Chapter 3
Exponential and Logarithmic Functions

Algebra Toolbox

1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

2. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

3. $-4^{-3} = -\left(\frac{1}{4^3}\right) = -\frac{1}{64}$

4. $-4^{-2} = -\left(\frac{1}{4^2}\right) = -\frac{1}{16}$

5. $\frac{1}{3^{-3}} = \frac{1}{\frac{1}{3^3}} = 1 \div \frac{1}{3^3} = 1 \bullet 3^3 = 27$

6. $\frac{1}{5^{-2}} = \frac{1}{\frac{1}{5^2}} = 1 \div \frac{1}{5^2} = 1 \bullet 5^2 = 25$

7. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

8. $\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

9. $10^{-2} \bullet 10^0 = \frac{1}{10^2} \bullet 1 = \frac{1}{100}$

10. $8^{-2} \bullet 8^0 = \frac{1}{8^2} \bullet 1 = \frac{1}{64}$

11. $\left(2^{-1}\right)^3 = 2^{-1 \bullet 3} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

12. $\left(4^{-2}\right)^2 = 4^{-2 \bullet 2} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$

13. $10^{5^0} = 10^{(5 \bullet 0)} = 10^0 = 1$

14. $4^{2^2} = 4^{2 \bullet 2} = 4^4 = 256$

15. $x^{-4} \bullet x^{-3} = x^{-4 + -3} = x^{-7} = \frac{1}{x^7}$

16. $y^{-5} \bullet y^{-3} = y^{-5 + -3} = y^{-8} = \frac{1}{y^8}$

17. $\left(c^{-6}\right)^3 = c^{-6 \bullet 3} = c^{-18} = \frac{1}{c^{18}}$

18. $\left(x^{-2}\right)^4 = x^{-2 \bullet 4} = x^{-8} = \frac{1}{x^8}$

19. $\frac{a^{-4}}{a^{-5}} = a^{-4-(-5)} = a^1 = a$

20. $\frac{b^{-6}}{b^{-8}} = b^{-6-(-8)} = b^2$

21. $\left(x^{-\frac{1}{2}}\right)\left(x^{\frac{2}{3}}\right) = x^{-\frac{1}{2}+\frac{2}{3}} = x^{-\frac{3}{6}+\frac{4}{6}} = x^{\frac{1}{6}}$

22. $\left(y^{-\frac{1}{3}}\right)\left(y^{\frac{2}{5}}\right) = y^{-\frac{1}{3}+\frac{2}{5}} = y^{-\frac{5}{15}+\frac{6}{15}} = y^{\frac{1}{15}}$

23. $\left(3a^{-3}b^{2}\right)\left(2a^{2}b^{-4}\right)$
$=6a^{-3+2}b^{2+-4}$
$=6a^{-1}b^{-2}$
$=\dfrac{6}{ab^{2}}$

24. $\left(4a^{-2}b^{3}\right)\left(-2a^{4}b^{-5}\right)$
$=-8a^{-2+4}b^{3+-5}$
$=-8a^{2}b^{-2}$
$=\dfrac{-8a^{2}}{b^{2}}$

25. $\left(\dfrac{2x^{-3}}{x^{2}}\right)^{-2}=\left(2x^{-3-2}\right)^{-2}$
$=\left(2x^{-5}\right)^{-2}$
$=\left(2\right)^{-2}\left(x^{-5}\right)^{-2}$
$=\dfrac{1}{2^{2}}x^{-5\bullet-2}$
$=\dfrac{1}{4}x^{10}$
$=\dfrac{x^{10}}{4}$

26. $\left(\dfrac{3y^{-4}}{2y^{2}}\right)^{-3}=\left(\dfrac{2y^{2}}{3y^{-4}}\right)^{3}$
$=\dfrac{\left(2y^{2}\right)^{3}}{\left(3y^{-4}\right)^{3}}$
$=\dfrac{8y^{6}}{27y^{-12}}$
$=\dfrac{8y^{6-(-12)}}{27}$
$=\dfrac{8y^{18}}{27}$

27. $\dfrac{28a^{4}b^{-3}}{-4a^{6}b^{-2}}=-7a^{4-6}b^{-3-(-2)}$
$=-7a^{-2}b^{-1}$
$=\dfrac{-7}{a^{2}b}$

28. $\dfrac{36x^{5}y^{-2}}{-6x^{6}y^{-4}}=-6x^{5-6}y^{-2-(-4)}$
$=-6x^{-1}y^{2}$
$=\dfrac{-6y^{2}}{x}$

29. 4.6×10^{7}

30. 8.62×10^{11}

31. 9.4×10^{-5}

32. 2.78×10^{-6}

33. $437,200$

34. $7,910,000$

35. 0.00056294

36. 0.0063478

37. $\left(6.25\times10^{7}\right)\left(5.933\times10^{-2}\right)$
$\left(6.25\bullet5.933\right)\times10^{7+-2}$
37.08125×10^{5}
Rewriting in scientific notation
3.708125×10^{6}

38. $\dfrac{2.961\times10^{-2}}{4.583\times10^{-4}}$

$\dfrac{2.961}{4.583}\times10^{-2-(-4)}$

0.6460833515×10^{2}

Rewriting in scientific notation

6.460833515×10^{1}

39. Simple Interest

$I = Prt$

$I = (2000)(0.06)(5)$

$I = 600$ or \$600

Future Value

$S = P + I$

$S = 2000 + 600$

$S = 2600$ or \$2600

40. Simple Interest

$I = Prt$

$I = (8500)(0.075)(10)$

$I = 6375$ or \$6375

Future Value

$S = P + I$

$S = 8500 + 6375$

$S = 14,875$ or \$14,875

41. Simple Interest

$I = Prt$

$I = (3500)(0.12)\left(\dfrac{6}{12}\right)$

$I = 210$ or \$210

Future Value

$S = P + I$

$S = 3500 + 210$

$S = 3710$ or \$3710

42. Simple Interest

$I = Prt$

$I = (5600)(0.06)\left(\dfrac{6}{12}\right)$

$I = 168$ or \$168

Future Value

$S = P + I$

$S = 5600 + 168$

$S = 5768$ or \$5768

Section 3.1 Skills Check

1. Functions c) and e) represent exponential functions. They both fit the form $y = a^x$, where a is a constant greater than zero and $a \neq 1$.

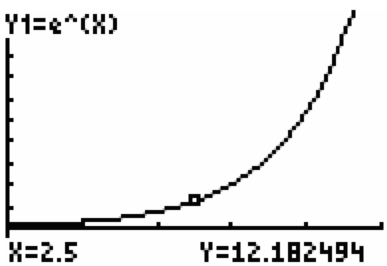
2. **a.** Growth. $k = 0.1 > 0$.

b. Decay. $k = -1.4 < 0$.

c. Decay. $k = -5 < 0$.

d. Growth. $k = 3 > 0$.

3. **a.**



[0, 5] by [–2, 100]

b. $f(1) = e^1 = e \approx 2.718$

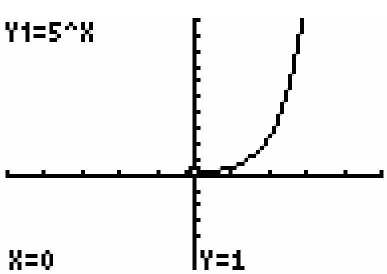
$f(-1) = e^{-1} = \frac{1}{e} \approx 0.368$

$f(4) = e^4 \approx 54.598$

c. $y = 0$

d. $(0,1)$ since $f(0) = 1$.

4. **a.**



[–5, 5] by [–60, 100]

b. $f(1) = 5^1 = 5$

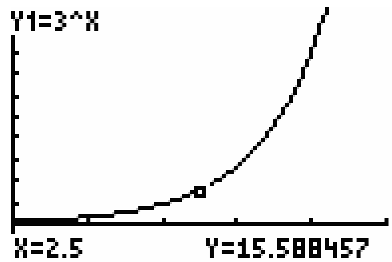
$f(3) = 5^3 = 125$

$f(-2) = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

c. $y = 0$

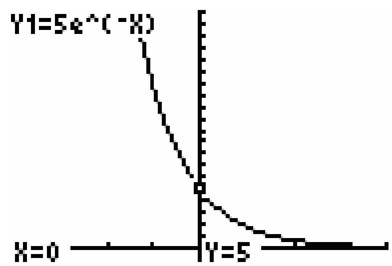
d. $(0,1)$ since $f(0) = 1$.

5.



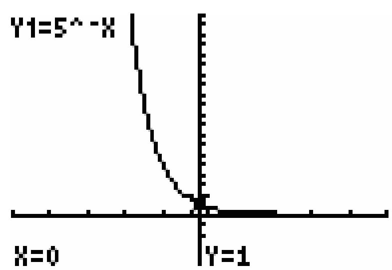
[0, 5] by [–2, 100]

6.

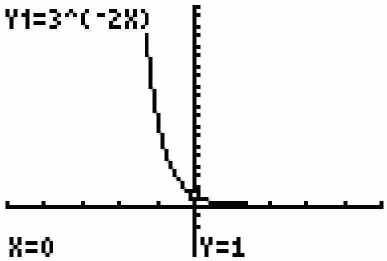


[–4, 4] by [–1, 20]

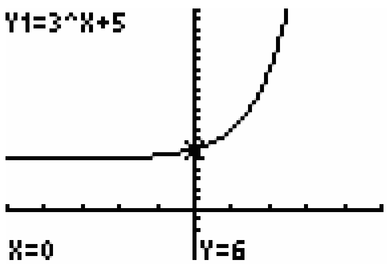
7.



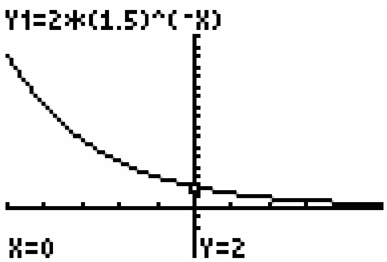
[–5, 5] by [–5, 20]

8. 

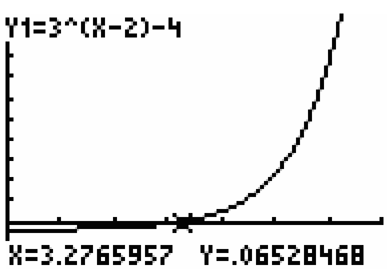
[–5, 5] by [–5, 20]

9. 

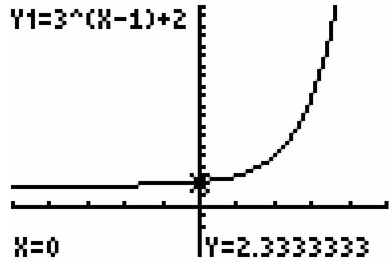
[–5, 5] by [–-5, 20]

10. 

[–5, 5] by [–5, 20]

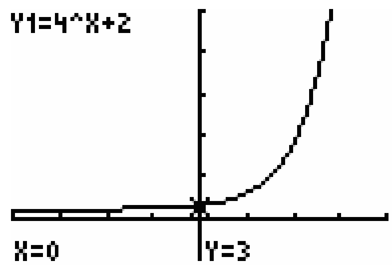
11. 

[0, 7] by [–6, 100]

12. 

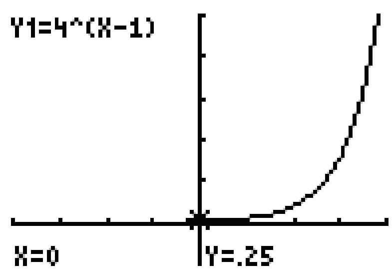
[–5, 5] by [–5, 20]

13. In comparison to 4^x, the graph has the same shape but shifted 2 units up.



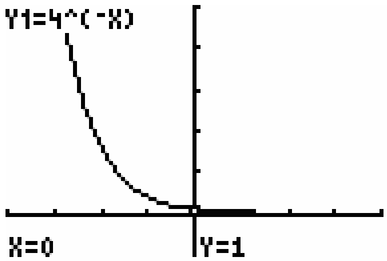
[–4, 4] by [–10, 50]

14. In comparison to 4^x, the graph has the same shape but shifted 1 unit right.



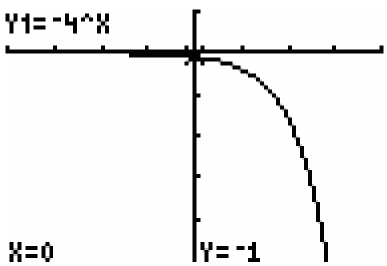
[–4, 4] by [–10, 50]

15. In comparison to 4^x, the graph has the same shape but reflected with respect to the *x*-axis.



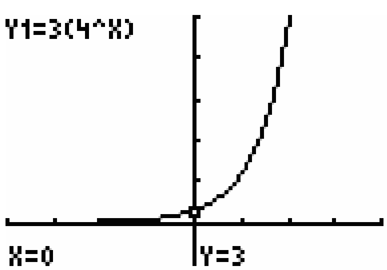
[–4, 4] by [–10, 50]

16. In comparison to 4^x, the graph has the same shape but reflected with respect to the *y*-axis.



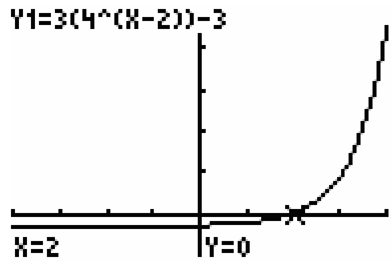
[–4, 4] by [–50, 10]

17. In comparison to 4^x, the graph is stretched vertically by a factor of 3.



[–4, 4] by [–10, 50]

18. In comparison to 4^x, the graph is stretched vertically by a factor of 3, shifted right 2 units, and shifted down 3 units.

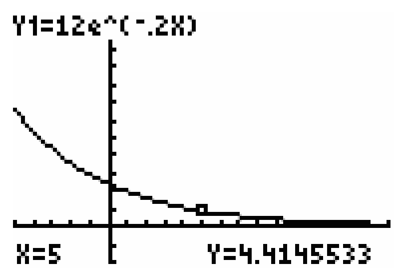


[–4, 4] by [–20, 50]

19. Both graphs are stretched vertically by a factor of 3 in comparison with 4^x. Therefore, the graph in Exercise 18 has the same shape as the graph in Exercise 17, but it is shifted 2 units right and 3 units down.

20. All the functions are increasing except for the functions in Exercises 15 and 16, which are decreasing.

21. a.



[–5, 15] by [–10, 60]

b. $f(10) = 12e^{-0.2(10)} = 12e^{-2} = \frac{12}{e^2} \approx 1.624$

$f(-10) = 12e^{-0.2(-10)} = 12e^{2} \approx 88.669$

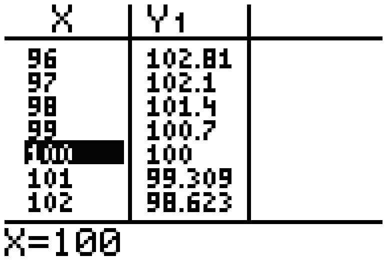
c. Since the function is decreasing, it represents decay.

22. a. $y = 200\left(2^{-0.01(20)}\right)$

$= 200\left(2^{-0.2}\right)$

≈ 174.110

b.



X	Y1	
96	102.81	
97	102.1	
98	101.4	
99	100.7	
100	100	
101	99.309	
102	98.623	

X=100

The value of x is 100.

Section 3.1 Exercises

23. a. Let $x = 0$ and solve for y.

$$y = 12{,}000\left(2^{-0.08 \cdot 0}\right)$$
$$= 12{,}000\left(2^{0}\right)$$
$$= 12{,}000(1)$$
$$= 12{,}000$$

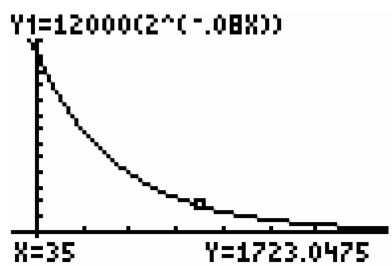
At the end of the ad campaign, sales were \$12,000 per week.

b. Let $x = 6$ and solve for y.

$$y = 12{,}000\left(2^{-0.08 \cdot 6}\right)$$
$$= 12{,}000\left(2^{-0.48}\right)$$
$$= 12{,}000(0.716977624)$$
$$\approx 8603.73$$

Six weeks after the ad campaign, sales were \$8603.73 per week.

c. No. Sales approach a level of zero but never actually reach that level. Consider the graph of the model below.



[–5, 75] by [–2000, 15,000]

24. a. Let $x = 0$ and solve for y.

$$y = 10{,}000\left(3^{-0.05 \cdot 0}\right)$$
$$= 10{,}000\left(3^{0}\right)$$
$$= 10{,}000(1)$$
$$= 10{,}000$$

At the end of the ad campaign, sales were \$10,000 per week.

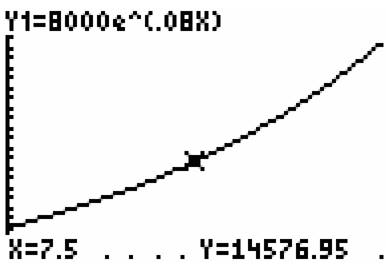
b. Let $x = 8$ and solve for y.

$$\begin{aligned} y &= 10{,}000\left(3^{-0.05\cdot 8}\right) \\ &= 10{,}000\left(3^{-0.40}\right) \\ &= 10{,}000(0.644394015) \\ &\approx 6443.94 \end{aligned}$$

Eight weeks after the ad campaign, sales were \$6,443.94 per week.

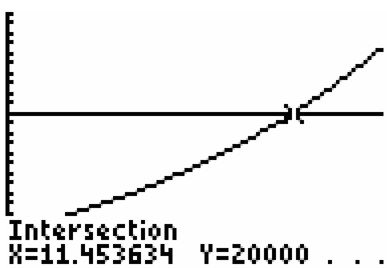
c. The equation is of the form $y = b^{kx}$, with $b = 3 > 0$ and $k = -0.05 < 0$. Since b is positive while k is negative, the function is decreasing.

25. a.



[0, 15] by [50, 30,000]

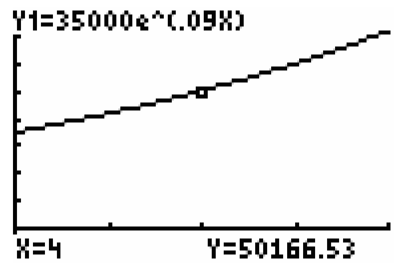
b.



[0, 15] by [50, 30,000]

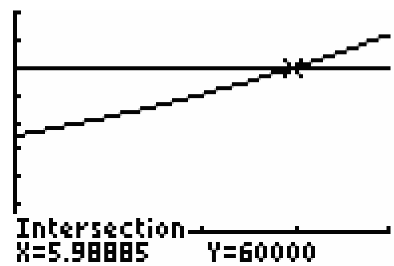
The future value will be \$20,000 in approximately 11.45 years.

26. a.



[0, 8] by [–10,000, 80,000]

b.



[0, 8] by [–10,000, 80,000]

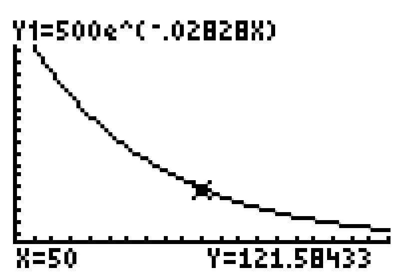
The future value will be \$60,000 in about 6 years.

27. a.

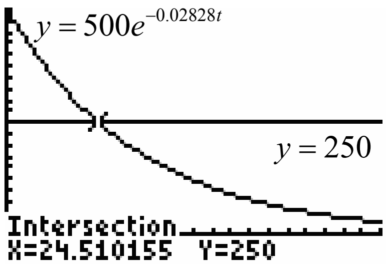
$$\begin{aligned} A(10) &= 500e^{-0.02828(10)} \\ &= 500e^{-0.2828} \\ &= 500(0.7536705069) \\ &\approx 376.84 \end{aligned}$$

Approximately 376.84 grams remain after 10 years.

b.



[0, 100] by [–50, 500]

c. 

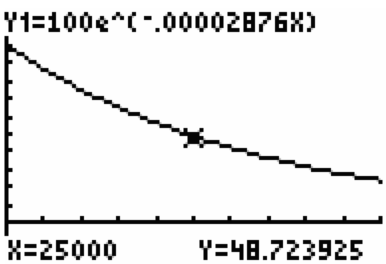
[0, 100] by [–50, 500]

The half-life is approximately 24.5 years.

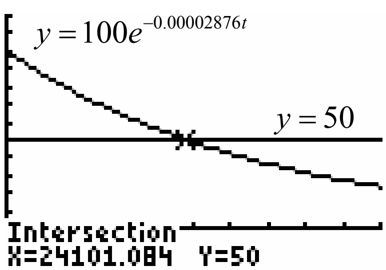
28. a.

$$\begin{aligned} A(100) &= 100e^{-0.00002876(100)} \\ &= 100e^{-0.002876} \\ &= 100(0.9971281317) \\ &\approx 99.71 \end{aligned}$$

Approximately 99.71 grams remain after 100 years.

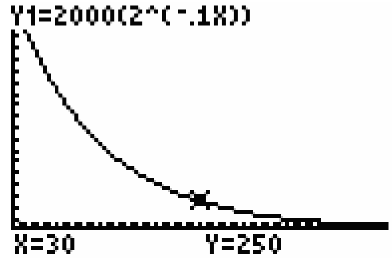
b. 

[0, 50,000] by [–20, 120]

c. 

[[0, 50,000] by [–20, 120]

The half-life is approximately 24,101 years.

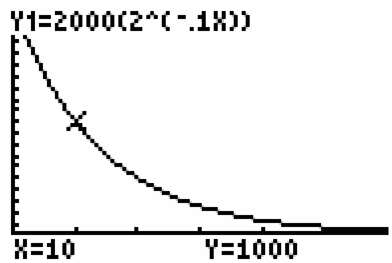
29. a. 

[0, 60] by [–200, 2000]

b.

X	Y1	Y2
6	1319.5	0
7	1231.1	0
8	1148.7	0
9	1071.8	0
10	1000	0
11	933.03	0
12	870.55	0

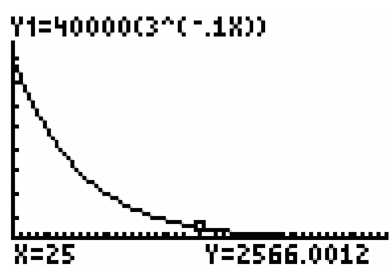
X=10



[0, 60] by [–200, 2000]

Ten weeks after the campaign ended, the weekly sales were $1000.

c. Weekly sales drop by half, from $2000 to $1000, ten weeks after the end of the ad campaign. It is important for this company to advertise.

30. a. 

[0, 50] by [0, 50,000]

b.

X	Y1
8	16610
9	14882
10	13333
11	11946
12	10703
13	9589.6
14	8591.9

X=10

Ten weeks after the campaign ended, weekly sales were $13,333.

c. Yes. Spending $5000 to boost sales to $40,000, especially considering the rapid drop in sales over just a few weeks, is a good idea.

31. a. $P = 40{,}000\left(0.95^{20}\right)$

$= 40{,}000(0.3584859224)$

$= 14{,}339.4369$

$\approx 14{,}339.44$

The purchasing power will be $14,339.44.

b. A person with $40,000 in retirement income at age 50 would be receiving the equivalent of approximately $14,339 twenty years later at age 70. Note that answers to part *b* could vary.

32. a. $P = 60{,}000\left(0.95^{4}\right)$

$= 60{,}000(0.81450625)$

$= 48{,}870.375$

After four years, the purchasing power drops to $48,870.38.

b.

X	Y1
8	39805
9	37815
10	35924
11	34128
12	32422
13	30801
14	29260

X=14

After 14 or more years, the purchasing power is below $30,000.

33. a. $y = 100{,}000e^{0.05(4)}$

$= 100{,}000e^{0.2}$

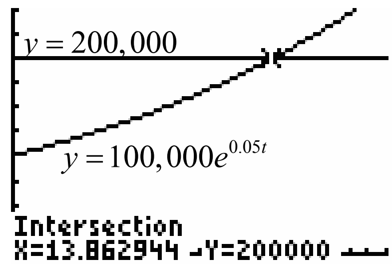
$= 100{,}000(1.221402758)$

$= 122{,}140.2758$

$\approx 122{,}140.28$

The value of property after 4 years will be $122,140.28.

b.



[0, 20] by [–5000, 250,000]

The value of the property doubles in 13.86 years or approximately 14 years.

34. a. $V(0) = 850\left(1.04^{0}\right) = 850(1) = 850$

The table was worth $850 in 1990.

b. $V(15) = 850\left(1.04^{15}\right)$

$= 850(1.800943506)$

≈ 1530.80

In 2005, the value of the antique table is $1530.80.

c.

X	Y1
13	1415.3
14	1471.9
15	1530.8
16	1592
17	1655.7
18	1721.9
19	1790.8

X=18

The antique table doubles in value in approximately 2008.

35. a. Increasing. The exponent is positive for all values of $t \geq 0$.

b. $$\begin{aligned} P(5) &= 53{,}000e^{0.015(5)} \\ &= 53{,}000e^{0.075} \\ &= 53{,}000(1.077884151) \\ &\approx 57{,}128 \end{aligned}$$

The population is 57,128 in 2005.

c. $$\begin{aligned} P(10) &= 53{,}000e^{0.015(10)} \\ &= 53{,}000e^{0.15} \\ &= 53{,}000(1.161834243) \\ &\approx 61{,}577 \end{aligned}$$

The populatin is 61,577 in 2010.

d. $$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= \frac{61{,}577 - 53{,}000}{10 - 0} \\ &= \frac{8577}{10} \\ &= 857.7 \end{aligned}$$

The average rate of growth in population between 2000 and 2010 is 857.7 people per year.

36. a. Since the coefficient of the variable exponent is negative, the model indicated that the population is decreasing.

b. $$\begin{aligned} P(7) &= 800{,}000e^{-0.020(7)} \\ &= 800{,}000e^{-0.14} \\ &\approx 695{,}486.59 \end{aligned}$$

The population in 2010 is 695,487.

c. $$\begin{aligned} P(17) &= 800{,}000e^{-0.020(17)} \\ &= 800{,}000e^{-0.34} \\ &\approx 596{,}416.26 \end{aligned}$$

The population in 2020 is 596,416.

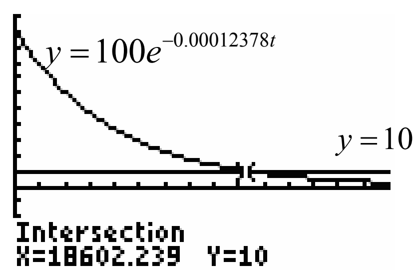
d. $$\begin{aligned} \frac{P(7) - P(0)}{7 - 0} &= \frac{695{,}487 - 800{,}000}{7} \\ &= \frac{-104{,}513}{7} \\ &= -14{,}930.43 \end{aligned}$$

The average rate of change is –14,930 people per year. The population decreases on average by 14,930 people per year.

37. a. $$\begin{aligned} y &= 100e^{-0.00012378(1000)} \\ &= 100e^{-0.12378} \\ &= 100(0.8835742058) \\ &\approx 88.36 \end{aligned}$$

Appriximately 88.36 atoms remain after 1000 years.

b.



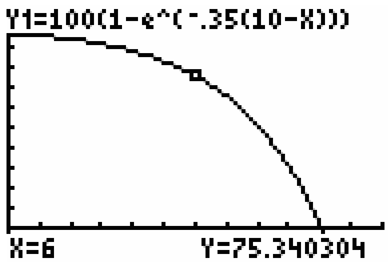
[0, 30,000] by [–50, 110]

After approximately 18,602 years, 10 grams of Carbon-14 remains.

38. a. $$\begin{aligned} y &= 100\left(1 - e^{-0.35(10-2)}\right) \\ &= 100\left(1 - e^{-2.8}\right) \\ &= 100(0.9391899374) \\ &\approx 93.92 \end{aligned}$$

After two hours, 93.92% of the drug remains in the bloodstream.

b.



[0, 12] by [–15, 110]

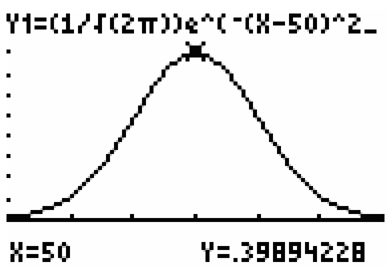
c.

X	Y1
8	50.341
9	29.531
10	0
11	-41.91
12	-101.4
13	-185.8
14	-305.5

X=10

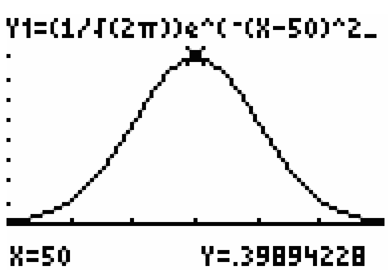
After 10 hours, the drug is totally gone from the bloodstream.

39. a.



[47, 53] by [0, 0.5]

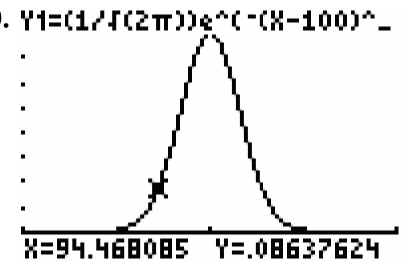
b.



[47, 53] by [0, 0.5]

The average score is 50.

40.



[80, 120] by [0, 0.4]

Section 3.2 Skills Check

1. $y = \log_3 x \Leftrightarrow 3^y = x$

2. $2y = \log_5 x \Leftrightarrow 5^{2y} = x$

3. $y = \ln(2x) = \log_e(2x) \Leftrightarrow e^y = 2x$

4. $y = \log(-x) = \log_{10}(-x) \Leftrightarrow 10^y = -x$

5. $x = 4^y \Leftrightarrow \log_4 x = y$

6. $m = 3^p \Leftrightarrow \log_3 m = p$

7. $32 = 2^5 \Leftrightarrow \log_2 32 = 5$

8. $9^{2x} = y \Leftrightarrow \log_9 y = 2x$

9. 0.845

10. 2.659

11. 4.454

12. Undefined.

13. 4.806

14. 2.303

15. $y = \log_2 32 \Leftrightarrow 2^y = 32$
Therefore, $y = 5$.

16. $y = \log_9 81 \Leftrightarrow 9^y = 81$
Therefore, $y = 2$.

17. $y = \log_3 27 \Leftrightarrow 3^y = 27$
Therefore, $y = 3$.

18. $y = \log_4 64 \Leftrightarrow 4^y = 64$
Therefore, $y = 3$.

19. $y = \log_5 625 \Leftrightarrow 5^y = 625$
$5^y = 5^4$
Therefore, $y = 4$.

20. $y = \log_2 64 \Leftrightarrow 2^y = 64$
$2^y = 2^6$
Therefore, $y = 6$.

21. $y = \log_9 27 \Leftrightarrow 9^y = 27$
$3^{2y} = 3^3$
$2y = 3$
$y = \frac{3}{2}$

22. $y = \log_4 2 \Leftrightarrow 4^y = 2$
$2^{2y} = 2^1$
$2y = 1$
$y = \frac{1}{2}$

23. $y = \ln(e^3) = \log_e(e^3) \Leftrightarrow e^y = e^3$
Therefore, $y = 3$.

24. $y = \log(100) = \log_{10}(100) \Leftrightarrow 10^y = 100$
Therefore, $y = 2$.

25. $y = \log_3\left(\frac{1}{27}\right) \Leftrightarrow 3^y = \frac{1}{27}$

$3^y = \frac{1}{3^3}$

$3^y = 3^{-3}$

Therefore, $y = -3$.

26. $y = \ln(1) = \log_e(1) \Leftrightarrow e^y = 1$

Therefore, $y = 0$.

27. $y = \ln(e) = \log_e(e) \Leftrightarrow e^y = e$

$e^y = e^1$

Therefore, $y = 1$.

28. $y = \log(0.0001) = \log_{10}(0.0001)$

$y = \log_{10}(0.0001) \Leftrightarrow 10^y = 0.0001$

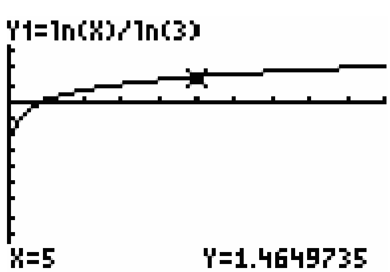
$10^y = \frac{1}{10,000}$

$10^y = \frac{1}{10^4}$

$10^y = 10^{-4}$

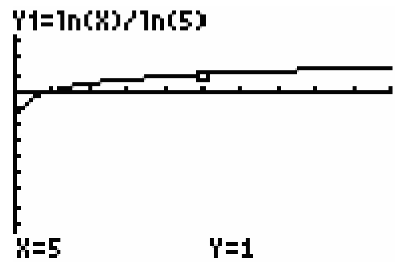
Therefore, $y = -4$.

29. $y = \log_3 x = \frac{\ln x}{\ln 3}$



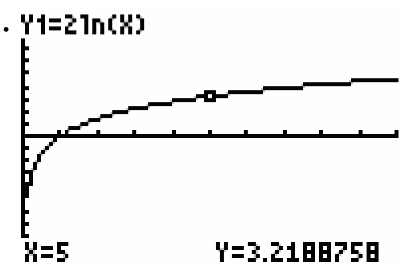
[0, 10] by [–10, 5]

29. $y = \log_5 x = \frac{\ln x}{\ln 5}$



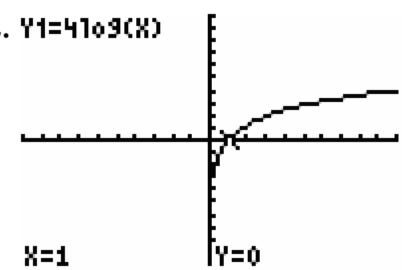
[0, 10] by [–10, 5]

31.



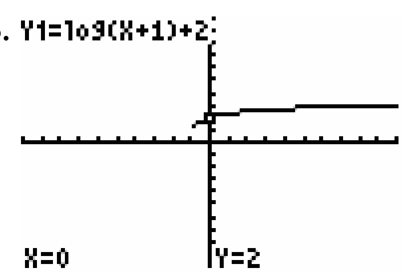
[0, 10] by [–10, 10]

32.



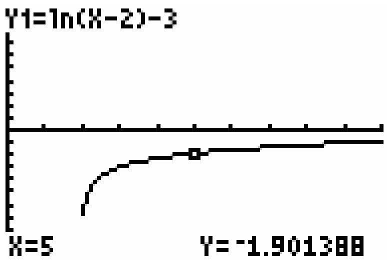
[0, 10] by [–10, 10]

33.



[0, 10] by [–10, 10]

34.



[0, 10] by [–10, 10]

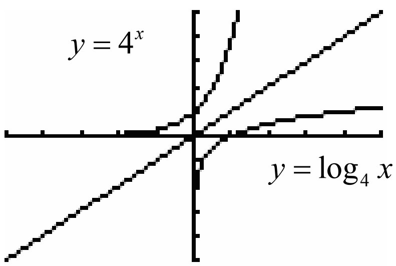
35. **a.** $y = 4^x$

$x = 4^y \Leftrightarrow \log_4 x = y$

Therefore, the inverse function is

$y = \log_4 x.$

b.



[–5, 5] by [–5, 5]

The graphs are symmetric with respect to the $y = x$ line.

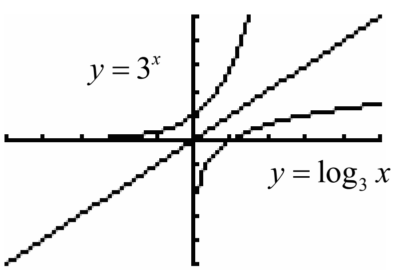
36. **a.** $y = 3^x$

$x = 3^y \Leftrightarrow \log_3 x = y$

Therefore, the inverse function is

$y = \log_3 x.$

b.



[–5, 5] by [–5, 5]

The graphs are symmetric with respect to the $y = x$ line.

37. $\log_a a = x \Leftrightarrow a^x = a$

If $a > 0$ and $a \neq 1,$ then $x = 1.$

38. $\log_a 1 = x \Leftrightarrow a^x = 1$

If $a > 0$ and $a \neq 1,$ then $x = 0.$

39. $\log_2 x = 3 \Leftrightarrow 2^3 = x$

$x = 8$

40. $\log_4 x = -2 \Leftrightarrow 4^{-2} = x$

$x = \frac{1}{4^2}$

$x = \frac{1}{16}$

41. $5 + 2\ln x = 8$

$2\ln x = 3$

$\ln x = \frac{3}{2}$

$\log_e x = \frac{3}{2} \Leftrightarrow e^{\frac{3}{2}} = x$

$x = e^{\frac{3}{2}} \approx 4.4817$

42. $4 + 3\log x = 10$

$3\log x = 6$

$\log x = 2$

$\log_{10} x = 2 \Leftrightarrow 10^2 = x$

$x = 100$

Section 3.2 Exercises

43. a. In 1925, $x = 1925 - 1900 = 25.$

$f(25) = 12.734\ln(25) + 17.875$

$f(25) = 58.8642$

In 1925, the expected life span is 59 years.

In 1996, $x = 1996 - 1900 = 96.$

$f(96) = 12.734\ln(96) + 17.875$

$f(96) = 75.9974$

In 1996, the expected life span is 76 years.

b. Based on the model, life span increased tremendously between 1925 and 1996. The increase could be due to multiple factors, including improved healthcare and nutrition.

44. In 1986, $x = 1986 - 1980 = 6.$

$y = 114.198 + 4.175\ln(6)$

$y \approx 121.679$

In 1986, the population of Japan is 121,679,000.

In 2000, $x = 2000 - 1980 = 20.$

$y = 114.198 + 4.175\ln(20)$

$y \approx 126.705$

In 2000, the population of Japan is 126,705,000.

45. a. In 1990, $x = 1990 - 1980 = 10.$

$f(10) = 282.1666 + 2771.0125\ln(10)$

$f(10) = 6662.66$

In 1990, the official single poverty level is approximately \$6662.

In 1999, $x = 1999 - 1980 = 19.$

$f(19) = 282.1666 + 2771.0125\ln(19)$

$f(19) = 8441.24$

In 1999, the official single poverty level is approximately \$8441.

b. Based on the solutions to part a), the function seems to be increasing.

c. Over time, inflation causes the minimum poverty level to rise.

46. a. In 1960, $x = 1960 - 1950 = 10.$

$y = -61.0399 + 34.326\ln(10)$

$y \approx 17.9986$

In 1960, the percentage is 18%.

In 1995, $x = 1995 - 1950 = 45.$

$y = -61.0399 + 34.326\ln(45)$

$y \approx 69.6276$

In 1995, the percentage is 69.6%.

b. Based on the solutions to part a), the function seems to be increasing.

47. $\dfrac{\ln 2}{0.10} \approx 6.9$ years

48. $\dfrac{\ln 2}{0.07} \approx 9.9$ years

49. $n = \dfrac{\log 2}{0.0086}$

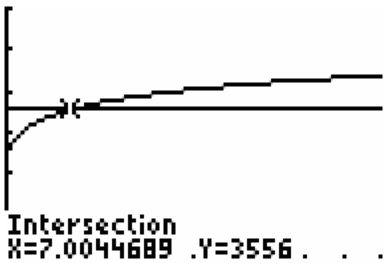
$n = 35.0035 \approx 35$

Since it takes approximately 35 quarters for an investment to double under this scenario, then in terms of years the time to double is approximately $\dfrac{35}{4} = 8.75$ years.

50. $n = \dfrac{\log 2}{0.0253} = 11.898 \approx 12$

Since the compounding is semi-annual, 12 compounding periods corresponds to approximately 6 years.

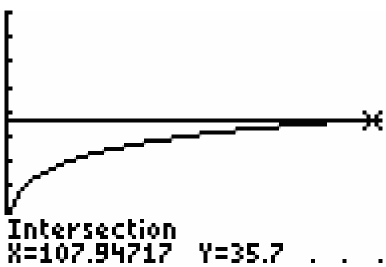
51.



[0, 40] by [3200, 3800]

When the cost is \$3556, approximately 7 units are produced.

52.



[0, 110] by [30, 40]

When the cost is \$35.70, approximately 108 units are supplied.

53. a.

$$f(x) = 12.734\ln(x) + 17.875$$
$$y = 12.734\ln(x) + 17.875$$
$$y - 17.875 = 12.734\ln(x)$$
$$\frac{y-17.875}{12.734} = \frac{12.734\ln(x)}{12.734}$$
$$\ln(x) = \frac{y-17.875}{12.734}$$

b. $\ln(x) = \frac{y-17.875}{12.734}$

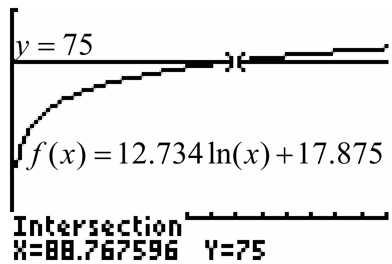
$$\log_e x = \frac{y-17.875}{12.734} \Leftrightarrow e^{\frac{y-17.875}{12.734}} = x$$

c. Let $y = 75$ and solve for x.

$$x = e^{\frac{y-17.875}{12.734}}$$
$$x = e^{\frac{(75)-17.875}{12.734}}$$
$$x = e^{4.486021674}$$
$$x = 88.76759607 \approx 89$$

In approximately 1989, the expected life span will be 75 years.

d.



[0, 150] by [–20, 100]

As expected, the answers to parts c) and d) are the same.

54. a.

$$w(x) = 1.11 + 16.94\ln x$$
$$y = 1.11 + 16.94\ln x$$
$$y - 1.11 = 16.94\ln x$$
$$\ln x = \frac{y-1.11}{16.94}$$

b. $\ln x = \frac{y-1.11}{16.94}$

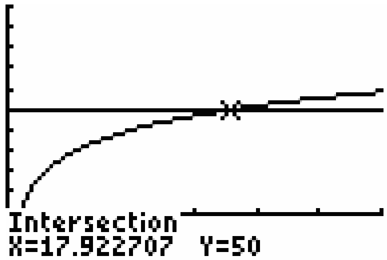
$$\log_e x = \frac{y-1.11}{16.94} \Leftrightarrow e^{\frac{y-1.11}{16.94}} = x$$

c. $e^{\frac{50-1.11}{16.94}} = x$

$$x = e^{2.886068477} \approx 17.923$$

The percentage reaches 50% in about 17.9 years or in 1988.

d.



[0, 30] by [–20, 100]

Yes. The methods yield the same solution.

55. $R = \log\left(\frac{I}{I_0}\right)$

$R = \log\left(\frac{25,000I_0}{I_0}\right)$

$R = \log(25,000) = 4.3979 \approx 4.4$

The earthquake measures 4.4 on the Richter scale.

56. a. $R = \log\left(\frac{I}{I_0}\right)$

$R = \log\left(\frac{250,000I_0}{I_0}\right)$

$R = \log(250,000)$

$= 5.397940009 \approx 5.4$

The earthquake measures 5.4 on the Richter scale.

b. Suppose one earthquake has a magnitude of AI_0, while another earthquake has a magnitude of $10AI_0$.

$R_1 = \log\left(\frac{AI_0}{I_0}\right) = \log(A)$

$R_2 = \log\left(\frac{10AI_0}{I_0}\right)$

$= \log(10A)$

$= \log 10 + \log A$

$= 1 + \log A$

$= 1 + R_1$

The stronger earthquake measures one more unit than the weaker earthquake on the Richter scale.

57. a. $R = \log\left(\frac{I}{I_0}\right) = \log_{10}\left(\frac{I}{I_0}\right)$

$R = \log_{10}\left(\frac{I}{I_0}\right) \Leftrightarrow 10^R = \frac{I}{I_0}$

b. $10^R = \frac{I}{I_0}$

$I = 10^R I_0$

$I = 10^{7.1} I_0$

$I = 12,589,254I_0$

58. $10^R = \frac{I}{I_0}$

$I = 10^R I_0$

$I = 10^{8.25} I_0$

$I = 177,827,941I_0$

59. The difference in the Richter scale measurements is $8.25 - 7.1 = 1.15$. Therefore, the intensity of the 1906 earthquake was $10^{1.15} \approx 14.13$ times stronger than the intensity of the 1989 earthquake.

60. The difference in the Richter scale measurements is $8.9 - 8.25 = 0.65$.

Therefore, the intensity of the largest earthquake ever recorded was $10^{0.65} \approx 4.47$ times stronger than the intensity of the 1906 earthquake.

61. $L = 10\log\left(\frac{I}{I_0}\right)$

$L = 10\log\left(\frac{20{,}000 I_0}{I_0}\right)$

$L = 10\log(20{,}000) \approx 43$

The decibel level is approximately 43.

62. Suppose the intensity of one sound is AI_0, while the intensity of a second sound is $100AI_0$. Then,

$L_1 = 10\log\left(\frac{AI_0}{I_0}\right) = 10\log(A)$

$L_2 = 10\log\left(\frac{100AI_0}{I_0}\right)$

$= 10\log(100A)$

$= 10(\log 100 + \log A)$

$= 20 + 10\log A$

$= 20 + L_1$

As a decibel level, the higher intensity sound measures 20 more than the lower intensity sound.

63. $L = 10\log\left(\frac{I}{I_0}\right)$

$40 = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\log_{10}\left(\frac{I}{I_0}\right) = 4 \Leftrightarrow 10^4 = \frac{I}{I_0}$

$\frac{I}{I_0} = 10^4$

$I = 10^4 I_0 = 10{,}000 I_0$

64. $L = 10\log\left(\frac{I}{I_0}\right)$

$140 = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\log_{10}\left(\frac{I}{I_0}\right) = 14 \Leftrightarrow 10^{14} = \frac{I}{I_0}$

$\frac{I}{I_0} = 10^{14}$

$I = 10^{14} I_0 = 100{,}000{,}000{,}000{,}000 I_0$

65. $L_1 = 10\log\left(\frac{115 I_0}{I_0}\right)$

$= 10\log(115)$

≈ 20.6

$L_2 = 10\log\left(\frac{9{,}500{,}000 I_0}{I_0}\right)$

$= 10\log(9{,}500{,}000)$

≈ 69.8

The decibel level on a busy street is approximately 49 more than the decibel level of a whisper.

66. $L = 10\log\left(\frac{I}{I_0}\right)$

Let $L = 140$.

$140 = 10\log\left(\frac{I}{I_0}\right)$

$14 = \log\left(\frac{I}{I_0}\right) \Leftrightarrow 10^{14} = \frac{I}{I_0}$

$I = 10^{14} I_0$

Let $L = 120$.

$120 = 10\log\left(\frac{I}{I_0}\right)$

$12 = \log\left(\frac{I}{I_0}\right) \Leftrightarrow 10^{12} = \frac{I}{I_0}$

$I = 10^{12} I_0$

Comparing the intensity levels:

$\frac{10^{14}}{10^{12}} = 10^2 = 100$

The decibel level of 140 is one hundred times as intense as a decibel level of 120.

67. $pH = -\log\left[H^+\right]$

$= -\log(0.0000631)$

$= 4.2$

68. $7.79 = -\log\left[H^+\right]$

$-7.79 = \log_{10}\left[H^+\right] \Leftrightarrow 10^{-7.79} = \left[H^+\right]$

$\left[H^+\right] = 10^{-7.79} \approx 0.0000000162$

69. $pH = -\log\left[H^+\right]$

$-pH = \log_{10}\left[H^+\right] \Leftrightarrow 10^{-pH} = H^+$

$H^+ = 10^{-pH}$

If $pH = 1, H^+ = 10^{-1} = \frac{1}{10} = 0.1$

If $pH = 14, H^+ = 10^{-14} = \frac{1}{10^{14}}$

$\frac{1}{10^{14}} \le H^+ \le \frac{1}{10}$

Section 3.3 Skills Check

1.
$$1600 = 10^x$$
$$\log(1600) = \log(10^x)$$
$$x = \log(1600) \approx 3.204$$

2.
$$4600 = 10^x$$
$$\log(4600) = \log(10^x)$$
$$x = \log(4600) \approx 3.663$$

3.
$$2500 = e^x$$
$$\ln(2500) = \ln(e^x)$$
$$x = \ln(2500) \approx 7.824$$

4.
$$54.6 = e^x$$
$$\ln(54.6) = \ln(e^x)$$
$$x = \ln(54.6) \approx 4.000$$

5.
$$8900 = e^{5x}$$
$$\ln(8900) = \ln(e^{5x})$$
$$5x = \ln(8900)$$
$$x = \frac{\ln(8900)}{5} \approx 1.819$$

6.
$$2400 = 10^{8x}$$
$$\log(2400) = \log(10^{8x})$$
$$8x = \log(2400)$$
$$x = \frac{\log(2400)}{8} \approx 0.423$$

7.
$$4000 = 200e^{8x}$$
$$20 = e^{8x}$$
$$\ln(20) = \ln(e^{8x})$$
$$8x = \ln(20)$$
$$x = \frac{\ln(20)}{8} \approx 0.374$$

8.
$$5200 = 13e^{12x}$$
$$\frac{5200}{13} = e^{12x}$$
$$400 = e^{12x}$$
$$\ln(400) = \ln(e^{12x})$$
$$12x = \ln(400)$$
$$x = \frac{\ln(400)}{12}$$
$$x \approx 0.499$$

9.
$$8000 = 500(10^x)$$
$$16 = 10^x$$
$$\log(16) = \log(10^x)$$
$$x = \log(16) \approx 1.204$$

10.
$$9000 = 400(10^x)$$
$$22.5 = 10^x$$
$$\log(22.5) = \log(10^{2x})$$
$$x = \log(22.5) \approx 1.352$$

11.
$$\log 10^{14} = \log_{10} 10^{14}$$
$$= 14 \log_{10} 10$$
$$= 14(1) = 14$$

12. $\ln(e^5) = 5\ln(e) = 5(1) = 5$

13. $10^{\log_{10} 12} = 12$

14. $6^{\log_6 25} = 25$

15. $\log_a(100) = \log_a(20\bullet 5)$
$= \log_a(20) + \log_a(5)$
$= 1.4406 + 0.7740$
$= 2.2146$

16. $\log_a(4) = \log_a\left(\frac{20}{5}\right)$
$= \log_a(20) - \log_a(5)$
$= 1.4406 - 0.7740$
$= 0.6666$

17. $\log_a 5^3 = 3\log_a 5$
$= 3(0.7740)$
$= 2.322$

18. $\log_a \sqrt{20} = \log_a(20)^{\frac{1}{2}}$
$= \frac{1}{2}\log_a(20)$
$= \frac{1}{2}(1.4406)$
$= 0.7203$

19. $\ln\left(\frac{3x-2}{x+1}\right) = \ln(3x-2) - \ln(x+1)$

20. $\log\left[x^3(3x-4)^5\right]$
$= \log(x^3) + \log(3x-4)^5$
$= 3\log x + 5\log(3x-4)$

21. $\log_3 \frac{\sqrt[4]{4x+1}}{4x^2}$
$= \log_3\left(\sqrt[4]{4x+1}\right) - \log_3(4x^2)$
$= \log_3\left[(4x+1)^{\frac{1}{4}}\right] - \left[\log_3(4) + \log_3(x^2)\right]$
$= \frac{1}{4}\log_3(4x+1) - \left[\log_3(4) + 2\log_3(x)\right]$
$= \frac{1}{4}\log_3(4x+1) - \log_3(4) - 2\log_3(x)$

22. $\log_3 \frac{\sqrt[3]{3x-2}}{5x^2}$
$= \log_3\left(\sqrt[3]{3x-2}\right) - \log_3(5x^2)$
$= \log_3\left[(3x-2)^{\frac{1}{3}}\right] - \left[\log_3(5) + \log_3(x^2)\right]$
$= \frac{1}{3}\log_3(3x-2) - \left[\log_3(5) + 2\log_3(x)\right]$
$= \frac{1}{4}\log_3(3x-2) - \log_3(5) - 2\log_3(x)$

23. $3\log_2 x + \log_2 y$
$= \log_2 x^3 + \log_2 y$
$= \log_2(x^3 y)$

24. $\log x - \frac{1}{3}\log y$
$= \log x - \log(y)^{\frac{1}{3}}$
$= \log x - \log\left(\sqrt[3]{y}\right)$
$= \log\left(\frac{x}{\sqrt[3]{y}}\right)$

25. $4\ln(2a) - \ln(b)$
$= \ln(2a)^4 - \ln(b)$
$= \ln\left(\frac{16a^4}{b}\right)$

26. $6\ln(5y)+2\ln x$
$= \ln(5y)^6 + \ln x^2$
$= \ln\left[(5y)^6 x^2\right]$
$= \ln\left(15{,}625x^2y^6\right)$

27. $\log_6(18) = \dfrac{\ln(18)}{\ln(6)} = 1.6131$

28. $\log_7(215) = \dfrac{\ln(215)}{\ln(7)} = 2.7600$

29. $\log_8\left(\sqrt{2}\right) = \log_8(2)^{\frac{1}{2}}$
$= \dfrac{1}{2}\log_8(2)$
$= \left(\dfrac{1}{2}\right)\left(\dfrac{\ln(2)}{\ln(8)}\right)$
$= 0.1667$

30. $\log_4\left(\sqrt[3]{10}\right) = \log_4(10)^{\frac{1}{3}}$
$= \dfrac{1}{3}\log_4(10)$
$= \left(\dfrac{1}{3}\right)\left(\dfrac{\ln(10)}{\ln(4)}\right)$
$= 0.5537$

31. $8^x = 1024$
$\ln(8^x) = \ln(1024)$
$x\ln(8) = \ln(1024)$
$x = \dfrac{\ln(1024)}{\ln(8)}$
$x = 3.\overline{3} = \dfrac{10}{3}$

32. $9^x = 2187$
$\ln(9^x) = \ln(2187)$
$x\ln(9) = \ln(2187)$
$x = \dfrac{\ln(2187)}{\ln(9)}$
$x = 3.5$

33. $2\left(5^{3x}\right) = 31{,}250$
$5^{3x} = 15{,}625$
$\ln\left(5^{3x}\right) = \ln(15{,}625)$
$3x\ln(5) = \ln(15{,}625)$
$x = \dfrac{\ln(15{,}625)}{3\ln(5)}$
$x = 2$

34. $2\left(6^{2x}\right) = 2592$
$6^{2x} = 1296$
$\ln\left(6^{2x}\right) = \ln(1296)$
$2x\ln(6) = \ln(1296)$
$x = \dfrac{\ln(1296)}{2\ln(6)}$
$x = 2$

35. $5^{x-2} = 11.18$
$\ln\left(5^{x-2}\right) = \ln(11.18)$
$(x-2)\ln(5) = \ln(11.18)$
$x-2 = \dfrac{\ln(11.18)}{\ln(5)}$
$x = \dfrac{\ln(11.18)}{\ln(5)} + 2$
$x \approx 3.5$

36.
$$3^{x-4}=140.3$$
$$\ln\left(3^{x-4}\right)=\ln(140.3)$$
$$(x-4)\ln(3)=\ln(140.3)$$
$$x-4=\frac{\ln(140.3)}{\ln(3)}$$
$$x=\frac{\ln(140.3)}{\ln(3)}+4$$
$$x\approx 8.5$$

37.
$$18,000=30\left(2^{12x}\right)$$
$$600=2^{12x}$$
$$\log(600)=\log\left(2^{12x}\right)$$
$$12x\log(2)=\log(600)$$
$$x=\frac{\log(600)}{12\log(2)}$$
$$x\approx 0.769$$

38.
$$5880=21\left(2^{3x}\right)$$
$$\frac{5880}{21}=2^{3x}$$
$$280=2^{3x}$$
$$\log(280)=\log\left(2^{3x}\right)$$
$$3x\log(2)=\log(280)$$
$$x=\frac{\log(280)}{3\log(2)}$$
$$x\approx 2.710$$

39.
$$5+\ln(8x)=23-2\ln(x)$$
$$\ln(8x)+2\ln(x)=23-5$$
$$\ln(8x)+\ln(x)^2=18$$
$$\ln\left(8x\bullet x^2\right)=18$$
$$e^{\ln\left(8x^3\right)}=e^{18}$$
$$8x^3=e^{18}$$
$$x=\sqrt[3]{\frac{e^{18}}{8}}$$
$$x=\frac{e^6}{2}$$
$$x\approx 201.7$$

40.
$$3\ln x+8=\ln(3x)+12.18$$
$$3\ln(x)-\ln(3x)=12.18-8$$
$$\ln\left(x^3\right)-\ln(3x)=4.18$$
$$\ln\left(\frac{x^3}{3x}\right)=4.18$$
$$e^{\ln\left(\frac{x^2}{3}\right)}=e^{4.18}$$
$$\frac{x^2}{3}=e^{4.18}$$
$$x=\sqrt{3e^{4.18}}$$
$$x\approx 14$$

41. $2\log(x)-2=\log(x-25)$

$\log(x^2)-\log(x-25)=2$

$\log\left(\frac{x^2}{x-25}\right)=2$

$10^{\log\left(\frac{x^2}{x-25}\right)}=10^2$

$\frac{x^2}{x-25}=100$

$x^2=100(x-25)$

$x^2=100x-2500$

$x^2-100x+2500=0$

$(x-50)(x-50)=0$

$x=50$

42. $\ln(x-6)+54=\ln x-1000$

$\ln(x-6)-\ln x=-1000-54$

$\ln\left(\frac{x-6}{x}\right)=-1054$

$e^{\ln\left(\frac{x-6}{x}\right)}=e^{-1054}$

$\frac{x-6}{x}=e^{-1054}$

$x-6=e^{-1054}x$

$x-e^{-1054}x=6$

$x\left(1-e^{-1054}\right)=6$

$x=\frac{6}{1-e^{-1054}}$

$x\approx 6$

Note that $x=6$ does not check in the original problem, since $\ln(x-6)=\ln(6-6)=\ln(0)$, which is undefined. However, x is not exactly 6. The exact solution, $x=\frac{6}{1-e^{-1054}}$, does check.

43. $3^x<243$

$\ln(3^x)<\ln(243)$

$x\ln(3)<\ln(243)$

$x<\frac{\ln(243)}{\ln(3)}$

$x<5$

44. $7^x\geq 2401$

$\ln(7^x)\geq\ln(2401)$

$x\ln(7)\geq\ln(2401)$

$x\geq\frac{\ln(2401)}{\ln(7)}$

$x\geq 4$

45. $5(2^x)\geq 2560$

$\ln(2^x)\geq\ln\left(\frac{2560}{5}\right)$

$x\ln(2)\geq\ln(512)$

$x\geq\frac{\ln(512)}{\ln(2)}$

$x\geq 9$

46. $15(4^x)\leq 15{,}360$

$\ln(4^x)\leq\ln\left(\frac{15{,}360}{15}\right)$

$x\ln(4)\leq\ln(1024)$

$x\leq\frac{\ln(1024)}{\ln(4)}$

$x\leq 5$

Section 3.3 Exercises

47. $10,880 = 340\left(2^q\right)$

$$2^q = \frac{10,880}{340}$$

$$2^q = 32$$

$$\ln\left(2^q\right) = \ln\left(32\right)$$

$$q\ln(2) = \ln(32)$$

$$q = \frac{\ln(32)}{\ln(2)}$$

$$q = 5$$

When the price is $10,880, the quantity supplied is 5.

48. $256.60 = 4000\left(3^{-q}\right)$

$$3^{-q} = \frac{256.60}{4000}$$

$$3^{-q} = 0.06415$$

$$\ln\left(3^{-q}\right) = \ln\left(0.06415\right)$$

$$-q\ln(3) = \ln\left(0.06415\right)$$

$$q = \frac{\ln\left(0.06415\right)}{-\ln(3)}$$

$$q = 2.5$$

When the price is $256.60, the quantity supplied is 2.5 thousand.

49. a. $s = 25,000e^{-0.072x}$

$$\frac{s}{25,000} = e^{-0.072x}$$

$$\Leftrightarrow \log_e\left(\frac{s}{25,000}\right) = -0.072x$$

$$\ln\left(\frac{s}{25,000}\right) = -0.072x$$

b. $\ln\left(\frac{16,230}{25,000}\right) = -0.072x$

$$x = \frac{\ln\left(\frac{16,230}{25,000}\right)}{-0.072}$$

$$x = 6$$

Six weeks after the completion of the campaign, the sales fell to $16,230.

50. a. $S = 3200e^{-0.08x}$

$$\frac{S}{3200} = e^{-0.08x} \Leftrightarrow \log_e\left(\frac{S}{3200}\right) = -0.08x$$

$$x = \frac{\ln\left(\frac{S}{3200}\right)}{-0.08}$$

b. $x = \frac{\ln\left(\frac{S}{3200}\right)}{-0.08} = \frac{\ln\left(\frac{2145}{3200}\right)}{-0.08} = 5$

After five days, the sales have fallen to $2145.

51. a. $S = 3200e^{-0.08(0)} = 3200e^0 = 3200$

At the end of the ad campaign, daily sales were $3200.

b. $S = 3200e^{-0.08x}$

$$1600 = 3200e^{-0.08x}$$

$$\frac{1}{2} = e^{-0.08x}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.08x}\right)$$

$$-0.08x = \ln\left(\frac{1}{2}\right)$$

$$x = \frac{\ln\left(\frac{1}{2}\right)}{-0.08} = 8.664$$

Approximately 9 days after the completion of the ad campaign, sales dropped below half their level the day the campaign ended.

52. a.
$$\begin{aligned} S &= 25{,}000e^{-0.072(0)} \\ &= 25{,}000e^{0} \\ &= 25{,}000 \end{aligned}$$

At the end of the campaign, sales were $25,000.

b.

X	Y_1
8	14054
9	13077
10	12169
11	11323
12	10537
13	9804.8
14	9123.7

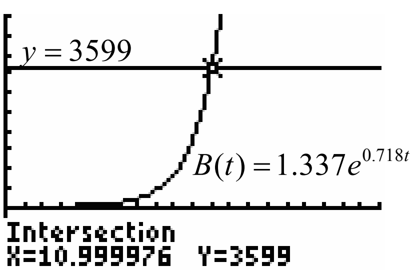
X=10

In the tenth week, sales dropped below half the initial amount of $25,000.

53. a.
$$\begin{aligned} B(10) &= 1.337e^{0.718(10)} \\ &= 1.337e^{7.18} \\ &= 1755.358341 \\ &\approx 1755.36 \end{aligned}$$

Based on the model, in 1995, Snapple's revenue was approximately $1755.36 million.

b. Applying the intersection of graphs method



[0, 20] by [–1500, 5000]

Revenues would reach $3599 in 1996.

54.

X	Y_1
6	57991
7	58868
8	59757
9	60660
10	61577
11	62508
12	63453

X=9

The population reaches 60,000 in between 8 and 9 years, which corresponds to 2009.

55.
$$\begin{aligned} 20{,}000 &= 40{,}000e^{-0.05t} \\ e^{-0.05t} &= 0.5 \\ \ln\left(e^{-0.05t}\right) &= \ln(0.5) \\ -0.05t &= \ln(0.5) \\ t &= \frac{\ln(0.5)}{-0.05} \\ t &= 13.86294361 \end{aligned}$$

It will take approximately 13.86 years for the $40,000 pension to decrease to $20,000 in purchasing power.

56.
$$\begin{aligned} 30{,}000 &= 60{,}000e^{-0.05t} \\ 60{,}000e^{-0.05t} &= 30{,}000 \\ e^{-0.05t} &= 0.5 \\ \ln\left(e^{-0.05t}\right) &= \ln(0.5) \\ -0.05t &= \ln(0.5) \\ t &= \frac{\ln(0.5)}{-0.05} \\ t &= 13.86294361 \end{aligned}$$

It will take approximately 13.86 years for the $60,000 in purchasing power to decrease to $30,000 in purchasing power.

57. $200,000 = 100,000e^{0.03t}$

$$2 = e^{0.03t}$$
$$\ln(2) = \ln\left(e^{0.03t}\right)$$
$$\ln(2) = 0.03t$$
$$t = \frac{\ln(2)}{0.03}$$
$$t = 23.1049$$

It will take approximately 23.1 years for the value of the property to double.

58. $254,250 = 200,000e^{0.05t}$

$$1.27125 = e^{0.05t}$$
$$\ln(1.27125) = \ln\left(e^{0.05t}\right)$$
$$\ln(1.27125) = 0.05t$$
$$t = \frac{\ln(1.27125)}{0.05}$$
$$t = 4.8$$

It will take approximately 4.8 years for the value of the property to reach $254,250.

59. $S = Pe^{rt}$

Note that the initial investment is P and that double the initial investment is $2P$.

$$2P = Pe^{rt}$$
$$2 = e^{rt}$$
$$\ln(2) = \ln\left(e^{rt}\right)$$
$$rt = \ln(2)$$
$$t = \frac{\ln(2)}{r}$$

The time to double is $\ln(2)$ divided by the interest rate.

60. See Exercise 59 for more information.

$$\frac{\ln(2)}{0.10} = 6.93 \approx 7$$

It will take approximately 7 years to double the investment.

61. **a.** $n = \log_{1.02} 2$

$$n = \frac{\log 2}{\log 1.02}$$
$$n = 35.0027 \approx 35$$

b. Since it takes approximately 35 quarters for an investment to double under this scenario, then in terms of years the time to double is $\frac{35}{4} = 8.75$ years.

62. **a.** $n = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06} \approx 11.9$

b. Since the compounding is semi-annual, 11.9 compounding periods corresponds to approximately 6 years.

63. $t = \log_{1.05} 2$

$$t = \frac{\log 2}{\log 1.05}$$
$$t = 14.20669908$$
$$t \approx 14.2$$

The future value will be $40,000 in approximately 14.2 years.

64. $t = \log_{1.08} 3.4$

$$t = \frac{\log 3.4}{\log 1.08}$$
$$t = 15.9012328$$
$$t \approx 15.9$$

The future value will be $30,000 in approximately 15.9 years.

65.

$$40{,}000 = 10{,}000\left(1+\frac{0.08}{12}\right)^{12t}$$
$$4 = \left(1+\frac{0.08}{12}\right)^{12t}$$
$$\ln(4) = \ln\left[\left(1+\frac{0.08}{12}\right)^{12t}\right]$$
$$12t\ln\left(1.00\overline{6}\right) = \ln(4)$$
$$t = \frac{\ln(4)}{12\ln\left(1.00\overline{6}\right)}$$
$$t \approx 17.3864$$

It will take approximately 17.4 years for the initial investment of \$10,000 to grow to \$40,000.

66.

$$60{,}000 = 25{,}000\left(1+\frac{0.12}{4}\right)^{4t}$$
$$2.4 = (1.03)^{4t}$$
$$\ln(2.4) = \ln\left[(1.03)^{4t}\right]$$
$$4t\ln(1.03) = \ln(2.4)$$
$$t = \frac{\ln(2.4)}{4\ln(1.03)}$$
$$t \approx 7.4046729$$

It will take approximately 7.4 years for the initial investment of \$25,000 to grow to \$60,000.

67. a. $A(0) = 500e^{-0.02828(0)} = 500e^{0} = 500$

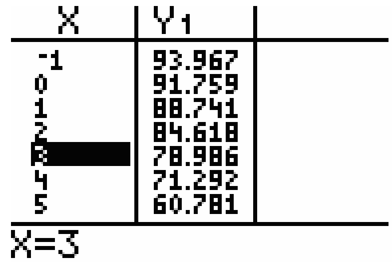
The initial quantity is 500 grams.

b.

$$250 = 500e^{-0.02828t}$$
$$0.5 = e^{-0.02828t}$$
$$\ln(0.5) = \ln\left(e^{-0.02828t}\right)$$
$$-0.02828t = \ln(0.5)$$
$$t = \frac{\ln(0.5)}{-0.02828} = 24.51$$

The half-life, the time it takes the initial quantity to become half, is approximately 24.5 years.

68.

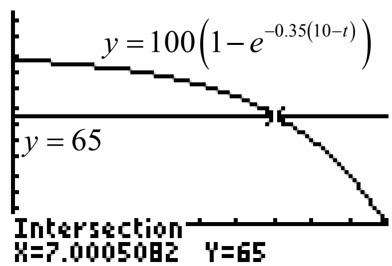


X	Y1
-1	93.967
0	91.759
1	88.741
2	84.618
3	78.986
4	71.292
5	60.781

X=3

The concentration reaches 79% in about 3 hours.

69. Applying the intersection of graphs method:



[0, 10] by [–20, 125]

After approximately 7 hours, the percent of the maximum dosage present is 65%.

70.

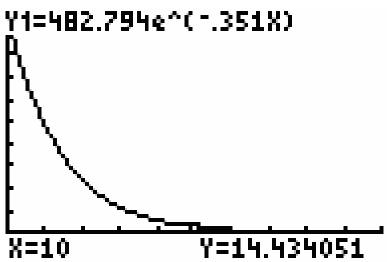
$$318 = 500e^{-0.02828t}$$
$$e^{-0.02828t} = \frac{318}{500}$$
$$\ln\left(e^{-0.02828t}\right) = \ln\left(\frac{318}{500}\right)$$
$$-0.02828t = \ln\left(\frac{318}{500}\right)$$
$$t = \frac{\ln\left(\frac{318}{500}\right)}{-0.02828}$$
$$t \approx 16$$

The amount of thorium reaches 318 grams in about 16 years.

71.
$$
\begin{aligned}
50 &= 100e^{-0.00002876t}\\
0.5 &= e^{-0.00002876t}\\
\ln(0.5) &= \ln\left(e^{-0.00002876t}\right)\\
-0.00002876t &= \ln(0.5)\\
t &= \frac{\ln(0.5)}{-0.00002876}\\
t &\approx 24{,}101
\end{aligned}
$$

The half-life is approximately 24,101 years.

72. a.



[0, 20] by [–50, 500]

b. The number of vinyl records sold is approaching zero.

c.

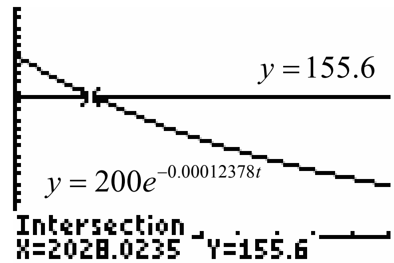
X	Y1
8	29.125
9	20.503
10	14.434
11	10.161
12	7.1534
13	5.0359
14	3.5452

X=13

Sales declined to 5 million after approximately 13 years or in 1993.

d. Vinyl albums have been replaced by compact discs. Note that answers will vary.

73. Applying the intersection of graphs method:



[0, 6000] by [0, 400]

After approximately 2028 years 155.6 grams of carbon-14 remains.

74. a.
$$
\begin{aligned}
16{,}230 &= 25{,}000e^{-0.072x}\\
e^{-0.072x} &= \frac{16{,}230}{25{,}000}\\
\ln\left(e^{-0.072x}\right) &= \ln\left(\frac{16{,}230}{25{,}000}\right)\\
-0.072x &= \ln\left(\frac{16{,}230}{25{,}000}\right)\\
x &= \frac{\ln\left(\frac{16{,}230}{25{,}000}\right)}{-0.072}\\
x &\approx 6
\end{aligned}
$$

Sales fell below \$16,230 approximately 6 weeks after the end of the campaign.

b.

X	Y1
5	17442
6	16230
7	15103
8	14054
9	13077
10	12169
11	11323

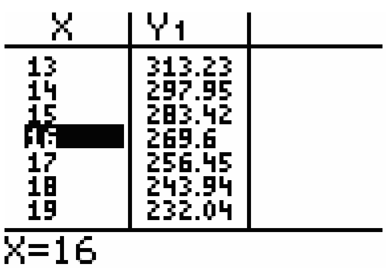
X=6

75. a.

$$S = 600e^{-0.05x}$$
$$\frac{S}{600} = e^{-0.05x}$$
$$\ln\left(\frac{S}{600}\right) = \ln\left(e^{-0.05x}\right)$$
$$-0.05x = \ln\left(\frac{S}{600}\right)$$
Let $S = 269.60$
$$-0.05x = \ln\left(\frac{269.60}{600}\right)$$
$$x = \frac{\ln\left(\frac{269.60}{600}\right)}{-0.05}$$
$$x \approx 16$$

Sixteen weeks after the end of the campaign, sales dropped below \$269.60 thousand.

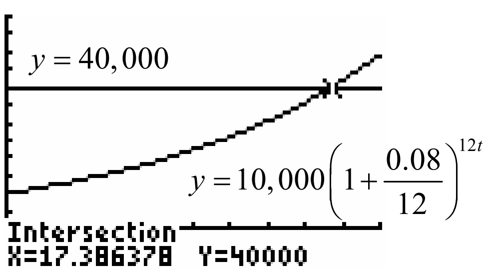
b.



X	Y1
13	313.23
14	297.95
15	283.42
16	269.6
17	256.45
18	243.94
19	232.04

X=16

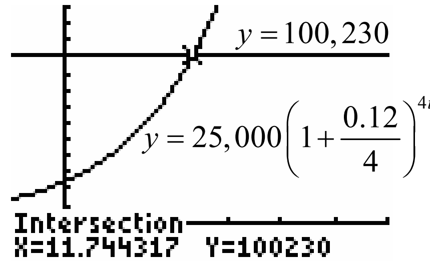
76. Applying the intersection of graphs method:



[0, 20] by [–10,000, 60,000]

For the first 17 years, the future value of the investment will be below \$40,000.

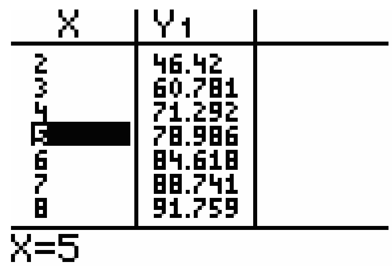
77. Applying the intersection of graphs method:



[–5, 30] by [–20,000, 130,000]

After 12 years, the future value of the investment will be greater than \$100,230.

78.



X	Y1
2	46.42
3	60.781
4	71.292
5	78.986
6	84.618
7	88.741
8	91.759

X=5

For the first five hours, the drug dosage remains below 79%.

79.

$$S = P(1.10)^n$$
$$P(1.10)^n = S$$
$$1.10^n = \frac{S}{P}$$
$$\log\left(1.10^n\right) = \log\left(\frac{S}{P}\right)$$
$$n\log(1.10) = \log\left(\frac{S}{P}\right)$$

$$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1.10)}$$

Let $S = 2P,$ since the investment doubles.

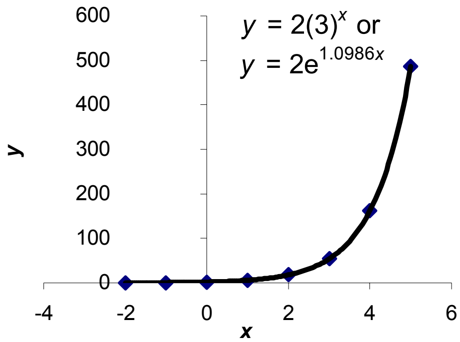
$$n = \frac{\log\left(\frac{2P}{P}\right)}{\log(1.10)}$$

$$n = \frac{\log(2)}{\log(1.10)}$$

$$n = \log_{1.10} 2$$

Section 3.4 Skills Check

1.



2.

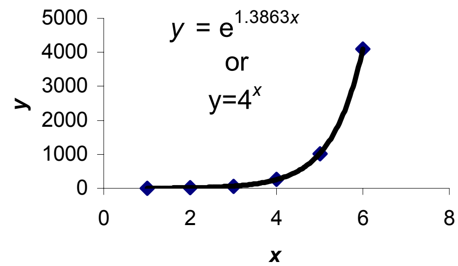
x	$f(x)$	First Differences	Percent Change
1	4		
2	16	12	300.00%
3	64	48	300.00%
4	256	192	300.00%
5	1024	768	300.00%
6	4096	3072	300.00%

x	$g(x)$	First Differences	Percent Change
1	2.5		
2	6	3.5	140.00%
3	8.5	2.5	41.67%
4	10	1.5	17.65%
5	8	–2	–20.00%
6	6	–2	–25.00%

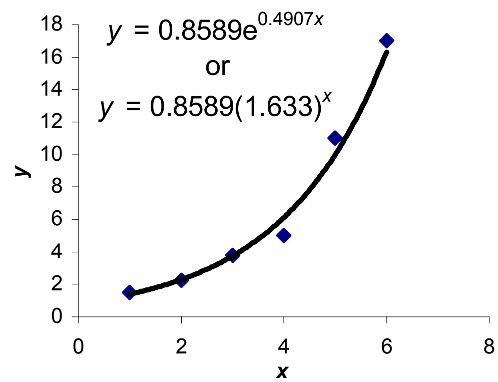
x	$h(x)$	First Differences	Percent Change
1	1.5		
2	2.25	0.75	50.00%
3	3.8	1.55	68.89%
4	5	1.2	31.58%
5	11	6	120.00%
6	17	6	54.55%

Since the percent change in the first table is constant, *f(x)* is exactly exponential. Since the percent change in the second table is both positive and negative, *g(x)* is not exponential. Since the percent change in the third table is approximately 50%, except for the 120% increase from 5 to 11, *h(x)* is approximately exponential.

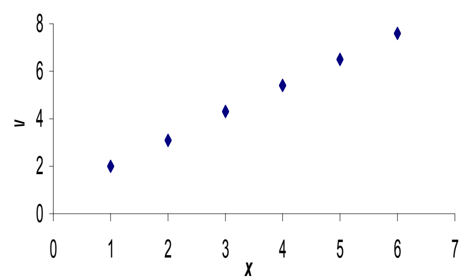
3.



4.



5. a.

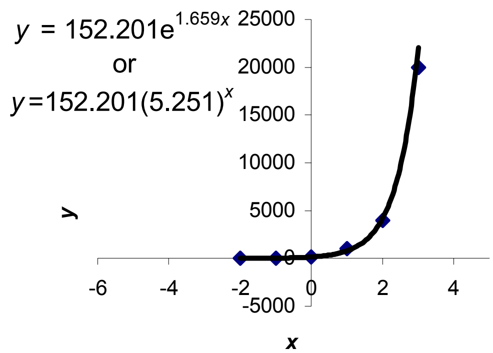


b.

x	y	First Differences	Percent Change
1	2		
2	3.1	1.1	55.00%
3	4.3	1.2	38.71%
4	5.4	1.1	25.58%
5	6.5	1.1	20.37%
6	7.6	1.1	16.92%

Considering the scatter plot from part a) and the chart above, a linear model fits the data best. The first differences are very close to being constant.

6.

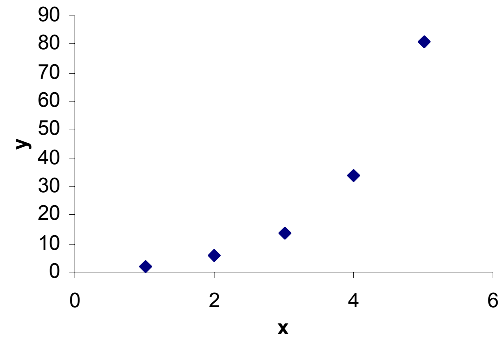


7.

x	y	First Differences	Percent Change
1	2		
2	6	4	200.00%
3	14	8	133.33%
4	34	20	142.86%
5	81	47	138.24%

Since the percent change is approximately constant and the first differences vary, an exponential function will fit the data best.

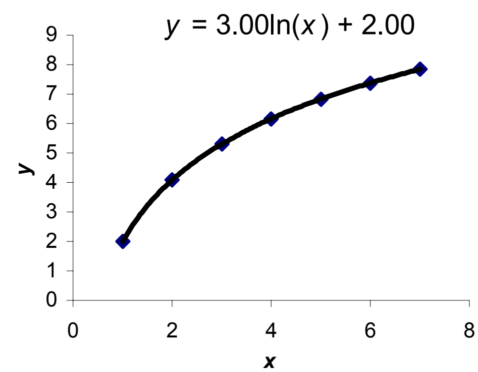
8.



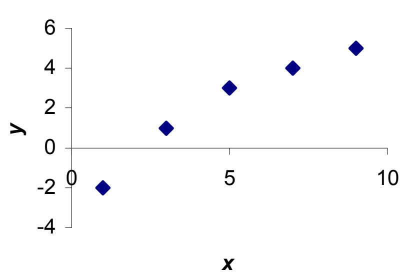
An exponential model is clearly the better fit based on the scatter plot.

9. Using technology yields, $y = 0.876e^{0.914x}$ or $y = 0.876(2.494)^x$.

10.

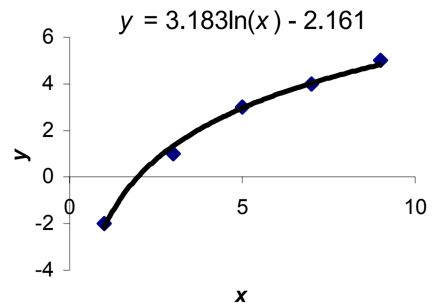


11. a.

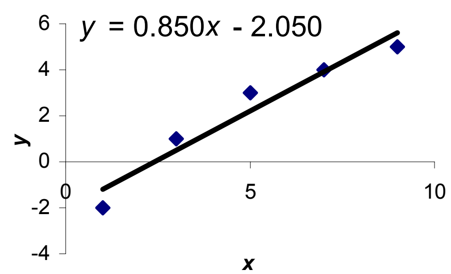


b. Based on the scatter plot, it appears that a logarithmic model fits the data best.

12. a.



b.



c. The logarithmic model is a much better fit based on the two scatter plots.

Section 3.4 Exercises

13. a. $y = a(1+r)^x$

$y = 30,000(1+0.04)^x$

$y = 30,000(1.04)^x$

b. $y = 30,000(1.04)^t$

$y = 30,000(1.04)^{15} \approx 54,028.31$

In 2010, the retail price of the automobile is predicted to be \$54,028.31.

14. a. $y = a(1+r)^x$

$y = 190,000(1+0.03)^x$

$y = 19,000(1.03)^x$

b. $y = 190,000(1.03)^x$

$= 190,000(1.03)^{10}$

$\approx 255,344.11$

In 2010, the population is predicted to be 255,344.

15. a. $y = a(1+r)^x$

$y = 20,000(1-0.02)^x$

$y = 20,000(0.98)^x$

b. $y = 20,000(0.98)^t$

$= 20,000(0.98)^5$

$= 18,078.42$

In five weeks the sales are predicted to decline to \$18,078.42.

16. a. $y = a(1+r)^x$

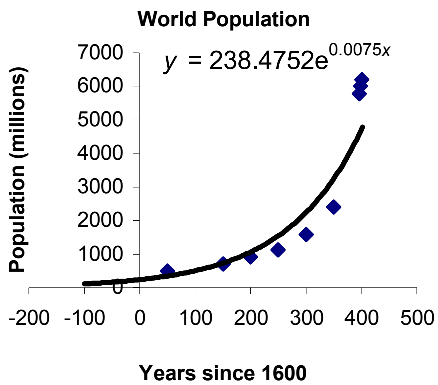
$y = 220{,}000(1+0.03)^x$

$y = 220{,}000(1.03)^x$

b. $y = 220{,}000(1.03)^5 \approx 255{,}040.30$

In 2005, the value of the home is predicted to increase to $255,040.

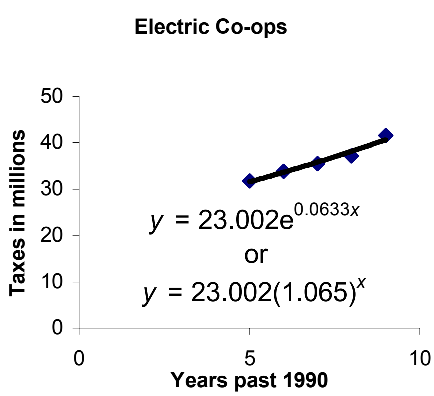
17. a.



The model is
$y = 238.4752e^{0.0075x}$ or
$y = 238.4752(1.0075)^x$.

b. See part a).

18. a.



b. $y = 23.002(1.065)^{15}$

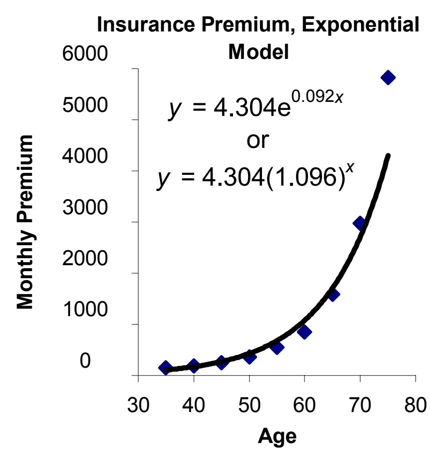
Substituting into the unrounded model yields $y \approx 59.46$.

```
15→X
                15
23.00226253459*
1.065358722808З^
X
     59.45777011
```

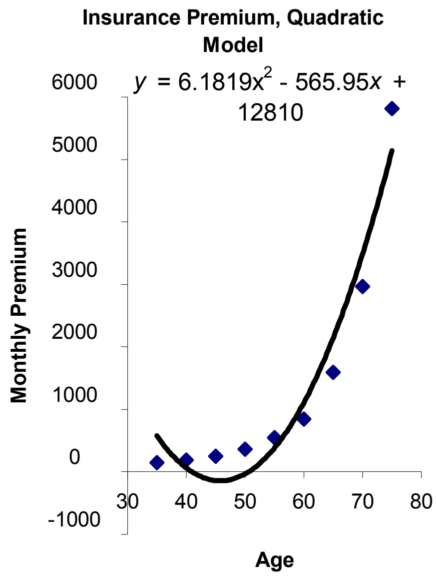
In 2005, the taxes paid are approximately $59.46 million. The result is not necessarily reliable because 2005 is beyond the range of the collected data. The calculation is an extrapolation.

c. See part a) above.

19. a.

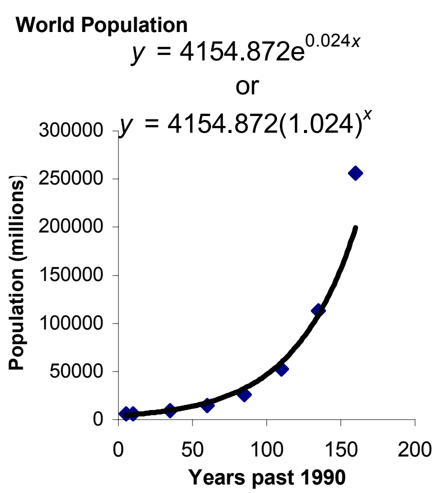


b.

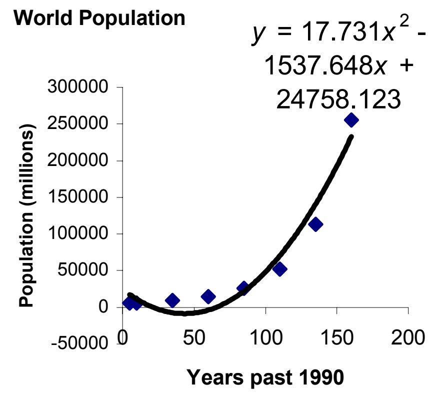


c. Considering parts a) and b), the exponential model is the best fit.

20. a.

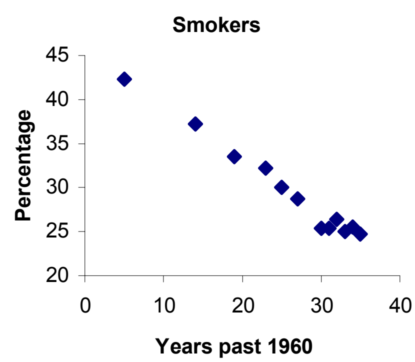


b.



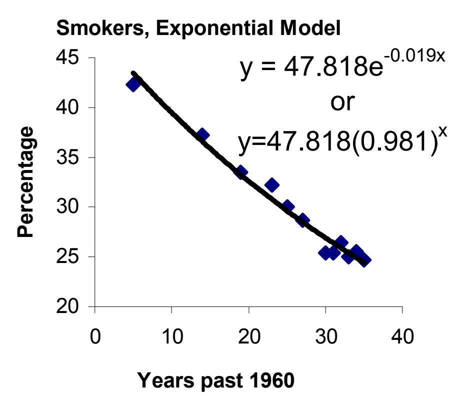
c. The exponential function seems to be the better fit, based on the graphs in parts a) and b).

21. a.

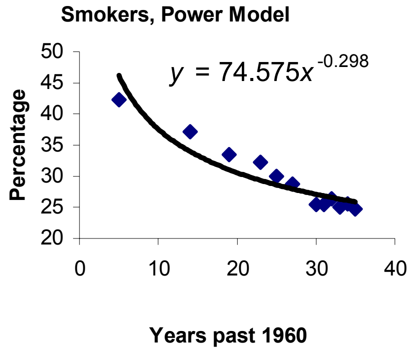


b. Yes, an exponential function could be used. A linear function would also fit the data well.

c.

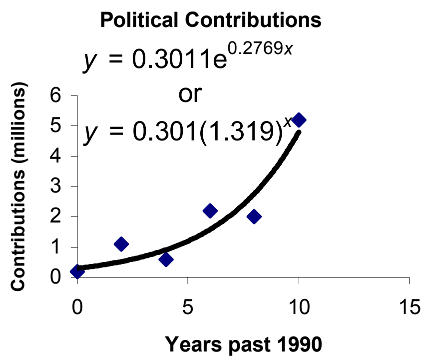


d.

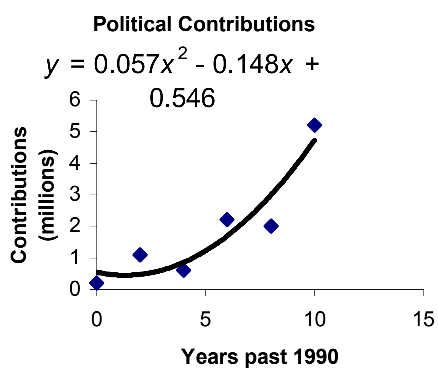


e. The exponential model appears to be best.

22. a.

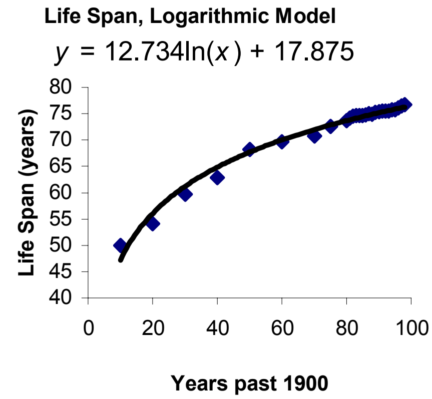


b.

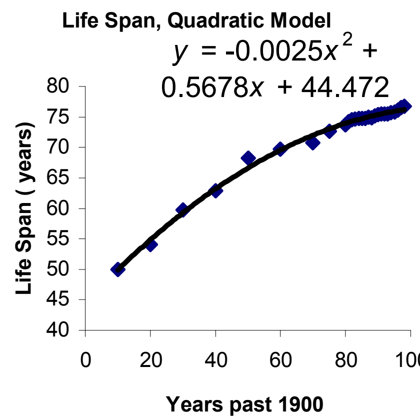


c. Both models seem to fit the data reasonably well.

23. a.



b.



c. Based on the graphs in parts a) and b), it appears that the quadratic function is the better fit.

d. In 2010, $x = 110$.
Using the logarithmic function,
$y = 12.734\ln(110) + 17.875$.

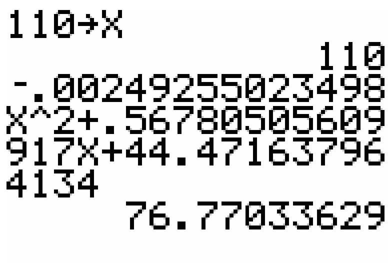
Substituting into the unrounded model yields $y \approx 76.73$.

```
110→X
                 110
17.87476746397 9+
12.7344139134211
n(X)
         77.73263003
```

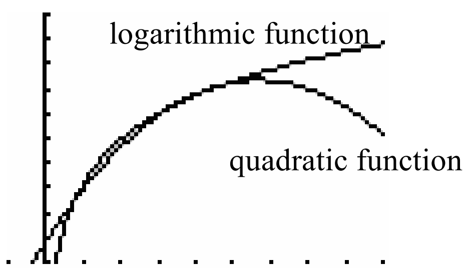
Using the quadratic function,

$$y = -0.0025(110)^2 + 0.5678(110) + 44.472.$$

Substituting into the unrounded model yields $y \approx 76.77$.

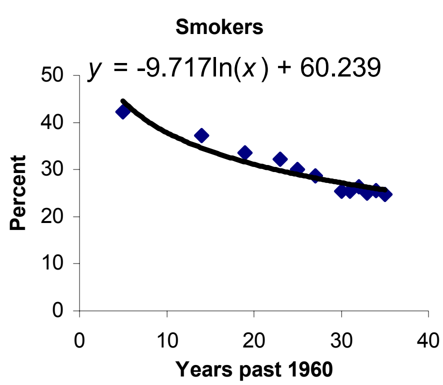


The logarithmic function produces a better prediction. For years beyond 2010, the logarithmic function continues to produce better predictions. The logarithmic function increases while the quadratic function begins to decrease.



[–20, 200] by [40, 90]

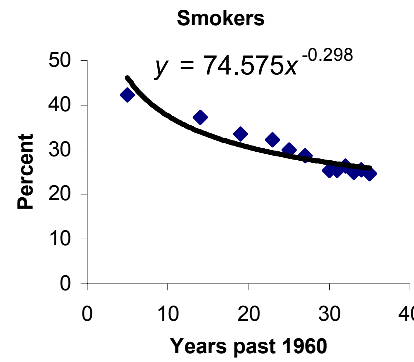
24. a.



b. $y = 60.239 - 9.717\ln(29) \approx 27.52$

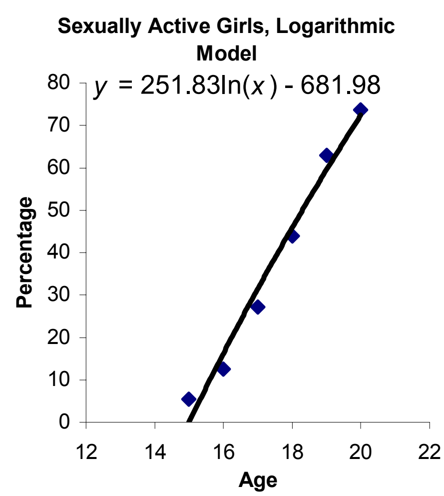
The percentage of smokers in 1989 is approximately 27.52%.

c.



Both functions seem to model the data reasonably well.

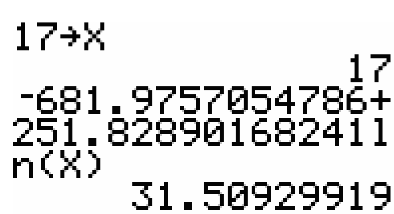
25. a.



b. $y = 251.83\ln(x) - 681.98$

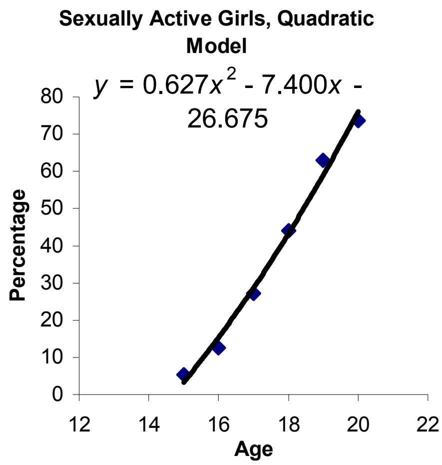
$y = 251.83\ln(17) - 681.98$

Substituting into the unrounded model yields $y \approx 31.5$.



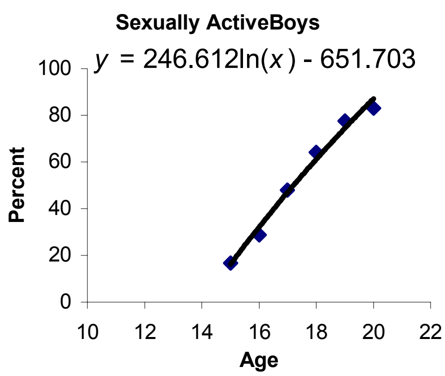
The percentage of girls 17 or younger who have been sexually active is 31.5%.

c.



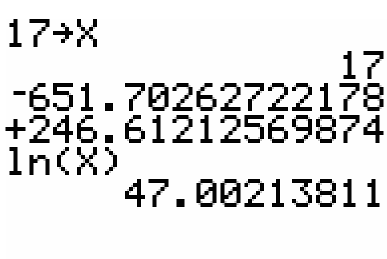
d. Based on the graphs in parts a) and c), the quadratic function seems to be the better fit.

26. a.



b. $y = -651.703 + 246.612\ln(17)$

Substituting into the unrounded model yields $y \approx 47.021$.



The percentage of boys 17 or younger who have been sexually active is 47.0%.

c. Based on the answers to problems 13 and 14, it seems that more males are sexually active at given ages than females.

Section 3.5 Skills Check

1. $15{,}000e^{0.06(20)}$

$=15{,}000e^{1.2}$

$=15{,}000(3.320116923)$

$=49{,}801.75$

2. $8000e^{0.05(10)}$

$=8000e^{0.5}$

$=8000(1.648721271)$

$=13{,}189.77$

3. $3000(1.06)^{30}$

$=3000(5.743491173)$

$=17{,}230.47$

4. $20{,}000(1.07)^{20}$

$=20{,}000(3.869684462)$

$=77{,}393.69$

5. $12{,}000\left(1+\frac{0.10}{4}\right)^{(4)(8)}$

$=12{,}000(1+.025)^{32}$

$=12{,}000(1.025)^{32}$

$=12{,}000(2.203756938)$

$=26{,}445.08$

6. $23{,}000\left(1+\frac{0.08}{12}\right)^{(12)(20)}$

$=23{,}000(1.00\overline{6})^{240}$

$=23{,}000(4.926802771)$

$=113{,}316.46$

7. $P\left(1+\frac{r}{k}\right)^{kn}$

$=3000\left(1+\frac{0.08}{2}\right)^{(2)(18)}$

$=3000(1.04)^{36}$

$=12{,}311.80$

8. $P\left(1+\frac{r}{k}\right)^{kn}$

$=8000\left(1+\frac{0.12}{12}\right)^{(12)(8)}$

$=8000(1.01)^{96}$

$=20{,}794.18$

9. $300\left[\frac{1.02^{240}-1}{0.02}\right]$

$=300\left[\frac{115.8887352-1}{0.02}\right]$

$=300\left[\frac{114.8887352}{0.02}\right]$

$=300[5744.436758]$

$=1{,}723{,}331.03$

10. $2000\left[\frac{1.10^{12}-1}{0.10}\right]$

$=2000\left[\frac{3.138428377-1}{0.10}\right]$

$=2000\left[\frac{2.138428377}{0.10}\right]$

$=2000[21.38428377]$

$=42{,}768.57$

11. $g(2.5)=1123.60$

$g(3)=1191.00$

$g(3.5)=1191.00$

12. $f(2) = 300$

$f(1.99) = 200$

$f(2.1) = 300$

13. $S = P\left(1+\frac{r}{k}\right)^{kn}$

$P\left(1+\frac{r}{k}\right)^{kn} = s$

$P = \frac{S}{\left(1+\frac{r}{k}\right)^{kn}}$

$P = S\left(1+\frac{r}{k}\right)^{-kn}$

14. $S = P(1+i)^n$

$P = \frac{S}{(1+i)^n}$

$P = S(1+i)^{-n}$

Section 3.5 Exercises

15. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.08,\ k = 1,\ t = 8$

$S = 8800\left(1+\frac{0.08}{1}\right)^{(1)(8)}$

$S = 8800(1.08)^8$

$S = 16,288.19$

The future value is \$16,288.19.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.08,\ k = 1,\ t = 30$

$S = 8800\left(1+\frac{0.08}{1}\right)^{(1)(30)}$

$S = 8800(1.08)^{30}$

$S = 88,551.38$

The future value is \$88,551.38.

16. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 6400,\ r = 0.07,\ k = 1,\ t = 10$

$S = 6400\left(1+\frac{0.07}{1}\right)^{(1)(10)}$

$S = 6400(1.07)^{10}$

$S = 12,589.77$

The future value is \$12,589.77.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 6400,\ r = 0.07,\ k = 1,\ t = 30$

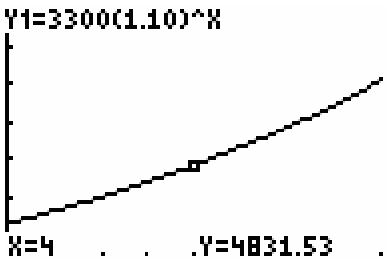
$S = 6400\left(1+\frac{0.07}{1}\right)^{(1)(30)}$

$S = 6400(1.07)^{30}$

$S = 48{,}718.43$

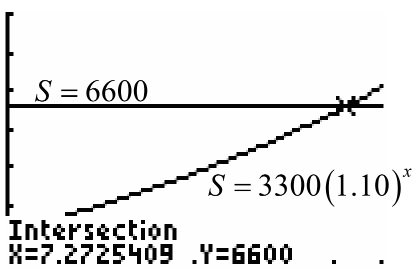
The future value is $48,718.43.

17. a.



[0, 8] by [2500, 9000]

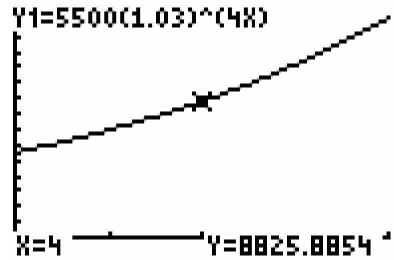
b.



[0, 8] by [2500, 9000]

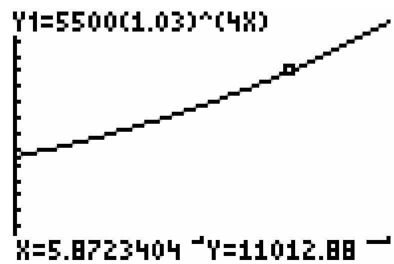
The initial investment doubles in approximately 7.3 years. After 8 years compounded annually, the initial investment will be more than doubled.

18. a.



[0, 8] by [–1000, 15,000]

b.



[0, 8] by [0, 20,000]

Based on the graph it takes about 5.8 years for the initial investment to double. Since compounding occurs quarterly, the time to double is 6 years.

19. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10{,}000,\ r = 0.12,\ k = 4,\ t = 10$

$S = 10{,}000\left(1+\frac{0.12}{4}\right)^{(4)(10)}$

$S = 10{,}000(1.03)^{40}$

$S = 32{,}620.38$

The future value is $32,620.38.

20. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.06,\ k = 2,\ t = 10$

$S = 8800\left(1+\frac{0.06}{2}\right)^{(2)(10)}$

$S = 8800(1.03)^{20}$

$S = 15,893.78$

The future value is $15,893.78.

21. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10,000,\ r = 0.12,\ k = 365,\ t = 10$

$S = 10,000\left(1+\frac{0.12}{365}\right)^{(365)(10)}$

$S = 10,000(1.0003287671233)^{3650}$

$S = 33,194.62$

The future value is $33,194.62.

b. Since the compounding occurs more often in Exercise 21 than in Exercise 19, the future value in Exercise 21 is greater.

22. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 8800,\ r = 0.06,\ k = 365,\ t = 10$

$S = 8800\left(1+\frac{0.06}{365}\right)^{(365)(10)}$

$S = 8800(1.000164384)^{3650}$

$S = 16,033.85$

The future value is $16,033.85.

b. The answers are different. Changing the number of compounds per year affects the future value of the investment.

23. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10,000,\ r = 0.12,\ k = 12,\ t = 15$

$S = 10,000\left(1+\frac{0.12}{12}\right)^{(12)(15)}$

$S = 10,000(1.01)^{180}$

$S = 59,958.02$

The future value is $59,958.02. The interest earned is the future value minus the present value. In this case, 59,958.02 – 10,000 = $49,958.02.

24. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 20,000,\ r = 0.08,\ k = 4,\ t = 25$

$S = 20,000\left(1+\frac{0.08}{4}\right)^{(4)(25)}$

$S = 20,000(1.02)^{100}$

$S = 144,892.92$

The future value is $144,892.92.

25. a. $S = Pe^{rt}$

$P = 10,000,\ r = 0.06,\ t = 12$

$S = 10,000e^{(0.06)(12)}$

$S = 10,000e^{0.72}$

$S = 20,544.33$

The future value is $20,544.33.

b. $S = Pe^{rt}$

$P = 10,000,\ r = 0.06,\ t = 18$

$S = 10,000e^{(0.06)(18)}$

$S = 10,000e^{1.08}$

$S = 29,446.80$

The future value is $29,446.80.

26. a. $S = Pe^{rt}$

$P = 42,000,\ r = 0.07,\ t = 10$

$S = 42,000e^{(0.07)(10)}$

$S = 42,000e^{0.7}$

$S = 84,577.61$

The future value is $84,577.61.

b. $S = Pe^{rt}$

$P = 42,000,\ r = 0.07,\ t = 20$

$S = 42,000e^{(0.07)(20)}$

$S = 42,000e^{1.4}$

$S = 170,318.40$

The future value is $170,318.40.

27. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 10,000,\ r = 0.06,\ k = 1,\ t = 18$

$S = 10,000\left(1+\frac{0.06}{1}\right)^{(1)(18)}$

$S = 10,000(1.06)^{18}$

$S \approx 28,543.39$

The future value is $28,543.39.

b. Continuous compounding yields a higher future value, 29,446.80 – 28,543.39 = 903.41 additional dollars.

28. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 42,000,\ r = 0.07,\ k = 1,\ t = 20$

$S = 42,000\left(1+\frac{0.07}{1}\right)^{(1)(20)}$

$S = 42,000(1.07)^{20}$

$S \approx 126,526.75$

The future value is $126,526.75.

b. Continuous compounding yields a higher future value, 170,318.40 – 126,526.75 = 43,791.65 additional dollars.

29. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

Doubling the investment implies $S = 2P$.

$2P = P\left(1+\frac{r}{k}\right)^{kt}$

$\frac{2P}{P} = \frac{P\left(1+\frac{r}{k}\right)^{kt}}{P}$

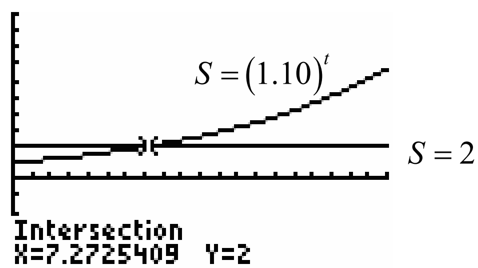
$2 = \left(1+\frac{r}{k}\right)^{kt}$

$k = 1,\ r = 0.10$

$2 = \left(1+\frac{0.10}{1}\right)^{(1)t}$

$2 = (1.10)^t$

Applying the intersection of graphs method:



[0, 20] by [–5, 10]

The time to double is approximately 7.3 years. In terms of discrete units, the time to double is 8 years.

b. $S = Pe^{rt}$

Doubling the investment implies

$S = 2P.$

$2P = Pe^{rt}$

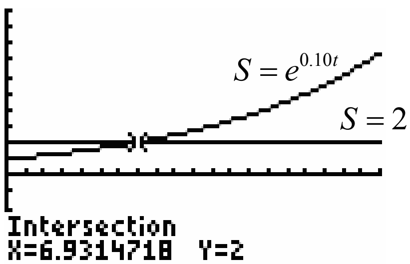
$\frac{2P}{P} = \frac{Pe^{rt}}{P}$

$2 = e^{rt}$

$r = 0.10$

$2 = e^{0.10t}$

Applying the intersection of graphs method:



[0, 20] by [–5, 10]

The time to double is approximately 6.9 years. In terms of discrete units, the time to double is 7 years.

30. a. $S = P\left(1 + \frac{r}{k}\right)^{kt}$

Doubling the investment implies

$S = 2P.$

$2P = P\left(1 + \frac{r}{k}\right)^{kt}$

$\frac{2P}{P} = \frac{P\left(1 + \frac{r}{k}\right)^{kt}}{P}$

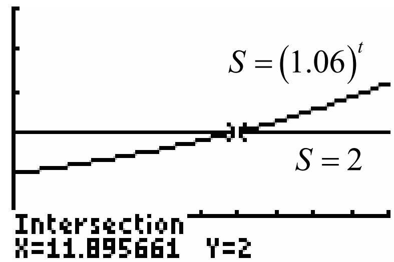
$2 = \left(1 + \frac{r}{k}\right)^{kt}$

$k = 1, r = 0.06$

$2 = \left(1 + \frac{0.06}{1}\right)^{(1)t}$

$2 = (1.06)^t$

Applying the intersection of graphs method:



[0, 20] by [–1, 5]

The time to double is approximately 11.9 years. In terms of discrete units, the time to double is 12 years.

b. $S = Pe^{rt}$

Doubling the investment implies

$S = 2P.$

$2P = Pe^{rt}$

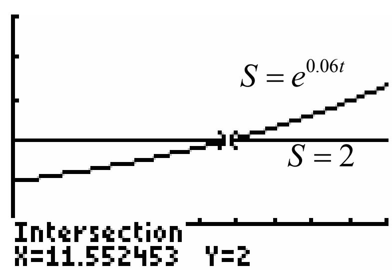
$\frac{2P}{P} = \frac{Pe^{rt}}{P}$

$2 = e^{rt}$

$r = 0.06$

$2 = e^{0.06t}$

Applying the intersection of graphs method:



[0, 20] by [–1, 5]

The time to double is approximately 11.55 years. In terms of discrete units, the time to double is 12 years.

31. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 2000,\ r = 0.05,\ k = 1,\ t = 8$

$S = 2000\left(1+\frac{0.05}{1}\right)^{(1)(8)}$

$S = 2000(1.05)^8$

$S = 2954.91$

The future value is $2954.91.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 2000,\ r = 0.05,\ k = 1,\ t = 18$

$S = 2000\left(1+\frac{0.05}{1}\right)^{(1)(18)}$

$S = 2000(1.05)^{18}$

$S = 4813.24$

The future value is $4813.24.

32. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 12{,}000,\ r = 0.08,\ k = 4,\ t = \frac{1}{2}$

$S = 12{,}000\left(1+\frac{0.08}{4}\right)^{(4)\left(\frac{1}{2}\right)}$

$S = 12{,}000(1.02)^2$

$S = 12{,}484.80$

The future value is $12,484.80.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 12{,}000,\ r = 0.08,\ k = 4,\ t = 10$

$S = 12{,}000\left(1+\frac{0.08}{4}\right)^{(4)(10)}$

$S = 12{,}000(1.02)^{40}$

$S = 26{,}496.48$

The future value is $26,496.48.

33. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 3000,\ r = 0.06,\ k = 12,\ t = 112$

$S = 3000\left(1+\frac{0.06}{12}\right)^{(12)(12)}$

$S = 3000(1.005)^{144}$

$S = 6152.25$

The future value is $6152.25.

34. a. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 9000,\ r = 0.08,\ k = 4,\ t = 0.5$

$S = 9000\left(1+\frac{0.08}{4}\right)^{(4)(0.5)}$

$S = 9000(1.02)^2$

$S = 9363.60$

The future value is $9363.60.

b. $S = P\left(1+\frac{r}{k}\right)^{kt}$

$P = 9000,\ r = 0.08,\ k = 4,\ t = 15$

$S = 9000\left(1+\frac{0.08}{4}\right)^{(4)(15)}$

$S = 9000(1.02)^{60}$

$S = 29{,}529.28$

The future value is \$29,529.28.

35. a.

Years	Future Value
0	1000
7	2000
14	4000
21	8000
28	16,000

b. $S = 1000\left(1+\frac{0.10}{4}\right)^{4t}$

$S = 1000(1.025)^{4t}$

$S = 1000\left((1.025)^4\right)^t$

$S = 1000(1.104)^t$

c. After five years, the investment is worth $S = 1000(1.104)^5 = \$1640.01$.
After 10.5 years, the investment is worth $S = 1000(1.104)^{10.5} = \$2826.02$.

36. a.

Years	Future Value
0	1000
6	2000
12	4000
18	8000
24	16,000

b. $S = 1000\left(1+\frac{0.116}{12}\right)^{12t}$

$S = 1000(1.009\overline{6})^{12t}$

$S = 1000\left((1.009\overline{6})^{12}\right)^t$

$S = 1000(1.122)^t$

c. After two months, the value of the investment is
$S = 1000(1.122)^{\left(\frac{2}{12}\right)} = \$1019.37$.

After four years, the investment is worth $S = 1000(1.122)^4 = \$1584.79$.

After 12.5 years, the investment is worth $S = 1000(1.122)^{12.5} = \$4216.10$.

Section 3.6 Skills Check

1. $S = P(1+i)^n$

$$\frac{S}{(1+i)^n} = \frac{P(1+i)^n}{(1+i)^n}$$

$$P = \frac{S}{(1+i)^n}$$

2. Considering the answer to Exercise 1 and continuing the algebra yields

$$P = \frac{S}{(1+i)^n} = S(1+i)^{-n}.$$

3. $A \bullet i = R\left[1-(1+i)^{-n}\right]$

$$\frac{A \bullet i}{i} = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

4. $2000\left[\frac{1-(1+0.01)^{-240}}{0.01}\right]$

$$= 2000\left[\frac{1-(1.01)^{-240}}{0.01}\right]$$

$$= 2000\left[\frac{1-0.0918058365}{0.01}\right]$$

$$= 2000\left[\frac{0.9081941635}{0.01}\right]$$

$$= 2000[90.81941635]$$

$$= 181,638.83$$

5. $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$i \bullet A = i\left(R\left[\frac{1-(1+i)^{-n}}{i}\right]\right)$$

$$iA = R\left[1-(1+i)^{-n}\right]$$

$$\frac{iA}{\left[1-(1+i)^{-n}\right]} = \frac{R\left[1-(1+i)^{-n}\right]}{\left[1-(1+i)^{-n}\right]}$$

$$R = \frac{iA}{\left[1-(1+i)^{-n}\right]} = A\left[\frac{i}{1-(1+i)^{-n}}\right]$$

6. $240,000\left[\frac{0.01}{1-(1+0.10)^{-120}}\right]$

$$= 240,000\left[\frac{0.01}{1-(1.10)^{-120}}\right]$$

$$= 240,000\left[\frac{0.01}{1-(1.078643128 \times 10^{-5})}\right]$$

$$= 240,000\left[\frac{0.01}{0.9999892136}\right]$$

$$= 240,000[0.0100001079]$$

$$= 2400.03$$

Section 3.6 Exercises

7. $$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$10{,}000 = P\left(1 + \frac{0.06}{1}\right)^{(1)(10)}$$

$$10{,}000 = P(1.06)^{10}$$

$$P = \frac{10{,}000}{(1.06)^{10}}$$

$$P = 5583.94$$

An initial amount of $5583.94 will grow to $10,000 in 10 years if invested at 6% compounded annually.

8. $$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$30{,}000 = P\left(1 + \frac{0.07}{1}\right)^{(1)(15)}$$

$$30{,}000 = P(1.07)^{15}$$

$$P = \frac{30{,}000}{(1.07)^{15}}$$

$$P = 10{,}873.38$$

An initial amount of $10,873.38 will grow to $30,000 in 15 years if invested at 7% compounded annually.

9. $$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$30{,}000 = P\left(1 + \frac{0.10}{12}\right)^{(12)(18)}$$

$$30{,}000 = P\left(1.008\overline{3}\right)^{216}$$

$$P = \frac{30{,}000}{\left(1.008\overline{3}\right)^{216}}$$

$$P = 4996.09$$

An initial amount of $4996.09 will grow to $30,000 in 18 years if invested at 10% compounded monthly.

10. $$S = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$1{,}000{,}000 = P\left(1 + \frac{0.11}{12}\right)^{(12)(50)}$$

$$1{,}000{,}000 = P\left(1.0091\overline{6}\right)^{600}$$

$$P = \frac{1{,}000{,}000}{\left(1.0091\overline{6}\right)^{600}}$$

$$P = 4190.46$$

An initial amount of $4190.46 will grow to $1,000,000 in 50 years if invested at 11% compounded monthly.

11. $$A = R\left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

$$A = 1000\left[\frac{1 - (1 + 0.07)^{-10}}{0.07}\right]$$

$$A = 1000\left[\frac{1 - (1.07)^{-10}}{0.07}\right]$$

$$A = 1000\left[\frac{1 - (0.5083492921)}{0.07}\right]$$

$$A = 1000[7.023581541]$$

$$A = 7023.58$$

Investing $7023.58 initially will produce an income of $1000 per year for 10 years if the interest rate is 7% compounded annually.

12. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$A = 500\left[\dfrac{1-(1+0.09)^{-20}}{0.09}\right]$

$A = 500\left[\dfrac{1-(1.09)^{-20}}{0.09}\right]$

$A = 500\left[\dfrac{1-(0.1784308898)}{0.09}\right]$

$A = 500[9.128545669]$

$A \approx 4564.27$

Investing \$4564.27 initially will produce an income of \$500 per year for 20 years if the interest rate is 9% compounded annually.

13. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$A = 50{,}000\left[\dfrac{1-(1+0.08)^{-19}}{0.08}\right]$

$A = 50{,}000\left[\dfrac{1-(1.08)^{-19}}{0.08}\right]$

$A = 50{,}000\left[\dfrac{1-(0.231712064)}{0.08}\right]$

$A = 50{,}000[9.6035992]$

$A = 480{,}179.96$

The formula above calculates the present value of the annuity given the payment made at the end of each period. Twenty total payments were made, but only nineteen occurred at the end of a compounding period. The first payment of \$50,000 was made up front. Therefore, the total value of the lottery winnings is $50{,}000 + 480{,}179.96 = \$530{,}179.96$.

14. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$i = \dfrac{0.08}{2} = 0.04,\ n = 4 \bullet 2 = 8$

$A = 3000\left[\dfrac{1-(1+0.04)^{-8}}{0.04}\right]$

$A = 3000\left[\dfrac{1-(1.04)^{-8}}{0.04}\right]$

$A = 3000[6.732744875]$

$A \approx 20{,}198.23$

A lump sum of \$20,198.23 is required to generate the annuity.

15. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$A = 3000\left[\dfrac{1-\left(1+\dfrac{0.09}{12}\right)^{-(30)(12)}}{\dfrac{0.09}{12}}\right]$

$A = 3000\left[\dfrac{1-(1.0075)^{-360}}{0.0075}\right]$

$A = 3000\left[\dfrac{1-0.0678860074}{0.0075}\right]$

$A = 3000[124.2818657]$

$A = 372{,}845.60$

The disabled man should seek a lump sum payment of \$372,845.60.

16. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$A = 400\left[\dfrac{1-\left(1+\dfrac{0.08}{12}\right)^{-(12)(4)}}{\dfrac{0.08}{12}}\right]$$

$$A = 400\left[\dfrac{1-(1.00\overline{6})^{-48}}{0.00\overline{6}}\right]$$

$$A = 400[40.96191296]$$

$$A = 16{,}384.77$$

A fair offer for the car would be \$16,384.77.

17. a. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$A = 122{,}000\left[\dfrac{1-(1+0.10)^{-9}}{0.10}\right]$$

$$A = 122{,}000\left[\dfrac{1-(1.10)^{-9}}{0.10}\right]$$

$$A = 122{,}000\left[\dfrac{1-0.4240976184}{0.10}\right]$$

$$A = 122{,}000[5.759023816]$$

$$A = 702{,}600.91$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. Ten total payments were made, but only nine occurred at the end of a compounding period. The first payment of \$100,000 was made up front. Therefore, the total value of the sale is $100{,}000 + 702{,}600.91 = \$802{,}600.91$.

b. $R = A\left[\dfrac{i}{1-(1+i)^{-n}}\right]$

$$R = 700{,}000\left[\dfrac{0.10}{1-(1+0.10)^{-9}}\right]$$

$$R = 700{,}000\left[\dfrac{0.10}{1-(1.10)^{-9}}\right]$$

$$R = 700{,}000\left[\dfrac{0.10}{1-0.4240976184}\right]$$

$$R = 700{,}000[0.1736405391]$$

$$R = 121{,}548.38$$

The annuity payment is \$121,548.38.

c. The \$100,000 plus the annuity yields a higher present value and therefore would be the better choice. Over the nine year annuity period, the \$100,000 cash plus \$122,000 annuity yields \$452 more per year than investing \$700,000 in cash.

18. a. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$A = 250{,}000\left[\dfrac{1-(1+0.07)^{-5}}{0.07}\right]$$

$$A = 250{,}000\left[\dfrac{1-(1.07)^{-5}}{0.07}\right]$$

$$A = 250{,}000[4.100197436]$$

$$A = 1{,}025{,}049.36$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. Six total payments were made, but only five occurred at the end of a compounding period. The first payment of \$200,000 was made up front. Therefore, the total value of the sale is $200{,}000 + 1{,}025{,}049.36 = \$1{,}225{,}049.36$.

b. $R = A\left[\dfrac{i}{1-(1+i)^{-n}}\right]$

$R = 1{,}000{,}000\left[\dfrac{0.07}{1-(1+0.07)^{-5}}\right]$

$R = 1{,}000{,}000\left[\dfrac{0.07}{1-(1.07)^{-5}}\right]$

$R = 1{,}000{,}000[0.2438906944]$

$R = 243{,}890.69$

The annuity payment is $243,890.69.

c. The present value of the all cash transaction is $1,200,000, while the present value of the cash plus annuity transaction is $1,225,049. The cash plus annuity is better. Over the 5-year annuity period, the $200,000 cash plus $250,000 annuity yields approximately $6109 per month more than investing 1,000,000 in cash.

19. a. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$A = 1600\left[\dfrac{1-\left(1+\dfrac{0.09}{12}\right)^{-(30)(12)}}{\dfrac{0.09}{12}}\right]$

$A = 1600\left[\dfrac{1-(1.0075)^{-360}}{0.0075}\right]$

$A = 1600\left[\dfrac{1-0.0678860074}{0.0075}\right]$

$A = 1600[124.2818657]$

$A = 198{,}850.99$

The couple can afford to pay $198,850.99 for a house.

b. $(\$1600 \text{ per month})\times(12 \text{ months})$
$\times(30 \text{ years}) = \$576{,}000$

c. $576{,}000 - 198{,}850.99 = \$377{,}149.01$

20. a. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$A = 400\left[\dfrac{1-\left(1+\dfrac{0.12}{12}\right)^{-(4)(12)}}{\dfrac{0.12}{12}}\right]$

$A = 400\left[\dfrac{1-(1.01)^{-48}}{0.01}\right]$

$A = 400[37.97395949]$

$A = 15{,}189.58$

A total of $15,189.58 can be paid for the car in order for the payment to remain $400 per month.

b. $(\$400 \text{ per month})\times(48 \text{ months})$
$= \$19{,}200$

c. $19{,}200 - 15{,}189.58 = \$4010.42$
The interest is $4010.42.

21. a. $\dfrac{8}{4} = 2\%$

b. $(4 \text{ years})\times(4 \text{ payments per year})$
$= 16 \text{ payments}$

c. $R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$

$R = 10,000\left[\frac{0.02}{1-(1+0.02)^{-16}}\right]$

$R = 10,000\left[\frac{0.02}{1-(1.02)^{-16}}\right]$

$R = 10,000\left[\frac{0.02}{1-0.7284458137}\right]$

$R = 10,000[0.0736501259]$

$R = 736.50$

The quarterly payment is $736.50.

22. a. $\frac{6}{12} = 0.5\%$

b. $(6 \text{ years}) \times (12 \text{ payments per year})$
$= 72$ payments

c. $R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$

$R = 36,000\left[\frac{0.005}{1-(1+0.005)^{-72}}\right]$

$R = 36,000\left[\frac{0.005}{1-(1.005)^{-72}}\right]$

$R = 36,000[0.0165728879]$

$R = 596.62$

The monthly car payment is $596.62.

23. a. $i = \frac{0.06}{12} = 0.005,\ n = 360$

$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$

$R = 250,000\left[\frac{0.005}{1-(1+0.005)^{-360}}\right]$

$R = 250,000\left[\frac{0.005}{1-0.166041928}\right]$

$R = 250,000\left[\frac{0.005}{0.833958072}\right]$

$R = 250,000[0.0059955053]$

$R = 1498.88$

The monthly mortgage payment is $1498.88.

b. $(30 \text{ years}) \times (12 \text{ payments per year})$
$\times(\$1498.88) = \$539,596.80$
Including the down payment, the total cost of the house is $639,596.80.

c. $639,596.80 - 350,000 = \$289,596.80$

24. a. $i = \frac{0.08}{4} = 0.02,\ n = 100$

$R = A\left[\frac{i}{1-(1+i)^{-n}}\right]$

$R = 450,000\left[\frac{0.02}{1-(1+0.02)^{-100}}\right]$

$R = 450,000\left[\frac{0.02}{1-(1.02)^{-100}}\right]$

$R = 450,000[0.0232027435]$

$R = 10,441.23$

The monthly payment is $10,441.23.

b. $(25 \text{ years}) \times (4 \text{ payments per year})$
$\times(\$10,441.23) = \$1,044,123$
Including the down payment, the total cost of the restaurant is $1,344,123.

c. $1,344,123 - 750,000 = \$594,123$

Section 3.7 Skills Check

1. $\dfrac{79.514}{1+0.835e^{-0.0298(80)}}$

$=\dfrac{79.514}{1+0.835e^{-2.384}}$

$=\dfrac{79.514}{1+0.835(0.0921811146)}$

$=\dfrac{79.514}{1.076971231}$

$=73.83112727$

≈ 73.83

2. a. $y=\dfrac{79.514}{1+0.835e^{-0.0298x}}$

$y=\dfrac{79.514}{1+0.835e^{-0.0298(10)}}$

$y=\dfrac{79.514}{1+0.835(0.7423013397)}$

$y=\dfrac{79.514}{1.619821619}$

$y=49.08812124$

$y\approx 49.09$

b. $y=\dfrac{79.514}{1+0.835e^{-0.0298x}}$

$y=\dfrac{79.514}{1+0.835e^{-0.0298(50)}}$

$y=\dfrac{79.514}{1+0.835(0.2253726555)}$

$y=\dfrac{79.514}{1.188186167}$

$y=66.92048955$

$y\approx 66.92$

3. $1000(0.06)^{0.2^t}$

Let $t=4$.

$1000(0.06)^{0.2^4}=1000(0.06)^{0.0016}$

$=1000(0.9955086592)$

≈ 995.51

Let $t=6$.

$1000(0.06)^{0.2^6}=1000(0.06)^{0.000064}$

$=1000(0.9998199579)$

≈ 999.82

4. $2000(0.004)^{0.5^t}$

Let $t=5$.

$2000(0.004)^{0.5^5}=2000(0.004)^{0.03125}$

$=2000(0.8415198695)$

≈ 1683.04

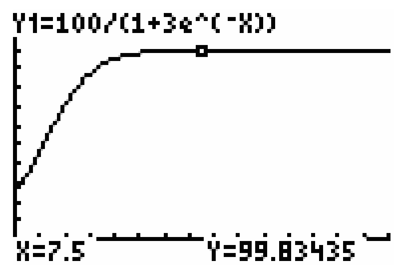
Let $t=10$.

$2000(0.004)^{0.5^{10}}=2000(0.004)^{0.0009765625}$

$=2000(0.9946224593)$

≈ 1989.24

5. a.



[0, 15] by [0, 120]

b.

X	Y1
0	25
5	98.019
10	99.986
15	100
20	100
25	100
30	100

X=0

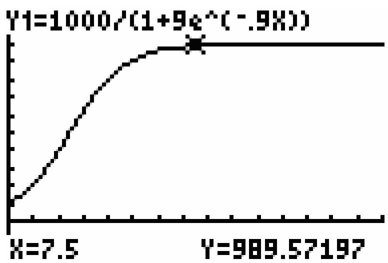
$f(0) = 25$

$f(10) = 99.986$

c. The graph is increasing.

d. Based on the graph, the y-values of the function approach 100. Therefore the limiting value of the function is 100. $y = c = 100$ is a horizontal asymptote of the function.

6. a.



[0, 15] by [0, 1200]

b.

X	Y1	
0	100	
1	214.63	
2	401.98	
3	623.11	
4	802.62	
5	909.11	
6	960.94	

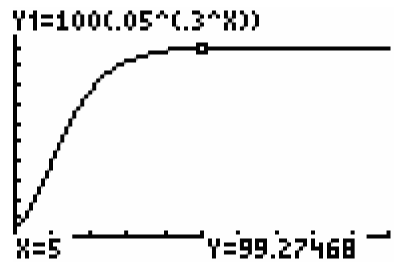
X=2

$f(2) = 401.98$

$f(5) = 909.11$

c. Based on the graph, the y-values of the function approach 1000. Therefore the limiting value of the function is 1000. $y = c = 1000$ is a horizontal asymptote.

7. a.



[0, 10] by [0, 120]

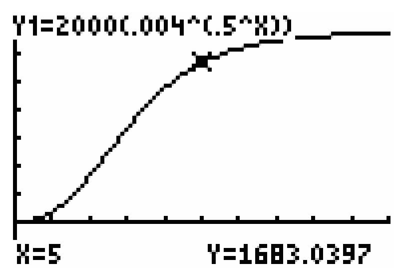
b. Let $x = 0$, and solve for y.

$y = 100(0.05)^{0.3^0} = 100(0.05)^1 = 5$

The initial value is 5.

c. The maximum value is c. In this case, $c = 100$.

8. a.



[0, 10] by [0, 2200]

b. Let $t = 0$, and solve for N.

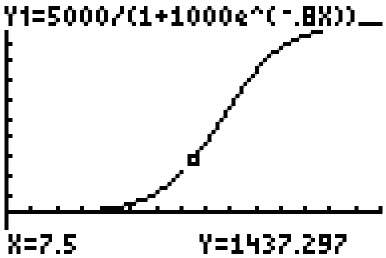
$N = 2000(0.004)^{0.5^0} = 2000(0.004)^1 = 8$

The initial value is 8.

c. The maximum value is c. In this case, $c = 2000$.

Section 3.7 Exercises

9. a.

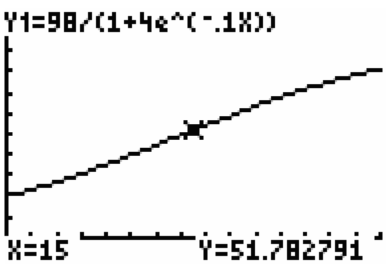


[0, 15] by [–1000, 5500]

b. At $x = 0$, the number of infected students is the value of the y-intercept of the function. The y-intercept is $\frac{c}{1+a} = \frac{5000}{1+1000} = 4.995 \approx 5$.

c. The upper limit is $c = 5000$.

10. a.



[0, 30] by [–10, 110]

b. $p(10) = \frac{98}{1+4e^{-0.1(10)}}$

$= \frac{98}{1+4e^{-1}} \approx 39.652$

The population in 1998 is approximately 39,652 people.

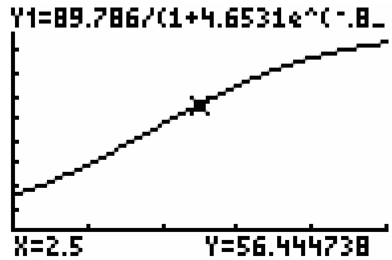
c. $p(100) = \frac{98}{1+4e^{-0.1(100)}}$

$= \frac{98}{1+4e^{-10}} \approx 97.982$

The population in 2088 is approximately 97,982 people.

d. The upper limit is $c = 98$ or 98,000 people.

11. a.



[0, 5] by [0, 100]

b.

X	Y1
0	15.883
1	29.555
2	47.442
3	64.552
4	76.661
5	83.523
6	86.931

X=1

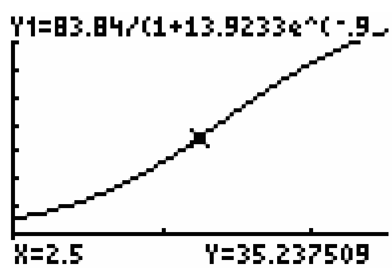
The model indicates that 29.56% of 16-year old boys have been sexually active

c. Consider the table in part b) above.

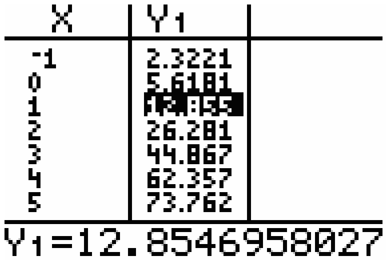
The model indicates that 86.93% of 21-year old boys have been sexually active

d. The upper limit is $c = 89.786$.

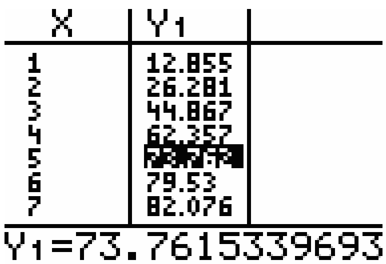
12. a.



[0, 5] by [–10, 80]

b. 

The model indicates that approximately 12.85% of 16-year old girls have been sexually active.

c. 

The model indicates that 73.76% of 20-year old girls have been sexually active.

d. The upper limit is $c = 83.84$.

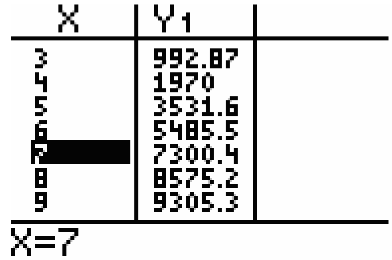
13. a. Let $t = 1$ and solve for N.

$$\begin{aligned} N &= \frac{10,000}{1+100e^{-0.8(1)}} \\ &= \frac{10,000}{1+100e^{-0.8}} \\ &= \frac{10,000}{45.393289641} \\ &\approx 218 \end{aligned}$$

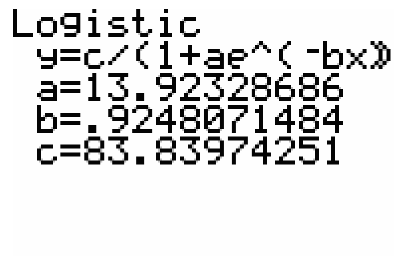
Approximately 218 people have heard the rumor after the first day.

b. Let $t = 4$ and solve for N.

$$\begin{aligned} N &= \frac{10,000}{1+100e^{-0.8(4)}} \\ &= \frac{10,000}{1+100e^{-3.2}} \\ &= \frac{10,000}{5.076220398} \\ &\approx 1970 \end{aligned}$$

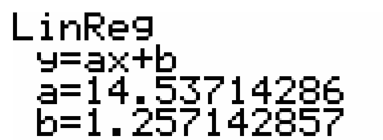
c. 

After seven days, 7300 people have heard the rumor.

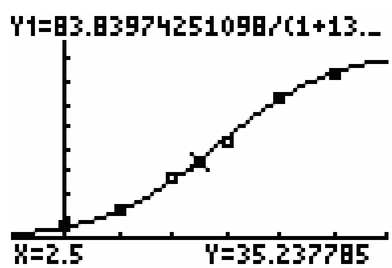
14. a. 

$$y = \frac{83.84}{1+13.9233e^{-0.9248x}}$$

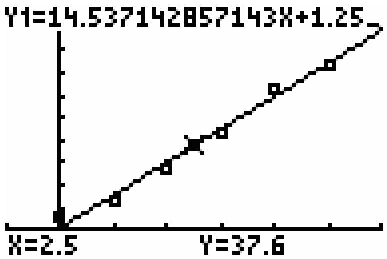
b. Yes, the models are the same.

c. 

$y = 14.537x + 1.257$

d. 

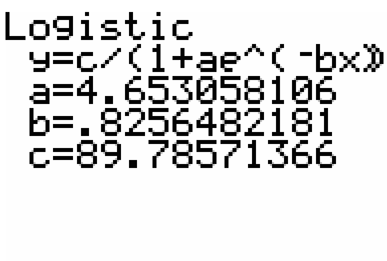
[–1, 6] by [–10, 100]



[–1, 6] by [–10, 100]

The logistic function is a better model of the data.

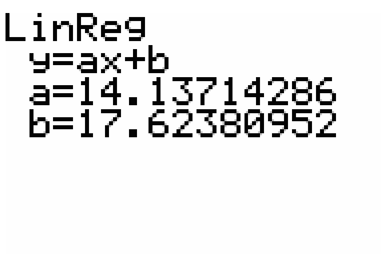
15. a.



$$y = \frac{89.786}{1 + 4.653e^{-0.826x}}$$

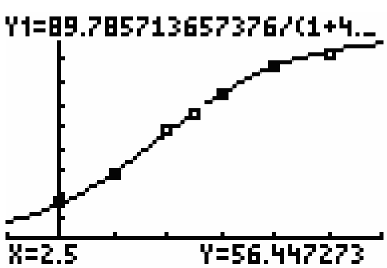
b. Yes.

c.

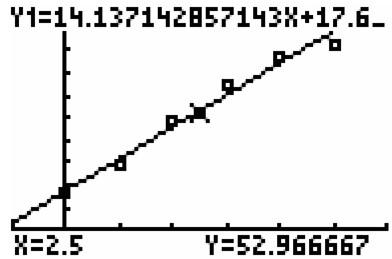


$y = 14.137x + 17.624$

d.



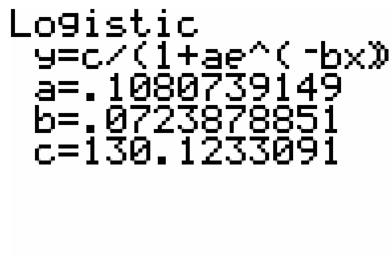
[–1, 6] by [–10, 100]



[–1, 6] by [–10, 100]

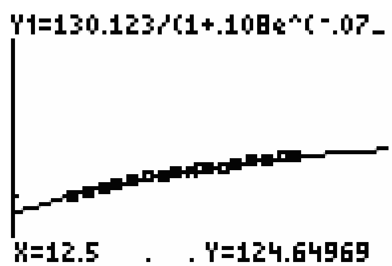
The logistic model is a much better fit.

16. a.



$$y = \frac{130.123}{1 + 0.108e^{-0.072x}}$$

b.



[0, 25] by [100, 150]

The model fits the data very well.

17. a. Let $t = 0$ and solve for N.

$$\begin{aligned} N &= 10{,}000(0.4)^{0.2^0} \\ &= 10{,}000(0.4)^1 \\ &= 10{,}000(0.4) \\ &= 4000 \end{aligned}$$

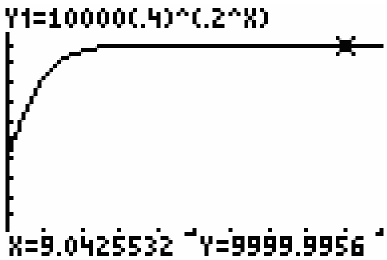
The initial population size is 4000 students.

b. Let $t = 4$ and solve for N.

$$\begin{aligned} N &= 10{,}000(0.4)^{0.2^4} \\ &= 10{,}000(0.4)^{0.0016} \\ &= 10{,}000(0.998535009) \\ &= 9985.35009 \\ &\approx 9985 \end{aligned}$$

After four years, the population is approximately 9985 students.

c.



[0, 10] by [0, 12,000]

The upper limit appears to be 10,000.

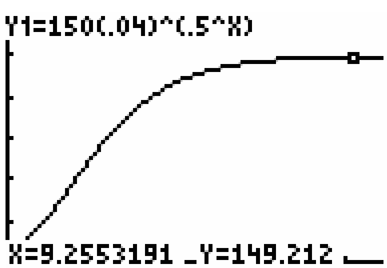
18. a.

X	Y1
4	122.66
5	135.65
6	142.64
7	146.27
8	148.13
9	149.06
10	149.53

X=8

In eight years the number of employees is approximately 148.

b.



[0, 10] by [0, 180]

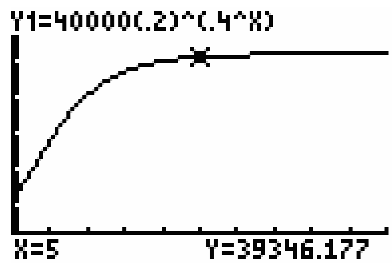
As the time increases, the number of employees approaches 150.

19. a. Let $t = 1$ and solve for N.

$$\begin{aligned} N &= 40{,}000(0.2)^{0.4^1} \\ &= 40{,}000(0.2)^{0.4} \\ &= 40{,}000(0.5253055609) \\ &= 21{,}012.22244 \\ &\approx 21{,}012 \end{aligned}$$

After one month, the approximately 21,012 units will be sold.

b.



[0, 10] by [0, 50,000]

c. The upper limit appears to be 40,000.

20. a. Let $t = 0$ and solve for N.

$$\begin{aligned} N &= 1600(0.6)^{0.2^0} \\ &= 1600(0.6)^1 \\ &= 1600(0.6) \\ &= 960 \end{aligned}$$

Initially the company has 960 employees.

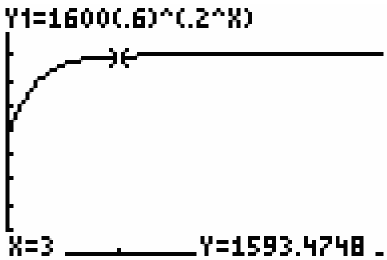
b. Let $t = 3$ and solve for N.

$$\begin{aligned} N &= 1600(0.6)^{0.2^3} \\ &= 1600(0.6)^{0.008} \\ &= 1600(0.9959217338) \\ &\approx 1593.47 \end{aligned}$$

After three years, the company had approximately 1593 employees.

c. The upper limit is 1600 employees.

d.



[0, 10] by [0, 2000]

21. a. Let $t = 0$ and solve for N.

$$\begin{aligned} N &= 1000(0.01)^{0.5^0} \\ &= 1000(0.01)^1 \\ &= 1000(0.01) \\ &= 10 \end{aligned}$$

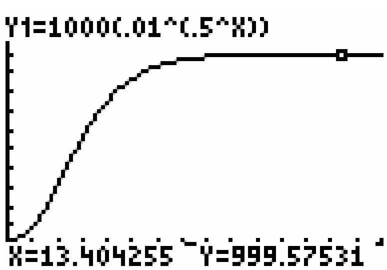
Initially the company had 10 employees.

b. Let $t = 1$ and solve for N.

$$\begin{aligned} N &= 1000(0.01)^{0.5^1} \\ &= 1000(0.01)^{0.5} \\ &= 1000(0.1) \\ &= 100 \end{aligned}$$

After one year, the company had 100 employees.

c. The upper limit is 1000 employees.



[0, 15] by [0, 1200]

d.

X	Y1
3	562.34
4	749.89
5	865.96
6	930.57
7	964.66
8	982.17
9	991.05

X=6

In the sixth year 930 people were employed by the company.

22. a. Let $t = 0$ and solve for N.

$$\begin{aligned} N &= 8000(0.1)^{0.3^0} \\ &= 8000(0.1)^1 \\ &= 8000(0.1) \\ &= 800 \end{aligned}$$

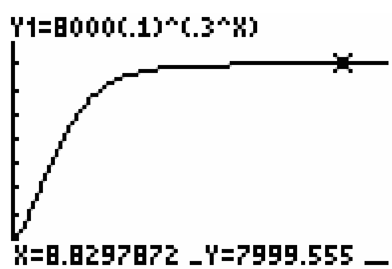
Initially the company sold 800 units.

b. Let $t = 1$ and solve for N

$$\begin{aligned} N &= 8,000(0.1)^{0.3^1} \\ &= 8,000(0.1)^{0.3} \\ &= 8,000(0.5011872336) \approx 4009.50 \end{aligned}$$

After three weeks, the company sold approximately 4009 units.

c. The upper limit is 8000 units.



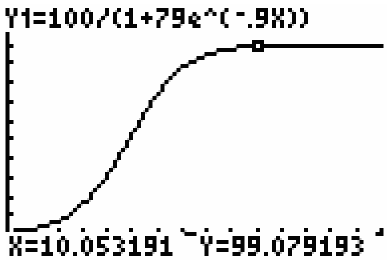
[0, 10] by [0, 10,000]

d.

X	Y1
0	800
1	4009.5
2	6502.6
3	7517.8
4	7852.2
5	7955.4
6	7986.6

X=2

In the second week, approximately 6500 units were sold by the company.

23. 

[0, 15] by [0, 120]

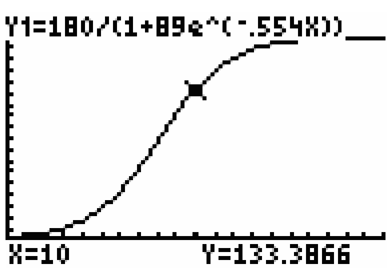
After 10 days, 99 people are infected.

24.

X	Y1
6.75	5105.3
7	5601
7.25	6111.7
7.5	6632.2
7.75	7156.9
8	7679.8
8.25	8195.1

X=7.75

In about eight weeks, half the community has been reached by the advertisement.

25. 

[0, 20] by [–20, 200]

X	Y1
7	63.345
8	87.452
9	111.93
10	133.39
11	149.9
12	161.38
13	168.81

X=11

In approximately 11 years, the deer population reaches a level of 150.

26.

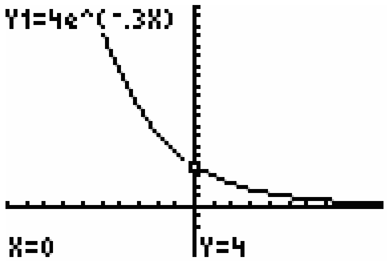
X	Y1
6.75	281.95
7	324.25
7.25	368.39
7.5	413.32
7.75	457.92
8	501.09
8.25	541.88

X=8

Five hundred students in the elementary school will be infected in approximately 8 days.

Chapter 3 Skills Check

1. a.



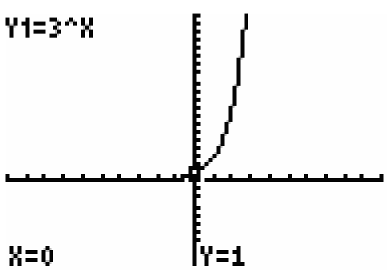
[–10, 10] by [–5, 20]

b. $f(-10) = 4e^{-0.3(-10)}$
$= 4e^{3}$
≈ 80.342

$f(10) = 4e^{-0.3(10)}$
$= 4e^{-3}$
≈ 0.19915

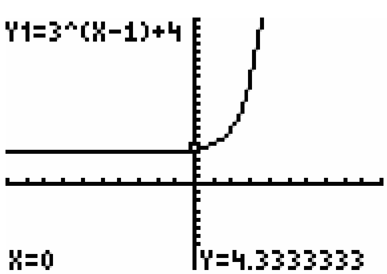
2. The function in Exercise 1, $f(x) = 4e^{-0.3x}$, is decreasing.

3.



[–10, 10] by [–10, 20]

4.



[–10, 10] by [–10, 20]

5. The graph in Exercise 4 is shift one unit right and three units up in comparison with the graph in Exercise 3.

6. The function is increasing. See the solution to Exercise 4.

7. a. $y = 1000(2)^{-0.1x}$
$= 1000(2)^{-0.1(10)}$
$= 1000(2)^{-1}$
$= 500$

b.

X	Y1
15	353.55
16	329.88
17	307.79
18	287.17
19	267.94
20	250
21	233.26

X=20

When $y = 250$, $x = 20$.

8. $x = 6^{y} \Leftrightarrow \log_6 x = y$

9. $y = 7^{3x} \Leftrightarrow \log_7 y = 3x$

10. $y = \log_4 x \Leftrightarrow x = 4^{y}$

11. $y = \log(x) = \log_{10} x$
$y = \log_{10} x \Leftrightarrow x = 10^{y}$

12. $y = \ln x = \log_e x$
$y = \log_e x \Leftrightarrow x = e^{y}$

13. $y = 4^x$

$x = 4^y$

$x = 4^y \Leftrightarrow \log_4 x = y$

Therefore, the inverse function is
$y = \log_4 x$.

14. $\log 22 = \log_{10} 22 = 1.3424$

15. $\ln 56 = \log_e 56 = 4.0254$

16. $\log 10 = \log_{10} 10 = 1$

17. $\log_2 16$

$y = \log_2 16 \Leftrightarrow 2^y = 16$

$y = 4$

18. $\ln\left(e^4\right) = \log_e\left(e^4\right)$

$y = \log_e\left(e^4\right) \Leftrightarrow e^y = e^4$

$y = 4$

19. $\log(0.001) = \log_{10}\left(\frac{1}{1000}\right)$

$y = \log_{10}\left(\frac{1}{1000}\right) \Leftrightarrow 10^y = \frac{1}{1000}$

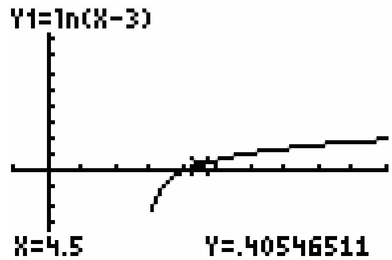
$10^y = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

$y = -3$

20. $\log_3(54) = \frac{\ln(54)}{\ln(3)} = 3.6039$

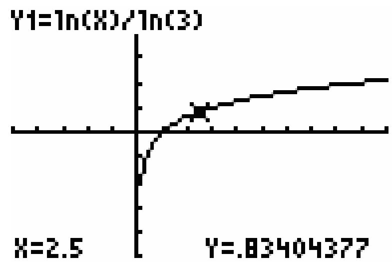
21. $\log_8(56) = \frac{\ln(56)}{\ln(8)} = 1.9358$

22.



[–1, 10] by [–5, 10]

23.



[–5, 10] by [–5, 5]

24. $340 = e^x$

$\ln(340) = \ln\left(e^x\right)$

$x = \ln(340)$

$x \approx 5.8289$

25. $1500 = 300e^{8x}$

$\frac{1500}{300} = \frac{300e^{8x}}{300}$

$5 = e^{8x}$

$\ln(5) = \ln\left(e^{8x}\right)$

$8x = \ln(5)$

$x = \frac{\ln(5)}{8}$

$x \approx 0.2012$

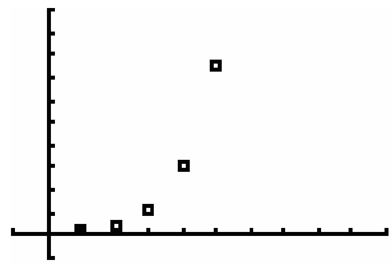
26.
$$\begin{aligned}9200 &= 23\left(2^{3x}\right)\\ \frac{9200}{23} &= \frac{23\left(2^{3x}\right)}{23}\\ 2^{3x} &= 400\\ \ln\left(2^{3x}\right) &= \ln(400)\\ 3x\ln(2) &= \ln(400)\\ x &= \frac{\ln(400)}{3\ln(2)}\\ x &\approx 2.8813\end{aligned}$$

27.
$$\begin{aligned}4\left(3^{x}\right) &= 36\\ 3^{x} &= 9\\ \log\left(3^{x}\right) &= \log(9)\\ x\log(3) &= \log(9)\\ x &= \frac{\log(9)}{\log(3)}\\ x &= 2\end{aligned}$$

28. $\ln\left[\frac{(2x-5)^3}{x-3}\right]$
$= \ln(2x-5)^3 - \ln(x-3)$
$= 3\ln(2x-5) - \ln(x-3)$

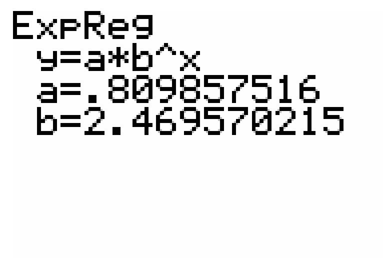
29. $6\log_4 x - 2\log_4 y$
$= \log_4 x^6 - \log_4 y^2$
$= \log_4\left(\frac{x^6}{y^2}\right)$

30.



[–1, 10] by [–10, 100]

The data is best modeled by an exponential function.

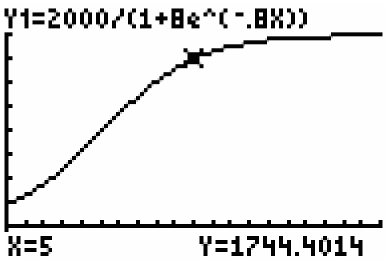


$y = 0.810(2.470)^x$

31. $P\left(1+\frac{r}{k}\right)^{kn}$
$= 1000\left(1+\frac{0.08}{12}\right)^{(12)(20)}$
$= 1000\left(1.00\overline{6}\right)^{240}$
≈ 4926.80

32. $1000\left[\frac{1-1.03^{-240+120}}{0.03}\right]$
$= 1000\left[\frac{1-1.03^{-120}}{0.03}\right]$
$= 1000\left[\frac{0.9711906782}{0.03}\right]$
$= 1000[32.37302261]$
$= 32,373.02$

33. a.



[0, 10] by [0, 2300]

b. $f(0) = \dfrac{2000}{1+8e^{-0.8(0)}}$

$= \dfrac{2000}{1+8e^{0}}$

$= \dfrac{2000}{9}$

≈ 222.22

$f(8) = \dfrac{2000}{1+8e^{-0.8(8)}}$

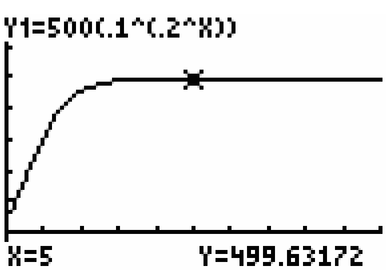
$= \dfrac{2000}{1+8e^{-6.4}}$

$= \dfrac{2000}{1.013292458}$

≈ 1973.76

c. The limiting value of the function is 2000.

34. a.



[0, 10] by [0, 700]

b. $y = 500(0.1)^{0.2^0}$

$= 500(0.1)^1$

$= 500(0.1)$

$= 50$

c. The limiting value is 500.

Chapter 3 Review Exercises

35. Let $x = 30$.

$$\begin{aligned} y &= 84.7518(1.0746)^{30} \\ &= 733.75997 \\ &\approx 733.760 \end{aligned}$$

In 1990, the total number of prisoners was approximately 733,760.

36. Let $x = 4$.

$$\begin{aligned} y &= 2000(2)^{-0.1(4)} \\ &= 2000(2)^{-0.4} \\ &= 2000(0.7578582833) \\ &\approx 1515.72 \end{aligned}$$

Four weeks after the end of the advertising campaign, the daily sales in dollars will be \$1515.72.

37.

X	Y1
0	1.337
1	2.7413
2	5.6205
3	11.524
4	23.628
5	48.445
6	99.328

X=3

Annual revenue exceeded \$10 million during 1988.

38. a. $R = \log\left(\dfrac{I}{I_0}\right)$

$R = \log\left(\dfrac{1000I_0}{I_0}\right)$

$R = \log(1000) = 3$

The earthquake measures 3 on the Richter scale.

b. $10^R = \dfrac{I}{I_0}$

$I = 10^R I_0$

$I = 10^{6.5} I_0$

$I = 3{,}162{,}277.66 I_0$

39. The difference in the Richter scale measurements is $7.9 - 4.8 = 3.1$. Therefore the intensity of the Indian earthquake was $10^{3.1} \approx 1259$ times stronger than the intensity of the U.S. earthquake.

40. $t = \log_{1.12} 3 = \dfrac{\ln 3}{\ln 1.12} \approx 9.69$

The investment will triple in approximately 10 years.

41. a. $S = 1000(2)^{\left(\frac{x}{7}\right)}$

$\dfrac{S}{1000} = (2)^{\left(\frac{x}{7}\right)} \Leftrightarrow \log_2\left(\dfrac{S}{1000}\right) = \dfrac{x}{7}$

$x = 7\log_2\left(\dfrac{S}{1000}\right)$

b.

$$\begin{aligned} x &= 7\log_2\left(\frac{19{,}504}{1000}\right) \\ &= 7\log_2(19.504) \\ &= 7\left(\frac{\ln 19.504}{\ln 2}\right) \\ &\approx 29.99989 \end{aligned}$$

In about 30 years the future value will be \$19,504.

42.
$$1000 = 2000(2)^{-0.1x}$$
$$0.5 = 2^{-0.1x}$$
$$\ln(0.5) = \ln\left(2^{-0.1x}\right)$$
$$\ln(0.5) = -0.1x\ln 2$$
$$x = \frac{\ln 0.5}{-0.1\ln 2}$$
$$x = 10$$

In 10 days, sales will decay by half.

43.

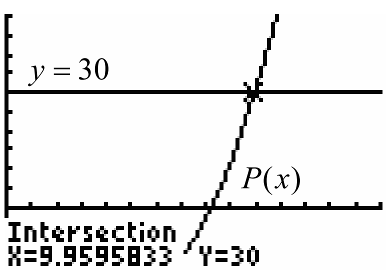
X	Y1
24	201.81
25	214.7
26	228.41
27	242.99
28	258.51
29	275.02
30	292.58

X=29

In 1989, the rate is 275 per 100,000.

44. a.
$$P(x) = R(x) - C(x)$$
$$P(x) = 10(1.26)^x - (2x+50)$$
$$= 10(1.26)^x - 2x - 50$$

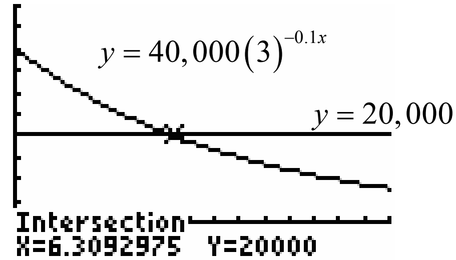
b. Applying the intersection of graphs method:



[0, 15] by [–15, 50]

Selling at least 10 mobile homes produces a profit of at least $30,000.

45. Applying the intersection of graphs method:



[0, 15] by [–7500, 50,000]

After seven weeks, sales will be less than half.

46. a.
$$y = 100e^{-0.00012378(5000)}$$
$$= 100e^{-0.6189}$$
$$= 100(0.5385365021)$$
$$\approx 53.85$$

After 5000 years, approximately 53.85 grams of carbon-14 remains.

b.
$$36\% y_0 = y_0 e^{-0.00012378t}$$
$$0.36 = e^{-0.00012378t}$$
$$\ln(0.36) = \ln\left(e^{-0.00012378t}\right)$$
$$\ln(0.36) = -0.00012378t$$
$$-0.00012378t = \ln(0.36)$$
$$t = \frac{\ln(0.36)}{-0.00012378}$$
$$t \approx 8253.77$$

The wood was cut approximately 8254 years ago.

47.

X	Y1
13	31323
14	29795
15	28342
16	26960
17	25645
18	24394
19	23204

X=14

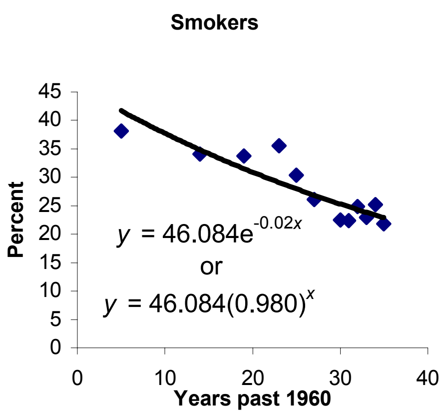
After approximately 14 years, the purchasing power will be less than half of the original $60,000 income.

48. $S = 2000e^{0.08(10)} = 2000e^{0.8} \approx 4451.08$
The future value is approximately $4451.08 after 10 years.

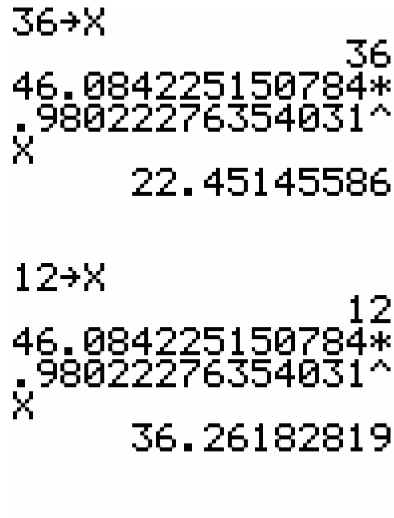
49.
$$13{,}784.92 = 3300(1.10)^x$$
$$(1.10)^x = 4.177248485$$
$$\ln\left[(1.10)^x\right] = \ln[4.177248485]$$
$$x\ln(1.10) = \ln(4.177248485)$$
$$x = \frac{\ln(4.177248485)}{\ln(1.10)}$$
$$x \approx 15$$

The investment reaches the indicated value in 15 years.

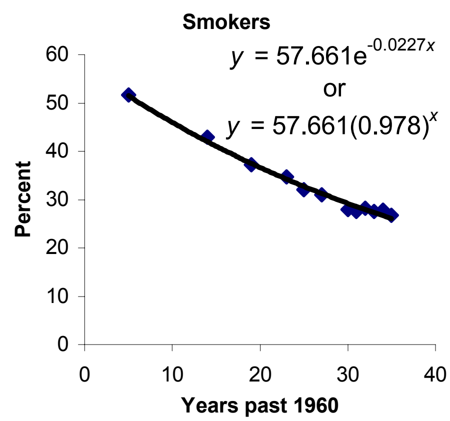
50. a.



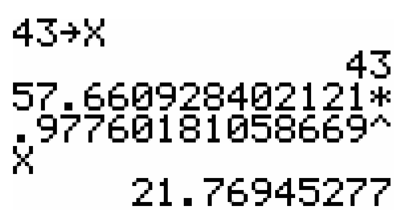
b. In 1972, the percentage of black female smokers is 36.3%. In 1996, the percentage of black female smokers is 22.5%.



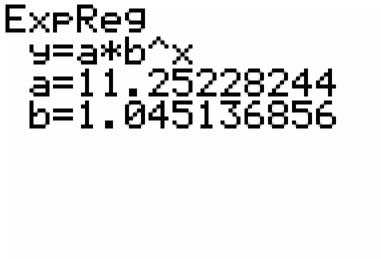
51. a.



b. In 2003 the percentage of U.S. smokers is 21.8%.

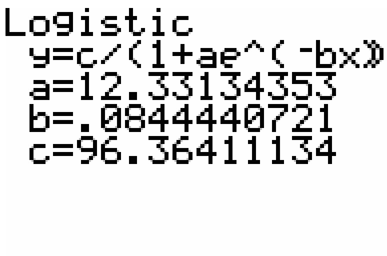


52. a.



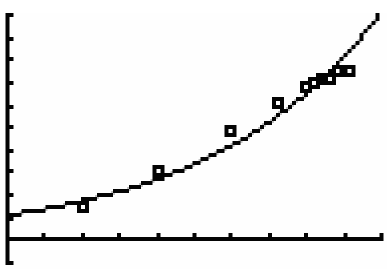
$y = 11.252(1.045)^x$

b.



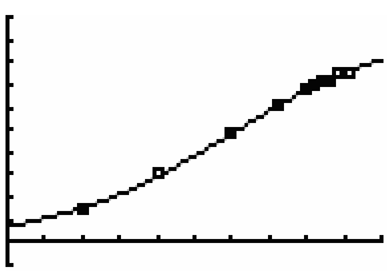
$$y = \frac{96.364}{1 + 12.331e^{-0.084x}}$$

c. exponential model



[0, 50] by [–10, 100]

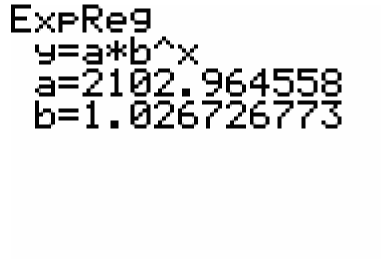
logistic model



[0, 50] by [–10, 100]

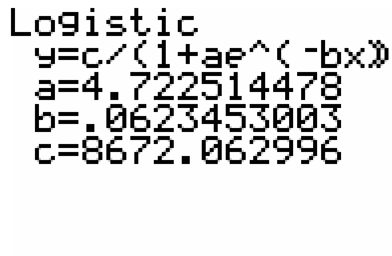
It appears the logistic model fits the data points better.

53. a.



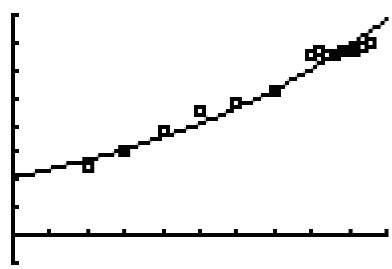
$y = 2102.96(1.0267)^x$

b.



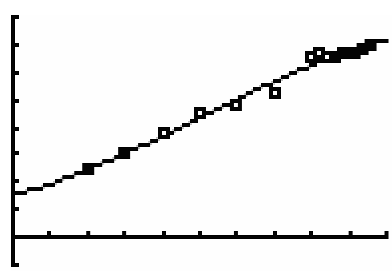
$$y = \frac{8672.06}{1 + 4.723e^{-0.0623x}}$$

c. exponential model



[0, 50] by [–1000, 8000]

logistic model



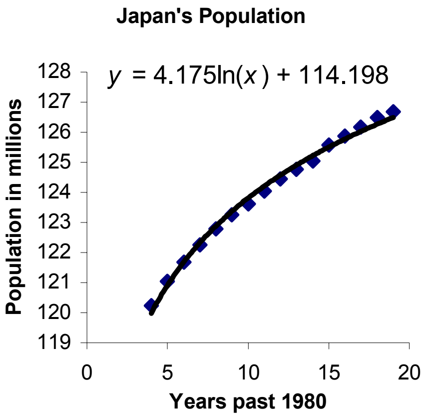
[0, 50] by [–1000, 8000]

The logistic model fits the data best.

54.
```
LnReg
 y=a+blnx
 a=27.94521723
 b=8.840292063
```

$y = 27.945 + 8.84 \ln x$

55. a.



b. $y = 114.198 + 4.175 \ln(2004 - 1980)$

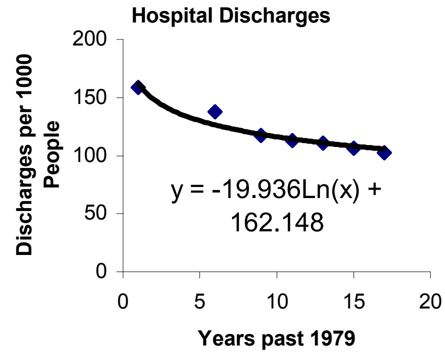
$= 114.198 + 4.175 \ln(24)$

≈ 127.4663747

In 2004 Japan's population is approximately 127,466,375. Using the unrounded model yields 127,466,485.

```
24→X
                   24
114.19830610294+
4.1749382567168l
n(X)
       127.4664846
```

56. a.



b. See part a) above.

c.
```
26→X
                   26
162.14845792466+
-19.936339398596
ln(X)
       97.19393955
```

In 2005 the number of discharges per 1000 people is approximately 97.

57. $S = Pe^{rt}$

$S = 12{,}500e^{(0.05)(10)}$

$= 12{,}500e^{0.5} \approx 20{,}609.02$

The future value is \$20,609.02.

58. $S = P\left(1 + \frac{r}{k}\right)^{kt}$

$S = 20{,}000\left(1 + \frac{0.06}{1}\right)^{(1)(7)}$

$S = 20{,}000(1.06)^7 \approx 30{,}072.61$

The future value is \$30,072.61.

59. $S = R\left[\dfrac{(1+i)^n - 1}{i}\right]$

$$S = 1000\left[\dfrac{\left(1+\dfrac{0.12}{4}\right)^{(4)(6)} - 1}{\dfrac{0.12}{4}}\right]$$

$$S = 1000\left[\dfrac{(1.03)^{24} - 1}{0.03}\right]$$

$$S = 1000\left[\dfrac{(1.03)^{24} - 1}{0.03}\right]$$

$$S = 1000(34.42647022) \approx 34,426.47$$

The future value is $34,426.47.

60. $S = R\left[\dfrac{(1+i)^n - 1}{i}\right]$

$$S = 1500\left[\dfrac{\left(1+\dfrac{0.08}{12}\right)^{(12)(10)} - 1}{\dfrac{0.08}{12}}\right]$$

$$S = 1500\left[\dfrac{(1.00\overline{6})^{120} - 1}{0.00\overline{6}}\right]$$

$$S = 1500(182.9460352)$$

$$S \approx 274,419.05$$

The future value is $274,419.05.

61. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$A = 2000\left[\dfrac{1-\left(1+\dfrac{0.08}{12}\right)^{-(12)(15)}}{\dfrac{0.08}{12}}\right]$$

$$A = 2000\left[\dfrac{1-(1.00\overline{6})^{-180}}{0.00\overline{6}}\right]$$

$$A = 2000[104.6405922] \approx 209,281.18$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. The present value is $209,218.18.

62. $A = R\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$A = 500\left[\dfrac{1-\left(1+\dfrac{0.10}{2}\right)^{-(2)(12)}}{\dfrac{0.10}{2}}\right]$$

$$A = 500\left[\dfrac{1-(1.05)^{-24}}{0.05}\right]$$

$$A = 500[13.79864179]$$

$$A \approx 6899.32$$

The formula above calculates the present value of the annuity given the payment made at the end of each period. The present value is $6899.32.

63. $R = A\left[\dfrac{i}{1-(1+i)^{-n}}\right]$

$R = 2000\left[\dfrac{\frac{0.12}{12}}{1-\left(1+\frac{0.12}{12}\right)^{-36}}\right]$

$R = 2000\left[\dfrac{0.01}{1-(1.01)^{-36}}\right]$

$R = 2000[0.0332143098]$

$R \approx 66.43$

The monthly payment is \$66.43.

64. $R = A\left[\dfrac{i}{1-(1+i)^{-n}}\right]$

$R = 120,000\left[\dfrac{\frac{0.06}{12}}{1-\left(1+\frac{0.06}{12}\right)^{-(12)(25)}}\right]$

$R = 120,000\left[\dfrac{0.005}{1-(1.005)^{-300}}\right]$

$R = 120,000[0.006443014]$

$R \approx 773.16$

The monthly payment is \$773.16.

65. a. In 1990, $x = 1990 - 1950 = 40$.

$y = \dfrac{96.3641}{1+12.3313e^{-0.0844(40)}}$

$= \dfrac{96.3641}{1+12.3313e^{-3.376}}$

$= \dfrac{96.3641}{1+0.4215321394}$

≈ 67.79

Based on the model the percentage of out-of-wedlock births in 1990 is 67.79%.

In 1996, $x = 1996 - 1950 = 46$.

$y = \dfrac{96.3641}{1+12.3313e^{-0.0844(46)}}$

$= \dfrac{96.3641}{1+12.3313e^{-3.8824}}$

$= \dfrac{96.3641}{1+0.2540410897}$

≈ 76.84

Based on the model the percentage of out-of-wedlock births in 1996 is 76.84%.

b. The upper limit on the percentage of out-of-wedlock births is 96.3641%.

66. a. Let $x = 14$.

$y = \dfrac{1400}{1+200e^{-0.5(14)}}$

$= \dfrac{1400}{1+200e^{-7}}$

$= \dfrac{1400}{1+0.1823763931}$

≈ 1184.06

After 14 days approximately 1184 students are infected.

b.

X	Y1	
13	1076.4	
14	1184.1	
15	1260.6	
16	1312	
17	1345.3	
18	1366.3	
19	1379.4	

X=16

After 16 days 1312 students are infected.

67. a. $N = 4000(0.06)^{0.4^{(2-1)}}$

$= 4000(0.06)^{0.4^1}$

$= 4000(0.06)^{0.4}$

≈ 1298.13

After two years, the enrollment will be approximately 1298 students.

b. $N = 4000(0.06)^{0.4^{(10-1)}}$

$= 4000(0.06)^{0.4^9}$

$= 4000(0.06)^{0.000262144}$

≈ 3997.05

After ten years the enrollment will be approximately 3997 students.

c. The upper limit on the number of students based on the model is 4000.

68. a. $N = 18,000(0.03)^{0.4^{10}}$

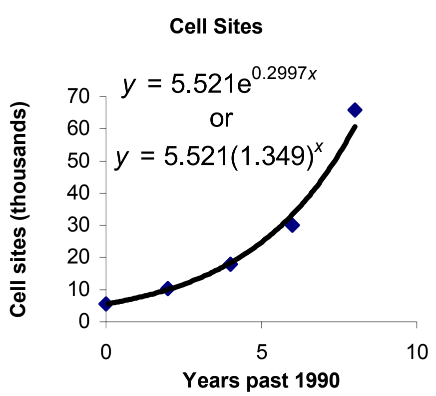
$= 18,000(0.03)^{0.0001048576}$

$\approx 17,993.38$

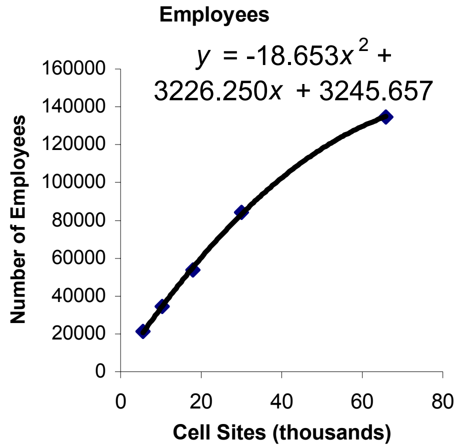
After ten months the number of units sold in a month will be approximately 17,993.

b. The upper limit on the number of units sold per month is 18,000.

69. a.



b.



c. $C(t) = 5.521(1.349)^t$

$E(C) = -18.653C^2 + 3226.250C + 3245.657$

$E(C(t)) = -18.653\left(5.521(1.349)^t\right)^2 + 3226.250\left(5.521(1.349)^t\right) + 3245.657$

$E(C(t))$ calculates the number of employees given the number of years past 1990.

d.

$$\begin{aligned} E(C(7)) &= -18.653\left(5.521(1.349)^7\right)^2 + 3226.250\left(5.521(1.349)^7\right) + 3245.657 \\ &= -18.653(44.88501709)^2 + 3226.250(44.88501709) + 3245.657 \\ &= -18.653(2014.664759) + 3226.250(44.88501709) + 3245.657 \\ &= 110{,}476.4016 \end{aligned}$$

e. In 1997 the cellular telephone industry employed approximately 110,476 people.

Group Activity/Extended Applications

1. The first person on the list receives \$36. Each of the original six people on the list sends their letter to six people. Therefore, 36 people receive letters with the original six names, and each of the 36 forwards a dollar to the first person on the original list.

2. The 36 people receiving the first letter place their name on the bottom of the list, shift up the second person to first place. The 36 people send out six letters each, for a total of $36 \bullet 6 = 216$ letters. Therefore the second person on the original list receives \$216.

3.

Cycle Number	Money Sent to the Person on Top of the List
1	$6^2 = 36$
2	$6^3 = 216$
3	$6^4 = 1296$
4	$6^5 = 7776$
5	$6^6 = 46,656$

4. Position 5 generates the most money!

5.

```
QuadReg
 y=ax²+bx+c
 a=5914.285714
 b=⁻25405.71429
 c=22356
```

```
PwrReg
 y=a*x^b
 a=20.33965715
 b=4.338874682
```

```
ExpReg
 y=a*b^x
 a=6
 b=6
```

The exponential model, $y = 6(6)^x = 6^{x+1}$, fits the data exactly.

6. $y = 6^{6+1} = 6^7 = 279,936$
The sixth person on the original list receives \$279,936.

7. The total number of responses on the sixth cycle would be
$6 + 36 + 216 + 1296 + 7776 + 46,656 +$
$279,936 = 335,922$

8. $y = 6^{10+1} = 6^{11} = 362,797,056$
On the tenth cycle 362,797,056 people receive the chain letter and are suppose to respond with \$1.00 to the first name on the list.

9. The answer to problem 8 is larger than the U.S. population. There is no unsolicited person in the U.S. to whom to send the letter.

10. Chain letters are illegal since people entering lower on the chain have a very small chance of earning money from the scheme.