frac{3}{x+1}$$

28.

$$\begin{array}{r}
a^3+a^2 \\
a+2\overline{)a^4+3a^3+2a^2} \\
\underline{a^4+2a^3} \qquad\quad \\
a^3+2a^2 \\
\underline{a^3+2a^2} \\
0
\end{array}$$

Thus, the quotient is:

a^3+a^2 with remainder 0.

29.

$$\begin{array}{r}
3x^3 - x^2 + 6x - 2 \\
x^2 - 2 \overline{)\, 3x^5 - x^4 + 0x^3 + 0x^2 + 5x - 1} \\
\underline{3x^5 \qquad - 6x^3} \qquad\qquad\qquad \\
- x^4 + 6x^3 + 0x^2 \qquad\quad \\
\underline{- x^4 \qquad + 2x^2} \qquad\quad \\
6x^3 - 2x^2 + 5x \\
\underline{6x^3 \qquad - 12x} \\
- 2x^2 + 17x - 1 \\
\underline{-2x^2 \qquad + 4} \\
17x - 5
\end{array}$$

Thus, the quotient is:

$\left(3x^3 - x^2 + 6x - 2\right)$ with rem $(17x - 5)$

or

$3x^3 - x^2 + 6x - 2 + \dfrac{17x - 5}{x^2 - 2}$

30.

$$\begin{array}{r}
x^2 + 1 \\
3x^2 - 1 \overline{)\, 3x^4 + 0x^3 + 2x^2 + 0x + 1} \\
\underline{3x^4 \qquad - x^2} \qquad\qquad \\
3x^2 + 0x + 1 \\
\underline{3x^2 \qquad - 1} \\
2
\end{array}$$

Thus, the quotient is:

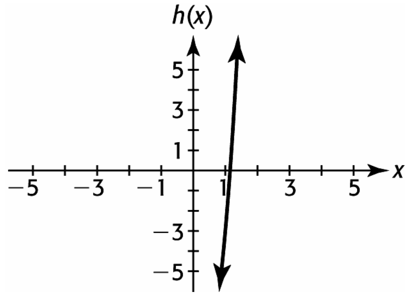
$x^2 + 1$ with remainder 2

or

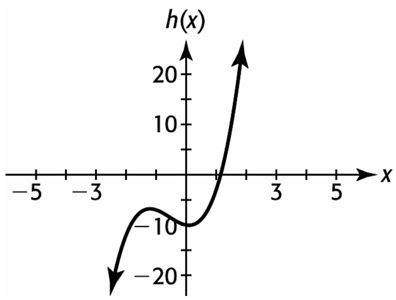
$x^2 + 1 + \dfrac{2}{3x^2 - 1}$

Section 6.1 Skills Check

1. **a.** $h(x)=3x^3+5x^2-x-10$

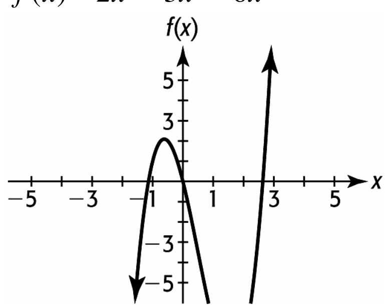


b.



Window b) gives a complete graph.

2. **a.** $f(x)=2x^3-3x^2-6x$



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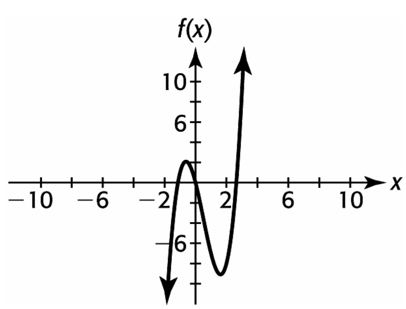
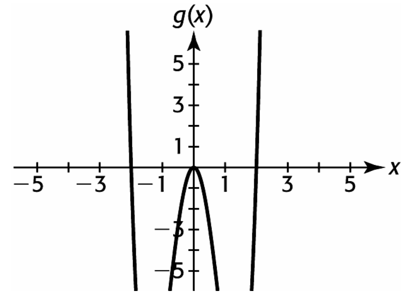
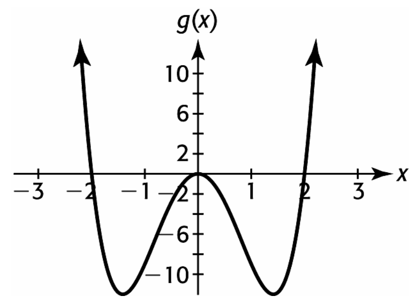


Window b) gives a complete graph.

3. **a.** $g(x)=3x^4-12x^2$

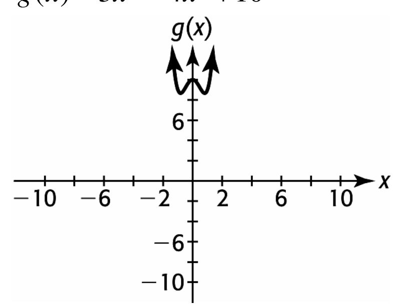


b.

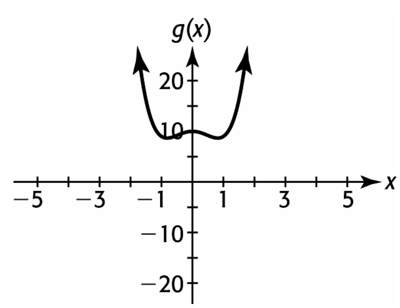


Window b) gives a complete graph.

4. **a.** $g(x)=3x^4-4x^2+10$



b.



Window b) gives a complete graph.

5. a. The x-intercepts appear to be -2, 1, and 2.

b. The leading coefficient is positive since the graph rises to the right.

c. The function is cubic since the end behavior is "one end up and one end down".

6. a. The x-intercepts appear to be -1, 2, and 3.

b. The leading coefficient is negative since the graph falls to the right.

c. The function is quartic since the end behavior is "both ends opening down".

7. a. The x-intercepts appear to be -1, 1, and 5.

b. The leading coefficient is negative since the graph falls to the right.

c. The function is cubic since the end behavior is "one end up and one end down".

8. a. The x-intercepts appear to be -1, 2, and 5.

b. The leading coefficient is positive since the graph rises to the right.

c. The function is quartic since the end behavior is "both ends opening up".

9. a. The x-intercepts appear to be -1.5 and 1.5.

b. The leading coefficient is positive since the graph rises to the right.

c. The function is quartic since the end behavior is "both ends opening up".

10. a. The x-intercepts appear to be -2, and 3.

b. The leading coefficient is negative since the graph falls to the right.

c. The function is quartic since the end behavior is "both ends opening down".

11. matches with graph C since it is cubic with a positive leading coefficient.

12. matches with graph A since it is cubic with a negative leading coefficient and y-intercept (0, 2).

13. matches with graph E since it is cubic with a negative leading coefficient and y-intercept $(0, -6)$.

14. matches with graph B since it is quartic with a positive leading coefficient and y-intercept (0, 12).

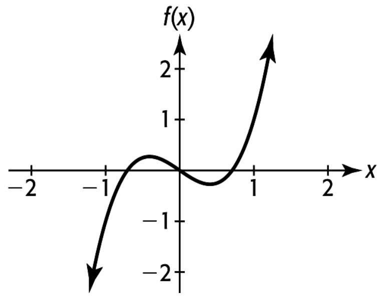
15. matches with graph F since it is quartic with a positive leading coefficient and y-intercept (0, 3).

16. matches with graph D since it is quartic with a negative leading coefficient.

17. a. The polynomial is 3^{rd} degree, and the leading coefficient is 2.

b. The graph rises right and falls left because the leading coefficient is positive and the function is cubic.

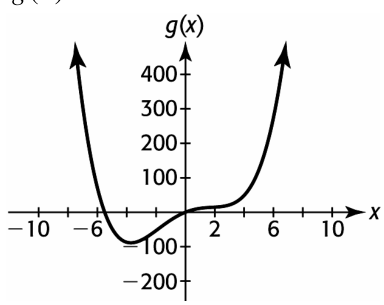
c. $f(x) = 2x^3 - x$



18. a. The polynomial is 4th degree, and the leading coefficient is 0.3.

b. The graph rises right and rises left because the leading coefficient is positive and the function is quartic.

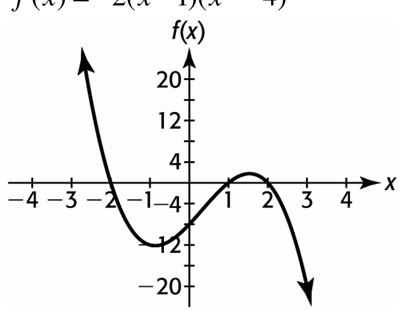
c. $g(x) = 0.3x^4 - 6x^2 + 17x$



19. a. The polynomial is 3rd degree, and the leading coefficient is –2.

b. The graph falls right and rises left because the leading coefficient is negative and the function is cubic.

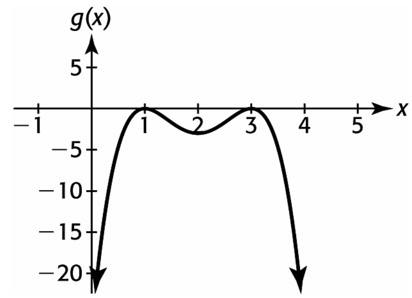
c. $f(x) = -2(x-1)(x^2-4)$



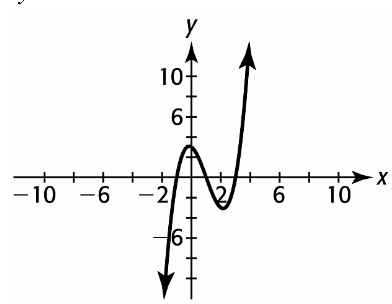
20. a. The polynomial is 4th degree, and the leading coefficient is –3.

b. The graph falls right and falls left because the leading coefficient is negative and the function is quartic.

c. $g(x) = -3(x-3)^2(x-1)^2$

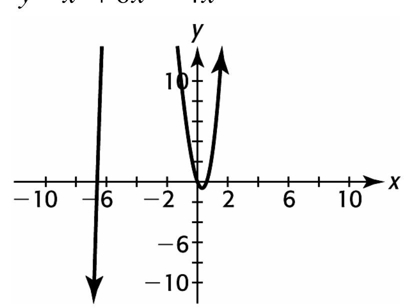


21. a. $y = x^3 - 3x^2 - x + 3$



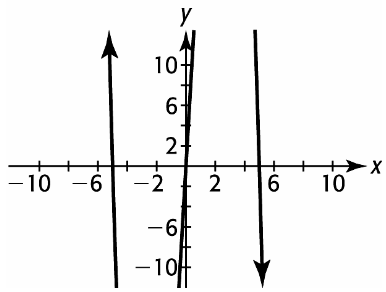
b. Yes, the graph is complete. As suggested by the degree of the cubic function, three x-intercepts show, along with the y-intercept.

22. a. $y = x^3 + 6x^2 - 4x$

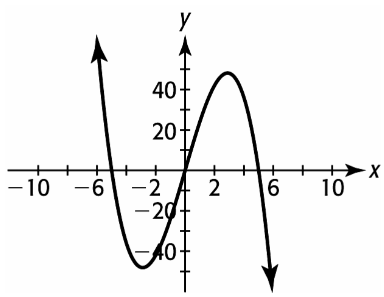


b. No. One turning point does not show.

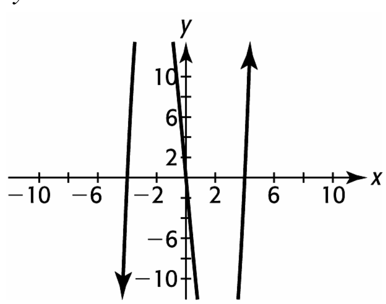
23. a. $y = 25x - x^3$



b.



24. a. $y = x^3 - 16x$



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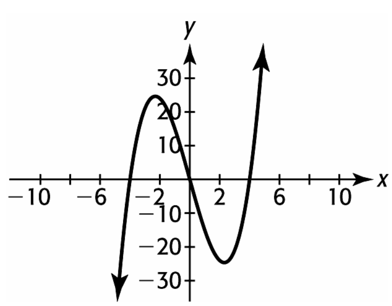
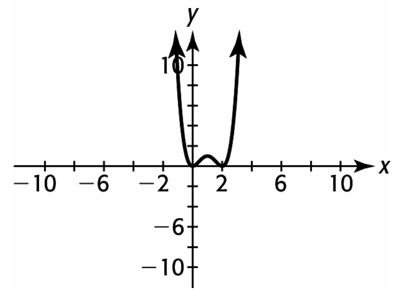
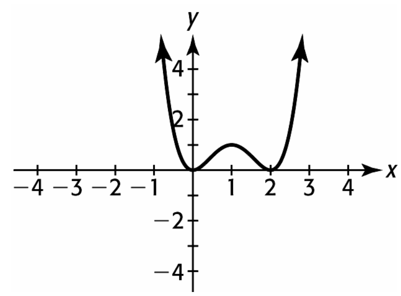


25. a. $y = x^4 - 4x^3 + 4x^2$

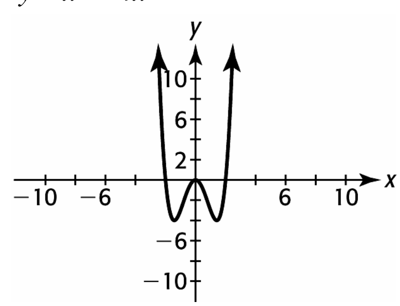


b.



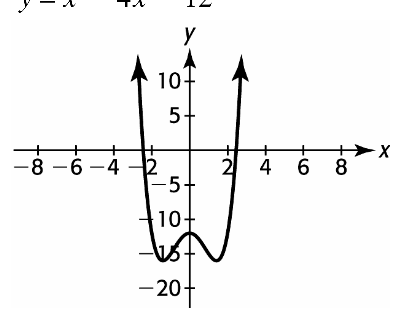
c. The window in part b) yields the best view of the turning points.

26. a. $y = x^4 - 4x^2$



b. Yes. The graph is complete.

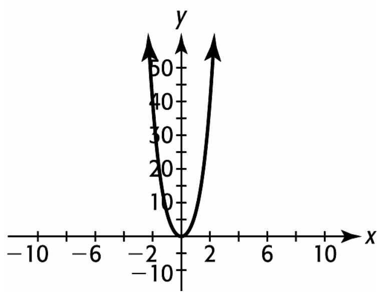
27. a. $y = x^4 - 4x^2 - 12$



b. The graph has three turning points.

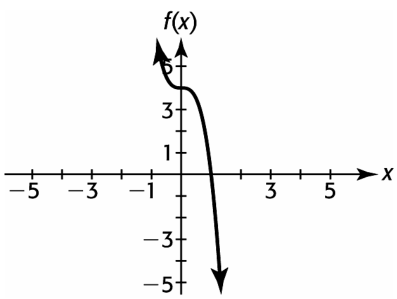
c. No, since the polynomial is degree 4, it has at most three turning points.

28. a. $y = x^4 + 6x^2$



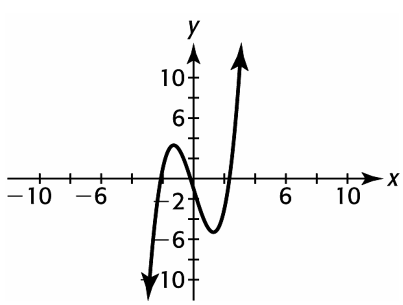
b. Yes, since the polynomial is degree 4, it has at most three turning points. It could have 3 or 1 turning points.

29.



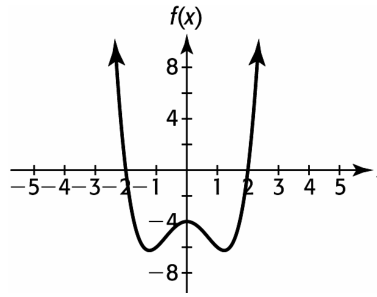
Answers will vary. One such graph is for the function, $f(x) = -4x^3 + 4$, as shown.

30.



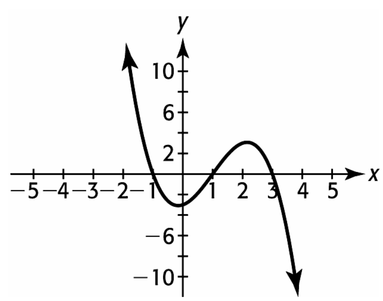
Answers will vary. One such graph is for the function, $f(x) = x^3 - 5x - 1$, as shown.

31.



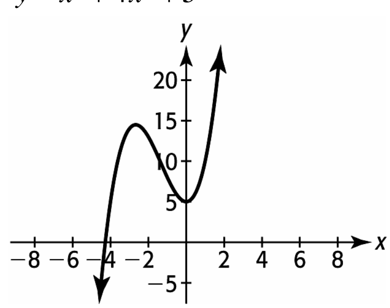
Answers will vary. One such graph is for the function, $f(x) = x^4 - 3x^2 - 4$, as shown.

32.

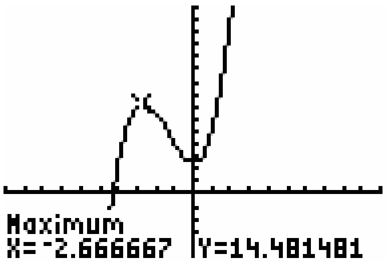


Answers will vary. One such graph is for the function, $f(x) = -x^3 + 3x^2 + x - 3$, as shown.

33. a. $y = x^3 + 4x^2 + 5$



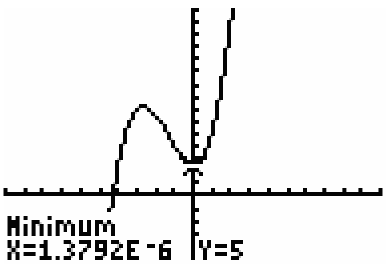
b.



[–10, 10] by [–10, 30]

The local maximum is approximately $(-2.67, 14.48)$.

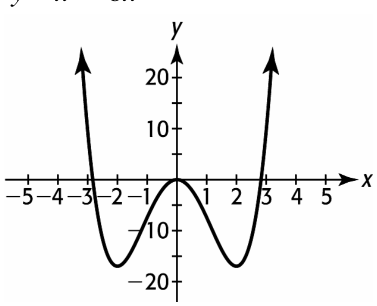
c.



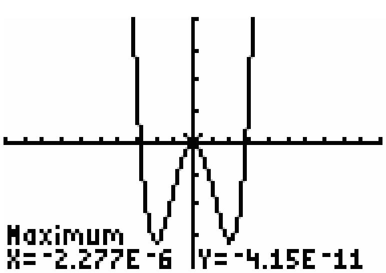
[–10, 10] by [–10, 30]

The local minimum is $(0,5)$.

34. a. $y = x^4 - 8x^2$



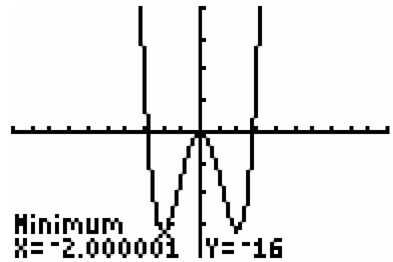
b.



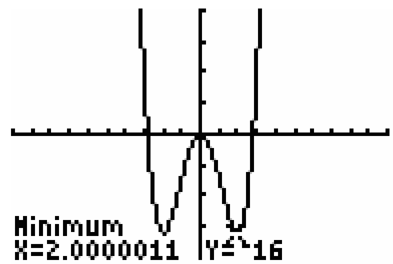
[–10, 10] by [–20, 20]

The local maximum is $(0,0)$.

c.



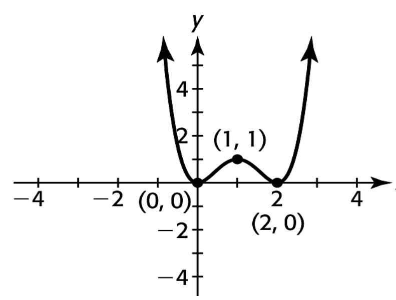
[–10, 10] by [–20, 20]



[–10, 10] by [–20, 20]

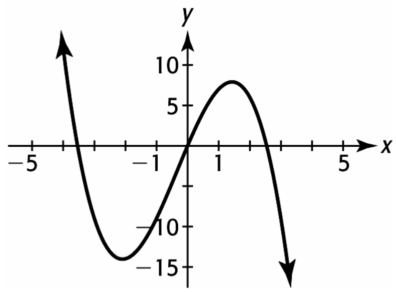
The local minima are $(-2,-16)$ and $(2,-16)$.

35. $y = x^4 - 4x^3 + 4x^2$

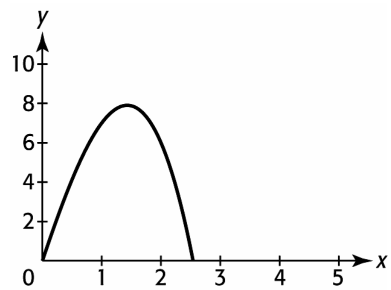


The local maximum is $(1,1)$. The local minima are $(0,0)$ and $(2,0)$.

36. a. $y = -x^3 - x^2 + 9x$



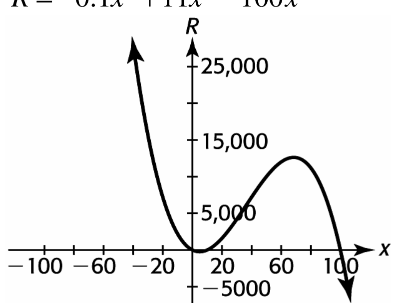
b.



c. The graph in part b) resembles a 2nd degree (quadratic) function.

Section 6.1 Exercises

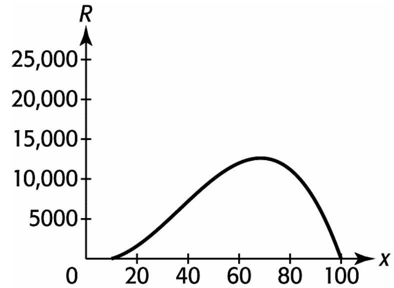
37. a. $R = -0.1x^3 + 11x^2 - 100x$



There are two turning points.

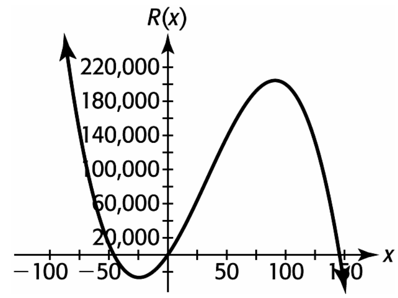
b. Based on the physical context of the problem, both *x* and *R* should be nonnegative.

c.



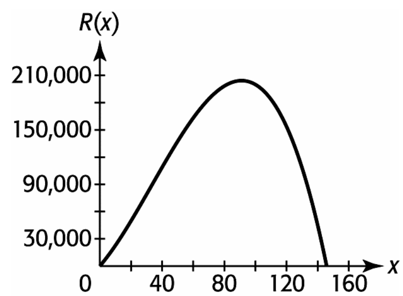
d. Fifty units yield revenue of $10,000.

38. a. $R(x) = 2000x + 30x^2 - 0.3x^3$



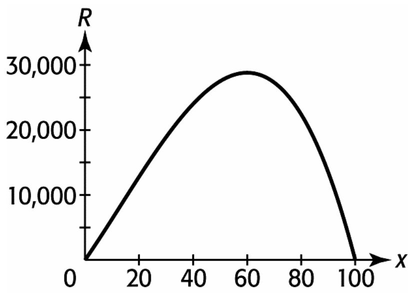
b. Based on the physical context of the problem, both *x* and *R(x)* should be nonnegative.

c.



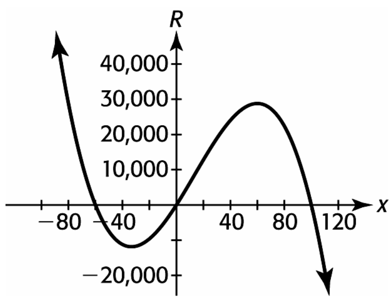
d. Sixty units yield revenue of $163,200.

39. a.



b. Selling 60 units yield a maximum daily revenue of $28,800.

c. $R = 600x - 0.1x^3 + 4x^2$

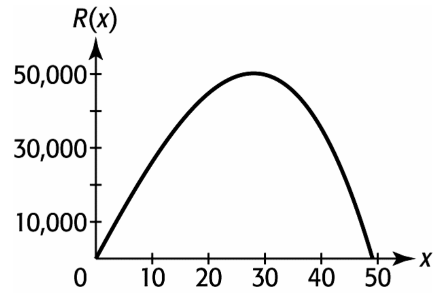


Answers will vary for the window.

d. The graph in part a) represents the physical situation better since both the number of units produced and the revenue must be nonnegative.

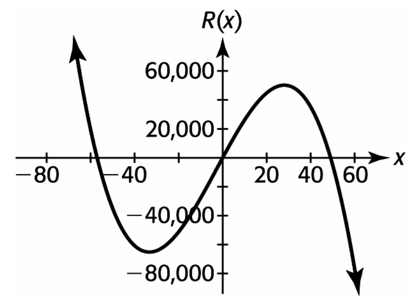
e. As shown in part a), the graph is increasing on the interval $(0, 60)$.

40. a.



b. Selling 28 units yield a maximum weekly revenue of $50,176.

c. $R(x) = 2800x - 8x^2 - x^3$



Answers will vary for the window.

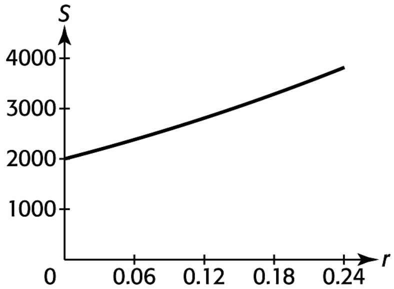
d. The graph in part a) represents the physical situation better since both the number of units produced and the revenue must be nonnegative.

e. As shown in part a), the graph is increasing on the interval $(0, 28)$.

41. a. $S = 2000(1 + r)^3$

Rate, r	Future Value, $S(\$)$
0.00	2,000.00
0.05	2,315.25
0.10	2,662.00
0.15	3,041.75
0.20	3,456.00

b.



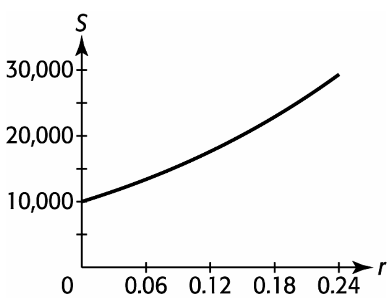
c. At the 20% rate, the investment yields $3456. At the 10% rate, the investment yields $2662. Therefore, the 20% rate yields $794 more.

d. The 10% rate is more realistic.

42. a. $S = 10000(1+r)^5$

Rate, *r*	Future Value, *S($)*
0.00	10,000.00
0.05	12,762.82
0.07	14,025.52
0.12	17,623.42
0.18	22,877.58

b.

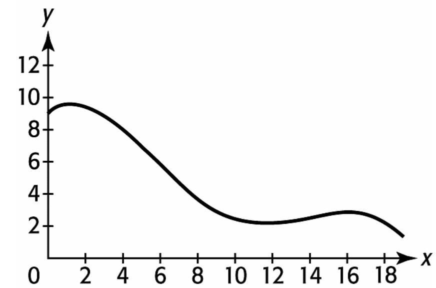


c. At the 24% rate, the investment yields $29,316.25. At the 10% rate, the investment yields $16,105.10. Therefore, the 24% rate yields $13,211.15 more.

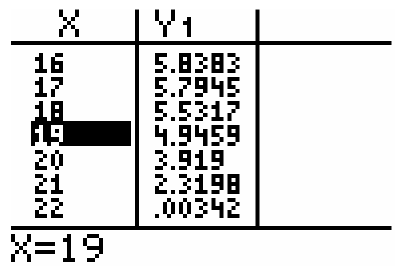
Rate, *r*	Future Value, *S($)*
0.10	16,105.10
0.24	29,316.25

d. The 10% rate is more realistic.

43. a. $y = -0.000564x^4 + 0.0221x^3 - 0.258x^2 + 0.558x + 9.399$

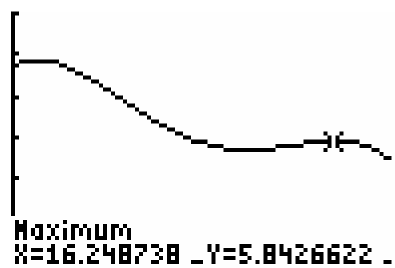


b.



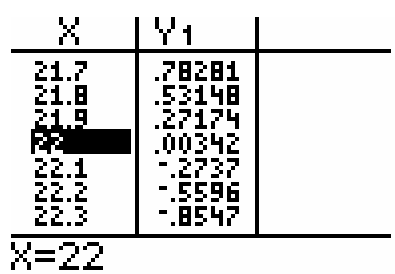
In 2009, when $x = 19$, the homicide rate was approximately 4.9 per 100,000 people.

c.



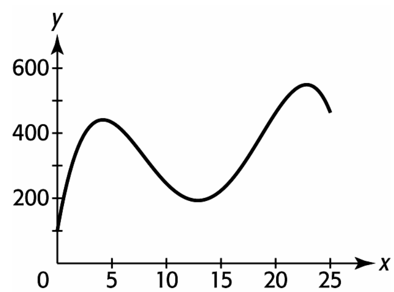
An x-value of 16.2 corresponds with the year 2007. After the year 2000, the number of homicides was at a maximum in 2007.

d.



The model should be valid for the years 1990 – 2009 as stated in the original problem. The graph seems to confirm this, but would definitely be invalid beginning with the year 2012 ($x = 22$) since the rate shown by the table at that point becomes negative.

44. a. $y = -0.0395x^4 + 2.101x^3 - 35.079x^2 + 194.109x + 100.148$



b.

X	Y1
22	510.65
23	517.01
24	492.33
25	426.94
26	310.2
27	130.56
28	-124.5

X=25

In 2005, when $x = 25$, there were 426.94 drunk driving crashes in South Carolina.

c.

X	Y1
22.4	516.34
22.5	517.14
22.6	517.68
22.7	517.95
22.8	517.93
22.9	517.62
23	517.01

X=22.7

An x-value of 22.7 corresponds with the year 2003. The number of fatalities from drunk driving crashes (518) was at a maximum in 2003.

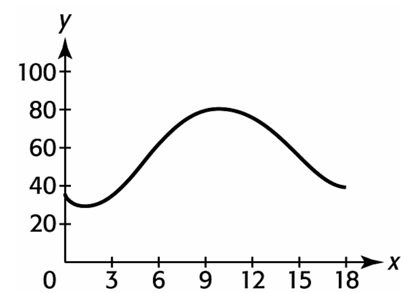
d.

X	Y1
10	239
11	209
12	190
13	183
14	190
15	210
16	243

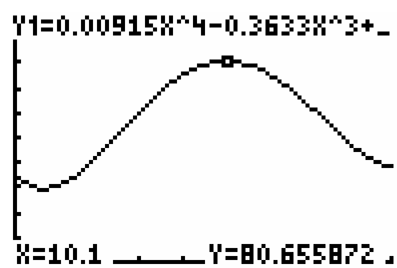
X=13

An x-value of 13 corresponds with the year 1993. The number of fatalities from drunk driving crashes (183) was at a minimum in 1993.

45. a. $y = 0.00915x^4 - 0.3633x^3 + 4.09979x^2 - 9.3062x + 35.5220$



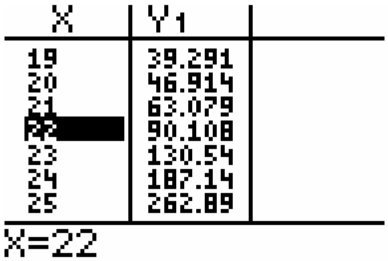
b.



[0, 18] by [0, 100]

The maximum number of executions occurred in the year 2001 when $x = 10.1$.

c.

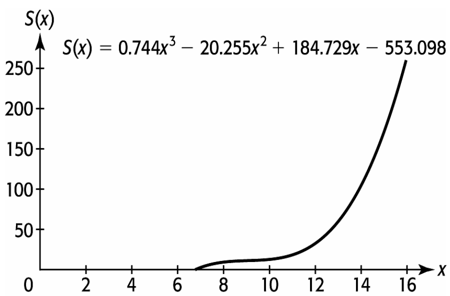


X	Y1
19	39.291
20	46.914
21	63.079
22	90.108
23	130.54
24	187.14
25	262.89

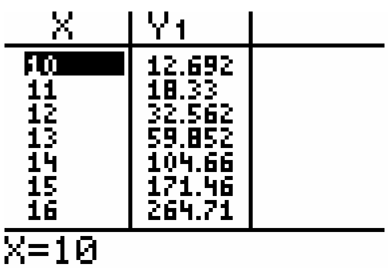
X=22

In 2012, when $x = 22$, the number of executions is estimated to be approximately 90.

46. a.



b.

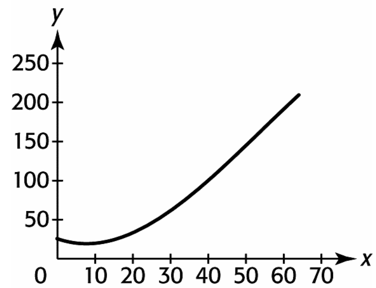


X	Y1
10	12.692
11	18.33
12	32.562
13	59.852
14	104.66
15	171.46
16	264.71

X=10

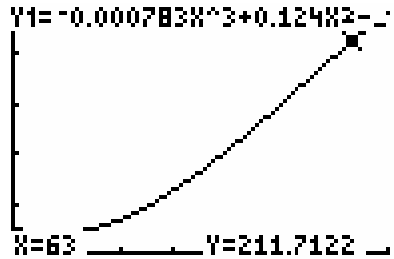
In the year 2000, when $x = 10$, the total hybrid electric passenger vehicle sales was 12.692 thousand (12,692). In 2006, when $x = 16$, the sales figure was 264.71 thousand (264,710).

47. a. $y = -0.000783x^3 + 0.124x^2 - 1.743x + 25.152$



b. No, there should be two turning points.

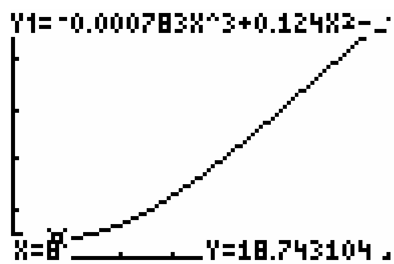
c.



[0, 70] by [0, 250]

For the year 2008, $x = 63$. The CPI in 2008 was 211.71.

d.

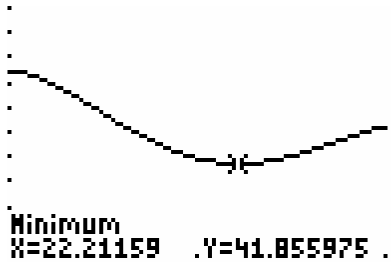


[0, 70] by [0, 250]

The minimum CPI of 18.7 occurred when $x = 8$, in 1953 (1945 + 8).

48. a.

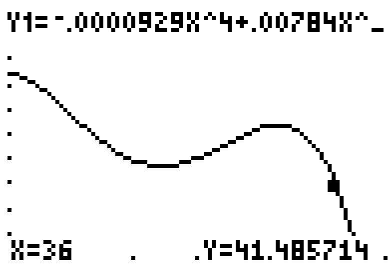
$$y = -0.0000929x^4 + 0.00784x^3 - 0.226x^2 + 2.508x + 34.347$$



[10, 30] by [40, 45]

The minimum median salary between 1980 and 2000 is $41,856 and occurred in the year 1993.

b.

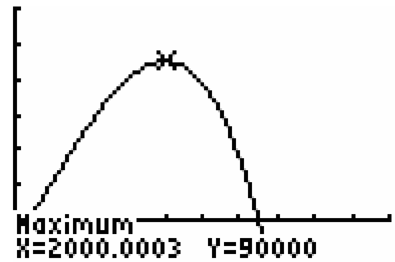


[10, 40] by [40, 45]

No, the model gives a lower median salary in 2006 ($41,486).

49. a.

$$P(x) = R(x) - C(x)$$
$$= \left(120x - 0.015x^2\right) -$$
$$\left(10{,}000 + 60x - 0.03x^2 + 0.00001x^3\right)$$
$$= -0.00001x^3 + 0.015x^2 +$$
$$60x - 10{,}000$$



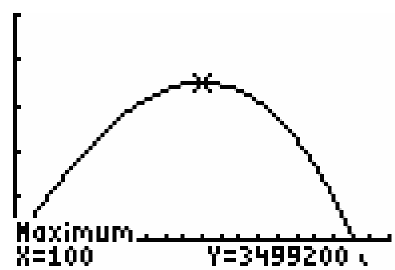
[0, 5000] by [–20,000, 120,000]

2000 units produced and sold yields a maximum profit.

b. The maximum profit is $90,000.

50. a.

$$P(x) = R(x) - C(x)$$
$$= \left(60{,}000x - 50x^2\right) -$$
$$\left(800 + 100x^2 + x^3\right)$$
$$= -x^3 - 150x^2 + 60{,}000x - 800$$



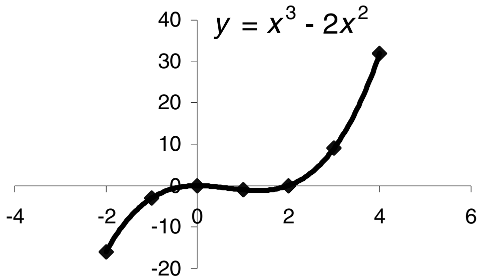
[0, 200] by [–500,000, 5,000,000]

The maximum of the function occurs when $x = 100$. Therefore, the maximum profit occurs when 100,000 units are produced and sold.

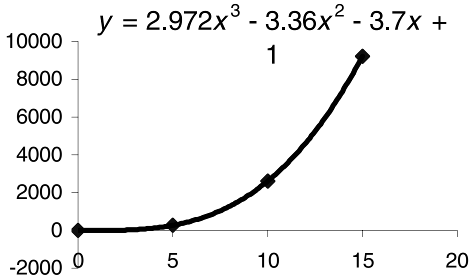
b. The maximum profit is $3,499,200.

Section 6.2 Skills Check

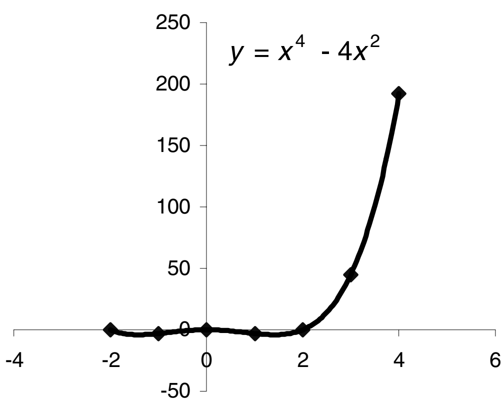
1.



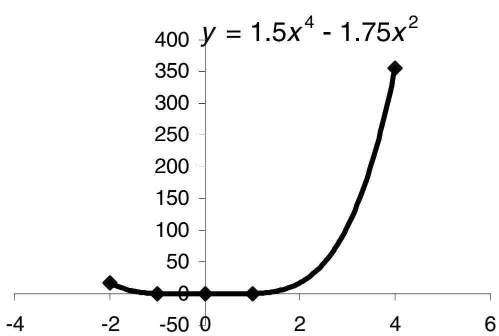
2.



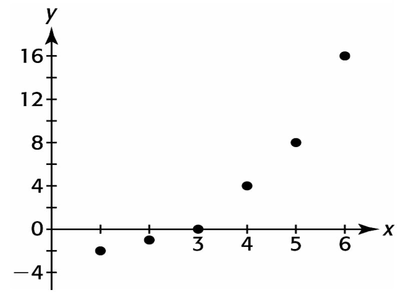
3.



4.

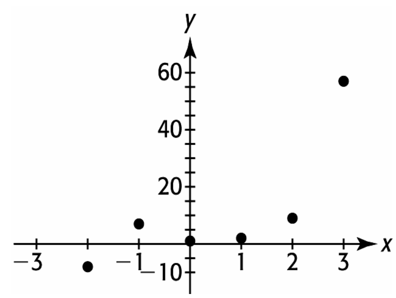


5. a.



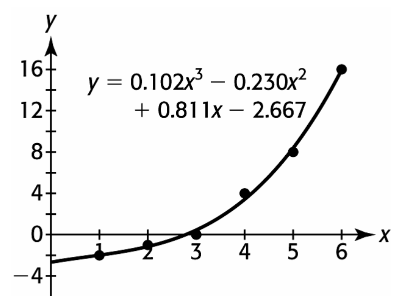
b. It appears that a cubic model will fit the data better.

6. a.

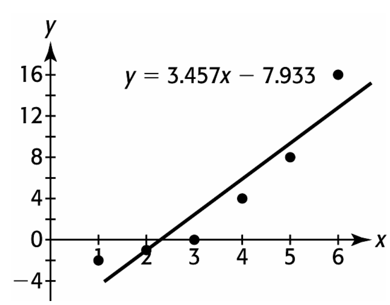


b. It appears that a cubic model will fit the data better.

7. a.

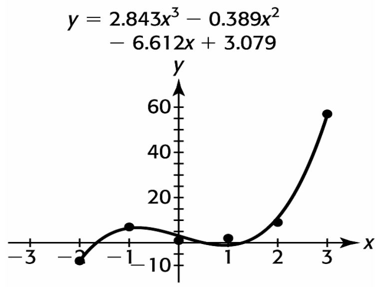


b.

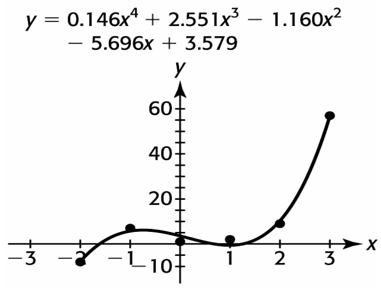


c. It appears that a cubic model will fit the data better.

8. a.

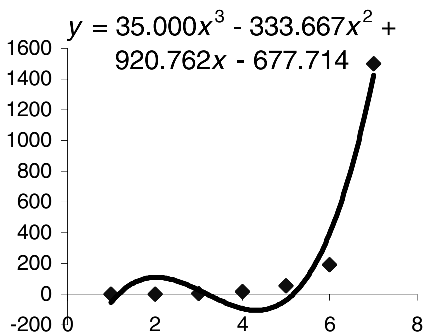


b.

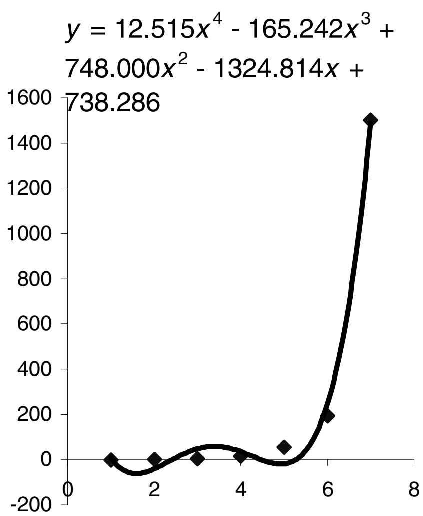


c. Both models fit the data equally well.

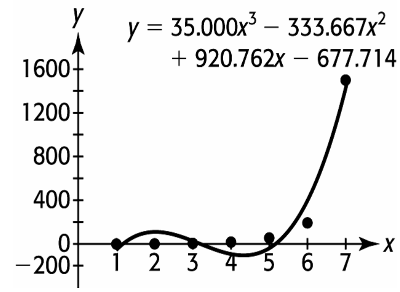
9. a.

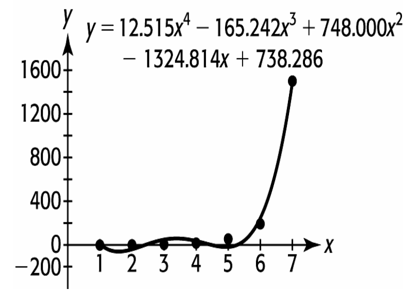


b.



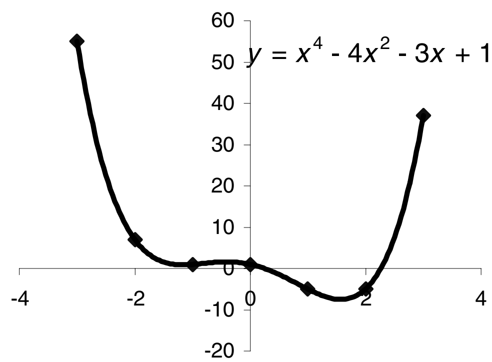
10. a.





b. The quartic model appears to be the better fit.

11.



12. Yes. The model found in Exercise 11 is a 4th degree polynomial, and fits the data and scatter plot exactly.

13.

x	$f(x)$	First Difference	Second Difference	Third Difference
0	0			
1	1	1		
2	5	4	3	
3	24	19	15	12
4	60	36	17	2
5	110	50	14	–3

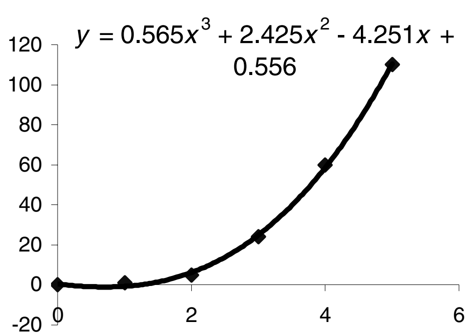
The function $f(x)$ is not exactly cubic.

14.

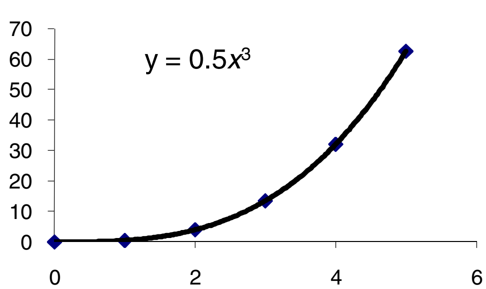
x	$g(x)$	First Difference	Second Difference	Third Difference
0	0			
1	0.5	0.5		
2	4	3.5	3	
3	13.5	9.5	6	3
4	32	18.5	9	3
5	62.5	30.5	12	3

The function $g(x)$ is exactly cubic.

15.



16.



Section 6.2 Exercises

17. a. The quartic equation is

$$y = 0.0041x^4 - 0.222x^3 + 4.287x^2 - 34.840x + 101.342$$

b. For the year 2012, $x = 22$. Using the rounded model in part a), the percent change in the GDP for 2012 is estimated to be 6.36%.

c. No, according to the model, the percent change in the GDP for 2014 is estimated to be 25.85%, which is unlikely.

18. a. The equation is

$$y = -0.613x^3 + 23.835x^2 - 179.586x + 385.670$$

b.

$$y = -0.613x^3 + 23.835x^2 - 179.586x + 385.670$$

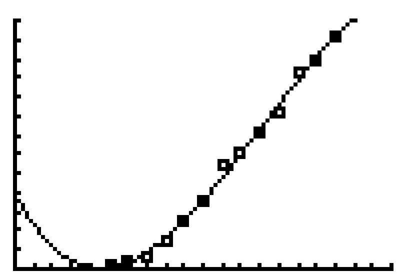
For the year 2013, $x = 23$

$$y = -0.613(23)^3 + 23.835(23)^2 - 179.586(23) + 385.670$$

$y = 1{,}405{,}000{,}000.$

The number of world-wide users of the Internet in 2013 is predicted to be 1405 million.

c.



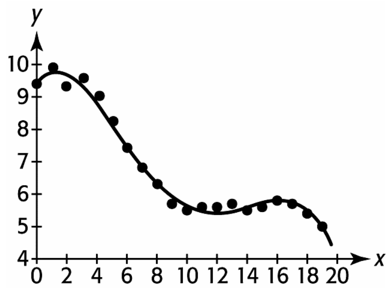
[0, 20] by [0, 1300]

It is a good fit to the data.

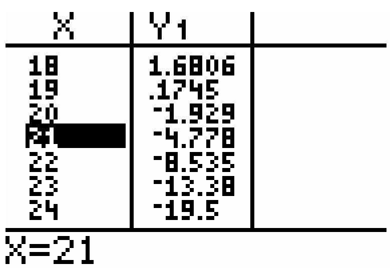
19. a. The quartic function is:

$$y = -0.0006x^4 + 0.0221x^3 - 0.2582x^2 + 0.5576x + 9.3990$$

b.



c.



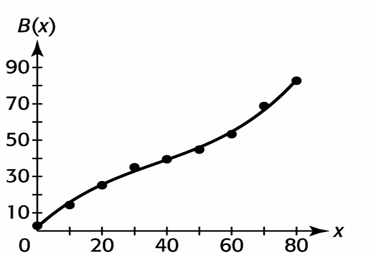
X	Y1
18	1.6806
19	.1745
20	-1.929
21	-4.778
22	-8.535
23	-13.38
24	-19.5

X=21

The rounded model estimates the homicide rate in 2011, when $x = 21$, to be –4.8 per 100,000 people. No, a negative value is not possible.

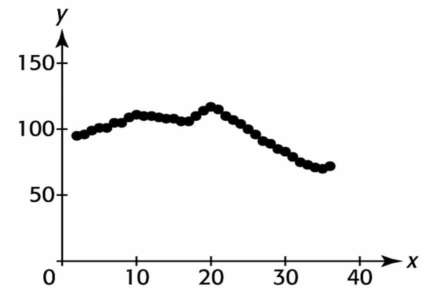
20. a. $y = 0.0002x^3 - 0.0264x^2 + 1.6019x + 2.1990$

b.



c. Yes. It appears the model fits the data well.

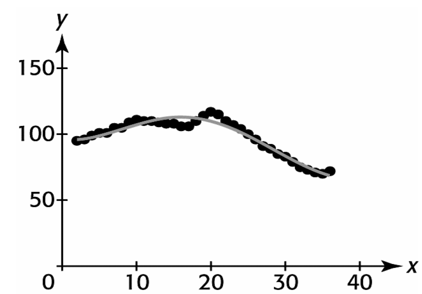
21. a.



b. The quartic function is:

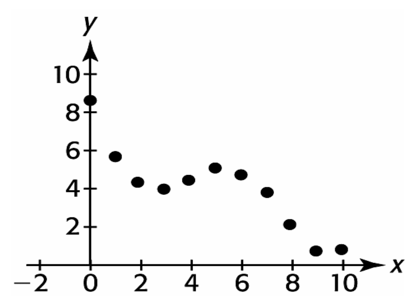
$$y = 0.0003x^4 - 0.0217x^3 + 0.4178x^2 - 1.3912x + 98.4661$$

c.



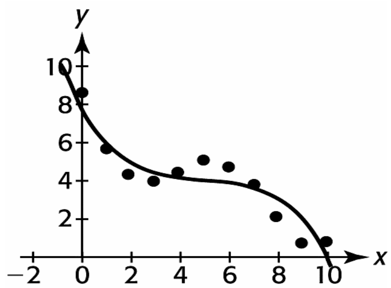
d. For $x = 39$ in the rounded model in part b), the number of pregnancies per thousand in 2009 was 86.

22. a.



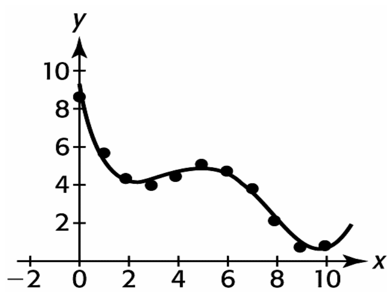
b. The cubic function is:

$$y = -0.0307x^3 + 0.4519x^2 - 2.2708x + 8.0540$$



c. The quartic function is:

$$y = 0.0111x^4 - 0.2523x^3 + 1.8365x^2 - 5.0400x + 8.8515$$



d. See parts b) and c).

e. The cubic is a better choice for prediction after 2010 since it is likely that ad revenue will continue to decrease.

23. a. The cubic function is:

$$y = 0.0790x^3 - 2.103x^2 + 10.695x + 179.504$$

b. In 2008, $x = 18$, so

$$y = 0.0790(18)^3 - 2.103(18)^2 + 10.695(18) + 179.504 = 151.37$$

In 2008, the number of births was approximately 151,370.

24. a. The cubic function is:

$$y = -0.0505x^3 + 0.671x^2 - 1.686x + 36.795$$

b. In 2012, $x = 13$, so

$$y = -0.0505(13)^3 + 0.671(13)^2 - 1.686(13) + 36.795 = 17.3$$

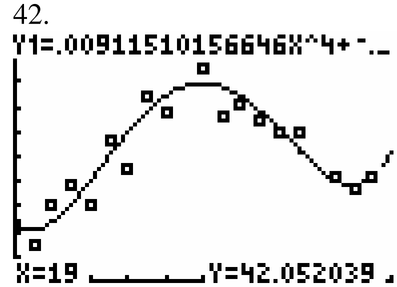
In 2012, the accidental death rate is predicted to be 17.3 per 100,000 residents.

c. When $x \geq 14.5$, or after 2013, the model begins to give negative results and is no longer valid.

25. a. The quartic function is:

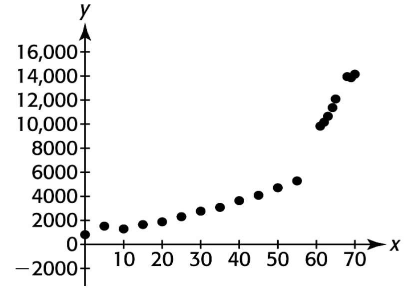
$$y = 0.00912x^4 - 0.34812x^3 + 3.63997x^2 - 4.95490x + 22.00010$$

b. Using the unrounded model for $x = 19$, the number of executions in 2009 was 42.

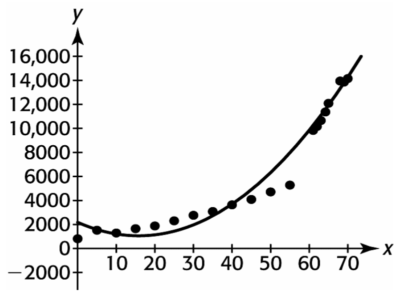


[0, 20] by [0, 100]

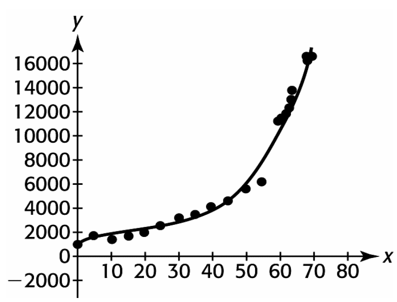
26. a.



b. The quadratic function is:
$$y = 4.492x^2 - 141.801x + 2177.694$$



c. The cubic function is:
$$y = 0.096x^3 - 5.816x^2 + 148.108x + 747.767$$

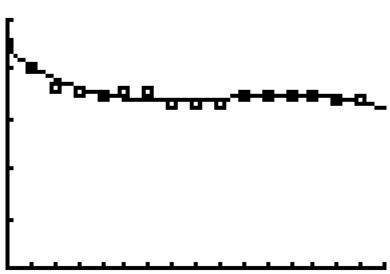


d. The cubic model is a better fit for the data.

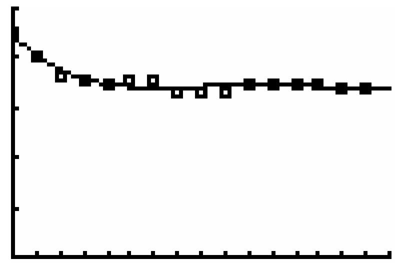
27. a. The cubic function is:
$$y = -0.008x^3 + 0.221x^2 - 1.903x + 22.052$$

b. The quartic function is:
$$y = 0.001x^4 - 0.025x^3 + 0.386x^2 - 2.416x + 22.326$$

c.



[0, 16] by [0, 25]



[0, 16] by [0, 25]

It appears that both models are nearly equal.

28. a. $y = -0.004x^3 + 0.502x^2 - 6.097x + 85.688$

b. This model predicts the 2012 consumer price index to be 660.4

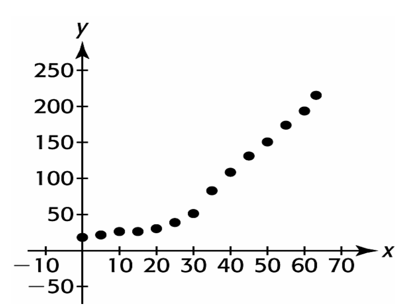
29. a. The quartic function is:
$$y = -0.000000313x^4 + 0.0000776x^3 - 0.00502x^2 + 0.0712x + 21.646$$

b. In 2012, when $x = 112$, the age at first marriage for women will be 26.6 years.

30. a. The cubic function is:
$$y = 0.00000675x^3 + 0.000169x^2 - 0.0774x + 25.975$$

b. In 2012, when $x = 112$, the age at first marriage for men will be 28.9 years.

31. a.



b. Based on the scatter plot, it appears that a cubic model will fit the data well.

The cubic function is:

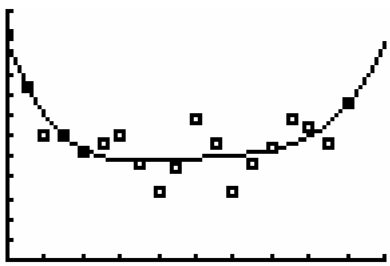
$$y = -0.00082x^3 + 0.125x^2 - 1.7446x + 25.0583$$

c. In 2015, when $x = 70$, the CPI is estimated to be 235.2.

32. a. The quartic function is:

$$y = 0.00028x^4 - 0.01155x^3 + 0.17835x^2 - 1.18426x + 5.21895$$

b.



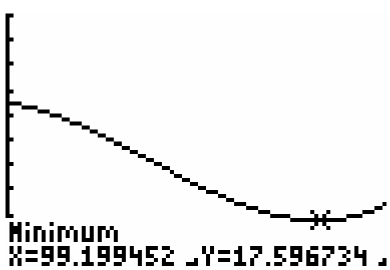
[0, 20] by [0, 6]

The fit is only fair.

33. a. The cubic function is:

$$y = 0.000078x^3 - 0.01069x^2 - 0.1818x + 64.6848$$

b.



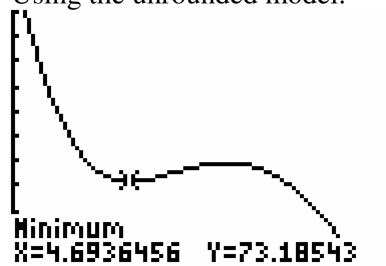
[0, 120] by [0, 100]

In 2000, when $x = 99.2$, the per cent of elderly men in the work force reached its minimum.

34. a. The quartic function is:

$$y = 0.002x^4 - 0.075x^3 + 1.002x^2 - 5.177x + 82.325$$

b. Using the unrounded model:



[0, 15] by [70, 80]

c. Since the minimum point in part b) is at (4.7, 73.2) and $x = 4.7$ represents the year 1995, the model suggests the percent of 12th graders who used alcohol in the last 12 months was lower in 1995 than it was in 1994 and 1996.

d. Using the Table feature, an absolute minimum appears to be at located at $x = 18$, which corresponds to the year 2008.

35. a. The quartic function is:

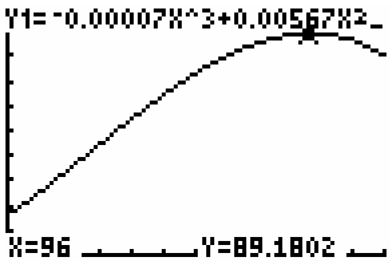
$$y = 0.258x^4 - 8.676x^3 + 87.476x^2 - 244.413x - 150.928$$

b. Using the unrounded model, the estimate for 2008, when $x = 18$, is \$286.228 billion. The rounded model yields \$277.238 billion.

36. a. The cubic function is:

$$y = -0.00007x^3 + 0.00567x^2 + 0.863x + 16.009$$

b.



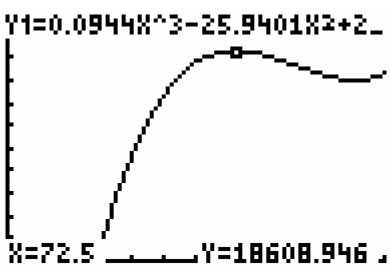
[0, 120] by [0, 100]

In 2046, when $x = 96$, the number of women in the work force will reached its maximum.

37. a. The cubic function is:

$$y = 0.0944x^3 - 25.9401x^2 + 2273.2513x - 45{,}827.8973$$

b.

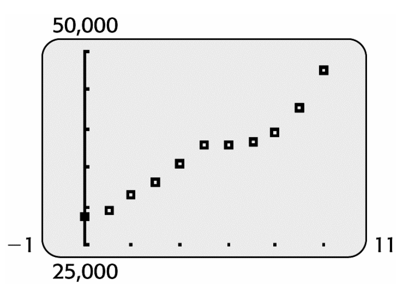


[0, 120] by [0, 22,000]

When $x = 72.5$, which corresponds to the year 1973, the number of U.S. workers who were union members was maximized.

c. The relative maximum from the model agrees fairly well with the data.

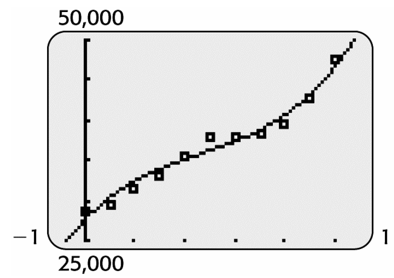
38. a.



b. The cubic function is:

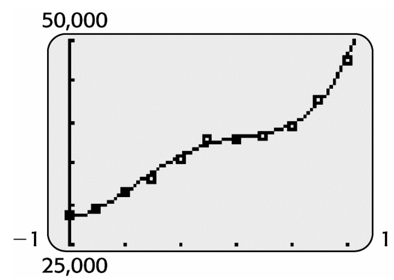
$$y = 33.7496x^3 - 467.3421x^2 + 3223.9841x + 27{,}576.4546$$

c.



d. The quartic function is:

$$y = 14.2241x^4 - 250.7317x^3 + 1310.6664x^2 - 332.0328x + 28{,}600.5874$$



e. The cubic model appears to be the better fit for extrapolation of the data after 2009 since it has a more modest increase each year.

Section 6.3 Skills Check

1. $(2x-3)(x+1)(x-6)=0$

$2x-3=0, x+1=0, x-6=0$

$x=3/2, x=-1, x=6$

2. $(3x+1)(2x-1)(x+5)=0$

$3x+1=0, 2x-1=0, x+5=0$

$x=-1/3, x=1/2, x=-5$

3. $(x+1)^2(x-4)(2x-5)=0$

$x+1=0, x-4=0, 2x-5=0$

$x=-1, x=4, x=5/2$

4. $(2x+3)^2(5-x)^2=0$

$2x+3=0, 5-x=0$

$x=-3/2, x=5$

5. $x^3-16x=0$

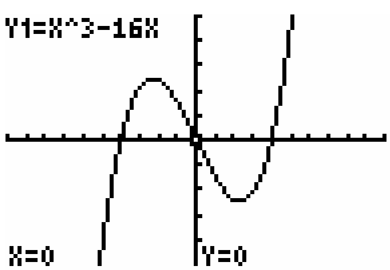
$x\left(x^2-16\right)=0$

$x\left(x+4\right)\left(x-4\right)=0$

$x=0, x+4=0, x-4=0$

$x=0, x=-4, x=4$

Checking graphically



[–10, 10] by [–50, 50]

6. $2x^3-8x=0$

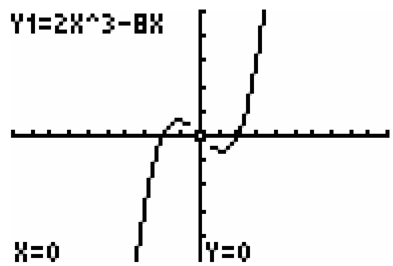
$2x\left(x^2-4\right)=0$

$2x\left(x+2\right)\left(x-2\right)=0$

$2x=0, x+2=0, x-2=0$

$x=0, x=-2, x=2$

Checking graphically



[–10, 10] by [–50, 50]

7. $x^4-4x^3+4x^2=0$

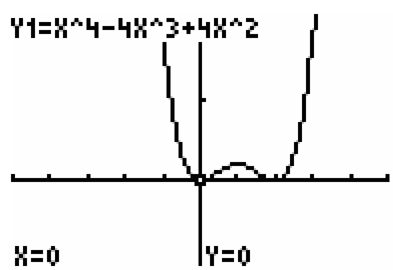
$x^2\left(x^2-4x+4\right)=0$

$x^2\left(x-2\right)\left(x-2\right)=0$

$x^2=0 \Rightarrow x=0, x-2=0$

$x=0, x=2$

Checking graphically



[–5, 5] by [–5, 10]

8. $x^4-6x^3+9x^2=0$

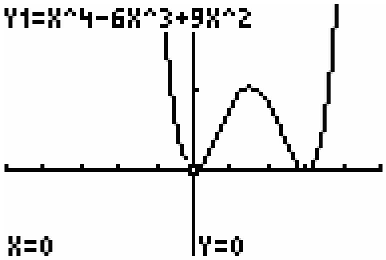
$x^2\left(x^2-6x+9\right)=0$

$x^2\left(x-3\right)\left(x-3\right)=0$

$x^2=0 \Rightarrow x=0, x-3=0$

$x=0, x=3$

Checking graphically



[–5, 5] by [–5, 10]

9. $4x^3 - 4x = 0$

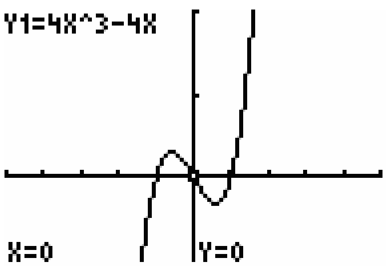
$4x(x^2 - 1) = 0$

$4x(x+1)(x-1) = 0$

$4x = 0, x+1 = 0, x-1 = 0$

$x = 0, x = -1, x = 1$

Checking graphically



[–5, 5] by [–5, 10]

10. $x^4 - 3x^3 + 2x^2 = 0$

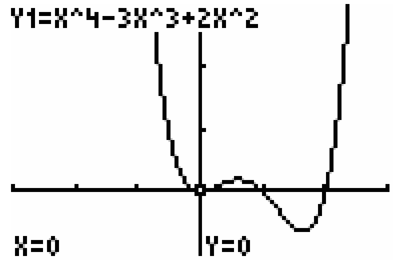
$x^2(x^2 - 3x + 2) = 0$

$x^2(x-2)(x-1) = 0$

$x^2 = 0 \Rightarrow x = 0, x-2 = 0, x-1 = 0$

$x = 0, x = 2, x = 1$

Checking graphically



[–3, 3] by [–1, 3]

11. $x^3 - 4x^2 - 9x + 36 = 0$

$(x^3 - 4x^2) + (-9x + 36) = 0$

$x^2(x-4) + (-9)(x-4) = 0$

$(x-4)(x^2 - 9) = 0$

$(x-4)(x+3)(x-3) = 0$

$x-4 = 0, x+3 = 0, x-3 = 0$

$x = 4, x = -3, x = 3$

12. $x^3 + 5x^2 - 4x - 20 = 0$

$(x^3 + 5x^2) + (-4x - 20) = 0$

$x^2(x+5) + (-4)(x+5) = 0$

$(x+5)(x^2 - 4) = 0$

$(x+5)(x+2)(x-2) = 0$

$x+5 = 0, x+2 = 0, x-2 = 0$

$x = -5, x = -2, x = 2$

13. $3x^3 - 4x^2 - 12x + 16 = 0$

$(3x^3 - 4x^2) + (-12x + 16) = 0$

$x^2(3x-4) + (-4)(3x-4) = 0$

$(3x-4)(x^2 - 4) = 0$

$(3x-4)(x+2)(x-2) = 0$

$x = \frac{4}{3}, x = -2, x = 2$

14. $4x^3 + 8x^2 - 36x - 72 = 0$

$4\left(x^3 + 2x^2 - 9x - 18\right) = 0$

$4\left[\left(x^3 + 2x^2\right) + \left(-9x - 18\right)\right] = 0$

$4\left[x^2\left(x+2\right) + \left(-9\right)\left(x+2\right)\right] = 0$

$4[(x+2)(x^2 - 9)] = 0$

$4(x+2)(x+3)(x-3) = 0$

$x = -2, x = -3, x = 3$

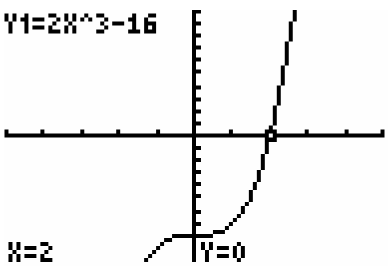
15. $2x^3 - 16 = 0$

$2x^3 = 16$

$x^3 = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$



[–5, 5] by [–20, 20]

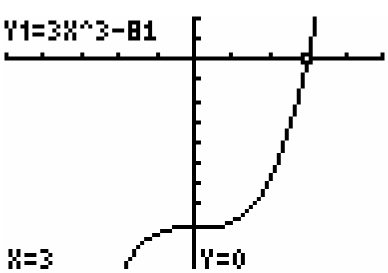
16. $3x^3 - 81 = 0$

$3x^3 = 81$

$x^3 = 27$

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$x = 3$



[–5, 5] by [–100, 20]

17. $\frac{1}{2}x^4 - 8 = 0$

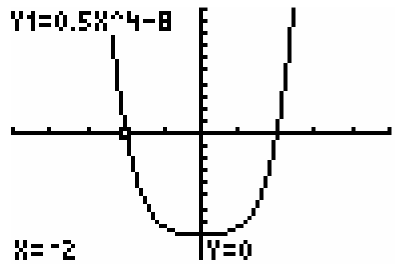
$\frac{1}{2}x^4 = 8$

$2\left(\frac{1}{2}x^4\right) = 2\left(8\right)$

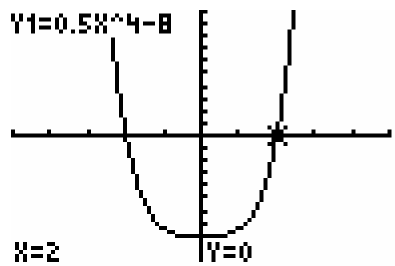
$x^4 = 16$

$\sqrt[4]{x^4} = \pm\sqrt[4]{16}$

$x = \pm 2$



[–5, 5] by [–10, 10]



[–5, 5] by [–10, 10]

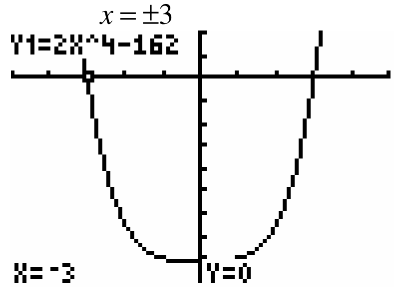
18. $2x^4 - 162 = 0$

$2x^4 = 162$

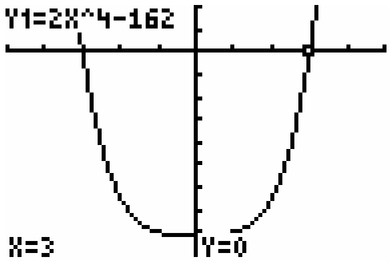
$x^4 = 81$

$\sqrt[4]{x^4} = \pm\sqrt[4]{81}$

$x = \pm 3$



[–5, 5] by [–180, 40]



[–5, 5] by [–180, 40]

19. $4x^4 - 8x^2 = 0$

$4x^2(x^2 - 2) = 0$

$4x^2 = 0,\ x^2 - 2 = 0$

$4x^2 = 0 \Rightarrow x = 0$

$x^2 - 2 = 0 \Rightarrow x^2 = 2$

$\sqrt{x^2} = \pm\sqrt{2}$

$x = \pm\sqrt{2}$

$x = \pm\sqrt{2},\ x = 0$

20.

$3x^4 - 24x^2 = 0$

$3x^2(x^2 - 8) = 0$

$3x^2 = 0,\ x^2 - 8 = 0$

$3x^2 = 0 \Rightarrow x = 0$

$x^2 - 8 = 0 \Rightarrow x^2 = 8$

$\sqrt{x^2} = \pm\sqrt{8} = \pm\sqrt{4\times 2}$

$x = \pm 2\sqrt{2}$

$x = 0,\ x = \pm 2\sqrt{2}$

21. $0.5x^3 - 12.5x = 0$

$0.5x(x^2 - 25) = 0$

$0.5x(x+5)(x-5) = 0$

$x = 0, x = -5, x = 5$

22.

$0.2x^3 - 24x = 0$

$0.2x(x^2 - 120) = 0$

$0.2x = 0,\ x^2 - 120 = 0$

$0.2x = 0 \Rightarrow x = 0$

$x^2 - 120 = 0 \Rightarrow x^2 = 120$

$\sqrt{x^2} = \pm\sqrt{120} = \pm\sqrt{4\times 30}$

$x = \pm 2\sqrt{30}$

$x = 0,\ x = \pm 2\sqrt{30}$

23. $x^4 - 6x^2 + 9 = 0$

$(x^2 - 3)(x^2 - 3) = 0$

$x^2 - 3 = 0,\ x^2 - 3 = 0$

$x^2 - 3 = 0 \Rightarrow x^2 = 3$

$\sqrt{x^2} = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

24. $x^4 - 10x^2 + 25 = 0$

$(x^2 - 5)(x^2 - 5) = 0$

$x^2 - 5 = 0,\ x^2 - 5 = 0$

$x^2 - 5 = 0 \Rightarrow x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

25. **a.** $f(x) = 0$ implies $x = -3, x = 1, x = 4$. Note that the x-intercepts are the solutions.

b. The factors are $(x + 3)$, $(x - 1)$, and $(x - 4)$.

26. **a.** $f(x) = 0$ implies $x = -2, x = 0.5, x = 8$. Note that the x-intercepts are the solutions.

b. The factors are $(x + 2)$, $(x - ½)$, and $(x - 8)$.

27. a. The x-intercepts appear to be at –1, 2, and 3. Since the graph only touches at $x = -1$, the factor $x + 1$ will be squared.

b. The factors are $(x + 1)^2$, $(x - 2)$, and $(x - 3)$.

28. a. The x-intercepts appear to be at –1, 2, and 5. Since the graph only touches at $x = -1$, the factor $x + 1$ will be squared.

b. The factors are $(x + 1)^2$, $(x - 2)$, and $(x - 5)$.

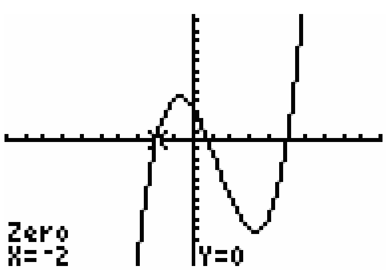
29. a. The x-intercepts appear to be at –1, 1, and 5.

b. The factors are $(x + 1)$, $(x - 1)$, and $(x - 5)$.

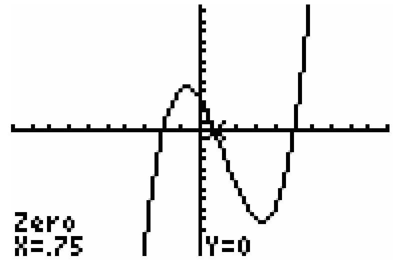
30. a. The x-intercepts appear to be at –2, 1, and 2.

b. The factors are $(x + 2)$, $(x - 1)$, and $(x - 2)$.

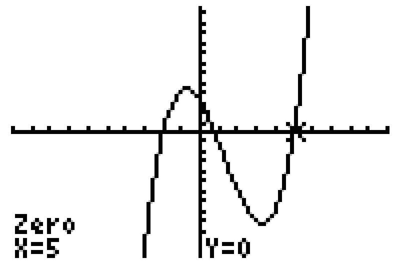
31. The x-intercepts (zeros) are the solutions of $4x^3 - 15x^2 - 31x + 30 = 0$.



[–10, 10] by [–125, 125]



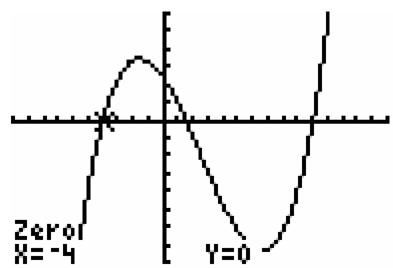
[–10, 10] by [–125, 125]



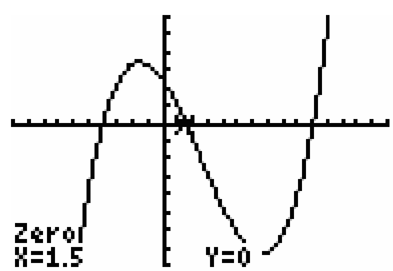
[–10, 10] by [–125, 125]

$x = -2, x = 0.75 \text{ or } \dfrac{3}{4}, x = 5$

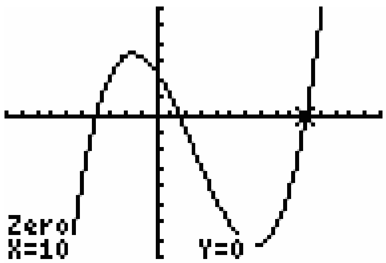
32. The x-intercepts (zeros) are the solutions of $2x^3 - 15x^2 - 62x + 120 = 0$.



[–10, 15] by [–400, 300]



[–10, 15] by [–400, 300]



[–10, 15] by [–400, 300]

$x = -4,\ x = 1.5,\ x = 10$

Section 6.3 Exercises

33. a. $R = 400x - x^3$

$400x - x^3 = 0$

$x(400 - x^2) = 0$

$x(20 - x)(20 + x) = 0$

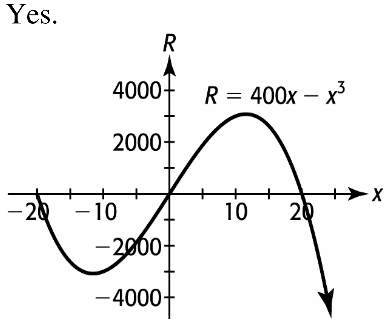
$x = 0,\ 20 - x = 0,\ 20 + x = 0$

$x = 0,\ -x = -20,\ x = -20$

$x = 0,\ x = 20,\ x = -20$

In the physical context of the problem, selling zero units or selling 20 units will yield revenue of zero dollars. -20 units is not possible and so is eliminated.

b. Yes.



34. a. $R = 12{,}000x - 0.003x^3$

$12{,}000x - 0.003x^3 = 0$

$0.003x(4{,}000{,}000 - x^2) = 0$

$0.003x = 0 \qquad 4{,}000{,}000 - x^2 = 0$

$x = 0 \qquad x^2 = 4{,}000{,}000$

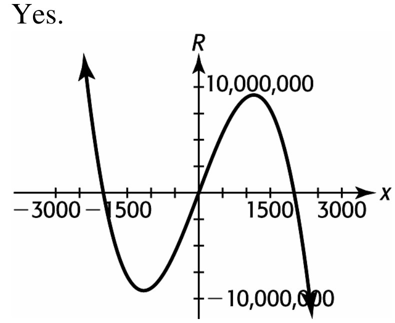
$x = \pm\sqrt{4{,}000{,}000}$

$x = \pm 2000$

$x = 0,\ x = 2000,\ x = -2000$

In the physical context of the problem, selling zero units or selling 2000 units will yield revenue of zero dollars. -2000 units is not possible and so is eliminated.

b. Yes.



35. a. $R = (100{,}000 - 0.1x^2)x$

$(100{,}000 - 0.1x^2)x = 0$

$x = 0,\ 100{,}000 - 0.1x^2 = 0$

$-0.1x^2 = -100{,}000$

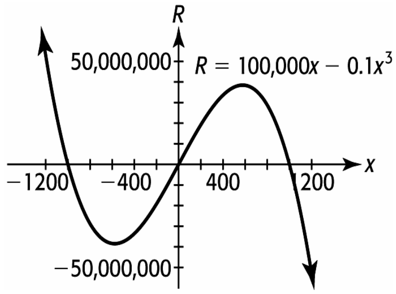
$x^2 = 1{,}000{,}000$

$x = \pm\sqrt{1{,}000{,}000}$

$x = 0,\ x = 1000,\ x = -1000$

In the physical context of the problem, selling zero units or selling 1000 units will yield revenue of zero dollars. -1000 units is not possible and so is eliminated.

b. Yes.



36. a. $R = (100x - x^2)x$

$(100x - x^2)x = 0$

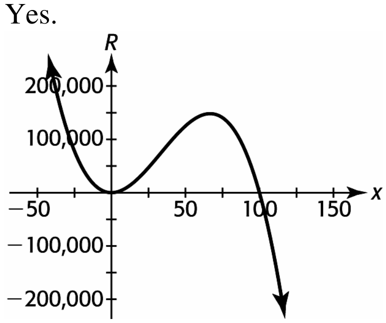
$x^2(100 - x) = 0$

$x^2 = 0,\ 100 - x = 0$

$x = 0, x = 100$

In the physical context of the problem, selling zero units or selling 100 units will yield revenue of zero dollars.

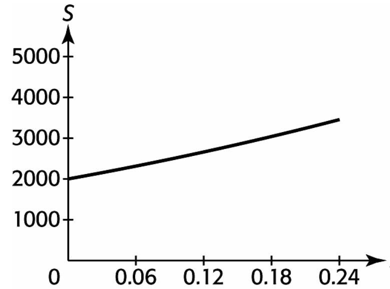
b. Yes.



37. a. Complete the table:

Rate	Future Value
4%	\$2249.73
5%	\$2315.25
7.25%	\$2467.30
10.5%	\$2698.47

b.



c. $2662 = 2000(1+r)^3$

$(1+r)^3 = \frac{2662}{2000} = 1.331$

$\sqrt[3]{(1+r)^3} = \sqrt[3]{1.331} = 1.10$

$1 + r = 1.10$

$r = 1.10 - 1$

$r = 0.10 = 10\%$

d. $3456 = 2000(1+r)^3$

$(1+r)^3 = \frac{3456}{2000} = 1.728$

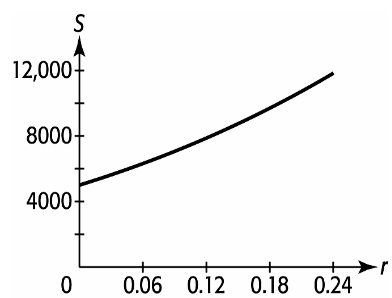
$\sqrt[3]{(1+r)^3} = \sqrt[3]{1.728} = 1.20$

$1 + r = 1.20$

$r = 1.20 - 1$

$r = 0.20 = 20\%$

38. a.



b. $10{,}368 = 5000(1+r)^4$

$(1+r)^4 = \dfrac{10{,}368}{5000} = 2.0736$

$\sqrt[4]{(1+r)^4} = \pm\sqrt[4]{2.0736} = \pm 1.20$

$1+r = \pm 1.20$

$r = \pm 1.20 - 1$

$r = 0.20 \text{ or } -2.2$

Since the negative solution does not make sense in the context of the problem, $r = 20\%$.

c. $(5000 + 2320.50) = 5000(1+r)^4$

$(1+r)^4 = \dfrac{7320.50}{5000} = 1.4641$

$\sqrt[4]{(1+r)^4} = \pm\sqrt[4]{1.4641} = \pm 1.10$

$1+r = \pm 1.10$

$r = \pm 1.10 - 1$

$r = 0.10 \text{ or } -2.1$

Since the negative solution does not make sense in the context of the problem, $r = 10\%$.

39. a. The height is x inches, since the distance cut is x units and that distance when folded forms the height of the box.

b. The length and width of box will be what is left after the corners are cut. Since each corner measures x inches square, the length and the width are $18 - 2x$.

c. $V = lwh$

$V = (18-2x)(18-2x)x$

$V = (324 - 36x - 36x + 4x^2)x$

$V = 324x - 72x^2 + 4x^3$

d. $V = 0$

$0 = 324x - 72x^2 + 4x^3$

From part c) above:

$0 = (18-2x)(18-2x)x$

$18 - 2x = 0,\ x = 0$

$18 - 2x = 0 \Rightarrow 2x = 18 \Rightarrow x = 9$

$x = 0,\ x = 9$

e. A box will not exist for either of the values calculated in part d) above. For both values of x, no tab will exist to fold up to form the box.

40. a. $0 = 144x - 48x^2 + 4x^3$

$4x(36 - 12x + x^2) = 0$

$4x(x^2 - 12x + 36) = 0$

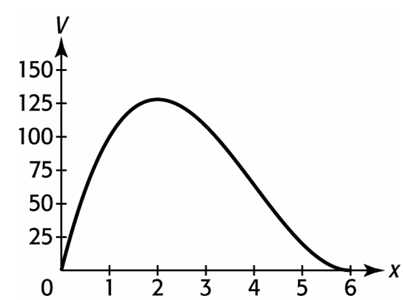
$4x(x-6)(x-6) = 0$

$x = 0, x = 6$

b. For the values calculated in part a) no box can be formed. The calculated values of x yield no tabs that can be folded up to form the box.

c. A box can be created as long as $0 < x < 6$.

d.



41. Since the profit is given in hundreds of dollars, \$40,000 should be represented as 400 hundreds. Thus,

$$400 = -x^3 + 2x^2 + 400x - 400$$
$$0 = -x^3 + 2x^2 + 400x - 800$$
$$x^3 - 2x^2 - 400x + 800 = 0$$
$$\left(x^3 - 2x^2\right) + \left(-400x + 800\right) = 0$$
$$x^2\left(x-2\right) + \left(-400\right)\left(x-2\right) = 0$$
$$\left(x-2\right)\left(x^2 - 400\right) = 0$$
$$\left(x-2\right)\left(x+20\right)\left(x-20\right) = 0$$
$$x = 2, x = -20, x = 20$$

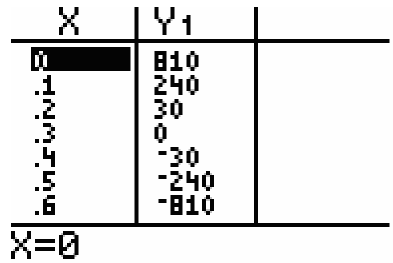
The negative answer does not make sense in the physical context of the problem. Producing and selling 2 units or 20 units yields a profit of \$40,000.

42. Since the cost is given in hundreds of dollars, \$120,000 should be represented as 1200 hundreds. Thus,

$$1200 = 3x^3 - 6x^2 - 300x + 1800$$
$$3x^3 - 6x^2 - 300x + 1800 - 1200 = 0$$
$$3x^3 - 6x^2 - 300x + 600 = 0$$
$$3\left(x^3 - 2x^2 - 100x + 200\right) = 0$$
$$3\left[\left(x^3 - 2x^2\right) + \left(-100x + 200\right)\right] = 0$$
$$3\left[x^2\left(x-2\right) + \left(-100\right)\left(x-2\right)\right] = 0$$
$$3\left(x-2\right)\left(x^2 - 100\right) = 0$$
$$3\left(x-2\right)\left(x+10\right)\left(x-10\right) = 0$$
$$x = 2, x = -10, x = 10$$

The negative answer does not make sense in the physical context of the problem. Producing and selling 2 units or 10 units yields a cost of \$120,000.

43. a. $s = 30\left(3 - 10t\right)^3$



X	Y1
0	810
.1	240
.2	30
.3	0
.4	-30
.5	-240
.6	-810

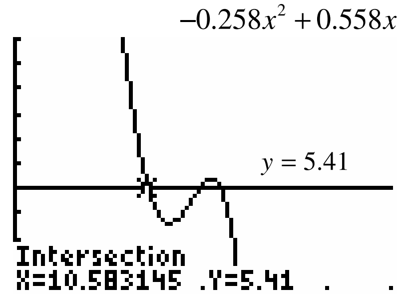
X=0

t	0	0.1	0.2	0.3
s (cm/sec)	810	240	30	0

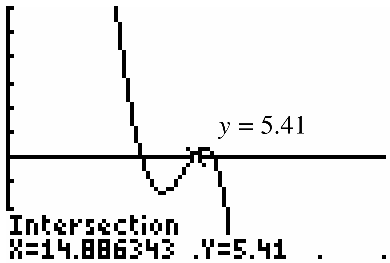
b.
$$0 = 30\left(3 - 10t\right)^3$$
$$\left(3 - 10t\right)^3 = \frac{0}{30} = 0$$
$$\sqrt[3]{\left(3 - 10t\right)^3} = \sqrt[3]{0}$$
$$3 - 10t = 0$$
$$t = \frac{-3}{-10}$$
$$t = 0.3$$

The solution in the table is the same as the solution found by the root method.

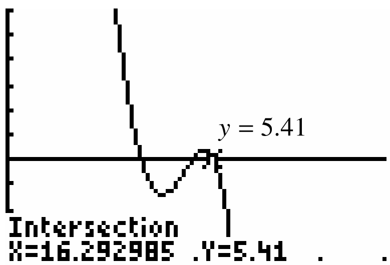
44. Applying the intersection of graphs method for $y = -0.000564x^4 + 0.022x^3 - 0.258x^2 + 0.558x + 9.399$



[0, 30] by [5, 6]



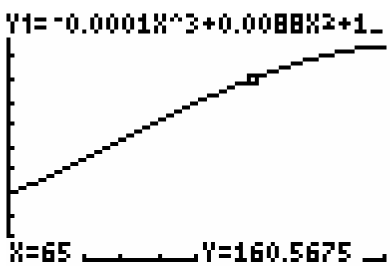
[0, 30] by [5, 6]



[0, 30] by [5, 6]

After 1990, the homicide rate was 5.41 per 100,000 people in the years 2001, 2005, and 2007.

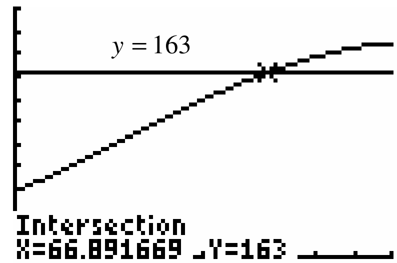
45. a. $y = -0.0001x^3 + 0.0088x^2 + 1.43x + 57.9$



[0, 100] by [0, 220]

When $x = 65$, in the year 2015, the civilian labor force is projected to be 160.6 million people.

b.

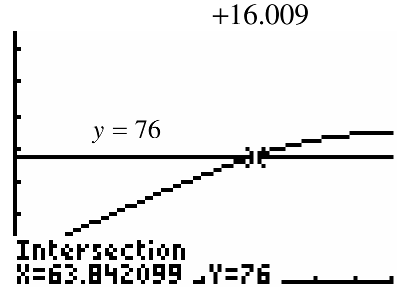


[0, 100] by [0, 220]

When $x = 66.9$, in the year 2017, the projected civilian work force will be 163 million.

46. Applying the intersection of graphs method for

$$y = -0.00007x^3 + 0.00567x^2 + 0.863x + 16.009$$

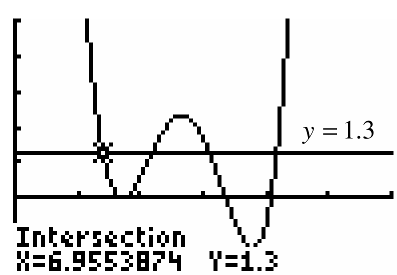


[0, 100] by [0, 150]

When $x = 63.8$, in the year 2014, the number of women in the work force is estimated to be 76 million.

47. a. Applying the intersection of graphs method for

$$y = 0.0041x^4 - 0.222x^3 + 4.287x^2 - 34.8398x + 101.33417$$



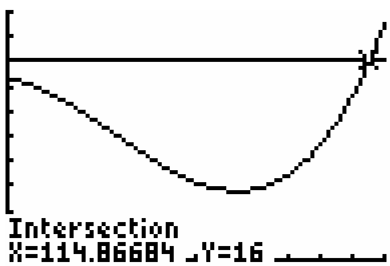
[0, 30] by [–2, 5]

After 6.96 years, in 1997, the percent change is 1.3%.

b. The model will be 1.3% again. Note that there are three additional intersection points on the graph in part a), in the years 2001 (x = 10.9), 2006 (x = 15.4), and 2011 (x = 20.9).

48. Applying the intersection of graphs method for

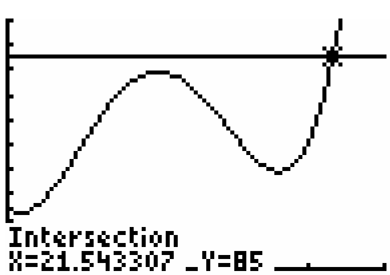
$$y = 0.0000384x^3 - 0.00397x^2 - 0.03829x + 14.58102$$



[0, 120] by [0, 20]

In 2015 (x = 114.9) the percentage of the U.S. population that was foreign born is estimated to be 16%.

49. Applying the intersection of graphs method for $y = 0.00894x^4 + 0.344x^3 + 3.653x^2 - 5.648x + 25.077$



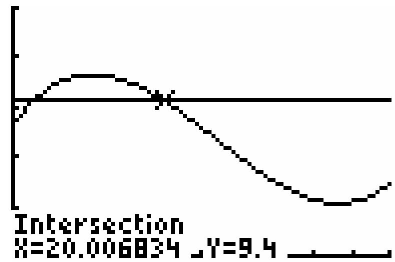
[0, 25] by [0, 100]

In 2012 (x = 21.5) the number of executions carried out in the U.S. is estimated to be 85.

50. a. The cubic function is:

$$y = 0.000447x^3 - 0.0361x^2 + 0.615x + 7.966$$

b.



[0, 50] by [0, 15]

According to the model, the personal savings rate was 9.4% in the year 1980.

c. Yes, this agrees with the data in the table for year 1980.

Section 6.4 Skills Check

1.

$$\begin{array}{r|rrrrr} 3 & 1 & -4 & 0 & 3 & 10 \\ & & 3 & -3 & -9 & -18 \\ \hline & 1 & -1 & -3 & -6 & -8 \end{array}$$

$$x^3 - x^2 - 3x - 6 - \frac{8}{x-3}$$

2.

$$\begin{array}{r|rrrrr} -4 & 1 & 2 & -3 & 0 & 1 \\ & & -4 & 8 & -20 & 80 \\ \hline & 1 & -2 & 5 & -20 & 81 \end{array}$$

$$x^3 - 2x^2 + 5x - 20 + \frac{81}{x+4}$$

3.

$$\begin{array}{r|rrrrr} 1 & 2 & -3 & 0 & 1 & -7 \\ & & 2 & -1 & -1 & 0 \\ \hline & 2 & -1 & -1 & 0 & -7 \end{array}$$

$$2x^3 - x^2 - x - \frac{7}{x-1}$$

4.

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 0 & 0 & -1 \\ & & -1 & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -1 & 0 \end{array}$$

$$x^3 - x^2 + x - 1$$

5.

$$\begin{array}{r|rrrrr} 3 & 2 & -4 & 0 & 3 & 18 \\ & & 6 & 6 & 18 & 63 \\ \hline & 2 & 2 & 6 & 21 & 81 \end{array}$$

Since the remainder is not zero, 3 is not a solution of the equation.

6.

$$\begin{array}{r|rrrrr} -5 & 1 & 3 & -10 & 8 & 40 \\ & & -5 & 10 & 0 & -40 \\ \hline & 1 & -2 & 0 & 8 & 0 \end{array}$$

Since the remainder is zero, -5 is a solution of the equation.

7.

$$\begin{array}{r|rrrrr} -3 & -1 & 0 & -9 & 3 & 0 \\ & & 3 & -9 & 54 & -171 \\ \hline & -1 & 3 & -18 & 57 & -171 \end{array}$$

Since the remainder is not zero, $x+3$ is not a factor.

8.

$$\begin{array}{r|rrrrr} -2 & 2 & 5 & 0 & -6 & -4 \\ & & -4 & -2 & 4 & 4 \\ \hline & 2 & 1 & -2 & -2 & 0 \end{array}$$

Since the remainder is zero, $x+2$ is a factor.

9.

$$\begin{array}{r|rrrr} -1 & -1 & 1 & 1 & -1 \\ & & 1 & -2 & 1 \\ \hline & -1 & 2 & -1 & 0 \end{array}$$

One solution is $x=-1$. The new polynomial is $-x^2+2x-1$.

Solve $-x^2+2x-1=0$.

$x^2-2x+1=0$

$(x-1)(x-1)=0$

$x=1, x=1$

The remaining solution is $x=1$ (a double solution).

10.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -1 & -4 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

One solution is $x=1$. The new polynomial is x^2+5x+4.

Solve $x^2+5x+4=0$.

$(x+1)(x+4)=0$

$x=-1, x=-4$

The remaining solutions are $x=-1$, $x=-4$.

11.

−5)	1	2	−21	−22	40
		−5	15	30	−40
	1	−3	−6	8	0

One solution is $x = -5$. The new polynomial is $x^3 - 3x^2 - 6x + 8$.

Synthetically dividing by the 2nd given solution in the new polynomial yields:

1)	1	−3	−6	8
		1	−2	−8
	1	−2	−8	0

The 2nd solution is $x = 1$. The new polynomial is $x^2 - 2x - 8$.

Solve $x^2 - 2x - 8 = 0$.

$(x-4)(x+2) = 0$

$x = 4, x = -2$

The remaining solutions are $x = 4$, $x = -2$.

12.

3)	2	−17	51	−63	27
		6	−33	54	−27
	2	−11	18	−9	0

One solution is $x = 3$. The new polynomial is $2x^3 - 11x^2 + 18x - 9$.

Synthetically dividing by the 2nd given solution in the new polynomial yields:

1)	2	−11	18	−9
		2	−9	9
	2	−9	9	0

The 2nd solution is $x = 1$. The new polynomial is $2x^2 - 9x + 9$.

Solve $2x^2 - 9x + 9 = 0$.

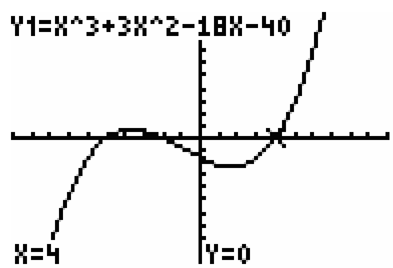
$(2x-3)(x-3) = 0$

$x = \frac{3}{2}, x = 3$

The remaining solutions are $x = \frac{3}{2}, x = 3$.

Note that $x = 3$ is a double solution.

13. Applying the x-intercept method:



[–10, 10] by [–250, 250]

One solution appears to be $x = 4$.

4)	1	3	−18	−40
		4	28	40
	1	7	10	0

The new polynomial is $x^2 + 7x + 10$.

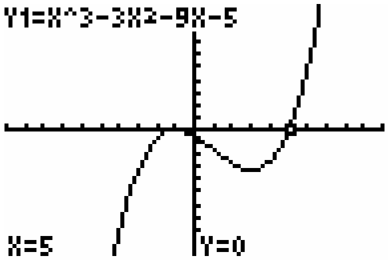
Solve $x^2 + 7x + 10 = 0$.

$(x+2)(x+5) = 0$

$x = -2, x = -5$

The remaining solutions are $x = -5, x = -2$.

14. Applying the x-intercept method:



[–10, 10] by [–100, 100]

One solution appears to be $x = 5$.

5)	1	–3	–9	–5
		5	10	5
	1	2	1	0

The new polynomial is $x^2 + 2x + 1$.

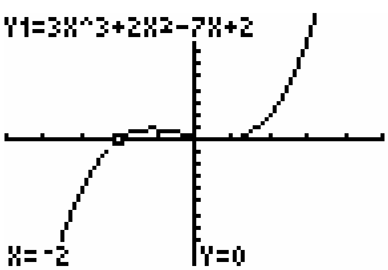
Solve $x^2 + 2x + 1 = 0$.

$(x+1)(x+1) = 0$

$x = -1, x = -1$

The remaining solution is $x = -1$, a double solution.

15. Applying the x-intercept method:



[–5, 5] by [–100, 100]

One solution appears to be $x = –2$.

–2)	3	2	–7	2
		–6	8	–2
	3	–4	1	0

The new polynomial is $3x^2 - 4x + 1$.

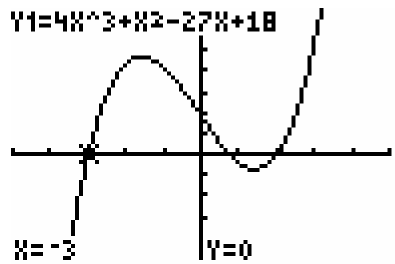
Solve $3x^2 - 4x + 1 = 0$.

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}, x = 1$

The remaining solutions are $x = 1, x = \frac{1}{3}$.

16. Applying the x-intercept method:



[–5, 5] by [–50, 70]

One solution appears to be $x = –3$.

–3)	4	1	–27	18
		–12	33	–18
	4	–11	6	0

The new polynomial is $4x^2 - 11x + 6$.

Solve $4x^2 - 11x + 6 = 0$.

$(4x-3)(x-2) = 0$

$x = \frac{3}{4}, x = 2$

The remaining solutions are $x = 2, x = \frac{3}{4}$.

17. $x^3 - 6x^2 + 5x + 12 = 0$

$$\frac{p}{q} = \pm\left(\frac{1,2,3,4,6,12}{1}\right) = \pm(1,2,3,4,6,12)$$

18. $4x^3+3x^2-9x+2=0$

$$\frac{p}{q}=\pm\left(\frac{1,2}{1,2,4}\right)=\pm\left(1,2,\frac{1}{2},\frac{1}{4}\right)$$

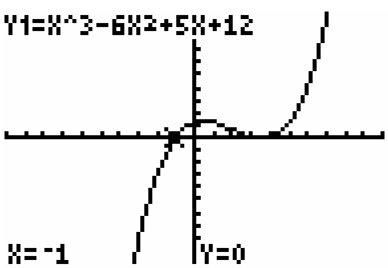
19. $9x^3+18x^2+5x-4=0$

$$\frac{p}{q}=\pm\left(\frac{1,2,4}{1,3,9}\right)$$
$$=\pm\left(1,2,4,\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{1}{9},\frac{2}{9},\frac{4}{9}\right)$$

20. $6x^4-x^3-42x^2-29x+6=0$

$$\frac{p}{q}=\pm\left(\frac{1,2,3,6}{1,2,3,6}\right)$$
$$=\pm\left(1,2,3,6,\frac{1}{2},\frac{3}{2},\frac{1}{3},\frac{2}{3},\frac{1}{6}\right)$$

21. Applying the x-intercept method:



[–10, 10] by [–100, 100]

One solution appears to be $x=-1$.

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 5 & 12 \\ & & -1 & 7 & -12 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

The new polynomial is $x^2-7x+12$.

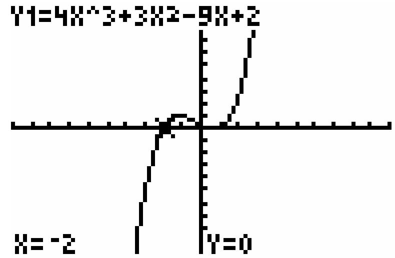
Solve $x^2-7x+12=0$.

$(x-3)(x-4)=0$

$x=3,\ x=4$

The remaining solutions are $x=3, x=4$.

22. Applying the x-intercept method:



[–10, 10] by [–100, 100]

One solution appears to be $x=-2$.

$$\begin{array}{r|rrrr} -2 & 4 & 3 & -9 & 2 \\ & & -8 & 10 & -2 \\ \hline & 4 & -5 & 1 & 0 \end{array}$$

The new polynomial is $4x^2-5x+1$.

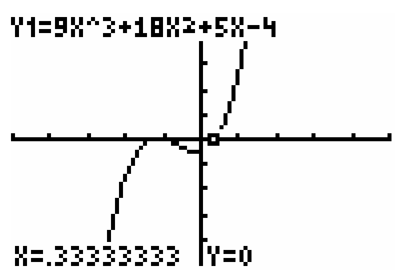
Solve $4x^2-5x+1=0$.

$(4x-1)(x-1)=0$

$x=\frac{1}{4},\ x=1$

The remaining solutions are $x=\frac{1}{4}, x=1$.

23. Applying the x-intercept method:



[–5, 5] by [–50, 50]

One solution appears to be $x=\frac{1}{3}$.

$\frac{1}{3}$	9	18	5	–4
		3	7	4
	9	21	12	0

The new polynomial is $9x^2 + 21x + 12$.

Solve $9x^2 + 21x + 12 = 0$.

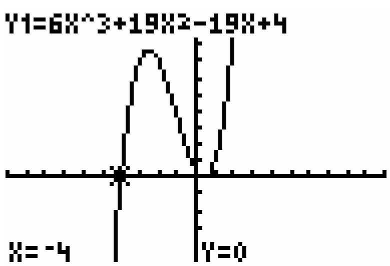
$3(3x^2 + 7x + 4) = 0$

$3(3x + 4)(x + 1) = 0$

$x = -\frac{4}{3}, x = -1$

The remaining solutions are $x = -\frac{4}{3}, x = -1$.

24. Applying the *x*-intercept method



[–10, 10] by [–50, 100]

One solution appears to be $x = -4$.

–4	6	19	–19	4
		–24	20	–4
	6	–5	1	0

The new polynomial is $6x^2 - 5x + 1$.

Solve $6x^2 - 5x + 1 = 0$.

$(3x - 1)(2x - 1) = 0$

$x = \frac{1}{3}, x = \frac{1}{2}$

The remaining solutions are $x = \frac{1}{3}, x = \frac{1}{2}$.

25. Factoring the common *x* term:

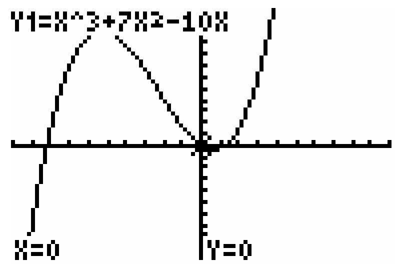
$x^3 = 10x - 7x^2$

$x^3 + 7x^2 - 10x = 0$

$x(x^2 + 7x - 10) = 0$

$x = 0, x^2 + 7x - 10 = 0$

One solution is $x = 0$.



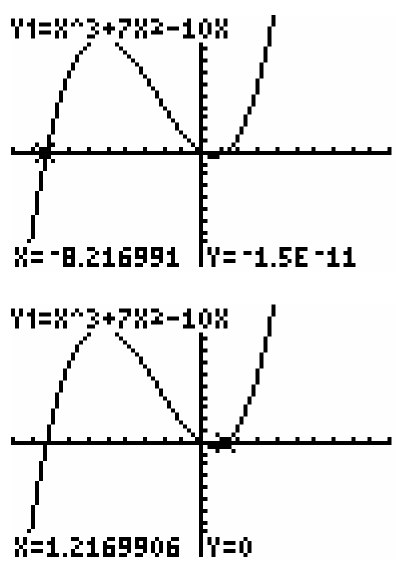
[–10, 10] by [–100, 120]

Applying the quadratic formula:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{89}}{2} \approx -8.217,\ 1.217$$

The remaining solutions are $x = \frac{-7 \pm \sqrt{89}}{2}$, both real numbers, which show on the graph.



[–10, 10] by [–100, 120]

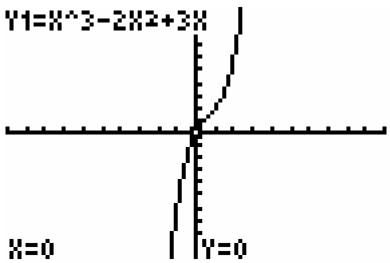
26. Factoring the common t term:

$t^3 - 2t^2 + 3t = 0$

$t(t^2 - 2t + 3) = 0$

$t = 0, t^2 - 2t + 3 = 0$

One solution is $t = 0$.



[–10, 10] by [–10, 10]

Applying the quadratic formula:

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

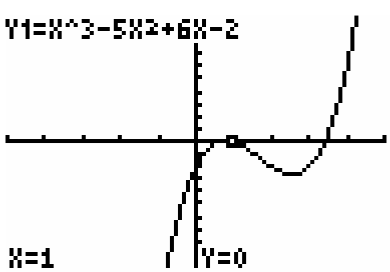
$$t = \frac{2 \pm \sqrt{-8}}{2}$$

$$t = \frac{2 \pm 2i\sqrt{2}}{2} = \frac{2(1 \pm i\sqrt{2})}{2}$$

$$t = 1 \pm i\sqrt{2}$$

The remaining solutions are $t = 1 \pm i\sqrt{2}$, both imaginary, which do not show on the graph.

27. Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that w = 1 is a zero.

1)	1	–5	6	–2
		1	–4	2
	1	–4	2	0

The new polynomial is $w^2 - 4w + 2$.

Applying the quadratic formula:

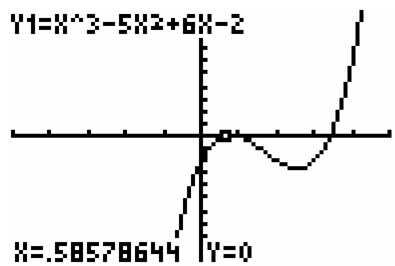
$$w = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

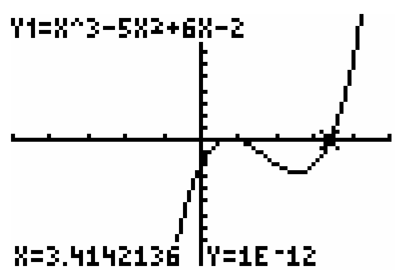
$$w = \frac{4 \pm \sqrt{8}}{2}$$

$$w = \frac{4 \pm 2\sqrt{2}}{2} = \frac{2(2 \pm \sqrt{2})}{2}$$

$$w = 2 \pm \sqrt{2} \approx 0.586,\ 3.414$$

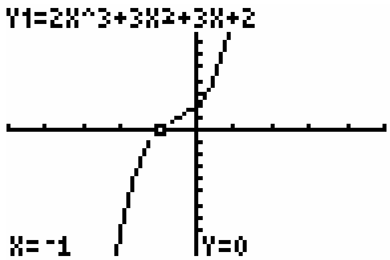
The remaining solutions are $w = 2 \pm \sqrt{2}$ both real numbers, which show on the graph.





[–5, 5] by [–10, 10]

28. Applying the *x*-intercept method:



[–5, 5] by [–10, 10]

It appears that $w = -1$ is a zero.

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 3 & 2 \\ & & -2 & -1 & -2 \\ \hline & 2 & 1 & 2 & 0 \end{array}$$

The new polynomial is $2w^2 + w + 2$.

Applying the quadratic formula:

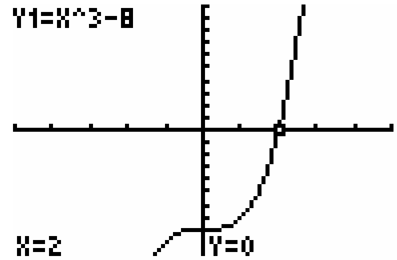
$$w = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(2)}}{2(2)}$$

$$w = \frac{-1 \pm \sqrt{-15}}{4}$$

$$w = \frac{-1 \pm i\sqrt{15}}{4}$$

The remaining solutions are $w = \dfrac{-1 \pm i\sqrt{15}}{4}$, both imaginary, which do not show on the graph.

29. Applying the *x*-intercept method:



[–5, 5] by [–10, 10]

It appears that $z = 2$ is a zero.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

The new polynomial is $z^2 + 2z + 4$.

Applying the quadratic formula:

$$z = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

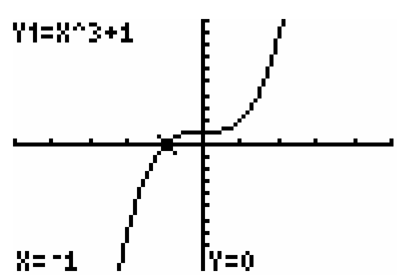
$$z = \frac{-2 \pm \sqrt{-12}}{2}$$

$$z = \frac{-2 \pm 2i\sqrt{3}}{2} = \frac{\cancel{2}(-1 \pm i\sqrt{3})}{\cancel{2}}$$

$$z = -1 \pm i\sqrt{3}$$

The remaining solutions are $z = -1 \pm i\sqrt{3}$ both imaginary, which do not show on the graph..

30. Applying the *x*-intercept method:



[–5, 5] by [–10, 10]

It appears that $x = -1$ is a zero.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

The new polynomial is $x^2 - x + 1$.

Applying the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

The remaining solutions are $x = \frac{1 \pm i\sqrt{3}}{2}$, both imaginary, which do not show on the graph.

Section 6.4 Exercises

31. a.

$$\begin{array}{r|rrrr} 50 & -0.2 & 66 & -1600 & -60{,}000 \\ & & -10 & 2800 & 60{,}000 \\ \hline & -0.2 & 56 & 1200 & 0 \end{array}$$

The quadratic factor of $P(x)$ is $-0.2x^2 + 56x + 1200$.

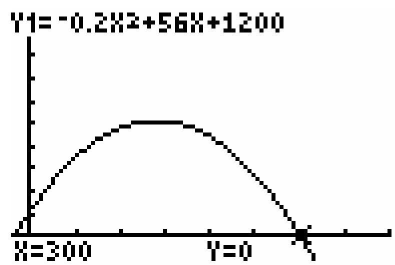
b. $-0.2x^2 + 56x + 1200 = 0$

$-0.2(x^2 - 280x - 6000) = 0$

$-0.2(x + 20)(x - 300) = 0$

$x = -20,\ x = 300$

In the context of the problem, only the positive solution is reasonable. Producing and selling 300 units results is break-even for the product.



[–20, 400] by [–1000, 10,000]

32. a.

$$\begin{array}{r|rrrr} 10 & -1 & 98 & -700 & -1800 \\ & & -10 & 880 & 1800 \\ \hline & -1 & 88 & 180 & 0 \end{array}$$

The quadratic factor of $P(x)$ is $-1x^2 + 88x + 180$.

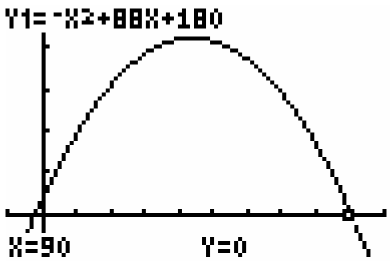
b. $-x^2 + 88x + 180 = 0$

$-1(x^2 - 88x - 180) = 0$

$-1(x - 90)(x + 2) = 0$

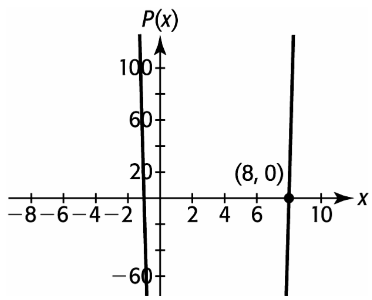
$x = 90,\ x = -2$

In the context of the problem, only the positive solution is reasonable. Producing and selling 90 units results is break-even for the product.



[–10, 100] by [–500, 2500]

33. a.



b. Based on the graph in part a), one x-intercept appears to be $x = 8$.

c.

8)	–0.1	50.7	–349.2	–400
		–0.8	399.2	400
	–0.1	49.9	50	0

The quadratic factor of $P(x)$ is $-0.1x^2 + 49.9x + 50$.

d. Based on parts b) and c), one zero is $x = 8$. To find more zeros, solve $-0.1x^2 + 49.9x + 50 = 0$.

$-0.1\left(x^2 - 499 - 500\right) = 0$

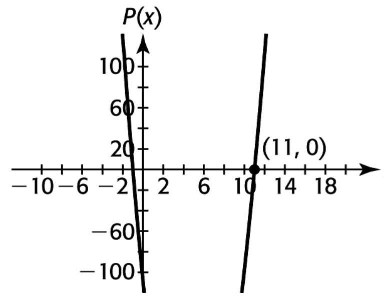
$-0.1(x - 500)(x + 1) = 0$

$x = 500,\ x = -1$

The zeros are $x = 500,\ x = -1,\ x = 8$.

e. Based on the context of the problem, producing and selling 8 units or 500 units results in break-even for the product.

34. a.



b. Based on the graph in part a), one x-intercept appears to be $x = 11$.

c.

11)	–0.1	10.9	–97.9	–108.9
		–1.1	107.8	108.9
	–0.1	9.8	9.9	0

The quadratic factor of $P(x)$ is $-0.1x^2 + 9.8x + 9.9$.

d. Based on parts b) and c), one zero is $x = 11$. To find more zeros, solve $-0.1x^2 + 9.8x + 9.9 = 0$.

$-0.1\left(x^2 - 98 - 99\right) = 0$

$-0.1(x - 99)(x + 1) = 0$

$x = 99,\ x = -1$

The zeros are $x = 99, x = -1, x = 11$.

e. Based on the context of the problem, producing and selling 11 units or 99 units results in break-even.

35. $R(x) = 9000$

$1810x - 81x^2 - x^3 = 9000$

$x^3 + 81x^2 - 1810x + 9000 = 0$

Since $x = 9$ is a solution,

9)	1	81	−1810	9000
		9	810	−9000
	1	90	−1000	0

The quadratic factor of $R(x)$ is $x^2 + 90x - 1000$. To determine more solutions, solve $x^2 + 90x - 1000 = 0$.

$(x+100)(x-10) = 0$

$x = -100, x = 10$

Revenue of \$9000 is also achieved by selling 10 units.

36. $R(x) = 1000$

$250x - 5x^2 - x^3 = 1000$

$x^3 + 5x^2 - 250x + 1000 = 0$

Since $x = 5$ is a solution,

5)	1	5	−250	1000
		5	50	−1000
	1	10	−200	0

The quadratic factor of $R(x)$ is $x^2 + 10x - 200$. To determine more solutions, solve $x^2 + 10x - 200 = 0$.

$(x+20)(x-10) = 0$

$x = -20, x = 10$

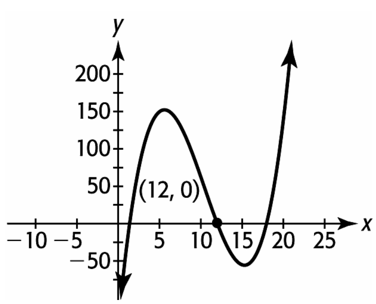
Revenue of \$1000 is also achieved by selling 10 units.

37. a. $y = 244$

$0.4566x^3 - 14.3085x^2 + 117.2978x + 107.8456 = 244$

$0.4566x^3 - 14.3085x^2 + 117.2978x - 136.1544 = 0$

b.



It appears that $x = 12$ is a solution.

c.

12)	0.4566	−14.3085	117.2978	−136.1544
		5.4792	−105.9516	136.1544
	0.4566	−8.8293	11.3462	0

The quadratic factor of $P(x)$ is $0.4566x^2 - 8.8293x + 11.3462$.

d. $0.4566x^2 - 8.8293x + 11.3462 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8.8293) \pm \sqrt{(-8.8293)^2 - 4(0.4566)(11.3462)}}{2(0.4566)}$$

$$x = \frac{8.8293 \pm \sqrt{57.23383881}}{0.9132}$$

$x = 17.953,\ x = 1.384$

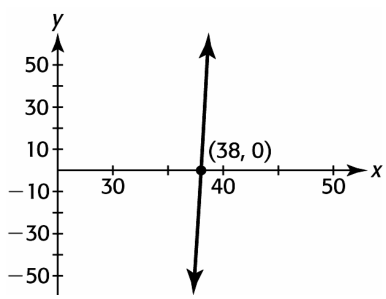
e. Based on the solutions in previous parts, the number of fatalities was 244 in 1982, 1992, and 1998.

38. a. $y = 2862$

$0.20x^3 - 13.71x^2 + 265.06x + 1612.56 = 2862$

$0.20x^3 - 13.71x^2 + 265.06x - 1249.44 = 0$

b.



It appears that $x = 38$ is a solution.

c.

38)	0.20	−13.71	265.06	−1249.44
		7.6	−232.18	1249.44
	0.20	−6.11	32.88	0

The quadratic factor of $P(x)$ is $0.2x^2 - 6.11x + 32.88$.

d. $0.2x^2 - 6.11x + 32.88 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

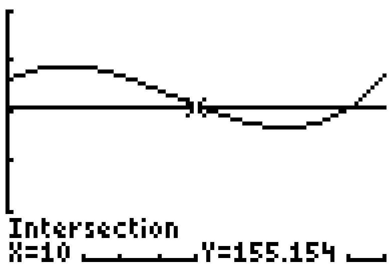
$$x = \frac{-(-6.11) \pm \sqrt{(-6.11)^2 - 4(0.2)(32.88)}}{2(0.2)}$$

$$x = \frac{6.11 \pm \sqrt{11.0281}}{0.4}$$

$$x = 23.58, x = 6.97$$

e. Based on the solutions in previous parts, college enrollment is 2862 thousand (2,862,000) in 1967, 1984, and 1998.

39. Applying the intersection of graphs method for $y = 0.0790x^3 - 2.103x^2 + 10.695x + 179.504$

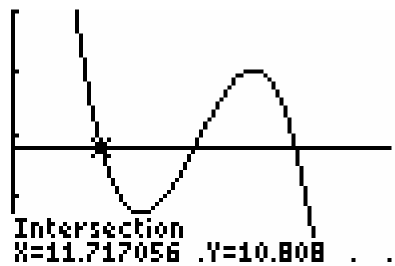


[0, 20] by [0, 250]

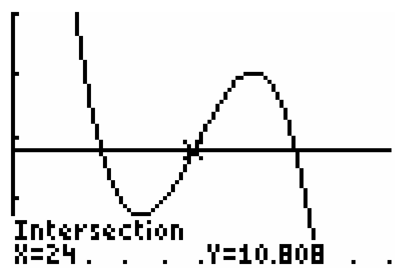
[0, 20] by [0, 250]

Based on the graphs, 155,154 births occurred in 2009, in addition to 2000.

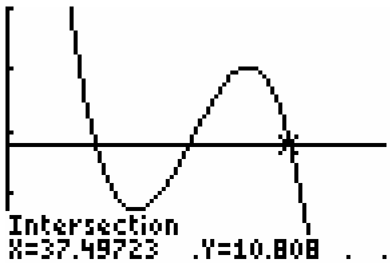
40. Applying the intersection of graphs method for $y = -0.0014x^3 + 0.1025x^2 - 2.2687x + 25.5704$



[0, 50] by [9, 13]



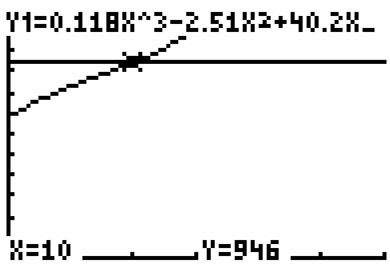
[0, 50] by [9, 13]



[0, 50] by [9, 13]

Based on the graphs, 10,808 births occurred in 1982 and 2008, in addition to 1994.

41. a.



[0, 30] by [0, 1200]

Ten years after 2000, in 2010, the U.S. per capita out-of-pocket cost for healthcare was $946.

b.

$y = 946$

$0.118x^3 - 2.51x^2 + 40.2x + 677 = 946$

$0.118x^3 - 2.51x^2 + 40.2x - 269 = 0$

Since $x = 10$ is a solution,

10)	0.118	−2.51	40.2	−269
		1.18	−13.3	269
	0.118	−1.33	26.9	0

The new polynomial is

$0.118x^2 - 1.33x + 26.9$.

To determine more solutions,

solve $0.118x^2 - 1.33x + 26.9 = 0$.

Applying the quadratic formula:

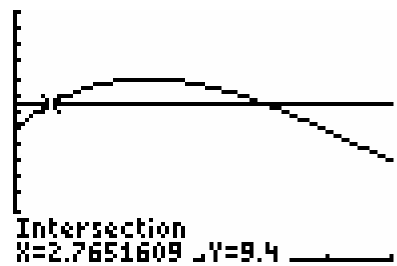
$$t = \frac{-(-1.33) \pm \sqrt{(-1.33)^2 - 4(0.118)(26.9)}}{2(0.118)}$$

$$t = \frac{1.33 \pm \sqrt{-10.9279}}{0.236}$$

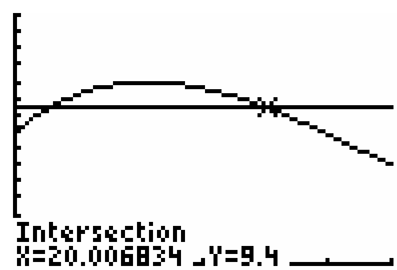
$t =$ a non-real solution

Thus, there are no other years when the U.S. per capita out-of-pocket cost is $946.

42. Applying the intersection of graphs method for $y = 0.000447x^3 - 0.0361x^2 + 0.615x + 7.966$



[0, 30] by [0, 15]



[0, 30] by [0, 15]

Thus, according to the model, since 1960, the Americans' personal savings rate was 9.4% in 1963 and in 1980.

Section 6.5 Skills Check

1. a. To find the vertical asymptote let
$x-5=0$
$x=5$ is the vertical asymptote.

b. The degree of the numerator is less than the degree of the denominator. Therefore, $y=0$ is the horizontal asymptote.

2. a. To find the vertical asymptote let
$x-4=0$
$x=4$ is the vertical asymptote.

b. The degree of the numerator is less than the degree of the denominator. Therefore, $y=0$ is the horizontal asymptote.

3. a. To find the vertical asymptote let
$5-2x=0$
$-2x=-5$
$x=\frac{-5}{-2}=\frac{5}{2}$ is the vertical asymptote.

b. The degree of the numerator is equal to the degree of the denominator.
Therefore, $y=\frac{1}{-2}=-\frac{1}{2}$ is the horizontal asymptote.

4. a. To find the vertical asymptote let
$3-x=0$
$-x=-3$
$x=3$ is the vertical asymptote.

b. The degree of the numerator is equal to the degree of the denominator.
Therefore, $y=\frac{2}{-1}=-2$ is the horizontal asymptote.

5. a. To find the vertical asymptote let
$x^2-1=0$
$(x+1)(x-1)=0$
$x=-1, x=1$ are the vertical asymptotes.

b. The degree of the numerator is greater than the degree of the denominator. Therefore, there is not a horizontal asymptote.

6. a. To find the vertical asymptote let
$x^2+3=0$
$x^2=-3$
$x=\pm\sqrt{-3}=\pm i\sqrt{3}$
Since there is not a real number solution to the equation, there is not a vertical asymptote.

b. The degree of the numerator is equal to the degree of the denominator.
Therefore, $y=\frac{1}{1}=1$ is the horizontal asymptote.

7. The function in part c) does not have a vertical asymptote. Its denominator cannot be zero. Parts a), b), and d) all have denominators that can equal zero for specific x-values.

8. The function in part c) does not have a horizontal asymptote. The degree of the numerator is greater than the degree of the denominator.

9. matches with graph E that has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 0$.

10. matches with graph B that has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 1$.

11. matches with graph F that has two vertical asymptotes at $x = 4$ and $x = -2$ (a result of factoring the function's denominator) , and a horizontal asymptote at $y = 0$.

12. matches with graph A that has two vertical asymptotes at $x = 2$ and $x = -2$ (a result of factoring the function's denominator), and a horizontal asymptote at $y = 0$.

13. matches with graph C that has a vertical asymptote at $x = 3$, but there is no horizontal asymptote since the degree of the numerator is greater than the degree of the denominator.

14. matches with graph D that has a vertical asymptote at $x = 4$, but there is no horizontal asymptote since the degree of the numerator is greater than the degree of the denominator.

15. a. The degree of the numerator is equal to the degree of the denominator.

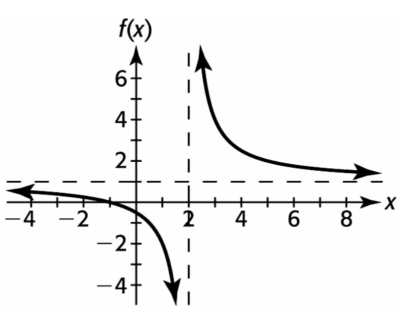
Therefore, $y = \frac{1}{1} = 1$ is the horizontal asymptote.

b. To find the vertical asymptote let

$x - 2 = 0$

$x = 2$ is the vertical asymptote.

c.



16. a. The degree of the numerator is equal to the degree of the denominator.

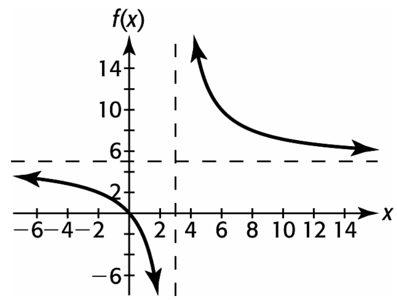
Therefore, $y = \frac{5}{1} = 5$ is the horizontal asymptote.

b. To find the vertical asymptote let

$x - 3 = 0$

$x = 3$ is the vertical asymptote.

c.



17. a. The degree of the numerator is less than the degree of the denominator. Therefore, $y = 0$ is the horizontal asymptote.

b. To find the vertical asymptote let

$1 - x^2 = 0$

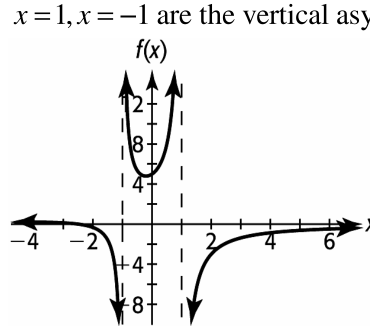
$x^2 = 1$

$\sqrt{x^2} = \pm\sqrt{1}$

$x = \pm 1$

$x = 1, x = -1$ are the vertical asymptotes.

c.



18. a. The degree of the numerator is greater than the degree of the denominator. Therefore, there is not a horizontal asymptote.

b. To find the vertical asymptote let

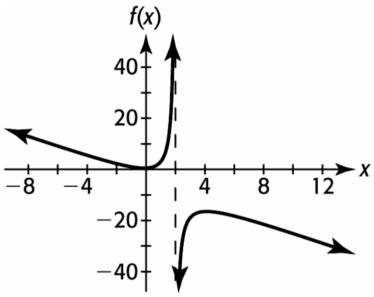
$2 - x = 0$

$-x = -2$

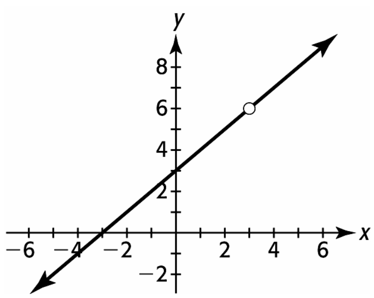
$x = 2$

$x = 2$ is the vertical asymptote.

c.



19. $y = \dfrac{x^2 - 9}{x - 3}$

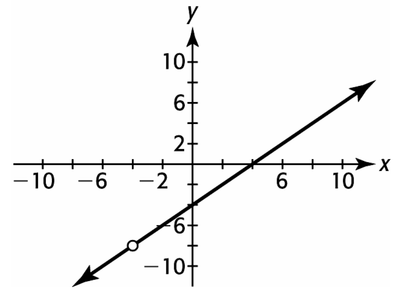


X	Y1
0	3
1	4
2	5
3	ERROR
4	7
5	8
6	9

X=3

There is a hole in the graph at $x = 3$.

20. $y = \dfrac{x^2 - 16}{x + 4}$

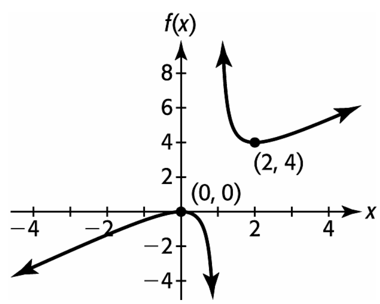


X	Y1
-6	-10
-5	-9
-4	ERROR
-3	-7
-2	-6
-1	-5
0	-4

X= -4

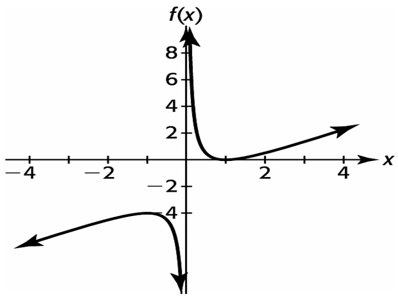
There is a hole in the graph at $x = -4$.

21. $f(x) = \dfrac{x^2}{x - 1}$



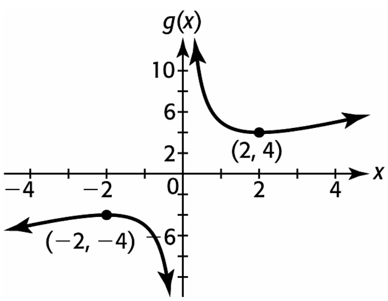
Based on the model, the turning points appear to be (0, 0) and (2, 4).

22. $f(x)=\dfrac{(x-1)^2}{x}$



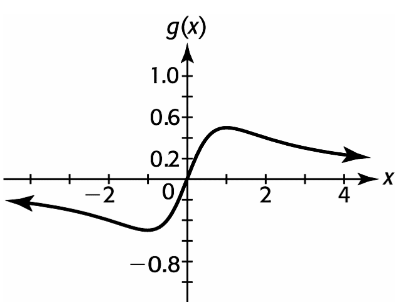
Based on the model, the turning points appear to be (–1, –4) and (1, 0).

23. $g(x)=\dfrac{x^2+4}{x}$



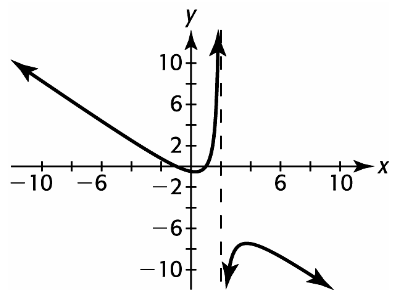
Based on the model, the turning points appear to be (–2, –4) and (2, 4).

24. $g(x)=\dfrac{x}{x^2+1}$



Based on the model, the turning points appear to be (–1, –0.5) and (1, 0.5).

25. a. $y=\dfrac{1-x^2}{x-2}$



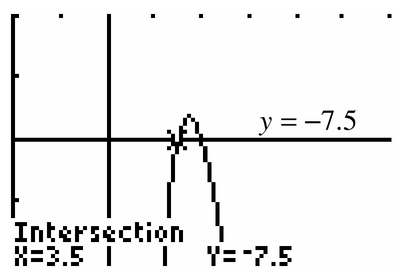
b.

X	Y1
-2	.75
-1	0
0	-.5
1	0
2	ERROR
3	-8
4	-7.5

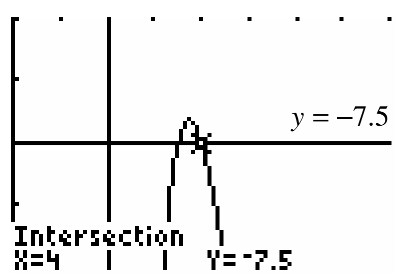
X=1

Based on the model and the table, when $x = 1$, $y = 0$ and when $x = 3$, $y = -8$.

c.



[0, 8] by [–7.7, –7.3]



[0, 8] by [–7.7, –7.3]

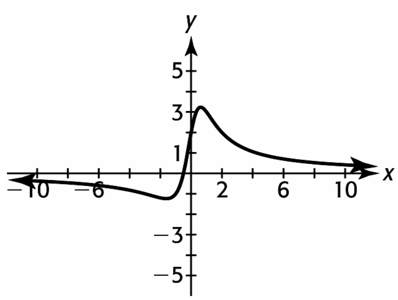
Based on the models, it appears that when $y=-7.5$, then $x = 3.5$ or $x = 4$.

d.

$$-7.5=\frac{1-x^2}{x-2} \qquad \text{LCD}: x-2$$
$$-7.5(x-2)=\left(\frac{1-x^2}{x-2}\right)(x-2)$$
$$-7.5x+15=1-x^2$$
$$x^2-7.5x+14=0$$
$$10(x^2-7.5x+14)=10(0)$$
$$10x^2-75x+140=0$$
$$(x-4)(10x-35)=0$$
$$x-4=0, \quad 10x-35=0$$
$$x=4, x=3.5$$

Both solutions check.

26. a. $y=\frac{2+4x}{x^2+1}$



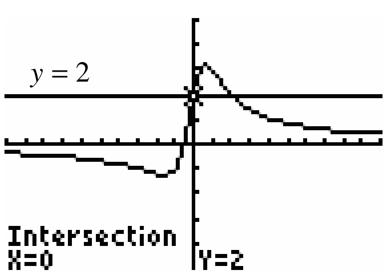
b.

X	Y1	
-3	-1	
-2	-1.2	
-1	-1	
0	2	
1	3	
2	2	
3	1.4	

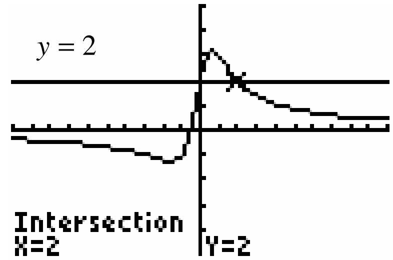
X=3

Based on the model and the table, when $x = -3$, $y = -1$, and when $x = 3$, $y = 1.4$.

c.



[–10, 10] by [–5, 5]



[–10, 10] by [–5, 5]

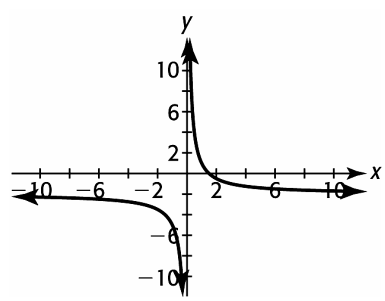
Based on the models, it appears that when $y=2$, then $x = 0$ or $x = 2$.

d.

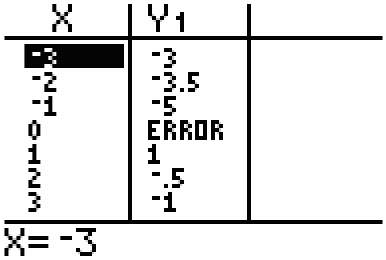
$$2=\frac{2+4x}{x^2+1} \qquad \text{LCD}: x^2+1$$
$$2(x^2+1)=\left(\frac{2+4x}{x^2+1}\right)(x^2+1)$$
$$2x^2+2=2+4x$$
$$2x^2-4x=0$$
$$2x(x-2)=0$$
$$x=0, \quad x=2$$

Both solutions check.

27. a. $y=\frac{3-2x}{x}$

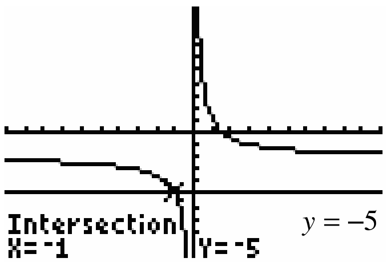


b.



Based on the model and the table, when $x = -3$, $y = -3$ and when $x = 3$, $y = -1$.

c.



[–10, 10] by [–10, 10]

Based on the graph, it appears that when $y = -5$, then $x = -1$.

d. $-5 = \frac{3-2x}{x}$ LCD: x

$$-5x = \left(\frac{3-2x}{x}\right)x$$

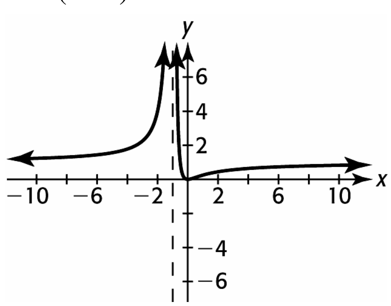
$$-5x = 3 - 2x$$

$$-3x = 3$$

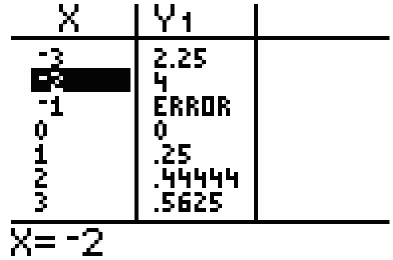
$$x = -1$$

The solution checks.

28. a. $y = \frac{x^2}{(x+1)^2}$

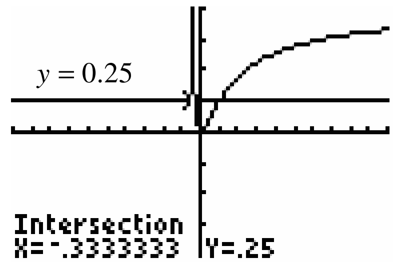


b.

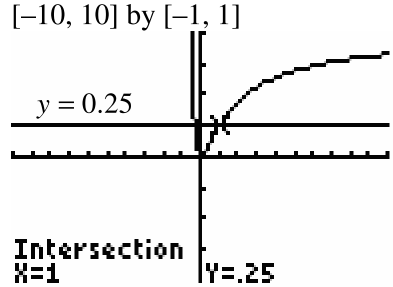


Based on the model and the table, when $x = -2$, $y = 4$ and when $x = 0$, $y = 0$.

c.



[–10, 10] by [–1, 1]



[–10, 10] by [–1, 1]

Based on the models, it appears that when $y = 0.25$, then $x = -\frac{1}{3}$ or $x = 1$.

d.

$$0.25=\frac{x^2}{(x+1)^2}$$

$$\frac{1}{4}=\frac{x^2}{(x+1)^2} \qquad \text{LCD}: 4(x+1)^2$$

$$\frac{1}{4}\left[4(x+1)^2\right]=\left[\frac{x^2}{(x+1)^2}\right]\left[4(x+1)^2\right]$$

$$(x+1)^2=4x^2$$

$$x^2+2x+1=4x^2$$

$$3x^2-2x-1=0$$

$$(3x+1)(x-1)=0$$

$$3x+1=0, \quad x-1=0$$

$$x=-\frac{1}{3}, \quad x=1$$

Both solutions check.

29.

$$\frac{x^2+1}{x-1}+x=2+\frac{2}{x-1} \qquad \text{LCD: } x-1$$

$$(x-1)\left(\frac{x^2+1}{x-1}+x\right)=(x-1)\left(2+\frac{2}{x-1}\right)$$

$$x^2+1+x(x-1)=2(x-1)+2$$

$$x^2+1+x^2-x=2x-2+2$$

$$2x^2-x+1=2x$$

$$2x^2-3x+1=0$$

$$(2x-1)(x-1)=0$$

$$x=\frac{1}{2}, \quad x=1$$

$x=1$ does not check since the denominator $x-1=0$.

The only solution is $x=\frac{1}{2}$.

30.

$$\frac{x}{x-2}-x=1+\frac{2}{x-2} \qquad \text{LCD: } x-2$$

$$(x-2)\left(\frac{x}{x-2}-x\right)=(x-2)\left(1+\frac{2}{x-2}\right)$$

$$x-x(x-2)=1(x-2)+2$$

$$x-x^2+2x=x-2+2$$

$$-x^2+3x=x$$

$$-x^2+2x=0$$

$$-x(x-2)=0$$

$$-x=0, \quad x-2=0$$

$$x=0, \quad x=2$$

$x=2$ does not check since the denominator $x-2=0$.

The only solution is $x=0$.

31. $y=\frac{k}{x^4}$ is the inverse function format.

$$5=\frac{k}{(-1)^4}$$

$$k=5$$

$$y=\frac{5}{(0.5)^4}=80$$

32. $S=\frac{k}{\sqrt{T}}$ is the inverse function format.

$$4=\frac{k}{\sqrt{4}}=\frac{k}{2}$$

$$k=8$$

$$S=\frac{8}{\sqrt{16}}=\frac{8}{4}=2$$

Section 6.5 Exercises

33. a. $\overline{C} = \dfrac{400 + 50(500) + 0.01(500)^2}{500}$

$\overline{C} = \dfrac{27{,}900}{500} = 55.8$

The average cost is \$55.80 per unit.

b. $\overline{C} = \dfrac{400 + 50(60) + 0.01(60)^2}{60}$

$\overline{C} = \dfrac{3436}{60} = 57.2\overline{6}$

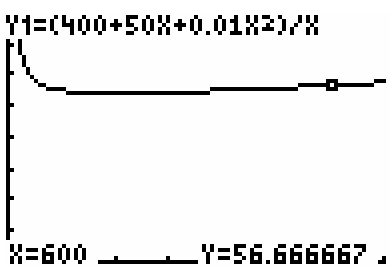
The average cost is \$57.27 per unit.

c. $\overline{C} = \dfrac{400 + 50(100) + 0.01(100)^2}{100}$

$\overline{C} = \dfrac{5500}{100} = 55$

The average cost is \$55 per unit.

d. No. Consider the graph of the function where x = 600 units. The average cost per unit is then \$56.67.



[0, 700] by [0, 80]

34. a. $\overline{C} = \dfrac{1000 + 30(30) + 0.1(30)^2}{30}$

$\overline{C} = \dfrac{1990}{30} = 66.\overline{3}$

The average cost is \$66.33 per unit.

b. $\overline{C} = \dfrac{1000 + 30(300) + 0.1(300)^2}{300}$

$\overline{C} = \dfrac{19000}{300} = 63.\overline{3}$

The average cost is \$63.33 per unit.

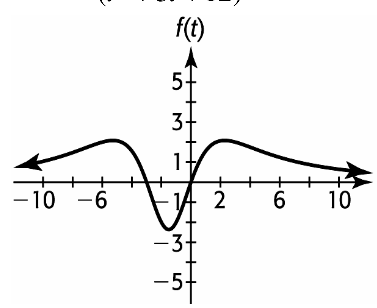
c. When $x = 0$, the function is undefined. If no units are produced, then there can be no average cost per unit.

35. a. $y = \dfrac{400(5)}{5 + 20} = \dfrac{2000}{25} = 80$

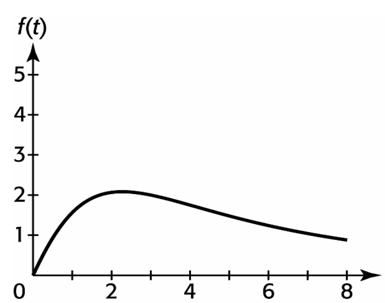
\$5000 in monthly advertising expenditures results in a monthly sales volume of \$80,000.

b. When $x = -20$, the denominator is zero and the function is undefined. Since advertising expenditures cannot be negative, x cannot be –20 in the context of the problem.

36. a. $f(t) = \dfrac{100(t^2 + 3t)}{(t^2 + 3t + 12)^2}$



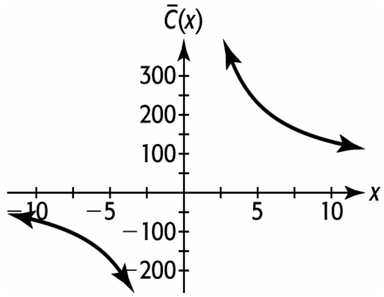
b.



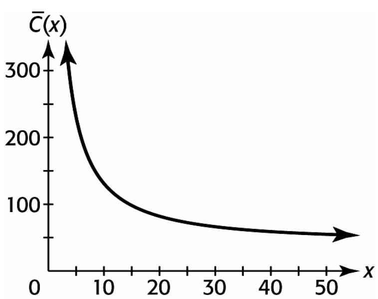
c. The graph in part b) is a better model of the physical situation. It displays the function over its domain of $0 \le t \le 8$.

d. Based on the graph, productivity is higher around lunch than at quitting time.

37. a. $\bar{C}(x) = \dfrac{1000 + 30x + 0.1x^2}{x}$

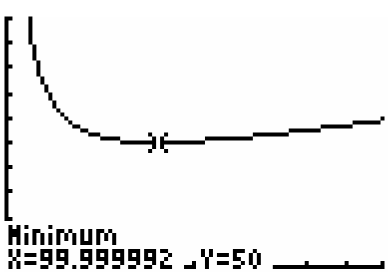


b.



c. The window in part b) fits the context of the problem where both x and $\bar{C}(x)$ are greater than zero.

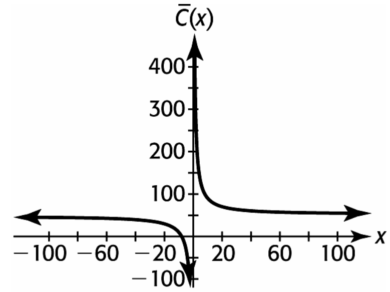
d.



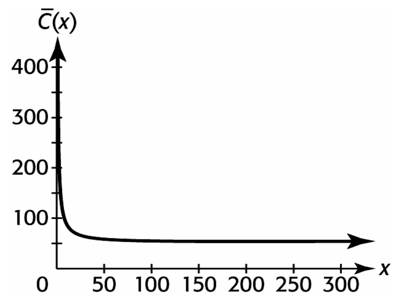
[0, 250] by [0, 300]

The minimum average cost of $50 occurs when $x = 100$. Since x is measured in hundreds of units produced, 10,000 units are produced.

38. a. $\bar{C}(x) = \dfrac{400 + 50x + 0.01x^2}{x}$

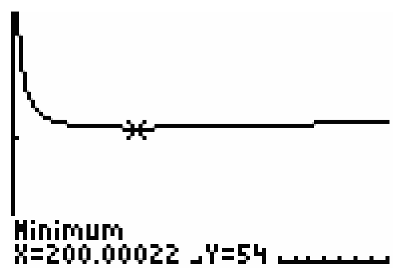


b.



c. The graph in part b) is more appropriate, since producing a negative number of units does not make sense in the context of the problem.

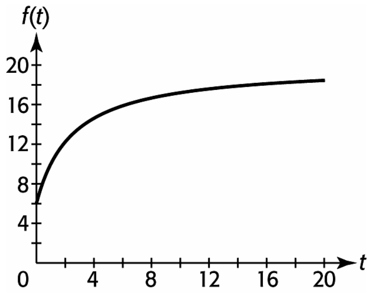
d.



[0, 600] by 0, 100]

The minimum average cost of $54 occurs when x = 200. Since x is measured in hundreds of units produced 20,000 units are produced.

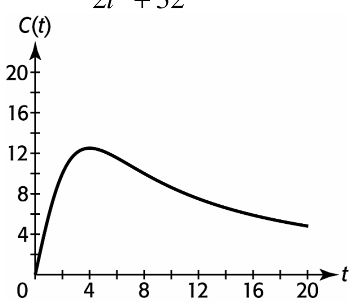
39. a. $f(t)=\dfrac{30+40t}{5+2t}$



b. $f(0)=6$. The initial number of employees for the startup company is 6.

c. $f(12)\approx 17.586$. After 12 months, the number of employees for the startup company is approximately 18.

40. a. $f(t)=\dfrac{200t}{2t^2+32}$



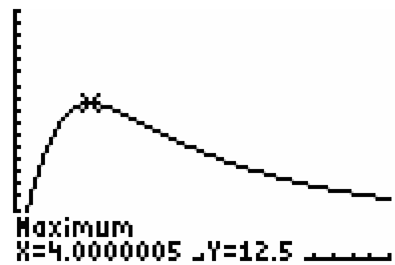
b.

X	Y1
0	0
1	5.8824
2	10
3	12
4	12.5
5	12.195
6	11.538

X=5

One hour after the injection, the drug concentration is 5.88%. Five hours after the injection, the drug concentration is 12.20%.

c.



[0, 20] by [0, 20]

The maximum drug concentration is 12.5% occurring 4 hours after the injection.

d. After four hours, the drug concentration begins to drop until it approaches a level of zero as time continues to pass.

41. a. To find the vertical asymptote let $100-p=0$.

$-p=-100$

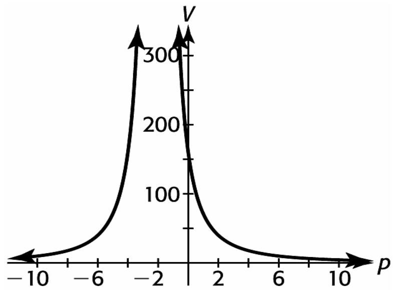
$p=100$ is the vertical asymptote.

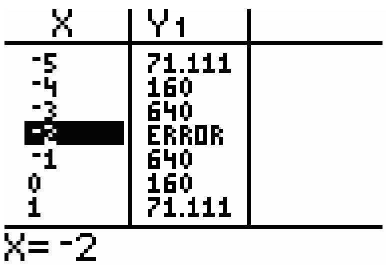
b. Since $p\neq 100$, 100% of the impurities can not be removed from the waste water.

42. a. If spending is allowed to increase without bound, the function will approach its horizontal asymptote. Since the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is $p=\dfrac{100}{1}=100$. Therefore as spending approaches infinity, the percentage of pollution removed approaches 100%.

b. No. 100% of the pollution cannot be removed. To do so would require spending an infinite amount of money.

43. a. $V = \dfrac{640}{(p+2)^2}$





X	Y1
-5	71.111
-4	160
-3	640
-2	ERROR
-1	640
0	160
1	71.111

X= -2

Based on the graph and the table, the vertical asymptote occurs at $p = -2$.

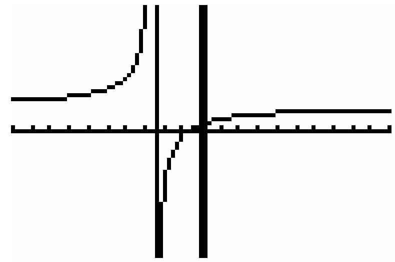
b.

Price per Unit($)	Weekly Sales Volume (in 1000's)
5	13,061
20	1322
50	237
100	62
200	16
500	3

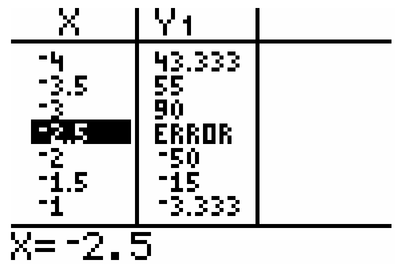
c. The domain of the function in the context of the problem is $p \geq 0$. There is no vertical asymptote on the restricted domain.

d. The horizontal asymptote is $V = 0$. As the price grows without bound, the sales volume approaches zero units.

44. a. $N = \dfrac{30+40t}{5+2t}$



[–10, 10] by [–100, 100]



X	Y1
-4	43.333
-3.5	55
-3	90
-2.5	ERROR
-2	-50
-1.5	-15
-1	-3.333

X= -2.5

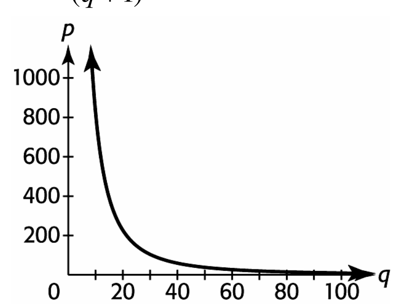
There is a vertical asymptote at $t = -2.5$.

b. No. The only vertical asymptote is at $t = -2.5$.

c. Yes. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $N = \dfrac{40}{2} = 20.$

d. As the number of months grows without bound, the number of employees approaches 20.

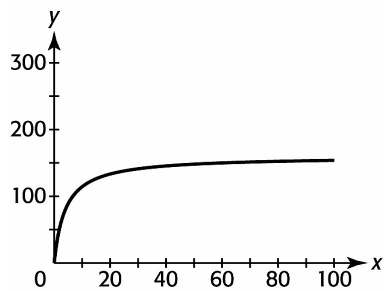
45. a. $p = \dfrac{100{,}000}{(q+1)^2}$



b. The horizontal asymptote is $p = 0$.

c. As the price falls, the quantity demanded increases.

46. a. $y = \dfrac{800x}{20 + 5x}$



b. Since the degree of the numerator equals the degree of the denominator the horizontal asymptote is $y = \dfrac{800}{5} = 160$.

c.

Advertising expenses	Weekly sales
0	0
50	14,814.81
100	15,384.62
200	15,686.27
300	15,789.47
500	15,873.02

d. If an unlimited amount of money is spent on advertising, then the maximum weekly sales will approach \$16,000, represented by the horizontal asymptote of the function.

47. a.

$$S = \frac{40}{x} + \frac{x}{4} + 10 \qquad \text{LCD: } 4x$$

$$\left(\frac{40}{x}\right)\left(\frac{4}{4}\right) + \frac{x}{4}\left(\frac{x}{x}\right) + \left(\frac{10}{1}\right)\left(\frac{4x}{4x}\right)$$

$$\frac{160}{4x} + \frac{x^2}{4x} + \frac{40x}{4x}$$

$$\frac{x^2 + 40x + 160}{4x}$$

$$S = \frac{x^2 + 40x + 160}{4x}$$

b.

$$21 = \frac{x^2 + 40x + 160}{4x}$$

$$21(4x) = \left(\frac{x^2 + 40x + 160}{4x}\right)(4x)$$

$$84x = x^2 + 40x + 160$$

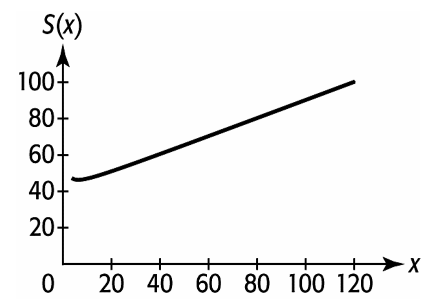
$$x^2 - 44x + 160 = 0$$

$$(x - 40)(x - 4) = 0$$

$$x = 40, \quad x = 4$$

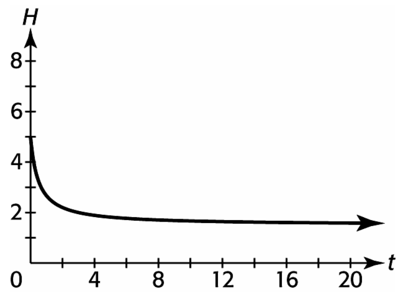
After 4 hours or 40 hours of training, the monthly sales will be \$21,000.

48. a. $S(x) = \dfrac{20}{x} + 40 + \dfrac{x}{2}$



b. Based on the model, 20 hours of training corresponds to sales of \$51,000.

49. a. $H = \dfrac{5+3t}{2t+1}$



b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $H = \dfrac{3}{2}$. As the amount of training increases, the time it takes to assemble one unit approaches 1.5 hours.

c.

X	Y1
14.5	1.6167
15	1.6129
15.5	1.6094
16	1.6061
16.5	1.6029
17	1.6
17.5	1.5972

X=17

It takes 17 days of training to reduce the time it takes to assemble one unit to 1.6 hours.

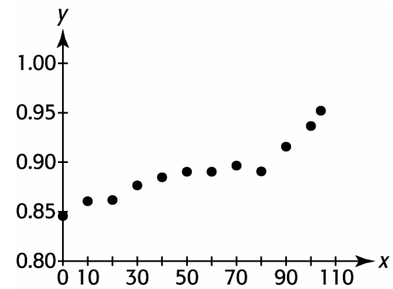
50.

X	Y1
26	138.67
26.5	139.02
27	139.35
27.5	139.68
28	140
28.5	140.31
29	140.61

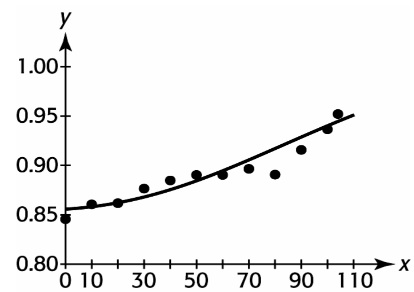
X=28

Advertising expenses of \$2800 produces weekly sales of \$14,000.

51. a.



b.



c. The fit of the scatter plot to the function appears to be good.

52. a. Let $A =$ Area.

$A = x \bullet y$

$x \bullet y = 51{,}200$

b. Let $L =$ Fence Perimeter.

$L = x + 2y$

c. $L = x + 2y$

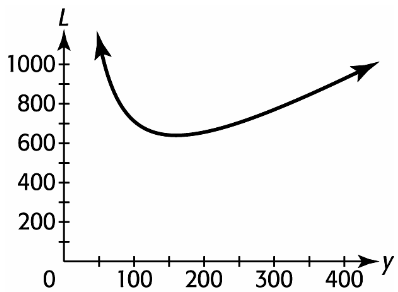
$A = x \bullet y = 51{,}200$

$x = \dfrac{51{,}200}{y}$

Substituting for x in the formula for L:

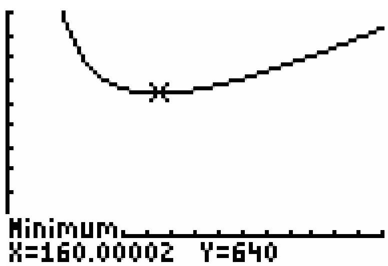
$L = \dfrac{51{,}200}{y} + 2y$

d.



Note that in this case, the x-axis represents the variable y, and the y-axis represents the variable L.

e.



[0, 400] by [0, 1000]

Note that in this case, the x-axis represents the variable y, and the y-axis represents the variable L.

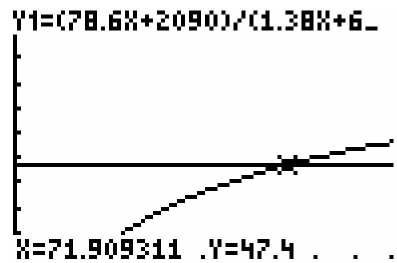
The minimum length of fence occurs when $y = 160$. Therefore, the total length of the fence is $L = \frac{51,200}{160} + 2(160) = 640$ feet. The dimensions of the rectangular field are $y = 160$ feet and $x = \frac{51,200}{160} = 320$ feet.

53. a. $p(t) = \frac{78.6t + 2090}{1.38t + 64.1}$ with t equal to the number of years after 1950.

For x = 80 (2030 – 1950), $p(80)$ = 48.0%. Yes, this agrees with the given data for the year 2030.

b. The maximum possible percent of women in the workforce, according to the model, would occur at the horizontal asymptote. Since the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is at $y = \frac{78.6}{1.38} = 56.957 \approx 57\%$.

c. The percent will reach 47.4% when x = 71.9, in the year 2022 (1950 + 72).



[0, 100] by [40, 60]

54. $f(x) = \frac{289.173 - 58.5731x}{x+1}$, with $x \geq 5$.

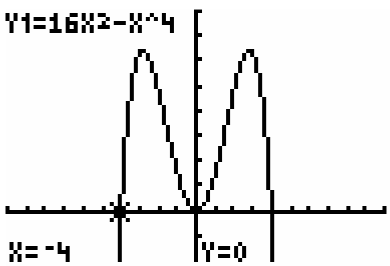
a. Yes, the vertical asymptote would occur when $x + 1 = 0$ or at $x = -1$.

b. No, since the domain is $x \geq 5$, there is no vertical asymptote to consider.

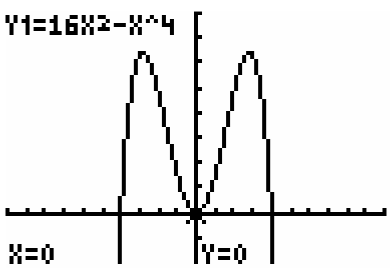
c. Yes, since the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is at $y = \frac{-58.5731}{1} = -58.5731$.

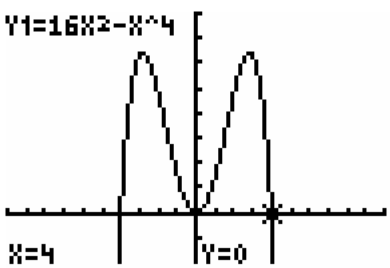
Section 6.6 Skills Check

1. $16x^2 - x^4 \ge 0$
Applying the x-intercept method:



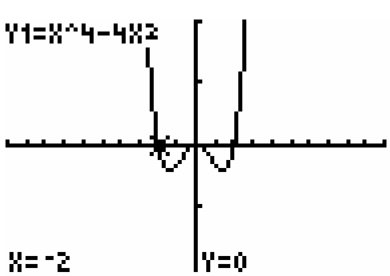
[–10, 10] by [–20, 80]



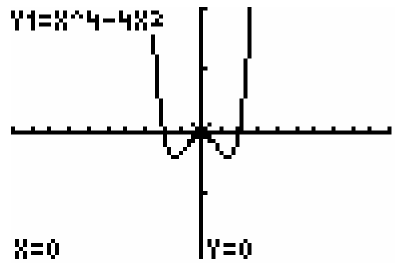


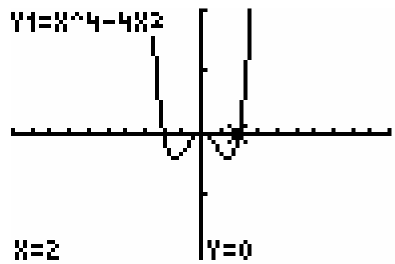
The function is greater than or equal to zero on the interval $\left[-4,4\right]$ or when $-4 \le x \le 4$.

2. $x^4 - 4x^2 \le 0$
Applying the x-intercept method:



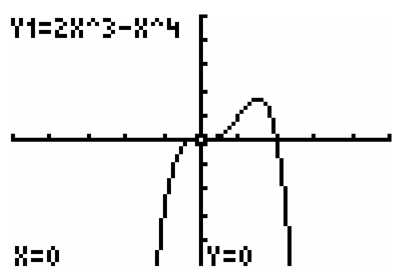
[–10, 10] by [–20, 20]



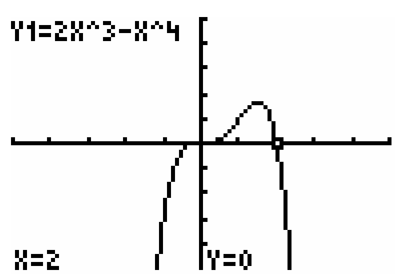


The function is less than or equal to zero on the interval $\left[-2,2\right]$ or when $-2 \le x \le 2$.

3. $2x^3 - x^4 < 0$
Applying the x-intercept method:



[–5, 5] by [–5, 5]

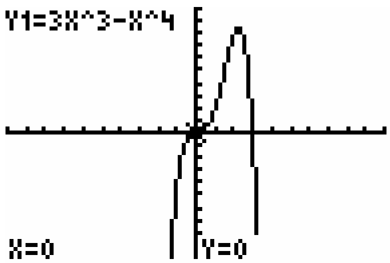


The function is less than zero on the interval $\left(-\infty,0\right)\cup\left(2,\infty\right)$ or when $x < 0$ or $x > 2$.

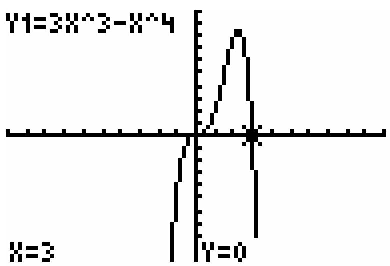
4. $3x^3 \ge x^4$

$3x^3 - x^4 \ge 0$

Applying the x-intercept method:



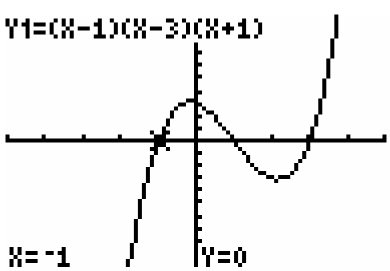
[–10, 10] by [–10, 10]



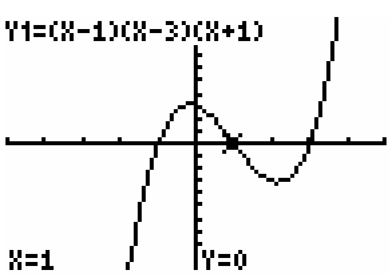
The function is greater than or equal to zero on the interval $[0,3]$ or when $0 \le x \le 3$.

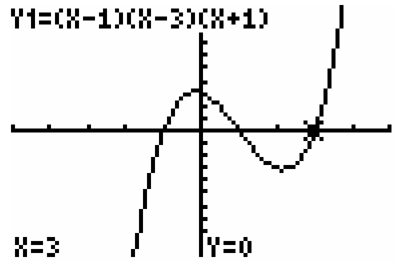
5. $(x-1)(x-3)(x+1) \ge 0$

Applying the x-intercept method:



[–5, 5] by [–10, 10]

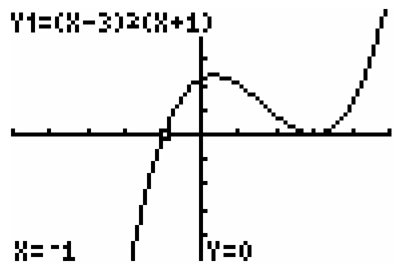




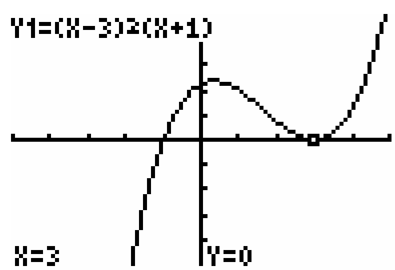
The function is greater than or equal to zero on the interval $[-1,1] \cup [3,\infty)$ or when $-1 \le x \le 1$ or $x \ge 3$.

6. $(x-3)^2(x+1) < 0$

Applying the x-intercept method:



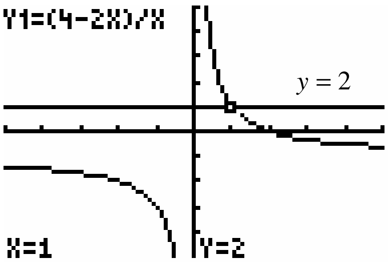
[–5, 5] by [–20, 20]



The function is less than zero on the interval $(-\infty,-1)$ or when $x < -1$.

7. $\dfrac{4-2x}{x} > 2$

Applying the intersection of graphs method:

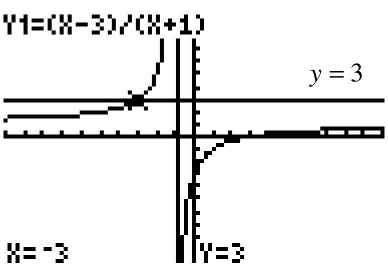


[–5, 5] by [–10, 10]

Note that the graphs intersect when $x = 1$. Also note that a vertical asymptote occurs at $x = 0$. Therefore, $\dfrac{4-2x}{x} > 2$ on the interval $(0,1)$ or when $0 < x < 1$.

8. $\dfrac{x-3}{x+1} \geq 3$

Applying the intersection of graphs method:

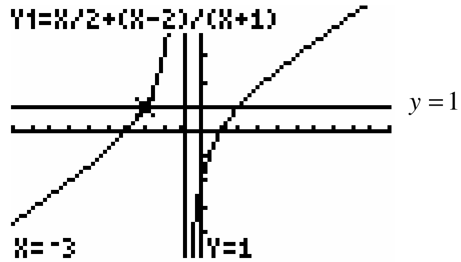


[–10, 10] by [–10, 10]

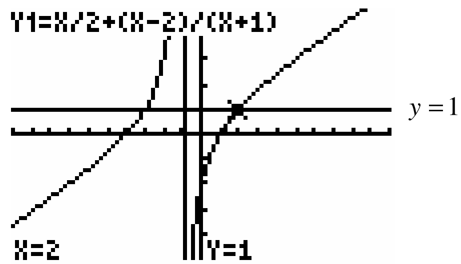
Note that the graphs intersect when $x = -3$. Also note that a vertical asymptote occurs at $x+1=0$ or $x=-1$. Therefore, $\dfrac{x-3}{x+1} \geq 3$ on the interval $[-3,-1)$ or when $-3 \leq x < -1$.

9. $\dfrac{x}{2} + \dfrac{x-2}{x+1} \leq 1$

Applying the intersection of graphs method:



[–10, 10] by [–5, 5]



Note that the graphs intersect when $x = -3$ and $x = 2$. Also note that a vertical asymptote occurs at $x+1=0$ or $x=-1$. Therefore, $\dfrac{x}{2} + \dfrac{x-2}{x+1} \leq 1$ on the interval $(-\infty,-3] \cup (-1,2]$ or when $x \leq -3$ or $-1 < x \leq 2$.

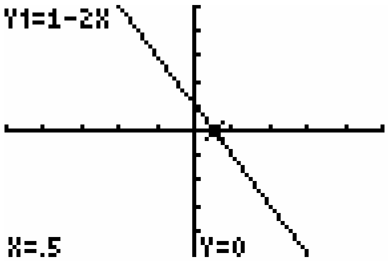
10. $\dfrac{x}{x-1} \leq 2x + \dfrac{1}{x-1}$

$\dfrac{x}{x-1} - 2x - \dfrac{1}{x-1} \leq 0$

$\dfrac{x-1}{x-1} - 2x \leq 0$

$1 - 2x \leq 0, \quad x \neq 1$

Applying the *x*-intercept method after simplifying the original inequality:



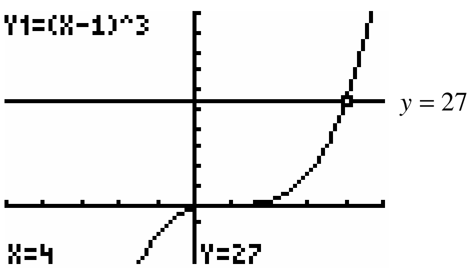
[–5, 5] by [–5, 5]

The inequality is true for all x in the domain of the function such that x is in the interval $\left[\frac{1}{2},\infty\right)$ or $x \geq \frac{1}{2}$. Recall that $x \neq 1$.

Therefore, the solution is $\left[\frac{1}{2},1\right)\cup(1,\infty)$.

11. $(x-1)^3 > 27$

Applying the intersection of graphs method:

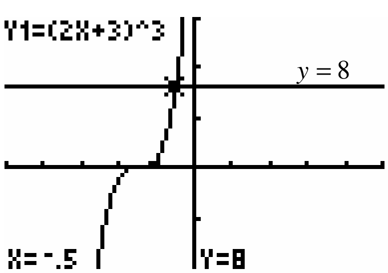


[–5, 5] by [–15, 50]

Note that the graphs intersect when $x = 4$. Therefore $(x-1)^3 > 27$ on the interval $(4,\infty)$ or when $x > 4$.

12. $(2x+3)^3 \leq 8$

Applying the intersection of graphs method:



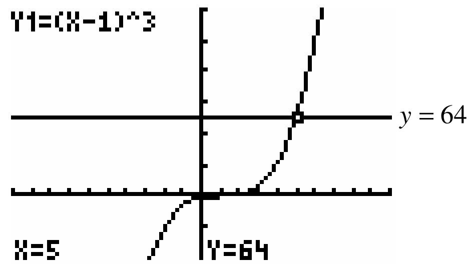
[–5, 5] by [–10, 15]

Note that the graphs intersect when $x = -\frac{1}{2}$.

Therefore, $(2x+3)^3 \leq 8$ on the interval $\left(-\infty,-\frac{1}{2}\right]$ or when $x \leq -\frac{1}{2}$.

13. $(x-1)^3 < 64$

Applying the intersection of graphs method:



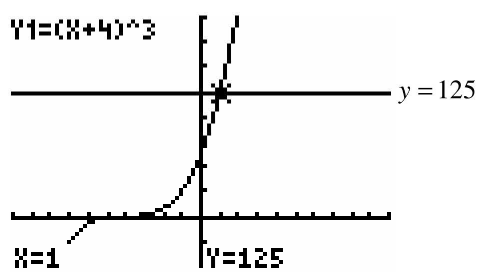
[–10, 10] by [–50, 150]

Note that the graphs intersect when $x = 5$. Therefore, $(x-1)^3 < 64$ on the interval $(-\infty,5)$ or when $x < 5$.

14. $(x+4)^3 - 125 \geq 0$

$(x+4)^3 \geq 125$

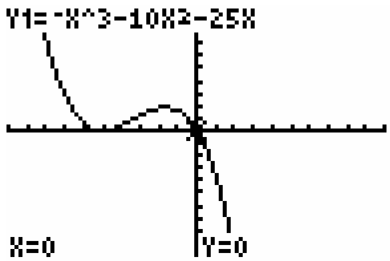
Applying the intersection of graphs method:



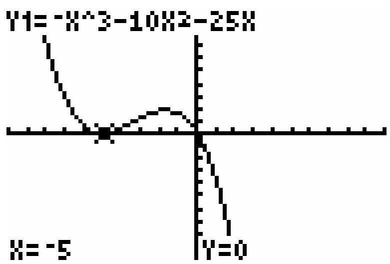
[–10, 10] by [–50, 200]

Note that the graphs intersect when $x = 1$. Therefore, $(x+4)^3 - 125 \geq 0$ on the interval $[1,\infty)$ or when $x \geq 1$.

15. $-x^3 - 10x^2 - 25x \le 0$
Applying the x-intercept method:

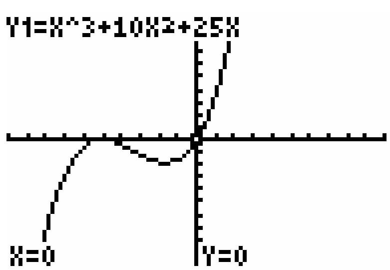


[–10, 10] by [–100, 100]

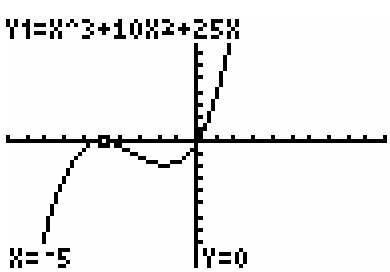


Note that the x-intercepts are $x = -5$ and $x = 0$. Therefore, $-x^3 - 10x^2 - 25x \le 0$ on the interval $\{-5\} \cup [0, \infty)$ or when $x = -5$ or $x \ge 0$.

16. $x^3 + 10x^2 + 25x < 0$
Applying the x-intercept method:



[–10, 10] by [–100, 100]



Note that the x-intercepts are $x = 0$ and $x = -5$. Therefore, $x^3 + 10x^2 + 25x < 0$ on the interval $(-\infty, -5) \cup (-5, 0)$ or when $x < 0$ and $x \ne -5$.

17. a. $f(x) < 0 \Rightarrow x < -3$ or $0 < x < 2$

b. $f(x) \ge 0 \Rightarrow -3 \le x \le 0$ or $x \ge 2$

18. a. $f(x) < 0 \Rightarrow x < 3$

b. $f(x) \ge 0 \Rightarrow x \ge 3$

19. a. $f(x) \ge 2 \Rightarrow \frac{1}{2} \le x \le 3$

20. a. $f(x) < 0 \Rightarrow 1 < x < 3$

b. $f(x) \ge 0 \Rightarrow x < 1$ or $x \ge 3$

Section 6.6 Exercises

21.

$R = 400x - x^3$

$= x(400 - x^2)$

$= x(20 - x)(20 + x)$

To find the zeros, let $R = 0$ and solve for x.

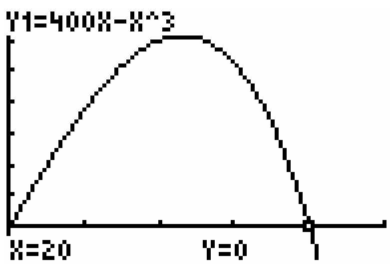
$R = 0$

$x(20 - x)(20 + x) = 0$

$x = 0,\ 20 - x = 0,\ 20 + x = 0$

$x = 0,\ x = 20,\ x = -20$

Note that since x represents product sales, only positive values of x make sense in the context of the problem.



[0, 25] by [–500, 3500]

Based on the graph and the zeros calculated above, the revenue is positive, $R > 0$, in the interval $(0, 20)$ or when $0 < x < 20$. Selling between 0 and 20 units, not inclusive, generates positive revenue.

22. a. Revenue

$= (\text{Price per unit})(\text{Number of units})$

$R(x) = (1000 - 0.1x^2)(x)$

$= 1000x - 0.1x^3$

b.

To find the zeros, set the revenue function equal zero and solve for x.

$1000x - 0.1x^3 = 0$

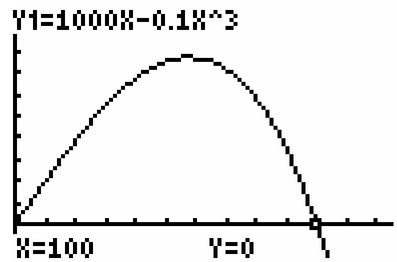
$-0.1x(10{,}000 - x^2) = 0$

$-0.1x(100 - x)(100 + x) = 0$

$-0.1x = 0,\ 100 - x = 0,\ 100 + x = 0$

$x = 0,\ x = 100,\ x = -100$

Note that since x represents product sales, only positive values of x make sense in the context of the problem.



[0, 125] by [–7500, 50,000]

Based on the graph and the zeros calculated above, the revenue is positive in the interval $(0,100)$ or when $0 < x < 100$. Selling between 0 and 100 units, not inclusive, generates positive revenue.

23. a. $V > 0$

$1296x - 144x^2 + 4x^3 > 0$

Find the zeros:

$1296x - 144x^2 + 4x^3 = 0$

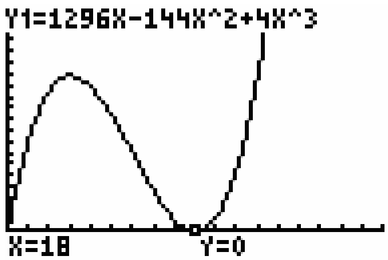
$4x^3 - 144x^2 + 1296x = 0$

$4x(x^2 - 36x + 324) = 0$

$4x(x - 18)(x - 18) = 0$

$4x = 0,\ x - 18 = 0,\ x - 18 = 0$

$x = 0,\ x = 18,\ x = 18$



[0, 36] by [–500, 5000]

Based on the graph and the zeros calculated above, the volume is positive in the interval $(0,18)\cup(18,\infty)$ or when $0<x<18$ or $x>18$.

b. In the context of the problem, the largest possible cut is 18 centimeters. Therefore, to generate a positive volume, the size of the cut, *x*, must be in the interval $(0,18)$ or $0<x<18$.

24. $V>0$

$192x-56x^2+4x^3>0$

Find the zeros:

$192x-56x^2+4x^3=0$

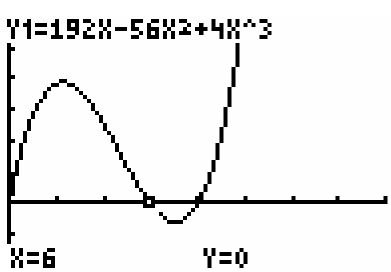
$4x^3-56x^2+192x=0$

$4x\left(x^2-14x+48\right)=0$

$4x(x-6)(x-8)=0$

$4x=0,\quad x-6=0,\quad x-8=0$

$x=0,\quad x=6,\quad x=8$



[0, 16] by [–100, 300]

Based on the graph and the zeros calculated above, the volume is positive in the interval $(0,6)\cup(8,\infty)$ or when $0<x<6$ or $x>8$.
In the physical context of the problem, the largest possible cut is 6 inches. A cut of larger than 6 inches would not be possible on the side of the cardboard measuring 12 inches. Therefore, to generate a positive volume, the size of the cut, *x*, must be in the interval $(0,6)$ or $0<x<6$.

25. $C(x)\ge 1200$

$3x^3-6x^2-300x+1800\ge 1200$

$3x^3-6x^2-300x+600\ge 0$

Find the zeros:

$3x^3-6x^2-300x+600=0$

$3\left(x^3-2x^2-100x+200\right)=0$

$3\left[\left(x^3-2x^2\right)+(-100x+200)\right]=0$

$3\left[x^2(x-2)+(-100)(x-2)\right]=0$

$3(x-2)\left(x^2-100\right)=0$

$3(x-2)(x+10)(x-10)=0$

$x-2=0,\quad x+10=0,\quad x-10=0$

$x=2,\quad x=-10,\quad x=10$

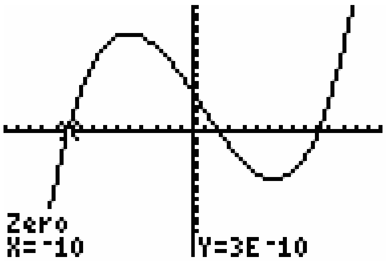
Sign chart:

Function	---	+++	---	+++
3	+++	+++	+++	+++
$(x-10)$	---	---	---	+++
$(x+10)$	---	+++	+++	+++
$(x-2)$	---	---	+++	+++
	–10	2	10	

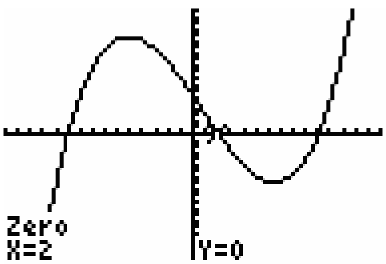
Based on the sign chart, the function is greater than zero on the intervals $(-10,2)$ and $(10,\infty)$. Considering the context of the problem, the number of units cannot be negative. The endpoints of the interval would be part of the solution because the question uses the phrase "at least." Therefore, total cost is at least $120,000 if

$0 \le x \le 2$ or if $x \ge 10$. In interval notation the solution is $[0,2] \cup [10,\infty]$.

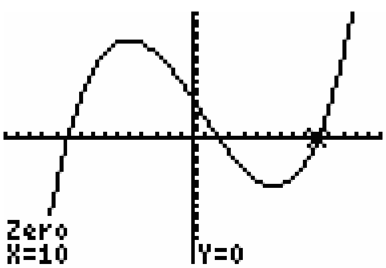
Applying the *x*-intercept method:



[–15, 15] by [–2000, 2000]



[–15, 15] by [–2000, 2000]



[–15, 15] by [–2000, 2000]

In context of the problem, the number of units must be greater than or equal to zero. Therefore, based on the graphs, the total cost is greater than or equal to $120,000 on the intervals $[0,2] \cup [10,\infty]$ or when $0 \le x \le 2$ or $x \ge 10$.

26. $P(x) \ge 400$

$-x^3 + 2x^2 + 400x - 400 \ge 400$

$-x^3 + 2x^2 + 400x - 800 \ge 0$

Find the zeros:

$-x^3 + 2x^2 + 400x - 800 = 0$

$-1\left(x^3 - 2x^2 - 400x + 800\right) = 0$

$-1\left[\left(x^3 - 2x^2\right) + \left(-400x + 800\right)\right] = 0$

$-1\left[x^2\left(x-2\right) + \left(-400\right)\left(x-2\right)\right] = 0$

$-1\left(x-2\right)\left(x^2 - 400\right) = 0$

$-1\left(x-2\right)\left(x+20\right)\left(x-20\right) = 0$

$x - 2 = 0, \quad x + 20 = 0, \quad x - 20 = 0$

$x = 2, \quad x = -20, \quad x = 20$

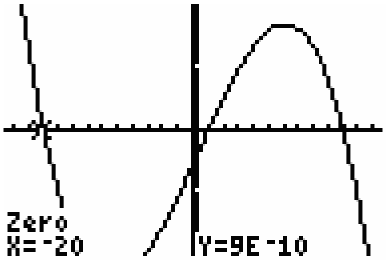
Sign chart:

Function	+++	---	+++	---
–1	---	---	---	---
$(x-20)$	---	---	---	+++
$(x+20)$	---	+++	+++	+++
$(x-2)$	---	---	+++	+++

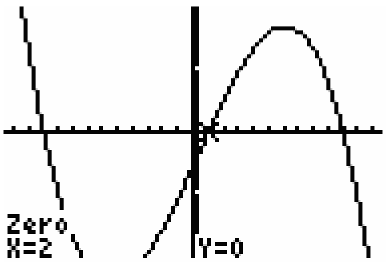
–20 2 20

Based on the sign chart, the function is greater than zero on the intervals $(-\infty,-20)$ and $(2,20)$. Considering the context of the problem, the number of units cannot be negative. The endpoints of the interval would be part of the solution because the question uses the phrase "at least." Therefore, total cost is at least $40,000 if $2 \le x \le 20$. In interval notation the solution is $[2,20]$.

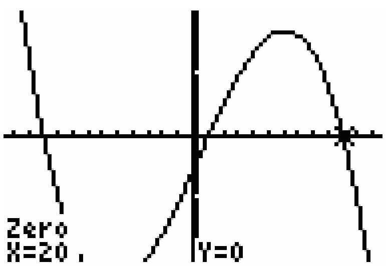
Applying the x-intercept method:



[–25, 25] by [–3000, 3000]



[–25, 25] by [–3000, 3000]

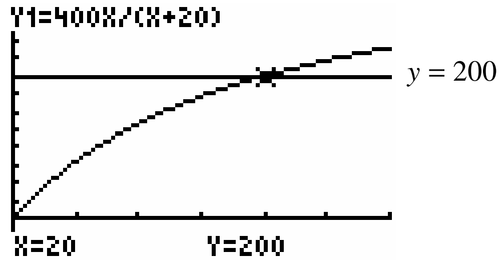


[–25, 25] by [–3000, 3000]

In context of the problem, the number of units must be greater than or equal to zero. Therefore, based on the graphs, the total cost is greater than or equal to \$40,000 on the interval $[2, 20]$ or when $2 \le x \le 20$.

27. $y = \dfrac{400x}{x+20}$

Applying the intersection of graphs method:



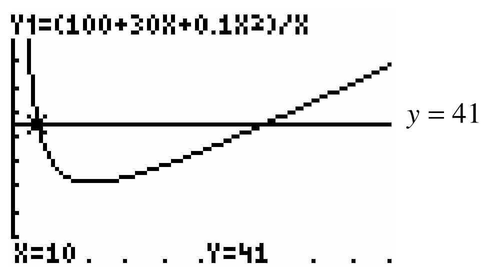
[0, 30] by [–50, 300]

Note that the graphs intersect when $x = 20$. Therefore, $\dfrac{400x}{x+20} \ge 200$ on the interval $[20, \infty)$ or when $x \ge 20$.

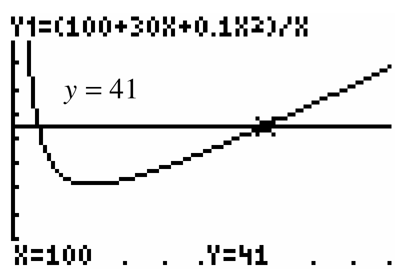
Therefore, since x is in thousands of dollars, spending \$20,000 or more on advertising results in sales of at least \$200,000.

28. $\overline{C} = \dfrac{100 + 30x + 0.1x^2}{x}$

Applying the intersection of graphs method:



[0, 150] by [30, 50]



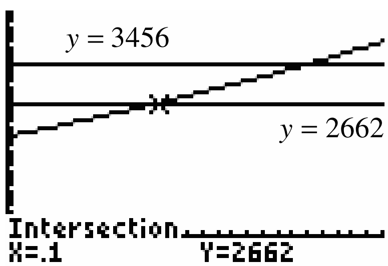
Note that the graphs intersect when $x = 10$ or $x = 100$. Therefore,

$$\overline{C} = \frac{100 + 30x + 0.1x^2}{x} \leq 41$$ on the interval $[10,100]$ or when $10 \leq x \leq 100$.

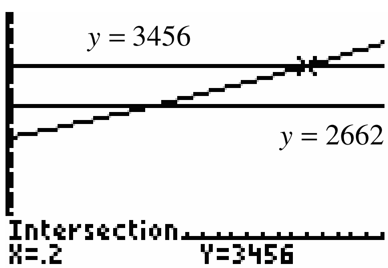
Therefore, since x is in hundreds of units, producing between 1000 units and 10,000 units, inclusive, generates an average cost of at most $41 per unit.

29. $S = 2000(1+r)^3$

Applying the intersection of graphs method



[0, 0.25] by [–500, 4500]



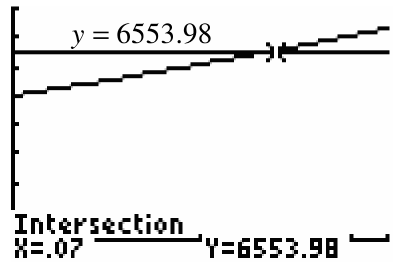
Note that the graphs intersect when $r = 0.10$ or $r = 0.20$. Therefore,

$2662 \leq 2000(1+r)^3 \leq 3456$ on the interval $[0.10, 0.20]$ or when $0.10 \leq r \leq 0.20$.

Therefore, interest rates between 10% and 20%, inclusive, generate future values between $2662 and $3456, inclusive.

30. $S = 5000(1+r)^4$

Applying the intersection of graphs method:



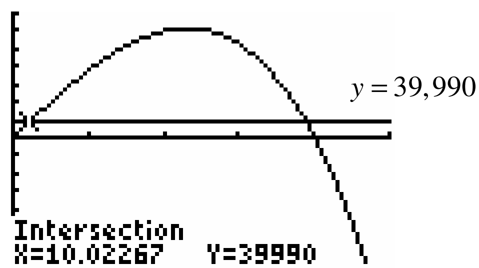
[0, 0.10] by [–500, 8000]

Note that the graphs intersect when $r = 0.07$. Therefore, $5000(1+r)^4 \geq 6553.98$ on the interval $[0.07, \infty)$ or when $r \geq 0.07$.

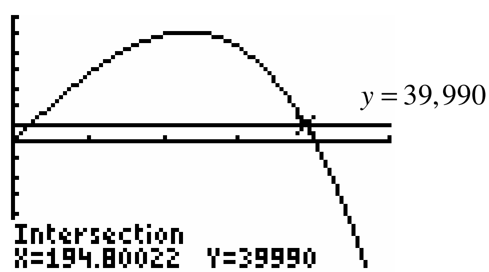
Therefore, interest rates greater than or equal to 7% generate future values of at least $6553.98.

31. $R = 4000x - 0.1x^3$

Applying the intersection of graphs method:



[0, 250] by [–350,000, 350,000]



Note that the graphs intersect when $x \approx 10.02$ and when $x \approx 194.80$.

Therefore, $R = 4000x - 0.1x^3 \ge 39{,}990$ on the interval $[10.02, 194.80]$ or when $10.02 \le x \le 194.80$.

Therefore, producing and selling between 10 units and 194 units, inclusive, generates a revenue of at least \$39,990.

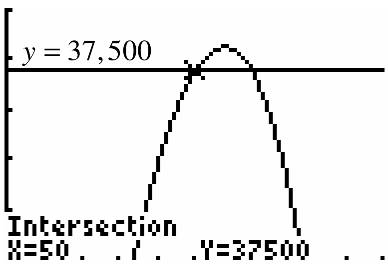
32. a. Revenue =

$(\text{Price per unit})(\text{Number of units})$

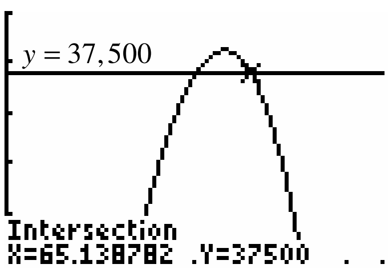
$R(x) = (1000 - 0.1x^2)(x)$

$R(x) = 1000x - 0.1x^3$

b. Applying the intersection of graphs method:



[0, 100] by [30,000, 40,000]



Note that the graphs intersect when $x = 50$ and when $x \approx 65.139$. Therefore, $R(x) = 1000x - 0.1x^3 \le 37{,}500$ on the interval $[0, 50]$ or $[65.139, \infty)$ or when $0 \le x \le 50$ or $x \ge 65.139$.

Therefore, producing and selling between 0 units and 50 units, inclusive, or more than 65 units generates a revenue of at most \$37,500.

33.

Considering the supply function and solving for q:

$6p - q = 180$

$-q = 180 - 6p$

$q = 6p - 180$

Considering the demand function and solving for q:

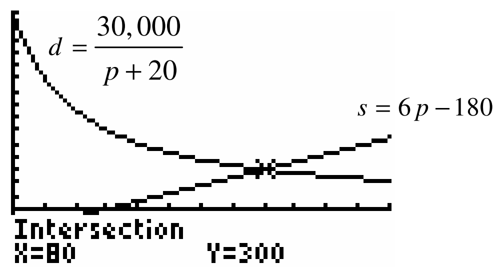
$(p + 20)q = 30{,}000$

$q = \dfrac{30{,}000}{p + 20}$

Supply > Demand

$6p - 180 > \dfrac{30{,}000}{p + 20}$

Applying the intersection of graphs method:

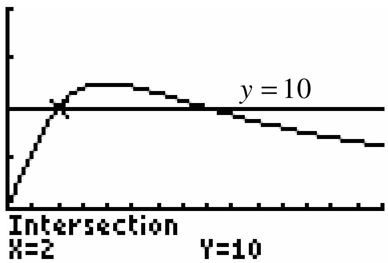


[0, 120] by [–400, 1500]

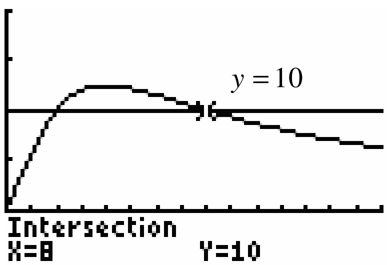
When the price is above \$80, supply exceeds demand.

34. $C(t)=\dfrac{200t}{2t^2+32}$

Applying the intersection of graphs method:



[0, 15] by [–5, 20]



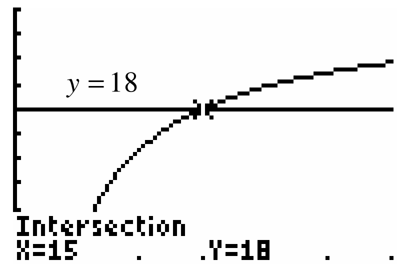
Note that the graphs intersect when $t=2$ and when $t=8$. Therefore,

$C(t)=\dfrac{200t}{2t^2+32}\geq 10$ on the interval $\left[2,8\right]$ or when $2\leq t\leq 8$.

Therefore, the drug concentration remains at least 10% between 2 hours and 8 hours, inclusive. The results contradict the claim of the of the drug company that the drug remains at a 10% for at least 8 hours. The calculations suggest that the concentration is at least 10% for only 6 hours, $8-2=6$.

35. a. $f(t)=\dfrac{30+40t}{5+2t}$

Applying the intersection of graphs method:



[0, 30] by [15, 20]

Note that the graphs intersect when $t=15$. Therefore, $f(t)=\dfrac{30+40t}{5+2t}<18$ on the interval $\left[0,15\right)$ or when $0\leq t<15$.

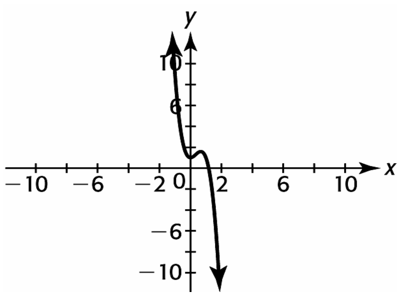
b. For the first 15 months of the operation, the number of employees is below 18.

Chapter 6 Skills Check

1. The degree of the polynomial is the highest exponent. In this case, the degree of the polynomial is 4.

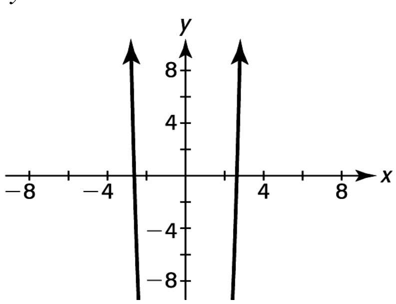
2. A fourth degree polynomial function is called a quartic function.

3. $y = -4x^3 + 4x^2 + 1$

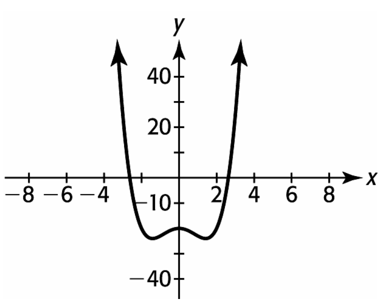


Yes. The graph is complete on the given viewing window.

4. a. $y = x^4 - 4x^2 - 20$

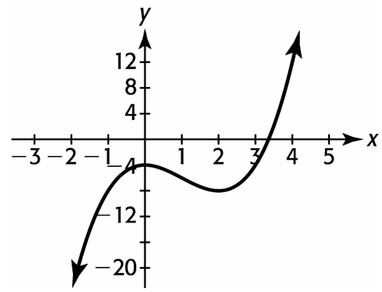


b.

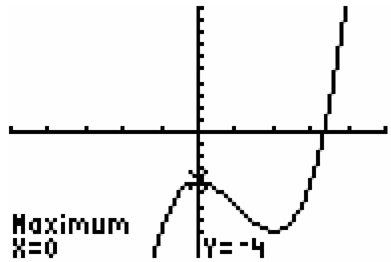


Viewing windows may vary.

5. a. $y = x^3 - 3x^2 - 4$



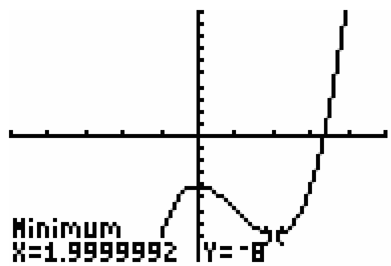
b.



[–5, 5] by [–10, 10]

The local maximum is $(0,-4)$.

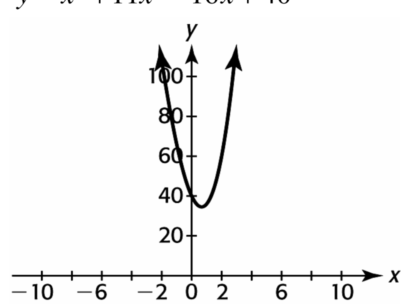
c.



[–5, 5] by [–10, 10]

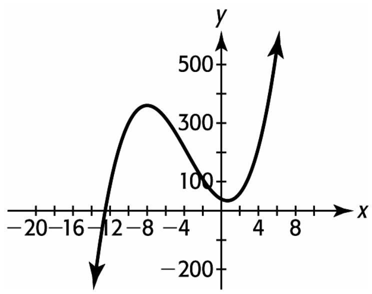
The local minimum is $(2,-8)$.

6. a. $y = x^3 + 11x^2 - 16x + 40$



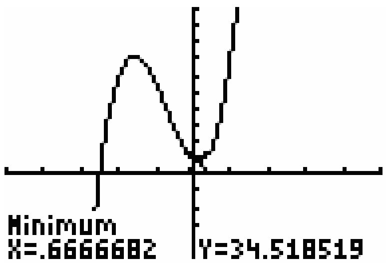
b. No. The graph is not complete.

c.

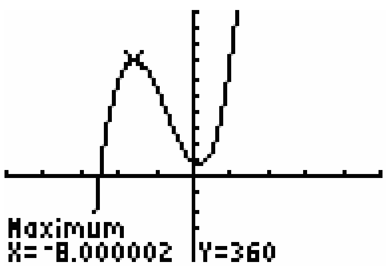


Viewing windows may vary.

d.



[–25, 25] by [–250, 500]



[–25, 25] by [–250, 500]

7. $x^3 - 16x = 0$

$x(x^2 - 16) = 0$

$x(x+4)(x-4) = 0$

$x = 0, \quad x+4 = 0, \quad x-4 = 0$

$x = 0, \quad x = -4, \quad x = 4$

8. $2x^4 - 8x^2 = 0$

$2x^2(x^2 - 4) = 0$

$2x^2(x+2)(x-2) = 0$

$2x^2 = 0, \quad x+2 = 0, \quad x-2 = 0$

$x = 0, \quad x = -2, \quad x = 2$

9. $x^4 - x^3 - 20x^2 = 0$

$x^2(x^2 - x - 20) = 0$

$x^2(x-5)(x+4) = 0$

$x^2 = 0, \quad x-5 = 0, \quad x+4 = 0$

$x = 0, \quad x = 5, \quad x = -4$

10. $x^3 - 15x^2 + 56x = 0$

$x(x^2 - 15x + 56) = 0$

$x(x-7)(x-8) = 0$

$x = 0, \quad x-7 = 0, \quad x-8 = 0$

$x = 0, \quad x = 7, \quad x = 8$

11. $4x^3 - 20x^2 - 4x + 20 = 0$

$4(x^3 - 5x^2 - x + 5) = 0$

$4\left[(x^3 - 5x^2) + (-x + 5)\right] = 0$

$4\left[x^2(x-5) + -1(x-5)\right] = 0$

$4(x-5)(x^2 - 1) = 0$

$4(x-5)(x+1)(x-1) = 0$

$x-5 = 0, \quad x+1 = 0, \quad x-1 = 0$

$x = 5, \quad x = -1, \quad x = 1$

12. $12x^3 - 9x^2 - 48x + 36 = 0$

$3(4x^3 - 3x^2 - 16x + 12) = 0$

$3\left[(4x^3 - 3x^2) + (-16x + 12)\right] = 0$

$3\left[x^2(4x - 3) + (-4)(4x - 3)\right] = 0$

$3(4x - 3)(x^2 - 4) = 0$

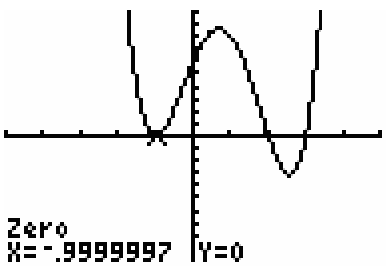
$3(4x - 3)(x + 2)(x - 2) = 0$

$4x - 3 = 0, \quad x + 2 = 0, \quad x - 2 = 0$

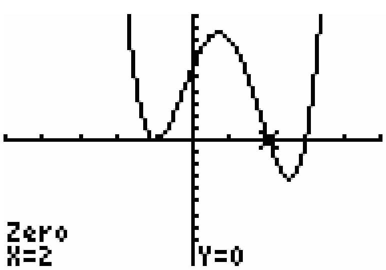
$x = \frac{3}{4}, \quad x = -2, \quad x = 2$

13. $y = x^4 - 3x^3 - 3x^2 + 7x + 6$

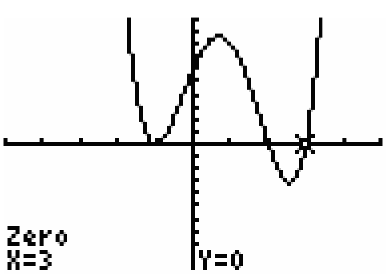
Applying the x-intercept method:



[–5, 5] by [–10, 10]



[–5, 5] by [–10, 10]

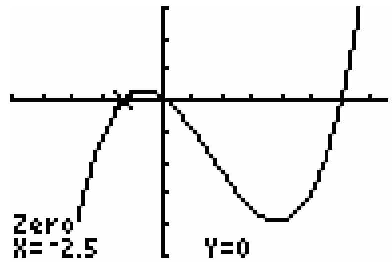


[–5, 5] by [–10, 10]

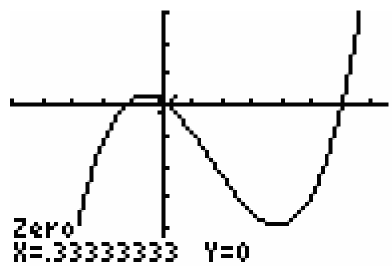
Based on the graphs, the solutions are $x = -1, x = 2, x = 3$.

14. $y = 6x^3 - 59x^2 - 161x + 60$

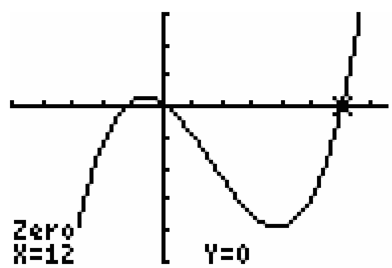
Applying the x-intercept method:



[–10, 15] by [–2500, 1500]



[–10, 15] by [–2500, 1500]



[–10, 15] by [–2500, 1500]

Based on the graphs, the solutions are $x = -2.5, x = \frac{1}{3}, x = 12.$

15. $(x-4)^3 = 8$

$\sqrt[3]{(x-4)^3} = \sqrt[3]{8}$

$x-4=2$

$x=6$

16. $5(x-3)^4 = 80$

$(x-3)^4 = 16$

$\sqrt[4]{(x-3)^4} = \pm\sqrt[4]{16}$

$x-3=\pm 2$

$x=5, x=1$

17.

$$\begin{array}{r|rrrrr} 2 & 4 & -3 & 0 & 2 & -8 \\ & & 8 & 10 & 20 & 44 \\ \hline & 4 & 5 & 10 & 22 & 36 \end{array}$$

$4x^3+5x^2+10x+22+\dfrac{36}{x-2}$

18.

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -11 & 4 \\ & & 2 & 7 & -4 \\ \hline & 2 & 7 & -4 & 0 \end{array}$$

The new polynomial is $2x^2+7x-4$.

Set the polynomial equal to zero and solve.

$2x^2+7x-4=0$

$(2x-1)(x+4)=0$

$2x-1=0, x+4=0$

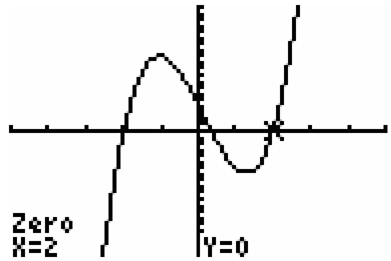
$2x-1=0 \Rightarrow 2x=1 \Rightarrow x=\dfrac{1}{2}$

$x+4=0 \Rightarrow x=-4$

The solutions are $x=1, x=-4, x=\dfrac{1}{2}$.

19. $y=3x^3-x^2-12x+4$

Applying the x-intercept method:



[–5, 5] by [–20, 20]

It appears that $x=2$ is a zero.

$$\begin{array}{r|rrrr} 2 & 3 & -1 & -12 & 4 \\ & & 6 & 10 & -4 \\ \hline & 3 & 5 & -2 & 0 \end{array}$$

The new polynomial is $3x^2+5x-2$.

Applying the quadratic formula:

$x=\dfrac{-(5)\pm\sqrt{(5)^2-4(3)(-2)}}{2(3)}$

$x=\dfrac{-5\pm\sqrt{49}}{6}$

$x=\dfrac{-5\pm 7}{6}$

$x=\dfrac{-5+7}{6}=\dfrac{2}{6}=\dfrac{1}{3}$

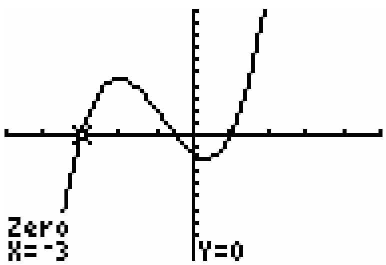
or

$x=\dfrac{-5-7}{6}=\dfrac{-12}{6}=-2$

The solutions are $x=2, x=-2, x=\dfrac{1}{3}$

20. $y = 2x^3 + 5x^2 - 4x - 3$

Applying the *x*-intercept method:



[–5, 5] by [–20, 20]

It appears that $x = -3$ is a zero.

−3)	2	5	−4	−3
		−6	3	3
	2	−1	−1	0

The new polynomial is $2x^2 - x - 1$.

Applying the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1 \pm 3}{4}$$

$$x = \frac{1+3}{4} = \frac{4}{4} = 1$$

or

$$x = \frac{1-3}{4} = \frac{-2}{4} = -\frac{1}{2}$$

The solutions are $x = -3, x = -\frac{1}{2}, x = 1$.

21. a. $y = \dfrac{1 - x^2}{x + 2}$

To find the y-intercept, let $x = 0$ and solve for *y*.

$$y = \frac{1-(0)^2}{0+2} = \frac{1}{2}$$

$$\left(0, \frac{1}{2}\right)$$

To find x-intercepts, let the numerator equal zero and solve for *x*.

$$1 - x^2 = 0$$

$$x^2 = 1$$

$$\sqrt{x^2} = \pm\sqrt{1}$$

$$x = \pm 1$$

$$(-1, 0), (1, 0)$$

b. To find the vertical asymptote let

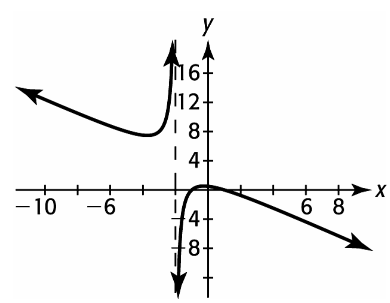
$$x + 2 = 0$$

$$x = -2$$

$x = -2$ is the vertical asymptote.

The degree of the numerator is greater than the degree of the denominator. Therefore, there is not a horizontal asymptote.

c.



22. a. $y = \dfrac{3x-2}{x-3}$

To find the y-intercept, let $x = 0$ and solve for y.

$y = \dfrac{3(0)-2}{0-3} = \dfrac{-2}{-3} = \dfrac{2}{3}$

$\left(0, \dfrac{2}{3}\right)$

To find x-intercepts, let the numerator equal zero and solve for x.

$3x - 2 = 0$

$3x = 2$

$x = \dfrac{2}{3}$

$\left(\dfrac{2}{3}, 0\right)$

b. To find the vertical asymptote let

$x - 3 = 0$

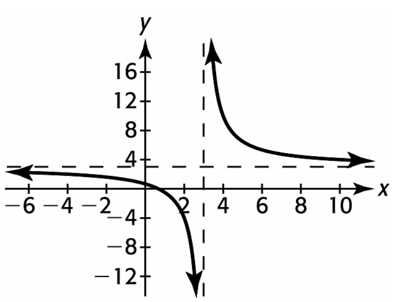
$x = 3$

$x = 3$ is the vertical asymptote.

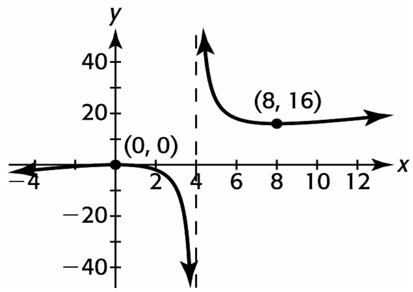
The degree of the numerator equals the degree of the denominator. Therefore, the ratio of the leading coefficients is the horizontal asymptote.

$y = \dfrac{3}{1} = 3$

c.

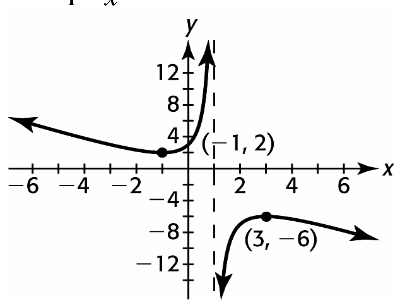


23. $y = \dfrac{x^2}{x-4}$



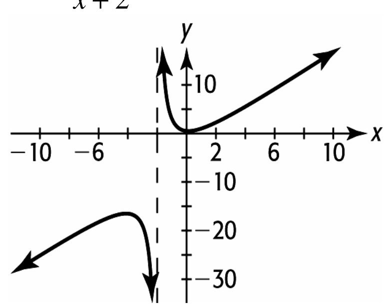
The local maximum is $(0,0)$, while the local minimum is $(8,16)$.

24. $y = \dfrac{x^2+3}{1-x}$

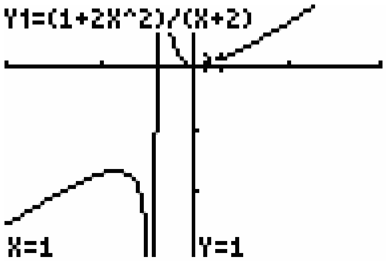


The local maximum is $(3,-6)$, while the local minimum is $(-1,2)$.

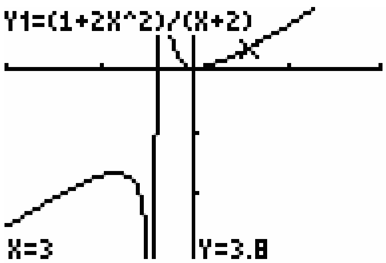
25. a. $y = \dfrac{1+2x^2}{x+2}$



b.

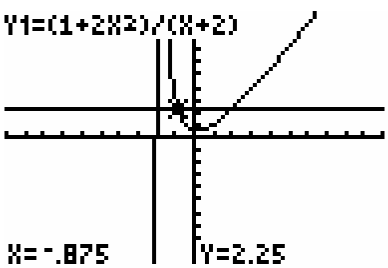


[–10, 10] by [–30, 10]

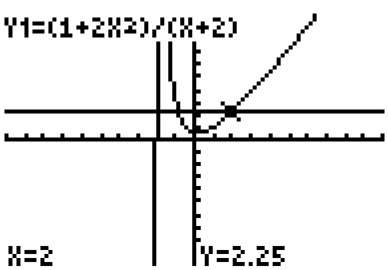


When $x = 1$, $y = 1$. When $x = 3$, $y = 3.8$.

c.



[–10, 10] by [–10, 10]



If $y = 2.25$, then $x = –0.875$ or $x = 2$.

d.

$$\frac{9}{4} = \frac{1+2x^2}{x+2} \qquad \text{LCD: } 4(x+2)$$

$$4(x+2)\left(\frac{9}{4}\right) = 4(x+2)\left(\frac{1+2x^2}{x+2}\right)$$

$$9(x+2) = 4(1+2x^2)$$

$$9x+18 = 4+8x^2$$

$$8x^2 - 9x - 14 = 0$$

$$(8x+7)(x-2) = 0$$

$$8x+7 = 0 \Rightarrow 8x = -7 \Rightarrow x = -\frac{7}{8}$$

$$x-2 = 0 \Rightarrow x = 2$$

The solutions are $x = 2, x = -\frac{7}{8}$.

26. $x^4 - 13x^2 + 36 = 0$

$$(x^2-9)(x^2-4) = 0$$

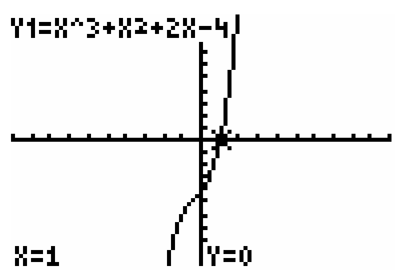
$$(x+3)(x-3)(x+2)(x-2) = 0$$

$$x+3=0, \quad x-3=0, \quad x+2=0, \quad x-2=0$$

$$x=-3,\ x=3,\ x=-2,\ x=2$$

27. $y = x^3 + x^2 + 2x - 4$

Applying the x-intercept method:



[–10, 10] by [–10, 10]

It appears that $x = 1$ is a zero.

1)	1	1	2	–4
		1	2	4
	1	2	4	0

The new polynomial is $x^2 + 2x + 4$.

Set the new polynomial equal to zero and solve.

$$x^2 + 2x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

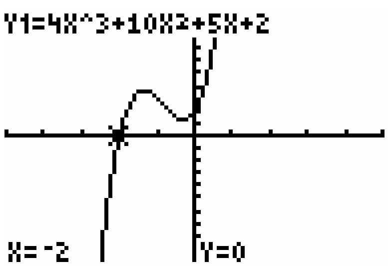
$$x = \frac{-2 \pm 2i\sqrt{3}}{2} = \frac{2(-1 \pm 1i\sqrt{3})}{2}$$

$$x = -1 \pm i\sqrt{3}$$

The solutions are $x = 1,\ x = -1 \pm i\sqrt{3}$.

28. $y = 4x^3 + 10x^2 + 5x + 2$

Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that $x = –2$ is a zero.

$-2)$	4	10	5	2
		-8	-4	-2
	4	2	1	0

The new polynomial is $4x^2 + 2x + 1$.

Set the new polynomial equal to zero and solve.

$$4x^2 + 2x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{-12}}{8}$$

$$x = \frac{-2 \pm 2i\sqrt{3}}{8}$$

$$x = \frac{-1 \pm i\sqrt{3}}{4}$$

$$x = \frac{-1}{4} \pm \frac{\sqrt{3}}{4}i$$

The solutions are $x = -2,\ x = \frac{-1}{4} + \frac{\sqrt{3}}{4}i,$ $x = \frac{-1}{4} - \frac{\sqrt{3}}{4}i.$

29. $x^3 - 5x^2 \ge 0$

$$x^2(x-5) \ge 0$$

$$x^2(x-5) = 0$$

$$x - 5 = 0, \quad x^2 = 0$$

$$x = 5, \quad x = 0$$

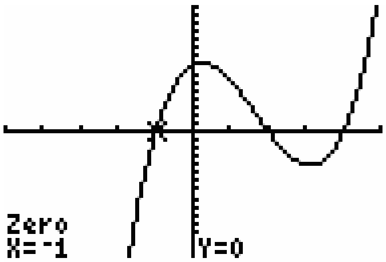
Sign chart:

Function	---	---	+++
x^2	+++	+++	+++
$(x-5)$	---	---	+++
		0	5

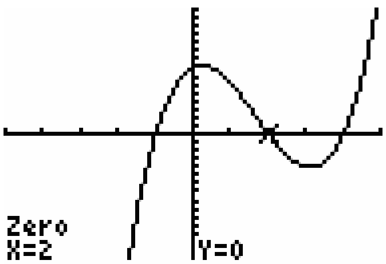
Based on the sign chart, the function is greater than or equal to zero on the interval $[5, \infty)$ or when $x \ge 5$. In addition, the function is equal to zero when $x = 0$.

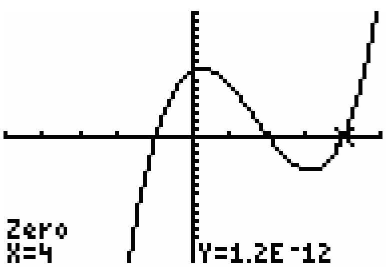
30. $y = x^3 - 5x^2 + 2x + 8$

Applying the x-intercept method:



[–5, 5] by [–15, 15]



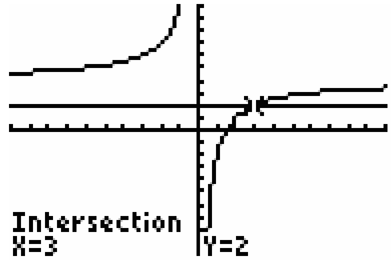


Based on the graphs, the function is greater than or equal to zero on the intervals $[-1, 2]$ or $[4, \infty)$ or when $x \geq 4$ or $-1 \leq x \leq 2$.

31. $2 < \dfrac{4x-6}{x}$

$\dfrac{4x-6}{x} > 2$

Applying the intersection of graphs method:

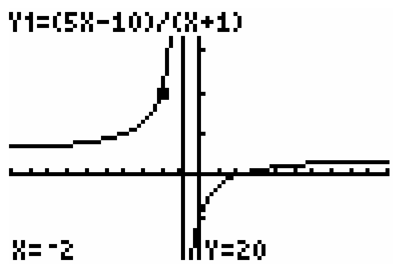


[–10, 10] by [–10, 10]

Note that the graphs intersect when $x = 3$. Also note that a vertical asymptote occurs at $x = 0$. Therefore, $\dfrac{4x-6}{x} > 2$ on the interval $(-\infty, 0) \cup (3, \infty)$ or when $x < 0$ or $x > 3$.

32. $\dfrac{5x-10}{x+1} \geq 20$

Applying the intersection of graphs method:

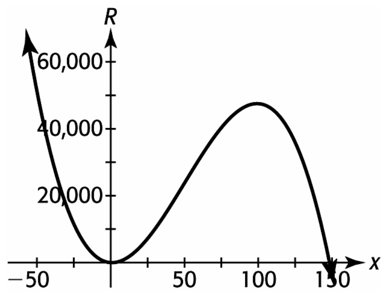


[–10, 10] by [–20, 40]

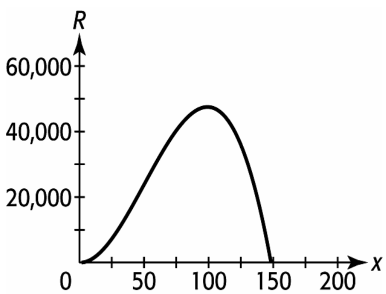
Note that the graphs intersect when $x = -2$. Also note that a vertical asymptote occurs at $x = -1$. Therefore, $\dfrac{5x-10}{x+1} \geq 20$ on the interval $[-2, -1)$ or when $-2 \leq x < -1$.

Chapter 6 Review Exercises

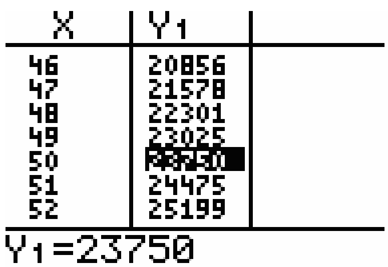
33. a. $R = -0.1x^3 + 15x^2 - 25x$



b.

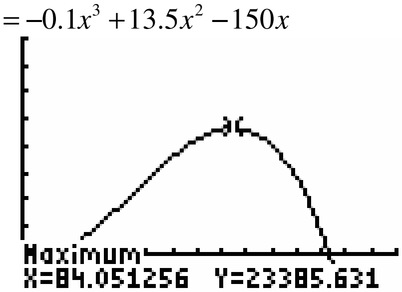


c.



When 50,000 units are produced and sold, the revenue is $23,750.

34. $R = -0.1x^3 + 13.5x^2 - 150x$



[0, 150] by [–6000, 40,000]

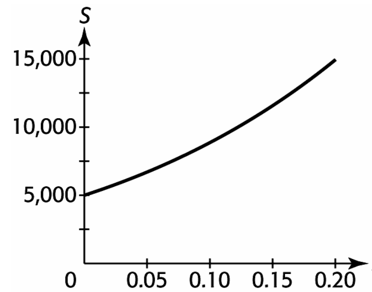
When 84,051 units are sold, the maximum revenue of $23,385.63 is generated.

35. a. $S = 5000(1 + r)^6$

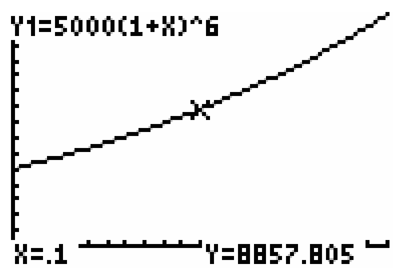
Using the table feature of a TI-83 graphing calculator:

Rate, r	Future Value, $S(\$)$
0.01	5307.60
0.05	6700.48
0.10	8857.81
0.15	11,565.30

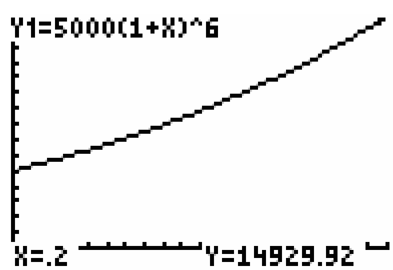
b.



c.



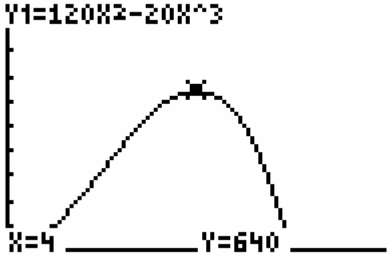
[0, 0.2] by [–1000, 15,000]



[0, 0.2] by [–1000, 15,000]

The difference in the future values is $14,929.92 - 8857.81 = \$6072.11$.

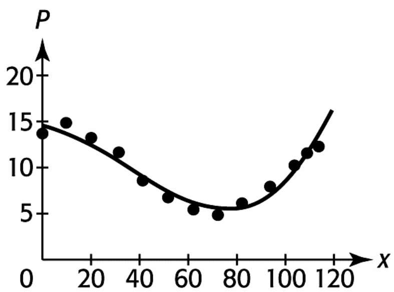
36. $y = 120x^2 - 20x^3$



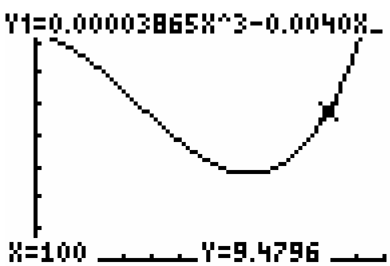
[0, 8] by [0, 1000]

An intensity level of 4 allows the maximum amount of photosynthesis to take place.

37. a. $P = 0.00003865x^3 - 0.0040x^2 - 0.0375x + 14.5796$

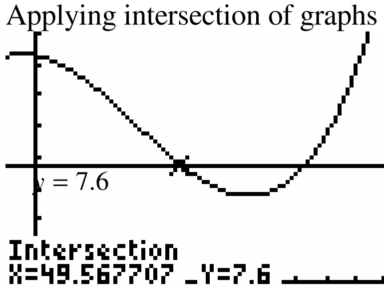


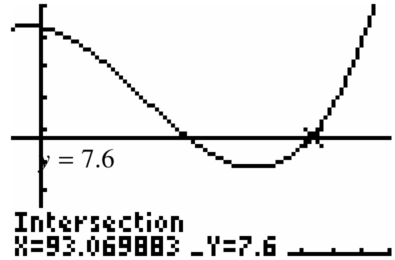
b.



In 2000, the model indicates the percentage was 9.5%. This value is not very close to the actual value of 10.4%.

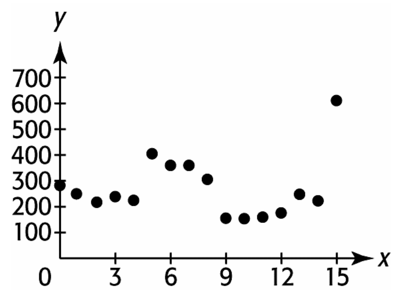
c. Applying intersection of graphs method:





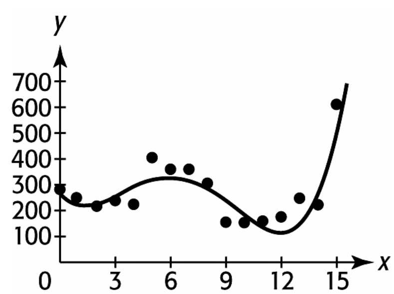
Based on the graphs, the percent of the U.S. population that is foreign born was 7.6% in approximately 1950, and again in 1994.

38. a.



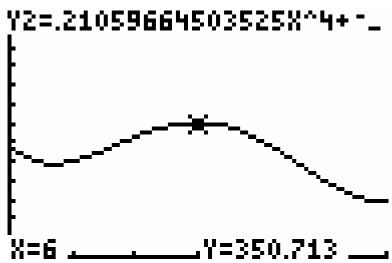
b. $y = 0.211x^4 - 5.372x^3 + 40.038x^2 - 83.244x + 296.203$

c.



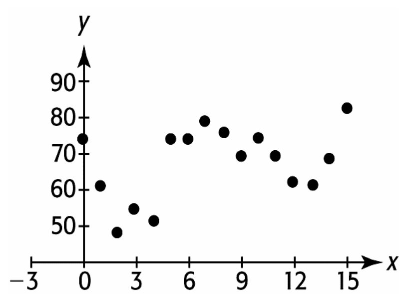
d. Using the unrounded model, the approximate debt to the United Nations in 1998 was $304.89 million.

e. Between the years 1990 and 2002, the maximum debt occurred in the year 1996.



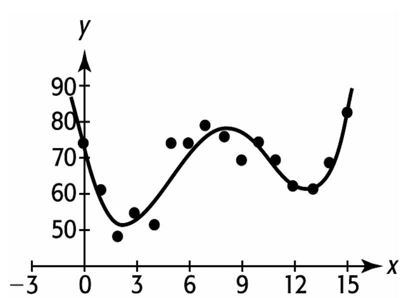
[0, 12] by [0, 650]

39. a.



b. $y = 0.0274x^4 - 0.834x^3 + 7.999x^2$
$-24.203x + 74.135$

c.



40.

$$9261 = 8000(1+r)^3$$
$$(1+r)^3 = \frac{9261}{8000}$$
$$\sqrt[3]{(1+r)^3} = \sqrt[3]{\frac{9261}{8000}}$$
$$1 + r = 1.05$$
$$r = 1.05 - 1$$
$$r = 0.05$$

An interest rate of 5% creates a future value of \$9261 after 3 years.

41. a.

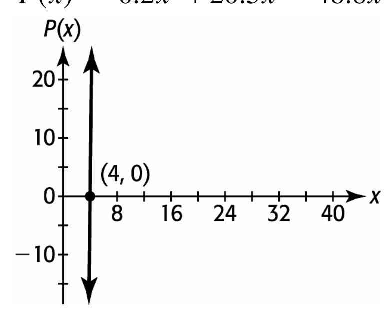
$$V = 0$$
$$324x - 72x^2 + 4x^3 = 0$$
$$4x^3 - 72x^2 + 324x = 0$$
$$4x(x^2 - 18x + 81) = 0$$
$$4x(x-9)(x-9) = 0$$
$$4x = 0, \quad x - 9 = 0, \quad x - 9 = 0$$
$$x = 0, \quad x = 9, \quad x = 9$$

b. If the values of *x* from part a) are used to cut squares from corners of a piece of tin, no box can be created. Either no square is cut or the squares encompass all the tin. Therefore, the volume of the box is zero.

c. Reasonable values of *x* would allow for a box to be created. An *x*-value larger than zero and less than half the length of the edge of the piece of tin would allow for a box to be created. Therefore, reasonable values are

$$0 < x < \frac{18}{2} \text{ or } 0 < x < 9.$$

42. a. $P(x) = -0.2x^3 + 20.5x^2 - 48.8x - 120$



The *x*-intercept is $(4, 0)$.

b.

4	−0.2	20.5	−48.8	−120
		−0.8	78.8	120
	−0.2	19.7	30	0

The remaining quadratic factor is $-0.2x^2 + 19.7x + 30$.

c. Set the remaining polynomial equal to zero and solve.

$-0.2x^2 + 19.7x + 30 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(19.7) \pm \sqrt{(19.7)^2 - 4(-0.2)(30)}}{2(-0.2)}$

$x = \frac{-19.7 \pm \sqrt{412.09}}{-0.4}$

$x = \frac{-19.7 \pm 20.3}{-0.4}$

$x = \frac{-19.7 + 20.3}{-0.4}, \; x = \frac{-19.7 - 20.3}{-0.4}$

$x = -1.5, \; x = 100$

The solutions are $x = 4, \; x = -1.5,$ and $x = 100.$

d. Since negative solutions do not make sense in the context of the question, break-even occurs when 400 units are produced or when 10,000 units are produced.

43. a. The quartic function is:

$y = 37.792x^4 - 745.250x^3$
$+5349.708x^2 - 16512.250x$
$+18885$

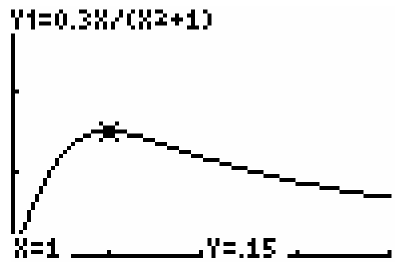
b. The model estimates that Greenland's ice sheet loss in 2010 was 21,400 billion tons.

44. $C = \frac{0.3t}{t^2 + 1}$

a. Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $C = 0.$

b. As the time increases, the concentration of the drug approaches zero percent.

c.



[0, 4] by [0, 0.3]

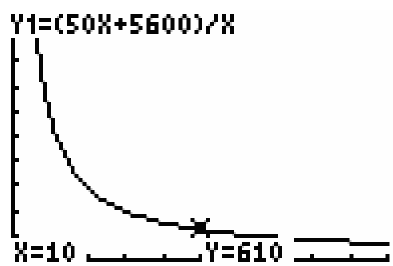
The maximum drug concentration is 15%, occurring after one hour.

45. $\overline{C}(x) = \frac{50x + 5600}{x}$

a. $\overline{C}(0)$ does not exist. If no units are produced, an average cost per unit cannot be calculated.

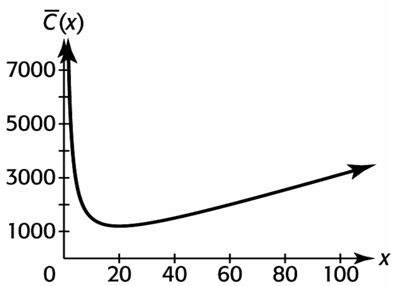
b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $\overline{C}(x) = \frac{50}{1} = 50.$ As the number of units produced increases without bound, the average cost per unit approaches \$50.

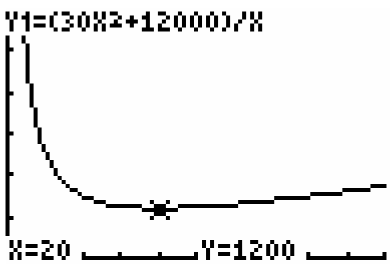
c. The function decreases as x increases.



[0, 20] by [0, 5000]

46. a. $\overline{C} = \dfrac{30x^2 + 12{,}000}{x}$

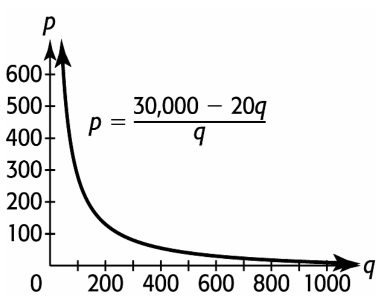


b. 

[0, 50] by [0, 6000]

The minimum average cost is $1200, occurring when 20 units are produced.

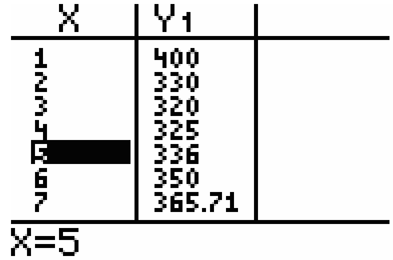
47.



48. a. $C(x) = 200 + 20x + \dfrac{180}{x}$

$$C(x) = \frac{200x}{x} + \frac{20x^2}{x} + \frac{180}{x}$$

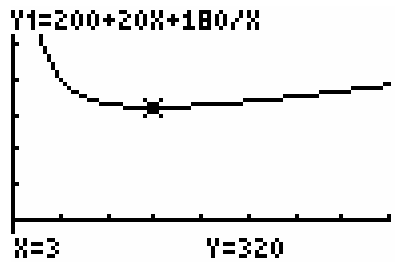
$$C(x) = \frac{20x^2 + 200x + 180}{x}$$

b. 

X	Y1
1	400
2	330
3	320
4	325
5	336
6	350
7	365.71

X=5

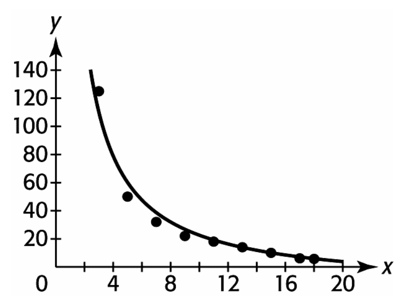
Using 5 plates creates a cost of $336.

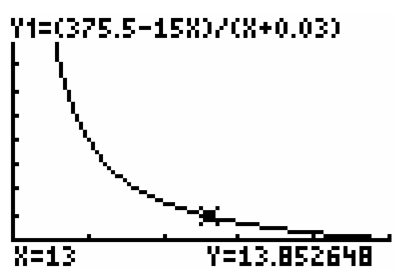
c. 

[0, 8] by [–100, 600]

Using 3 plates creates a minimum cost of $320.

49. a. $y = \dfrac{375.5 - 15x}{x + 0.03}$

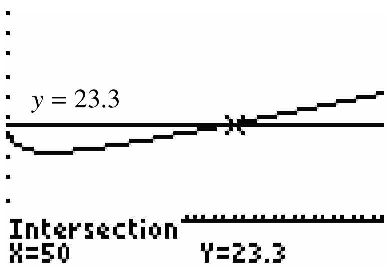


b. 

During the 1993-94 school year, the model indicates 13.85 students per computer. The value is close to the actual data of 14 students per computer, as displayed in the table.

50. $S = \dfrac{40}{x} + \dfrac{x}{4} + 10$

Applying the intersection of graphs method:

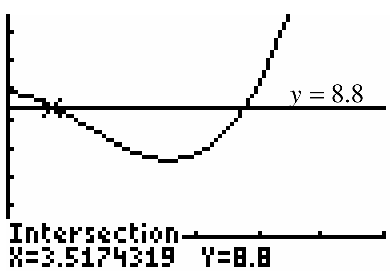


[4, 80] by [–10, 50]

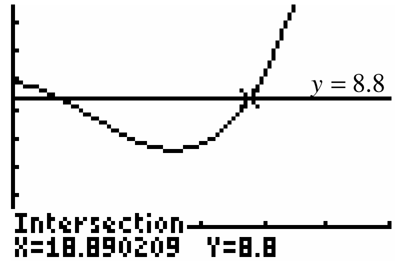
Note that $S \geq 23.3$ occurs on the interval $[50, \infty)$ or when $x \geq 50$. Fifty or more hours of training results in sales greater than \$23,300.

51. $y = 0.00380x^3 - 0.0704x^2 - 0.0780x + 9.780$

Applying the intersection of graphs method:



[0, 30] by [–2, 15]

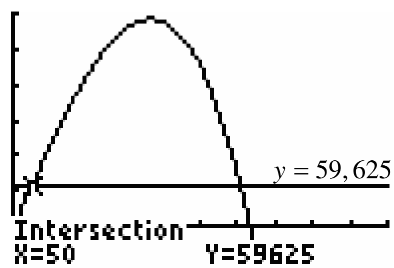


[0, 30] by [–2, 15]

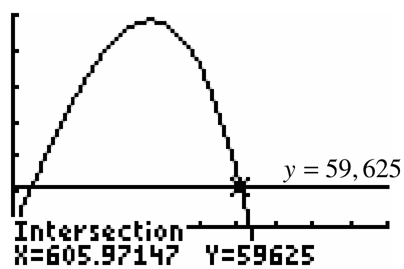
Note that $y \leq 8.8$ when $3.517 \leq x \leq 18.89$. Between 1994 and 2009 inclusive, the homicide rate is no more than 8.8 per 100,000 people.

52. $y = 1200x - 0.003x^3$

Applying the intersection of graphs method:



[0, 1000] by [–50,000, 300,000]

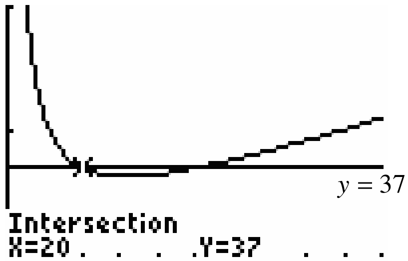


[0, 1000] by [–50,000, 300,000]

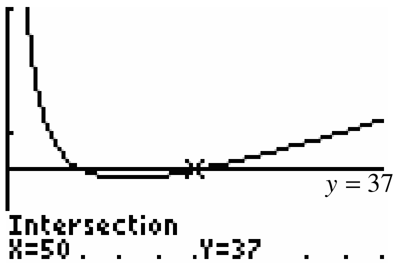
Note that $R \geq 59,625$ when $x \geq 50$ and $x \leq 605.97$. Selling 50 or more units but no more than 605 units creates a revenue stream of at least \$59,625.

53. $\overline{C} = \dfrac{100 + 30x + 0.1x^2}{x}$

Applying the intersection of graphs method



[0, 100] by [30, 50]

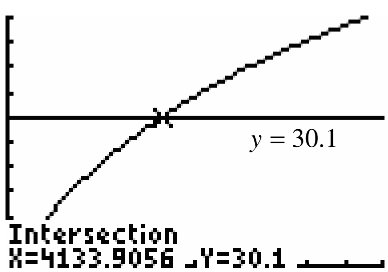


[0, 100] by [30, 50]

$\overline{C} \le 37$ when $20 \le x \le 50$. The average cost is at most $37 when between 20 and 50 units, inclusive, are produced.

54. $p = \dfrac{100C}{9600 + C}$

Applying the intersection of graphs method:



[0, 10,000] by [0, 50]

Note that $p \ge 30.1$ when $x \ge 4133.91$. To remove at least 30.1% of the particulate pollution will cost at least $4133.91.

Group Activity/Extended Applications

1. Global Climate Change

1. **a.** $p \neq 100$

b. D: [0, 100)
R: [0, 242,000)

2. C(60) = \$3630 hundred = \$363,000.
C(80) = \$9680 hundred = 968,000. This result means the annual cost in dollars of removing 80% of the particulate pollution from the smokestack of a power plant is \$968,000.

3. The vertical asymptote of this function is $p = 100$, and means it is impossible to remove 100% of the particulate pollution. Algebraically, the horizontal asymptote is $y = 0$, but in the context of the problem (within the domain), there is no horizontal asymptote that makes any sense.

4. The y-intercept is (0, 0) and means that it would cost \$0 if no particulate pollution amount is removed.

5. The CFO of this company should recommend to pay the fine since the cost to remove 80% of the pollution (\$968,000) is higher than the fine of \$700,000.

6. If the company has already paid \$363,000 to remove 60% of the pollution, the difference to remove 20% more is only \$605,000 which is less than the \$700,000 fine. Advise the company to remove the 20% difference rather than pay the fine.

7. Answer may vary, but at least \$1,000,000 would be higher than the cost to remove 80% of the pollution.

8. The cost to remove 90% is \$2,178,000 which is significantly higher than paying the current fine. It would seem it is not worth the cost.

2. Printing

A. **1.** Assuming the printer uses 10 plates, then $1000 \cdot 10 = 10{,}000$ impressions can be made per hour. If 10,000 impressions are made per hour, it will take $\frac{100{,}000}{10{,}000} = 10$ hours to complete all the invitations.

2. Since it costs \$128 per hour to run the press, the cost of using 10 plates is $10 \cdot 128 = \$1280$.

3. The 10 plates cost $10 \cdot 8 = \$80$.

4. The total cost of finishing the job is $1280 + 80 = \$1360$

B. **1.** Let x = number of plates. Then, the cost of the plates is $8x$.

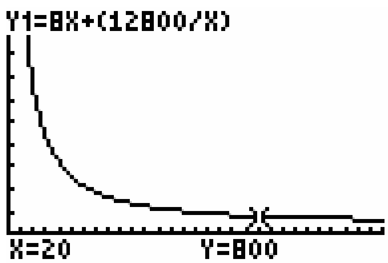
2. Using x plates implies x invitations can be made per impression.

3. $1000x$ invitations per hour
Creating all 100,000 invitations would require $\frac{100{,}000}{1000x} = \frac{100}{x}$ hours.

4. $C(x) = 8x + 128\left(\frac{100}{x}\right)$

$C(x) = 8x + \left(\frac{12{,}800}{x}\right)$, where

x represents the number of plates and $C(x)$ represents the cost of the 100,000 invitations in dollars.

5. 

[0, 30] by [0, 10,000]

Producing 20 plates minimizes the cost, since the number of plates, x, that can be produced is between 1 and 20 inclusive.

6. Considering a table of values yields

X	Y1
1	12808
2	6416
3	4290.7
4	3232
5	2600
6	2181.3
7	1884.6

X=1

X	Y1
8	1664
9	1494.2
10	1360
11	1251.6
12	1162.7
13	1088.6
14	1026.3

X=8

X	Y1
14	1026.3
15	973.33
16	928
17	888.94
18	855.11
19	825.68
20	800

X=20

Producing 20 plates creates a minimum cost of $800 for printing the 100,000 invitations.