CHAPTER 7
Systems of Equations and Matrices

Chapter 7 Toolbox

1. The constant of proportionality is 12.

2. $y = 10x$

$x = \frac{y}{10}$

$x = \frac{1}{10}y$

The constant of proportionality is $\frac{1}{10}$.

3. Is $\frac{5}{6} = \frac{10}{18}$?

Is $5 \times 18 = 6 \times 10$?

$90 \neq 60$

No. The ratios are not proportional.

4. Is $\frac{8}{3} = \frac{24}{9}$?

Is $8 \times 9 = 3 \times 24$?

$72 = 72$

Yes. The ratios are proportional.

5. Does $\frac{4}{6} = \frac{2}{3}$?

$4 \times 3 = 6 \times 2$

$12 = 12$

Yes. The pairs are proportional.

6. Does $\frac{5.1}{2.3} = \frac{51}{23}$?

$5.1 \times 23 = 2.3 \times 51$

$117.3 = 117.3$

Yes. The pairs are proportional.

7. $y = kx$

$18 = k(2)$

$2k = 18$

$k = 9$

$y = 9x$

Let $x = 11$.

$y = 9(11)$

$y = 99$

8.

$x = ky$

$3 = k(18)$

$18k = 3$

$k = \frac{3}{18} = \frac{1}{6}$

$x = \frac{1}{6}y$

Let $x = 13$.

$13 = \frac{1}{6}y$

$6(13) = 6\left(\frac{1}{6}y\right)$

$y = 78$

9. If the pairs of triples are proportional, then there exists a number k such that $y = kx$ for all pairs (x, y).

$y = kx$

$20 = k(8)$

$k = \frac{20}{8} = 2.5$

$y = 2.5x$

Checking the other pairs:

$12.5 = 2.5(6)$

$12.5 \neq 15$

The pairs of triples are not proportional.

10. If the pairs of triples are proportional, then there exists a number k such that $y = kx$ for all pairs (x, y).

$$\begin{aligned} y &= kx \\ 8.2 &= k(4.1) \\ k &= \frac{8.2}{4.1} = 2 \\ y &= 2x \end{aligned}$$

Checking the other pairs:
$13.6 = 6.8(2)$
$13.6 = 13.6$
and
$18.6 = 2(9.3)$
$18.6 = 18.6$

The pairs of triples are proportional.

11. If the pairs of triples are proportional, then there exists a number k such that $y = kx$ for all pairs (x, y).

$$\begin{aligned} y &= kx \\ -3.4 &= k(-1) \\ k &= \frac{-3.4}{-1} = 3.4 \\ y &= 3.4x \end{aligned}$$

Checking the other pairs yields
$10.2 = 3.4(6)$
$10.2 \neq 20.4$

The pairs of triples are not proportional.

12. If the pairs of triples are proportional, then there exists a number k such that $y = kx$ for all pairs (x, y).

$$\begin{aligned} y &= kx \\ 2 &= k(5) \\ k &= \frac{2}{5} = 0.4 \\ y &= 0.4x \end{aligned}$$

Checking the other pairs:
$-3 = 0.4(-12)$
$-3 \neq -4.8$

The pairs of triples are not proportional.

13. Substituting yields:

$$\begin{aligned} x - 2y + z &= 8 \\ 1 - 2(-2) + 3 &= 8 \\ 8 &= 8 \end{aligned}$$

and

$$\begin{aligned} 2x - y + 2z &= 10 \\ 2(1) - (-2) + 2(3) &= 10 \\ 2 + 2 + 6 &= 10 \\ 10 &= 10 \end{aligned}$$

and

$$\begin{aligned} 3x - 2y + z &= 5 \\ 3(1) - 2(-2) + 3 &= 5 \\ 10 &\neq 5 \end{aligned}$$

The given values do not satisfy the system of equations.

14. Substituting yields:

$$\begin{aligned} x-2y+2z&=-9\\ -1-2(4)+2(0)&=-9\\ -9&=-9\\ &\text{and}\\ 2x-3y+z&=-14\\ 2(-1)-3(4)+0&=-14\\ -14&=-14\\ &\text{and}\\ x+2y-3z&=7\\ -1+2(4)-3(0)&=7\\ 7&=7 \end{aligned}$$

The given values satisfy the system of equations.

15. Substituting yields:

$$\begin{aligned} x+y-z&=4\\ 5+(-2)-(-1)&=4\\ 4&=4\\ &\text{and}\\ 2x+3y-z&=5\\ 2(5)+3(-2)-(-1)&=5\\ 5&=5\\ &\text{and}\\ 3x-2y+5z&=14\\ 3(5)-2(-2)+5(-1)&=14\\ 15+4-5&=14\\ 14&=14 \end{aligned}$$

The given values do satisfy the system of equations.

16. Substituting yields:

$$\begin{aligned} x-2y-3z&=2\\ 2-2(3)-3(-2)&=2\\ 2&=2\\ &\text{and}\\ 2x+3y+3z&=19\\ 2(2)+3(3)+3(-2)&=19\\ 4+9-6&=5\\ 7&\neq 19 \end{aligned}$$

The given values do not satisfy the system of equations.

17. Since the coefficients of the variables are proportional, but the constants are not, the planes are parallel.

18. Since the coefficients of the variables are proportional and the constants are in the same proportion, the planes are the same.

19. Since the coefficients of the variables are not proportional, the planes are different. They intersect along a line. The planes are neither the same nor parallel.

20. Since the coefficients of the variables are proportional and the constants are in the same proportion, the planes are the same.

Section 7.1 Skills Check

1. Since z is isolated, back substitution yields:

$$y+3(3)=11$$
$$y+9=11$$
$$y=2$$
and
$$x+2y-z=3$$
$$x+2(2)-3=3$$
$$x+1=3$$
$$x=2$$

The solutions are $x=2,\ y=2,\ z=3.$

2. Since z is isolated, back substitution yields:

$$y-2(4)=11$$
$$y-8=11$$
$$y=19$$
and
$$x-4y-3z=3$$
$$x-4(19)-3(4)=3$$
$$x-76-12=3$$
$$x-88=3$$
$$x=91$$

The solutions are $x=91,\ y=19,\ z=4.$

3. Since z is isolated, back substitution yields:

$$y+3(-2)=3$$
$$y-6=3$$
$$y=9$$
and
$$x+2y-z=6$$
$$x+2(9)-(-2)=6$$
$$x+18+2=6$$
$$x+20=6$$
$$x=-14$$

The solutions are $x=-14,\ y=9,\ z=-2.$

4. Since z is isolated, back substitution yields:

$$y+3(-5)=21$$
$$y-15=21$$
$$y=36$$
and
$$x+2y-z=22$$
$$x+2(36)-(-5)=22$$
$$x+72+5=22$$
$$x+77=22$$
$$x=-55$$

The solutions are $x=-55,\ y=36,\ z=-5.$

5. $$\begin{cases} x-y-4z=0 \\ y+2z=4 \\ 3y+7z=22 \end{cases} \xrightarrow{-3Eq2+Eq3\to Eq3}$$

$$\begin{cases} x-y-4z=0 \\ y+2z=4 \\ z=10 \end{cases}$$

Since z is isolated, back substitution yields:

$$y+2(10)=4$$
$$y+20=4$$
$$y=-16$$
and
$$x-y-4z=0$$
$$x-(-16)-4(10)=0$$
$$x+16-40=0$$
$$x-24=0$$
$$x=24$$

The solutions are $x=24,\ y=-16,\ z=10.$

6. $\begin{cases} x+4y-11z=33 \\ y-3z=11 \\ 2y+7z=-4 \end{cases} \xrightarrow{-2Eq2+Eq3\to Eq3}$

$\begin{cases} x+4y-11z=33 \\ y-3z=11 \\ 13z=-26 \end{cases} \xrightarrow{\frac{1}{13}Eq3\to Eq3}$

$\begin{cases} x+4y-11z=33 \\ y-3z=11 \\ z=-2 \end{cases}$

Since z is isolated, back substitution yields:

$$y-3(-2)=11$$
$$y+6=11$$
$$y=5$$

and

$$x+4y-11z=33$$
$$x+4(5)-11(-2)=33$$
$$x+20+22=33$$
$$x+42=33$$
$$x=-9$$

The solutions are $x=-9,\ y=5,\ z=-2.$

7. $\begin{cases} x-2y+3z=0 \\ y-2z=-1 \\ y+5z=6 \end{cases} \xrightarrow{-1Eq2+Eq3\to Eq3}$

$\begin{cases} x-2y+3z=0 \\ y-2z=-1 \\ 7z=7 \end{cases} \xrightarrow{\frac{1}{7}Eq3\to Eq3}$

$\begin{cases} x-2y+3z=0 \\ y-2z=-1 \\ z=1 \end{cases}$

Since z is isolated, back substitution yields:

$$y-2(1)=-1$$
$$y-2=-1$$
$$y=1$$

and

$$x-2y+3z=0$$
$$x-2(1)+3(1)=0$$
$$x-2+3=0$$
$$x+1=0$$
$$x=-1$$

The solutions are $x=-1,\ y=1,\ z=1.$

8. $\begin{cases} x-2y+3z=-10 \\ y-2z=7 \\ y-3z=6 \end{cases} \xrightarrow{-1Eq2+Eq3\to Eq3}$

$\begin{cases} x-2y+3z=-10 \\ y-2z=7 \\ -z=-1 \end{cases} \xrightarrow{-1Eq3\to Eq3}$

$\begin{cases} x-2y+3z=-10 \\ y-2z=7 \\ z=1 \end{cases}$

Since z is isolated, back substitution yields:

$$y-2(1)=7$$
$$y-2=7$$
$$y=9$$

and

$$x-2y+3z=-10$$
$$x-2(9)+3(1)=-10$$
$$x-18+3=-10$$
$$x-15=-10$$
$$x=5$$

The solutions are $x=5,\ y=9,\ z=1.$

9. $\begin{cases} x+2y-2z=0 \\ x-y+4z=3 \\ x+2y+2z=3 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-1Eq1+Eq2\to Eq2}$

$\begin{cases} x+2y-2z=0 \\ -3y+6z=3 \\ 4z=3 \end{cases} \xrightarrow{\frac{1}{4}Eq3\to Eq3}$

$\begin{cases} x+2y-2z=0 \\ -3y+6z=3 \\ z=\dfrac{3}{4} \end{cases}$

Since z is isolated, back substitution yields:

$$-3y+6\left(\frac{3}{4}\right)=3$$
$$-3y+\frac{9}{2}=3$$
$$-3y=-\frac{3}{2}$$
$$y=\frac{1}{2}$$

and

$$x+2y-2z=0$$
$$x+2\left(\frac{1}{2}\right)-2\left(\frac{3}{4}\right)=0$$
$$x+1-\frac{3}{2}=0$$
$$x-\frac{1}{2}=0$$
$$x=\frac{1}{2}$$

The solutions are $x=\frac{1}{2},\ y=\frac{1}{2},\ z=\frac{3}{4}$.

10. $\begin{cases} x+4y-14z=22 \\ x+5y+z=-2 \\ x+4y-z=9 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-1Eq1+Eq2\to Eq2}$

$\begin{cases} x+4y-14z=22 \\ y+15z=-24 \\ 13z=-13 \end{cases} \xrightarrow{\frac{1}{13}Eq3\to Eq3}$

$\begin{cases} x+4y-14z=22 \\ y+15z=-24 \\ z=-1 \end{cases}$

Since z is isolated, back substitution yields:

$$y+15(-1)=-24$$
$$y-15=-24$$
$$y=-9$$

and

$$x+4y-14z=22$$
$$x+4(-9)-14(-1)=22$$
$$x-36+14=22$$
$$x-22=22$$
$$x=44$$

The solutions are $x=44,\ y=-9,\ z=-1$.

11.

$\begin{cases} x+3y-z=0 \\ x-2y+z=8 \\ x-6y+2z=6 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-1Eq1+Eq2\to Eq2}$

$\begin{cases} x+3y-z=0 \\ -5y+2z=8 \\ -9y+3z=6 \end{cases} \xrightarrow{9Eq2-5Eq3\to Eq3}$

$\begin{cases} x+3y-z=0 \\ -5y+2z=8 \\ 3z=42 \end{cases} \xrightarrow{\frac{1}{3}Eq3\to Eq3}$

$\begin{cases} x+3y-z=0 \\ -5y+2z=8 \\ z=14 \end{cases}$

Since z is isolated, back substitution yields:

$$\begin{aligned} -5y+2(14)&=8\\ -5y+28&=8\\ -5y&=-20\\ y&=4 \end{aligned}$$

and

$$\begin{aligned} x+3y-z&=0\\ x+3(4)-14&=0\\ x+12-14&=0\\ x&=2 \end{aligned}$$

The solutions are $x=2,\ y=4,\ z=14.$

12. $\begin{cases} x-5y-2z=7\\ x-3y+4z=21\\ x-5y+2z=19 \end{cases} \xrightarrow[-1Eq1+Eq2\to Eq3]{-1Eq1+Eq2\to Eq2}$

$\begin{cases} x-5y-2z=7\\ 2y+6z=14\\ 4z=12 \end{cases} \xrightarrow{\frac{1}{4}Eq3\to Eq3}$

$\begin{cases} x-5y-2z=7\\ 2y+6z=14\\ z=3 \end{cases}$

Since z is isolated, back substitution yields:

$$\begin{aligned} 2y+6(3)&=14\\ 2y+18&=14\\ 2y&=-4\\ y&=-2 \end{aligned}$$

and

$$\begin{aligned} x-5y-2z&=7\\ x-5(-2)-2(3)&=7\\ x+4&=7\\ x&=3 \end{aligned}$$

The solutions are $x=3,\ y=-2,\ z=3.$

13. $\begin{cases} 2x+4y-14z=0\\ 3x+5y+z=19\\ x+4y-z=12 \end{cases} \xrightarrow[Eq1-2Eq3\to Eq3]{-3Eq1+2Eq2\to Eq2}$

$\begin{cases} 2x+4y-14z=0\\ -2y+44z=38\\ -4y-12z=-24 \end{cases} \xrightarrow{-2Eq2+Eq3\to Eq3}$

$\begin{cases} 2x+4y-14z=0\\ -2y+44z=38\\ -100z=-100 \end{cases} \xrightarrow{-\frac{1}{100}Eq3\to Eq3}$

$\begin{cases} 2x+4y-14z=0\\ -2y+44z=38\\ z=1 \end{cases}$

Since z is isolated, back substitution yields:

$$\begin{aligned} -2y+44(1)&=38\\ -2y+44&=38\\ -2y&=-6\\ y&=3 \end{aligned}$$

and

$$\begin{aligned} 2x+4y-14z&=0\\ 2x+4(3)-14(1)&=0\\ 2x+12-14&=0\\ 2x&=2\\ x&=1 \end{aligned}$$

The solutions are $x=1,\ y=3,\ z=1.$

14. $\begin{cases} 3x-6y+6z=24 \\ 2x-6y+2z=6 \\ x-3y+2z=4 \end{cases} \xrightarrow[Eq1-3Eq3\to Eq3]{-2Eq1+3Eq2\to Eq2}$

$\begin{cases} 3x-6y+6z=24 \\ -6y-6z=-30 \\ 3y=12 \end{cases} \xrightarrow{\frac{1}{3}Eq3\to Eq3}$

$\begin{cases} 3x-6y+6z=24 \\ -6y-6z=-30 \\ y=4 \end{cases}$

Since y is isolated, back substitution yields:

$$\begin{aligned} -6(4)-6(z)&=-30 \\ -24-6z&=-30 \\ -6z&=-6 \\ z&=1 \end{aligned}$$

and

$$\begin{aligned} 3x-6y+6z&=24 \\ 3x-6(4)+6(1)&=24 \\ 3x-24+6&=24 \\ 3x&=42 \\ x&=14 \end{aligned}$$

The solutions are $x=14,\ y=4,\ z=1.$

15. $\begin{cases} 3x-4y+6z=10 \\ 2x-4y-5z=-14 \\ x+2y-3z=0 \end{cases} \xrightarrow[Eq1-3Eq3\leftrightarrow Eq3]{-2Eq1+3Eq2\leftrightarrow Eq2}$

$\begin{cases} 3x-4y+6z=10 \\ -4y-27z=-62 \\ -10y+15z=10 \end{cases} \xrightarrow{5Eq2-2Eq3\leftrightarrow Eq3}$

$\begin{cases} 3x-4y+6z=10 \\ -4y-27z=-62 \\ -165z=-330 \end{cases} \xrightarrow{-\frac{1}{165}Eq3\leftrightarrow Eq3}$

$\begin{cases} 3x-4y+6z=10 \\ -4y-27z=-62 \\ z=2 \end{cases}$

Since z is isolated, back substitution yields:

$$\begin{aligned} -4y-27(2)&=-62 \\ -4y-54&=-62 \\ -4y&=-8 \\ y&=2 \end{aligned}$$

and

$$\begin{aligned} 3x-4y+6z&=10 \\ 3x-4(2)+6(2)&=10 \\ 3x-8+12&=10 \\ 3x&=6 \\ x&=2 \end{aligned}$$

The solutions are $x=2,\ y=2,\ z=2.$

16. $\begin{cases} x-3y+4z=7 \\ 2x+2y-3z=-3 \\ x-3y+z=-2 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-2Eq1+Eq2\to Eq2}$

$\begin{cases} x-3y+4z=7 \\ 8y-11z=-17 \\ -3z=-9 \end{cases} \xrightarrow{-\frac{1}{3}Eq3\to Eq3}$

$\begin{cases} x-3y+4z=7 \\ 8y-11z=-17 \\ z=3 \end{cases}$

Since z is isolated, back substitution yields

$$\begin{aligned} 8y-11(3)&=-17 \\ 8y-33&=-17 \\ 8y&=16 \\ y&=2 \end{aligned}$$

and

$$\begin{aligned} x-3y+4z&=7 \\ x-3(2)+4(3)&=7 \\ x-6+12&=7 \\ x&=1 \end{aligned}$$

The solutions are $x=1,\ y=2,\ z=3.$

17. $\begin{cases} x-3y+z=-2 \\ \quad y-2z=-4 \end{cases}$

Let z be any real number.

$y=2z-4$

and

$$\begin{aligned} x &= 3y-z-2 \\ &= 3(2z-4)-z-2 \\ &= 6z-12-z-2 \\ &= 5z-14 \end{aligned}$$

There are infinitely many solutions to the system all fitting the form

$x=5z-14,\ \ y=2z-4,\ \ z.$

18. $\begin{cases} x+2y+2z=4 \\ \quad y+3z=6 \end{cases}$

Let z be any real number.

$y=6-3z$

and

$$\begin{aligned} x &= -2y-2z+4 \\ &= -2(6-3z)-2z+4 \\ &= -12+6z-2z+4 \\ &= 4z-8 \end{aligned}$$

There are infinitely many solutions to the system all fitting the form

$x=4z-8,\ \ y=6-3z,\ \ z.$

19. $\begin{cases} x-y+2z=4 \\ \quad y+8z=16 \end{cases}$

Let z be any real number.

$y=16-8z$

and

$$\begin{aligned} x &= y-2z+4 \\ &= (16-8z)-2z+4 \\ &= 20-10z \end{aligned}$$

There are infinitely many solutions to the system all fitting the form

$x=20-10z,\ \ y=16-8z,\ \ z.$

20. Since $0\neq 4$, the system is inconsistent and has no solution.

21. $\begin{cases} x-y+2z=-1 \\ 2x+2y+3z=-3 \\ x+3y+z=-2 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-2Eq1+Eq2\to Eq2}$

$\begin{cases} x-y+2z=-1 \\ \quad 4y-z=-1 \\ \quad 4y-z=-1 \end{cases} \xrightarrow{-1Eq2+Eq3\to Eq3}$

$\begin{cases} x-y+2z=-1 \\ \quad 4y-z=-1 \\ \quad\quad 0=0 \end{cases}$

Let z be any real number.

$4y-z=-1$

$4y=z-1$

$y=\frac{1}{4}z-\frac{1}{4}$

and

$$\begin{aligned} x &= y-2z-1 \\ &= \left(\frac{1}{4}z-\frac{1}{4}\right)-2z-1 \\ &= -\frac{7}{4}z-\frac{5}{4} \end{aligned}$$

There are infinitely many solutions to the system all fitting the form

$x=-\frac{7}{4}z-\frac{5}{4},\ \ y=\frac{1}{4}z-\frac{1}{4},\ \ z.$

22.

$\begin{cases} x+5y-2z=5 \\ x+3y-2z=23 \\ x+4y-2z=14 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-1Eq1+Eq2\to Eq2}$

$\begin{cases} x+5y-2z=5 \\ \quad -2y=18 \\ \quad -y=9 \end{cases} \xrightarrow[-\frac{1}{2}Eq2+Eq3\to Eq3]{-\frac{1}{2}Eq2\to Eq2}$

$\begin{cases} x+5y-2z=5 \\ \quad y=-9 \\ \quad 0=0 \end{cases}$

Let z be any real number.

$y=-9$

$$\begin{aligned} x &= -5y+2z+5 \\ &= -5(-9)+2z+5 \\ &= 2z+50 \end{aligned}$$

There are infinitely many solutions to the system all fitting the form $x=2z+50,\ \ y=-9,\ z.$

23. $\begin{cases} 3x-4y-6z=10 \\ 2x-4y-5z=-14 \\ x \qquad -z=0 \end{cases} \xrightarrow[Eq1-3Eq3\to Eq3]{Eq2-2Eq3\to Eq2}$

$\begin{cases} 3x-4y-6z=10 \\ -4y-3z=-14 \\ -4y-3z=10 \end{cases} \xrightarrow{-1Eq2+Eq3\to Eq3}$

$\begin{cases} 3x-4y-6z=10 \\ -4y-3z=-14 \\ 0=24 \end{cases}$

Since $0\neq 24$, the system is inconsistent and has no solution.

24. $\begin{cases} 3x-9y+4z=24 \\ 2x-6y+2z=6 \\ x-3y+2z=20 \end{cases} \xrightarrow[Eq1-3Eq3\to Eq3]{Eq2-2Eq3\to Eq2}$

$\begin{cases} 3x-9y+4z=24 \\ -2z=-34 \\ -2z=-36 \end{cases} \xrightarrow{-1Eq2+Eq3\to Eq3}$

$\begin{cases} 3x-9y+4z=24 \\ -2z=-34 \\ 0=-2 \end{cases}$

Since $0\neq -2$, the system is inconsistent and has no solution.

25. $\begin{cases} 2x+3y-14z=16 \\ 4x+5y-30z=34 \\ x+y-8z=9 \end{cases} \xrightarrow[Eq1-2Eq3\to Eq3]{Eq2-4Eq3\to Eq2}$

$\begin{cases} 2x+3y-14z=16 \\ y+2z=-2 \\ y+2z=-2 \end{cases} \xrightarrow{-1Eq2+Eq3\to Eq3}$

$\begin{cases} 2x+3y-14z=16 \\ y+2z=-2 \\ 0=0 \end{cases}$

Let z be any real number.

$y=-2z-2$

and

$$\begin{aligned} x &= -y+8z+9 \\ &= -(-2z-2)+8z+9 \\ &= 10z+11 \end{aligned}$$

There are infinitely many solutions to the system all fitting the form $x=10z+11,\ \ y=-2z-2,\ \ z.$

26. $\begin{cases} 2x+4y-4z=6 \\ x+5y-3z=3 \\ 3x+3y-5z=9 \end{cases} \xrightarrow[3Eq1-2Eq3\to Eq3]{-3Eq2+Eq3\to Eq2}$

$\begin{cases} 2x+4y-4z=6 \\ -12y+4z=0 \\ 6y-2z=0 \end{cases} \xrightarrow{Eq2+2Eq3\to Eq3}$

$\begin{cases} 2x+4y-4z=6 \\ -12y+4z=0 \\ 0=0 \end{cases}$

Let z be any real number.

$-12y+4z=0$

$-12y=-4z$

$y=\frac{-4z}{-12}=\frac{1}{3}z$

and

$2x+4y-4z=6$

$2x=-4y+4z+6$

$x=\frac{-4\left(\frac{1}{3}z\right)+4z+6}{2}$

$x=\frac{\frac{8}{3}z+6}{2}$

$x=\frac{4}{3}z+3$

There are infinitely many solutions to the system all fitting the form

$x=\frac{4}{3}z+3,\ \ y=\frac{1}{3}z,\ \ z.$

Section 7.1 Exercises

27. $\begin{cases} x+y+z=60 \\ 15{,}000x+25{,}000y+45{,}000z=1{,}400{,}000 \\ 30x+40y+50z=2200 \end{cases} \xrightarrow[-30Eq1+Eq3\to Eq3]{-15{,}000Eq1+Eq2\to Eq2}$

$\begin{cases} x+y+z=60 \\ 10{,}000y+30{,}000z=500{,}000 \\ 10y+20z=400 \end{cases} \xrightarrow{Eq2-1000Eq3\to Eq3}$

$\begin{cases} x+y+z=60 \\ 10{,}000y+30{,}000z=500{,}000 \\ 10{,}000z=100{,}000 \end{cases} \xrightarrow{\frac{1}{10{,}000}Eq3\to Eq3}$

$\begin{cases} x+y+z=60 \\ 10{,}000y+30{,}000z=500{,}000 \\ z=10 \end{cases}$

Since z is isolated, back substitution yields:

$$\begin{aligned} 10{,}000y+30{,}000(10)&=500{,}000 \\ 10{,}000y+300{,}000&=500{,}000 \\ 10{,}000y&=200{,}000 \\ y&=20 \end{aligned}$$

and

$$\begin{aligned} x+y+z&=60 \\ x+20+10&=60 \\ x&=30 \end{aligned}$$

The solution to the system is $x=30,\ y=20,\ z=10.$ The agency should purchase 30 compact cars, 20 midsize cars, and 10 luxury cars.

28. Note that $x=2(y+z)$ can be rewritten as $x-2y-2z=0.$

$\begin{cases} x+y+z=1800 \\ x-2y-2z=0 \\ 20x+35y+50z=48{,}000 \end{cases} \xrightarrow[-20Eq1+Eq3\to Eq3]{-1Eq1+Eq2\to Eq2}$

$\begin{cases} x+y+z=1800 \\ -3y-3z=-1800 \\ 15y+30z=12{,}000 \end{cases} \xrightarrow{5Eq2+Eq3\to Eq3}$

$$\begin{cases} x+y+z=1800 \\ -3y-3z=-1800 \\ 15z=3000 \end{cases} \xrightarrow{\frac{1}{15}Eq3 \to Eq3}$$

$$\begin{cases} x+y+z=1800 \\ -3y-3z=-1800 \\ z=200 \end{cases}$$

Since z is isolated, back substitution yields:

$$\begin{aligned} -3y-3(200)&=-1800 \\ -3y-600&=-1800 \\ -3y&=-1200 \\ y&=400 \\ &\text{and} \\ x+y+z&=1800 \\ x+400+200&=1800 \\ x&=1200 \end{aligned}$$

The solution to the system is $x=1200$, $y=400$, $z=200$. The theater owner should sell 1200 \$20 tickets, 400 \$35 tickets, and 200 \$50 tickets.

29. **a.** $x+y=2600$

b. $40x$

c. $60y$

d. $40x+60y=120{,}000$

e. $$\begin{cases} x+y=2600 \\ 40x+60y=120{,}000 \end{cases} \xrightarrow{-40Eq1+Eq2 \to Eq2}$$

$$\begin{cases} x+y=2600 \\ 20y=16{,}000 \end{cases} \xrightarrow{\frac{1}{20}Eq2 \to Eq2}$$

$$\begin{cases} x+y=2600 \\ y=800 \end{cases}$$

Since y is isolated, back substitution yields:

$$\begin{aligned} x+y&=2600 \\ x+800&=2600 \\ x&=1800 \end{aligned}$$

The solution to the system is $x = 1800,\ y = 800$. The concert promoter must sell 1800 \$40 tickets and 800 \$60 tickets.

30. a. $x + y + z = 500{,}000$

b. $8\%x + 10\%y + 14\%z = 49{,}000$, or rewriting
$0.08x + 0.10y + 0.14z = 49{,}000$

c. $x = y + z$, or rewriting
$x - y - z = 0$

d.
$$\begin{cases} x + y + z = 500{,}000 \\ 0.08x + 0.10y + 0.14z = 49{,}000 \\ x - y - z = 0 \end{cases} \xrightarrow[-1Eq1+Eq3 \to Eq3]{-0.08Eq1+Eq2 \to Eq2}$$

$$\begin{cases} x + y + z = 500{,}000 \\ 0.02y + 0.06z = 9000 \\ -2y - 2z = -500{,}000 \end{cases} \xrightarrow{100Eq2+Eq3 \to Eq3}$$

$$\begin{cases} x + y + z = 500{,}000 \\ 0.02y + 0.06z = 9000 \\ 4z = 400{,}000 \end{cases} \xrightarrow{\frac{1}{4}Eq3 \to Eq3}$$

$$\begin{cases} x + y + z = 500{,}000 \\ 0.02y + 0.06z = 9000 \\ z = 100{,}000 \end{cases}$$

Since z is isolated, back substitution yields:

$$\begin{aligned} 0.02y + 0.06(100{,}000) &= 9000 \\ 0.02y + 6000 &= 9000 \\ 0.02y &= 3000 \\ y &= 150{,}000 \end{aligned}$$

and

$$\begin{aligned} x + y + z &= 500{,}000 \\ x + 150{,}000 + 100{,}000 &= 500{,}000 \\ x &= 250{,}000 \end{aligned}$$

The solution to the system is $x = 250{,}000,\ y = 150{,}000,\ z = 100{,}000$. To satisfy the specified investment conditions, \$250,000 should be invested at 8%, \$150,000 invested at 10%, and \$100,000 invested at 14%.

31. a. $x+y+z=400,000$

b. $7.5\%x+8\%y+9\%z=33,700$, or rewriting
$0.075x+0.08y+0.09z=33,700$

c. $z=x+y$, or rewriting
$x+y-z=0$

d.
$$\begin{cases} x+y+z=400,000 \\ 0.075x+0.08y+0.09z=33,700 \\ x+y-z=0 \end{cases} \xrightarrow[-1Eq1+Eq3\to Eq3]{-0.075Eq1+Eq2\to Eq2}$$

$$\begin{cases} x+y+z=400,000 \\ 0.005y+0.015z=3700 \\ -2z=-400,000 \end{cases} \xrightarrow{-\frac{1}{2}Eq3\to Eq3}$$

$$\begin{cases} x+y+z=400,000 \\ 0.005y+0.015z=3700 \\ z=200,000 \end{cases}$$

Since z is isolated, back substitution yields:

$$\begin{aligned} 0.005y+0.015(200,000)&=3700 \\ 0.005y+3000&=3700 \\ 0.005y&=700 \\ y&=140,000 \end{aligned}$$

and

$$\begin{aligned} x+y+z&=400,000 \\ x+140,000+200,000&=400,000 \\ x&=60,000 \end{aligned}$$

The solution to the system is $x=60,000,\ y=140,000,\ z=200,000.$ \$60,000 is invested in the property returning 7.5%, \$140,000 is invested in the property returning 8%, and \$200,000 is invested in the property returning 9%.

32. a. Let $x=$ cost of property 1, $y=$ cost of property 2, and $z=$ cost of property 3.
$x+y+z=285,000$

b. $x=y+45,000$, or rewriting
$x-y=45,000$

c. $z = 2(x + y)$, or rewriting

$z = 2x + 2y$

$2x + 2y - z = 0$

d. $$\begin{cases} x + y + z = 285{,}000 \\ x - y = 45{,}000 \\ 2x + 2y - z = 0 \end{cases} \xrightarrow[-2Eq1+Eq3 \to Eq3]{-1Eq1+Eq2 \to Eq2}$$

$$\begin{cases} x + y + z = 285{,}000 \\ -2y - z = -240{,}000 \\ -3z = -570{,}000 \end{cases} \xrightarrow{-\frac{1}{3}Eq3 \to Eq3}$$

$$\begin{cases} x + y + z = 285{,}000 \\ -2y - z = -240{,}000 \\ z = 190{,}000 \end{cases}$$

Since z is isolated, back substitution yields:

$$\begin{aligned} -2y - 190{,}000 &= -240{,}000 \\ -2y &= -50{,}000 \\ y &= 25{,}000 \end{aligned}$$

and

$$\begin{aligned} x + y + z &= 285{,}000 \\ x + 25{,}000 + 190{,}000 &= 285{,}000 \\ x &= 70{,}000 \end{aligned}$$

The solution to the system is $x = 70{,}000$, $y = 25{,}000$, $z = 190{,}000$. $70,000 is invested in the first property, $25,000 is invested in the second property, and $190,000 is invested in the third property.

33. Let A = the number of units of product A, B = the number of units of product B, and C = the number of units of product C.

$$\begin{cases} 24A + 20B + 40C = 8000 \\ 40A + 30B + 60C = 12{,}400 \\ 150A + 180B + 200C = 52{,}600 \end{cases} \xrightarrow[-25Eq1+4Eq3 \to Eq3]{-5Eq1+3Eq2 \to Eq2}$$

$$\begin{cases} 24A + 20B + 40C = 8000 \\ -10B - 20C = -2800 \\ 220B - 200C = 10{,}400 \end{cases} \xrightarrow{22Eq2+Eq3 \to Eq3}$$

$$\begin{cases} 24A+20B+40C=8000 \\ -10B-20C=-2800 \\ -640C=-51,200 \end{cases} \xrightarrow{-\frac{1}{640}Eq3 \to Eq3}$$

$$\begin{cases} 24A+20B+40C=8000 \\ -10B-20C=-2800 \\ C=80 \end{cases}$$

Since C is isolated, back substitution yields:

$$\begin{aligned} -10B-20(80) &= -2800 \\ -10B-1600 &= -2800 \\ -10B &= -1200 \\ B &= 120 \end{aligned}$$

and

$$\begin{aligned} 24A+20B+40C &= 8000 \\ 24A+20(120)+40(80) &= 8000 \\ 24A+2400+3200 &= 8000 \\ 24A &= 2400 \\ A &= 100 \end{aligned}$$

The solution to the system is $A=100,\ B=120,\ C=80$. To meet the restrictions imposed on volume, weight, and value, 100 units of product A, 120 units of product B, and 80 of product C can be carried.

34. a. $$\begin{cases} 50x+108y+127z=393 \\ 0.1y+5.5z=5.7 \\ 22x+25.5y+18.0z=91.0 \end{cases}$$

b. $$\begin{cases} 50x+108y+127z=393 \\ 0.1y+5.5z=5.7 \\ 22x+25.5y+18.0z=91.0 \end{cases} \xrightarrow{-11Eq1+25Eq3 \to Eq3}$$

$$\begin{cases} 50x+108y+127z=393 \\ 0.1y+5.5z=5.7 \\ -550.5y-947z=-2048 \end{cases} \xrightarrow{5505Eq2+Eq3 \to Eq3}$$

$$\begin{cases} 50x+108y+127z=393 \\ 0.1y+5.5z=5.7 \\ 29,330.5z=29,330.5 \end{cases} \xrightarrow{\frac{1}{29,330.5}Eq3 \to Eq3}$$

$$\begin{cases} 50x+108y+127z=393 \\ 0.1y+5.5z=5.7 \\ z=1 \end{cases}$$

Since z is isolated, back substitution yields:

$$0.1y + 5.5(1) = 5.7$$
$$0.1y + 5.5 = 5.7$$
$$0.1y = 0.2$$
$$y = 2$$
and
$$50x + 108y + 127z = 393$$
$$50x + 108(2) + 127(1) = 393$$
$$50x + 216 + 127 = 393$$
$$50x + 343 = 393$$
$$50x = 50$$
$$x = 1$$

The solution to the system is $x = 1,\ y = 2,\ z = 1$. The combination consists of 1 ounce of All Bran, 2 ounces of Frosted Flakes, and 1 ounce of Natural Mixed Grain.

35. a. $x + y + z = 500{,}000$ means that the sum of the money invested in the three accounts is $500,000. $0.08x + 0.10y + 0.14z = 49{,}000$ means that the sum of the interest earned on the three accounts in one year is $49,000.

b. $$\begin{cases} x + y + z = 500{,}000 \\ 0.08x + 0.10y + 0.14z = 49{,}000 \end{cases} \xrightarrow{-0.8Eq1 + Eq2 \to Eq2}$$
$$\begin{cases} x + y + z = 500{,}000 \\ 0.02y + 0.06z = 9{,}000 \end{cases}$$
Let $z = z$.
$$0.02y + 0.06z = 9000$$
$$0.02y = 9000 - 0.06z$$
$$y = 450{,}000 - 3z$$
and
$$x + (450{,}000 - 3z) + z = 500{,}000$$
$$x - 2z + 450{,}000 = 500{,}000$$
$$x = 2z + 50{,}000$$
The solution is $x = 2z + 50{,}000,\ y = 450{,}000 - 3z,\ z = z$.

c. $z \ge 0$ implies that $x = 2z + 50{,}000 \ge 0$.

For $y \ge 0$, $450{,}000 - 3z \ge 0$.

Solving for z,

$-3z \ge -450{,}000$

$z \le \dfrac{-450{,}000}{-3}$

$z \le 150{,}000$

For all investments to be non-negative, $0 \le z \le 150{,}000$.

36. a. $\begin{cases} x + y + z = 200{,}000 \\ z = 45{,}000 + (x + y) \end{cases}$

or

$\begin{cases} x + y + z = 200{,}000 \\ -x - y + z = 45{,}000 \end{cases}$

b. Since there are more variables than equations, the system will not have a unique solution.

c. $\begin{cases} x + y + z = 200{,}000 \\ -x - y + z = 45{,}000 \end{cases} \xrightarrow{Eq1+Eq2 \to Eq2}$

$\begin{cases} x + y + z = 200{,}000 \\ 2z = 245{,}000 \end{cases} \xrightarrow{\frac{1}{2}Eq2 \to Eq2}$

$\begin{cases} x + y + z = 200{,}000 \\ z = 122{,}500 \end{cases}$

$z = 122{,}500$

Let $y = y$.

$x + y = 77{,}500$

$x = 77{,}500 - y$

The solution is $x = 77{,}500 - y$, $y = y$, $z = 122{,}500$. The largest loan is $122,500.

Note that $0 \le y \le 77{,}500$. Otherwise x is negative.

d. The sum of the other two loans is $77,500.

37. Let x = the number of Acclaim units produced, y = the number of Bestfrig units produced, and z = the number of Cool King units produced.

$$\begin{cases} 5x+4y+4.5z=300 \\ 2x+1.4y+1.7z=120 \\ 1.4x+1.2y+1.3z=210 \end{cases} \xrightarrow[-0.7Eq2+Eq3\to Eq3]{-2Eq1+5Eq2\to Eq2}$$

$$\begin{cases} 5x+4y+4.5z=300 \\ -1y-0.5z=0 \\ 0.22y+0.11z=126 \end{cases} \xrightarrow{0.22Eq2+Eq3\to Eq3}$$

$$\begin{cases} 5x+4y+4.5z=300 \\ -1y-0.5z=0 \\ 0=126 \end{cases}$$

Since $0 \neq 126$, the system is inconsistent and has no solution. It is not possible to satisfy the given manufacturing conditions.

38. Let x = the number of units of product A, y = the number of units of product B, and z = the number of units of product C.

$$\begin{cases} 24x+20y+50z=4000 \\ 36x+30y+75z=6000 \\ 150x+180y+120z=24,450 \end{cases} \xrightarrow[-25Eq1+4Eq3\to Eq3]{-3Eq1+2Eq2\to Eq2}$$

$$\begin{cases} 24x+20y+50z=4000 \\ 0=0 \\ 220y-770z=-2200 \end{cases} \xrightarrow{Eq2\leftrightarrow Eq3}$$

$$\begin{cases} 24x+20y+50z=4000 \\ 220y-770z=-2200 \\ 0=0 \end{cases}$$

Let $z = z$.

$220y-770z=-2200$

$220y=770z-2200$

$y=3.5z-10$

and

$24x+20(3.5z-10)+50z=4000$

$24x+70z-200+50z=4000$

$24x+120z-200=4000$

$24x=4200-120z$

$x=175-5z$

The solution is $x=175-5z,\ y=3.5z-10,\ z=z$.

Note that $z \ge 0$. $y \ge 0$ implies that

$$3.5z - 10 \ge 0$$
$$3.5z \ge 10$$
$$z \ge \frac{10}{3.5} \approx 3.$$

$x \ge 0$ implies that

$$175 - 5z \ge 0$$
$$-5z \ge -175$$
$$z \le \frac{175}{5}$$
$$z \le 35.$$

Therefore, $3 \le z \le 35$.

39. Let x = grams of food I, y = grams of food II, and z = grams of food III.

$$\begin{cases} 12\%x + 14\%y + 8\%z = 6.88 \\ 8\%x + 6\%y + 16\%z = 6.72 \\ 10\%x + 10\%y + 12\%z = 6.80 \end{cases}$$

or

$$\begin{cases} 0.12x + 0.14y + 0.08z = 6.88 \\ 0.08x + 0.06y + 0.16z = 6.72 \\ 0.10x + 0.10y + 0.12z = 6.80 \end{cases}$$

$$\begin{cases} 0.12x + 0.14y + 0.08z = 6.88 \\ 0.08x + 0.06y + 0.16z = 6.72 \\ 0.10x + 0.10y + 0.12z = 6.80 \end{cases} \xrightarrow[-5Eq1+6Eq3 \to Eq3]{-2Eq1+3Eq2 \to Eq2}$$

$$\begin{cases} 0.12x + 0.14y + 0.08z = 6.88 \\ \quad -0.10y + 0.32z = 6.40 \\ \quad -0.10y + 0.32z = 6.40 \end{cases} \xrightarrow{-1Eq2+Eq3 \to Eq3}$$

$$\begin{cases} 0.12x + 0.14y + 0.08z = 6.88 \\ \quad -0.10y + 0.32z = 6.40 \\ \quad 0 = 0 \end{cases}$$

Let $z = z$.

$-0.10y + 0.32z = 6.40$

$-0.10y = 6.40 - 0.32z$

$y = 3.2z - 64$

and

$0.12x + 0.14y + 0.08z = 6.88$

$0.12x + 0.14(3.2z - 64) + 0.08z = 6.88$

$0.12x + 0.448z - 8.96 + 0.08z = 6.88$

$0.12x + 0.528z - 8.96 = 6.88$

$0.12x = 15.84 - 0.528z$

$x = \dfrac{15.84 - 0.528z}{0.12}$

$x = 132 - 4.4z$

The solution is $x = 132 - 4.4z$, $y = 3.2z - 64$, $z = z$.

Note that $z \geq 0$. $y \geq 0$ implies that $3.2z - 64 \geq 0$.

$$3.2z \geq 64$$
$$z \geq \frac{64}{3.2}$$
$$z \geq 20$$

$x \geq 0$ implies that $132 - 4.4z \geq 0$.

$$-4.4z \geq -132$$
$$z \leq \frac{-132}{-4.4}$$
$$z \leq 30$$

Therefore, $20 \leq z \leq 30$.

40. a. Let x = amount invested in the 8% account, y = amount invested in the 7.5% account, and z = amount invested in the 10% account.

$$\begin{cases} x + y + z = 1,400,000 \\ 0.08x + 0.075y + 0.10z = 120,000 \end{cases} \xrightarrow{-0.08Eq1 + Eq2 \to Eq2}$$

$$\begin{cases} x + y + z = 1,400,000 \\ -0.005y + 0.02z = 8000 \end{cases}$$

Let $z = z$.

$-0.005y + 0.02z = 8000$

$-0.005y = -0.02z + 8000$

$y = \dfrac{-0.02z + 8000}{-0.005}$

$y = 4z - 1{,}600{,}000$

and

$x + y + z = 1{,}400{,}000$

$x + (4z - 1{,}600{,}000) + z = 1{,}400{,}000$

$x = 3{,}000{,}000 - 5z$

The solution is $x = 3{,}000{,}000 - 5z,\ \ y = 4z - 1{,}600{,}000,\ \ z = z.$

b. To minimize investment risk, minimize the amount of money invested in the 10% account.

$$z \ge 0$$

and

$$x \ge 0$$
$$3{,}000{,}000 - 5z \ge 0$$
$$-5z \ge -3{,}000{,}000$$
$$z \le 600{,}000$$

and

$$y \ge 0$$
$$4z - 1{,}600{,}000 \ge 0$$
$$z \ge 400{,}000$$

Therefore, $400{,}000 \le z \le 600{,}000.$

To minimize risk, invest \$400,000 in the 10% account, \$0 in the 7.5% account, and the remainder (\$1,000,000) in the 8% account.

Section 7.2 Skills Check

1. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 1 & -2 & -1 & -2 \\ 2 & 2 & 1 & 11 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 3 & -4 & 6 & 10 \\ 2 & -4 & -5 & -14 \\ 1 & 2 & -3 & 0 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 5 & -3 & 2 & 12 \\ 3 & 6 & -9 & 4 \\ 2 & 3 & -4 & 9 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 7 \\ 2 & 2 & -3 & -3 \\ 1 & -3 & 1 & -2 \end{array}\right]$

5. Since the matrix is in reduced row-echelon form, the solution is $x=-1,\ y=4,\ z=-2$.

6. Since the matrix is in reduced row-echelon form, the solution is $x=-2,\ y=4,\ z=8$.

7. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 1 & -2 & -1 & -2 \\ 2 & 2 & 1 & 11 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{-1R_1+R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -3 & 0 & -6 \\ 0 & 0 & 3 & 3 \end{array}\right] \xrightarrow[\left(\frac{1}{3}\right)R_3\to R_3]{\left(-\frac{1}{3}\right)R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x+y-z=4 \\ y=2 \\ z=1 \end{cases}$$

$x+2-1=4$

$x+1=4$

$x=3$

The solutions are $x=3,\ y=2,\ z=1$.

8. $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 2 & 3 & -1 & 11 \\ 4 & 2 & -4 & 12 \end{array}\right] \xrightarrow[-4R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0 & 5 & -7 & 3 \\ 0 & 6 & -16 & -4 \end{array}\right] \xrightarrow{\frac{1}{5}R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0 & 1 & -\frac{7}{5} & \frac{3}{5} \\ 0 & 6 & -16 & -4 \end{array}\right] \xrightarrow{-6R_2+R_3\to R_3}$

$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0 & 1 & -\frac{7}{5} & \frac{3}{5} \\ 0 & 0 & -\frac{38}{5} & -\frac{38}{5} \end{array}\right] \xrightarrow{\left(-\frac{5}{38}\right)R_3\to R_3}$

$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 0 & 1 & -\frac{7}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 1 \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x - y + 3z = 4 \\ y - \frac{7}{5}z = \frac{3}{5} \\ z = 1 \end{cases}$$

$y - \frac{7}{5}(1) = \frac{3}{5}$

$y = \frac{10}{5}$

$y = 2$

and

$x - (2) + 3(1) = 4$

$x + 1 = 4$

$x = 3$

The solutions are $x = 3,\ y = 2,\ z = 1.$

9. $\left[\begin{array}{ccc|c} 2 & -3 & 4 & 13 \\ 1 & -2 & 1 & 3 \\ 2 & -3 & 1 & 4 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 2 & -3 & 4 & 13 \\ 2 & -3 & 1 & 4 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 1 & -1 & -2 \end{array}\right] \xrightarrow{-1R_2+R_3\to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & -3 & -9 \end{array}\right] \xrightarrow{\left(-\frac{1}{3}\right)R_3\to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x - 2y + z = 3 \\ y + 2z = 7 \\ z = 3 \end{cases}$$

$y + 2(3) = 7$

$y + 6 = 7$

$y = 1$

and

$x - 2(1) + (3) = 3$

$x + 1 = 3$

$x = 2$

The solutions are $x = 2,\ y = 1,\ z = 3.$

10. $\left[\begin{array}{ccc|c} 3 & 1 & 2 & 1 \\ 2 & 3 & -4 & -20 \\ 2 & 4 & 8 & 14 \end{array}\right] \xrightarrow{R_1 - R_2 \to R_1}$

$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 21 \\ 2 & 3 & -4 & -20 \\ 2 & 4 & 8 & 14 \end{array}\right] \xrightarrow[-2R_1 + R_3 \to R_3]{-2R_1 + R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 21 \\ 0 & 7 & -16 & -62 \\ 0 & 8 & -4 & -28 \end{array}\right] \xrightarrow{-1R_2 + R_3 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 21 \\ 0 & 1 & 12 & 34 \\ 0 & 8 & -4 & -28 \end{array}\right] \xrightarrow{-8R_2 + R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 21 \\ 0 & 1 & 12 & 34 \\ 0 & 0 & -100 & -300 \end{array}\right] \xrightarrow{\left(-\frac{1}{100}\right)R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & 6 & 21 \\ 0 & 1 & 12 & 34 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x - 2y + 6z = 21 \\ \quad y + 12z = 34 \\ \qquad z = 3 \end{cases}$$

$y + 12(3) = 34$

$y + 36 = 34$

$y = -2$

and

$x - 2(-2) + 6(3) = 21$

$x + 22 = 21$

$x = -1$

The solutions are $x = -1,\ y = -2,\ z = 3.$

11. Since the third row of the given augmented matrix implies $0 = 1$, the system is inconsistent and has no solution.

12. Since the third row of the augmented matrix states $0 = 0$, the system has infinitely many solutions. Let z be any real number. Then,

$y + 3z = 5$

$y = 5 - 3z$

and

$x + 2z = 1$

$x = 1 - 2z$

There are infinitely many solutions to the system of the form $x = 1 - 2z,\ \ y = 5 - 3z,\ z.$

13. Since the third row of the augmented matrix states $0 = 0$, the system has infinitely many solutions. Let z be any real number. Then,

$y - 5z = 5$

$y = 5z + 5$

and

$x + 3z = 2$

$x = 2 - 3z$

There are infinitely many solutions to the system of the form

$x = 2 - 3z,\ y = 5z + 5,\ z.$

14.

Since the third row of the augmented matrix states $0 = 0$, the system has infinitely many solutions. Let z be any real number. Then,

$y + 2z = -2$

$y = -2 - 2z$

and

$x - z = 3$

$x = z + 3$

There are infinitely many solutions to the system of the form

$x = z + 3,\ y = -2 - 2z,\ z.$

15. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -2 & -1 & 6 \\ 2 & 2 & 1 & 3 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{-1R_1+R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 0 & 6 \\ 0 & 0 & 3 & 3 \end{array}\right] \xrightarrow[\left(\frac{1}{3}\right)R_3\to R_3]{\left(-\frac{1}{3}\right)R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$

Back substituting to find the solutions

$\begin{cases} x+y-z=0 \\ y=-2 \\ z=1 \end{cases}$

$x-2-1=0$

$x-3=0$

$x=3$

The solutions are $x=3,\ y=-2,\ z=1.$

16. $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 2 & -1 & 2 & 6 \\ 3 & 2 & -1 & 1 \end{array}\right] \xrightarrow[-3R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 0 & 3 & 0 & 16 \\ 0 & 8 & -4 & 16 \end{array}\right] \xrightarrow[\left(-\frac{1}{4}\right)R_3\to R_3]{\left(\frac{1}{3}\right)R_2\to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 0 & 1 & 0 & \frac{16}{3} \\ 0 & -2 & 1 & -4 \end{array}\right] \xrightarrow{2R_2+R_3\to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 0 & 1 & 0 & \frac{16}{3} \\ 0 & 0 & 1 & \frac{20}{3} \end{array}\right]$

Back substituting to find the solutions

$\begin{cases} x-2y+z=-5 \\ y=\frac{16}{3} \\ z=\frac{20}{3} \end{cases}$

$x-2\left(\frac{16}{3}\right)+\left(\frac{20}{3}\right)=-5$

$x-\frac{32}{3}+\frac{20}{3}=-5$

$x-\frac{12}{3}=-5$

$x-4=-5$

$x=-1$

The solutions are $x=-1,\ \ y=\frac{16}{3},\ \ z=\frac{20}{3}.$

17. $\left[\begin{array}{ccc|c} 3 & -2 & 5 & 15 \\ 1 & -2 & -2 & -1 \\ 2 & -2 & 0 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1}$

$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 3 & -2 & 5 & 15 \\ 2 & -2 & 0 & 0 \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{-3R_1+R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 0 & 4 & 11 & 18 \\ 0 & 2 & 4 & 2 \end{array}\right] \xrightarrow[\left(\frac{1}{2}\right)R_3 \to R_3]{\left(\frac{1}{4}\right)R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 0 & 1 & \frac{11}{4} & \frac{9}{2} \\ 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{-1R_2+R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 0 & 1 & \frac{11}{4} & \frac{9}{2} \\ 0 & 0 & \frac{-3}{4} & -\frac{7}{2} \end{array}\right] \xrightarrow{\left(-\frac{4}{3}\right)R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 0 & 1 & \frac{11}{4} & \frac{9}{2} \\ 0 & 0 & 1 & \frac{14}{3} \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x-2y-2z=-1 \\ y+\frac{11}{4}z=\frac{9}{2} \\ z=\frac{14}{3} \end{cases}$$

$y+\frac{11}{4}\left(\frac{14}{3}\right)=\frac{9}{2}$

$y+\frac{77}{6}=\frac{27}{6}$

$y=-\frac{50}{6}$

$y=-\frac{25}{3}$

and

$x-2\left(-\frac{25}{3}\right)-2\left(\frac{14}{3}\right)=-1$

$x+\frac{50}{3}-\frac{28}{3}=-1$

$x+\frac{22}{3}=-\frac{3}{3}$

$x=-\frac{25}{3}$

The solutions are $x=-\frac{25}{3},\ y=-\frac{25}{3},\ z=\frac{14}{3}$.

18. $\left[\begin{array}{ccc|c} 1 & 3 & 5 & 5 \\ 2 & 4 & 3 & 9 \\ 2 & 3 & 1 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 5 \\ 0 & -2 & -7 & -1 \\ 0 & -3 & -9 & -9 \end{array}\right] \xrightarrow[\left(-\frac{1}{2}\right)R_2 \to R_3]{\left(-\frac{1}{3}\right)R_3 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 5 \\ 0 & 1 & 3 & 3 \\ 0 & 1 & \frac{7}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{-1R_2+R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 5 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & \frac{1}{2} & -\frac{5}{2} \end{array}\right] \xrightarrow{2R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 5 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x+3y+5z=5 \\ \quad y+3z=3 \\ \qquad z=-5 \end{cases}$$

$y+3(-5)=3$

$y-15=3$

$y=18$

and

$x+3(18)+5(-5)=5$

$x+54-25=5$

$x+29=5$

$x=-24$

The solutions are $x=-24,\ y=18,\ z=-5$.

19. $\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ 2 & 1 & 1 & 1 \end{array}\right] \xrightarrow[-1R_1+R_3 \to R_3]{-3R_1+R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 0 & -2 & -4 & -6 \\ 0 & -2 & -3 & -4 \end{array}\right] \xrightarrow[\left(-\frac{1}{2}\right)R_3 \to R_3]{\left(-\frac{1}{2}\right)R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & \frac{3}{2} & 2 \end{array}\right] \xrightarrow{-1R_2+R_3 \to R_3}$

$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -\frac{1}{2} & -1 \end{array}\right] \xrightarrow[-2R_3 \to R_3]{\left(\frac{1}{2}\right)R_1 \to R_1}$

$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 2 & \frac{5}{2} \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$

Back substituting to find the solutions

$$\begin{cases} x+\frac{3}{2}y+2z=\frac{5}{2} \\ y+2z=3 \\ z=2 \end{cases}$$

$y+2(2)=3$

$y+4=3$

$y=-1$

and

$x+\frac{3}{2}(-1)+2(2)=\frac{5}{2}$

$x-\frac{3}{2}+4=\frac{5}{2}$

$x+\frac{5}{2}=\frac{5}{2}$

$x=0$

The solutions are $x=0,\ y=-1,\ z=2$.

20. $$\left[\begin{array}{ccc|c} 4 & 3 & 8 & 1 \\ 2 & 3 & 8 & 5 \\ 2 & 5 & 5 & 6 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 8 & 5 \\ 4 & 3 & 8 & 1 \\ 2 & 5 & 5 & 6 \end{array}\right] \xrightarrow[-1R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 8 & 5 \\ 0 & -3 & -8 & -9 \\ 0 & 2 & -3 & 1 \end{array}\right] \xrightarrow[\left(\frac{1}{2}\right)R_3 \to R_3]{\left(-\frac{1}{3}\right)R_2 \to R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 8 & 5 \\ 0 & 1 & \frac{8}{3} & 3 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{-1R_2+R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 8 & 5 \\ 0 & 1 & \frac{8}{3} & 3 \\ 0 & 0 & -\frac{25}{6} & -\frac{5}{2} \end{array}\right] \xrightarrow{\left(-\frac{6}{25}\right)R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 8 & 5 \\ 0 & 1 & \frac{8}{3} & 3 \\ 0 & 0 & 1 & \frac{3}{5} \end{array}\right]$$

Back substituting to find the solutions

$$\begin{cases} 2x+3y+8z=5 \\ y+\frac{8}{3}z=3 \\ z=\frac{3}{5} \end{cases}$$

$$y+\frac{8}{3}\left(\frac{3}{5}\right)=3$$

$$y+\frac{8}{5}=\frac{15}{5}$$

$$y=\frac{7}{5}$$

and

$$2x+3\left(\frac{7}{5}\right)+8\left(\frac{3}{5}\right)=5$$

$$2x+\frac{21}{5}+\frac{24}{5}=5$$

$$2x+\frac{45}{5}=5$$

$$2x+9=5$$

$$2x=-4$$

$$x=-2$$

The solutions are $x=-2,\ y=\frac{7}{5},\ z=\frac{3}{5}$.

21. $\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 2 & 0 & 4 & 1 & 5 \\ 2 & -3 & 1 & 0 & -5 \\ 0 & 1 & 2 & 20 & 4 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 2 & 2 & 3 & 9 \\ 0 & -1 & -1 & 2 & -1 \\ 0 & 1 & 2 & 20 & 4 \end{array}\right] \xrightarrow[R_3+R_4\to R_4]{2R_3+R_2\to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 0 & 0 & 7 & 7 \\ 0 & -1 & -1 & 2 & -1 \\ 0 & 0 & 1 & 22 & 3 \end{array}\right] \xrightarrow{\substack{R_3\to R_2 \\ R_2\to R_4 \\ R_4\to R_3}}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & -1 & -1 & 2 & -1 \\ 0 & 0 & 1 & 22 & 3 \\ 0 & 0 & 0 & 7 & 7 \end{array}\right] \xrightarrow{\substack{-1R_2\to R_2 \\ \left(\frac{1}{7}\right)R_4\to R_4}}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & 22 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Back substituting to find the solutions

$$\begin{cases} x-y+z-w=-2 \\ \quad y+z-2w=1 \\ \qquad z+22w=3 \\ \qquad\qquad w=1 \end{cases}$$

$z+22(1)=3$

$z=-19$

and

$y+(-19)-2(1)=1$

$y-19-2=1$

$y-21=1$

$y=22$

and

$x-(22)+(-19)-(1)=-2$

$x-22-19-1=-2$

$x-42=-2$

$x=40$

The solutions are $x=40,\ y=22,$ $z=-19,\ w=1.$

22.

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 2 & -3 & 4 & 1 & 12 \\ 2 & -3 & 1 & -4 & 7 \\ 1 & -1 & 1 & 1 & 4 \end{array}\right] \xrightarrow[\substack{-2R_1+R_3\to R_3 \\ -1R_1+R_4\to R_4}]{-2R_1+R_2\to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 0 & 1 & 2 & 7 & -8 \\ 0 & 1 & -1 & 2 & -13 \\ 0 & 1 & 0 & 4 & -6 \end{array}\right] \xrightarrow[-1R_2+R_4\to R_4]{-1R_2+R_3\to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 0 & 1 & 2 & 7 & -8 \\ 0 & 0 & -3 & -5 & -5 \\ 0 & 0 & -2 & -3 & 2 \end{array}\right] \xrightarrow{\left(-\frac{1}{3}\right)R_3\to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 0 & 1 & 2 & 7 & -8 \\ 0 & 0 & 1 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & -2 & -3 & 2 \end{array}\right] \xrightarrow{2R_3+R_4\to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 0 & 1 & 2 & 7 & -8 \\ 0 & 0 & 1 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{16}{3} \end{array}\right] \xrightarrow{3R_4\to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -3 & 10 \\ 0 & 1 & 2 & 7 & -8 \\ 0 & 0 & 1 & \frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 & 16 \end{array}\right]$$

Back substituting to find the solutions

$$\begin{cases} x-2y+z-3w=10 \\ \quad y+2z+7w=-8 \\ \qquad z+\frac{5}{3}w=\frac{5}{3} \\ \qquad\quad w=16 \end{cases}$$

$z+\frac{5}{3}(16)=\frac{5}{3}$

$z+\frac{80}{3}=\frac{5}{3}$

$z=-\frac{75}{3}=-25$

and

$y+2(-25)+7(16)=-8$

$y-50+112=-8$

$y+62=-8$

$y=-70$

and

$x-2(-70)+(-25)-3(16)=10$

$x+140-25-48=10$

$x+67=10$

$x=-57$

The solutions are $x=-57$, $y=-70$, $z=-25$, $w=16$.

23. $\left[\begin{array}{ccc|c} -2 & 3 & 2 & 13 \\ -2 & -2 & 3 & 0 \\ 4 & 1 & 4 & 11 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
      [[1 0 0 0]
       [0 1 0 3]
       [0 0 1 2]]
```

The solution to the system is $x=0, y=3, z=2.$

24. $\left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 6 & 7 & 8 & 9 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
    [[1 0 -1  -2]
     [0 1 2   3 ]
     [0 0 0   0 ]]
```

Since the third row of the augmented matrix states $0=0$, the system has infinitely many solutions. Let z be any real number. Then,

$y+2z=3$

$y=3-2z$

and

$x-z=-2$

$x=z-2$

There are infinitely many solutions to the system of the form $x=z-2$, $y=3-2z$, z.

25. $\left[\begin{array}{ccc|c} 2 & 5 & 6 & 6 \\ 3 & -2 & 2 & 4 \\ 5 & 3 & 8 & 10 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
[[1 0 22/19 32/…
 [0 1 14/19 10/…
 [0 0 0     0  …
```

or

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{22}{19} & \frac{32}{19} \\ 0 & 1 & \frac{14}{19} & \frac{10}{19} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Since the third row of the augmented matrix states $0 = 0,$ the system has infinitely many solutions. Let z be any real number. Then,

$$y + \frac{14}{19}z = \frac{10}{19}$$
$$y = \frac{10}{19} - \frac{14}{19}z$$

and

$$x + \frac{22}{19}z = \frac{32}{19}$$
$$x = \frac{32}{19} - \frac{22}{19}z$$

There are infinitely many solutions to the system of the form

$$x = \frac{32}{19} - \frac{22}{19}z, \quad y = \frac{10}{19} - \frac{14}{19}z, \quad z.$$

26. $\left[\begin{array}{ccc|c} -1 & 5 & -3 & 10 \\ 3 & 7 & 2 & 5 \\ 4 & 12 & -1 & 15 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
[[1 0 0 0      ]
 [0 1 0 35/31  ]
 [0 0 1 -45/31]]
```

The solution to the system is

$$x = 0, \quad y = \frac{35}{31}, \quad z = -\frac{45}{31}.$$

27. $\left[\begin{array}{ccc|c} -1 & -5 & 3 & -2 \\ 3 & 7 & 2 & 5 \\ 4 & 12 & -1 & 7 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
…1 0 31/8   11/8…
…0 1 -11/8 1/8 …
…0 0 0      0   …
```

or

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{31}{8} & \frac{11}{8} \\ 0 & 1 & -\frac{11}{8} & \frac{1}{8} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Since the third row of the augmented matrix states $0 = 0,$ the system has infinitely many solutions. Let z be any real number. Then,

$$y - \frac{11}{8}z = \frac{1}{8}$$
$$y = \frac{11}{8}z + \frac{1}{8}$$

and

$$x + \frac{31}{8}z = \frac{11}{8}$$
$$x = \frac{11}{8} - \frac{31}{8}z$$

There are infinitely many solutions to the system of the form

$$x = \frac{11}{8} - \frac{31}{8}z, \quad y = \frac{11}{8}z + \frac{1}{8}, \quad z.$$

28. $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 12 \\ 2 & -6 & 1 & 7 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([B])▸Frac
  [[1  -3 0 2/3 ]
   [0 0  1 17/3]]
```

Since the augmented matrix contains two equations with three variables, the system has infinitely many solutions.

$z = \frac{17}{3}$

Let y be any real number. Then,

$x - 3y = \frac{2}{3}$

$x = 3y + \frac{2}{3}$

There are infinitely many solutions to the system of the form

$x = 3y + \frac{2}{3},\ \ y,\ \ z = \frac{17}{3}.$

29. $\left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 4 & 1 & -3 & 6 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([B])▸Frac
…1 0  -1/2 23/14…
…0 1  -1    -4/7 …
```

or

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & \frac{23}{14} \\ 0 & 1 & -1 & -\frac{4}{7} \end{array}\right]$$

Since the augmented matrix contains two equations with three variables, the system has infinitely many solutions. Let z be any real number. Then,

$y - z = -\frac{4}{7}$

$y = z - \frac{4}{7}$

and

$x - \frac{1}{2}z = \frac{23}{14}$

$x = \frac{1}{2}z + \frac{23}{14}$

There are infinitely many solutions to the system of the form

$x = \frac{1}{2}z + \frac{23}{14},\ \ y = z - \frac{4}{7},\ \ z.$

30. $\left[\begin{array}{ccc|c} 3 & 2 & -1 & 4 \\ 2 & -3 & 1 & 3 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
[[1 0  -1/13 18/…
 [0 1  -5/13  -1/…
```

or

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{13} & \frac{18}{13} \\ 0 & 1 & -\frac{5}{13} & -\frac{1}{13} \end{array}\right]$$

Since the augmented matrix contains two equations with three variables, the system has infinitely many solutions. Let z be any real number. Then,

$$y-\frac{5}{13}z=-\frac{1}{13}$$
$$y=\frac{5}{13}z-\frac{1}{13}$$

and

$$x-\frac{1}{13}z=\frac{18}{13}$$
$$x=\frac{1}{13}z+\frac{18}{13}$$

There are infinitely many solutions to the system of the form

$$x=\frac{1}{13}z+\frac{18}{13},\quad y=\frac{5}{13}z-\frac{1}{13},\quad z.$$

31. $\left[\begin{array}{cccc|c} 1 & 0 & -3 & -3 & -2 \\ 1 & 1 & 1 & 3 & 2 \\ 2 & 1 & -2 & -2 & 0 \\ 3 & 2 & -1 & 1 & 2 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([C])▸Frac
 [[1 0 -3 0 -2]
  [0 1 4  0 4 ]
  [0 0 0  1 0 ]
  [0 0 0  0 0 ]]
```

Since the fourth row of the augmented matrix states $0=0$, the system has infinitely many solutions.

$w=0$

Let z be any real number. Then,

$y+4z=4$

$y=4-4z$

and

$x-3z=-2$

$x=3z-2$

There are infinitely many solutions to the system of the form

$x=3z-2,\ y=4-4z,\ z,\ w=0.$

32. $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 5 & 10 \\ 1 & 2 & 1 & 6 & 16 \\ 1 & 1 & 2 & 7 & 11 \\ 2 & 3 & 3 & 13 & 27 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([C])▸Frac
  [[1 0 0 2 3]
   [0 1 0 1 6]
   [0 0 1 2 1]
   [0 0 0 0 0]]
```

Since the fourth row of the augmented matrix states $0 = 0,$ the system has infinitely many solutions.
Let w be any real number. Then,

$$z + 2w = 1$$
$$z = 1 - 2w$$

and

$$y + w = 6$$
$$y = 6 - w$$

and

$$x + 2w = 3$$
$$x = 3 - 2w$$

There are infinitely many solutions to the system of the form
$x = 3 - 2w, \;\; y = 6 - w, \;\; z = 1 - 2w, \;\; w.$

Section 7.2 Exercises

33. Let x, y, and z represent the number of tickets in each section.

Note that $x = 2(y + z)$, which can be rewritten as $x - 2y - 2z = 0.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3600 \\ 1 & -2 & -2 & 0 \\ 40 & 70 & 100 & 192{,}000 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
  [[1 0 0 2400]
   [0 1 0 800 ]
   [0 0 1 400 ]]
```

The solution to the system is $x = 2400, \;\; y = 800, \;\; z = 400.$ The theater owner should sell 2400 \$40 tickets, 800 \$70 tickets, and 400 \$100 tickets.

34. Let x, y, and z represent the number of compact, midsize, and luxury cars respectively.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 90 \\ 18{,}000 & 25{,}000 & 40{,}000 & 2{,}270{,}000 \\ 25 & 35 & 55 & 3150 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
    [[1 0 0 40]
     [0 1 0 30]
     [0 0 1 20]]
```

The solution to the system is $x = 40, \;\; y = 30, \;\; z = 20.$ The agency should purchase 40 compact cars, 30 midsize cars, and 20 luxury cars.

35. a. Let x = the points per each true-false question, y = the points per each multiple-choice question, and $z =$ the points per each essay question.

$15x + 10y + 5z = 100$

and

$2x = y,$ or rewriting

$2x - y = 0$

and

$3x = z,$ or rewriting

$3x - z = 0$

The system is

$$\begin{cases} 15x + 10y + 5z = 100 \\ 2x - y = 0 \\ 3x - z = 0 \end{cases}$$

b. $\left[\begin{array}{ccc|c} 15 & 10 & 5 & 100 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
      [[1 0 0 2]
       [0 1 0 4]
       [0 0 1 6]]
```

The solution to the system is $x = 2,\ \ y = 4,\ \ z = 6.$ On the exam, true-false questions are worth two points, multiple-choice questions are worth four points, and essay questions are worth six points.

36. a. Let x = the number of true-false questions, y = the number of multiple-choice questions, and $z =$ the number of essay questions.

$x + y + z = 35$

and

$2z = y,$ or rewriting

$y - 2z = 0$

and

$2y = x,$ or rewriting

$x - 2y = 0$

The system is

$$\begin{cases} x + y + z = 35 \\ \quad y - 2z = 0 \\ \quad x - 2y = 0 \end{cases}$$

b. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 35 \\ 0 & 1 & -2 & 0 \\ 1 & -2 & 0 & 0 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
    [[1 0 0 20]
     [0 1 0 10]
     [0 0 1 5 ]]
```

The solution to the system is $x = 20,\ \ y = 10,\ \ z = 5.$ The exam consists of 20 true-false questions, 10 multiple-choice questions, and 5 essay questions.

37. Let x = the number of Plan I units, y = the number of Plan II units, and $z =$ the number of Plan III units.

$$\begin{cases} 4x + 8y + 14z = 42 \\ 2x + 4y + 6z = 20 \\ 6y + 6z = 18 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & 8 & 14 & 42 \\ 2 & 4 & 6 & 20 \\ 0 & 6 & 6 & 18 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
     [[1 0 0 3]
      [0 1 0 2]
      [0 0 1 1]]
```

The solution to the system is $x = 3, \ y = 2, \ z = 1$. The client needs to purchase 3 units of Plan I, 2 units of Plan II, and 1 unit of Plan III to achieve the investment objectives.

38. Let x = number of Deluxe models, y = number of Premium models, and z = number of Ultimate models.

$$\begin{cases} 0.8x + 1y + 1.4z = 55 \\ 1x + 1.5y + 2z = 75 \\ 0.6x + 0.75y + 1z = 40 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0.8 & 1 & 1.4 & 55 \\ 1 & 1.5 & 2 & 75 \\ 0.6 & 0.75 & 1 & 40 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A]
    [[1 0 0 25]
     [0 1 0 0 ]
     [0 0 1 25]]
```

The solution to the system is $x = 25, \ y = 0, \ z = 25$. The manufacturer can produce 25 Deluxe models, no Premium models, and 25 Ultimate models.

39. Let x = grams of Food I, y = grams of Food II, and z = grams of Food III.

$$\begin{cases} 12\%x + 15\%y + 28\%z = 3.74 \\ 8\%x + 6\%y + 16\%z = 2.04 \\ 15\%x + 2\%y + 6\%z = 1.35 \end{cases}$$

or

$$\begin{cases} 0.12x + 0.15y + 0.28z = 3.74 \\ 0.08x + 0.06y + 0.16z = 2.04 \\ 0.15x + 0.02y + 0.06z = 1.35 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0.12 & 0.15 & 0.28 & 3.74 \\ 0.08 & 0.06 & 0.16 & 2.04 \\ 0.15 & 0.02 & 0.06 & 1.35 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
     [[1 0 0 5]
      [0 1 0 6]
      [0 0 1 8]]
```

The solution to the system is $x = 5, y = 6, z = 8$. The psychologist recommends 5 grams of Food I, 6 grams of Food II, and 8 grams of Food III each day.

40. Let x = the amount borrowed at 6%, y = the amount borrowed at 8%, and z = the amount borrowed at 10%.

$$\begin{cases} x + y + z = 440{,}000 \\ x + y = 3z \\ 6\%x + 8\%y + 10\%z = 34{,}400 \end{cases}$$

or

$$\begin{cases} x + y + z = 440{,}000 \\ x + y - 3z = 0 \\ 0.06x + 0.08y + 0.10z = 34{,}400 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 440{,}000 \\ 1 & 1 & -3 & 0 \\ 0.06 & 0.08 & 0.10 & 34{,}400 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
[[1 0 0 150000]
 [0 1 0 180000]
 [0 0 1 110000]]
```

The solution to the system is $x = 150{,}000,\ \ y = 180{,}000,\ \ z = 110{,}000.$ The company borrowed \$150,000 at 6%, \$180,000 at 8%, and \$110,000 at 10%.

41. a. Let x = the amount invested at 8%, y = the amount invested at 10%, and z = the amount invested at 12%.

$$\begin{cases} x + y + z = 400{,}000 \\ 8\%x + 10\%y + 12\%z = 42{,}400 \end{cases}$$

or

$$\begin{cases} x + y + z = 400{,}000 \\ 0.08x + 0.10y + 0.12z = 42{,}400 \end{cases}$$

b. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 400{,}000 \\ 0.08 & 0.10 & 0.12 & 42{,}400 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
…1 0 -1 -120000…
…0 1 2  520000 …
```

Let $z = z$.

$y + 2z = 520{,}000$

$y = 520{,}000 - 2z$

and

$x - z = -120{,}000$

$x = z - 120{,}000$

The solution is

$x = z - 120{,}000,$

$y = 520{,}000 - 2z,\ \ z = z.$

The largest possible investment in the 12% also produces the largest possible investment in the 8% account. The largest investment at 12% must also keep $y \geq 0$. Therefore,

$520{,}000 - 2z \geq 0$

$-2z \geq -520{,}000$

$z \leq \dfrac{-520{,}000}{-2}$

$z \leq 260{,}000$

The maximum value of z is \$260,000. Therefore, the maximum values of x and y are

$x = 260{,}000 - 120{,}000$

$\quad = \$140{,}000$

and

$y = 520{,}000 - 2z$

$\quad = 520{,}000 - 2(260{,}000)$

$\quad = \$0.$

42. a. Let x = the amount invested at 7.5%, y = the amount invested at 10%, and z = the amount invested at 8%.

$$\begin{cases} x + y + z = 235{,}000 \\ 7.5\%x + 10\%y + 8\%z = 18{,}000 \end{cases}$$

or

$$\begin{cases} x + y + z = 235{,}000 \\ 0.075x + 0.10y + 0.08z = 18{,}000 \end{cases}$$

b. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 235{,}000 \\ 0.075 & 0.10 & 0.08 & 18{,}000 \end{array}\right]$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
…1 0 .8 220000]
…0 1 .2 15000 ]]
```

Let $z = z$.

$y + 0.2z = 15{,}000$

$y = 15{,}000 - 0.2z$

and

$x + 0.8z = 220{,}000$

$x = 220{,}000 - 0.8z$

The solution to the system is

$x = 220{,}000 - 0.8z$,

$y = 15{,}000 - 0.2z,\ \ z = z$.

c. Let $z = 60{,}000$.

$x = 220{,}000 - 0.8(60{,}000)$

$x = 172{,}000$

and

$y = 15{,}000 - 0.2(60{,}000)$

$y = 3000$

If \$60,000 is invested in the 8% property, then \$172,000 is invested in the 7.5% property, and \$3000 is invested in the 10% property.

43. a. Let $x =$ the number of portfolio I units, $y =$ the number of portfolio II units, and $z =$ the number of portfolio III units.

$$\begin{cases} 10x + 12y + 10z = 180 \\ 2x + 8y + 6z = 140 \\ 3x + 5y + 4z = 110 \end{cases}$$

b. $$\left[\begin{array}{ccc|c} 10 & 12 & 10 & 180 \\ 2 & 8 & 6 & 140 \\ 3 & 5 & 4 & 110 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields a last row containing all zeros along with an augmented 1.

```
rref([A])▸Frac
   [[1 0 1/7 0]
    [0 1 5/7 0]
    [0 0 0   1]]
```

Therefore, the system is inconsistent and has no solution. The client cannot achieve the desired investment results.

44. Let $x =$ the number of Plan I units, $y =$ the number of Plan II units, and $z =$ the number of Plan III units.

$$\begin{cases} 14x + 4y + 18z = 58 \\ 4x + 2y + 6z = 20 \\ 6x + 6z = 18 \end{cases}$$

$$\left[\begin{array}{ccc|c} 14 & 4 & 18 & 58 \\ 4 & 2 & 6 & 20 \\ 6 & 0 & 6 & 18 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
      [[1 0 1 3]
       [0 1 1 4]
       [0 0 0 0]]
```

Since the last row consists entirely of zeros with a zero as the augment, the system is dependent and has infinitely many solutions.

Let $z = z$

$y + z = 4$

$y = 4 - z$

and

$x + z = 3$

$x = 3 - z$

The solution to the system is

$x = 3 - z,\ \ y = 4 - z,\ \ z = z$.

The table represents all the potential investment choices based on units per plan.

Plan I (x)	Plan II (y)	Plan III (z)
3	4	0
2	3	1
1	2	2
0	1	3

45. a. Let x = the number of \$40,000 cars, y = the number of \$30,000 cars, and z = the number of \$20,000.

$$\begin{cases} x+y+z=4 \\ 40{,}000x+30{,}000y+20{,}000z=100{,}000 \end{cases}$$

b. Since the system has more variables than equations, the system can not have a unique solution.

c. $$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 40{,}000 & 30{,}000 & 20{,}000 & 100{,}000 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
   [[1 0 -1  -2]
    [0 1 2   6 ]]
```

Let $z = z$.

$y + 2z = 6$

$y = 6 - 2z$

and

$x - z = -2$

$x = z - 2$

The solution to the system is

$x = z - 2,\ \ y = 6 - 2z,\ \ z = z.$

d. The only values of z that make sense in the context of the problem are $z = 2$ and $z = 3$. Other values of z create negative solutions for x and y. If $z = 2$, the young man purchases two \$20,000 cars, two \$30,000 cars, and zero \$40,000 cars. If $z = 3$, the young man purchases three \$20,000 cars, zero \$30,000 cars, and one \$40,000 car.

46. Let x = the number of type A clients, y = the number of type B clients, and z = the number of type C clients.

$$\begin{cases} x+y+z=500 \\ 400x+1000y+600z=300{,}000 \\ 600x+400y+200z=200{,}000 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 400 & 1000 & 600 & 300{,}000 \\ 600 & 400 & 200 & 200{,}000 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])►Frac
   [[1 0 0 200]
    [0 1 0 100]
    [0 0 1 200]]
```

200 type A clients, 100 type B clients, and 200 type C clients can be served under the given conditions.

47. $$\begin{cases} x_1 - x_2 = -550 \\ x_2 - x_3 = -1300 \\ x_3 - x_4 = 1200 \\ x_1 - x_4 = -650 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -550 \\ 0 & 1 & -1 & 0 & -1300 \\ 0 & 0 & 1 & -1 & 1200 \\ 1 & 0 & 0 & -1 & -650 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([C])►Frac
…1 0 0 -1  -650]
…0 1 0 -1  -100]
…0 0 1 -1  1200]
…0 0 0 0   0   ]]
```

Let $x_4 =$ any real number, then

$x_1 - x_4 = -650$

$x_1 = x_4 - 650$

and

$x_2 - x_4 = -100$

$x_2 = x_4 - 100$

and

$x_3 - x_4 = 1200$

$x_3 = x_4 + 1200$

The solution is

$x_1 = x_4 - 650, \;\; x_2 = x_4 - 100,$

$x_3 = x_4 + 1200, \;\; x_4.$

Since all the variables must be positive in the physical context of the problem, $x_4 \geq 650.$

48. a. Traffic flow at B is $x_1 + 503 = x_2 + 583.$
Traffic flow at C is $x_2 + 651 = x_3 + 601.$
Traffic flow at D is $x_3 + 350 = x_4 + 450.$

b. Rewriting the equations in part a) yields

$x_4 + 470 = x_1 + 340$

$470 - 340 = x_1 - x_4$

$x_1 - x_4 = 130$

and

$x_1 + 503 = x_2 + 583$

$x_1 - x_2 = 583 - 503$

$x_1 - x_2 = 80$

and

$x_2 + 651 = x_3 + 601$

$x_2 - x_3 = 601 - 651$

$x_2 - x_3 = -50$

and

$x_3 + 350 = x_4 + 450$

$x_3 - x_4 = 450 - 350$

$x_3 - x_4 = 100$

The system is

$$\begin{cases} x_1 - x_4 = 130 \\ x_1 - x_2 = 80 \\ x_2 - x_3 = -50 \\ x_3 - x_4 = 100 \end{cases}.$$

The matrix is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 130 \\ 1 & -1 & 0 & 0 & 80 \\ 0 & 1 & -1 & 0 & -50 \\ 0 & 0 & 1 & -1 & 100 \end{array}\right].$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
[[1 0 0 -1 130]
 [0 1 0 -1 50 ]
 [0 0 1 -1 100]
 [0 0 0 0  0  ]]
```

Let $x_4 =$ any real number.

$x_3 - x_4 = 100$

$x_3 = x_4 + 100$

and

$x_2 - x_4 = 50$

$x_2 = x_4 + 50$

and

$x_1 - x_4 = 130$

$x_1 = x_4 + 130$

The solution is

$x_1 = x_4 + 130, \;\; x_2 = x_4 + 50,$

$x_3 = x_4 + 100, \;\; x_4.$

Traffic from intersection A to intersection B is 130 plus the traffic from intersection D to intersection A. Traffic from intersection B to intersection C is 50 plus the traffic from intersection D to intersection A. Traffic from intersection C to intersection D is 100 plus the traffic from intersection D to intersection A. Traffic is measured by the number of cars moving from one intersection to another.

49. a. Water flow at A is $x_1 + x_2 = 400,000$.
Water flow at B is $x_1 = 100,000 + x_4$.
Water flow at D is $x_3 + x_4 = 100,000$.

Rewriting the equations yields the following system

$$\begin{cases} x_1 + x_2 = 400,000 \\ x_1 - x_4 = 100,000 \\ x_3 + x_4 = 100,000 \\ x_2 - x_3 = 200,000 \end{cases}$$

The matrix is

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 400,000 \\ 1 & 0 & 0 & -1 & 100,000 \\ 0 & 0 & 1 & 1 & 100,000 \\ 0 & 1 & -1 & 0 & 200,000 \end{array}\right]$$

b. Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
[[1 0 0 -1 1000…
 [0 1 0 1  3000…
 [0 0 1 1  1000…
 [0 0 0 0  0    …
```

```
rref([A])
…0 0 -1 100000]
…1 0 1  300000]
…0 1 1  100000]
…0 0 0  0      ]]
```

The system has infinitely many solutions. Let x_4 be any number. Then,
$x_3 = 100,000 - x_4$,
$x_2 = 300,000 - x_4$, and
$x_1 = 100,000 + x_4$.

Water flow from A to B is 100,000 plus the water flow from B to D. Water flow from A to C is 300,000 minus the water flow from B to D. Water flow from C to D is 100,000 minus the water flow from B to D. Water flow is measured by the number of gallons of water moving from one intersection to another.

Section 7.3 Skills Check

1. Only matrices of the same dimensions can be added. Therefore, matrices *A*, *D*, and *E* can be added, since they all have 3 rows and 3 columns. Furthermore, matrices B and F can be added, since they both have 2 rows and 3 columns. Note that a matrix can always be added to itself. Therefore, the following sums can be calculated:

$A+D,\ A+E,\ D+E$

$B+F$

$A+A,\ B+B,\ C+C,\ D+D,\ E+E,\ F+F$

2. To multiply two matrices, the number of columns in the first matrix must equal the number of rows in the second matrix. Therefore, the following products can be calculated:

AC, AD, AE

BA, BC, BD, BE

CB, CF

DA, DC, DE

EA, EC, ED

FA, FC, FD, FE

Additionally, square matrices can be multiplied by themselves. Therefore, $AA,\ DD,\ EE$ can be calculated.

3. To add the matrices, add the corresponding entries.

$$\begin{bmatrix} 1 & 3 & -2 \\ 3 & 1 & 4 \\ -5 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+2 & 3+3 & (-2)+1 \\ 3+3 & 1+4 & 4+(-1) \\ -5+2 & 3+5 & 6+1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & -1 \\ 6 & 5 & 3 \\ -3 & 8 & 7 \end{bmatrix}$$

4. **a.** $D+E$

$$= \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 2 & -7 \\ -5 & 0 & 5 \\ 7 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 3+2 & 1+(-7) \\ 3+(-5) & 4+0 & -1+5 \\ 2+7 & 5+(-4) & 1+(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 5 & -6 \\ -2 & 4 & 4 \\ 9 & 1 & 0 \end{bmatrix}$$

$E+D$

$$= \begin{bmatrix} 9 & 2 & -7 \\ -5 & 0 & 5 \\ 7 & -4 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 5 & -6 \\ -2 & 4 & 4 \\ 9 & 1 & 0 \end{bmatrix}$$

b. Both sums are equal.

5. $3A$

$$=\begin{bmatrix} 3(1) & 3(3) & 3(-2) \\ 3(3) & 3(1) & 3(4) \\ 3(-5) & 3(3) & 3(6) \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 9 & -6 \\ 9 & 3 & 12 \\ -15 & 9 & 18 \end{bmatrix}$$

6. $-4F$

$$=\begin{bmatrix} -4(2) & -4(1) & -4(3) \\ -4(4) & -4(0) & -4(1) \end{bmatrix}$$

$$=\begin{bmatrix} -8 & -4 & -12 \\ -16 & 0 & -4 \end{bmatrix}$$

7. $2D-4A$

$$=2\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix}-4\begin{bmatrix} 1 & 3 & -2 \\ 3 & 1 & 4 \\ -5 & 3 & 6 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6 & 2 \\ 6 & 8 & -2 \\ 4 & 10 & 2 \end{bmatrix}+\begin{bmatrix} -4 & -12 & 8 \\ -12 & -4 & -16 \\ 20 & -12 & -24 \end{bmatrix}$$

$$=\begin{bmatrix} 4+(-4) & 6+(-12) & 2+8 \\ 6+(-12) & 8+(-4) & -2+(-16) \\ 4+20 & 10+(-12) & 2+(-24) \end{bmatrix}$$

$$=\begin{bmatrix} 0 & -6 & 10 \\ -6 & 4 & -18 \\ 24 & -2 & -22 \end{bmatrix}$$

8. $2B-4F$

$$=2\begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & 4 \end{bmatrix}-4\begin{bmatrix} 2 & 1 & 3 \\ 4 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 2 & -2 \\ 6 & 4 & 8 \end{bmatrix}+\begin{bmatrix} -8 & -4 & -12 \\ -16 & 0 & -4 \end{bmatrix}$$

$$=\begin{bmatrix} 4+(-8) & 2+(-4) & -2+(-12) \\ 6+(-16) & 4+0 & 8+(-4) \end{bmatrix}$$

$$=\begin{bmatrix} -4 & -2 & -14 \\ -10 & 4 & 4 \end{bmatrix}$$

9. a. *AD*

$$=\begin{bmatrix}1 & 3 & -2\\ 3 & 1 & 4\\ -5 & 3 & 6\end{bmatrix}\begin{bmatrix}2 & 3 & 1\\ 3 & 4 & -1\\ 2 & 5 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2+9-4 & 3+12-10 & 1-3-2\\ 6+3+8 & 9+4+20 & 3-1+4\\ -10+9+12 & -15+12+30 & -5-3+6\end{bmatrix}$$

$$=\begin{bmatrix}7 & 5 & -4\\ 17 & 33 & 6\\ 11 & 27 & -2\end{bmatrix}$$

DA

$$=\begin{bmatrix}2 & 3 & 1\\ 3 & 4 & -1\\ 2 & 5 & 1\end{bmatrix}\begin{bmatrix}1 & 3 & -2\\ 3 & 1 & 4\\ -5 & 3 & 6\end{bmatrix}$$

$$=\begin{bmatrix}2+9-5 & 6+3+3 & -4+12+6\\ 3+12+5 & 9+4-3 & -6+16-6\\ 2+15-5 & 6+5+3 & -4+20+6\end{bmatrix}$$

$$=\begin{bmatrix}6 & 12 & 14\\ 20 & 10 & 4\\ 12 & 14 & 22\end{bmatrix}$$

b. The products are different.

c. The dimensions of the matrices are the same. Both matrices are 3×3.

10. a. *BC*

$$=\begin{bmatrix}2 & 1 & -1\\ 3 & 2 & 4\end{bmatrix}\begin{bmatrix}1 & 3\\ 2 & 1\\ 3 & -1\end{bmatrix}$$

$$=\begin{bmatrix}2+2-3 & 6+1+1\\ 3+4+12 & 9+2-4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 8\\ 19 & 7\end{bmatrix}$$

CB

$$=\begin{bmatrix}1 & 3\\ 2 & 1\\ 3 & -1\end{bmatrix}\begin{bmatrix}2 & 1 & -1\\ 3 & 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2+9 & 1+6 & -1+12\\ 4+3 & 2+2 & -2+4\\ 6-3 & 3-2 & -3-4\end{bmatrix}$$

$$=\begin{bmatrix}11 & 7 & 11\\ 7 & 4 & 2\\ 3 & 1 & -7\end{bmatrix}$$

10. b. No. The products are different.

10. c. The matrices have different dimensions. BC is 2×2, while CB is 3×3.

11. a.
$$DE=\begin{bmatrix}2&3&1\\3&4&-1\\2&5&1\end{bmatrix}\begin{bmatrix}9&2&-7\\-5&0&5\\7&-4&-1\end{bmatrix}$$
$$=\begin{bmatrix}18-15+7&4+0-4&-14+15-1\\27-20-7&6+0+4&-21+20+1\\18-25+7&4+0+-4&-14+25-1\end{bmatrix}$$
$$=\begin{bmatrix}10&0&0\\0&10&0\\0&0&10\end{bmatrix}$$

$$ED=\begin{bmatrix}9&2&-7\\-5&0&5\\7&-4&-1\end{bmatrix}\begin{bmatrix}2&3&1\\3&4&-1\\2&5&1\end{bmatrix}$$
$$=\begin{bmatrix}18+6-14&27+8-35&9-2-7\\-10+0+10&-15+0+25&-5+0+5\\14-12-2&21-16-5&7+4-1\end{bmatrix}$$
$$=\begin{bmatrix}10&0&0\\0&10&0\\0&0&10\end{bmatrix}$$

Note that $DE=ED$.

b. $\frac{1}{10}DE$
$$=\frac{1}{10}\begin{bmatrix}10&0&0\\0&10&0\\0&0&10\end{bmatrix}$$
$$=\begin{bmatrix}\frac{1}{10}(10)&\frac{1}{10}(0)&\frac{1}{10}(0)\\\frac{1}{10}(0)&\frac{1}{10}(10)&\frac{1}{10}(0)\\\frac{1}{10}(0)&\frac{1}{10}(0)&\frac{1}{10}(10)\end{bmatrix}$$
$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Note that the solution is the 3×3 identity matrix.

12. The identity matrix multiplied by a given matrix produces the original matrix. In this case, $ID=DI=D$.

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}2&3&1\\3&4&-1\\2&5&1\end{bmatrix}=\begin{bmatrix}2&3&1\\3&4&-1\\2&5&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}2&3&1\\3&4&-1\\2&5&1\end{bmatrix}$$

13. $A+B$

$$=\begin{bmatrix}1 & 5\\ 3 & 2\end{bmatrix}+\begin{bmatrix}2a & 3b\\ -c & -2d\end{bmatrix}$$

$$=\begin{bmatrix}1+2a & 5+3b\\ 3-c & 2-2d\end{bmatrix}$$

14. $A-B$

$$=\begin{bmatrix}a & b\\ c & d\\ f & g\end{bmatrix}-\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}a-1 & b-2\\ c-3 & d-4\\ f-5 & g-6\end{bmatrix}$$

15. $3A-2B$

$$=3\begin{bmatrix}a & b\\ c & d\end{bmatrix}-2\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}3a & 3b\\ 3c & 3d\end{bmatrix}+\begin{bmatrix}-2 & -4\\ -6 & -8\end{bmatrix}$$

$$=\begin{bmatrix}3a-2 & 3b-4\\ 3c-6 & 3d-8\end{bmatrix}$$

16. $2A-3B$

$$=2\begin{bmatrix}1 & -3 & 2\\ 2 & 2 & -1\\ 3 & 4 & 2\end{bmatrix}-3\begin{bmatrix}2 & 2 & 2\\ 3 & -2 & -1\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2 & -6 & 4\\ 4 & 4 & -2\\ 6 & 8 & 4\end{bmatrix}+\begin{bmatrix}-6 & -6 & -6\\ -9 & 6 & 3\\ -3 & -3 & -6\end{bmatrix}$$

$$=\begin{bmatrix}-4 & -12 & -2\\ -5 & 10 & 1\\ 3 & 5 & -2\end{bmatrix}$$

17. The new matrix will be $m\times k$.

18. AB does not equal BA in general. However, sometimes AB and BA are equal. See Exercises 9, 10, and 11a for examples.

19. a. The product of two matrices can be calculated if the number of columns of the first matrix is equal to the number of rows of the second matrix. Therefore, BA can be calculated since B has 2 columns and A has 2 rows.

b. The matrix formed by the product will by 4×3.

20. CD is 3×3.
DC is 4×4.

21. AB

$$=\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}a+3b+5c & 2a+4b+6c\\ d+3e+5f & 2d+4e+6f\end{bmatrix}$$

BA

$$=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}$$

$$=\begin{bmatrix}a+2d & b+2e & c+2f\\ 3a+4d & 3b+4e & 3c+4f\\ 5a+6d & 5b+6e & 5c+6f\end{bmatrix}$$

22. EF

$$=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}e & f\\ g & h\end{bmatrix}$$

$$=\begin{bmatrix}ae+bg & af+bh\\ ce+dg & cf+dh\end{bmatrix}$$

FE

$$=\begin{bmatrix} e & f \\ g & h \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$=\begin{bmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{bmatrix}$$

23. AB

$$=\begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 2-5 & 3-10 \\ 6-2 & 9-4 \end{bmatrix}$$

$$=\begin{bmatrix} -3 & -7 \\ 4 & 5 \end{bmatrix}$$

BA

$$=\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 2+9 & 10+6 \\ -1-6 & -5-4 \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 16 \\ -7 & -9 \end{bmatrix}$$

24. AB

$$=\begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -1 & 3 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 4-4 & 2+12 & 2+4 \\ 12+1 & 6-3 & 6-1 \\ -8-2 & -4+6 & -4+2 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 14 & 6 \\ 13 & 3 & 5 \\ -10 & 2 & -2 \end{bmatrix}$$

BA

$$=\begin{bmatrix} 4 & 2 & 2 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 4+6-4 & 16-2+4 \\ -1+9-2 & -4-3+2 \end{bmatrix}$$

$$=\begin{bmatrix} 6 & 18 \\ 6 & -5 \end{bmatrix}$$

25. AB

$$=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 3 \\ -2 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 3-1-4 & 1-3+2 \\ 9+4-8 & 3+12+4 \end{bmatrix}$$

$$=\begin{bmatrix} -2 & 0 \\ 5 & 19 \end{bmatrix}$$

BA

$$=\begin{bmatrix} 3 & 1 \\ 1 & 3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 3+3 & -3+4 & 6+4 \\ 1+9 & -1+12 & 2+12 \\ -2+3 & 2+4 & -4+4 \end{bmatrix}$$

$$=\begin{bmatrix} 6 & 1 & 10 \\ 10 & 11 & 14 \\ 1 & 6 & 0 \end{bmatrix}$$

26. a. AB

$$=\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}\begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 2-\frac{1}{2}-\frac{1}{2} & 2-1-1 & 2-\frac{3}{2}-\frac{1}{2} \\ -1+0+1 & -1+0+2 & -1+0+1 \\ 0+\frac{1}{2}-\frac{1}{2} & 0+1-1 & 0+\frac{3}{2}-\frac{1}{2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

BA

$$=\begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 3 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

$$=\begin{bmatrix} 2-1+0 & -1+0+1 & -\frac{1}{2}+1-\frac{1}{2} \\ 1-1+0 & -\frac{1}{2}+0+\frac{3}{2} & -\frac{1}{4}+1-\frac{3}{4} \\ 2-2+0 & -1+0+1 & -\frac{1}{2}+2-\frac{1}{2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b. Yes. In this case, $AB = BA$.

Section 7.3 Exercises

27. a. $A = \begin{bmatrix} 71 & 76 & 13 & 14 & 72 \\ 14 & 16 & 24 & 10 & 68 \end{bmatrix}$

$B = \begin{bmatrix} 256 & 198 & 66 & 8 & 11 \\ 20 & 14 & 16 & 1 & 1 \end{bmatrix}$

b. $A + B$

$= \begin{bmatrix} 71 & 76 & 13 & 14 & 72 \\ 14 & 16 & 24 & 10 & 68 \end{bmatrix} + \begin{bmatrix} 256 & 198 & 66 & 8 & 11 \\ 20 & 14 & 16 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 71+256 & 76+198 & 13+66 & 14+8 & 72+11 \\ 14+20 & 16+14 & 24+16 & 10+1 & 68+1 \end{bmatrix}$

$= \begin{bmatrix} 327 & 274 & 79 & 22 & 83 \\ 34 & 30 & 40 & 11 & 69 \end{bmatrix}$

28. a. $C = B - A$

$= \begin{bmatrix} 256 & 198 & 66 & 8 & 11 \\ 20 & 14 & 16 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 71 & 76 & 13 & 14 & 72 \\ 14 & 16 & 24 & 10 & 68 \end{bmatrix}$

$= \begin{bmatrix} 256-71 & 198-76 & 66-13 & 8-14 & 11-72 \\ 20-14 & 14-16 & 16-24 & 1-10 & 1-68 \end{bmatrix}$

$= \begin{bmatrix} 185 & 122 & 53 & -6 & -61 \\ 6 & -2 & -8 & -9 & -67 \end{bmatrix}$

The elements in the matrix represent the difference between the number of foreign endangered and threatened wildlife and the number of those wildlife within the U.S.

b. The negative entries in the matrix represent species of endangered and threatened wildlife that are more abundant in the U.S. than outside the U.S.

29. a. $A = \begin{bmatrix} 41,192 & 69,497 \\ 54,681 & 51,134 \\ 27,572 & 28,612 \end{bmatrix}$

b. $B = \begin{bmatrix} 243,470 & 296,374 \\ 138,004 & 95,804 \\ 43,781 & 39,216 \end{bmatrix}$

c. $C = A - B$

$$= \begin{bmatrix} 41,192 & 69,497 \\ 54,681 & 51,134 \\ 27,572 & 28,612 \end{bmatrix} - \begin{bmatrix} 243,470 & 296,374 \\ 138,004 & 95,804 \\ 43,781 & 39,216 \end{bmatrix}$$

$$= \begin{bmatrix} -202,278 & -226,877 \\ -83,323 & -44,670 \\ -16,209 & -10,604 \end{bmatrix}$$

d. Referring to the matrix in part *c,* since all answers are negative, there is no positive trade balance with the three Asian countries.

e. Referring to the matrix in part *c*, the largest trade deficit for the U.S. occurs in 2009 with China.

30. a.

$$A = \begin{bmatrix} 41,192 \\ 54,681 \\ 27,572 \end{bmatrix}$$

$$B = \begin{bmatrix} 69,497 \\ 51,134 \\ 28,612 \end{bmatrix}$$

$$C = \begin{bmatrix} 243,470 \\ 138,004 \\ 43,781 \end{bmatrix}$$

$$D = \begin{bmatrix} 296,374 \\ 95,804 \\ 39,216 \end{bmatrix}$$

b. The trade balance in 2005 is represented by $E = A - C$.

$$E = \begin{bmatrix} 41,192 \\ 54,681 \\ 27,572 \end{bmatrix} - \begin{bmatrix} 243,470 \\ 138,004 \\ 43,781 \end{bmatrix}$$

$$= \begin{bmatrix} -202,278 \\ -83,323 \\ -16,209 \end{bmatrix}$$

c. The trade balance in 2009 is represented by $F = B - D$.

$$F = \begin{bmatrix} 69,497 \\ 51,134 \\ 28,612 \end{bmatrix} - \begin{bmatrix} 296,374 \\ 95,804 \\ 39,216 \end{bmatrix}$$

$$= \begin{bmatrix} -226,877 \\ -44,670 \\ -10,604 \end{bmatrix}$$

d. $E - F$

$$= \begin{bmatrix} -202,278 \\ -83,323 \\ -16,209 \end{bmatrix} - \begin{bmatrix} -226,877 \\ -44,670 \\ -10,604 \end{bmatrix}$$

$$= \begin{bmatrix} 24,599 \\ -38,653 \\ -5,605 \end{bmatrix}$$

e. Yes, for China, the trade balance in 2005 is better than the trade balance in 2009.

31. The original matrix is

$$A = \begin{bmatrix} 857 & 695 \\ 629 & 605 \\ 567 & 510 \\ 947 & 719 \end{bmatrix}.$$

If the median income increases by 12%, then the new matrix is $A + 12\%A$ or $(1 + 12\%)A$ or $1.12A$.

$$1.12A = \begin{bmatrix} 1.12(857) & 1.12(695) \\ 1.12(629) & 1.12(605) \\ 1.12(567) & 1.12(510) \\ 1.12(947) & 1.12(719) \end{bmatrix} = \begin{bmatrix} 960 & 778 \\ 704 & 678 \\ 635 & 571 \\ 106 & 805 \end{bmatrix}$$

32. The original matrix is $A = \begin{bmatrix} 1.8361 & 113.907 \\ 1.5613 & 116.299 \end{bmatrix}$.

Reducing the currency returned by 5% implies that 95% (i.e., 100% – 5%) of the value will be returned. Therefore the new matrix would be 95%A.

$$0.95A = \begin{bmatrix} 0.95(1.8361) & 0.95(113.907) \\ 0.95(1,5613) & 0.95(116.299) \end{bmatrix} = \begin{bmatrix} 1.7443 & 108.2116 \\ 1.4832 & 110.4840 \end{bmatrix}$$

33. To compute the required matrix, the costs of TV, radio, and newspaper advertisements need to be multiplied by the number of each type advertisement targeted to the various audiences. The matrix is represented by BA.

$$BA = \begin{bmatrix} 30 & 45 & 35 \\ 25 & 32 & 40 \\ 22 & 12 & 30 \end{bmatrix} \begin{bmatrix} 12 \\ 15 \\ 5 \end{bmatrix}$$

Using technology to calculate the product yields:

```
[B][A]
        [[1210]
         [980 ]
         [594 ]]
```

Therefore, the cost of advertising to singles is \$1210, the cost of advertising to males 35–55 is \$980, and the cost of advertising to females 65+ is \$594.

34. a. $BA = \begin{bmatrix} 45 & 50 & 55 \\ 20 & 20 & 20 \end{bmatrix} \begin{bmatrix} 10 & 24 \\ 22 & 10 \\ 33 & 3 \end{bmatrix}$

Using technology to calculate the product yields

```
[B][A]
   [[3365 1745]
    [1300 740 ]]
```

	Men	Women
Robes	3365	1745
Hoods	1300	740

b. The cost of the robes for the men is \$3365. The cost of the robes for the women is \$1745.

35. a. To calculate the total cost of various products per department, for each product multiply the quantity needed by the unit cost and add the results. The matrix form of the multiplication corresponds to

$$\begin{bmatrix} 60 & 40 & 20 \\ 40 & 20 & 40 \end{bmatrix} \begin{bmatrix} 600 & 560 \\ 300 & 200 \\ 300 & 400 \end{bmatrix}.$$

Using technology to calculate the product yields

```
[A][B]
 [[54000 49600]
  [42000 42400]]
```

$$\begin{array}{l} \\ \text{Department A} \\ \text{Department B} \end{array}\begin{array}{c} \begin{array}{cc} \text{DeTuris} & \text{Marriott} \end{array} \\ \begin{bmatrix} 54{,}000 & 49{,}600 \\ 42{,}000 & 42{,}400 \end{bmatrix} \end{array}$$

b. To minimize the cost of the purchase, Department A should purchase the products from Marriott, while Department B should purchase the products from DeTuris.

36. To calculate the total raw material costs associated with manufacturing each style of chair, the number of units of each type material needed can be multiplied by the unit cost per material and the results added. The matrix form of the multiplication corresponds to

$$\begin{bmatrix} 5 & 20 & 0 & 10 \\ 10 & 9 & 0 & 0 \\ 5 & 10 & 10 & 10 \end{bmatrix}\begin{bmatrix} 15 \\ 12 \\ 14 \\ 30 \end{bmatrix}.$$

Using technology to calculate the product yields

```
[A][B]
           [[615]
            [258]
            [635]]
```

$$\begin{array}{l} \\ \text{Style A} \\ \text{Style B} \\ \text{Style C} \end{array}\begin{array}{c} \text{Manufacturing Cost} \\ \begin{bmatrix} 615 \\ 258 \\ 635 \end{bmatrix} \end{array}$$

37. $\begin{bmatrix} R \\ D \end{bmatrix} = \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$

Let $a = 0.50$ and $b = 0.50$.

$$\begin{bmatrix} R \\ D \end{bmatrix} = \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix}\begin{bmatrix} 0.50 \\ 0.50 \end{bmatrix}$$

Using technology to calculate the product yields

```
[A][B]
              [[.55]
               [.45]]
```

$$\begin{bmatrix} R \\ D \end{bmatrix} = \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix} = \begin{bmatrix} 55\% \\ 45\% \end{bmatrix}$$

Based on the model, in the next election Republicans will receive 55% of the vote, while Democrats will receive 45% of the vote.

38. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$

Let $a = 120{,}000$ and $b = 90{,}000$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 120{,}000 \\ 90{,}000 \end{bmatrix}$$

Using technology to calculate the product yields

```
[A][B]
       [[102000]
        [108000]]
```

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 102{,}000 \\ 108{,}000 \end{bmatrix}$$

Based on the model, Company X retains 102,000 of its customers, while Company Y retains 108,000 of its customers.

39. a. $\begin{bmatrix} 449 & 706 & 917 & 955 & 1003 & 768 \\ 436 & 663 & 737 & 722 & 742 & 598 \end{bmatrix}$

b. To generate a new matrix representing the median weekly earnings for men of 10% and the median weekly earnings for women of 25%, the matrix from part a) should be multiplied by the matrix $\begin{bmatrix} 1.10 & 0 \\ 0 & 1.25 \end{bmatrix}$.

$$\begin{bmatrix} 1.10 & 0 \\ 0 & 1.25 \end{bmatrix} \begin{bmatrix} 449 & 706 & 917 & 955 & 1003 & 768 \\ 436 & 663 & 737 & 722 & 742 & 598 \end{bmatrix}$$

Using technology to calculate the product yields:

$$[A][B] = \begin{bmatrix} 493.90 & 776.60 & 1008.70 & 1050.50 & 1103.30 & 844.80 \\ 545.00 & 828.75 & 921.25 & 902.50 & 927.50 & 747.50 \end{bmatrix}$$

40. a. $A = \begin{bmatrix} 606 & 358 \\ 518 & 49 \\ 324 & 50 \\ 207 & 48 \\ 170 & 566 \end{bmatrix}$

b. $B = \begin{bmatrix} 634 & 714 \\ 539 & 99 \\ 327 & 108 \\ 210 & 99 \\ 181 & 1326 \end{bmatrix}$

c. The increase in the number of libraries and the corresponding operating income is given by *B*–*A*.

$$B - A = \begin{bmatrix} 634 & 714 \\ 539 & 99 \\ 327 & 108 \\ 210 & 99 \\ 181 & 1326 \end{bmatrix} - \begin{bmatrix} 606 & 358 \\ 518 & 49 \\ 324 & 50 \\ 207 & 48 \\ 170 & 566 \end{bmatrix} = \begin{bmatrix} 28 & 356 \\ 21 & 50 \\ 3 & 58 \\ 3 & 51 \\ 11 & 760 \end{bmatrix}$$

d. Illinois has the largest increase in number of libraries.

e. California has the largest increase in operating income.

Section 7.4 Skills Check

1. a. AB

$$=\begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}1 & -0.5\\-2 & 1.5\end{bmatrix}$$

$$=\begin{bmatrix}3-2 & -1.5+1.5\\4-4 & -2+3\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

BA

$$=\begin{bmatrix}1 & -0.5\\-2 & 1.5\end{bmatrix}\begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3-2 & 1-1\\-6+6 & -2+3\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

b. Since $AB = BA = I$, A and B are inverses of one another.

2. a. AB

$$=\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{4}\\-\frac{1}{2} & 0 & \frac{1}{2}\\0 & \frac{1}{2} & -\frac{1}{4}\end{bmatrix}\begin{bmatrix}2 & 2 & 2\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2-\frac{1}{2}-\frac{1}{2} & 2-1-1 & 2-\frac{3}{2}-\frac{1}{2}\\-1+0+1 & -1+0+2 & -1+0+1\\0+\frac{1}{2}-\frac{1}{2} & 0+1-1 & 0+\frac{3}{2}-\frac{1}{2}\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

BA

$$=\begin{bmatrix}2 & 2 & 2\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{4}\\-\frac{1}{2} & 0 & \frac{1}{2}\\0 & \frac{1}{2} & -\frac{1}{4}\end{bmatrix}$$

$$=\begin{bmatrix}2-1+0 & -1+0+1 & -\frac{1}{2}+1-\frac{1}{2}\\1-1+0 & -\frac{1}{2}+0+\frac{3}{2} & -\frac{1}{4}+1-\frac{3}{4}\\2-2+0 & -1+0+1 & -\frac{1}{2}+2-\frac{1}{2}\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

b. Since $AB = BA = I$, A and B are inverses of one another.

3. Using technology to calculate AB yields

```
[A][B]
        [[1 0 0]
         [0 1 0]
         [0 0 1]]
```

Likewise, using technology to calculate BA yields

```
[B][A]
        [[1 0 0]
         [0 1 0]
         [0 0 1]]
```

Since $AB = BA = I$, A and B are inverses of one another.

4. Applying technology to calculate the products yields $CD = DC = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$.

Since $CD = DC = I$, C and D are inverses of one another.

5. $[A \mid I]$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \xrightarrow{-2R_1 + R_2 \to R_2}$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \xrightarrow{-3R_2 + R_1 \to R_1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\left[I \mid A^{-1}\right]$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

6. $[A \mid I]$

$$\left[\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow{-1R_1 + R_2 \to R_2}$$

$$\left[\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \xrightarrow{-4R_2 + R_1 \to R_1}$$

$$\left[\begin{array}{cc|cc} 2 & 0 & 5 & -4 \\ 0 & 1 & -1 & 1 \end{array}\right] \xrightarrow{\left(\frac{1}{2}\right)R_1 \to R_1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{2} & -2 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$\left[I \mid A^{-1}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{5}{2} & -2 \\ -1 & 1 \end{bmatrix}$$

7. Using technology to calculate A^{-1} yields

```
[A]⁻¹▸Frac
[[-1/6  -1/3  1  ]
 [-1/3  1/3   0  ]
 [1/3   2/3   -1]]
```

8. Using technology to calculate A^{-1} yields

```
Ans▸Frac
[[1/2  1/4   1/2]
 [1/4  -3/8  1/4]
 [1    -1/2  0  ]]
```

9. Using technology to calculate A^{-1} yields

```
[A]▸Frac
 [[-1/3  -1  1/3]
  [1     1   0  ]
  [-1/3  0   1/3]]
```

10. Using technology to calculate C^{-1} yields

```
[C]⁻¹▸Frac
       [[0 1 0]
        [1 0 1]
        [0 0 1]]
```

11. Using technology to calculate A^{-1} yields

```
[A]⁻¹
 [[.9   .2   -.7]
  [-.5  0    .5 ]
  [.7   -.4  -.1]]
```

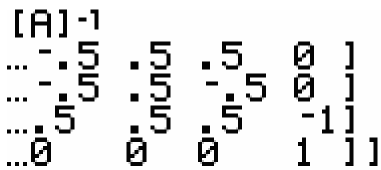
12. Using technology to calculate B^{-1} yields

```
[B]⁻¹
   [[.2 .3 .1 ]
    [.3 .4 -.1]
    [.2 .5 .1 ]]
```

13. Using technology to calculate C^{-1} yields

```
[C]⁻¹
     [[1 0 1 1]
      [0 1 1 1]
      [0 0 1 0]
      [0 0 0 1]]
```

14. Using technology to calculate A^{-1} yields



$$A^{-1} = \begin{bmatrix} -0.5 & 0.5 & 0.5 & 0 \\ -0.5 & 0.5 & -0.5 & 0 \\ 0.5 & 0.5 & 0.5 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

15. $AX = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$$A^{-1}(AX) = A^{-1}\left(\begin{bmatrix} 2 \\ 4 \end{bmatrix}\right)$$

$$IX = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 2+8 \\ 8+12 \end{bmatrix}$$

$$X = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

16. $AX = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

$$A^{-1}(AX) = A^{-1}\left(\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}\right)$$

$$IX = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 1 \\ 2 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1+4+2 \\ 0+4-2 \\ 2+0+4 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 \\ 2 \\ 6 \end{bmatrix}$$

17. $\begin{bmatrix} -1 & 1 & 0 \\ -2 & 3 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$

Let $A = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 3 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Applying technology to calculate A^{-1}

$$A^{-1}=\begin{bmatrix}1&1&2\\2&1&2\\2&0&1\end{bmatrix}$$

Solving for X

$$A^{-1}(AX)=A^{-1}\left(\begin{bmatrix}3\\5\\8\end{bmatrix}\right)$$

$$IX=\begin{bmatrix}1&1&2\\2&1&2\\2&0&1\end{bmatrix}\begin{bmatrix}3\\5\\8\end{bmatrix}$$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3+5+16\\6+5+16\\6+0+8\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}24\\27\\14\end{bmatrix}$$

$x=24,\ y=27,\ z=14$

Applying technology to calculate A^{-1}

$$A^{-1}=\begin{bmatrix}1&1&2\\3&2&4\\4&2&5\end{bmatrix}$$

Solving for X

$$A^{-1}(AX)=A^{-1}\left(\begin{bmatrix}2\\4\\8\end{bmatrix}\right)$$

$$IX=\begin{bmatrix}1&1&2\\3&2&4\\4&2&5\end{bmatrix}\begin{bmatrix}2\\4\\8\end{bmatrix}$$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2+4+16\\6+8+32\\8+8+40\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}22\\46\\56\end{bmatrix}$$

$x=22,\ y=46,\ z=56$

18. $\begin{bmatrix}-2&1&0\\-1&3&-2\\2&-2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\4\\8\end{bmatrix}$

Let $A=\begin{bmatrix}-2&1&0\\-1&3&-2\\2&-2&1\end{bmatrix}$

19. $\begin{bmatrix}4&-3&1\\-6&5&-2\\1&-1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-3\\1\end{bmatrix}$

Let $A=\begin{bmatrix}4&-3&1\\-6&5&-2\\1&-1&1\end{bmatrix}$

Applying technology to calculate A^{-1}

$$A^{-1}=\begin{bmatrix}3&2&1\\4&3&2\\1&1&2\end{bmatrix}$$

Solving for X

$$A^{-1}(AX) = A^{-1}\left(\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}\right)$$

$$IX = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6-6+1 \\ 8-9+2 \\ 2-3+2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x=1,\ y=1,\ z=1$

20. $$\begin{bmatrix} 5 & 3 & 1 \\ 4 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 9 \\ 2 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & 3 & 1 \\ 4 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$

Applying technology to calculate A^{-1}

$$A^{-1} = \begin{bmatrix} \frac{4}{3} & -\frac{5}{3} & 1 \\ -2 & 3 & -2 \\ \frac{1}{3} & -\frac{2}{3} & 1 \end{bmatrix}$$

Solving for X

$$A^{-1}(AX) = A^{-1}\left(\begin{bmatrix} 12 \\ 9 \\ 2 \end{bmatrix}\right)$$

$$IX = \begin{bmatrix} \frac{4}{3} & -\frac{5}{3} & 1 \\ -2 & 3 & -2 \\ \frac{1}{3} & -\frac{2}{3} & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 9 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16-15+2 \\ -24+27-4 \\ 4-6+2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

$x=3,\ y=-1,\ z=0$

21. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 2 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 2 \\ 2 & 1 & 2 \end{bmatrix}$

Applying technology to calculate A^{-1}

$$A^{-1} = \begin{bmatrix} \frac{6}{7} & -\frac{1}{7} & -\frac{2}{7} \\ \frac{2}{7} & \frac{2}{7} & -\frac{3}{7} \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1}(AX) = A^{-1}\left(\begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}\right)$$

$$IX = \begin{bmatrix} \frac{6}{7} & -\frac{1}{7} & -\frac{2}{7} \\ \frac{2}{7} & \frac{2}{7} & -\frac{3}{7} \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$$

Solving for X

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{24}{7} - \frac{4}{7} - \frac{6}{7} \\ \frac{8}{7} + \frac{8}{7} - \frac{9}{7} \\ -4 + 0 + 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ \frac{7}{7} \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$x = 2, \quad y = 1, \quad z = -1$

22. $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Applying technology to calculate A^{-1}

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & 1 \end{bmatrix}$$

Solving for X

$$A^{-1}(AX) = A^{-1}\left(\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}\right)$$

$$IX = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 + 2 + 6 \\ 1 + 2 + 0 \\ -1 + 6 + 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 11 \end{bmatrix}$$

$x = 7, \quad y = 3, \quad z = 11$

23. $$\begin{bmatrix}1&0&1&1\\0&1&1&1\\2&5&1&0\\0&3&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}90\\72\\108\\144\end{bmatrix}$$

Let $A=\begin{bmatrix}1&0&1&1\\0&1&1&1\\2&5&1&0\\0&3&0&1\end{bmatrix}$.

Applying technology to calculate A^{-1}

$$A^{-1}=\begin{bmatrix}\frac{7}{9}&-\frac{8}{9}&\frac{1}{9}&\frac{1}{9}\\-\frac{2}{9}&\frac{1}{9}&\frac{1}{9}&\frac{1}{9}\\-\frac{4}{9}&\frac{11}{9}&\frac{2}{9}&-\frac{7}{9}\\\frac{2}{3}&-\frac{1}{3}&-\frac{1}{3}&\frac{2}{3}\end{bmatrix}$$

Solving for X

$$A^{-1}(AX)=A^{-1}\left(\begin{bmatrix}90\\72\\108\\144\end{bmatrix}\right)$$

$$IX=\begin{bmatrix}\frac{7}{9}&-\frac{8}{9}&\frac{1}{9}&\frac{1}{9}\\-\frac{2}{9}&\frac{1}{9}&\frac{1}{9}&\frac{1}{9}\\-\frac{4}{9}&\frac{11}{9}&\frac{2}{9}&-\frac{7}{9}\\\frac{2}{3}&-\frac{1}{3}&-\frac{1}{3}&\frac{2}{3}\end{bmatrix}\begin{bmatrix}90\\72\\108\\144\end{bmatrix}$$

$$X=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}34\\16\\-40\\96\end{bmatrix}$$

$x_1=34,\ x_2=16,\ x_3=-40,\ x_4=96$

24. $$\begin{bmatrix}2&1&3&4\\1&1&1&-1\\1&1&1&1\\2&-1&3&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}1\\2\\10\\5\end{bmatrix}$$

Let $A=\begin{bmatrix}2&1&3&4\\1&1&1&-1\\1&1&1&1\\2&-1&3&-1\end{bmatrix}$.

Applying technology to calculate A^{-1}

$$A^{-1}=\begin{bmatrix}-2&-3&6&1\\\frac{1}{2}&\frac{5}{4}&-\frac{5}{4}&-\frac{1}{2}\\\frac{3}{2}&\frac{9}{4}&-\frac{17}{4}&-\frac{1}{2}\\0&-\frac{1}{2}&\frac{1}{2}&0\end{bmatrix}$$

Solving for X

$$A^{-1}(AX)=A^{-1}\left(\begin{bmatrix}1\\2\\10\\5\end{bmatrix}\right)$$

$$IX=\begin{bmatrix}-2&-3&6&1\\\frac{1}{2}&\frac{5}{4}&-\frac{5}{4}&-\frac{1}{2}\\\frac{3}{2}&\frac{9}{4}&-\frac{17}{4}&-\frac{1}{2}\\0&-\frac{1}{2}&\frac{1}{2}&0\end{bmatrix}\begin{bmatrix}1\\2\\10\\5\end{bmatrix}$$

$$X=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}57\\-12\\-39\\4\end{bmatrix}$$

$x_1=57,\ x_2=-12,\ x_3=-39,\ x_4=4$

Section 7.4 Exercises

25.

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Let $\text{A} = 150{,}000$ and $\text{B} = 120{,}000$.

$$\begin{bmatrix} 150{,}000 \\ 120{,}000 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Let $\text{D} = \begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}$. Using technology to calculate D^{-1} yields $\text{D}^{-1} = \begin{bmatrix} \frac{5}{2} & -\frac{5}{4} \\ -\frac{3}{2} & \frac{9}{4} \end{bmatrix}$.

Multiplying both sides by D^{-1} yields

$$\begin{bmatrix} \frac{5}{2} & -\frac{5}{4} \\ -\frac{3}{2} & \frac{9}{4} \end{bmatrix} \begin{bmatrix} 150{,}000 \\ 120{,}000 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & -\frac{5}{4} \\ -\frac{3}{2} & \frac{9}{4} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}.$$

Using technology to carry out the multiplication:

$$\begin{bmatrix} 225{,}000 \\ 45{,}000 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 225{,}000 \\ 45{,}000 \end{bmatrix}$$

Last year Company X "*a*" had 225,000 customers, and Company Y "*b*" had 45,000 customers.

26. $$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{2}{5} \\ \frac{1}{3} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Let $A = 90{,}000$ and $B = 85{,}000$.

$$\begin{bmatrix} 90{,}000 \\ 85{,}000 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{2}{5} \\ \frac{1}{3} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Let $D=\begin{bmatrix}\frac{2}{3} & \frac{2}{5}\\ \frac{1}{3} & \frac{3}{5}\end{bmatrix}$. Using technology to calculate D^{-1} yields $D^{-1}=\begin{bmatrix}\frac{9}{4} & -\frac{3}{2}\\ -\frac{5}{4} & \frac{5}{2}\end{bmatrix}$.

Multiplying both sides by D^{-1} yields

$$\begin{bmatrix}\frac{9}{4} & -\frac{3}{2}\\ -\frac{5}{4} & \frac{5}{2}\end{bmatrix}\begin{bmatrix}90{,}000\\ 85{,}000\end{bmatrix}=\begin{bmatrix}\frac{9}{4} & -\frac{3}{2}\\ -\frac{5}{4} & \frac{5}{2}\end{bmatrix}\begin{bmatrix}\frac{2}{3} & \frac{2}{5}\\ \frac{1}{3} & \frac{3}{5}\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}.$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix}75{,}000\\ 100{,}000\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}$$

$$\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}75{,}000\\ 100{,}000\end{bmatrix}$$

Last year the satellite company "*a*" had 75,000 customers, and the cable company "*b*" had 100,000 customers.

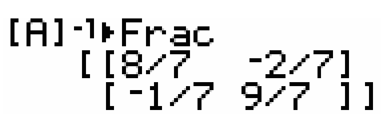
27. $\begin{bmatrix}R\\ D\end{bmatrix}=\begin{bmatrix}0.90 & 0.20\\ 0.10 & 0.80\end{bmatrix}\begin{bmatrix}r\\ d\end{bmatrix}$

Let $R=0.55$ and $D=0.45$.

$$\begin{bmatrix}0.55\\ 0.45\end{bmatrix}=\begin{bmatrix}0.90 & 0.20\\ 0.10 & 0.80\end{bmatrix}\begin{bmatrix}r\\ d\end{bmatrix}$$

Let $A=\begin{bmatrix}0.90 & 0.20\\ 0.10 & 0.80\end{bmatrix}$.

Using technology to calculate A^{-1} yields



$$A^{-1}=\begin{bmatrix}\frac{8}{7} & -\frac{2}{7}\\ -\frac{1}{7} & \frac{9}{7}\end{bmatrix}$$

Multiplying both sides by A^{-1} yields $\begin{bmatrix}\frac{8}{7} & -\frac{2}{7}\\ -\frac{1}{7} & \frac{9}{7}\end{bmatrix}\begin{bmatrix}0.55\\ 0.45\end{bmatrix}=\begin{bmatrix}\frac{8}{7} & -\frac{2}{7}\\ -\frac{1}{7} & \frac{9}{7}\end{bmatrix}\begin{bmatrix}0.90 & 0.20\\ 0.10 & 0.80\end{bmatrix}\begin{bmatrix}r\\ d\end{bmatrix}$.

Using technology to carry out the multiplication yields:

$$\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix}$$

$$\begin{bmatrix} r \\ d \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

In last year's election 50% of voters were Republicans and 50% were Democrats.

28.

$$\begin{bmatrix} D \\ R \\ C \end{bmatrix} = \begin{bmatrix} 50 & 40 & 30 \\ 40 & 50 & 30 \\ 10 & 10 & 40 \end{bmatrix} \begin{bmatrix} d \\ r \\ c \end{bmatrix}$$

Let $D = 42\ (42\%),\ R = 42\ (42\%),$ and $C = 16\ (16\%).$

$$\begin{bmatrix} 42 \\ 42 \\ 16 \end{bmatrix} = \begin{bmatrix} 50 & 40 & 30 \\ 40 & 50 & 30 \\ 10 & 10 & 40 \end{bmatrix} \begin{bmatrix} d \\ r \\ c \end{bmatrix}$$

Let $A = \begin{bmatrix} 50 & 40 & 30 \\ 40 & 50 & 30 \\ 10 & 10 & 40 \end{bmatrix}.$

Using technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} \frac{17}{300} & -\frac{13}{300} & -\frac{1}{100} \\ -\frac{13}{300} & \frac{17}{300} & -\frac{1}{100} \\ -\frac{1}{300} & -\frac{1}{300} & \frac{3}{100} \end{bmatrix}.$

Multiplying both sides by A^{-1} yields

$$\begin{bmatrix} \frac{17}{300} & -\frac{13}{300} & -\frac{1}{100} \\ -\frac{13}{300} & \frac{17}{300} & -\frac{1}{100} \\ -\frac{1}{300} & -\frac{1}{300} & \frac{3}{100} \end{bmatrix} \begin{bmatrix} 42 \\ 42 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{17}{300} & -\frac{13}{300} & -\frac{1}{100} \\ -\frac{13}{300} & \frac{17}{300} & -\frac{1}{100} \\ -\frac{1}{300} & -\frac{1}{300} & \frac{3}{100} \end{bmatrix} \begin{bmatrix} 50 & 40 & 30 \\ 40 & 50 & 30 \\ 10 & 10 & 40 \end{bmatrix} \begin{bmatrix} d \\ r \\ c \end{bmatrix}.$$

Using technology to carry out the multiplication:

$$\begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ r \\ c \end{bmatrix}$$

$$\begin{bmatrix} d \\ r \\ c \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \end{bmatrix}$$

In the last election, 40% ($d = 0.4$) voted for the Democratic candidate, 40% ($r = 0.4$) voted for the Republican candidate, and 20% ($c = 0.2$) voted for the Consumer candidate.

29. a. Let x represent the largest loan, y represent the medium size loan, and z represent the smallest loan.

$$\begin{cases} x + y + z = 400{,}000 \\ x = (y + z) + 100{,}000 \\ z = \frac{1}{2} y \end{cases}$$

or

$$\begin{cases} x + y + z = 400{,}000 \\ x - y - z = 100{,}000 \\ -\frac{1}{2} y + z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400{,}000 \\ 100{,}000 \\ 0 \end{bmatrix}$$

b. $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400,000 \\ 100,000 \\ 0 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$. Using technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{6} & -\frac{1}{6} & \frac{2}{3} \end{bmatrix}$.

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{6} & -\frac{1}{6} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{6} & -\frac{1}{6} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 400,000 \\ 100,000 \\ 0 \end{bmatrix}$$

Using technology to carry out the multiplication:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 250,000 \\ 100,000 \\ 50,000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 250,000 \\ 100,000 \\ 50,000 \end{bmatrix}$$

$x = 250,000,\ y = 100,000,\ z = 50,000$

The largest loan is \$250,000. The next medium size is \$100,000. The smallest loan is \$50,000.

30. Let x represent the units of Product A, and let y represent the units of Product B.

$$\begin{cases} 40x + 65y = 5200 \\ 320x + 360y = 31,360 \end{cases}$$

$$\begin{bmatrix} 40 & 65 \\ 320 & 360 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5200 \\ 31,360 \end{bmatrix}$$

Let $A = \begin{bmatrix} 40 & 65 \\ 320 & 360 \end{bmatrix}$. Using technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} -\frac{9}{160} & \frac{13}{1280} \\ \frac{1}{20} & -\frac{1}{160} \end{bmatrix}$.

$$\begin{bmatrix} -\frac{9}{160} & \frac{13}{1280} \\ \frac{1}{20} & -\frac{1}{160} \end{bmatrix}\begin{bmatrix} 40 & 65 \\ 320 & 360 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{9}{160} & \frac{13}{1280} \\ \frac{1}{20} & -\frac{1}{160} \end{bmatrix}\begin{bmatrix} 5200 \\ 31,360 \end{bmatrix}$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 64 \end{bmatrix}$$

$x = 26,\ \ y = 64$

The truck can carry 26 units of Product A and 64 units of Product B.

31. Let x represent the amount invested at 6%, y represent the amount invested at 8%, and z represent the amount invested at 10%.

$$\begin{cases} x + y + z = 400,000 \\ 2x = y \\ 0.06x + 0.08y + 0.10z = 36,000 \end{cases} \quad \text{or} \quad \begin{cases} x + y + z = 400,000 \\ 2x - y = 0 \\ 0.06x + 0.08y + 0.10z = 36,000 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 0.06 & 0.08 & 0.10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400,000 \\ 0 \\ 36,000 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 0.06 & 0.08 & 0.10 \end{bmatrix}$. Using technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} \frac{5}{4} & \frac{1}{4} & -\frac{25}{2} \\ \frac{5}{2} & -\frac{1}{2} & -25 \\ -\frac{11}{4} & \frac{1}{4} & \frac{75}{2} \end{bmatrix}$.

$$\begin{bmatrix} \frac{5}{4} & \frac{1}{4} & -\frac{25}{2} \\ \frac{5}{2} & -\frac{1}{2} & -25 \\ -\frac{11}{4} & \frac{1}{4} & \frac{75}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 0.06 & 0.08 & 0.10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & \frac{1}{4} & -\frac{25}{2} \\ \frac{5}{2} & -\frac{1}{2} & -25 \\ -\frac{11}{4} & \frac{1}{4} & \frac{75}{2} \end{bmatrix} \begin{bmatrix} 400,000 \\ 0 \\ 36,000 \end{bmatrix}$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 50,000 \\ 100,000 \\ 250,000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 50,000 \\ 100,000 \\ 250,000 \end{bmatrix}$$

$x = 50,000,\ y = 100,000,\ z = 250,000$

$50,000 is invested in the 6% account, $100,000 is invested in the 8% account, and $250,000 is invested in the 10% account.

32. Let x represent the number of orchestra seats, y represent the number of main seats, and z represent the number of balcony seats.

$$\begin{cases} x + y + z = 1000 \\ 80x + 50y + 40z = 50,000 \\ 80x + \frac{3}{4}(50y) + 40z = 42,500 \end{cases} \quad \text{or} \quad \begin{cases} x + y + z = 1000 \\ 80x + 50y + 40z = 50,000 \\ 80x + 37.5y + 40z = 42,500 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 80 & 50 & 40 \\ 80 & 37.5 & 40 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 50,000 \\ 42,500 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 80 & 50 & 40 \\ 80 & 37.5 & 40 \end{bmatrix}$. Using technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} -1 & \frac{1}{200} & \frac{1}{50} \\ 0 & \frac{2}{25} & -\frac{2}{25} \\ 2 & -\frac{17}{200} & \frac{3}{50} \end{bmatrix}$.

$$\begin{bmatrix} -1 & \frac{1}{200} & \frac{1}{50} \\ 0 & \frac{2}{25} & -\frac{2}{25} \\ 2 & -\frac{17}{200} & \frac{3}{50} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 80 & 50 & 40 \\ 80 & 37.5 & 40 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{200} & \frac{1}{50} \\ 0 & \frac{2}{25} & -\frac{2}{25} \\ 2 & -\frac{17}{200} & \frac{3}{50} \end{bmatrix}\begin{bmatrix} 1000 \\ 50,000 \\ 42,500 \end{bmatrix}$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 600 \\ 300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 600 \\ 300 \end{bmatrix}$$

$x = 100,\ y = 600,\ z = 300$

The theater has 100 orchestra seats, 600 main seats, and 300 balcony seats.

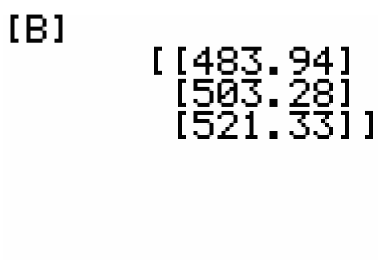
33. Let x represent the percentage of venture capital from business loans, y represent thepercentage of venture capital from auto loans, and z represent the percentage of venture capital from home loans.

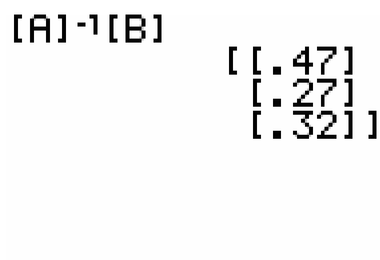
$$\begin{cases} 532x + 58y + 682z = 483.94 \\ 562x + 62y + 695z = 503.28 \\ 578x + 69y + 722z = 521.33 \end{cases}$$

$$\begin{bmatrix} 532 & 58 & 682 \\ 562 & 62 & 695 \\ 578 & 69 & 722 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 483.94 \\ 503.28 \\ 521.33 \end{bmatrix}$$

Let $A = \begin{bmatrix} 532 & 58 & 682 \\ 562 & 62 & 695 \\ 578 & 69 & 722 \end{bmatrix}$. Using technology to calculate A^{-1} and applying A^{-1} to both sides of the equation yields $A^{-1} \begin{bmatrix} 532 & 58 & 682 \\ 562 & 62 & 695 \\ 578 & 69 & 722 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 483.94 \\ 503.28 \\ 521.33 \end{bmatrix}$.

Using technology to carry out the multiplication yields





$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.47 \\ 0.27 \\ 0.32 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.47 \\ 0.27 \\ 0.32 \end{bmatrix}$$

$x = 0.47, \ y = 0.27, \ z = 0.32$

The percentage of venture capital from business loans is 47%, the percentage of venture capital from auto loans is 27%, and the percentage of venture capital from home loans is 32%.

34. Let x represent the the number of 4-shelf metal bookcases, y represent the number of 6-shelf metal bookcases, and z represent the number of 4-shelf wooden bookcases.
The fabrication time is $2x+3y+3z=124,$ the painting time is $1x+1.5y+2z=68,$ and the packaging time is $0.5x+0.5y+0.5z=24.$

$$\begin{cases} 2x+3y+3z=124 \\ x+1.5y+2z=68 \\ 0.5x+0.5y+0.5z=24 \end{cases}$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & 1.5 & 2 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 124 \\ 68 \\ 24 \end{bmatrix}$$

Let $A=\begin{bmatrix} 2 & 3 & 3 \\ 1 & 1.5 & 2 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$. Using technology to calculate A^{-1} and applying A^{-1} to both sides of the equation yields $A^{-1}\begin{bmatrix} 2 & 3 & 3 \\ 1 & 1.5 & 2 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=A^{-1}\begin{bmatrix} 124 \\ 68 \\ 24 \end{bmatrix}$.

Using technology to carry out the multiplication yields

```
[A]-1
     [[-1 0  6 ]
      [2  -2 -4]
      [-1 2  0 ]]
```

```
[B]
            [[124]
             [68 ]
             [24 ]]
```

```
[A]-1[B]
             [[20]
              [16]
              [12]]
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 20 \\ 16 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 20 \\ 16 \\ 12 \end{bmatrix}$$

$x=20,\ y=16,\ z=12$

Each day 20 4-shelf metal bookcases, 16 6-shelf metal bookcase, and 12 4-shelf wooden bookcases can be produced.

35. a. "Just do it" is represented by the 10, 21, 19, 20, 27, 4, 15, 27, 9, and 20. Recall that spaces are coded as 27.

b. Multiplying by the encoding matrix and applying technology to generate the solutions yields:

$$\begin{bmatrix}4 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}10\\21\end{bmatrix}=\begin{bmatrix}124\\52\end{bmatrix}$$

$$\begin{bmatrix}4 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}19\\20\end{bmatrix}=\begin{bmatrix}156\\59\end{bmatrix}$$

$$\begin{bmatrix}4 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}27\\4\end{bmatrix}=\begin{bmatrix}124\\35\end{bmatrix}$$

$$\begin{bmatrix}4 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}15\\27\end{bmatrix}=\begin{bmatrix}168\\69\end{bmatrix}$$

$$\begin{bmatrix}4 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}9\\20\end{bmatrix}=\begin{bmatrix}116\\49\end{bmatrix}$$

The pairs of coded numbers are 124, 52, 156, 59, 124, 35, 168, 69, 116, and 49.

36. a. "Call home" is represented by the 3, 1, 12, 12, 27, 8, 15, 13, 5, and 27. Recall that spaces are coded as 27. Since an even number of codes is needed to create pairs of numbers, an extra space is added after the final "e" in the phrase.

b. Multiplying by the encoding matrix and applying technology to generate the solutions yields:

$$\begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}9\\8\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}12\\12\end{bmatrix}=\begin{bmatrix}60\\48\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}27\\8\end{bmatrix}=\begin{bmatrix}78\\70\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}15\\13\end{bmatrix}=\begin{bmatrix}69\\56\end{bmatrix}$$

$$\begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}\begin{bmatrix}5\\27\end{bmatrix}=\begin{bmatrix}91\\64\end{bmatrix}$$

The pairs of coded numbers are 9, 8, 60, 48, 78, 70, 69, 56, 91, and 64.

37 "Neatness counts" is represented by the 14, 5, 1, 20, 14, 5, 19, 19, 27, 3, 15, 21, 14, 20, and 19. Recall that spaces are coded as 27.

Multiplying by the encoding matrix and applying technology to generate the solutions yields:

$$\begin{bmatrix}4 & 4 & 4\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}\begin{bmatrix}14\\5\\1\end{bmatrix}=\begin{bmatrix}80\\27\\50\end{bmatrix}$$

$$\begin{bmatrix}4 & 4 & 4\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}\begin{bmatrix}20\\14\\5\end{bmatrix}=\begin{bmatrix}156\\63\\106\end{bmatrix}$$

$$\begin{bmatrix}4 & 4 & 4\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}\begin{bmatrix}19\\19\\27\end{bmatrix}=\begin{bmatrix}260\\138\\168\end{bmatrix}$$

$$\begin{bmatrix}4 & 4 & 4\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}\begin{bmatrix}3\\15\\21\end{bmatrix}=\begin{bmatrix}156\\96\\108\end{bmatrix}$$

$$\begin{bmatrix}4 & 4 & 4\\1 & 2 & 3\\2 & 4 & 2\end{bmatrix}\begin{bmatrix}14\\20\\19\end{bmatrix}=\begin{bmatrix}212\\111\\146\end{bmatrix}$$

The triples of coded numbers are 80, 27, 50, 156, 63, 106, 260, 138, 168, 156, 96, 108, 212, 111, and 146.

38. "Meet for lunch" is represented by the 13, 5, 5, 20, 27, 6, 15, 18, 27, 12, 21, 14, 3, 8, and 27. Recall that spaces are coded as 27. Since an odd number of codes is needed to create triples of numbers, an extra space is added after the final "h" in the phrase

Multiplying by the encoding matrix and applying technology to generate the solutions yields:

$$\begin{bmatrix}4&4&4\\1&2&3\\2&4&2\end{bmatrix}\begin{bmatrix}13\\5\\5\end{bmatrix}=\begin{bmatrix}92\\38\\56\end{bmatrix}$$

$$\begin{bmatrix}4&4&4\\1&2&3\\2&4&2\end{bmatrix}\begin{bmatrix}20\\27\\6\end{bmatrix}=\begin{bmatrix}212\\92\\160\end{bmatrix}$$

$$\begin{bmatrix}4&4&4\\1&2&3\\2&4&2\end{bmatrix}\begin{bmatrix}15\\18\\27\end{bmatrix}=\begin{bmatrix}240\\132\\156\end{bmatrix}$$

$$\begin{bmatrix}4&4&4\\1&2&3\\2&4&2\end{bmatrix}\begin{bmatrix}12\\21\\14\end{bmatrix}=\begin{bmatrix}188\\96\\136\end{bmatrix}$$

$$\begin{bmatrix}4&4&4\\1&2&3\\2&4&2\end{bmatrix}\begin{bmatrix}3\\8\\27\end{bmatrix}=\begin{bmatrix}152\\100\\92\end{bmatrix}$$

The triples of coded numbers are 92, 38, 56, 212, 92, 160, 240, 132, 156, 188, 96, 136, 152, 100, and 92.

39.
```
[A]-1▸Frac
        [[1  1]
         [2  3]]
```

Multiplying pairs of codes by the inverse of the encoding matrix decodes the message. Multiplying by A^{-1} and using technology to simplify yields:

$$\begin{bmatrix}1&1\\2&3\end{bmatrix}\begin{bmatrix}51\\-29\end{bmatrix}=\begin{bmatrix}22\\15\end{bmatrix}$$

$$\begin{bmatrix}1&1\\2&3\end{bmatrix}\begin{bmatrix}55\\-35\end{bmatrix}=\begin{bmatrix}20\\5\end{bmatrix}$$

$$\begin{bmatrix}1&1\\2&3\end{bmatrix}\begin{bmatrix}76\\-49\end{bmatrix}=\begin{bmatrix}27\\5\end{bmatrix}$$

$$\begin{bmatrix}1&1\\2&3\end{bmatrix}\begin{bmatrix}-15\\16\end{bmatrix}=\begin{bmatrix}1\\18\end{bmatrix}$$

$$\begin{bmatrix}1&1\\2&3\end{bmatrix}\begin{bmatrix}11\\1\end{bmatrix}=\begin{bmatrix}12\\25\end{bmatrix}$$

The decoded message is "Vote early."

40.
```
[A]-1▸Frac
      [[2   -3/2]
       [-1  1   ]]
```

Multiplying pairs of codes by the inverse of the encoding matrix decodes the message. Multiplying by A^{-1} and using technology to simplify yields:

$$\begin{bmatrix}2&-\frac{3}{2}\\-1&1\end{bmatrix}\begin{bmatrix}59\\74\end{bmatrix}=\begin{bmatrix}7\\15\end{bmatrix}$$

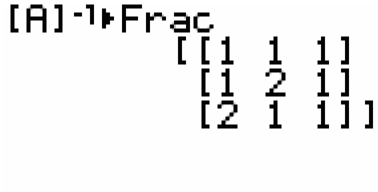
$$\begin{bmatrix}2&-\frac{3}{2}\\-1&1\end{bmatrix}\begin{bmatrix}72\\78\end{bmatrix}=\begin{bmatrix}27\\6\end{bmatrix}$$

$$\begin{bmatrix}2&-\frac{3}{2}\\-1&1\end{bmatrix}\begin{bmatrix}84\\102\end{bmatrix}=\begin{bmatrix}15\\18\end{bmatrix}$$

$$\begin{bmatrix}2&-\frac{3}{2}\\-1&1\end{bmatrix}\begin{bmatrix}81\\90\end{bmatrix}=\begin{bmatrix}27\\9\end{bmatrix}$$

$$\begin{bmatrix}2&-\frac{3}{2}\\-1&1\end{bmatrix}\begin{bmatrix}121\\148\end{bmatrix}=\begin{bmatrix}20\\27\end{bmatrix}$$

The decoded message is "Go for it."

41. 

Multiplying triples of codes by the inverse of the encoding matrix decodes the message. Multiplying by A^{-1} and using technology to simplify yields:

$$\begin{bmatrix}1&1&1\\1&2&1\\2&1&1\end{bmatrix}\begin{bmatrix}1\\-4\\16\end{bmatrix}=\begin{bmatrix}13\\9\\14\end{bmatrix}$$

$$\begin{bmatrix}1&1&1\\1&2&1\\2&1&1\end{bmatrix}\begin{bmatrix}21\\23\\-40\end{bmatrix}=\begin{bmatrix}4\\27\\25\end{bmatrix}$$

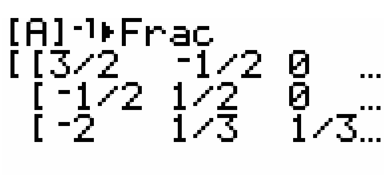
$$\begin{bmatrix}1&1&1\\1&2&1\\2&1&1\end{bmatrix}\begin{bmatrix}3\\6\\6\end{bmatrix}=\begin{bmatrix}15\\21\\18\end{bmatrix}$$

$$\begin{bmatrix}1&1&1\\1&2&1\\2&1&1\end{bmatrix}\begin{bmatrix}-26\\-14\\67\end{bmatrix}=\begin{bmatrix}27\\13\\1\end{bmatrix}$$

$$\begin{bmatrix}1&1&1\\1&2&1\\2&1&1\end{bmatrix}\begin{bmatrix}-9\\0\\23\end{bmatrix}=\begin{bmatrix}14\\14\\5\end{bmatrix}$$

$$\begin{bmatrix}1&1&1\\1&2&1\\2&1&1\end{bmatrix}\begin{bmatrix}9\\1\\8\end{bmatrix}=\begin{bmatrix}18\\19\\27\end{bmatrix}$$

The decoded message is "Mind your manners."

42. 

Multiplying triples of codes by the inverse of the encoding matrix decodes the message. Multiplying by A^{-1} and using technology to simplify yields:

$$\begin{bmatrix}\frac{3}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{1}{2}&0\\-2&\frac{1}{3}&\frac{1}{3}\end{bmatrix}\begin{bmatrix}16\\34\\128\end{bmatrix}=\begin{bmatrix}7\\9\\22\end{bmatrix}$$

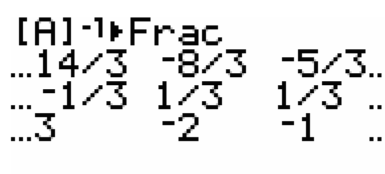
$$\begin{bmatrix}\frac{3}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{1}{2}&0\\-2&\frac{1}{3}&\frac{1}{3}\end{bmatrix}\begin{bmatrix}32\\86\\145\end{bmatrix}=\begin{bmatrix}5\\27\\13\end{bmatrix}$$

$$\begin{bmatrix}\frac{3}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{1}{2}&0\\-2&\frac{1}{3}&\frac{1}{3}\end{bmatrix}\begin{bmatrix}32\\86\\109\end{bmatrix}=\begin{bmatrix}5\\27\\1\end{bmatrix}$$

$$\begin{bmatrix}\frac{3}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{1}{2}&0\\-2&\frac{1}{3}&\frac{1}{3}\end{bmatrix}\begin{bmatrix}29\\33\\195\end{bmatrix}=\begin{bmatrix}27\\2\\18\end{bmatrix}$$

$$\begin{bmatrix}\frac{3}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{1}{2}&0\\-2&\frac{1}{3}&\frac{1}{3}\end{bmatrix}\begin{bmatrix}6\\8\\61\end{bmatrix}=\begin{bmatrix}5\\1\\11\end{bmatrix}$$

The decoded message is "Give me a break."

43. 

Multiplying triples of codes by the inverse of the encoding matrix decodes the message. Multiplying by A^{-1} and using technology to simplify yields:

$$\begin{bmatrix} -\frac{14}{3} & -\frac{8}{3} & -\frac{5}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 3 & -2 & -1 \end{bmatrix}\begin{bmatrix} 29 \\ -1 \\ 75 \end{bmatrix} = \begin{bmatrix} 13 \\ 15 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{14}{3} & -\frac{8}{3} & -\frac{5}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 3 & -2 & -1 \end{bmatrix}\begin{bmatrix} -19 \\ -66 \\ 50 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 25 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{14}{3} & -\frac{8}{3} & -\frac{5}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 3 & -2 & -1 \end{bmatrix}\begin{bmatrix} 46 \\ 41 \\ 47 \end{bmatrix} = \begin{bmatrix} 27 \\ 14 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{14}{3} & -\frac{8}{3} & -\frac{5}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 3 & -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -38 \\ 65 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 20 \end{bmatrix}$$

The decoded message is "Monday night."

44.

```
[A]-1▸Frac
[[26/11  -19/11  …
 [-1/11  2/11    …
 [19/11  -16/11  …
```

```
[A]-1▸Frac
… -19/11  -8/11]
… 2/11    2/11 ]
… -16/11  -5/11]]
```

Note that the inverse matrix is shown in two graphics, with the second column repeated in both graphics.

Multiplying triples of codes by the inverse of the encoding matrix decodes the message.

Multiplying by A^{-1} and using technology to simplify yields:

$$\begin{bmatrix} -\frac{26}{11} & -\frac{19}{11} & -\frac{8}{11} \\ -\frac{1}{11} & \frac{2}{11} & \frac{2}{11} \\ \frac{19}{11} & \frac{-16}{11} & \frac{-5}{11} \end{bmatrix}\begin{bmatrix} 20 \\ -33 \\ 142 \end{bmatrix} = \begin{bmatrix} 1 \\ 18 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{26}{11} & -\frac{19}{11} & -\frac{8}{11} \\ -\frac{1}{11} & \frac{2}{11} & \frac{2}{11} \\ \frac{19}{11} & \frac{-16}{11} & \frac{-5}{11} \end{bmatrix}\begin{bmatrix} 74 \\ 51 \\ 107 \end{bmatrix} = \begin{bmatrix} 9 \\ 22 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{26}{11} & -\frac{19}{11} & -\frac{8}{11} \\ -\frac{1}{11} & \frac{2}{11} & \frac{2}{11} \\ \frac{19}{11} & \frac{-16}{11} & \frac{-5}{11} \end{bmatrix}\begin{bmatrix} 67 \\ 87 \\ -26 \end{bmatrix} = \begin{bmatrix} 27 \\ 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{26}{11} & -\frac{19}{11} & -\frac{8}{11} \\ -\frac{1}{11} & \frac{2}{11} & \frac{2}{11} \\ \frac{19}{11} & \frac{-16}{11} & \frac{-5}{11} \end{bmatrix}\begin{bmatrix} 22 \\ -22 \\ 99 \end{bmatrix} = \begin{bmatrix} 18 \\ 12 \\ 25 \end{bmatrix}$$

The decoded message is "Arrive early."

45. Answers will vary.

Section 7.5 Skills Check

1. Isolating y in the first equation yields
$y = x^2$.
Substituting into the other equation yields
$3x + y = 0$
$3x + x^2 = 0$
$x(3 + x) = 0$
$x = 0,\ x = -3$
Back substituting to calculate y
$x = 0 \Rightarrow y = (0)^2 = 0$
$x = -3 \Rightarrow y = (-3)^2 = 9$
The solutions to the system are
$(0,0)$ and $(-3,9)$.

2. Isolating y in the second equation yields
$y = \frac{x}{2}$.
Substituting into the other equation yields
$x^2 - 2y = 0$
$x^2 - 2\left(\frac{x}{2}\right) = 0$
$x^2 - x = 0$
$x(x - 1) = 0$
$x = 0,\ x = 1$
Back substituting to calculate y
$x = 0 \Rightarrow y = \frac{0}{2} = 0$
$x = 1 \Rightarrow y = \frac{1}{2}$
The solutions to the system are
$(0,0)$ and $\left(1, \frac{1}{2}\right)$.

3. Isolating y in the second equation yields
$2x + 3y = 4$
$3y = 4 - 2x$
$y = \frac{4 - 2x}{3}$
Substituting into the other equation yields
$x^2 - 3y = 4$
$x^2 - 3\left(\frac{4 - 2x}{3}\right) = 4$
$x^2 - (4 - 2x) = 4$
$x^2 - 4 + 2x = 4$
$x^2 + 2x - 8 = 0$
$(x - 2)(x + 4) = 0$
$x = 2,\ x = -4$
Back substituting to calculate y
$x = 2 \Rightarrow y = \frac{4 - 2(2)}{3} = 0$
$x = -4 \Rightarrow y = \frac{4 - 2(-4)}{3} = 4$
The solutions to the system are
$(2,0)$ and $(-4,4)$.

4. Isolating y in the second equation yields
$3x + 5y = 8$
$5y = 8 - 3x$
$y = \frac{8 - 3x}{5}$
Substituting into the other equation yields
$x^2 - 5y = 2$
$x^2 - 5\left(\frac{8 - 3x}{5}\right) = 2$
$x^2 - (8 - 3x) = 2$
$x^2 - 8 + 3x = 2$
$x^2 + 3x - 10 = 0$
$(x - 2)(x + 5) = 0$
$x = 2,\ x = -5$
Back substituting to calculate y
$x = 2 \Rightarrow y = \frac{8 - 3(2)}{5} = \frac{2}{5}$
$x = -5 \Rightarrow y = \frac{8 - 3(-5)}{5} = \frac{23}{5}$
The solutions to the system are
$\left(2, \frac{2}{5}\right)$ and $\left(-5, \frac{23}{5}\right)$.

5. Substituting $y = 2x$ into the other equation yields

$x^2 + y^2 = 80$

$x^2 + (2x)^2 = 80$

$x^2 + 4x^2 = 80$

$5x^2 = 80$

$x^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

$x = 4,\ x = -4$

Back substituting to calculate y

$x = 4 \Rightarrow y = 2(4) = 8$

$x = -4 \Rightarrow y = 2(-4) = -8$

The solutions to the system are $(4,8)$ and $(-4,-8)$.

6. Isolating y in the second equation yields

$x + y = 0$

$y = -x$

Substituting into the other equation yields

$x^2 + y^2 = 72$

$x^2 + (-x)^2 = 72$

$x^2 + x^2 = 72$

$2x^2 = 72$

$x^2 = 36$

$x = \pm\sqrt{36} = \pm 6$

$x = 6,\ x = -6$

Back substituting to calculate y

$x = 6 \Rightarrow y = -(6) = -6$

$x = -6 \Rightarrow y = -(-6) = 6$

The solutions to the system are $(6,-6)$ and $(-6,6)$.

7. Isolating y in the first equation yields

$y = 8 - x$.

Substituting into the other equation yields

$xy = 12$

$x(8 - x) = 12$

$8x - x^2 = 12$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6,\ x = 2$

Back substituting to calculate y

$x = 6 \Rightarrow y = 8 - (6) = 2$

$x = 2 \Rightarrow y = 8 - (2) = 6$

The solutions to the system are $(6,2)$ and $(2,6)$.

8. Isolating y in the second equation yields

$2x - 3y = 2$

$-3y = 2 - 2x$

$y = \dfrac{2x-2}{3}$

Substituting into the other equation yields

$2xy + y = 36$

$2x\left(\dfrac{2x-2}{3}\right) + \left(\dfrac{2x-2}{3}\right) = 36$

$3\left[2x\left(\dfrac{2x-2}{3}\right) + \left(\dfrac{2x-2}{3}\right)\right] = 3[36]$

$2x(2x-2) + (2x-2) = 108$

$4x^2 - 4x + 2x - 2 = 108$

$4x^2 - 2x - 110 = 0$

$2(2x-11)(x+5) = 0$

$x = \dfrac{11}{2},\ x = -5$

Back substituting to calculate y

$x = \dfrac{11}{2} \Rightarrow y = \dfrac{2\left(\dfrac{11}{2}\right) - 2}{3} = \dfrac{9}{3} = 3$

$x = -5 \Rightarrow y = \dfrac{2(-5) - 2}{3} = \dfrac{-12}{3} = -4$

The solutions to the system are $\left(\dfrac{11}{2}, 3\right)$ and $(-5,-4)$.

9. Isolating y in the second equation yields
$2x - y + 4 = 0$ or $y = 2x + 4$.
Substituting into the other equation yields

$$x^2 + 5x - y = 6$$
$$x^2 + 5x - (2x + 4) = 6$$
$$x^2 + 5x - 2x - 4 = 6$$
$$x^2 + 3x - 10 = 0$$
$$(x - 2)(x + 5) = 0$$
$$x = 2,\ x = -5$$

Back substituting to calculate y

$$x = 2 \Rightarrow y = 2(2) + 4 = 8$$
$$x = -5 \Rightarrow y = 2(-5) + 4 = -6$$

The solutions to the system are
$(2,8)$ and $(-5,-6)$.

10. Isolating y in the second equation yields
$y = 18 - 10x$.
Substituting into the other equation yields

$$x^2 - y - 8x = 6$$
$$x^2 - (18 - 10x) - 8x = 6$$
$$x^2 - 18 + 10x - 8x = 6$$
$$x^2 + 2x - 24 = 0$$

Solving by factoring yields:

$$(x + 6)(x - 4) = 0$$
$$x = -6, x = 4$$

Back substituting to find y

$$x = -6 \Rightarrow$$
$$y = 18 - 10(-6) = 78$$
$$x = 4 \Rightarrow$$
$$y = 18 - 10(4) = -22$$

The solutions to the system are
$(-6,78)$ and $(4,-22)$.

11. Isolating y in the second equation yields
$2y - x = 61$ or $y = \dfrac{61 + x}{2}$.
Substituting into the other equation yields

$$2x^2 - 2y + 7x = 19$$
$$2x^2 - 2\left(\frac{61 + x}{2}\right) + 7x = 19$$
$$2x^2 - 61 - x + 7x = 19$$
$$2x^2 + 6x - 80 = 0$$
$$2(x^2 + 3x - 40) = 0$$
$$2(x - 5)(x + 8) = 0$$
$$x = 5,\ x = -8$$

Back substituting to calculate y

$$x = 5 \Rightarrow y = \frac{61 + (5)}{2} = 33$$
$$x = -8 \Rightarrow y = \frac{61 + (-8)}{2} = \frac{53}{2}$$

The solutions to the system are
$(5,33)$ and $\left(-8, \dfrac{53}{2}\right)$.

12. Isolating y in the second equation yields
$2x - y + 37 = 0$ or $y = 2x + 37$.
Substituting into the other equation yields

$$2x^2 + 4x - 2y = 24$$
$$2x^2 + 4x - 2(2x + 37) = 24$$
$$2x^2 + 4x - 4x - 74 = 24$$
$$2x^2 - 98 = 0$$
$$2(x^2 - 49) = 0$$
$$2(x - 7)(x + 7) = 0$$
$$x = 7,\ x = -7$$

Back substituting to calculate y

$$x = -7 \Rightarrow y = 2(-7) + 37 = 23$$
$$x = 7 \Rightarrow y = 2(7) + 37 = 51$$

The solutions to the system are
$(-7,23)$ and $(7,51)$.

13. Isolating y in both equations:

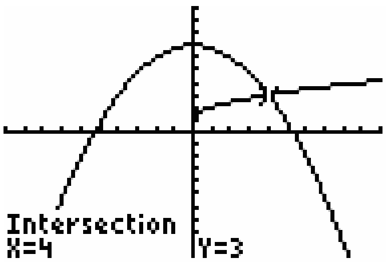
$$4y = 28 - x^2$$

$$y = \frac{28 - x^2}{4}$$

and

$$y = 1 + \sqrt{x}$$

Solving graphically by applying the intersection of graphs method:



[–10, 10] by [–10, 10]

The solution to the system is $(4,3)$.

14. Isolating y in both equations:

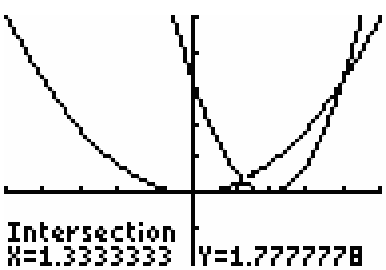
$$y = x^2$$

and

$$\sqrt{y} = 4 - 2x$$

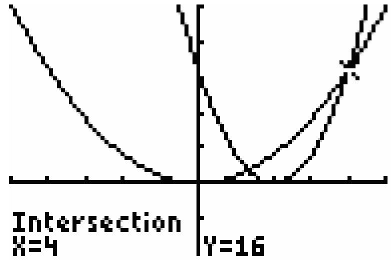
$$y = (4 - 2x)^2$$

Solving graphically by applying the intersection of graphs method:



[–5, 5] by [–10, 25]

Solving graphically by applying the intersection of graphs method:



[–5, 5] by [–10, 25]

The solutions to the system are $\left(\frac{4}{3}, \frac{16}{9}\right)$ and $(4,16)$ although (4, 16) does not satisfy both original equations.

15. Isolating y in the second equation:

$$4y = x + 8$$

$$y = \frac{x+8}{4}$$

Substituting into the other equation:

$$4xy + x = 10$$

$$4x\left(\frac{x+8}{4}\right) + x = 10$$

$$x^2 + 8x + x = 10$$

$$x^2 + 9x - 10 = 0$$

$$(x+10)(x-1) = 0$$

$$x = -10,\ x = 1$$

Back substituting to calculate y

$$x = -10 \Rightarrow y = \frac{(-10)+8}{4} = -\frac{1}{2}$$

$$x = 1 \Rightarrow y = \frac{(1)+8}{4} = \frac{9}{4}$$

The solutions to the system are $\left(-10, -\frac{1}{2}\right)$ and $\left(1, \frac{9}{4}\right)$.

16. Isolating y in the second equation:

$$3y = 2x + 4$$

$$y = \frac{2x+4}{3}$$

Substituting into the other equation:

$$xy + y = 4$$

$$x\left(\frac{2x+4}{3}\right)+\left(\frac{2x+4}{3}\right)=4$$

$$3\left[x\left(\frac{2x+4}{3}\right)+\left(\frac{2x+4}{3}\right)\right]=3[4]$$

$$x(2x+4)+2x+4=12$$

$$2x^2+4x+2x+4-12=0$$

$$2x^2+6x-8=0$$

$$2(x^2+3x-4)=0$$

$$2(x+4)(x-1)=0$$

$$x=-4, x=1$$

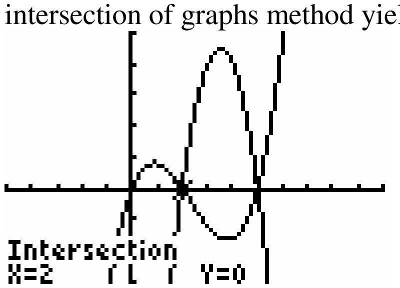
Back substituting to calculate y

$$x=-4 \Rightarrow y=\frac{2(-4)+4}{3}=-\frac{4}{3}$$

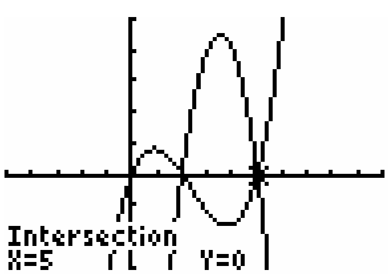
$$x=1 \Rightarrow y=\frac{2(1)+4}{3}=2$$

The solutions to the system are $\left(-4,-\frac{4}{3}\right)$ and $(1,2)$.

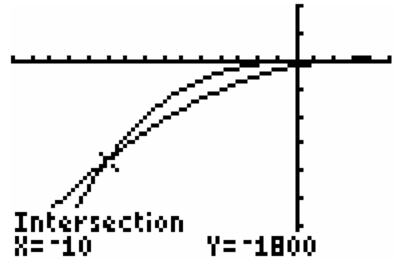
17. Solving graphically by applying the intersection of graphs method yields:



[–5, 10] by [–15, 25]



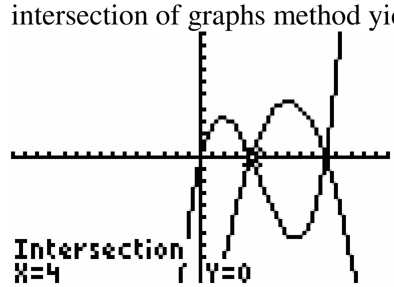
[–5, 10] by [–15, 25]



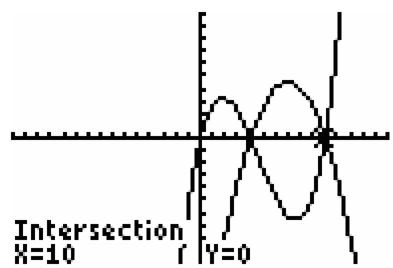
[–15, 5] by [–3500, 1000]

The solutions to the system are $(-10,-1800),(2,0),$ and $(5,0)$.

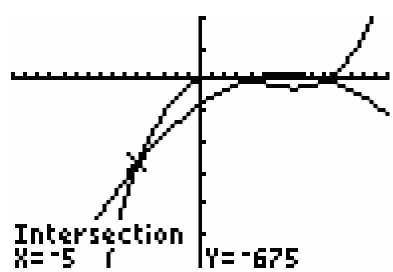
18. Solving graphically by applying the intersection of graphs method yields:



[–15, 15] by [–100, 100]



[–15, 15] by [–100, 100]



[–15, 15] by [–1500, 500]

The solutions to the system are $(-5,-675),(4,0),$ and $(10,0)$.

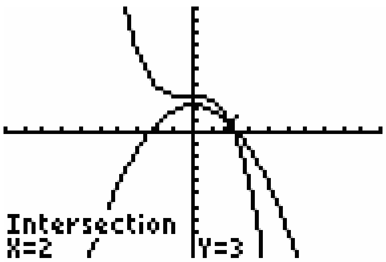
19. Isolating y in both equations:

$y = 15 - x^2 - x^3$

and

$y = 11 - 2x^2$

Solving graphically by applying the intersection of graphs method:



[–10, 10] by [–50, 50]

The solution to the system is $(2,3)$.

20. Isolating y in both equations:

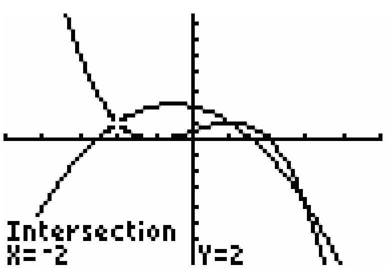
$2y = 2 + 3x - x^3$

$y = \dfrac{2 + 3x - x^3}{2}$

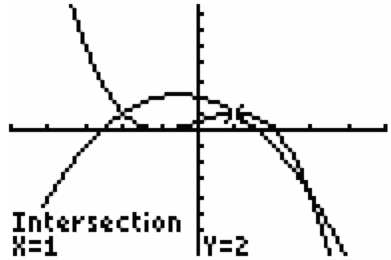
and

$y = 4 - x - x^2$

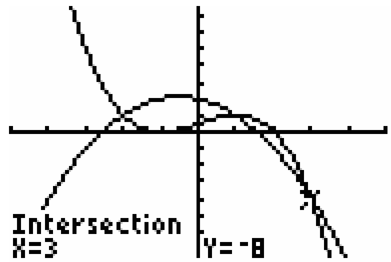
Solving graphically by applying the intersection of graphs method:



[–5, 5] by [–15, 15]



[–5, 5] by [–15, 15]



[–5, 5] by [–15, 15]

The solutions to the system are $(-2,2),(1,2),$ and $(3,-8)$.

21. a. $y = 11 - 2x^2$

Substituting into the other equation

$x^3 + x^2 + (11 - 2x^2) = 15$

$x^3 - x^2 - 4 = 0$

b.

2)	1	−1	0	−4
		2	2	4
	1	1	2	0

The new polynomial equation is $x^2 + x + 2$. It has no real number solutions.

Therefore, the only solution to the system is $(2,3)$.

22. a. $y = 4 - x - x^2$

Substituting into the other equation

$x^3 - 3x + 2\left(4 - x - x^2\right) = 2$

$x^3 - 3x + 8 - 2x - 2x^2 - 2 = 0$

$x^3 - 2x^2 - 5x + 6 = 0$

b.

1)	1	−2	−5	6
		1	−1	−6
	1	−1	−6	0

The new polynomial equation is

$x^2 - x - 6.$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3,\ x = -2$

Therefore, the three solutions to the system are $(-2, 2), (1, 2)$, and $(3, -8)$, as indicated by graphical methods in problem 20 above.

23. Isolating y in both equations:

$4y = 17 - x^2$

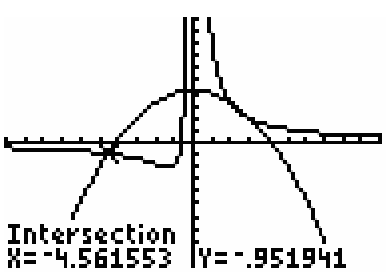
$y = \dfrac{17 - x^2}{4}$

and

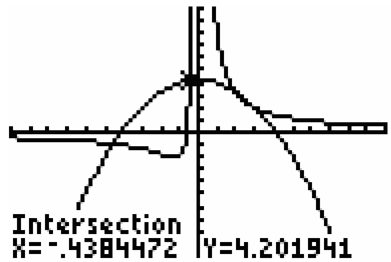
$x^2 y = 3 + 5x$

$y = \dfrac{3 + 5x}{x^2}$

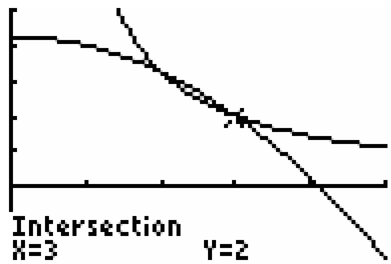
Solving graphically by applying the intersection of graphs method yields:



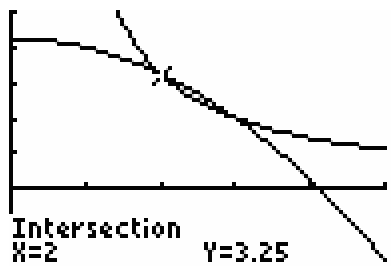
[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]



[0, 5] by [–2, 5]



[0, 5] by [–2, 5]

Pick any two of the following four solutions to the system:

Exact solutions

$(2, 3.25), (3, 2)$

Approximate solutions

$(-0.438, 4.20), (-4.562, -0.952)$

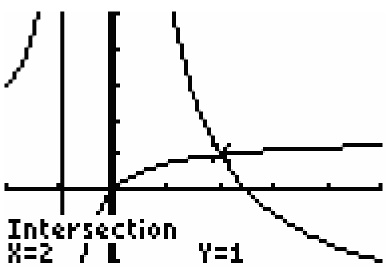
24. Isolating y in both equations:

$$xy = 10 - 4x$$
$$y = \frac{10-4x}{x}$$

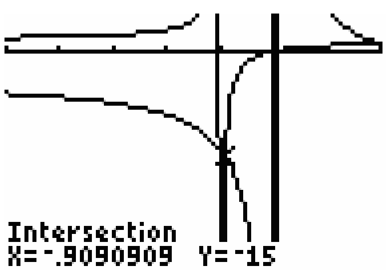
and

$$\frac{y}{x} + y = \frac{3}{2}$$
$$2x\left[\frac{y}{x} + y\right] = 2x\left[\frac{3}{2}\right]$$
$$2y + 2xy = 3x$$
$$y(2+2x) = 3x$$
$$y = \frac{3x}{2+2x}$$

Solving graphically by applying the intersection of graphs method yields:



[–2, 5] by [–2, 5]



[–5, 2] by [–30, 5]

The solutions to the system are $(2,1), (-0.\overline{90}, -15)$.

Section 7.5 Exercises

25. Equilibrium occurs when demand equals supply.

$$q^2 + 2q + 122 = 650 - 30q$$
$$q^2 + 32q - 528 = 0$$
$$(q+44)(q-12) = 0$$
$$q = -44,\ q = 12$$

Since $q \geq 0$ in the physical context of the question, $q = 12$.

Back substituting to find p

$$p = 650 - 30q$$
$$p = 650 - 30(12) = 290$$

Equilibrium occurs when the price is $290 and the demand is 1200 units.

26. Equilibrium occurs when demand equals supply.

$$q^2 + 500 = 1124 - 40q$$
$$q^2 + 40q - 624 = 0$$
$$(q+52)(q-12) = 0$$
$$q = -52,\ q = 12$$

Since $q \geq 0$ in the physical context of the question, $q = 12$.

Back substituting to find p

$$p = 1124 - 40q$$
$$p = 1124 - 40(12) = 644$$

Equilibrium occurs when the price is $644 and the demand is 1200 units.

27. Equilibrium occurs when demand equals supply.

$$0.1q^2 + 50q + 1027.50 = 6000 - 20q$$
$$0.1q^2 + 70q - 4972.50 = 0$$
$$q^2 + 700q - 49,725 = 0$$
$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$q = \frac{-700 \pm \sqrt{(700)^2 - 4(1)(-49,725)}}{2(1)}$$
$$q = \frac{-700 \pm \sqrt{490,000 + 198,900}}{2}$$
$$q = \frac{-700 \pm 830}{2}$$
$$q = \frac{-700 - 830}{2} = -765,$$
$$q = \frac{-700 + 830}{2} = 65$$

Since $q \geq 0$ in the physical context of the question, $q = 65$.

Back substituting to find p

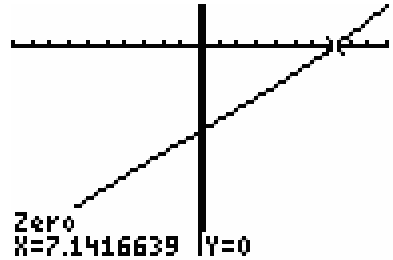
$$p = 6000 - 20q$$
$$p = 6000 - 20(65) = 4700$$

Equilibrium occurs when the price is \$4700 and the demand is 6500 units.

28. Equilibrium occurs when demand equals supply.

$$q^3 + 800q + 6000 = 12,200 - 10q - q^2$$
$$q^3 + q^2 + 810q - 6200 = 0$$

Solving graphically by applying the x-intercept method:



[–10, 10] by [–15,200, 3000]

Since $q \geq 0$ in the physical context of the question, $q \approx 7.14$.

Back substituting to find p

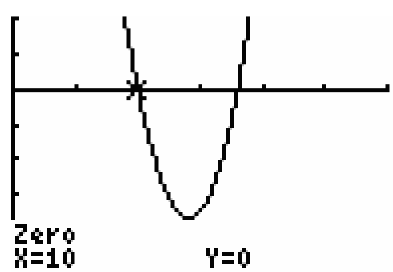
$$p = 12,200 - 10q - q^2$$
$$p = 12,200 - 10(7.14) - (7.14)^2$$
$$p = 12,077.6204 \approx 12,077.62$$

Equilibrium occurs when the price is approximately \$12,077.62 and the demand is 714 units. If the unrounded value found from the graph above is used, then p = \$12,077.58.

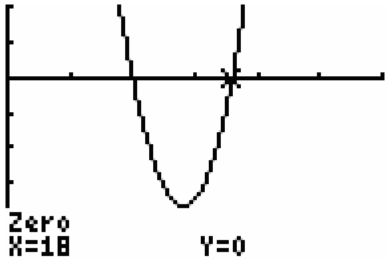
29. Break-even occurs when cost equals revenue.

$$C(x) = R(x)$$
$$2000x + 18,000 + 60x^2 = 4620x - 12x^2 - x^3$$
$$x^3 + 72x^2 - 2620x + 18,000 = 0$$

Solving graphically by applying the x-intercept method:



[0, 30] by [–2500, 1000]



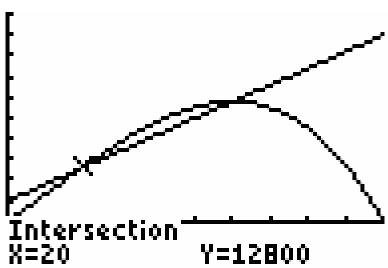
[0, 30] by [–2500, 1000]

Since $x \geq 0$ in the physical context of the question, negative solutions are not relevant.

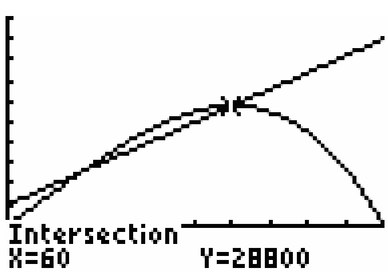
Break-even occurs when the number of thousands of units is 10 or 18. Producing and selling 10,000 units or 18,000 units results in revenue equaling cost.

30. Break-even occurs when cost equals revenue. $C(x) = R(x)$

Solving graphically by applying the intersection of graphs method yields:



[0, 100] by [–10,000, 50,000]



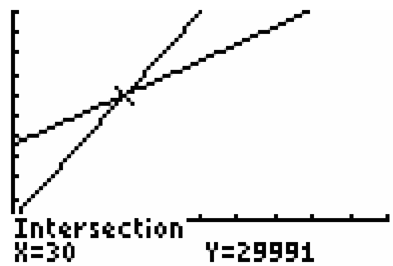
[0, 100] by [–10,000, 50,000]

Since $x \geq 0$ in the physical context of the question, negative solutions are not relevant.

Break-even occurs when the number of units is 20 or 60. Producing and selling 20 units or 60 units results in revenue equaling cost.

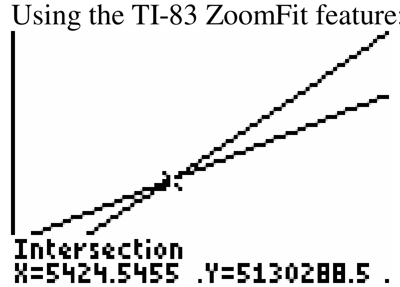
31. Break-even occurs when cost equals revenue. $C(x) = R(x)$

Solving graphically by applying the intersection of graphs method yields:



[0, 100] by [–10,000, 50,000]

Using the TI-83 ZoomFit feature:



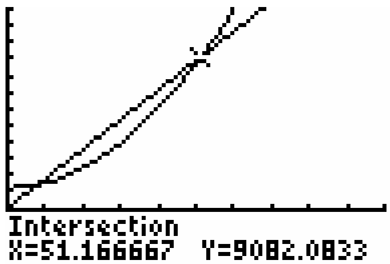
[5000, 6000] by [4,517,901, 6,017,901]

Since $x \geq 0$ in the physical context of the question, negative solutions are not relevant.

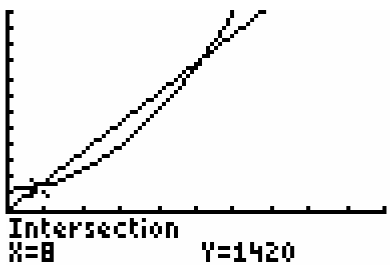
Break-even occurs when the number of units is 30 or approximately 5425. Producing and selling 30 units or approximately 5425 units results in revenue equaling cost.

32. Break-even occurs when cost equals revenue. $C(x)=R(x)$

Solving graphically by applying the intersection of graphs method yields



[0, 100] by [–3000, 12,000]



[0, 100] by [–3000, 12,000]

Break-even occurs when the number of hundreds of units is 8 or approximately 51.2. Producing and selling 800 units or approximately 5117 units results in revenue equaling cost.

33. Let $x=$ length and $y=$ width.

$$\begin{cases} xy=180 \\ 2(x-4)(y-4)=176 \end{cases}$$

Solving the first equation for y

$$y=\frac{180}{x}$$

Substituting

$$2(x-4)(y-4)=176$$

$$2(x-4)\left(\left[\frac{180}{x}\right]-4\right)=176$$

$$(2x-8)\left(\frac{180}{x}-4\right)=176$$

$$360-8x-\frac{1440}{x}+32=176$$

$$x\left[360-8x-\frac{1440}{x}+32\right]=x[176]$$

$$360x-8x^2-1440+32x-176x=0$$

$$-8x^2+216x-1440=0$$

$$-8\left(x^2-27x+180\right)=0$$

$$-8(x-15)(x-12)=0$$

$$x=15,\ x=12$$

The dimensions are 15 inches by 12 inches.

34. Let $x=$ length and $y=$ width.

$$\begin{cases} xy=512 \\ 4(x-8)(y-8)=768 \end{cases}$$

Solving the first equation for y

$y=\frac{512}{x}$

Substituting

$4(x-8)(y-8)=768$

$(4x-32)\left(\frac{512}{x}-8\right)=768$

$2048-32x-\frac{16,384}{x}+256=768$

$x\left[2048-32x-\frac{16,384}{x}+256\right]=x[768]$

$2048x-32x^2-16,384+256x-768x=0$

$-32x^2+1536x-16,384=0$

$-32(x^2-48x+512)=0$

$-16(x-16)(x-32)=0$

$x=16,\ x=32$

The dimensions are 16 centimeters by 32 centimeters.

35. Let x = length of the shorter side and y = length of the longer side.
Assuming that the box is open,

$V=(x)(x)(y)=x^2y.$

$$\text{Surface Area}=xy+2xy+2x^2=3xy+2x^2$$

$$\begin{cases} x^2y=2000 \\ 3xy+2x^2=800 \end{cases}$$

Solving the first equation for y

$y=\frac{2000}{x^2}$

Substituting

$3xy+2x^2=800$

$3x\left(\frac{2000}{x^2}\right)+2x^2=800$

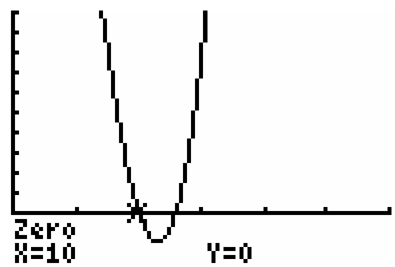
$\frac{6000}{x}+2x^2=800$

$x\left[\frac{6000}{x}+2x^2\right]=x[800]$

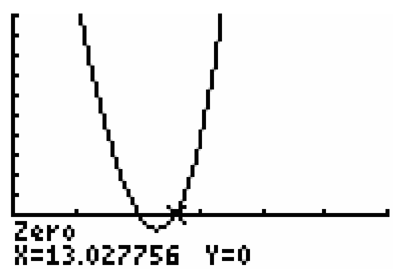
$6000+2x^3=800x$

$2x^3-800x+6000=0$

Solving graphically by applying the x-intercept method with $x\geq 0$ yields:



[0, 30] by [–250, 1000]



[0, 30] by [–250, 1000]

Substituting to find y

$x=10\Rightarrow y=\frac{2000}{(10)^2}=20$

$x\approx 13.03\Rightarrow y\approx\frac{2000}{(13.03)^2}=11.78$

In the second case, x is not the smaller side. Therefore, the solution is a box with dimension 10 cm by 10 cm by 20 cm.

36. Let x = length, y = width, and z = height. Note that $y = \frac{1}{2}x$.

Since the box has a top,

$$V = (x)(y)(z) = x\left(\frac{1}{2}x\right)z = \frac{1}{2}x^2z.$$

$$\begin{aligned}\text{Surface Area} &= 2xy + 2xz + 2yz \\ &= 2x\left(\frac{1}{2}x\right) + 2xz + 2\left(\frac{1}{2}x\right)z \\ &= x^2 + 2xz + xz \\ &= x^2 + 3xz\end{aligned}$$

$$\begin{cases}\frac{1}{2}x^2z = 6000 \\ x^2 + 3xz = 2200\end{cases}$$

Solving the first equation for z

$$\frac{1}{2}x^2z = 6000$$

$$z = \frac{12{,}000}{x^2}$$

Substituting

$$x^2 + 3xz = 2200$$

$$x^2 + 3x\left(\frac{12{,}000}{x^2}\right) = 2200$$

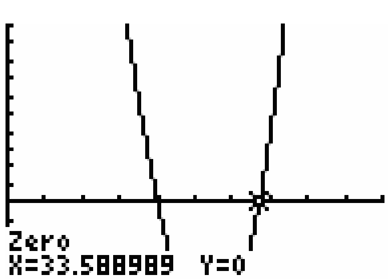
$$x^2 + \frac{36{,}000}{x} = 2200$$

$$x\left[x^2 + \frac{36{,}000}{x}\right] = x[2200]$$

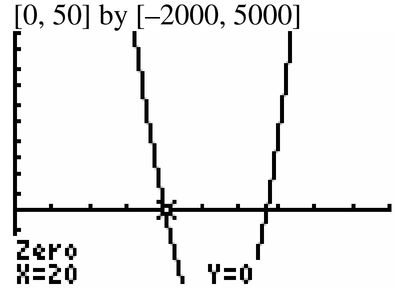
$$x^3 + 36{,}000 = 2200x$$

$$x^3 - 2200x + 36{,}000 = 0$$

Solving graphically by applying the x-intercept method with $x \geq 0$ yields



[0, 50] by [–2000, 5000]



[0, 50] by [–2000, 5000]

Substituting to find y and z yields:

$$x = 20 \Rightarrow y = \frac{1}{2}(20) = 10$$

$$\Rightarrow z = \frac{12{,}000}{(20)^2} = 30$$

$$x \approx 33.59 \Rightarrow y \approx \frac{1}{2}(33.59) = 16.80$$

$$\Rightarrow z \approx \frac{12{,}000}{(33.59)^2} = 10.64$$

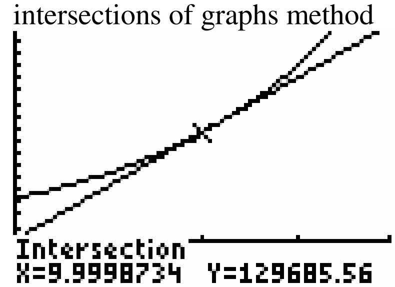
There are two possible boxes. One with dimensions 10 cm by 20 cm by 30 cm. and another with dimensions approximately 33.59 cm by 16.80 cm by 10.64 cm.

37. Let t = time in years and y = amount in dollars.

$$\begin{cases}y = 50{,}000(1.10)^t \\ y = 12{,}968.72t\end{cases}$$

$$50{,}000(1.10)^t = 12{,}968.72t$$

Solving graphically by applying the intersections of graphs method



[0, 20] by [–50,000, 250,000]

In approximately ten years the trust fund will equal the amount of money received from the second account.

38. Let x = length and width, and let y = height.
Assuming the box is open,
$V = (x)(x)(y) = x^2 y.$
$\text{Surface Area} = x^2 + 4xy$

$$\begin{cases} x^2 y = 500 \\ x^2 + 4xy = 500 \end{cases}$$

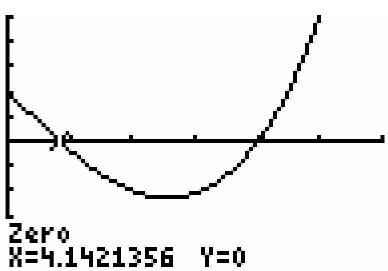
Solving the first equation for y

$$y = \frac{500}{x^2}$$

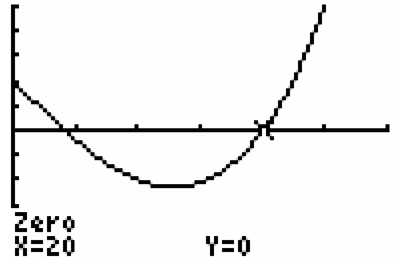
Substituting

$$x^2 + 4xy = 500$$
$$x^2 + 4x\left(\frac{500}{x^2}\right) = 500$$
$$x^2 + \frac{2000}{x} = 500$$
$$x\left[x^2 + \frac{2000}{x}\right] = x[500]$$
$$x^3 + 2000 = 500x$$
$$x^3 - 500x + 2000 = 0$$

Solving graphically by applying the x-intercept method with $x \ge 0$ yields



[0, 30] by [–5000, 5000]



[0, 30] by [–5000, 5000]

Substituting to find y and z yields:

$$x = 20 \Rightarrow y = \frac{500}{(20)^2} = \frac{500}{400} = 1.25$$
$$x \approx 4.14 \Rightarrow y \approx \frac{500}{(4.14)^2} = 29.17$$

There are two possible boxes. One with dimensions 20 cm by 20 cm by 1.25 cm. and another with dimensions approximately 4.14 cm by 4.14 cm by 29.17 cm.

If the unrounded value (4.1421356) is used, then y = 29.14.

Chapter 7 Skills Check

1. $$\begin{cases} 2x-3y+z=2 \\ 3x+2y-z=6 \\ x-4y+2z=2 \end{cases} \xrightarrow[-2Eq3+Eq1\to Eq3]{-3Eq3+Eq2\to Eq2}$$

$$\begin{cases} 2x-3y+z=2 \\ 14y-7z=0 \\ 5y-3z=-2 \end{cases} \xrightarrow{-5Eq2+14Eq3\to Eq3}$$

$$\begin{cases} 2x-3y+z=2 \\ 14y-7z=0 \\ -7z=-28 \end{cases} \xrightarrow{-\frac{1}{7}Eq3\to Eq3}$$

$$\begin{cases} 2x-3y+z=2 \\ 14y-7z=0 \\ z=4 \end{cases}$$

Since z is isolated, back substitution yields

$14y-7(4)=0$

$14y-28=0$

$14y=28$

$y=2$

and

$2x-3(2)+(4)=2$

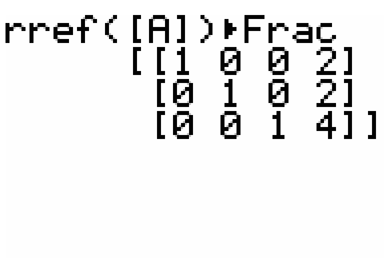
$2x-6+4=2$

$2x=4$

$x=2$

The solutions are $x=2,\ y=2,\ z=4$.

Applying technology yields



2. $$\begin{cases} 3x-2y-4z=9 \\ x+3y+2z=-1 \\ 2x+4y+4z=2 \end{cases} \xrightarrow[-2Eq2+Eq3\to Eq3]{-3Eq2+Eq1\to Eq2}$$

$$\begin{cases} 3x-2y-4z=9 \\ -11y-10z=12 \\ -2y=4 \end{cases} \xrightarrow{-\frac{1}{2}Eq3\to Eq3}$$

$$\begin{cases} 3x-2y-4z=9 \\ -11y-10z=12 \\ y=-2 \end{cases}$$

Since y is isolated, back substitution yields

$-11(-2)-10z=12$

$22-10z=12$

$-10z=-10$

$z=1$

and

$3x-2(-2)-4(1)=9$

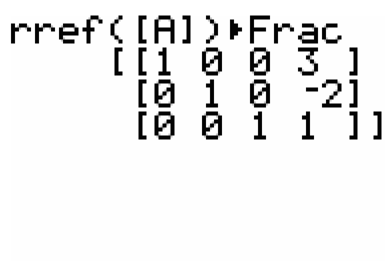
$3x+4-4=9$

$3x=9$

$x=3$

The solutions are $x=3,\ \ y=-2,\ \ z=1$.

Applying technology yields



3.
$$\begin{cases} 3x+2y-z=6 \\ 2x-4y-2z=0 \\ 5x+3y+6z=2 \end{cases} \xrightarrow[-5Eq2+2Eq3\to Eq3]{-3Eq2+2Eq1\to Eq2}$$

$$\begin{cases} 3x+2y-z=6 \\ 16y+4z=12 \\ 26y+22z=4 \end{cases} \xrightarrow[\frac{1}{2}Eq3\to Eq3]{\frac{1}{4}Eq2\to Eq2}$$

$$\begin{cases} 3x+2y-z=6 \\ 4y+z=3 \\ 13y+11z=2 \end{cases} \xrightarrow{-13Eq2+4Eq3\to Eq3}$$

$$\begin{cases} 3x+2y-z=6 \\ 4y+z=3 \\ 31z=-31 \end{cases} \xrightarrow{\frac{1}{31}Eq3\to Eq3}$$

$$\begin{cases} 3x+2y-z=6 \\ 4y+z=3 \\ z=-1 \end{cases}$$

Since z is isolated, back substitution yields

$4y+(-1)=3$

$4y=4$

$y=1$

and

$3x+2y-z=6$

$3x+2(1)-(-1)=6$

$3x+3=6$

$3x=3$

$x=1$

The solutions are $x=1,\ y=1,\ z=-1$.

Applying technology yields

```
rref([A])▸Frac
     [[1 0 0 1 ]
      [0 1 0 1 ]
      [0 0 1 -1]]
```

4.
$$\begin{cases} 3x+6y+9z=27 \\ 2x+3y-z=-2 \\ 4x+5y+z=6 \end{cases} \xrightarrow[-2Eq2+Eq3\to Eq3]{-3Eq2+2Eq1\to Eq2}$$

$$\begin{cases} 3x+6y+9z=27 \\ 3y+21z=60 \\ -y+3z=10 \end{cases} \xrightarrow{Eq2+3Eq3\to Eq3}$$

$$\begin{cases} 3x+6y+9z=27 \\ 3y+21z=60 \\ 30z=90 \end{cases} \xrightarrow{\frac{1}{30}Eq3\to Eq3}$$

$$\begin{cases} 3x+6y+9z=27 \\ 3y+21z=60 \\ z=3 \end{cases}$$

Since z is isolated, back substitution yields

$3y+21(3)=60$

$3y+63=60$

$3y=-3$

$y=-1$

and

$3x+6y+9z=27$

$3x+6(-1)+9(3)=27$

$3x+21=27$

$3x=6$

$x=2$

The solutions are $x=2,\ y=-1,\ z=3$.

Applying technology yields

```
rref([A])▸Frac
     [[1 0 0 2 ]
      [0 1 0 -1]
      [0 0 1 3 ]]
```

5. Writing the system as an augmented matrix and reducing yields

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 2 & -1 & 5 & 15 \\ 3 & -4 & 1 & 7 \end{array}\right] \xrightarrow[-3R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & -5 & 9 & 13 \\ 0 & -10 & 7 & 4 \end{array}\right] \xrightarrow{-2R_2+R_3\to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & -5 & 9 & 13 \\ 0 & 0 & -11 & -22 \end{array}\right] \xrightarrow[\left(-\frac{1}{11}\right)R_3\to R_3]{\left(-\frac{1}{5}\right)R_2\to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 1 & -\frac{9}{5} & -\frac{13}{5} \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z=2$

and

$y-\frac{9}{5}(2)=-\frac{13}{5}$

$y-\frac{18}{5}=-\frac{13}{5}$

$y=\frac{5}{5}=1$

and

$x+2(1)-2(2)=1$

$x+2-4=1$

$x-2=1$

$x=3$

The solutions are $x=3,\ \ y=1,\ \ z=2.$

Applying technology yields

```
rref([A])▸Frac
     [[1 0 0 3]
      [0 1 0 1]
      [0 0 1 2]]
```

6. Writing the system as an augmented matrix and reducing yields

$$\left[\begin{array}{ccc|c} -6 & 4 & -2 & 4 \\ 3 & -2 & 5 & -6 \\ 1 & -4 & 1 & -8 \end{array}\right] \xrightarrow{R_1\leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & -8 \\ 3 & -2 & 5 & -6 \\ -6 & 4 & -2 & 4 \end{array}\right] \xrightarrow[6R_1+R_3\to R_3]{-3R_1+R_2\to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & -8 \\ 0 & 10 & 2 & 18 \\ 0 & -20 & 4 & -44 \end{array}\right] \xrightarrow{2R_2+R_3\to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & -8 \\ 0 & 10 & 2 & 18 \\ 0 & 0 & 8 & -8 \end{array}\right] \xrightarrow{\left(\frac{1}{8}\right)R_3\to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & -8 \\ 0 & 10 & 2 & 18 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$z=-1$

and

$10y+2(-1)=18$

$10y-2=18$

$10y=20$

$y=2$

and

$x-4y+z=-8$

$x-4(2)+(-1)=-8$

$x-8-1=-8$

$x-9=-8$

$x=1$

The solutions are $x=1$, $y=2,\ z=-1.$

Applying technology yields

```
rref([A])▸Frac
     [[1 0 0 1 ]
      [0 1 0 2 ]
      [0 0 1 -1]]
```

7. Writing the system as an augmented matrix and reducing yields

$$\left[\begin{array}{ccc|c} 2 & 5 & 8 & 30 \\ 18 & 42 & 18 & 60 \end{array}\right] \xrightarrow{-9R_1+R_2 \to R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 8 & 30 \\ 0 & -3 & -54 & -210 \end{array}\right] \xrightarrow{\left(-\frac{1}{3}\right)R_2 \to R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 8 & 30 \\ 0 & 1 & 18 & 70 \end{array}\right]$$

Since the system has more variables than equations, the system is dependent and has infinitely many solutions.

Let $z = z =$ any real number

$y + 18z = 70$

$y = 70 - 18z$

and

$2x + 5y + 8z = 30$

$2x + 5(70 - 18z) + 8z = 30$

$2x + 350 - 90z + 8z = 30$

$2x + 350 - 82z = 30$

$2x = 82z - 320$

$x = 41z - 160$

The solution is

$x = 41z - 160,\ y = 70 - 18z,\ z = z.$

Applying technology yields

```
rref([B])▸Frac
[[1 0 -41 -160]
 [0 1 18  70  ]]
```

See the back substitution process above.

8. Writing the system as an augmented matrix and using technology to produce the reduced row-echelon form yields

$$\left[\begin{array}{ccc|c} 9 & 21 & 15 & 60 \\ 2 & 5 & 8 & 30 \\ 1 & 2 & -3 & -10 \end{array}\right]$$

```
rref([A])▸Frac
[[1 0 -31 -110]
 [0 1 14  50  ]
 [0 0 0   0   ]]
```

Note that the system has a last row of all zeros. Therefore, the system is dependent and has infinitely many solutions.

Let $z = z$

$y + 14z = 50$

$y = 50 - 14z$

and

$x - 31z = -110$

$x = 31z - 110$

The solution is

$x = 31z - 110,\ y = 50 - 14z,\ z = z$

9. Writing the system as an augmented matrix and using technology to produce the reduced row-echelon form yields

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 9 & 12 & 15 & 6 \\ 2 & 1 & 3 & -10 \end{array}\right]$$

```
rref([A])
   [[1 0 1.4 0]
    [0 1 .2  0]
    [0 0 0   1]]
```

Since the system has a last row of all zeros with a 1 as the augment, the system is inconsistent and has no solution.

10. Writing the system as an augmented matrix and using technology to produce the reduced row-echelon form yields

$$\left[\begin{array}{ccc|c} 5 & 7 & 10 & 6 \\ 2 & 5 & 6 & 1 \\ 3 & 2 & 4 & 6 \end{array}\right]$$

```
rref([A])►Frac
 [[1 0 8/11  0]
  [0 1 10/11 0]
  [0 0 0     1]]
```

Since the system has a last row of all zeros with a 1 as the augment, the system is inconsistent and has no solution.

11. Writing the system as an augmented matrix and using technology to produce the reduced row-echelon form yields

$$\left[\begin{array}{cccc|c} 3 & 2 & -1 & 1 & 12 \\ 1 & -4 & 3 & -1 & -18 \\ 1 & 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & -3 & -10 \end{array}\right]$$

```
rref([A])►Frac
  [[1 0 0 0 1 ]
   [0 1 0 0 3 ]
   [0 0 1 0 -2]
   [0 0 0 1 1 ]]
```

The solutions are $x=1,\ y=3,\ z=-2,$ and $w=1.$.

12. Writing the system as an augmented matrix and using technology to produce the reduced row-echelon form yields

$$\left[\begin{array}{cccc|c} 2 & 1 & -3 & 4 & 7 \\ 1 & -2 & 1 & -2 & 0 \\ 3 & 1 & 4 & 1 & -2 \\ 1 & 3 & 2 & 2 & -1 \end{array}\right]$$

```
rref([A])►Frac
  [[1 0 0 0 2 ]
   [0 1 0 0 1 ]
   [0 0 1 0 -2]
   [0 0 0 1 -1]]
```

The solutions are $x=2,\ y=1,\ z=-2,$ and $w=-1.$.

13. $B+D$

$$=\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}+\begin{bmatrix} -2 & 3 & 1 \\ -3 & 2 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 1-2 & 2+3 & 1+1 \\ 2-3 & -1+2 & 3+2 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & 5 & 2 \\ -1 & 1 & 5 \end{bmatrix}$$

14. $D-B$

$$=\begin{bmatrix} -2 & 3 & 1 \\ -3 & 2 & 2 \end{bmatrix}-\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} -2-1 & 3-2 & 1-1 \\ -3-2 & 2-(-1) & 2-3 \end{bmatrix}$$

$$=\begin{bmatrix} -3 & 1 & 0 \\ -5 & 3 & -1 \end{bmatrix}$$

15. $5C$

$$=5\begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 3 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 5(2) & 5(3) \\ 5(-1) & 5(2) \\ 5(3) & 5(-2) \end{bmatrix}$$

$$=\begin{bmatrix} 10 & 15 \\ -5 & 10 \\ 15 & -10 \end{bmatrix}$$

16. AB can not be calculated because the number of columns in matrix A is different from the number of rows in matrix B.

17. BA

$$=\begin{bmatrix}1 & 2 & 1\\ 2 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & 3 & -3\\ 2 & 4 & 1\\ -1 & 3 & 2\end{bmatrix}$$
$$=\begin{bmatrix}1+4-1 & 3+8+3 & -3+2+2\\ 2-2-3 & 6-4+9 & -6-1+6\end{bmatrix}$$
$$=\begin{bmatrix}4 & 14 & 1\\ -3 & 11 & -1\end{bmatrix}$$

18. CD

$$=\begin{bmatrix}2 & 3\\ -1 & 2\\ 3 & -2\end{bmatrix}\begin{bmatrix}-2 & 3 & 1\\ -3 & 2 & 2\end{bmatrix}$$
$$=\begin{bmatrix}-4-9 & 6+6 & 2+6\\ 2-6 & -3+4 & -1+4\\ -6+6 & 9-4 & 3-4\end{bmatrix}$$
$$=\begin{bmatrix}-13 & 12 & 8\\ -4 & 1 & 3\\ 0 & 5 & -1\end{bmatrix}$$

19. DC

$$=\begin{bmatrix}-2 & 3 & 1\\ -3 & 2 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 2\\ 3 & -2\end{bmatrix}$$
$$=\begin{bmatrix}-4-3+3 & -6+6-2\\ -6-2+6 & -9+4-4\end{bmatrix}$$
$$=\begin{bmatrix}-4 & -2\\ -2 & -9\end{bmatrix}$$

20. A^2

$$= A\times A$$
$$=\begin{bmatrix}1 & 3 & -3\\ 2 & 4 & 1\\ -1 & 3 & 2\end{bmatrix}\begin{bmatrix}1 & 3 & -3\\ 2 & 4 & 1\\ -1 & 3 & 2\end{bmatrix}$$
$$=\begin{bmatrix}1+6+3 & 3+12-9 & -3+3-6\\ 2+8-1 & 6+16+3 & -6+4+2\\ -1+6-2 & -3+12+6 & 3+3+4\end{bmatrix}$$
$$=\begin{bmatrix}10 & 6 & -6\\ 9 & 25 & 0\\ 3 & 15 & 10\end{bmatrix}$$

21. A^{-1}

$$\left[\begin{array}{ccc|ccc}1 & 3 & -3 & 1 & 0 & 0\\ 2 & 4 & 1 & 0 & 1 & 0\\ -1 & 3 & 2 & 0 & 0 & 1\end{array}\right]\xrightarrow[R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$$
$$=\left[\begin{array}{ccc|ccc}1 & 3 & -3 & 1 & 0 & 0\\ 0 & -2 & 7 & -2 & 1 & 0\\ 0 & 6 & -1 & 1 & 0 & 1\end{array}\right]\xrightarrow{3R_2+R_3\to R_3}$$
$$=\left[\begin{array}{ccc|ccc}1 & 3 & -3 & 1 & 0 & 0\\ 0 & -2 & 7 & -2 & 1 & 0\\ 0 & 0 & 20 & -5 & 3 & 1\end{array}\right]\xrightarrow{\left(\frac{1}{20}\right)R_3\to R_3}$$
$$=\left[\begin{array}{ccc|ccc}1 & 3 & -3 & 1 & 0 & 0\\ 0 & -2 & 7 & -2 & 1 & 0\\ 0 & 0 & 1 & -\frac{1}{4} & \frac{3}{20} & \frac{1}{20}\end{array}\right]\xrightarrow[3R_3+R_1\to R_1]{-7R_3+R_2\to R_2}$$

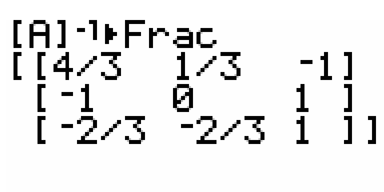
$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{1}{4} & \frac{9}{20} & \frac{3}{20} \\ 0 & -2 & 0 & -\frac{1}{4} & -\frac{1}{20} & -\frac{7}{20} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{3}{20} & \frac{1}{20} \end{array}\right] \xrightarrow{\left(-\frac{1}{2}\right)R_2 \to R_2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{1}{4} & \frac{9}{20} & \frac{3}{20} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{40} & \frac{7}{40} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{3}{20} & \frac{1}{20} \end{array}\right] \xrightarrow{(-3)R_2 + R_1 \to R_1}$$

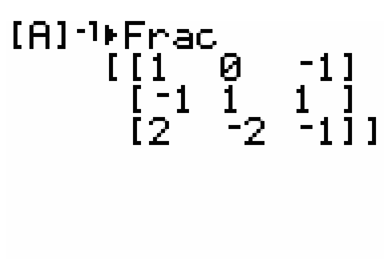
$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{8} & \frac{3}{8} & -\frac{3}{8} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{40} & \frac{7}{40} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{3}{20} & \frac{1}{20} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{8} & \frac{3}{8} & -\frac{3}{8} \\ \frac{1}{8} & \frac{1}{40} & \frac{7}{40} \\ -\frac{1}{4} & \frac{3}{20} & \frac{1}{20} \end{bmatrix}$$

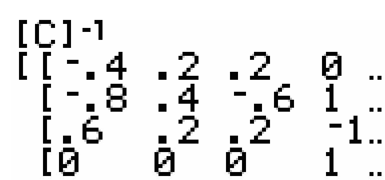
22. Applying technology to calculate A^{-1} yields



23. Applying technology to calculate A^{-1} yields



24. Applying technology to calculate A^{-1} yields



$$A^{-1} = \begin{bmatrix} -0.4 & 0.2 & 0.2 & 0 \\ -0.8 & 0.4 & -0.6 & 1 \\ 0.6 & 0.2 & 0.2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

25. $\begin{bmatrix} 1 & 1 & -3 \\ 2 & 4 & 1 \\ -1 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 15 \\ 5 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 4 & 1 \\ -1 & 3 & 2 \end{bmatrix}$. Applying technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{11}{30} & -\frac{13}{30} \\ \frac{1}{6} & \frac{1}{30} & \frac{7}{30} \\ -\frac{1}{3} & \frac{2}{15} & -\frac{1}{15} \end{bmatrix}$.

Multiplying both sides of the matrix equation by A^{-1} yields

$$\begin{bmatrix} -\frac{1}{6} & \frac{11}{30} & -\frac{13}{30} \\ \frac{1}{6} & \frac{1}{30} & \frac{7}{30} \\ -\frac{1}{3} & \frac{2}{15} & -\frac{1}{15} \end{bmatrix}\begin{bmatrix} 1 & 1 & -3 \\ 2 & 4 & 1 \\ -1 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} & \frac{11}{30} & -\frac{13}{30} \\ \frac{1}{6} & \frac{1}{30} & \frac{7}{30} \\ -\frac{1}{3} & \frac{2}{15} & -\frac{1}{15} \end{bmatrix}\begin{bmatrix} 8 \\ 15 \\ 5 \end{bmatrix}.$$

Using technology to carry out the multiplication yields:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$x = 2,\ y = 3,\ z = -1$

26. $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix}$. Applying technology to calculate A^{-1} yields $A^{-1} = \begin{bmatrix} 0.9 & 0.2 & -0.7 \\ -0.5 & 0 & 0.5 \\ 0.7 & -0.4 & -0.1 \end{bmatrix}$.

Multiplying both sides of the matrix equation by A^{-1} yields

$$\begin{bmatrix} 0.9 & 0.2 & -0.7 \\ -0.5 & 0 & 0.5 \\ 0.7 & -0.4 & -0.1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & -1 \\ 2 & 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.9 & 0.2 & -0.7 \\ -0.5 & 0 & 0.5 \\ 0.7 & -0.4 & -0.1 \end{bmatrix}\begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}.$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -16 \\ 20 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -16 \\ 20 \\ -8 \end{bmatrix}$$

$x = -16,\ y = 20,\ z = -8$

27. $$\begin{bmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 5 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 20 \\ 24 \end{bmatrix}$$

Let $A = \begin{bmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 5 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$. Applying technology to calculate A^{-1} yields

$$A^{-1} = \begin{bmatrix} -1.4 & 1.6 & -0.2 & -0.2 \\ 0.4 & -0.6 & 0.2 & 0.2 \\ 0.8 & -0.2 & 0.4 & -0.6 \\ -1.2 & 1.8 & -0.6 & 0.4 \end{bmatrix}.$$

Multiplying both sides of the matrix equation by A^{-1} yields

$$\begin{bmatrix} -1.4 & 1.6 & -0.2 & -0.2 \\ 0.4 & -0.6 & 0.2 & 0.2 \\ 0.8 & -0.2 & 0.4 & -0.6 \\ -1.2 & 1.8 & -0.6 & 0.4 \end{bmatrix}\begin{bmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 5 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1.4 & 1.6 & -0.2 & -0.2 \\ 0.4 & -0.6 & 0.2 & 0.2 \\ 0.8 & -0.2 & 0.4 & -0.6 \\ -1.2 & 1.8 & -0.6 & 0.4 \end{bmatrix}\begin{bmatrix} 6 \\ 12 \\ 20 \\ 24 \end{bmatrix}.$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -4 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -4 \\ 12 \end{bmatrix}$$

$x_1 = 2,\ x_2 = 4,\ x_3 = -4,\ x_4 = 12$

28. $$\begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 4 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Applying technology to calculate A^{-1} yields

$$A^{-1} = \begin{bmatrix} 0.25 & 0.25 & 0 & -0.75 \\ 0.5 & -0.5 & 0 & 0.5 \\ -0.25 & -0.25 & 1 & -0.25 \\ -0.5 & 0.5 & 0 & 0.5 \end{bmatrix}.$$

Multiplying both sides of the matrix equation by A^{-1} yields

$$\begin{bmatrix} 0.25 & 0.25 & 0 & -0.75 \\ 0.5 & -0.5 & 0 & 0.5 \\ -0.25 & -0.25 & 1 & -0.25 \\ -0.5 & 0.5 & 0 & 0.5 \end{bmatrix}\begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0 & -0.75 \\ 0.5 & -0.5 & 0 & 0.5 \\ -0.25 & -0.25 & 1 & -0.25 \\ -0.5 & 0.5 & 0 & 0.5 \end{bmatrix}\begin{bmatrix} 4 \\ 12 \\ 4 \\ 8 \end{bmatrix}.$$

Using technology to carry out the multiplication yields

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -2 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -2 \\ 8 \end{bmatrix}$$

$x_1 = -2,\ x_2 = 0,\ x_3 = -2,\ x_4 = 8$

29. Isolating y in the second equation

$4x - y = 4$

$y = 4x - 4$

Substituting into the other equation

$x^2 - y = x$

$x^2 - (4x - 4) = x$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4,\ x = 1$

Back substituting to calculate y

$x = 4 \Rightarrow y = 4(4) - 4 = 12$

$x = 1 \Rightarrow y = 4(1) - 4 = 0$

The solutions to the system are $(4,12)$ and $(1,0)$.

30. Isolating y in the second equation

$x^2 + 2y = 140$

$2y = 140 - x^2$

$y = \dfrac{140 - x^2}{2}$

Substituting into the other equation

$x^2 y = 2000$

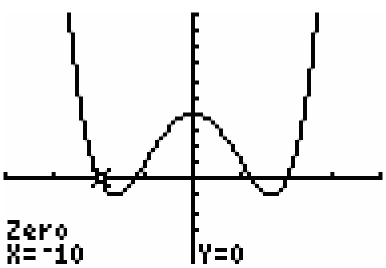
$x^2\left(\dfrac{140 - x^2}{2}\right) = 2000$

$2(x^2)\left(\dfrac{140 - x^2}{2}\right) = 2(2000)$

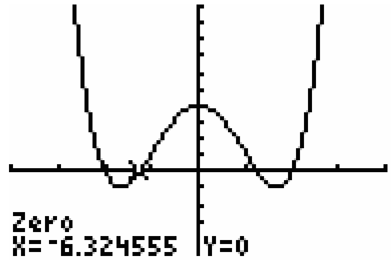
$140x^2 - x^4 = 4000$

$x^4 - 140x^2 + 4000 = 0$

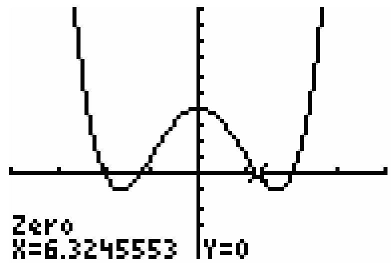
Solving graphically by applying the x-intercept method



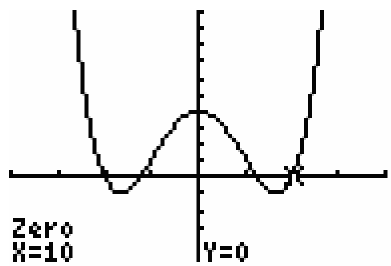
[–20, 20] by [–5000, 10,000]



[–20, 20] by [–5000, 10,000]



[–20, 20] by [–5000, 10,000]



[–20, 20] by [–5000, 10,000]

Back substituting to calculate y

$x = -10 \Rightarrow y = \dfrac{2000}{x^2} = \dfrac{2000}{(-10)^2} = 20$

$x = 10 \Rightarrow y = \dfrac{2000}{x^2} = \dfrac{2000}{(10)^2} = 20$

$x \approx -6.32 \Rightarrow y = \dfrac{2000}{x^2} = \dfrac{2000}{(-6.32)^2} \approx 50.07$

$x \approx 6.32 \Rightarrow y = \dfrac{2000}{x^2} = \dfrac{2000}{(6.32)^2} \approx 50.07$

The solutions to the system are $(-10,20), (-6.32,50.07), (6.32,50.07)$, and $(10,20)$.

Chapter 7 Review

31. Let x represent the number of \$40 tickets, y represent the number of \$60 tickets, and z represent the number of \$100 tickets.

$$\begin{cases} x+y+z=4000 \\ 40x+60y+100z=200{,}000 \\ y=\frac{1}{4}(x+z) \end{cases} \xrightarrow{4Eq3 \to Eq3}$$

$$\begin{cases} x+y+z=4000 \\ 40x+60y+100z=200{,}000 \\ x-4y+z=0 \end{cases} \xrightarrow[-1Eq1+Eq3 \to Eq3]{-40Eq1+Eq2 \to Eq2}$$

$$\begin{cases} x+y+z=4000 \\ 20y+60z=40{,}000 \\ -5y=-4000 \end{cases}$$

$-5y=-4000$

$y=800$

Substituting to find z

$20(800)+60z=40{,}000$

$16{,}000+60z=40{,}000$

$60z=24{,}000$

$z=400$

Substituting to find x

$x+(800)+(400)=4000$

$x=2800$

To generate \$200,000, the concert promoter needs to sell 2800 \$40 tickets, 800 \$60 tickets, and 400 \$100 tickets.

32. Let x represent the daily dosage of medication A, y represent the daily dosage of medication B, and z represent the daily dosage of medication C.

$$\begin{cases} 6x+2y+z=28.7 \\ z=\frac{1}{2}(x+y) \\ \frac{x}{y}=\frac{2}{3} \end{cases}$$

or

$$\begin{cases} 6x+2y+z=28.7 \\ x+y-2z=0 \\ 3x-2y=0 \end{cases}$$

$$\begin{cases} 6x+2y+z=28.7 \\ x+y-2z=0 \\ 3x-2y=0 \end{cases} \xrightarrow[-3Eq2+Eq3\to Eq3]{Eq1-6Eq2\to Eq2}$$

$$\begin{cases} 6x+2y+z=28.7 \\ -4y+13z=28.7 \\ -5y+6z=0 \end{cases} \xrightarrow{-5Eq2+4Eq3\to Eq3}$$

$$\begin{cases} 6x+2y+z=28.7 \\ -4y+13z=28.7 \\ -41z=-143.5 \end{cases} \xrightarrow{-\frac{1}{41}Eq3\to Eq3}$$

$$\begin{cases} 6x+2y+z=28.7 \\ -4y+13z=28.7 \\ z=3.5 \end{cases}$$

Substituting to find y

$-4y+13(3.5)=28.7$

$-4y+45.5=28.7$

$-4y=-16.8$

$y=4.2$

Substituting to find x

$6x+2(4.2)+(3.5)=28.7$

$6x+8.4+3.5=28.7$

$6x=16.8$

$x=2.8$

Each dosage of medication A contains 2.8 mg, each dosage of medication B contains 4.2 mg, and each dosage of medication C contains 3.5 mg.

33. Let x represent the amount invested in property I (12%), y represent the amount invested in property II (15%), and z represent the amount invested in property III (10%).

$$\begin{cases} x+y+z=750{,}000 \\ 0.12x+0.15y+0.10z=89{,}500 \\ z=\frac{1}{2}(x+y) \end{cases} \text{ or } \begin{cases} x+y+z=750{,}000 \\ 0.12x+0.15y+0.10z=89{,}500 \\ x+y-2z=0 \end{cases}$$

or

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 750{,}000 \\ 0.12 & 0.15 & 0.10 & 89{,}500 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

Using technology to solve the augmented matrix yields

```
rref([A])▸Frac
[[1 0 0 350000]
 [0 1 0 150000]
 [0 0 1 250000]]
```

To generate an annual return of \$89,500, \$350,000 needs to be invested in property I, \$150,000 needs to be invested in property II, and \$250,000 needs to be invested property III.

34. Let $x =$ the amount in 12% fund, $y =$ the amount in the 16% fund, and $z =$ the amount in the 8% fund.

$$\begin{cases} x+y+z=360{,}000 \\ 0.12x+0.16y+0.08z=35{,}200 \\ z=2(x+y) \end{cases} \quad \text{or} \quad \begin{cases} x+y+z=360{,}000 \\ 0.12x+0.16y+0.08z=35{,}200 \\ 2x+2y-z=0 \end{cases}$$

or

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 360{,}000 \\ 0.12 & 0.16 & 0.08 & 35{,}200 \\ 2 & 2 & -1 & 0 \end{array}\right]$$

Using technology to solve the augmented matrix yields

```
rref([A])▸Frac
[[1 0 0 80000 ]
 [0 1 0 40000 ]
 [0 0 1 240000]]
```

\$80,000 is invested in the 12% fund, \$40,000 is invested in the 16% fund, and \$240,000 is invested in the 8% fund.

35. Let x represent the cost of property I, y represent the cost of property II, and z represent the cost of property III.

$$\begin{cases} x+y+z=1{,}180{,}000 \\ x=y+75{,}000 \\ z=3(x+y) \end{cases} \quad \text{or} \quad \begin{cases} x+y+z=1{,}180{,}000 \\ x-y=75{,}000 \\ 3x+3y-z=0 \end{cases}$$

or

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}180{,}000 \\ 1 & -1 & 0 & 75{,}000 \\ 3 & 3 & -1 & 0 \end{array}\right]$$

Using technology to solve the augmented matrix yields

```
rref([A])▸Frac
[[1 0 0 185000]
 [0 1 0 110000]
 [0 0 1 885000]]
```

Property I costs $185,000, property II costs $110,000, and property III costs $885,000.

36. Let x = the number of Portfolio I units, y = the number of Portfolio II units, and z = the number of Portfolio III units.

$$\begin{cases} 10x+12y+10z=290 \\ 2x+8y+4z=138 \\ 3x+5y+8z=161 \end{cases} \quad \text{or} \quad \left[\begin{array}{ccc|c} 10 & 12 & 10 & 290 \\ 2 & 8 & 4 & 138 \\ 3 & 5 & 8 & 161 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
     [[1 0 0 5 ]
      [0 1 0 10]
      [0 0 1 12]]
```

The solution to the system is $x=5,\ y=10,\ z=12.$ The client needs to purchase 5 units of Portfolio I, 10 units of Portfolio II, and 12 units of Portfolio III to achieve the investment objectives.

37. $$\begin{cases} x+y+z=375{,}000 \\ x=y+50{,}000 \\ z=\frac{1}{2}(x+y) \end{cases}$$

Note that $x=y+50{,}000$ implies $x-y=50{,}000$. Likewise $z=\frac{1}{2}(x+y)$ implies that

$$2z=2\left(\frac{1}{2}(x+y)\right)$$
$$2z=x+y$$
$$x+y-2z=0.$$

Therefore, the system can be written as follows:

$$\begin{cases} x+y+z=375{,}000 \\ x-y=50{,}000 \\ x+y-2z=0 \end{cases}$$

Writing the system as an augmented matrix yields

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 375{,}000 \\ 1 & -1 & 0 & 50{,}000 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

Applying technology to produce the reduced row-echelon form of the matrix yields

```
rref([A])
[[1 0 0 150000]
 [0 1 0 100000]
 [0 0 1 125000]]
```

Therefore, $x = 150{,}000$, $y = 100{,}000$, and $z = 125{,}000$. The first property costs \$150,000, the second property costs \$100,000, and the third property costs \$125,000.

38. Let x = grams of Food I, y = grams of Food II, and z = grams of Food III.

$$\begin{cases} 10\%x + 11\%y + 18\%z = 12.5 \\ 12\%x + 9\%y + 10\%z = 9.1 \\ 14\%x + 12\%y + 8\%z = 9.6 \end{cases} \quad \text{or} \quad \begin{cases} 0.10x + 0.11y + 0.18z = 12.5 \\ 0.12x + 0.09y + 0.10z = 9.1 \\ 0.14x + 0.12y + 0.08z = 9.6 \end{cases}$$

Converting the system to an augmented matrix yields

$$\left[\begin{array}{ccc|c} 0.10 & 0.11 & 0.18 & 12.5 \\ 0.12 & 0.09 & 0.10 & 9.1 \\ 0.14 & 0.12 & 0.08 & 9.6 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
    [[1 0 0 20]
     [0 1 0 30]
     [0 0 1 40]]
```

The solution to the system is $x = 20$, $y = 30$, $z = 40$. The nutritionist recommends 20 grams of Food I, 30 grams of Food II, and 40 grams of Food III.

39. Let x = the number of passenger aircraft, y = the number of transport aircraft, and z = the number of jumbo aircraft.

$$\begin{cases} 200x + 200y + 200z = 2200 \\ 300x + 40y + 700z = 3860 \\ 40x + 130y + 70z = 920 \end{cases}$$

Converting the system to an augmented matrix yields

$$\left[\begin{array}{ccc|c} 200 & 200 & 200 & 2200 \\ 300 & 40 & 700 & 3860 \\ 40 & 130 & 70 & 920 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
     [[1 0 0 3]
      [0 1 0 4]
      [0 0 1 4]]
```

The solution to the system is $x = 3$, $y = 4$, $z = 4$. The delivery service needs to schedule 3 passenger planes, 4 transport planes, and 4 jumbo planes.

40. Let $x =$ amount invested in the tech fund, $y =$ amount invested in the balanced fund, and $z =$ amount invested in the utility fund.

$$\begin{cases} 180x + 210y + 120z = 210{,}000 \\ 18x + 42y + 18z = 12\%(210{,}000) \end{cases} \quad \text{or} \quad \begin{cases} 180x + 210y + 120z = 210{,}000 \\ 18x + 42y + 18z = 0.12(210{,}000) \end{cases}$$

Converting the system to an augmented matrix yields

$$\left[\begin{array}{ccc|c} 180 & 210 & 120 & 210{,}000 \\ 18 & 42 & 18 & 25{,}200 \end{array}\right]$$

Note that the system has more variables than equations. The system is dependent. Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])▸Frac
…1 0 1/3 2800/3…
…0 1 2/7 200    …
```

Let $z = z$.

$$y + \frac{2}{7}z = 200$$

$$y = 200 - \frac{2}{7}z$$

and

$$x + \frac{1}{3}z = \frac{2800}{3}$$

$$x = \frac{2800}{3} - \frac{1}{3}z$$

$$x = \frac{2800 - z}{3}$$

The solution is $x = \dfrac{2800 - z}{3},\ y = 200 - \dfrac{2}{7}z,\ z = z.$

Since y must be greater than or equal to zero, z must not exceed 700. Therefore, $0 \le z \le 700$.

41. Let x = the number of units of Product A, y = the number of units of Product B, and z = the number of units of Product C.

$$\begin{cases} 25x + 30y + 40z = 9260 \\ 30x + 36y + 60z = 12{,}000 \\ 150x + 180y + 200z = 52{,}600 \end{cases}$$

Converting the system to an augmented matrix yields

$$\left[\begin{array}{ccc|c} 25 & 30 & 40 & 9260 \\ 30 & 36 & 60 & 12{,}000 \\ 150 & 180 & 200 & 52{,}600 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
 [[1 1.2 0 252]
  [0 0   1 74 ]
  [0 0   0 0  ]]
```

Since the last row consists entirely of zeros with zero as the augment, the system is dependent and has infinitely many solutions.

$z = 74$

Let $y = y$.

$x + 1.2y = 252$

$x = 252 - 1.2y$

The solution is $x = 252 - 1.2y,\ \ y = y,\ \ z = 74$.

Note that $x \ge 0$. Furthermore, $252 - 1.2y \ge 0$.

$$-1.2y \ge -252$$

$$y \le \frac{-252}{-1.2}$$

$$y \le 210$$

Therefore, $0 \le y \le 210$.

42. Let x = the number of type A slugs, y = the number of type B slugs, and z = the number of type C slugs.

$$\begin{cases} 2x + 2y + 4z = 4000 & (\text{nutrient I}) \\ 6x + 8y + 20z = 16{,}000 & (\text{nutrient II}) \\ 2x + 4y + 12z = 8000 & (\text{nutrient III}) \end{cases}$$

Converting the system to an augmented matrix yields

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 4000 \\ 6 & 8 & 20 & 16{,}000 \\ 2 & 4 & 12 & 8000 \end{array}\right]$$

Using the calculator to generate the reduced row-echelon form of the matrix yields

```
rref([A])
 [[1 0 -2 0    ]
  [0 1 4  2000]
  [0 0 0  0    ]]
```

Since the last row consists entirely of zeros with zero as the augment, the system is dependent and has infinitely many solutions.

Let $z = z$

$y + 4z = 2000$

$y = 2000 - 4z$

and

$x - 2z = 0$

$x = 2z$

The solution is $x = 2z,\ \ y = 2000 - 4z,\ \ z = z$.

Note that $y \ge 0$. Furthermore, $2000 - 4z \ge 0$.

$$-4z \ge -2000$$

$$z \le \frac{-2000}{-4}$$

$$z \le 500$$

Therefore, $0 \le z \le 500$.

43. a.

$$A = \begin{bmatrix} 97{,}470 & 160{,}923 \\ 97{,}412 & 169{,}924 \\ 110{,}835 & 189{,}880 \\ 120{,}365 & 211{,}899 \\ 133{,}979 & 230{,}656 \\ 175{,}000 & 307{,}000 \\ 129{,}000 & 204{,}700 \end{bmatrix}$$

b.

$$B = \begin{bmatrix} 134{,}616 & 209{,}088 \\ 138{,}060 & 221{,}595 \\ 155{,}902 & 256{,}360 \\ 170{,}109 & 290{,}384 \\ 198{,}253 & 302{,}438 \\ 232{,}000 & 364{,}000 \\ 176{,}500 & 224{,}900 \end{bmatrix}$$

c.

Trade balance = Exports – Imports

$$A - B = \begin{bmatrix} 97{,}470 & 160{,}923 \\ 97{,}412 & 169{,}924 \\ 110{,}835 & 189{,}880 \\ 120{,}365 & 211{,}899 \\ 133{,}979 & 230{,}656 \\ 175{,}000 & 307{,}000 \\ 129{,}000 & 204{,}700 \end{bmatrix} - \begin{bmatrix} 134{,}616 & 209{,}088 \\ 138{,}060 & 221{,}595 \\ 155{,}902 & 256{,}360 \\ 170{,}109 & 290{,}384 \\ 198{,}253 & 302{,}438 \\ 232{,}000 & 364{,}000 \\ 176{,}500 & 224{,}900 \end{bmatrix}$$

$$= \begin{bmatrix} 97{,}470 - 134{,}616 & 160{,}923 - 209{,}088 \\ 97{,}412 - 138{,}060 & 169{,}924 - 221{,}595 \\ 110{,}835 - 155{,}902 & 189{,}880 - 256{,}360 \\ 120{,}365 - 170{,}109 & 211{,}899 - 290{,}384 \\ 133{,}979 - 198{,}253 & 230{,}656 - 302{,}438 \\ 175{,}000 - 232{,}000 & 307{,}000 - 364{,}000 \\ 129{,}000 - 176{,}500 & 204{,}700 - 224{,}900 \end{bmatrix}$$

$$= \begin{bmatrix} -37{,}146 & -48{,}165 \\ -40{,}648 & -51{,}671 \\ -45{,}067 & -66{,}480 \\ -49{,}744 & -78{,}485 \\ -64{,}274 & -71{,}782 \\ -57{,}000 & -57{,}000 \\ -47{,}500 & -20{,}200 \end{bmatrix}$$

d. No. The trend of the trade balances is getting worse for the U.S. over time.

e. In 2009, the trade balance is worse with Mexico.

44. a. Intersection A

$x_4 + 2250 = x_1 + 2900$

Intersection B

$x_1 + 3050 = x_2 + 2500$

Intersection C

$x_2 + 4100 = x_3 + 2000$

Intersection D

$x_3 + 1800 = x_4 + 3800$

b. Intersection A

$x_4 + 2250 = x_1 + 2900$

$x_1 - x_4 = -650$

Intersection B

$x_1 + 3050 = x_2 + 2500$

$x_1 - x_2 = -550$

Intersection C

$x_2 + 4100 = x_3 + 2000$

$x_2 - x_3 = -2100$

Intersection D

$x_3 + 1800 = x_4 + 3800$

$x_3 - x_4 = 2000$

The system is

$$\begin{cases} x_1 - x_4 = -650 \\ x_1 - x_2 = -550 \\ x_2 - x_3 = -2100 \\ x_3 - x_4 = 2000 \end{cases}$$

Writing the augmented matrix from the system yields

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -650 \\ 1 & -1 & 0 & 0 & -550 \\ 0 & 1 & -1 & 0 & -2100 \\ 0 & 0 & 1 & -1 & 2000 \end{array}\right]$$

Using technology to produce the reduced row-echelon form of the matrix yields

```
rref([A])
…1 0 0 -1  -650]
…0 1 0 -1  -100]
…0 0 1 -1  2000]
…0 0 0 0   0   ]]
```

Since the last row consists entirely of zeros with zero as the augment, the system is dependent and has infinitely many solutions.

Let $x_4 = x_4$.

$x_3 - x_4 = 2000$

$x_3 = 2000 + x_4$

and

$x_2 - x_4 = -100$

$x_2 = x_4 - 100$

and

$x_1 - x_4 = -650$

$x_1 = x_4 - 650$

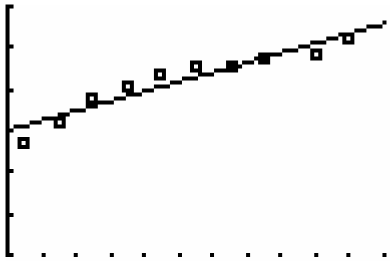
The solution is

$x_1 = x_4 - 650,\ x_2 = x_4 - 100,$

$x_3 = 2000 + x_4,\ x_4 = x_4.$

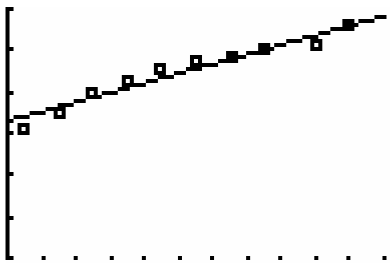
To ensure that all the variables are positive, $x_1 \geq 0,\ x_4 - 650 \geq 0$, and $x_4 \geq 650$.

45. a. $y = 0.586x + 450.086$



[0, 220] by [300, 600]

b. $y = 0.578x + 464.216$



[0, 220] by [300, 600]

c. The rate of change of critical reading SAT scores as a function of family income is an increase of 0.586 points per $1000 increase in family income, while the rate of change for math SAT scores is 0.578 points per $1000 increase in family income.

46. a. $x + y = 10$

b. $20\%x + 5\%y = 15.5\%(10)$

or

$0.20x + 0.05y = 1.55$

c. Applying the substitution method

$x + y = 10$

$x = 10 - y$

Substituting into the other equation

$0.20(10 - y) + 0.05y = 1.55$

$2 - 0.20y + 0.05y = 1.55$

$-0.15y + 2 = 1.55$

$-0.15y = -0.45$

$y = \dfrac{-0.45}{-0.15} = 3$

Substituting to find x

$x + y = 10$

$x + 3 = 10$

$x = 7$

To administer the desired concentration requires 7 cc of the 20% medication and 3 cc of the 5% medication.

47. $\begin{cases} p = q + 578 \\ p = 396 + q^2 \end{cases}$

$q + 578 = 396 + q^2$

$q^2 - q - 182 = 0$

$(q - 14)(q + 13) = 0$

$q = 14,\ q = -13$

Since $q \geq 0$ in the context of the question, then $q = 14$.

Substituting to calculate p

$p = 14 + 578$

$p = 592$

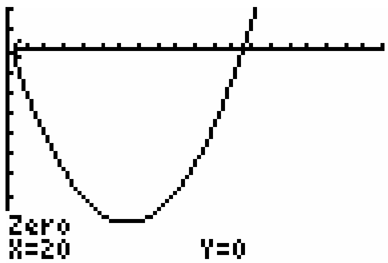
Equilibrium occurs when 14 units are produced and sold at a price of $592 per unit.

48. $C(x) = R(x)$

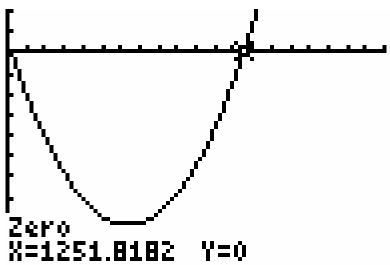
$2500x + x^2 + 27,540 = 3899x - 0.1x^2$

$1.1x^2 - 1399x + 27,540 = 0$

Solving graphically by applying the *x*-intercept method



[0, 2000] by [–500,000, 100,000]



[0, 2000] by [–500,000, 100,000]

Break-even occurs when 20 units are produced and sold or when approximately 1252 units are produced and sold.

Group Activity/Extended Application I

1. $A=\begin{bmatrix}108,481 & 138,921 & 121,312\\ 82,834 & 93,020 & 84,493\\ 78,390 & 91,563 & 69,969\end{bmatrix}$

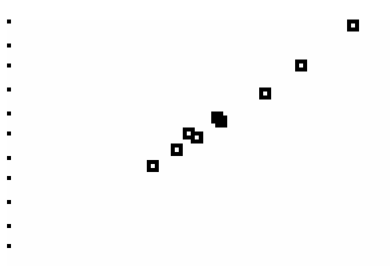
$B=\begin{bmatrix}98,552 & 127,542 & 109,720\\ 79,493 & 86,476 & 78,355\\ 73,372 & 87,570 & 66,427\end{bmatrix}$

2. $B-A$

$=\begin{bmatrix}-9,929 & -11,379 & -11,592\\ -3,341 & -6,544 & -6,138\\ -5,018 & -3,993 & -3,542\end{bmatrix}$

3. Based on the matrix in part 2, at all institutions female professors are paid less than male professors. Gender bias seems to exist at the institutions.

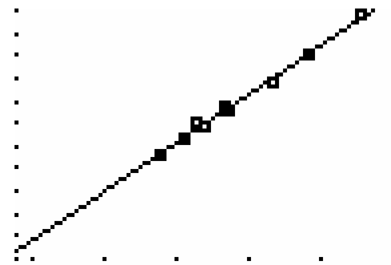
4.



[20,000, 150,000] by [20,000, 130,000]

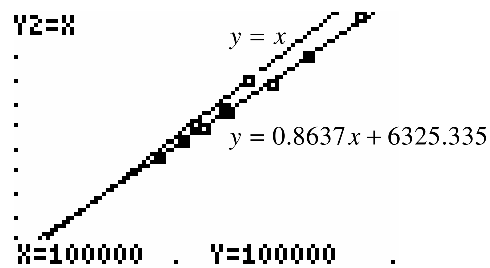
5. $y=0.8637x+6325.335$

6.



[20,000, 150,000] by [20,000, 130,000]

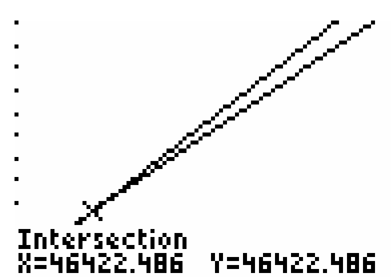
7.



[20,000, 150,000] by [20,000, 130,000]

8. Based on the graph in part 7, as salaries increase the gap between male and female salaries also increases.

9. Using the intersection of graphs method, it appears that the salary level at which male salaries become greater than female salaries is at $46,422.49.



[20,000, 150,000] by [20,000, 130,000]

Extended Application II

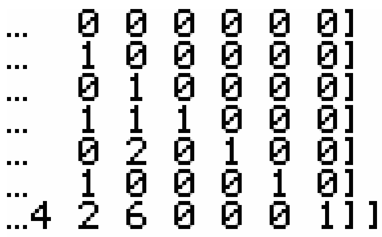
1. $D = \begin{bmatrix} 8 \\ 2 \\ 1 \\ 2 \\ 4 \\ 4 \\ 8 \end{bmatrix}$

2. $$I - P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 6 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & -6 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Applying technology to find the inverse of $I - P$:

```
[[1  0 0 0 0 0 ...
 [4  1 0 0 0 0 ...
 [1  0 1 0 0 0 ...
 [5  1 1 1 0 0 ...
 [2  0 2 0 1 0 ...
 [4  1 0 0 0 1 ...
 [14 2 6 0 0 0 ...
```



Note that the inverse matrix is shown in two graphics with several repeated columns. The inverse matrix is

$$(I-P)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 1 & 0 \\ 14 & 2 & 6 & 0 & 0 & 0 & 1 \end{bmatrix}$$

4. $(I-P)X = D$

$(I-P)^{-1}(I-P)X = (I-P)^{-1}D$

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 1 & 0 \\ 14 & 2 & 6 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \\ 1 \\ 2 \\ 4 \\ 4 \\ 8 \end{bmatrix}$$

Applying technology to carry out the multiplication yields

$$X = \begin{bmatrix} 8 \\ 34 \\ 9 \\ 45 \\ 22 \\ 38 \\ 130 \end{bmatrix}$$

5. Based on the calculations in part 4, the required number of pipes is 45, the required number of clamps is 22, the required number of braces is 38, and the required number of bolts is 130.