Chapter 4
Higher-Degree Polynomial and Rational Functions

Algebra Toolbox

1. **a.** The polynomial is 4^{th} degree.

b. The leading coefficient is 3.

2. **a.** The polynomial is 3^{rd} degree.

b. The leading coefficient is 5.

3. **a.** The polynomial is 5^{th} degree.

b. The leading coefficient is –14.

4. **a.** The polynomial is 6^{th} degree.

b. The leading coefficient is –8.

5. $4x^3 - 8x^2 - 140x$
$= 4x\left(x^2 - 2x - 35\right)$
$= 4x(x-7)(x+5)$

6. $4x^2 + 7x^3 - 2x^4$
$= -2x^4 + 7x^3 + 4x^2$
$= -1x^2\left(2x^2 - 7x - 4\right)$
$= -x^2(2x+1)(x-4)$

7. $x^4 - 13x^2 + 36$
$= \left(x^2 - 9\right)\left(x^2 - 4\right)$
$= (x+3)(x-3)(x+2)(x-2)$

8. $x^4 - 21x^2 + 80$
$= \left(x^2 - 16\right)\left(x^2 - 5\right)$
$= (x+4)(x-4)\left(x^2 - 5\right)$

9. $2x^4 - 8x^2 + 8$
$= 2\left(x^4 - 4x^2 + 4\right)$
$= 2\left(x^2 - 2\right)\left(x^2 - 2\right)$
$= 2\left(x^2 - 2\right)^2$

10. $3x^5 - 24x^3 + 48x$
$= 3x\left(x^4 - 8x^2 + 16\right)$
$= 3x\left(x^2 - 4\right)\left(x^2 - 4\right)$
$= 3x(x+2)(x-2)(x+2)(x-2)$
$= 3x(x+2)^2(x-2)^2$

11. $\dfrac{x-3y}{3x-9y} = \dfrac{x-3y}{3(x-3y)} = \dfrac{1}{3}$

12. $\dfrac{x^2-9}{4x+12} = \dfrac{(x+3)(x-3)}{4(x+3)} = \dfrac{x-3}{4}$

13. $\dfrac{2y^3-2y}{y^2-y}$
$= \dfrac{2y\left(y^2-1\right)}{y(y-1)}$
$= \dfrac{2y(y+1)(y-1)}{y(y-1)}$
$= 2(y+1)$

14. $\dfrac{4x^3-3x}{x^2-x}$

$=\dfrac{x(4x^2-3)}{x(x-1)}$

$=\dfrac{4x^2-3}{x-1}$

15. $\dfrac{x^2-6x+8}{x^2-16}$

$=\dfrac{(x-4)(x-2)}{(x+4)(x-4)}$

$=\dfrac{x-2}{x+4}$

16. $\dfrac{3x^2-7x-6}{x^2-4x+3}$

$=\dfrac{(3x+2)(x-3)}{(x-3)(x-1)}$

$=\dfrac{3x+2}{x-1}$

17. $\dfrac{x-3}{x^3}\bullet\dfrac{x(x-4)}{(x-4)(x-3)}$

$=\dfrac{x}{x^3}$

$=\dfrac{1}{x^2}$

18. $(x+2)(x-2)\left(\dfrac{2x-3}{x+2}\right)$

$=(x-2)(2x-3)$

$=3x^2-7x+6$

19. $\dfrac{4x+4}{x-4}\div\dfrac{8x^2+8x}{x^2-6x+8}$

$=\dfrac{4x+4}{x-4}\bullet\dfrac{x^2-6x+8}{8x^2+8x}$

$=\dfrac{4(x+1)}{x-4}\bullet\dfrac{(x-2)(x-4)}{8x(x+1)}$

$=\dfrac{x-2}{2x}$

20. $\dfrac{6x^2}{4x^2y-12xy}\div\dfrac{3x^2+12x}{x^2+x-12}$

$=\dfrac{6x^2}{4x^2y-12xy}\bullet\dfrac{x^2+x-12}{3x^2+12x}$

$=\dfrac{6x^2}{4xy(x-3)}\bullet\dfrac{(x+4)(x-3)}{3x(x+4)}$

$=\dfrac{1}{2y}$

21. $3+\dfrac{1}{x^2}-\dfrac{2}{x^3}$ LCM: x^3

$=\dfrac{3x^3}{x^3}+\dfrac{x}{x^3}-\dfrac{2}{x^3}$

$=\dfrac{3x^3+x-2}{x^3}$

22. $\dfrac{5}{x}-\dfrac{x-2}{x^2}+\dfrac{4}{x^3}$ LCM: x^3

$=\dfrac{5x^2}{x^3}-\dfrac{x(x-2)}{x^3}+\dfrac{4}{x^3}$

$=\dfrac{5x^2-(x^2-2x)+4}{x^3}$

$=\dfrac{5x^2-x^2+2x+4}{x^3}$

$=\dfrac{4x^2+2x+4}{x^3}$

23. $\frac{a}{a^2-2a}-\frac{a-2}{a^2}=\frac{a}{a(a-2)}-\frac{a-2}{a^2}$ LCM: $a^2(a-2)$

$$=\frac{a(a)}{a(a)(a-2)}-\frac{(a-2)(a-2)}{a^2(a-2)}$$

$$=\frac{a^2}{a^2(a-2)}-\frac{a^2-4a+4}{a^2(a-2)}$$

$$=\frac{a^2-(a^2-4a+4)}{a^2(a-2)}$$

$$=\frac{a^2-a^2+4a-4}{a^2(a-2)}$$

$$=\frac{4a-4}{a^2(a-2)}$$

$$=\frac{4(a-1)}{a^2(a-2)}$$

$$=\frac{4a-4}{a^3-2a^2}$$

24. $\frac{5x}{x^4-16}+\frac{8x}{x+2}=\frac{5x}{(x^2+4)(x^2-4)}+\frac{8x}{x+2}$

$$=\frac{5x}{(x^2+4)(x+2)(x-2)}+\frac{8x}{x+2}$$ LCM: $(x^2+4)(x+2)(x-2)$

$$=\frac{5x}{(x^2+4)(x+2)(x-2)}+\frac{8x(x^2+4)(x-2)}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{5x+8x(x^3-2x^2+4x-8)}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{5x+8x^4-16x^3+32x^2-64x}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{8x^4-16x^3+32x^2-59x}{(x^2+4)(x+2)(x-2)}$$

$$=\frac{8x^4-16x^3+32x^2-59x}{x^4-16}$$

25. $\dfrac{x-1}{x+1}-\dfrac{2}{x(x+1)}$

$\{\text{LCM: } x(x+1)\}$

$=\dfrac{x(x-1)}{x(x+1)}-\dfrac{2}{x(x+1)}$

$=\dfrac{x^2-x-2}{x(x+1)}$

$=\dfrac{(x-2)(x+1)}{x(x+1)}$

$=\dfrac{x-2}{x}$

26. $\dfrac{2x+1}{2(2x-1)}+\dfrac{5}{2x}-\dfrac{x+1}{x(2x-1)}$

$\{\text{LCM: } 2x(2x-1)\}$

$=\dfrac{x(2x+1)}{2x(2x-1)}+\dfrac{5(2x-1)}{2x(2x-1)}-\dfrac{2(x+1)}{2x(2x-1)}$

$=\dfrac{2x^2+x+(10x-5)-(2x+2)}{2x(2x-1)}$

$=\dfrac{2x^2+x+10x-5-2x-2}{2x(2x-1)}$

$=\dfrac{2x^2+9x-7}{2x(2x-1)}$

27.
$$
\begin{array}{r}
x^4-x^3+2x^2-2x+2 \\
x+1\overline{)\,x^5+0x^4+x^3+0x^2-0x-1} \\
\underline{-x^5-x^4} \\
-x^4+x^3 \\
\underline{+x^4+x^3} \\
2x^3+0x^2 \\
\underline{-2x^3-2x^2} \\
-2x^2-0x \\
\underline{2x^2+2x} \\
2x-1 \\
\underline{-2x-2} \\
-3
\end{array}
$$

$x^4-x^3+2x^2-2x+2\ R-3$

or

$x^4-x^3+2x^2-2x+2-\dfrac{3}{x+1}$

28.
$$
\begin{array}{r}
a^3+a^2 \\
a+2\overline{)\,a^4+3a^3+2a^2} \\
\underline{-a^4-2a^3} \\
a^3+2a^2 \\
\underline{-a^3-2a^2} \\
0
\end{array}
$$

a^3+a^2

29.

$$
\begin{array}{r}
3x^3 - x^2 + 6x - 2 \\
x^2 - 2\overline{)3x^5 - x^4 + 0x^3 + 0x^2 + 5x - 1} \\
\underline{-3x^5 \qquad + 6x^3} \qquad\qquad\qquad \\
-x^4 + 6x^3 + 0x^2 \qquad\quad \\
\underline{x^4 \qquad - 2x^2} \qquad\quad \\
6x^3 - 2x^2 + 5x \quad \\
\underline{-6x^3 \qquad + 12x} \quad \\
-2x^2 + 17x - 1 \\
\underline{2x^2 \qquad - 4} \\
17x - 5
\end{array}
$$

$\left(3x^3 - x^2 + 6x - 2\right) R \left(17x - 5\right)$

or

$3x^3 - x^2 + 6x - 2 + \dfrac{17x - 5}{x^2 - 2}$

30.

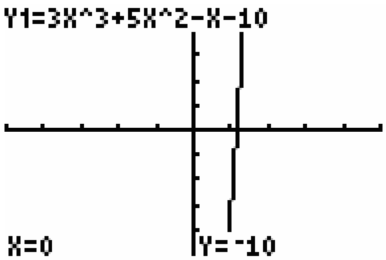
$$
\begin{array}{r}
x^2 + 1 \\
3x^2 - 1\overline{)3x^4 + 0x^3 + 2x^2 + 0x + 1} \\
\underline{-3x^4 \qquad + x^2} \qquad\qquad \\
3x^2 + 0x + 1 \\
\underline{-3x^2 \qquad + 1} \\
2
\end{array}
$$

$x^2 + 1 \; R \; 2$

or

$x^2 + 1 + \dfrac{2}{3x^2 - 1}$

Section 4.1 Skills Check

1. **a.** 

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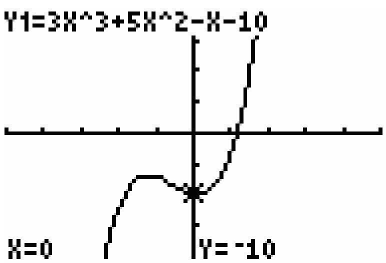
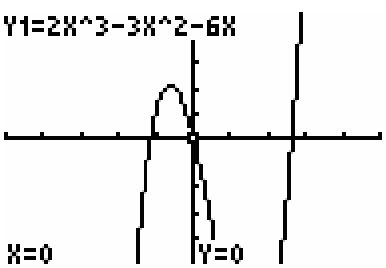
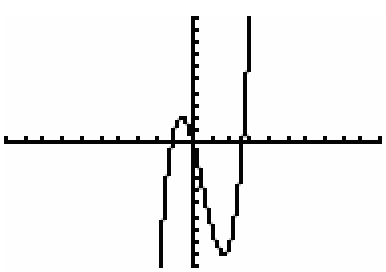
b. 

[–5, 5] by [–20, 20]

View b) is best.

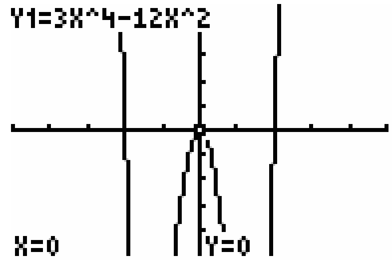
2. **a.** 

[–5, 5] by [–5, 5]

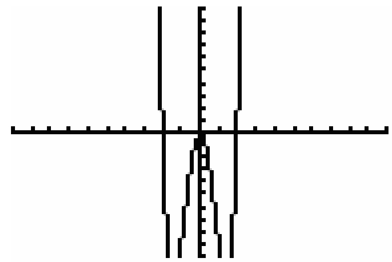
b. 

[–10, 10] by [–10, 10]

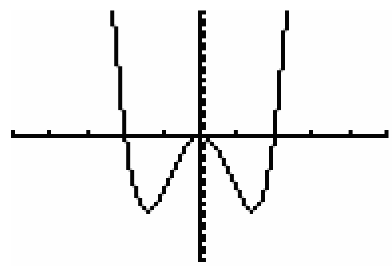
View b) is better.

3. **a.** 

[–5, 5] by [–5, 5]

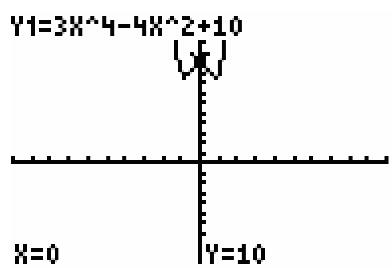
b. 

[–10, 10] by [–10, 10]

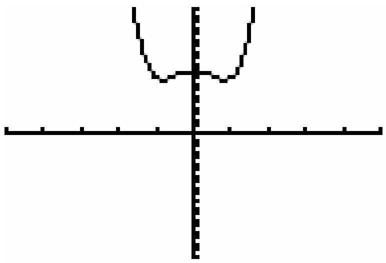
c. 

[–5, 5] by [–20, 20]

View c) is best.

4. **a.** 

[–10, 10] by [–10, 10]

b. 

[–5, 5] by [–20, 20]

View b) is best.

5. a. The x-intercepts appear to be –3, –1, and 2.

b. The leading coefficient is positive since the graph rises to the right.

c. The polynomial is at least 3rd degree since the curve has two turns and three x-intercepts.

6. a. The x-intercepts appear to be –1, 2, and 3.

b. The leading coefficient is negative since the graph falls to the right.

c. The polynomial is at least 4th degree since the curve has three turns and opens down in both directions.

7. a. The x-intercepts appear to be –1, 3, and 5.

b. The leading coefficient is negative since the graph falls to the right.

c. The polynomial is at least 3rd degree since the curve has two turns and three x-intercepts.

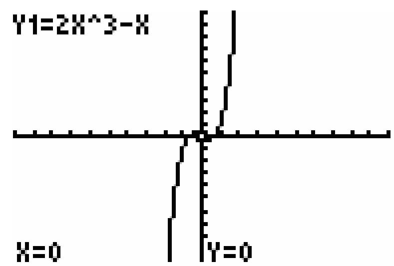
8. a. The x-intercepts appear to be –1, 0, 2, and 6.

b. The leading coefficient is positive since the graph rises to the right.

c. The polynomial is at least 4th degree since the curve has three turns and four x-intercepts.

9. a. The polynomial is 3rd degree, and the leading coefficient is 2.

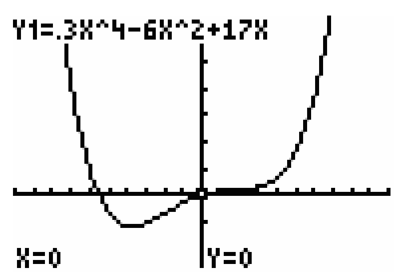
b. The graph rises right and falls left because the leading coefficient is positive and the function is cubic.

c. 

[–10, 10] by [–10, 10]

10. a. The polynomial is 4th degree, and the leading coefficient is 0.3.

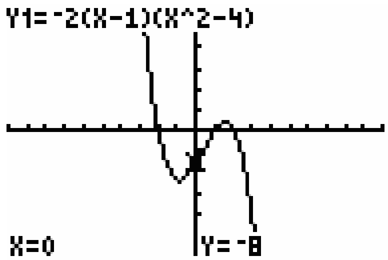
b. The graph rises right and rises left because the leading coefficient is positive and the function is quartic.

c. 

[–10, 10] by [–200, 500]

11. a. The polynomial is 3rd degree, and the leading coefficient is –2.

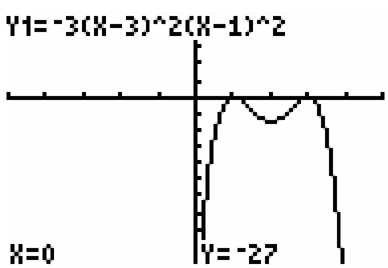
b. The graph falls right and rises left because the leading coefficient is negative and the function is cubic.

c. 

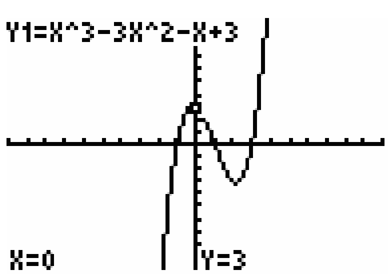
[–10, 10] by [–30, 30]

12. a. The polynomial is 4^{th} degree, and the leading coefficient is –3.

b. The graph falls right and falls left because the leading coefficient is negative and the function is quartic.

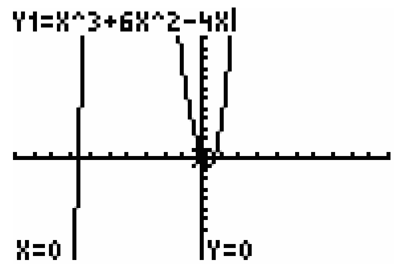
c. 

[–5, 5] by [–20, 10]

13. a. 

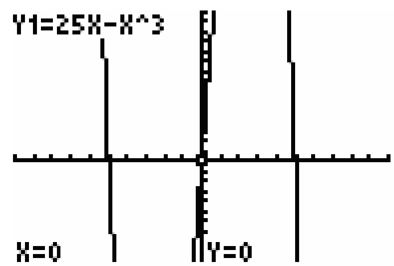
[–10, 10] by [–10, 10]

b. Yes, the graph is complete. As suggested by the degree of the cubic function, three *x*-intercepts, one *y*-intercept, and two turns are displayed on the graph.

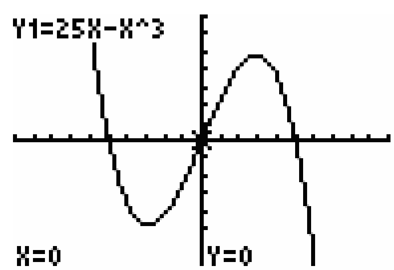
14. a. 

[–10, 10] by [–10, 10]

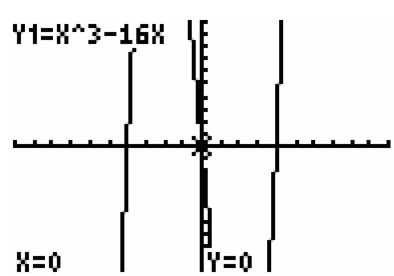
b. No. One turning point does not display properly in the given viewing window.

15. a. 

[–10, 10] by [–10, 10]

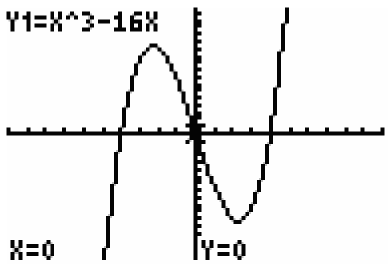
b. 

[–10, 10] by [–70, 70]

16. a. 

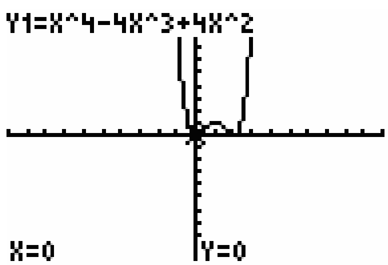
[–10, 10] by [–10, 10]

b.



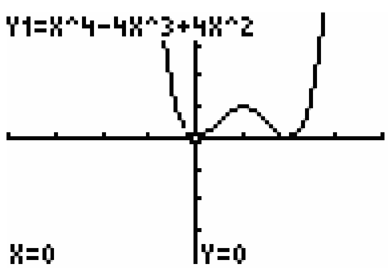
[–10, 10] by [–35, 35]

17. a.



[–10, 10] by [–10, 10]

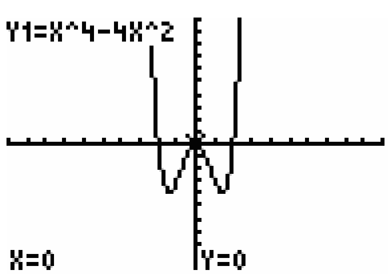
b.



[–4, 4] by [–4, 4]

c. The graph in part b) yields the best view of the turning points.

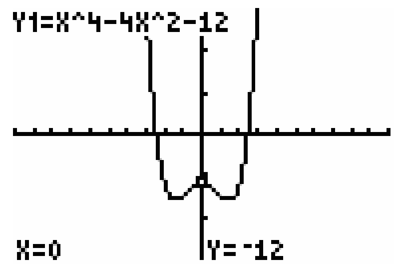
18. a.



[–10, 10] by [–10, 10]

b. Yes. The graph is complete.

19. a.

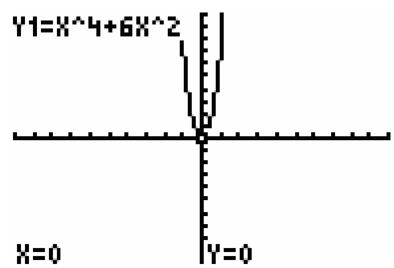


[–10, 10] by [–30, 30]

b. The graph has three turning points.

c. Since the polynomial is degree 4, it has at most three turning points.

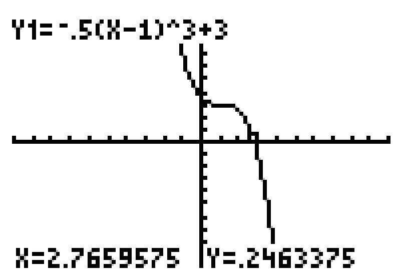
20. a.



[–10, 10] by [–10, 10]

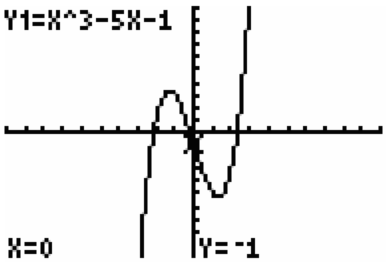
b. Since the polynomial is degree 4, it has at most three turning points. It could have fewer.

21.



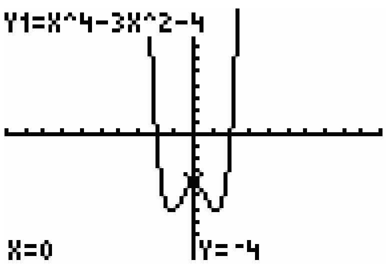
[–10, 10] by [–10, 10]

Answers will vary.

22. 

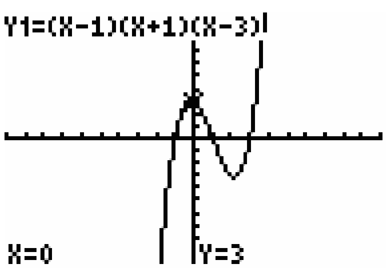
[–10, 10] by [–10, 10]

Answers will vary.

23. 

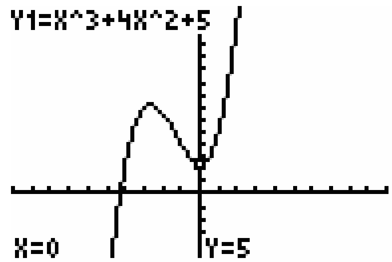
[–10, 10] by [–10, 10]

Answers will vary.

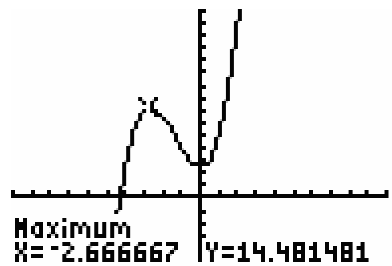
24. 

[–10, 10] by [–20, 20]

Answers will vary.

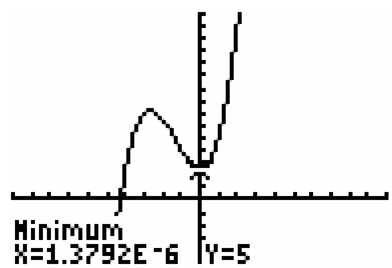
25. a. 

[–10, 10] by [–10, 30]

b. 

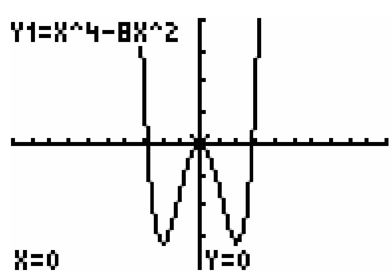
[–10, 10] by [–10, 30]

The local maximum is approximately $(-2.67, 14.48)$.

c. 

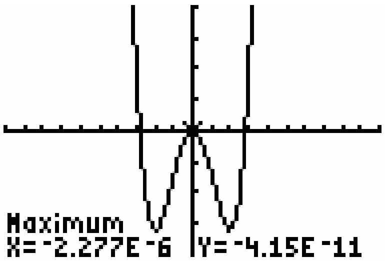
[–10, 10] by [–10, 30]

The local minimum is $(0,5)$.

26. a. 

[–10, 10] by [–20, 20]

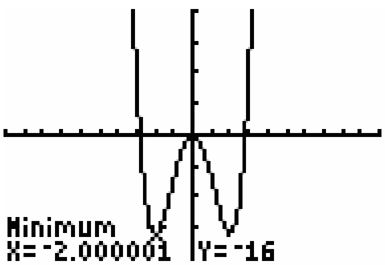
b.



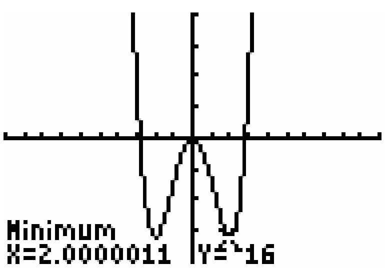
[–10, 10] by [–20, 20]

The local maximum is $(0,0)$.

c.



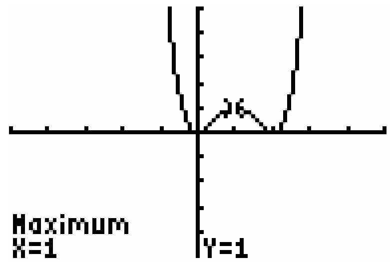
[–10, 10] by [–20, 20]



[–10, 10] by [–20, 20]

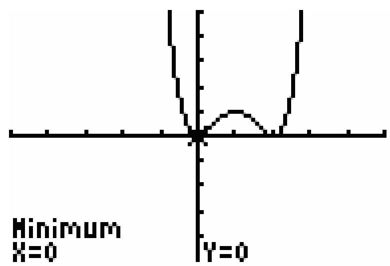
The local minimums are $(-2,-16)$ and $(2,-16)$.

27. $y = x^4 - 4x^3 + 4x^2$

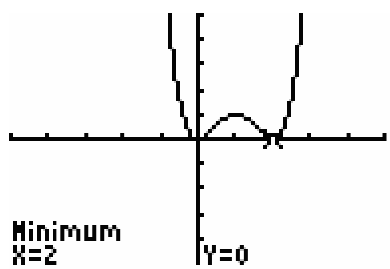


[–5, 5] by [–5, 5]

The local maximum is $(1,1)$.



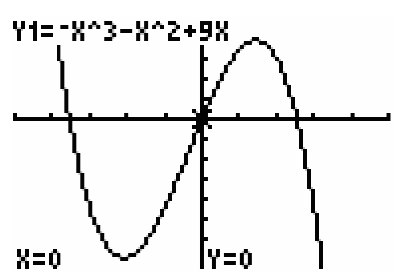
[–5, 5] by [–5, 5]



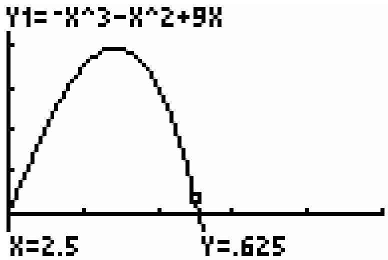
[–5, 5] by [–5, 5]

The local minimums are $(0,0)$ and $(2,0)$.

28. a.



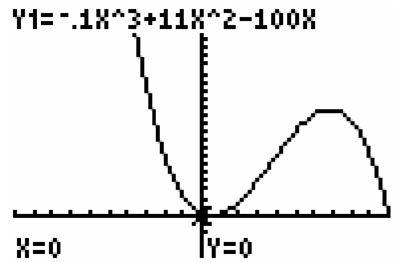
[–5, 5] by [–15, 10]

b. 

[0, 5] by [0, 10]

c. The graph in part b) resembles a 2nd degree or quadratic function.

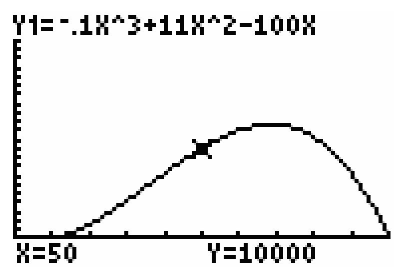
Section 4.1 Exercises

29. a. 

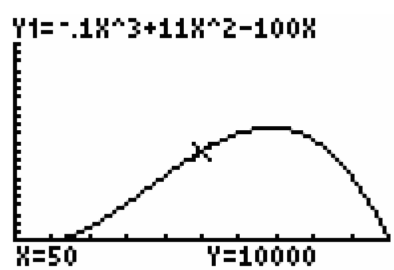
[–100, 100] by [–5000, 25,000]

There are two turning points.

b. Based on the physical context of the problem, both *x* and *y* need to be greater than or equal to zero.

c. 

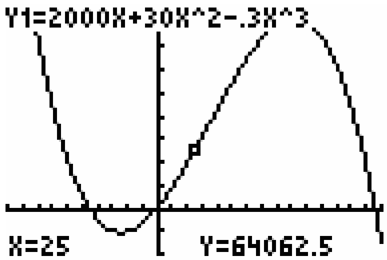
[0, 100] by [0, 25,000]

d. 

[0, 100] by [0, 25,000]

Fifty units produce revenue of $10,000.

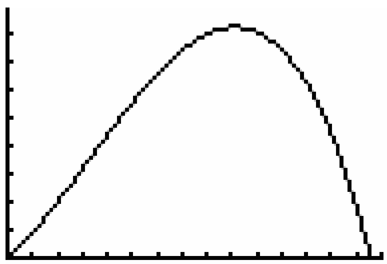
30. a.



[–100, 150] by [–30,000, 220,000]

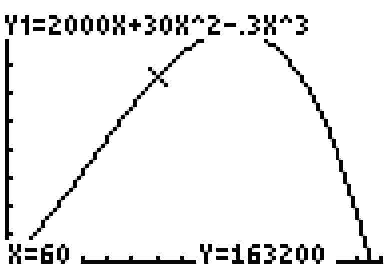
b. Based on the physical context of the problem, both *x* and *y* need to be greater than or equal to zero.

c.



[0, 150] by [0, 220,000]

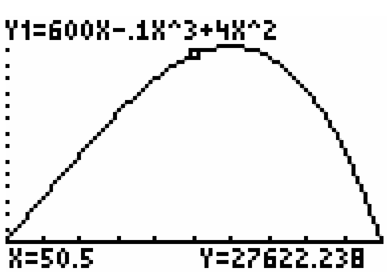
d.



[0, 150] by [0, 220,000]

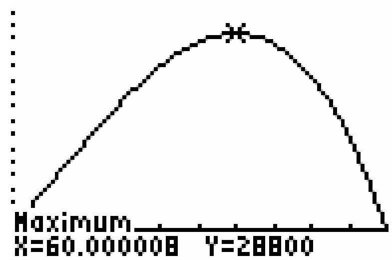
Sixty units produce revenue of $163,200.

31. a.



[0, 100] by [0, 30,000]

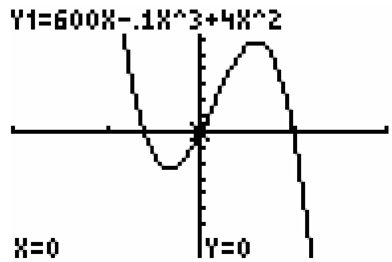
b.



[0, 100] by [0, 30,000]

Selling 60 units produces a maximum daily revenue of $28,800.

c.



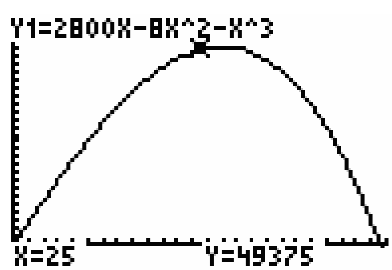
[–200, 200] by [–40,000, 40,000]

Answers will vary.

d. The graph in part a) best represents the physical situation. Both the number of units produced and the revenue must be greater than or equal to zero.

e. The graph is increasing on the interval $(0,60)$.

32. a.



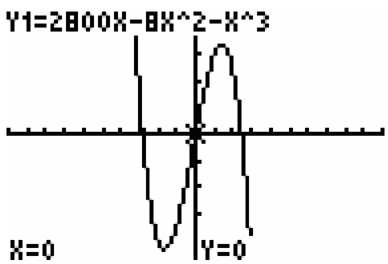
[0, 50] by [0, 51,000]

b.

[0, 50] by [0, 51,000]

Selling 28 units produces a maximum weekly revenue of $50,176.

c.



[–200, 200] by [–70,000, 70,000]

Answers will vary.

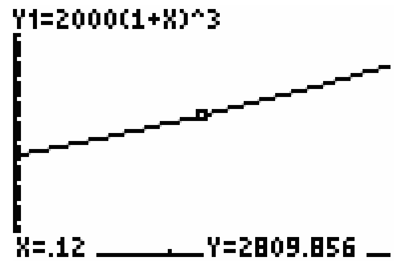
d. The graph in part a) best represents the physical situation. Both the number of units produced and the revenue must be greater than or equal to zero.

e. The graph is increasing on the interval $(0, 28)$.

33. a.

r, rate	S, future value
0	$2,000.00
5	$2,315.25
10	$2,662.00
15	$3,041.75
20	$3,456.00

b.



[0, 0.24] by [0, 5000]

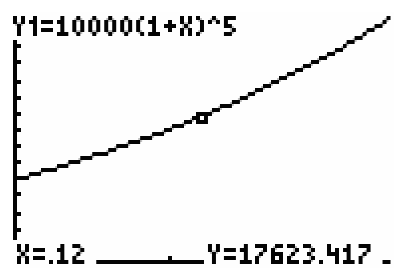
c. At the 20% rate the investment yields $3456. At the 10% rate the investment yields $2662. Therefore the 20% rate yields an extra $794.

d. The 10% rate is more realistic.

34. a.

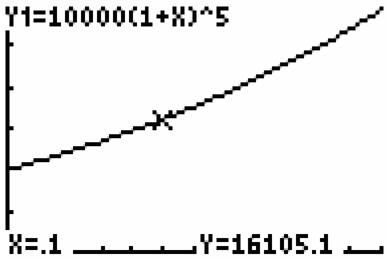
r, rate	S, future value
0	$10,000.00
5	$12,762.82
7	$14,025.52
12	$17,623.42
18	$22,877.58

b.

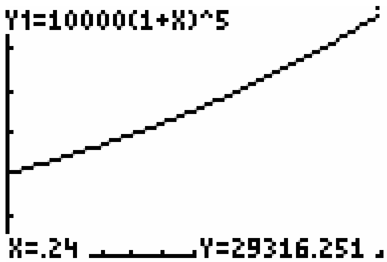


[0, 0.24] by [0, 30,000]

c. At the 24% rate the investment yields $29,316.25. At the 10% rate the investment yields $16,105.10. Therefore the 24% rate yields an extra $13,211.15.



[0, 0.24] by [0, 30,000]

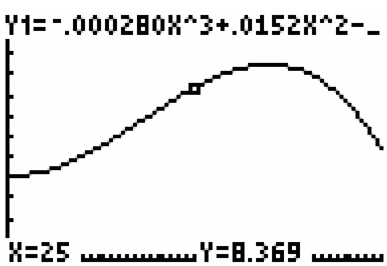


[0, 0.24] by [0, 30,000]

r, rate	*S*, future value
10	\$16,105.10
24	\$29,316.25

d. The 10% rate is more realistic.

35. a.



[0, 50] by [0, 12]

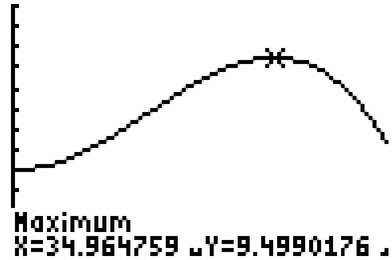
b.

X	Y1
35	9.499
36	9.4835
37	9.438
38	9.3606
39	9.2499
40	9.104
41	8.9213

X=40

In 1990, when $x = 40$, the homicide rate was approximately 9 per 100,000 people.

c.



[0, 50] by [0, 12]

An x-value of 35 corresponds with the year 1985. The number of homicides is at a maximum in 1985.

d. Consider the following table

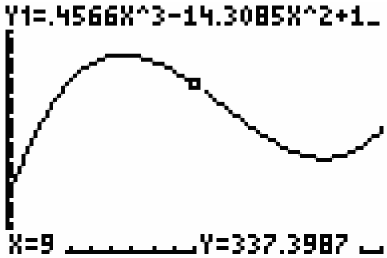
X	Y1
15	6.079
20	7.264
25	8.369
30	9.184
35	9.499
40	9.104
45	7.789

X=20

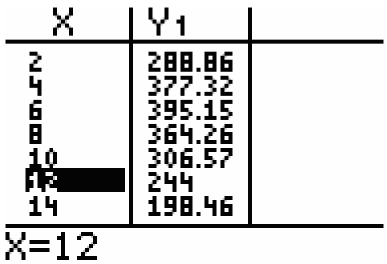
In 1970 the number of homicides per 100,000 people is approximately 7.264. In 1990 the number of homicides per 100,000 people is approximately 9.104. Calculating the average rate of change:

$$\frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{9.104 - 7.264}{40 - 20}$$
$$= \frac{1.84}{20}$$
$$= 0.092$$

The average rate of change for 1970-1990 is approximately 0.09 homicides per 100,000 people per year.

36. a. 

[0, 18] by [0, 500]

b. 

X	Y1
2	288.86
4	377.32
6	395.15
8	364.26
10	306.57
12	244
14	198.46

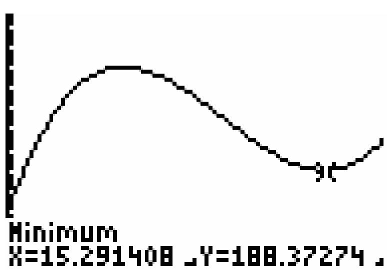
X=12

In 1992, when $x = 12$, there were 244 drunk driving crashes in South Carolina.

c. 

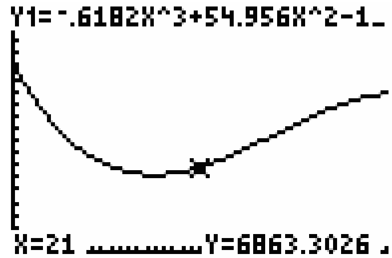
[0, 18] by [0, 500]

An x-value of 5.6 corresponds with the year 1986. The number of fatalities from drunk driving crashes was at a maximum in 1986.

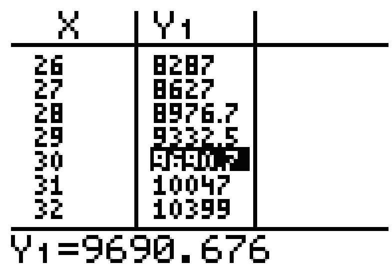
d. 

[0, 18] by [0, 500]

An x-value of 15.29 corresponds with the year 1995. The number of fatalities from drunk driving crashes was at a minimum in 1995.

37. a. 

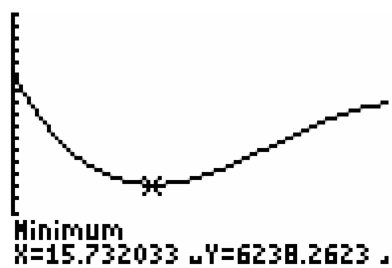
[0, 42] by [0, 20,000]

b. 

X	Y1
26	8287
27	8627
28	8976.7
29	9332.5
30	[illegible]
31	10047
32	10399

Y1=9690.676

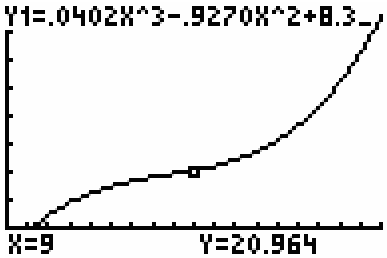
In 1980, when $x = 30$, the young adult population was approximately 9,690,676.

c. 

[0, 42] by [0, 20,000]

When x is 15.73, the year is 1966. The population reaches a minimum in 1966.

38. a.



[2, 18] by [0, 80]

b. The number of executions increased between 1982 and 1998.

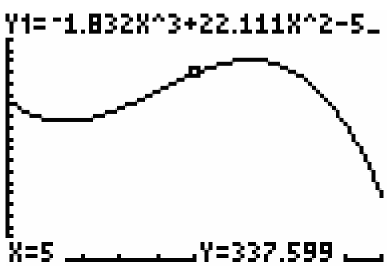
c.

X	Y_1
16	52.388
17	62.968
18	75.795
19	91.109
20	109.15
21	130.17
22	154.39

X=19

In 1999, when $x = 19$, the number of executions was approximately 91.

39. a.



[0, 10] by [0, 400]

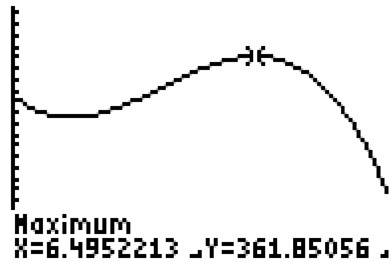
b.

X	Y_1
6	358.74
7	358.15
8	324.84
9	247.82
10	116.09
11	⁻81.34
12	⁻355.5

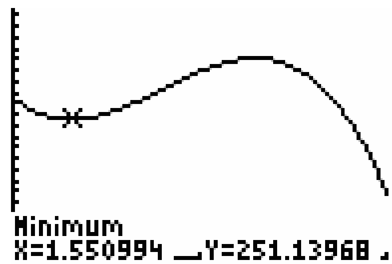
Y_1=324.843

In 1998, when $x = 8$, the national debt is approximately \$324.843 million.

c.



[0, 10] by [0, 400]



[0, 10] by [0, 400]

The minimum national debt of approximately \$251 million occurs in 1992, while the maximum national debt of approximately \$362 million occurs in 1997.

40.

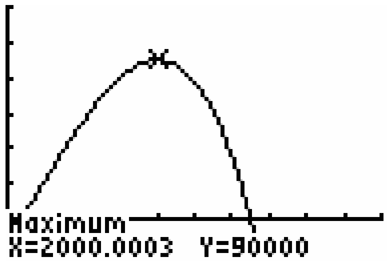
Plot1 Plot2 Plot3
\Y1=.0000591X^3-.00675X^2+.0523X+14.147
\Y2=
\Y3=
\Y4=
\Y5=

[0, 100] by [0, 20]

The minimum percentage occurs when $x = 72$ or the year 1972.

41. a. $P(x) = R(x) - C(x)$

$$= \left(120x - 0.015x^2\right) - \left(10{,}000 + 60x - 0.03x^2 + 0.00001x^3\right)$$
$$= -0.00001x^3 + 0.015x^2 + 60x - 10{,}000$$



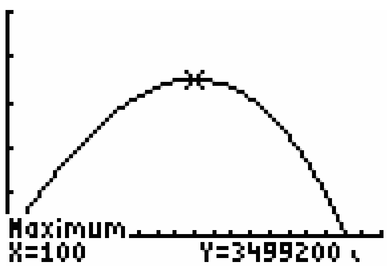
[0, 5000] by [–20,000, 120,000]

2000 units produced and sold produces a maximum profit.

b. The maximum profit is $90,000.

42. a. $P(x) = R(x) - C(x)$

$$= \left(60{,}000x - 50x^2\right) - \left(800 + 100x^2 + x^3\right)$$
$$= -x^3 - 150x^2 + 60{,}000x - 800$$



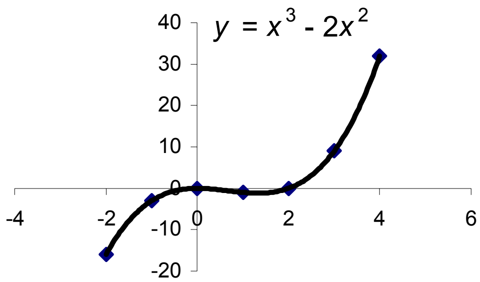
[0, 200] by [–500,000, 5,000,000]

100,000 units are produced and sold when $x = 100$. Therefore the maximum occurs when 100,000 units are produced and sold.

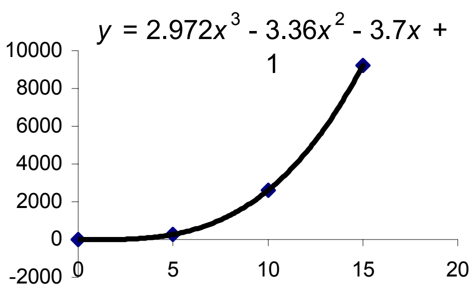
b. The maximum profit is $3,499,200.

Section 4.2 Skills Check

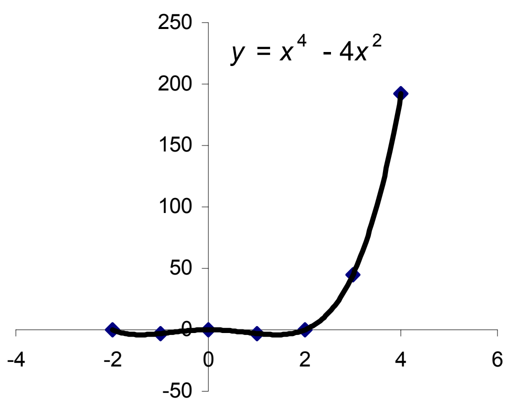
1.



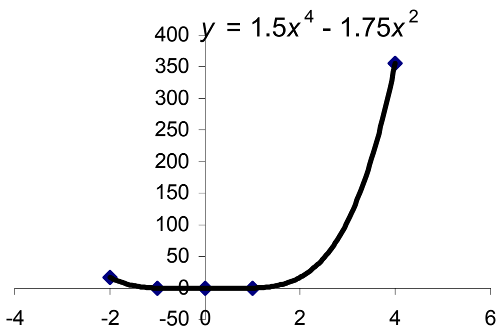
2.



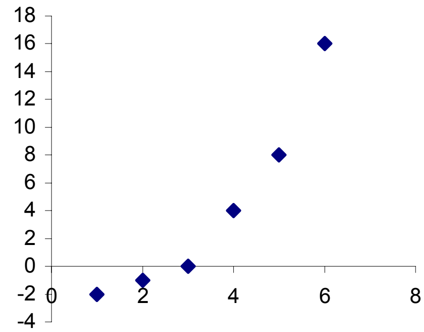
3.



4.

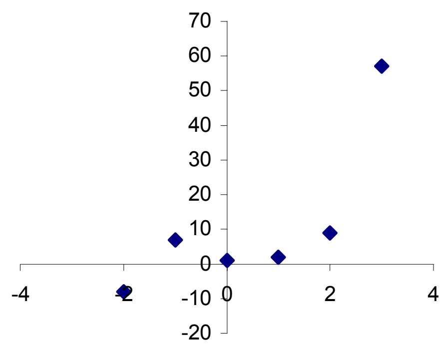


5. **a.**

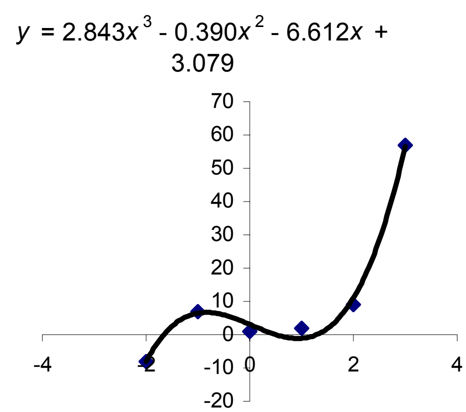


b. It appears based on the scatter plot that a cubic model will fit the data better.

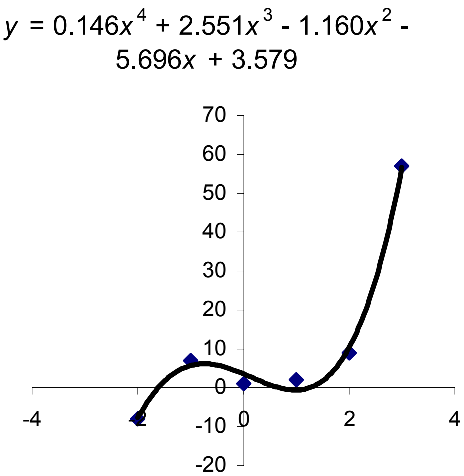
6. **a.**



b. Cubic model

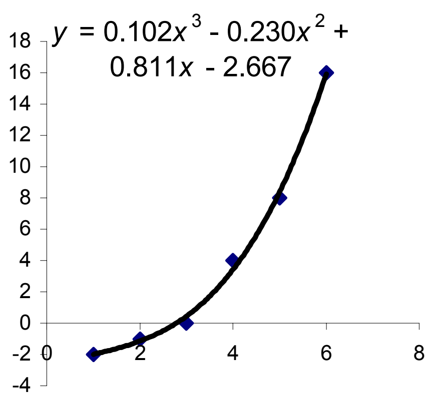


Quartic model

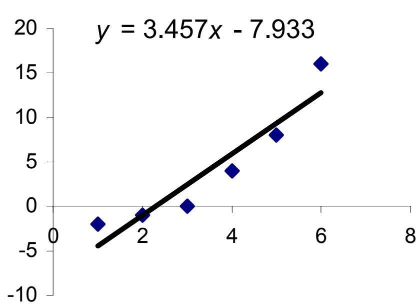


It appears the fit is nearly identical for both the cubic and quartic models.

7. a.

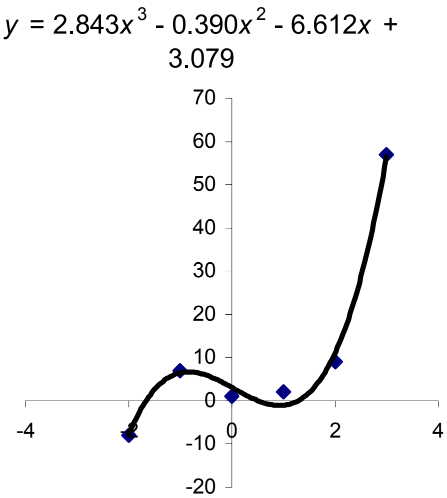


b.

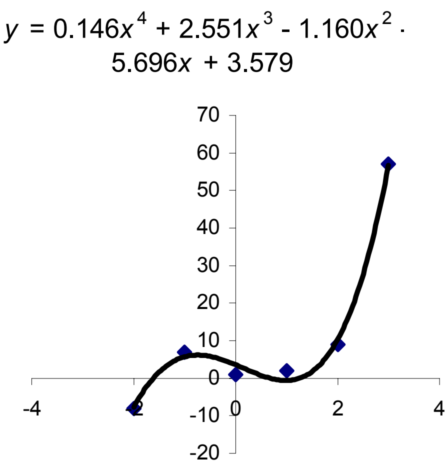


c. Clearly the cubic model is better.

8. a.

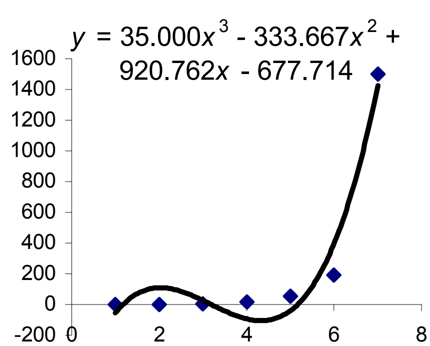


b.

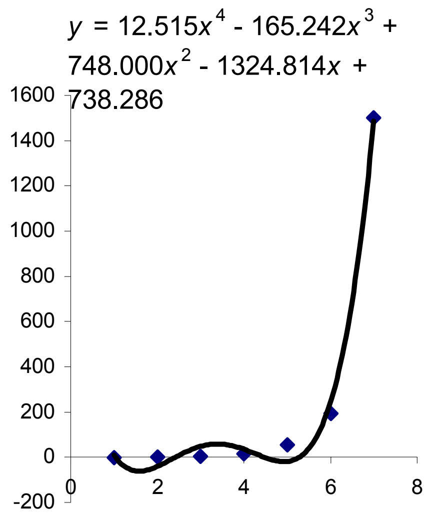


c. Both models fit the data equally well. Perhaps the quartic model is slightly better.

9. a.



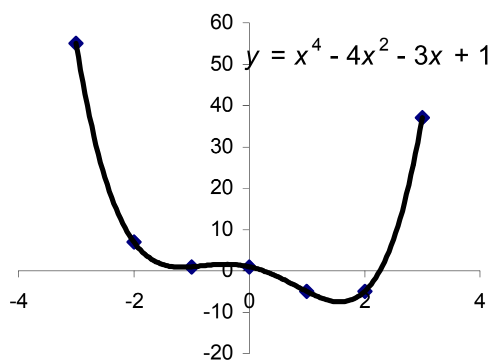
b.



10. a. See Exercises 5 a) and 5 b).

b. The quartic model appears to be better, based on the scatter plot graphs.

11.



12. Yes. The model found in Exercise 11 is a 4^{th} degree polynomial. Therefore, it is quartic.

13.

x	$f(x)$	First Difference	Second Difference	Third Difference
0	0			
1	1	1		
2	5	4	3	
3	24	19	15	12
4	60	36	17	2
5	110	50	14	–3

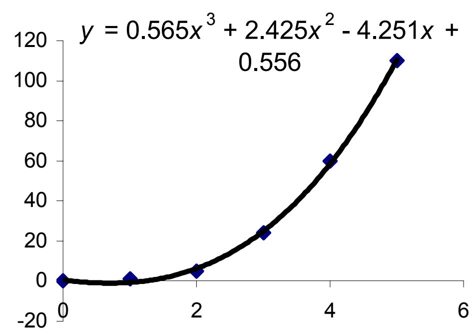
The function f is not exactly cubic.

14.

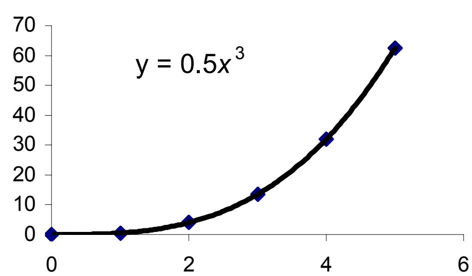
x	$g(x)$	First Difference	Second Difference	Third Difference
0	0			
1	0.5	0.5		
2	4	3.5	3	
3	13.5	9.5	6	3
4	32	18.5	9	3
5	62.5	30.5	12	3

The function g is exactly cubic.

15.

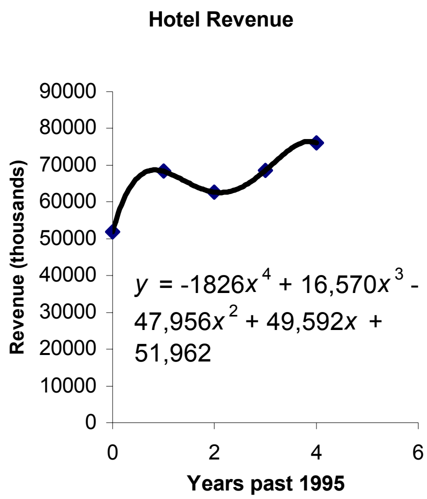


16.



Section 4.2 Exercises

17. a.



The equation is

$$y = -1826x^4 + 16{,}570x^3 - 47{,}956x^2 + 49{,}592x + 51{,}962.$$

b. $y = -1826x^4 + 16{,}570x^3 - 47{,}956x^2 + 49{,}592x + 51{,}962$

$y = -1826(3)^4 + 16{,}570(3)^3 - 47{,}956(3)^2 + 49{,}592(3) + 51{,}962$

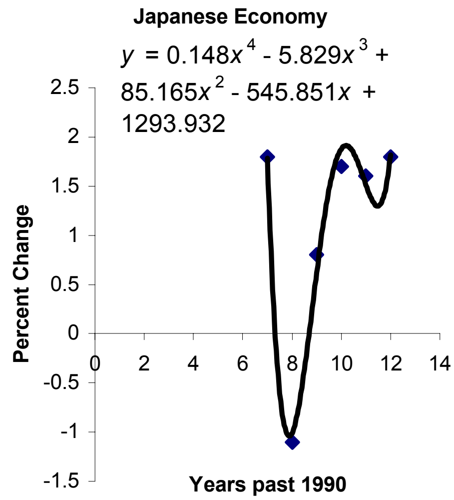
$y = 68{,}618$

The hotel revenue is predicted as \$68,618,000. The prediction is the same as the actual data point.

Using the unrounded model:

```
-1825.9999999952
X^4+16569.999999
962X^3+-47955.99
9999908X^2+49591
.999999934X+5196
2.000000002
                68618
```

18. a.

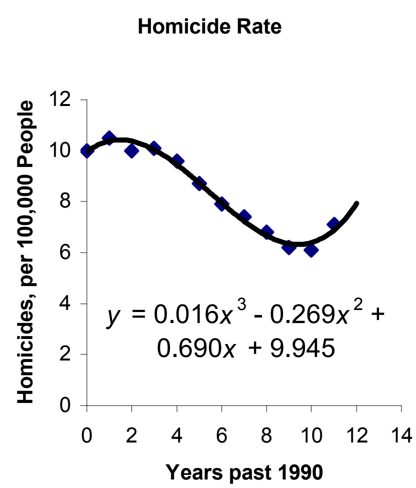


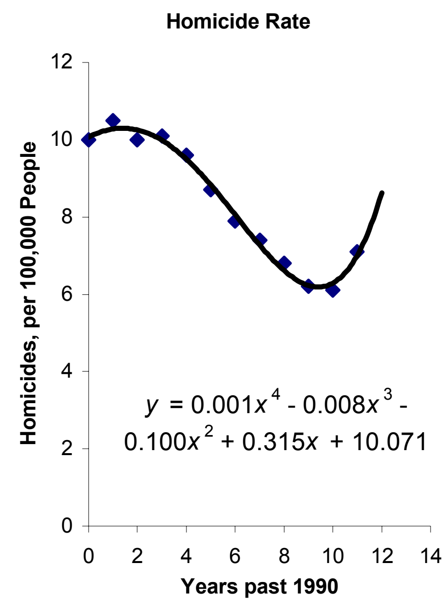
b. For 2002, the percent change in the gross domestic product is 1.818%

Using the unrounded model:

```
.147916666694X
^4+-5.8291666666
77X^3+85.1645833
33478X^2+-545.85
119047708X+1293.
9321428592
         1.817857143
```

19. a.





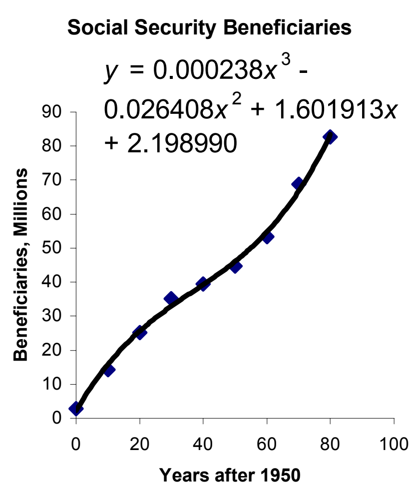
While both models appear to fit the data well, the quartic model is better.

b. Consider part a) above. The cubic model is

$y = 0.0165x^3 - 0.269x^2 + 0.690x$
$+9.945.$

c. Lower. The 9/11 terrorist attack skewed the number of deaths higher.

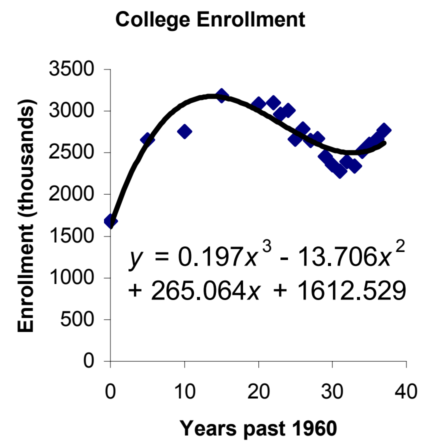
20. a.



b. Consider part a) above.

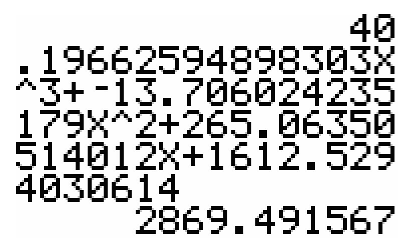
c. Yes. It appears the model fits the data well.

21. a.

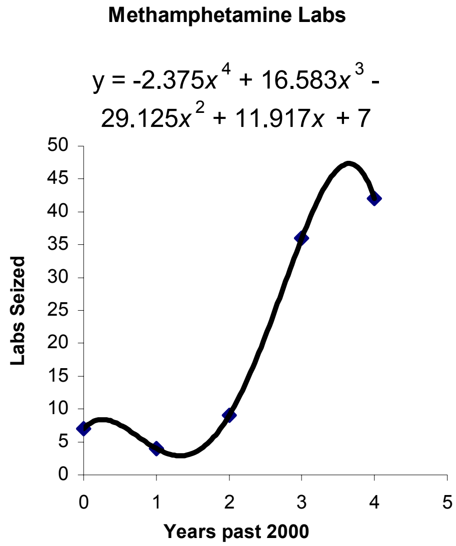


b. The predicted college enrollment in 2000 is 2,869,492.

Using the unrounded model:

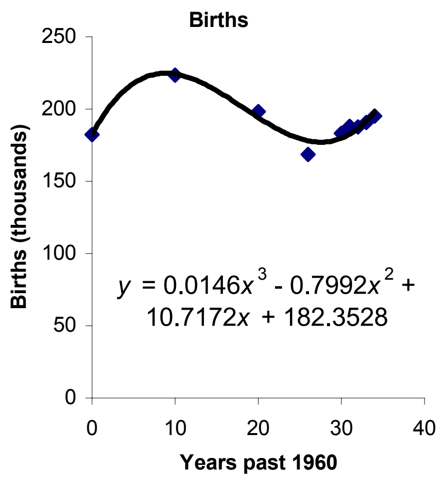


22. a.

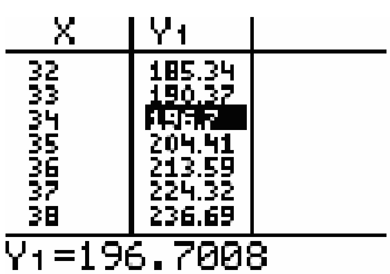


b. The model seems to fit the data points perfectly

23. a.



b.

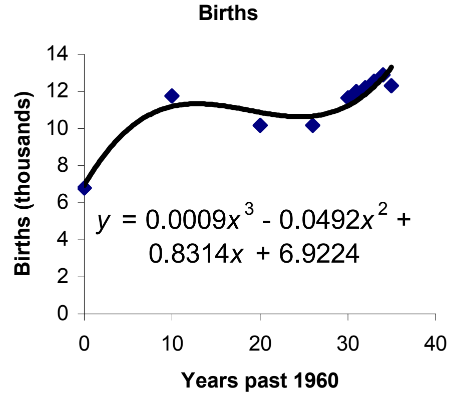


X	Y1
32	185.34
33	190.37
34	196.7
35	204.41
36	213.59
37	224.32
38	236.69

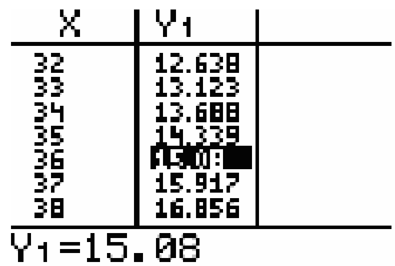
Y1=196.7008

In 1994 the number of births is approximately 196,701.

24. a.



b.

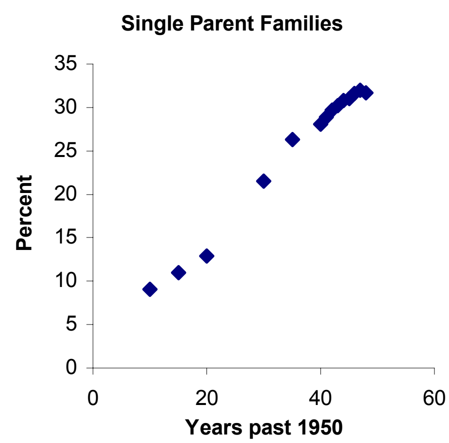


X	Y1
32	12.638
33	13.123
34	13.688
35	14.339
36	15.08
37	15.917
38	16.856

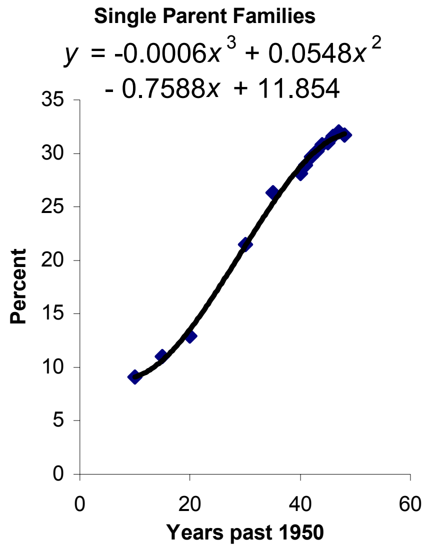
Y1=15.08

In 1996 the number of births is 15,080.

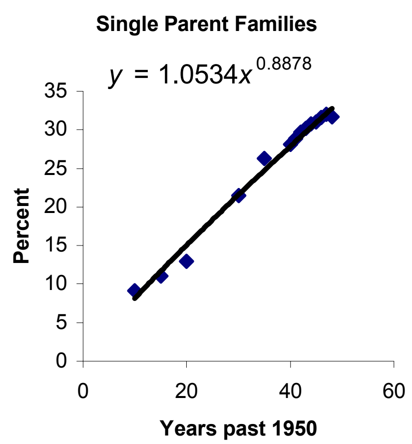
25. a.



b.

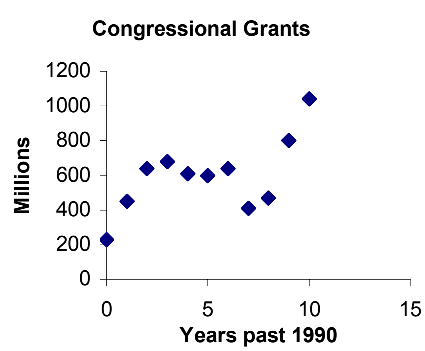


c.



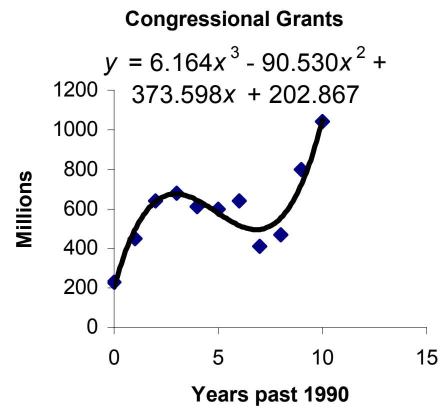
d. The cubic model appears to be a better fit.

26. a.



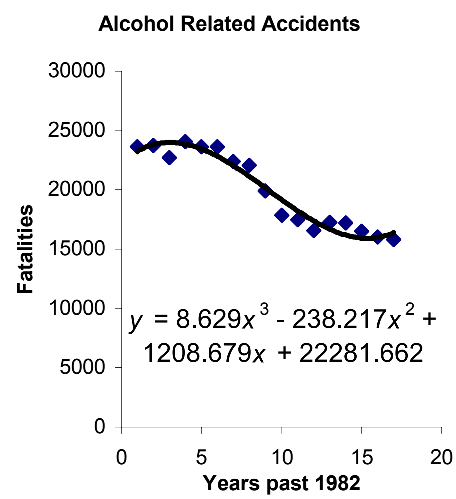
Based on the scatter plot it appears that a cubic model will fit the data well.

b.

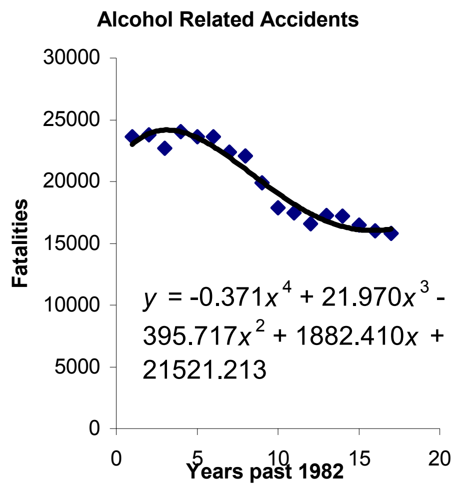


c. Based on part b), the model appears to be a good fit.

27. a.

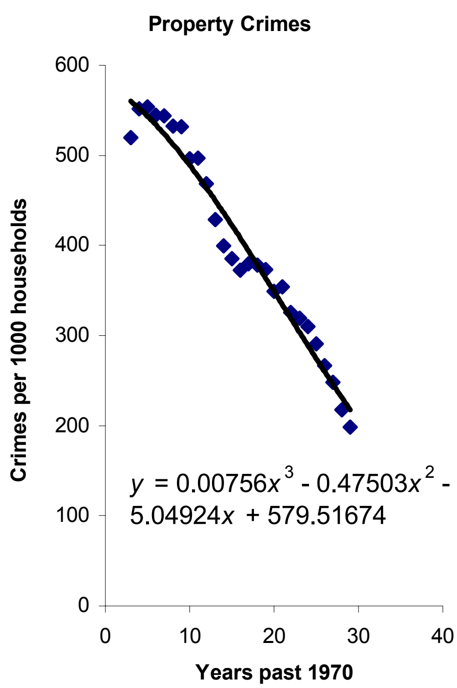


b.

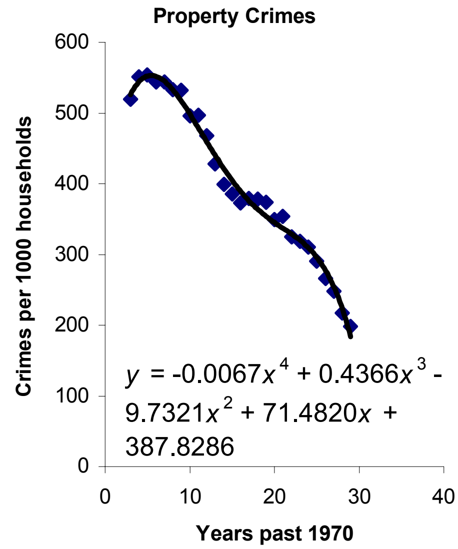


c. It appears that, based on parts a) and b), the quartic model is slightly better, although both models fit the data reasonably well.

28. a.



b.



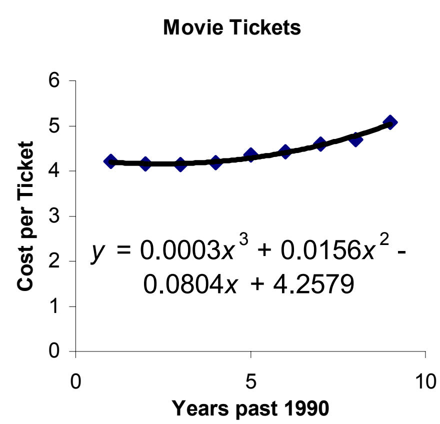
c. The quartic model is definitely better.

d.

```
                         30
.0075559157742 3X
^3+-.47502701640
28X^2+-5.0492389
326814X+579.5167
4259347
          204.5249858
```

The cubic model predicts approximately 205 crimes per 1000 in 2000.

29. a.



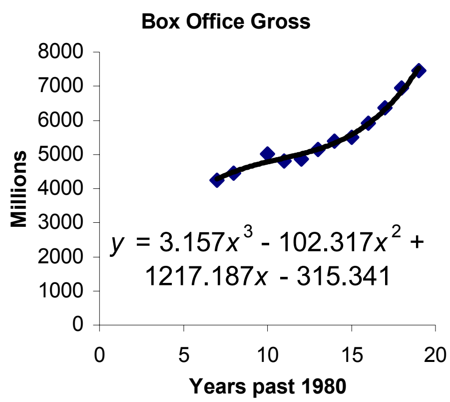
b.

X	Y_1	Y_2
8	4.7805	5.7
9	5.0364	5.7
10	5.3412	5.7
11	5.6969	5.7
12	6.1055	5.7
13	6.5689	5.7
14	7.0891	5.7

X=11

In about 11 years, in the year 2001, the price of a movie will be \$5.70.

30. a.



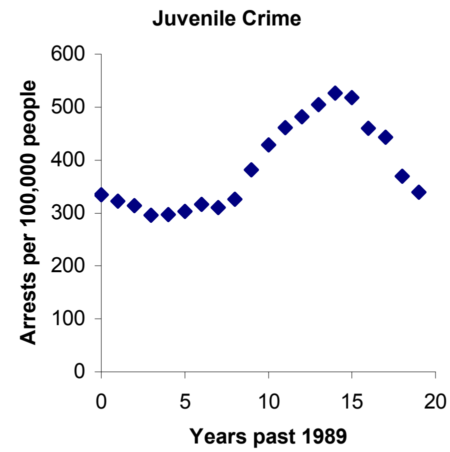
b.

X	Y_1	Y_2
6	3986.3	4653
7	4274.3	4653
8	4490.3	4653
9	4653.2	4653
10	4782	4653
11	4895.5	4653
12	5012.8	4653

X=9

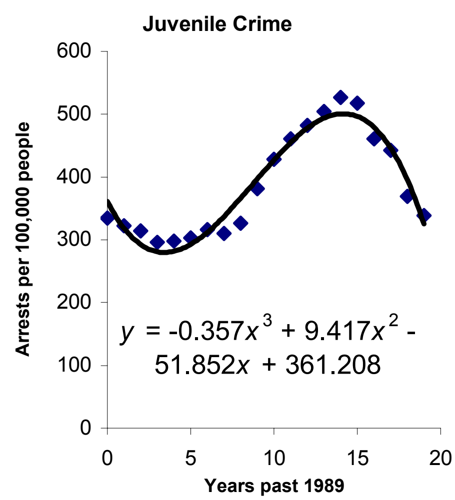
When $x = 9$, in the year 1989, the U.S. box office grosses were \$4653 million.

31. a.



Based on the scatter plot, it appears that cubic model will fit the data well.

b.



c. See part b) above.

d.

X	Y_1	
7	337.14	
10	427.13	
13	493.69	
16	478.96	
19	325.05	
22	-25.91	
25	-631.8	

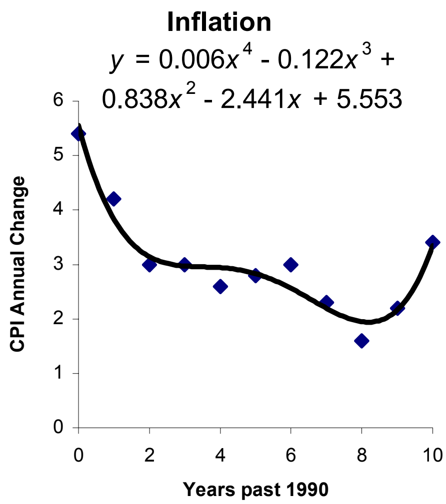
X=10

In 1990, $f(10) = 427.13$.

In 1999, $f(19) = 325.05$.

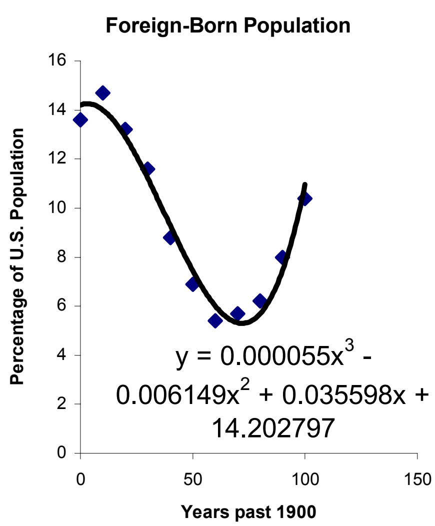
The ratio is $\dfrac{f(19)}{f(10)} = \dfrac{325.05}{427.13} \approx 76.1\%$

32. a.



b. The fit is good, but definitely not perfect.

33. a.

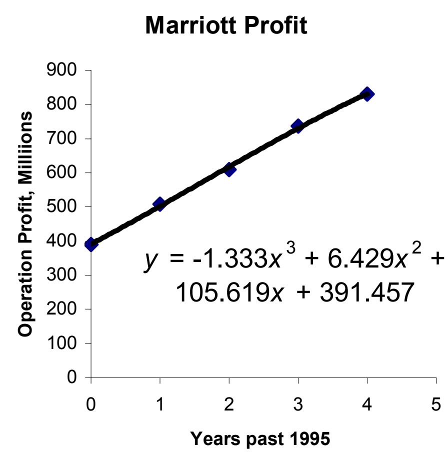


b.

```
                75
5.4681429681425E
-5X^3+-.00614860
13986X^2+.035598
29059826X+14.202
797202798
     5.355514277
```

In 1975 the percentage of the U.S. population that is foreign-born is approximately 5.36%

34. a.



b.

```
                 5
-1.33333333333264
X^3+6.4285714285
298X^2+105.61904
76191X+391.45714
285714
           913.6
```

In 2000, when $x = 5$, the operation profit for Marriot is predicted to be \$913.6 million.

Section 4.3 Skills Check

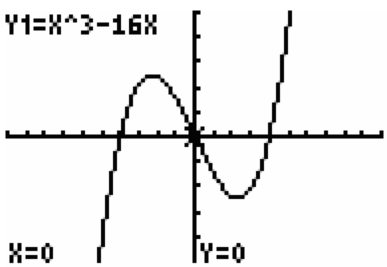
1. $x^3 - 16x = 0$

$x(x^2 - 16) = 0$

$x(x+4)(x-4) = 0$

$x = 0, x = -4, x = 4$

Checking graphically



[–10, 10] by [–50, 50]

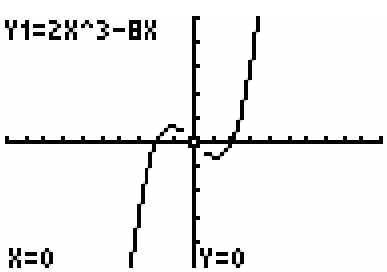
2. $2x^3 - 8x = 0$

$2x(x^2 - 4) = 0$

$2x(x+2)(x-2) = 0$

$x = 0, x = -2, x = 2$

Checking graphically



[–10, 10] by [–50, 50]

3. $x^4 - 4x^3 + 4x^2 = 0$

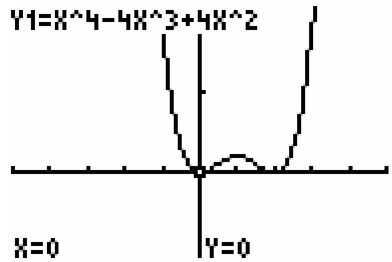
$x^2(x^2 - 4x + 4) = 0$

$x^2(x-2)(x-2) = 0$

$x^2 = 0 \Rightarrow x = 0$

$x = 0, x = 2$

Checking graphically



[–5, 5] by [–5, 10]

4. $x^4 - 6x^3 + 9x^2 = 0$

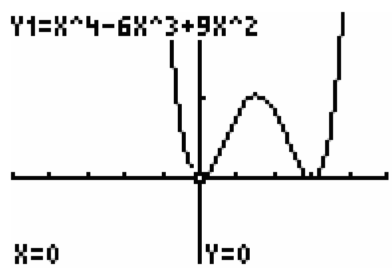
$x^2(x^2 - 6x + 9) = 0$

$x^2(x-3)(x-3) = 0$

$x^2 = 0 \Rightarrow x = 0$

$x = 0, x = 3$

Checking graphically



[–5, 5] by [–5, 10]

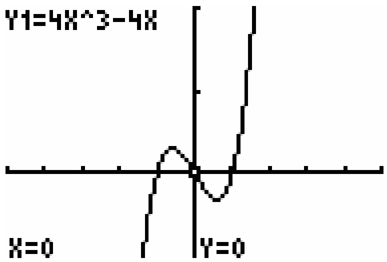
5. $4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$4x(x+1)(x-1) = 0$

$x = 0, x = -1, x = 1$

Checking graphically



[–5, 5] by [–5, 10]

6. $x^4 - 3x^3 + 2x^2 = 0$

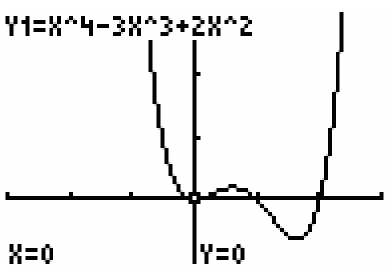
$x^2\left(x^2 - 3x + 2\right) = 0$

$x^2\left(x - 2\right)\left(x - 1\right) = 0$

$x^2 = 0 \Rightarrow x = 0$

$x = 0, x = 2, x = 1$

Checking graphically



[–3, 3] by [–1, 3]

7. $x^3 - 4x^2 - 9x + 36 = 0$

$\left(x^3 - 4x^2\right) + \left(-9x + 36\right) = 0$

$x^2\left(x - 4\right) + \left(-9\right)\left(x - 4\right) = 0$

$\left(x - 4\right)\left(x^2 - 9\right) = 0$

$\left(x - 4\right)\left(x + 3\right)\left(x - 3\right) = 0$

$x = 4, x = -3, x = 3$

8. $x^3 + 5x^2 - 4x - 20 = 0$

$\left(x^3 + 5x^2\right) + \left(-4x - 20\right) = 0$

$x^2\left(x + 5\right) + \left(-4\right)\left(x + 5\right) = 0$

$\left(x + 5\right)\left(x^2 - 4\right) = 0$

$\left(x + 5\right)\left(x + 2\right)\left(x - 2\right) = 0$

$x = -5, x = -2, x = 2$

9. $3x^3 - 4x^2 - 12x + 16 = 0$

$\left(3x^3 - 4x^2\right) + \left(-12x + 16\right) = 0$

$x^2\left(3x - 4\right) + \left(-4\right)\left(3x - 4\right) = 0$

$\left(3x - 4\right)\left(x^2 - 4\right) = 0$

$\left(3x - 4\right)\left(x + 2\right)\left(x - 2\right) = 0$

$x = \frac{4}{3}, x = -2, x = 2$

10. $4x^3 - 8x^2 - 36x - 72 = 0$

$4\left(x^3 - 2x^2 - 9x - 18\right) = 0$

$4\left[\left(x^3 - 2x^2\right) + \left(-9x - 18\right)\right] = 0$

$4\left[x^2\left(x - 2\right) + \left(-9\right)\left(x + 2\right)\right] = 0$

Does not factor

11. $2x^3 - 16 = 0$

$2x^3 = 16$

$x^3 = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

12. $3x^3 - 81 = 0$

$3x^3 = 81$

$x^3 = 27$

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$x = 3$

13. $\frac{1}{2}x^4 - 8 = 0$

$\frac{1}{2}x^4 = 8$

$2\left(\frac{1}{2}x^4\right) = 2(8)$

$x^4 = 16$

$\sqrt[4]{x^4} = \pm\sqrt[4]{16}$

$x = \pm 2$

14. $2x^4 - 162 = 0$

$2x^4 = 162$

$x^4 = 81$

$\sqrt[4]{x^4} = \pm\sqrt[4]{81}$

$x = \pm 3$

15. $4x^4 - 8x^2 = 0$

$4x^2\left(x^2 - 2\right) = 0$

$4x^2 = 0,\ x^2 - 2 = 0$

$4x^2 = 0 \Rightarrow x = 0$

$x^2 - 2 = 0 \Rightarrow x^2 = 2$

$\sqrt{x^2} = \pm\sqrt{2}$

$x = \pm\sqrt{2}$

$x = \pm\sqrt{2},\ x = 0$

16. $3x^4 - 24x^2 = 0$

$3x^2\left(x^2 - 8\right) = 0$

$3x^2 = 0,\ x^2 - 8 = 0$

$3x^2 = 0 \Rightarrow x = 0$

$x^2 - 8 = 0 \Rightarrow x^2 = 8$

$\sqrt{x^2} = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

$x = 0,\ x = \pm 2\sqrt{2}$

17. $0.5x^3 - 12.5x = 0$

$0.5x\left(x^2 - 25\right) = 0$

$0.5x(x+5)(x-5) = 0$

$x = 0, x = -5, x = 5$

18. $0.2x^3 - 24x = 0$

$0.2x\left(x^2 - 120\right) = 0$

$0.2x = 0,\ x^2 - 120 = 0$

$0.2x = 0 \Rightarrow x = 0$

$x^2 - 120 = 0 \Rightarrow x^2 = 120$

$\sqrt{x^2} = \pm\sqrt{120}$

$x = \pm 2\sqrt{30}$

$x = 0,\ x = \pm 2\sqrt{30}$

19. $x^4 - 6x^2 + 9 = 0$

$\left(x^2 - 3\right)\left(x^2 - 3\right) = 0$

$x^2 - 3 = 0,\ x^2 - 3 = 0$

$x^2 - 3 = 0 \Rightarrow x^2 = 3$

$\sqrt{x^2} = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

20. $x^4 - 10x^2 + 25 = 0$

$\left(x^2 - 5\right)\left(x^2 - 5\right) = 0$

$x^2 - 5 = 0,\ x^2 - 5 = 0$

$x^2 - 5 = 0 \Rightarrow x^2 = 5$

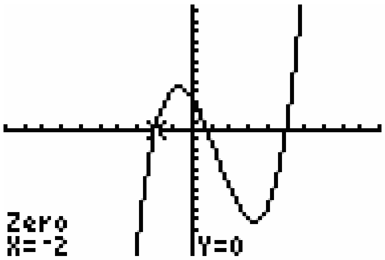
$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

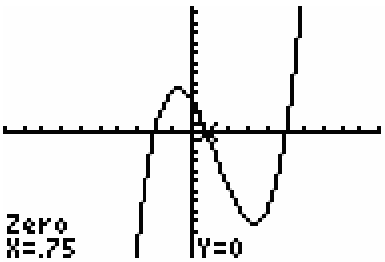
21. $f(x) = 0$ implies $x = -3, x = 1, x = 4$. Note that the x-intercepts are the solutions.

22. $f(x) = 0$ implies $x = -2, x = 0.5, x = 8$. Note that the x-intercepts are the solutions.

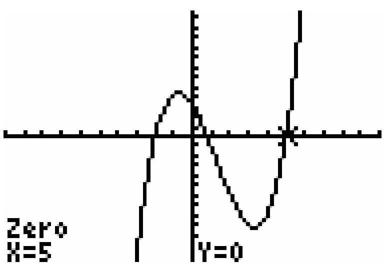
23. The *x*-intercepts (zeros) are the solutions.



[–10, 10] by [–125, 125]



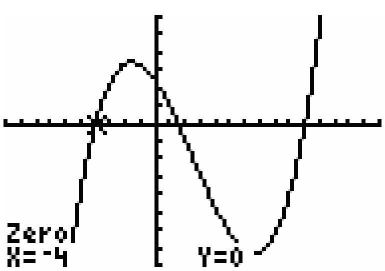
[–10, 10] by [–125, 125]



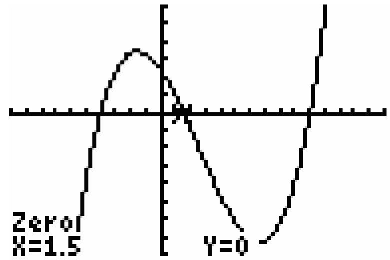
[–10, 10] by [–125, 125]

$x = -2, x = 0.75, x = 5$

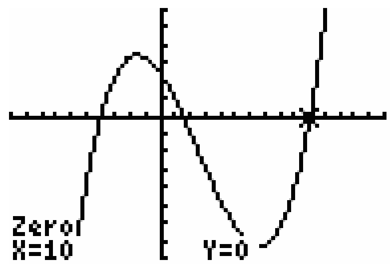
24. The *x*-intercepts (zeros) are the solutions.



[–10, 15] by [–400, 300]



[–10, 15] by [–400, 300]



[–10, 15] by [–400, 300]

$x = -4,\ x = 1.5,\ x = 10$

Section 4.3 Exercises

25. a. $R = 400x - x^3$

$400x - x^3 = 0$

$x\left(400 - x^2\right) = 0$

$x(20 - x)(20 + x) = 0$

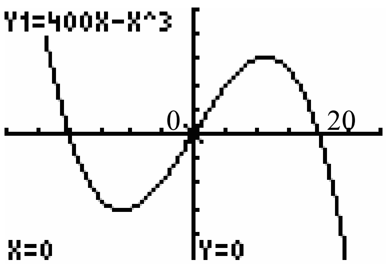
$x = 0,\ 20 - x = 0,\ 20 + x = 0$

$x = 0, -x = -20,\ x = -20$

$x = 0,\ x = 20,\ x = -20$

In the physical context of the problem, selling zero units or selling 20 units will yield revenue of zero dollars.

b. Yes.



[–30, 30] by [–5000, 5000]

26. a. $R = 12{,}000x - 0.003x^3$

$12{,}000x - 0.003x^3 = 0$

$0.003x\left(4{,}000{,}000 - x^2\right) = 0$

$0.003x = 0 \qquad 4{,}000{,}000 - x^2 = 0$

$x = 0 \qquad x^2 = 4{,}000{,}000$

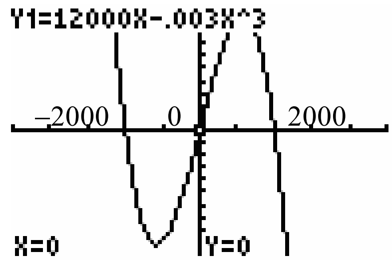
$x = \pm\sqrt{4{,}000{,}000}$

$x = \pm 2000$

$x = 0,\ x = 2000,\ x = -2000$

In the physical context of the problem, selling zero units or selling 2000 units will yield revenue of zero dollars.

b. Yes.



[–5000, 5000] by
[–10,000,000, 10,000,000]

27. a. $R = \left(100{,}000 - 0.1x^2\right)x$

$\left(100{,}000 - 0.1x^2\right)x = 0$

$x = 0,\ 100{,}000 - 0.1x^2 = 0$

$-0.1x^2 = -100{,}000$

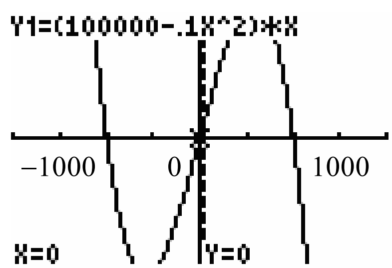
$x^2 = 1{,}000{,}000$

$x = \pm\sqrt{1{,}000{,}000}$

$x = 0,\ x = 1000,\ x = -1000$

In the physical context of the problem, selling zero units or selling 1000 units will yield revenue of zero dollars.

b. Yes.



[–2000, 2000] by
[–35,000,000, 35,000,000]

28. a. $R = \left(100x - x^2\right)x$

$\left(100x - x^2\right)x = 0$

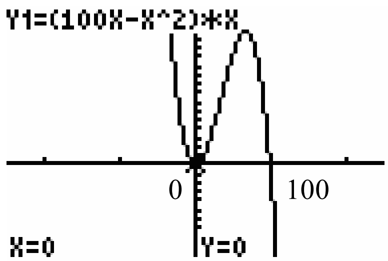
$x^2\left(100 - x\right) = 0$

$x^2 = 0,\ 100 - x = 0$

$x = 0, x = 100$

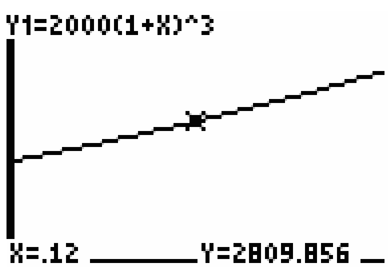
In the physical context of the problem, selling zero units or selling 100 units will yield revenue of zero dollars.

b. Yes.



[–250, 250] by [–100,000, 175,000]

29. a.



[0, 0.24] by [0, 5000]

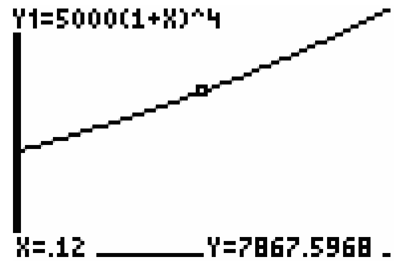
b.

$$2662 = 2000(1+r)^3$$
$$(1+r)^3 = \frac{2662}{2000}$$
$$\sqrt[3]{(1+r)^3} = \sqrt[3]{\frac{2662}{2000}}$$
$$1+r = \sqrt[3]{1.331}$$
$$r = \sqrt[3]{1.331} - 1$$
$$r \approx 0.10 = 10\%$$

c.

$$3456 = 2000(1+r)^3$$
$$(1+r)^3 = \frac{3456}{2000}$$
$$\sqrt[3]{(1+r)^3} = \sqrt[3]{\frac{3456}{2000}}$$
$$1+r = \sqrt[3]{1.728}$$
$$r = \sqrt[3]{1.728} - 1$$
$$r = 0.20 = 20\%$$

30. a.



[0, 0.24] by [0, 12,000]

b.

$$10{,}368 = 5000(1+r)^4$$
$$(1+r)^4 = \frac{10{,}368}{5000}$$
$$\sqrt[4]{(1+r)^4} = \sqrt[4]{\frac{10{,}368}{5000}}$$
$$1+r = \pm\sqrt[4]{2.0736}$$
$$r = \pm 1.2 - 1$$
$$r = 0.20 \text{ or } -2.2$$

Since the negative solution does not make sense in the context of the problem, $r = 20\%$.

c.

$$(5000 + 2320.50) = 5000(1+r)^4$$
$$(1+r)^4 = \frac{7320.50}{5000}$$
$$\sqrt[4]{(1+r)^4} = \sqrt[4]{\frac{7320.50}{5000}}$$
$$1+r = \pm\sqrt[4]{1.4641}$$
$$r = \pm 1.1 - 1$$
$$r = 0.10 \text{ or } -2.1$$

Since the negative solution does not make sense in the context of the problem, $r = 10\%$.

31. a. The height is x inches, since the distance cut is x units and that distance when folded forms the height of the box.

b. The length and width of box will be what is left after the corners are cut. Since each corner measures x inches square then the length and the width are $18-2x$.

c. $V = lwh$

$V = (18-2x)(18-2x)x$

$V = (324-36x-36x+4x^2)x$

$V = 324x-72x^2+4x^3$

d. $V = 0$

$0 = 324x-72x^2+4x^3$

From part c) above:

$0 = (18-2x)(18-2x)x$

$18-2x = 0,\ x = 0$

$18-2x = 0 \Rightarrow 2x = 18 \Rightarrow x = 9$

$x = 0,\ x = 9$

e. A box will not exist for either of the values calculated in part d) above. For both values of x, no tab will exist to fold up to form the box.

32. a. $0 = 144x-48x^2+4x^3$

$4x(36-12x+x^2) = 0$

$4x(x^2-12x+36) = 0$

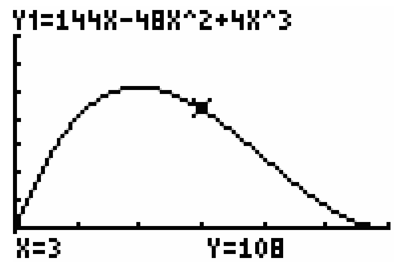
$4x(x-6)(x-6) = 0$

$x = 0, x = 6$

b. For the values calculated in part a) no box can be formed. The calculated values of x yield no tabs that can be folded up to form the box.

c. A box can be created as long as $0 < x < 6$.

d.



[0, 6] by [–25, 200]

33. $400 = -x^3+2x^2+400x-400$

$-1(x^3-2x^2-400x+400) = 400$

$x^3-2x^2-400x+400 = -400$

$x^3-2x^2-400x+800 = 0$

$(x^3-2x^2)+(-400x+800) = 0$

$x^2(x-2)+(-400)(x-2) = 0$

$(x-2)(x^2-400) = 0$

$(x-2)(x+20)(x-20) = 0$

$x = 2, x = -20, x = 20$

The negative answer does not make sense in the physical context of the question. Producing and selling 2 units or 20 units leads to a profit of $40,000.

34. $1200 = 3x^3-6x^2-300x+1800$

$3x^3-6x^2-300x+1800-1200 = 0$

$3x^3-6x^2-300x+600 = 0$

$3(x^3-2x^2-100x+200) = 0$

$3\left[(x^3-2x^2)+(-100x+200)\right] = 0$

$3\left[x^2(x-2)+(-100)(x-2)\right] = 0$

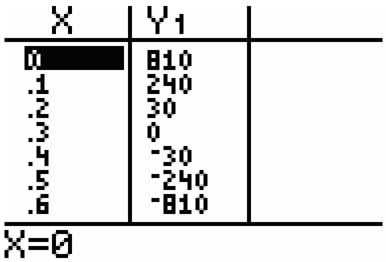
$3(x-2)(x^2-100) = 0$

$3(x-2)(x+10)(x-10) = 0$

$x = 2, x = -10, x = 10$

The negative answer does not make sense in the physical context of the question. Producing and selling 2 units or 10 units leads to a cost of $120,000.

35. a.



t	0	0.1	0.2	0.3
s (cm/sec)	810	240	30	0

b.

$$0 = 30(3-10t)^3$$

$$(3-10t)^3 = \frac{0}{30}$$

$$\sqrt[3]{(3-10t)^3} = \sqrt[3]{0}$$

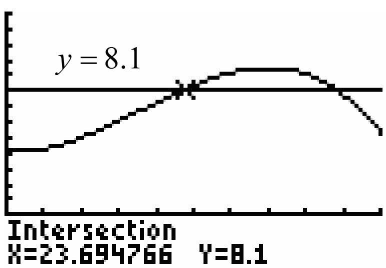
$$3-10t = 0$$

$$t = \frac{-3}{-10}$$

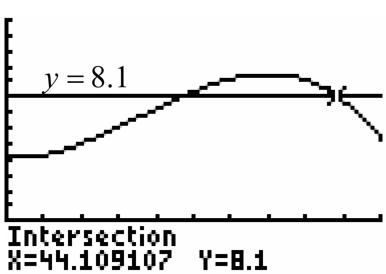
$$t = 0.3$$

The solution in the table is the same as the solution found by the root method.

36. Applying the intersection of graphs method yields:



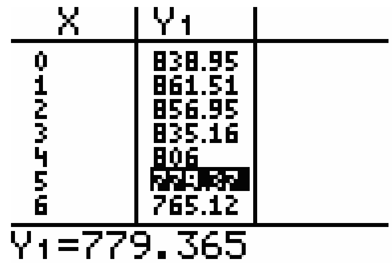
[0, 50] by [–3, 13]



[0, 50] by [–3, 13]

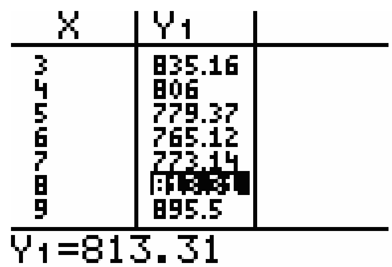
After approximately 23.6 years in 1974 and after approximately 44.1 years in 1994, the homicide rate is 8.1 per 100,000.

37. a.



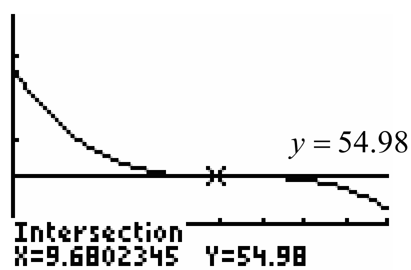
In 1995 the estimated number of arrests is 779,365.

b.



In 1998 the estimated number of arrests is 813,310.

38. a. Applying the intersection of graphs method:

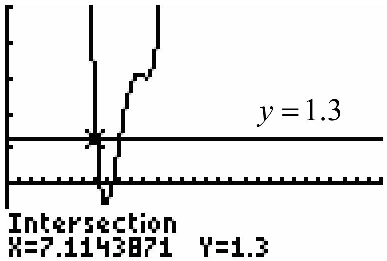


[0, 18] by –50, 250]

When $x \approx 9.68$, the stock price is \$54.98.

b. Since $x = 9.68$, after 10 months or in January 2000, the stock price is \$54.98.

39. a. Applying the intersection of graphs method:

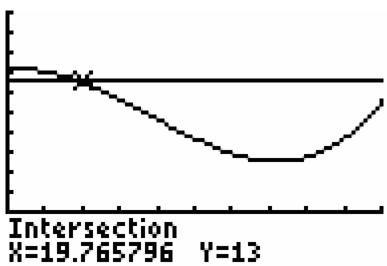


[0, 30] by [–2, 5]

After 7.11 years, in 1998, the percent change is 1.3%.

b. The model will be 1.3% again. Note that there is a second intersection point on the graph in part a).

40. Applying the intersection of graphs method:



[0, 100] by [–-5, 20]

In 1920 (x = 19.8) the percentage of the U.S. population that was foreign born was 13%.

Section 4.4 Skills Check

1.
$$\begin{array}{r|rrrrr} 3 & 1 & -4 & 0 & 3 & 10 \\ & & 3 & -3 & -9 & -18 \\ \hline & 1 & -1 & -3 & -6 & -8 \end{array}$$

$$x^3 - x^2 - 3x - 6 - \frac{8}{x-3}$$

2.
$$\begin{array}{r|rrrrr} -4 & 1 & 2 & -3 & 0 & 1 \\ & & -4 & 8 & -20 & 80 \\ \hline & 1 & -2 & 5 & -20 & 81 \end{array}$$

$$x^3 - 2x^2 + 5x - 20 + \frac{81}{x+4}$$

3.
$$\begin{array}{r|rrrrr} 1 & 2 & -3 & 0 & 1 & -7 \\ & & 2 & -1 & -1 & 0 \\ \hline & 2 & -1 & -1 & 0 & -7 \end{array}$$

$$2x^3 - x^2 - x - \frac{7}{x-1}$$

4.
$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 0 & 0 & -1 \\ & & -1 & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -1 & 0 \end{array}$$

$$x^3 - x^2 + x - 1$$

5.
$$\begin{array}{r|rrrrr} 3 & 2 & -4 & 0 & 3 & 18 \\ & & 6 & 6 & 18 & 63 \\ \hline & 2 & 2 & 6 & 21 & 81 \end{array}$$

Since the remainder is not zero, 3 is not a solution of the equation.

6.
$$\begin{array}{r|rrrrr} -5 & 1 & 3 & -10 & 8 & 40 \\ & & -5 & 10 & 0 & -40 \\ \hline & 1 & -2 & 0 & 8 & 0 \end{array}$$

Since the remainder is zero, -5 is a solution of the equation.

7.
$$\begin{array}{r|rrrrr} -3 & -1 & 0 & -9 & 3 & 0 \\ & & 3 & -9 & 54 & 171 \\ \hline & -1 & 3 & -18 & 57 & 171 \end{array}$$

Since the remainder is not zero, $x+3$ is not a factor.

8.
$$\begin{array}{r|rrrrr} -2 & 2 & 5 & 0 & -6 & -4 \\ & & -4 & -2 & 4 & 4 \\ \hline & 2 & 1 & -2 & -2 & 0 \end{array}$$

Since the remainder is zero, $x+2$ is a factor.

9.
$$\begin{array}{r|rrrr} -1 & -1 & 1 & 1 & -1 \\ & & 1 & -2 & 1 \\ \hline & -1 & 2 & -1 & 0 \end{array}$$

One factor is $x-1$. The new polynomial is $-x^2+2x-1$. Solve $-x^2+2x-1=0$.

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1, x = 1$

The solutions are $x=-1$ and $x=1$ (repeated two times).

10.
$$\begin{array}{r|rrrr} 1 & 1 & 4 & -1 & -4 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

One factor is $x-1$. The new polynomial is x^2+5x+4. Solve $x^2+5x+4=0$.

$(x+1)(x+4) = 0$

$x = -1, x = -4$

The solutions are $x=1$, $x=-1$, $x=-4$.

11. $-5\overline{)\begin{array}{rrrrr} 1 & 2 & -21 & -22 & 40 \\ & -5 & 15 & 30 & -40 \\ \hline 1 & -3 & -6 & 8 & 0 \end{array}}$

One factor is $x+5$. The new polynomial is x^3-3x^2-6x+8. Applying the rational solutions test yields:

$\frac{p}{q}=\pm\left(\frac{1,2,4,8}{1}\right)=\pm(1,2,4,8)$

$1\overline{)\begin{array}{rrrr} 1 & -3 & -6 & 8 \\ & 1 & -2 & -8 \\ \hline 1 & -2 & -8 & 0 \end{array}}$

One factor is $x-1$. The new polynomial is x^2-2x-8. Solve $x^2-2x-8=0$.

$(x-4)(x+2)=0$

$x=4, x=-2$

The solutions are $x=4,\ x=-2,\ x=1,$ and $x=-5$.

12. $3\overline{)\begin{array}{rrrrr} 2 & -17 & 51 & -63 & 27 \\ & 6 & -33 & 54 & -27 \\ \hline 2 & -11 & 18 & -9 & 0 \end{array}}$

One factor is $x+3$. The new polynomial is $2x^3-11x^2+18x-9$. Applying the rational solutions test yields:

$\frac{p}{q}=\pm\left(\frac{1,3,9}{1,2}\right)=\pm\left(1,3,9,\frac{1}{2},\frac{3}{2},\frac{9}{2}\right)$

$1\overline{)\begin{array}{rrrr} 2 & -11 & 18 & -9 \\ & 2 & -9 & 9 \\ \hline 2 & -9 & 9 & 0 \end{array}}$

One factor is $x-1$. The new polynomial is $2x^2-9x+9$. Solve $2x^2-9x+9=0$.

$(2x-3)(x-3)=0$

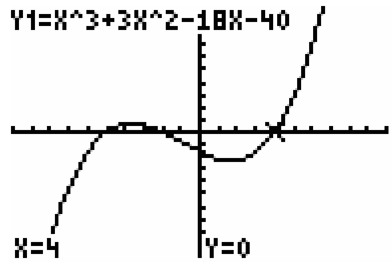
$2x-3=0, x-3=0$

$x=\frac{3}{2}, x=3$

The solutions are $x=\frac{3}{2}, x=3, x=1$.

Note that $x=3$ is a repeated solution.

13. Applying the x-intercept method:



[–10, 10] by [–250, 250]

One solution appears to be $x = 4$.

$4\overline{)\begin{array}{rrrr} 1 & 3 & -18 & -40 \\ & 4 & 28 & 40 \\ \hline 1 & 7 & 10 & 0 \end{array}}$

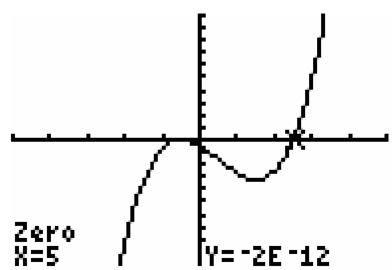
One factor is $x-4$. The new polynomial is $x^2+7x+10$. Solve $x^2+7x+10=0$.

$(x+2)(x+5)=0$

$x=-2, x=-5$

The solutions are $x=-5, x=-2, x=4$.

14. Applying the x-intercept method:



[–10, 10] by [–100, 100]

One solution appears to be $x = 5$.

5)	1	−3	−9	−5
		5	10	5
	1	2	1	0

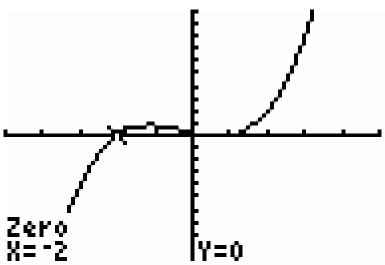
One factor is $x-5$. The new polynomial is x^2+2x+1. Solve $x^2+2x+1=0$.

$(x+1)(x+1)=0$

$x=-1, x=-1$

The solutions are $x=5, x=-1$.
Note that $x=-1$ is a repeated solution.

15. Applying the x-intercept method:



[–5, 5] by [–100, 100]

One solution appears to be $x = -2$.

−2)	3	2	−7	2
		−6	8	−2
	3	−4	1	0

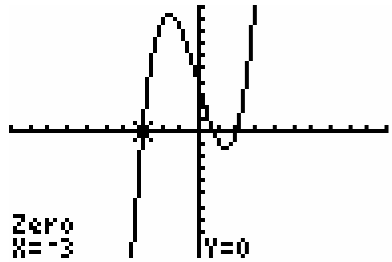
One factor is $x+2$. The new polynomial is $3x^2-4x+1$. Solve $3x^2-4x+1=0$.

$(3x-1)(x-1)=0$

$x=\frac{1}{3}, x=1$

The solutions are $x=-2, x=1, x=\frac{1}{3}$.

16. Applying the x-intercept method:



[–10, 10] by [–50, 50]

One solution appears to be $x = -3$.

−3)	4	1	−27	18
		−12	33	−18
	4	−11	6	0

One factor is $x+3$. The new polynomial is $4x^2-11x+6$. Solve $4x^2-11x+6=0$.

$(4x-3)(x-2)=0$

$x=\frac{3}{4}, x=2$

The solutions are $x=-3, x=2, x=\frac{3}{4}$.

17. $\frac{p}{q}=\pm\left(\frac{1,2,3,4,6,12}{1}\right)=\pm(1,2,3,4,6,12)$

18. $\frac{p}{q}=\pm\left(\frac{1,2}{1,2,4}\right)=\pm\left(1,2,\frac{1}{2},\frac{1}{4}\right)$

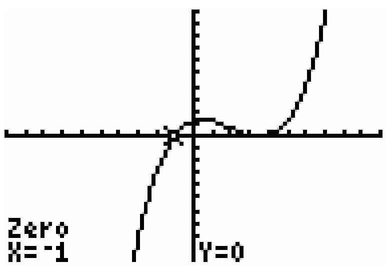
19. $\frac{p}{q}=\pm\left(\frac{1,2,4}{1,3,9}\right)$

$=\pm\left(1,2,4,\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{1}{9},\frac{2}{9},\frac{4}{9}\right)$

20. $\dfrac{p}{q} = \pm\left(\dfrac{1,2,3,6}{1,2,3,6}\right)$

$= \pm\left(1,2,3,6,\dfrac{1}{2},\dfrac{3}{2},\dfrac{1}{3},\dfrac{2}{3},\dfrac{1}{6}\right)$

21. Applying the x-intercept method:



[–10, 10] by [–100, 100]

One solution appears to be $x = -1$.

–1)	1	–6	5	12
		–1	7	–12
	1	–7	12	0

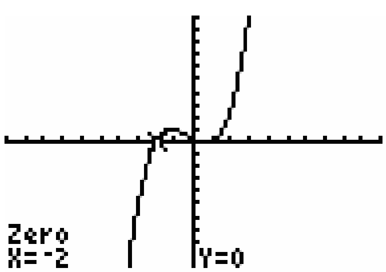
One factor is $x+1$. The new polynomial is $x^2-7x+12$. Solve $x^2-7x+12=0$.

$(x-3)(x-4)=0$

$x=3,\ x=4$

The solutions are $x=-1, x=3, x=4$.

22. Applying the x-intercept method:



[–10, 10] by [–100, 100]

One solution appears to be $x = -2$.

–2)	4	3	–9	2
		–8	10	–2
	4	–5	1	0

One factor is $x+2$. The new polynomial is $4x^2-5x+1$. Solve $4x^2-5x+1=0$.

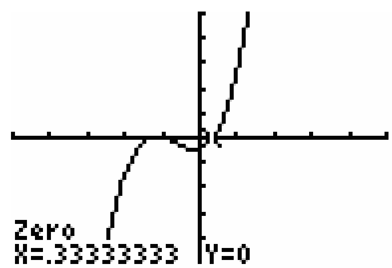
$(4x-1)(x-1)=0$

$4x-1=0, x-1=0$

$x=\dfrac{1}{4},\ x=1$

The solutions are $x=1, x=\dfrac{1}{4}, x=-2$.

23. Applying the x-intercept method:



[–5, 5] by [–50, 50]

One solution appears to be $x=\dfrac{1}{3}$.

$\frac{1}{3}$)	9	18	5	–4
		3	7	4
	9	21	12	0

One factor is $x-\dfrac{1}{3}$. The new polynomial is $9x^2+21x+12$. Solve $9x^2+21x+12=0$.

$3\left(3x^2+7x+4\right)=0$

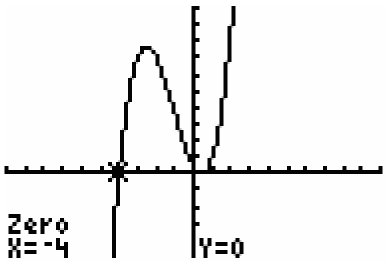
$3(3x+4)(x+1)=0$

$3x+4=0, x+1=0$

$x=-\dfrac{4}{3}, x=-1$

The solutions are $x=-\dfrac{4}{3}, x=-1, x=\dfrac{1}{3}$.

24. Applying the x-intercept method



[–10, 10] by [–50, 100]

One solution appears to be $x=-4$.

$$\begin{array}{r|rrrr} -4 & 6 & 19 & -19 & 4 \\ & & -24 & 20 & -4 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

One factor is $x+4$. The new polynomial is $6x^2-5x+1$. Solve $6x^2-5x+1=0$.

$(3x-1)(2x-1)=0$

$3x-1=0, 2x-1=0$

$x=\frac{1}{3}, x=\frac{1}{2}$

The solutions are $x=\frac{1}{3}, x=\frac{1}{2}, x=-4$.

25. $x^3=10x-7x^2$

$x^3+7x^2-10x=0$

$x\left(x^2+7x-10\right)=0$

$x=0, x^2+7x+10=0$

Applying the quadratic formula:

$$x=\frac{-7\pm\sqrt{7^2-4(1)(-10)}}{2(1)}$$

$$x=\frac{-7\pm\sqrt{89}}{2}$$

The solutions are $x=0, x=\frac{-7\pm\sqrt{89}}{2}$.

26. $t^3-2t^2+3t=0$

$t\left(t^2-2t+3\right)=0$

$t=0, t^2-2t+3=0$

Applying the quadratic formula:

$$t=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(3)}}{2(1)}$$

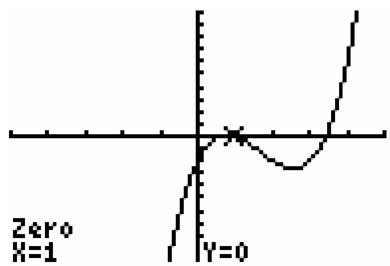
$$t=\frac{2\pm\sqrt{-8}}{2}$$

$$t=\frac{2\pm 2i\sqrt{2}}{2}$$

$t=1\pm i\sqrt{2}$

The solutions are $t=0$, $t=1\pm i\sqrt{2}$.

27. Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that $w=1$ is a zero.

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 6 & -2 \\ & & 1 & -4 & 2 \\ \hline & 1 & -4 & 2 & 0 \end{array}$$

The remaining quadratic factor is w^2-4w+2.

Applying the quadratic formula:

$$w = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

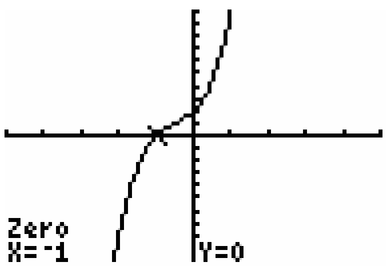
$$w = \frac{4 \pm \sqrt{8}}{2}$$

$$w = \frac{4 \pm 2\sqrt{2}}{2}$$

$$w = 2 \pm \sqrt{2}$$

The solutions are $w = 1,\ w = 2 \pm \sqrt{2}$.

28. Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that $w = -1$ is a zero.

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 3 & 2 \\ & & -2 & -1 & -2 \\ \hline & 2 & 1 & 2 & 0 \end{array}$$

The remaining quadratic factor is $2w^2 + w + 2$.

Applying the quadratic formula:

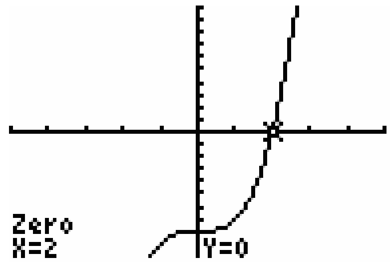
$$w = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(2)}}{2(2)}$$

$$w = \frac{-1 \pm \sqrt{-15}}{4}$$

$$w = \frac{-1 \pm i\sqrt{15}}{4}$$

The solutions are $w = -1, w = \frac{-1 \pm i\sqrt{15}}{4}$.

29. Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that $z = 2$ is a zero.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

The remaining quadratic factor is $z^2 + 2z + 4$.

Applying the quadratic formula:

$$z = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

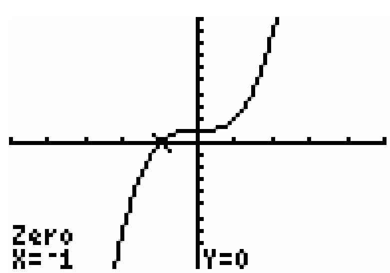
$$z = \frac{-2 \pm \sqrt{-12}}{2}$$

$$z = \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$z = -1 \pm i\sqrt{3}$$

The solutions are $z = 2, z = -1 \pm i\sqrt{3}$.

30. Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that $x = -1$ is a zero.

-1	1	0	0	1
		-1	1	-1
	1	-1	1	0

The remaining quadratic factor is $x^2 - x + 1$.

Applying the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

The solutions are $x = -1, x = \dfrac{1 \pm i\sqrt{3}}{2}$.

Section 4.4 Exercises

31. a.

50	-0.2	66	-1600	$-60{,}000$
		-10	2800	60,000
	-0.2	56	1200	0

The quadratic factor of $P(x)$ is $-0.2x^2 + 56x + 1200$.

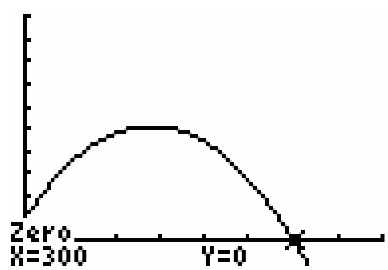
b. $-0.2x^2 + 56x + 1200 = 0$

$-0.2\left(x^2 - 280x - 6000\right) = 0$

$-0.2(x + 20)(x - 300) = 0$

$x = -20,\ x = 300$

In the context of the problem, only the positive solution is reasonable. Producing and selling 300 units results in break-even.



[–20, 400] by [–1000, 10,000]

32. a.

10	-1	98	-700	-1800
		-10	880	1800
	-1	88	180	0

The quadratic factor of $P(x)$ is $-1x^2 + 88x + 180$.

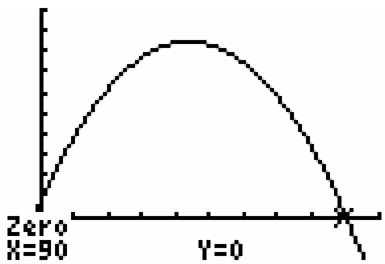
b. $-x^2 + 88x + 180 = 0$

$-1\left(x^2 - 88x - 180\right) = 0$

$-1(x-90)(x+2) = 0$

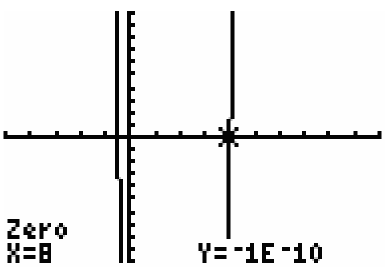
$x = 90,\ x = -2$

In the context of the problem only the positive solution is reasonable. Producing and selling 90 units results in break-even.



[–10, 100] by [–500, 2500]

33. a.



[–10, 20] by [–100, 100]

b. Based on the graph in part a), one x-intercept appears to be x = 8.

c.

8)	–0.1	50.7	–349.2	–400
		–0.8	399.2	400
	–0.1	49.9	50	0

The quadratic factor of $P(x)$ is $-0.1x^2 + 49.9x + 50.$

d. Based on parts b) and c), one zero is $x = 8.$ To find more zeros, solve $-0.1x^2 + 49.9x + 50 = 0.$

$-0.1\left(x^2 - 499 - 500\right) = 0$

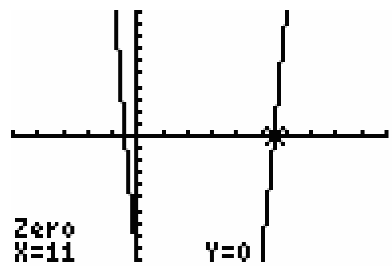
$-0.1(x-500)(x+1) = 0$

$x = 500,\ x = -1$

The zeros are $x = 500,\ x = -1,\ x = 8.$

e. Based on the context of the problem producing and selling 8 units or 500 units results in break-even.

34. a.



[–10, 20] by [–100, 100]

b. Based on the graph in part a), one x-intercept appears to be x = 11.

c.

11)	–0.1	10.9	–97.9	–108.9
		–1.1	107.8	108.9
	–0.1	9.8	9.9	0

The quadratic factor of $P(x)$ is $-0.1x^2 + 9.8x + 9.9.$

d. Based on parts b) and c), one zero is $x = 11.$ To find more zeros, solve $-0.1x^2 + 9.8x + 9.9 = 0.$

$-0.1\left(x^2 - 98 - 99\right) = 0$

$-0.1(x-99)(x+1) = 0$

$x = 99,\ x = -1$

The zeros are $x = 99, x = -1, x = 11.$

e. Based on the context of the problem producing and selling 11 units or 99 units results in break-even.

35. $R(x) = 9000$

$1810x - 81x^2 - x^3 = 9000$

$x^3 + 81x^2 - 1810x + 9000 = 0$

Since $x = 9$ is a solution,

9)	1	81	–1810	9000
		9	810	–9000
	1	90	–1000	0

The quadratic factor of $R(x)$ is $x^2 + 90x - 1000$. To determine more solutions, solve $x^2 + 90x - 1000 = 0$.

$(x + 100)(x - 10) = 0$

$x = -100, x = 10$

Revenue of $9000 is also achieved by selling 10 units.

36. $R(x) = 1000$

$250x - 5x^2 - x^3 = 1000$

$x^3 + 5x^2 - 250x + 1000 = 0$

Since $x = 5$ is a solution,

5)	1	5	–250	1000
		5	50	–1000
	1	10	–200	0

The quadratic factor of $R(x)$ is $x^2 + 10x - 200$. To determine more solutions, solve $x^2 + 10x - 200 = 0$.

$(x + 20)(x - 10) = 0$

$x = -20, x = 10$

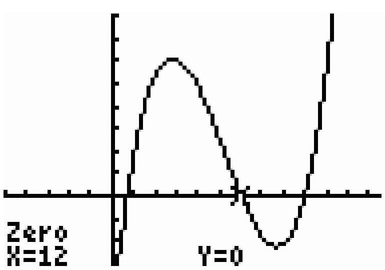
Revenue of $1000 is also achieved by selling 10 units.

37. a. $y = 244$

$0.4566x^3 - 14.3085x^2 + 117.2978x + 107.8456 = 244$

$0.4566x^3 - 14.3085x^2 + 117.2978x - 136.1544 = 0$

b.



[–10, 25] by [–75,200]

It appears that $x = 12$ is a zero.

c.

12)	0.4566	–14.3085	117.2978	–136.1544
		5.4792	–105.9516	136.1544
	0.4566	– 8.8293	11.3462	0

The quadratic factor of $P(x)$ is $0.4566x^2 - 8.8293x + 11.3462$.

d. $0.4566x^2 - 8.8293x + 11.3462 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8.8293) \pm \sqrt{(-8.8293)^2 - 4(0.4566)(11.3462)}}{2(0.4566)}$$

$$x = \frac{8.8293 \pm \sqrt{57.23383881}}{0.9132}$$

$x = 17.953,\ x = 1.384$

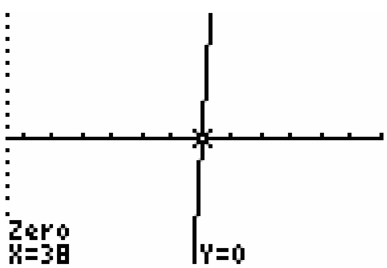
e. Based on the solutions in previous parts, the number of fatalities is 244 in 1982, 1992, and 1998.

38. a. $y = 2862$

$0.20x^3 - 13.71x^2 + 265.06x + 1612.56 = 2862$

$0.20x^3 - 13.71x^2 + 265.06x - 1249.44 = 0$

b.



[25, 50] by [–50,50]

It appears that $x = 38$ is a zero.

c.

38)	0.20	−13.71	265.06	−1249.44
		7.6	−232.18	1249.44
	0.20	−6.11	32.88	0

The quadratic factor of $P(x)$ is $0.2x^2 - 6.11x + 32.88$.

d. $0.2x^2 - 6.11x + 32.88 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

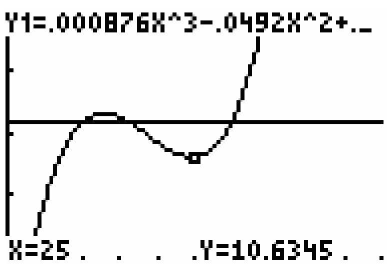
$$x = \frac{-(-6.11) \pm \sqrt{(-6.11)^2 - 4(0.2)(32.88)}}{2(0.2)}$$

$$x = \frac{6.11 \pm \sqrt{11.0281}}{0.4}$$

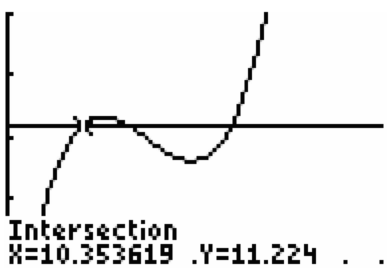
$$x = 23.58, x = 6.97$$

e. Based on the solutions in previous parts, college enrollment in thousands is 2862 in 1967, 1984, and 1998.

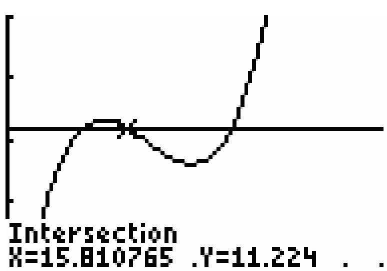
39. Applying the intersection of graphs method:



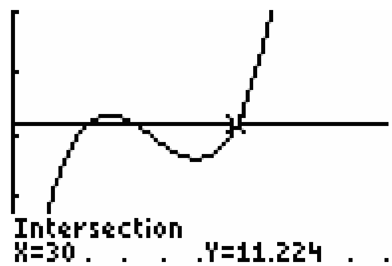
[0, 50] by [9, 13]



[0, 50] by [9, 13]



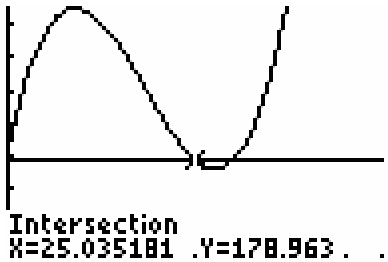
[0, 50] by [9, 13]



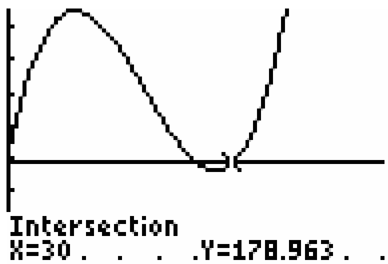
[0, 50] by [9, 13]

Based on the graphs, 11,224 births occur in 1971, 1976, and 1990.

40. Applying the intersection of graphs method:



[0, 50] by [150, 225]



[0, 50] by [150, 225]

Based on the graphs, 178,963 births occur in 1985 and 1990.

Section 4.5 Skills Check

1. **a.** To find the vertical asymptote let
$q(x)=0.$
$x-5=0$
$x=5$ is the vertical asymptote.

b. The degree of the numerator is less than the degree of the denominator. Therefore, $y=0$ is the horizontal asymptote.

2. **a.** To find the vertical asymptote let
$q(x)=0.$
$x-4=0$
$x=4$ is the vertical asymptote.

b. The degree of the numerator is less than the degree of the denominator. Therefore, $y=0$ is the horizontal asymptote.

3. **a.** To find the vertical asymptote let
$q(x)=0.$
$5-2x=0$
$-2x=-5$
$x=\frac{-5}{-2}=\frac{5}{2}$ is the vertical asymptote.

b. The degree of the numerator is equal to the degree of the denominator.
Therefore, $y=\frac{1}{-2}=-\frac{1}{2}$ is the horizontal asymptote.

4. **a.** To find the vertical asymptote let
$q(x)=0.$
$3-x=0$
$-x=-3$
$x=3$ is the vertical asymptote.

b. The degree of the numerator is equal to the degree of the denominator.
Therefore, $y=\frac{2}{-1}=-2$ is the horizontal asymptote.

5. **a.** To find the vertical asymptote let
$q(x)=0.$
$x^2-1=0$
$(x+1)(x-1)=0$
$x=-1, x=1$ are the vertical asymptotes.

b. The degree of the numerator is greater than the degree of the denominator. Therefore, there is not a horizontal asymptote.

6. **a.** To find the vertical asymptote let
$q(x)=0.$
$x^2+3=0$
$x^2=-3$
$x=\pm\sqrt{-3}=\pm i\sqrt{3}$
Since there is not a real number solution to the equation, there is not a vertical asymptote.

b. The degree of the numerator is equal to the degree of the denominator.
Therefore, $y=\frac{1}{1}=1$ is the horizontal asymptote.

7. The function in part c) does not have a vertical asymptote. Its denominator cannot be zero. Parts a), b), and d) all have denominators that can equal zero for specific x-values.

8. The function in part c) does not have a horizontal asymptote. The degree of the numerator is greater than the degree of the denominator. Note that a slant asymptote may exist.

9. a. The degree of the numerator is equal to the degree of the denominator.

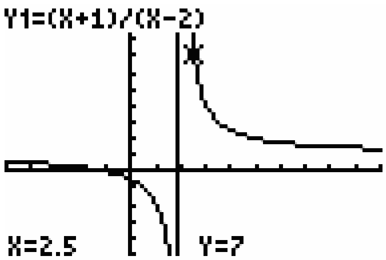
Therefore, $y = \frac{1}{1} = 1$ is the horizontal asymptote.

b. To find the vertical asymptote let $q(x) = 0$.

$x - 2 = 0$

$x = 2$ is the vertical asymptote.

c.



[–5, 10] by [–5, 10]

10. a. The degree of the numerator is equal to the degree of the denominator.

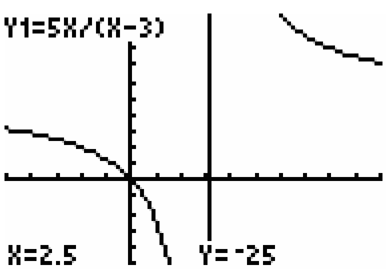
Therefore, $y = \frac{5}{1} = 5$ is the horizontal asymptote.

b. To find the vertical asymptote let $q(x) = 0$.

$x - 3 = 0$

$x = 3$ is the vertical asymptote.

c.



[–5, 10] by [–5, 10]

11. a. The degree of the numerator is less than the degree of the denominator.

Therefore, $y = 0$ is the horizontal asymptote.

b. To find the vertical asymptote let $q(x) = 0$.

$1 - x^2 = 0$

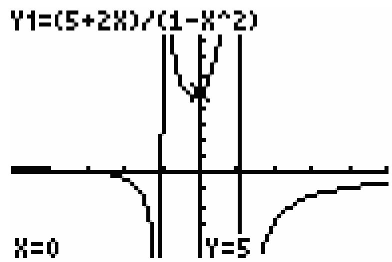
$x^2 = 1$

$\sqrt{x^2} = \pm\sqrt{1}$

$x = \pm 1$

$x = 1, x = -1$ are the vertical asymptotes.

c.



[–5, 5] by [–5, 10]

12. a. The degree of the numerator is greater than the degree of the denominator. Therefore, there is not a horizontal asymptote.

b. To find the vertical asymptote let $q(x) = 0$.

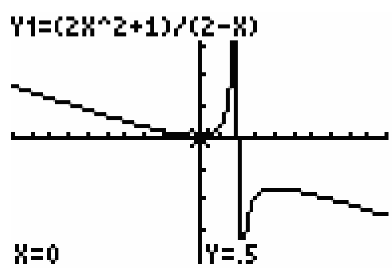
$2 - x = 0$

$-x = -2$

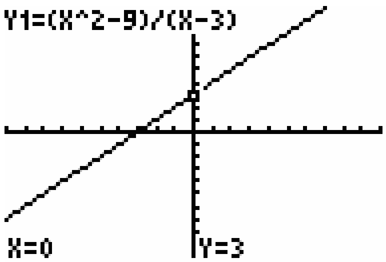
$x = 2$

$x = 2$ is the vertical asymptote.

c.



[–10, 10] by [–40, 40]

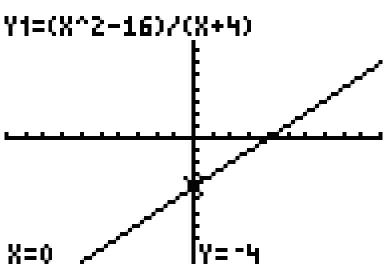
13. 

[–10, 10] by [–10, 10]

X	Y1
0	3
1	4
2	5
3	ERROR
4	7
5	8
6	9

X=3

There is a hole in the graph at $x = 3$.

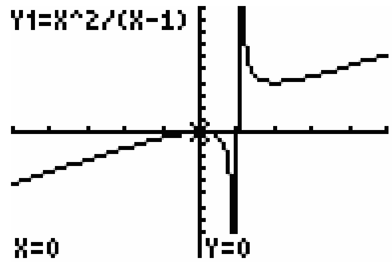
14. 

[–10, 10] by [–10, 10]

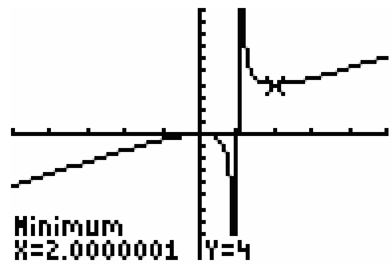
X	Y1
-6	-10
-5	-9
-4	ERROR
-3	-7
-2	-6
-1	-5
0	-4

X= -4

There is a hole in the graph at $x = –4$.

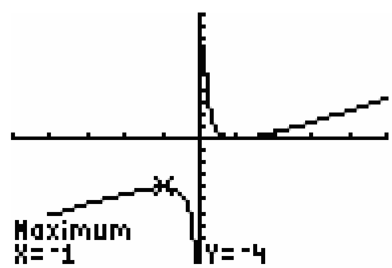
15. 

[–5, 5] by [–10, 10]

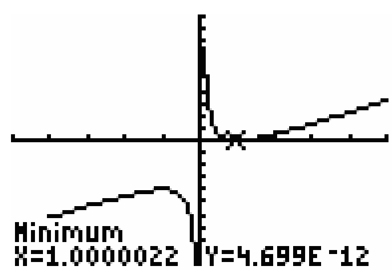


[–5, 5] by [–10, 10]

Based on the graphs, the turning points appear to be (0, 0) and (2, 4).

16. 

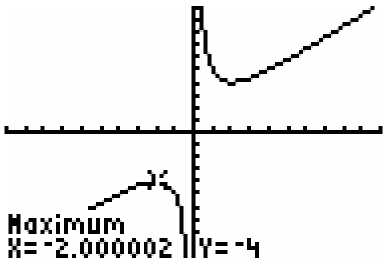
[–5, 5] by [–10, 10]



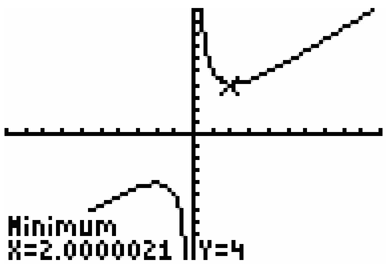
[–5, 5] by [–10, 10]

Based on the graphs, the turning points appear to be (–1, –4) and (1, 0).

17.



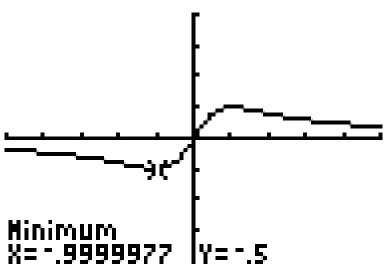
[–10, 10] by [–10, 10]



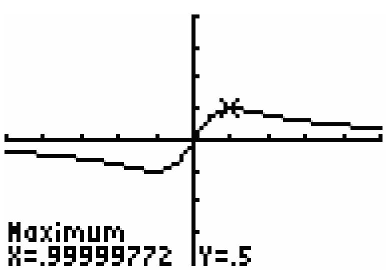
[–10, 10] by [–10, 10]

Based on the graphs, the turning points appear to be (–2, –4) and (2, 4).

18.



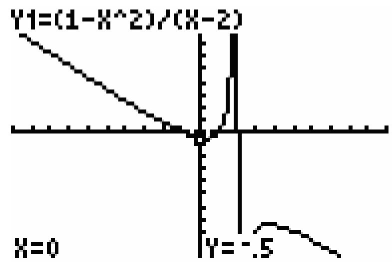
[–5, 5] by [–2, 2]



[–5, 5] by [–2, 2]

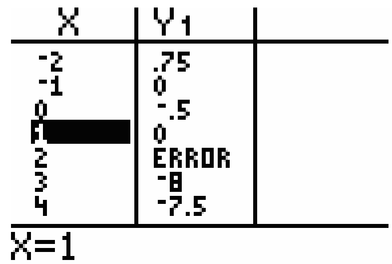
Based on the graphs, the turning points appear to be (–1, –0.5) and (1, 0.5).

19. a.



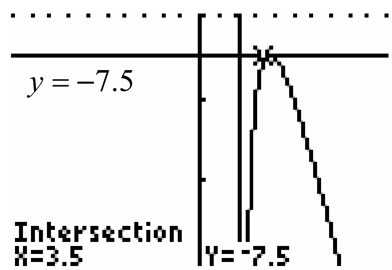
[–10, 10] by [–10, 10]

b.

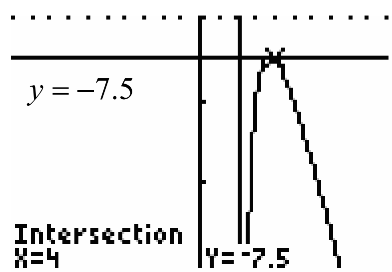


Based on the graph and the table, when $x = 1$, $y = 0$ and when $x = 3$, $y = -8$.

c.



[–10, 10] by [–10, –7]



[–10, 10] by [–10, –7]

Based on the graphs it appears that if $y = -7.5$, then $x = 3.5$ or $x = 4$.

d. $-7.5 = \dfrac{1-x^2}{x-2}$ LCM: $x-2$

$-7.5(x-2) = \left(\dfrac{1-x^2}{x-2}\right)(x-2)$

$-7.5x + 15 = 1 - x^2$

$x^2 - 7.5x + 14 = 0$

$10(x^2 - 7.5x + 14) = 10(0)$

$10x^2 - 75x + 140 = 0$

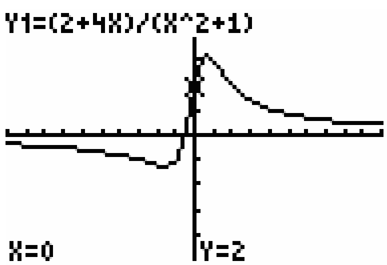
$(x-4)(10x-35) = 0$

$x - 4 = 0, \quad 10x - 35 = 0$

$x = 4, x = 3.5$

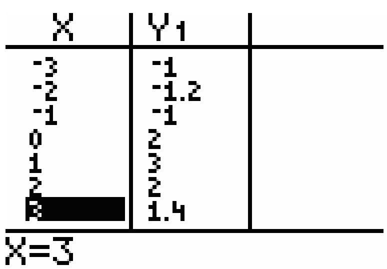
Both solutions check.

20. a.



[–10, 10] by [–5, 5]

b.

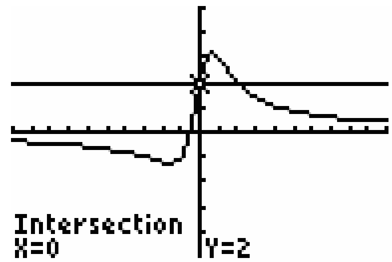


X	Y1
-3	-1
-2	-1.2
-1	-1
0	2
1	3
2	2
3	1.4

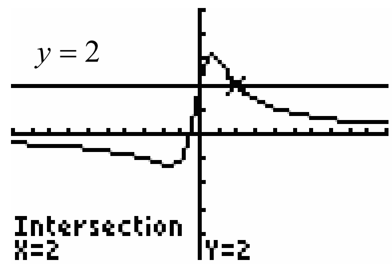
X=3

Based on the graph and the table, when $x = 3$, $y = 1.4$ and when $x = -3$, $y = -1$.

c.



[–10, 10] by [–5, 5]



[–10, 10] by [–5, 5]

Based on the graphs it appears that if $y = 2$, then $x = 0$ or $x = 2$.

d. $2 = \dfrac{2+4x}{x^2+1}$ LCM: x^2+1

$2(x^2+1) = \left(\dfrac{2+4x}{x^2+1}\right)(x^2+1)$

$2x^2 + 2 = 2 + 4x$

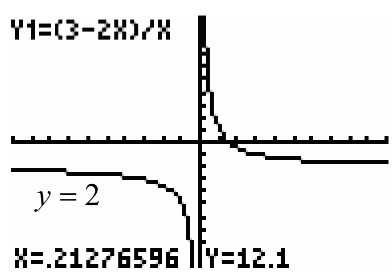
$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0, \quad x = 2$

Both solutions check.

21. a.



[–10, 10] by [–10, 10]

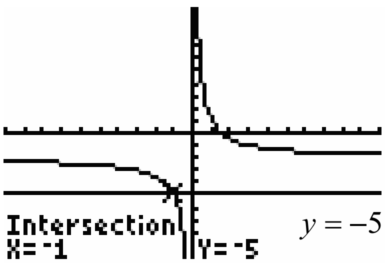
b.

X	Y_1
-3	-3
-2	-3.5
-1	-5
0	ERROR
1	1
2	-.5
3	-1

X= -3

Based on the graph and the table, when $x = 3$, $y = -1$ and when $x = -3$, $y = -3$.

c.



[–10, 10] by [–10, 10]

Based on the graph it appears that if $y = -5$, then $x = -1$.

d. $-5 = \dfrac{3-2x}{x}$ LCM : x

$$-5x = \left(\frac{3-2x}{x}\right)x$$

$$-5x = 3 - 2x$$

$$-3x = 3$$

$$x = -1$$

The solution checks.

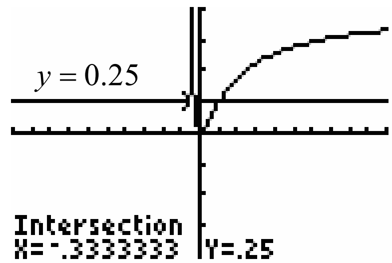
b.

X	Y_1
-3	2.25
-2	4
-1	ERROR
0	0
1	.25
2	.44444
3	.5625

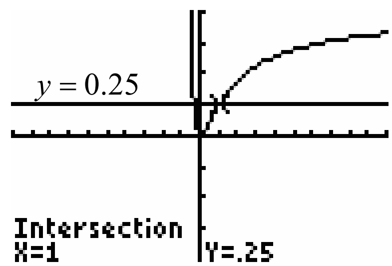
X= -2

Based on the graph and the table, when $x = 0$, $y = 0$ and when $x = -2$, $y = 4$.

c.



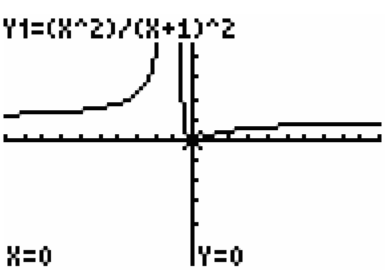
[–10, 10] by [–1, 1]



[–10, 10] by [–1, 1]

Based on the graphs it appears that if $y = 0.25$, then $x = -\dfrac{1}{3}$ or $x = 1$.

22. a.



[–10, 10] by [–6, 6]

d. $0.25 = \dfrac{x^2}{(x+1)^2}$

$\dfrac{1}{4} = \dfrac{x^2}{(x+1)^2}$ LCM : $4(x+1)^2$

$\dfrac{1}{4}\left[4(x+1)^2\right] = \left[\dfrac{x^2}{(x+1)^2}\right]\left[4(x+1)^2\right]$

$(x+1)^2 = 4x^2$

$x^2 + 2x + 1 = 4x^2$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$3x + 1 = 0, \quad x - 1 = 0$

$x = -\dfrac{1}{3}, \quad x = 1$

Both solutions check.

23. $\dfrac{x^2+1}{x-1} + x = 2 + \dfrac{2}{x-1}$ LCM: $x-1$

$(x-1)\left(\dfrac{x^2+1}{x-1} + x\right) = (x-1)\left(2 + \dfrac{2}{x-1}\right)$

$x^2 + 1 + x(x-1) = 2(x-1) + 2$

$x^2 + 1 + x^2 - x = 2x - 2 + 2$

$2x^2 - x + 1 = 2x$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$x = \dfrac{1}{2}, \quad x = 1$

$x = 1$ does not check.

The only solution is $x = \dfrac{1}{2}$.

24. $\dfrac{x}{x-2} - x = 1 + \dfrac{2}{x-2}$ LCM: $x-2$

$(x-2)\left(\dfrac{x}{x-2} - x\right) = (x-2)\left(1 + \dfrac{2}{x-2}\right)$

$x - x(x-2) = 1(x-2) + 2$

$x - x^2 + 2x = x - 2 + 2$

$-x^2 + 3x = x$

$-x^2 + 2x = 0$

$-x(x-2) = 0$

$-x = 0, \quad x - 2 = 0$

$x = 0, \quad x = 2$

$x = 2$ does not check.

The only solution is $x = 0$.

Section 4.5 Exercises

25. a. $\overline{C} = \dfrac{400 + 50(500) + 0.01(500)^2}{500}$

$\overline{C} = \dfrac{27,900}{500} = 55.8$

The average cost is $55.80 per unit.

b. $\overline{C} = \dfrac{400 + 50(60) + 0.01(60)^2}{60}$

$\overline{C} = \dfrac{3436}{60} = 57.2\overline{6}$

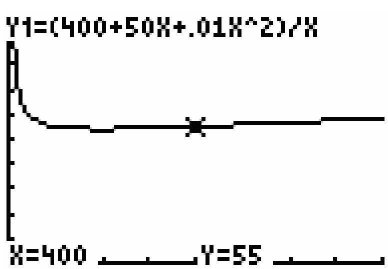
The average cost is $57.27 per unit.

c. $\overline{C} = \dfrac{400 + 50(100) + 0.01(100)^2}{100}$

$\overline{C} = \dfrac{5500}{100} = 55$

The average cost is $55 per unit.

d. No. Consider the graph of the function.



[0, 800] by [0, 100]

Note that the graph passes through a minimum and then begins to increase.

26. a. $\overline{C} = \dfrac{1000 + 30(30) + 0.1(30)^2}{30}$

$\overline{C} = \dfrac{1990}{30} = 66.\overline{3}$

The average cost is $66.33 per unit.

b. $\overline{C} = \dfrac{1000 + 30(300) + 0.1(300)^2}{300}$

$\overline{C} = \dfrac{19000}{300} = 63.\overline{3}$

The average cost is $63.33 per unit.

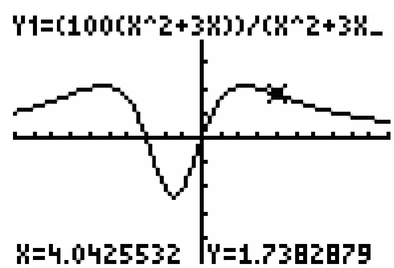
c. When $x = 0$, the function is undefined. If no units are produced, then there can be no average cost.

27. a. $y = \dfrac{400(5)}{5 + 20} = \dfrac{2000}{25} = 80$

$5000 in advertising expenditures results in sales volume of $80,000.

b. When $x = -20$, the denominator is zero and the function is undefined. Since advertising expenditures cannot be negative, x cannot be –20 in the context of the problem.

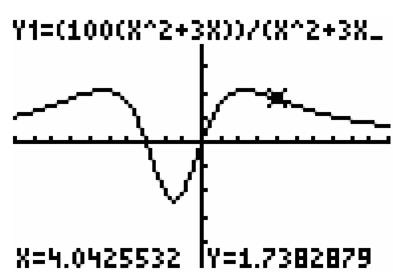
28. a.



[–10, 10] by [–5, 5]

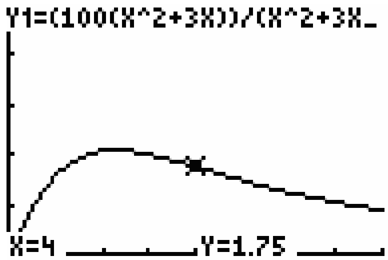
Based on the graph, productivity is higher around lunch than at quitting time.

b.



[–10, 10] by [–5, 5]

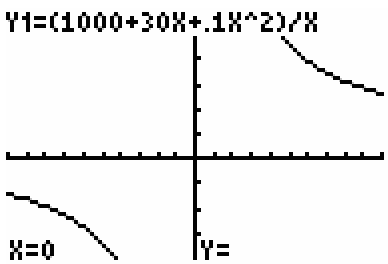
c.



[0, 8] by [0, 5]

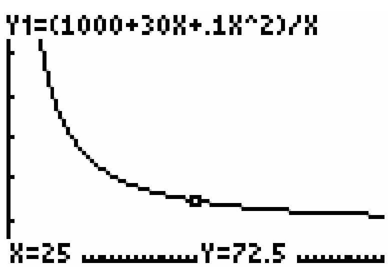
d. The graph in part c) is a better model of the physical situation. It displays the function over its domain of $0 \le t \le 8$.

29. a.



[–10, 10] by [–200, 300]

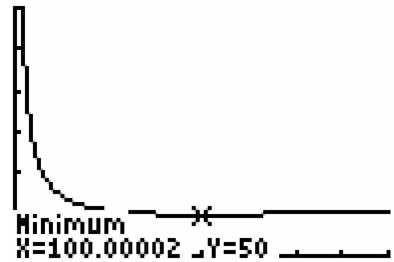
b.



[0, 50] by [0, 300]

c. The graph in part b) fits the context of the problem since both the viewing window for both x and $\overline{C}(x)$ are greater than or equal to zero.

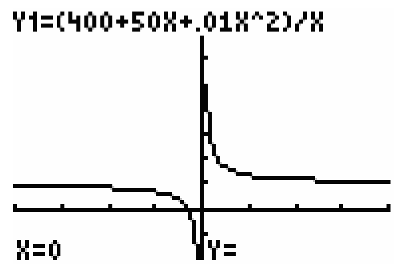
d.



[0, 200] by [0, 300]

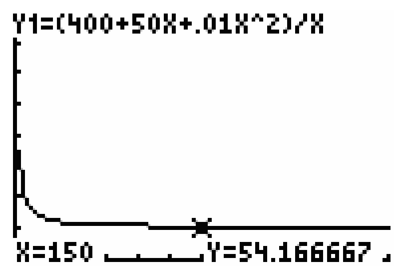
The minimum average cost of $50 occurs when 10,000 units are produced.

30. a.



[–100, 100] by [–100, 400]

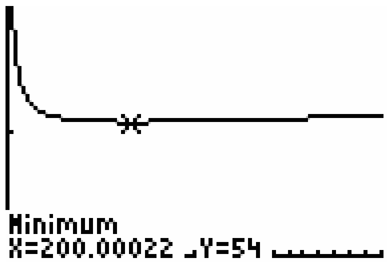
b.



[0, 300] by [0, 400]

c. The graph in part b) is more appropriate, since producing a negative number of units does not make sense in the context of the question.

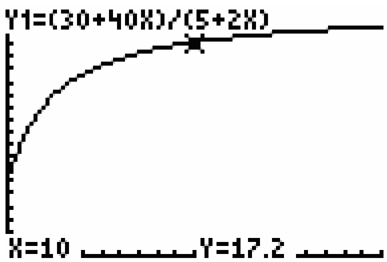
d.



[0, 600] by 0, 100]

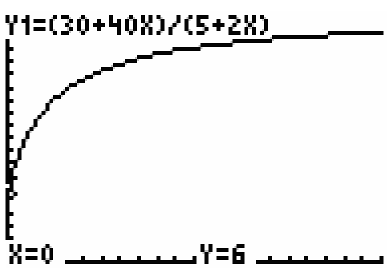
The minimum average cost of $54 occurs when 20,000 units are produced.

31. a.



[0, 20] by [0, 20]

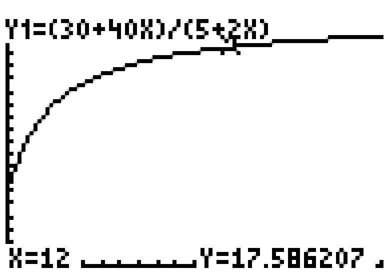
b.



[0, 20] by [0, 20]

$f(0)=6$. The initial number of employees for the startup company is 6.

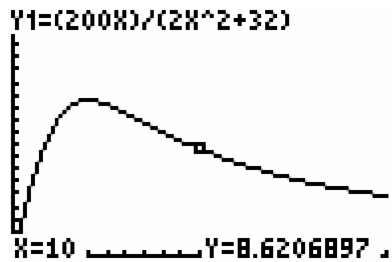
c.



[0, 20] by [0, 20]

$f(12)\approx 17.59$. After 12 months, the number of employees for the startup company is approximately 18.

32. a.



[0, 20] by [0, 20]

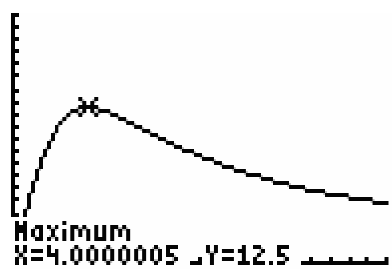
b.

X	Y1
0	0
1	5.8824
2	10
3	12
4	12.5
5	12.195
6	11.538

X=5

One hour after the injection the drug concentration is 5.88%. Five hours after the injection, the drug concentration is 12.20%.

c.



[0, 20] by [0, 20]

The maximum drug concentration is 12.5% occurring 4 hours after the injection.

d. After four hours, the drug concentration begins to drop until it reaches a level of zero. The drug concentration is zero when none of the drug remains in the system.

33. a. To find the vertical asymptote let

$q(x) = 0.$

$100 - p = 0$

$-p = -100$

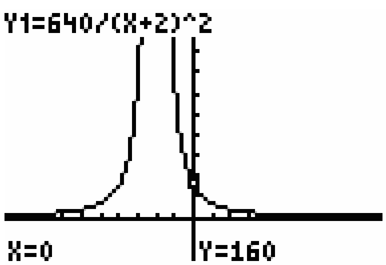
$p = 100$ is the vertical asymptote.

b. Since $p \neq 100$, 100% of the impurities can not be removed from the waste water.

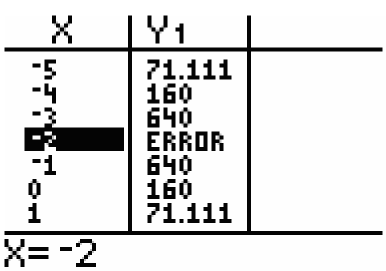
34. a. If spending is allowed to increase without bound, the function will approach its horizontal asymptote. Since the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is $p = \frac{100}{1} = 100.$ Therefore as spending approaches infinity, the percentage of pollution removed approaches 100%.

b. No. 100% of the pollution cannot be removed. To do so would require spending an infinite amount of money.

35. a.



[–10, 10] by [–200, 1000]



Based on the graph and the table, the vertical asymptote occurs at $p = -2.$

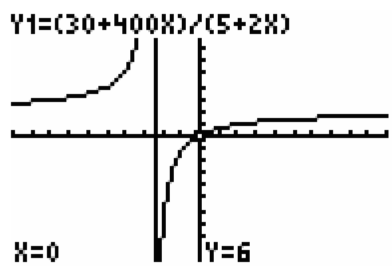
b.

Price	Sales Volume
5	13,061
20	1322
50	237
100	62
200	16
500	3

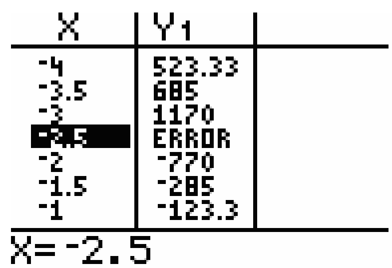
c. The domain of function in the context of the problem is $p \geq 0$. There is no vertical asymptote on the restricted domain.

d. The horizontal asymptote is $V = 0.$ As the price grows without bound, the sales volume approaches zero units.

36. a.



[–10, 10] by [–1000, 1000]



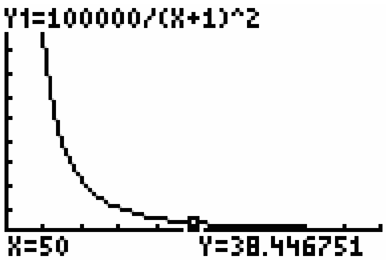
There is a vertical asymptote at $t = -2.5$.

b. No. The only vertical asymptote is at $t = -2.5$.

c. Yes. Since the degree of the numerator equals the degree of the denominator the horizontal asymptote is $N = \frac{400}{2} = 200.$

d. As the number of months grows without bound, the number of employees approaches 200.

37. a.

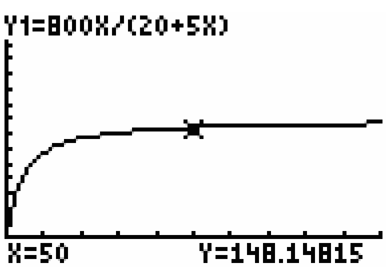


[0, 100] by [0, 1000]

b. The horizontal asymptote is $p = 0$.

c. As the price falls, the quantity demanded increases.

38. a.



[0, 100] by 0, 300]

b. Since the degree of the numerator equals the degree of the denominator the horizontal asymptote is $y = \frac{800}{5} = 160$.

c.

Advertising expenses	Weekly sales
0	0
50	14,814.81
100	15,384.62
200	15,686.27
300	15,789.47
500	15,873.02

d. If an unlimited amount of money is spent on advertising, then the weekly sales will approach \$16,000, represented by the horizontal asymptote of the function.

39. a. $S = \frac{40}{x} + \frac{x}{4} + 10$ LCM: $4x$

$$\left(\frac{40}{x}\right)\left(\frac{4}{4}\right) + \frac{x}{4}\left(\frac{x}{x}\right) + \left(\frac{10}{1}\right)\left(\frac{4x}{4x}\right)$$

$$\frac{160}{4x} + \frac{x^2}{4x} + \frac{40x}{4x}$$

$$\frac{x^2 + 40x + 160}{4x}$$

$$S = \frac{x^2 + 40x + 160}{4x}$$

b. $21 = \frac{x^2 + 40x + 160}{4x}$

$$21(4x) = \left(\frac{x^2 + 40x + 160}{4x}\right)(4x)$$

$$84x = x^2 + 40x + 160$$

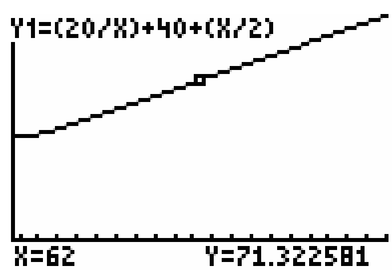
$$x^2 - 44x + 160 = 0$$

$$(x - 40)(x - 4) = 0$$

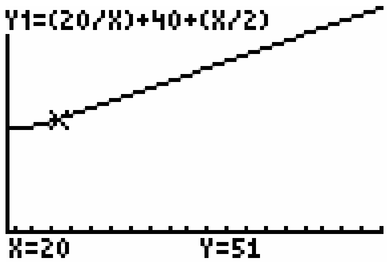
$$x = 40, \quad x = 4$$

After 4 hours or 40 hours of training, the monthly sales will be \$21,000.

40. a.

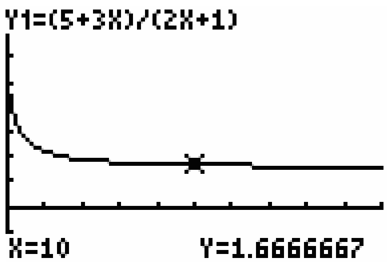


[4, 120] by [–10, 100]

b. 

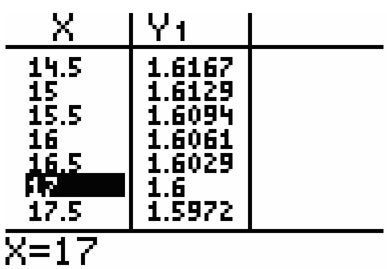
[4, 120] by [–10, 100]

Based on the model, 20 hours of training corresponds to sales of $51,000.

41. a. 

[0, 20] by [0, 8]

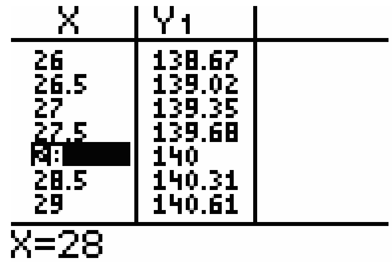
b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $H = \frac{3}{2}$. As the amount of training increases, the time it takes to assemble one unit approaches 1.5 hours.

c. 

X	Y1
14.5	1.6167
15	1.6129
15.5	1.6094
16	1.6061
16.5	1.6029
17	1.6
17.5	1.5972

X=17

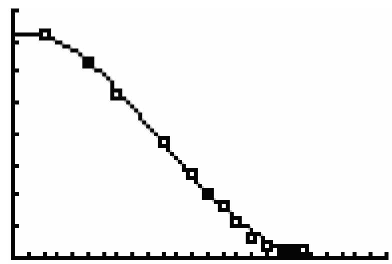
It takes 17 days of training to reduce the time it takes to assemble one unit to 1.6 hours.

42. 

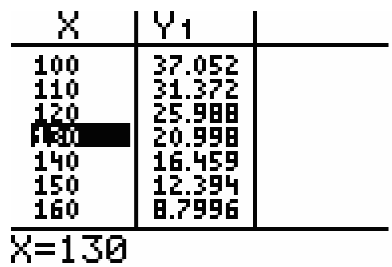
X	Y1
26	138.67
26.5	139.02
27	139.35
27.5	139.68
28	140
28.5	140.31
29	140.61

X=28

Advertising expenses of $2800 produces weekly sales of $14,000.

43. a. 

[0, 250] by [0, 80]

b. 

X	Y1
100	37.052
110	31.372
120	25.988
130	20.998
140	16.459
150	12.394
160	8.7996

X=130

The predicted value is 21.0%, while the actual value in the table is 21.2%. The values are relatively close.

44. a. Let A = Area.

$$A = x \bullet y$$
$$x \bullet y = 51,200$$

b. Let L = Fence Perimeter.

$$L = x + 2y$$

c. $L = x + 2y$

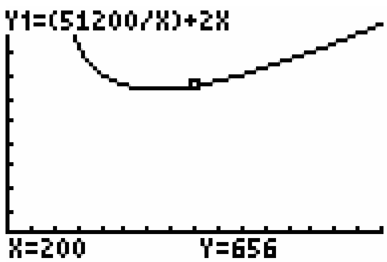
$A = x \cdot y = 51,200$

$x = \dfrac{51,200}{y}$

Substituting for x in the formula for L:

$L = \dfrac{51,200}{y} + 2y$

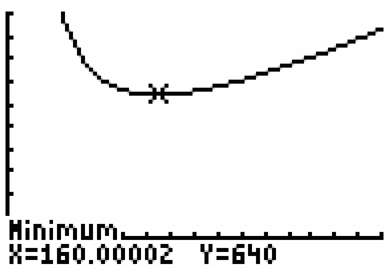
d.



[0, 400] by [0, 1000]

Note that in this case, the x-axis represents the variable y, and the y-axis represents the variable L.

e.



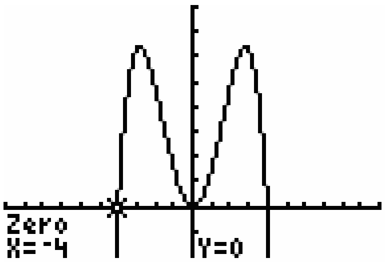
[0, 400] by [0, 1000]

Note that in this case, the x-axis represents the variable y, and the y-axis represents the variable L.

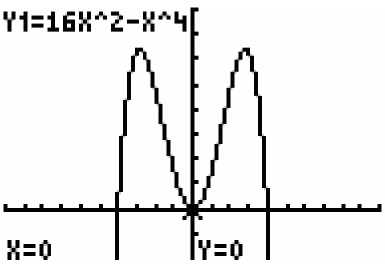
The minimum length of fence occurs when $y = 160$. Therefore, the total length of the fence is $L = \dfrac{51,200}{160} + 2(160) = 640$ feet. The dimensions of the rectangular field are $y = 160$ feet and $x = \dfrac{51,200}{160} = 320$ feet.

Section 4.6 Skills Check

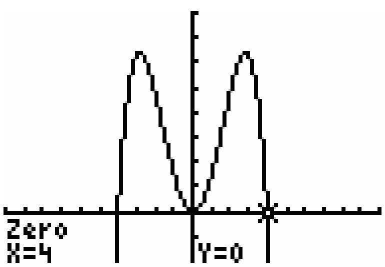
1. Applying the x-intercept method:



[–10, 10] by [–20, 80]



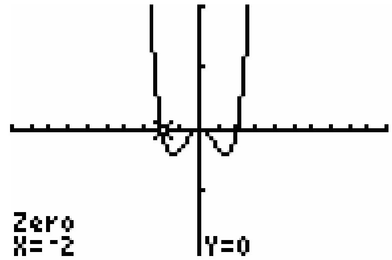
[–10, 10] by [–20, 80]



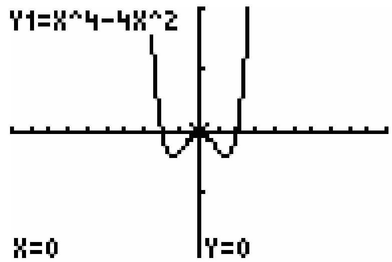
[–10, 10] by [–20, 80]

The function is greater than or equal to zero on the interval $[-4,4]$ or when $-4 \le x \le 4$.

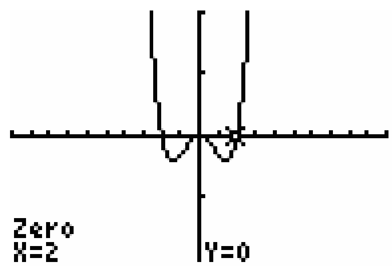
2. Applying the x-intercept method:



[–10, 10] by [–20, 20]



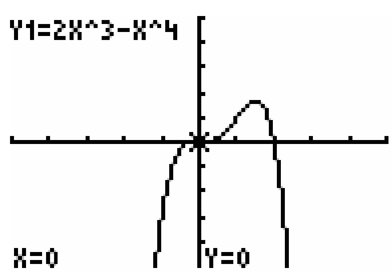
[–10, 10] by [–20, 20]



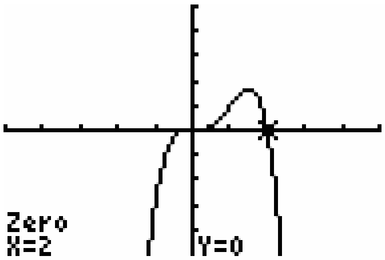
[–10, 10] by [–20, 20]

The function is less than or equal to zero on the interval $[-2,2]$ or when $-2 \le x \le 2$.

3. Applying the x-intercept method:



[–5, 5] by [–5, 5]



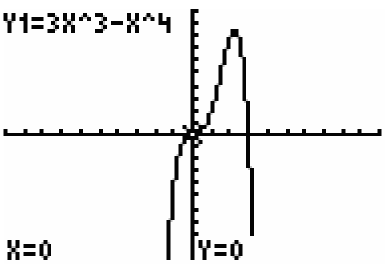
[–5, 5] by [–5, 5]

The function is less than zero on the interval $(-\infty,0)\cup(2,\infty)$ or when $x<0$ or $x>2$.

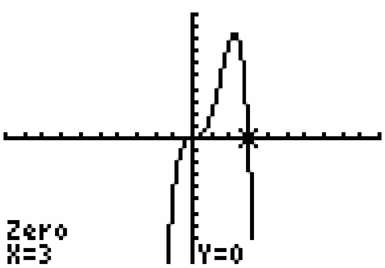
4. $3x^3 \ge x^4$

$3x^3 - x^4 \ge 0$

Applying the x-intercept method:



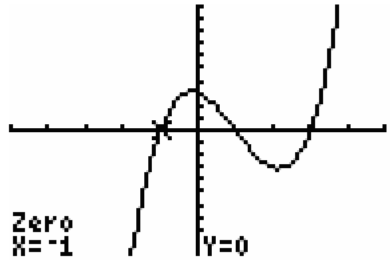
[–10, 10] by [–10, 10]



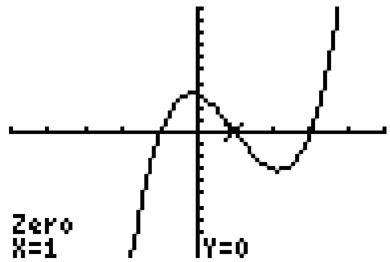
[–10, 10] by [–10, 10]

The function is greater than or equal to zero on the interval $[0,3]$ or when $0\le x\le 3$.

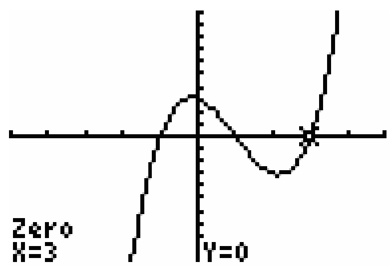
5. Applying the x-intercept method:



[–5, 5] by [–10, 10]



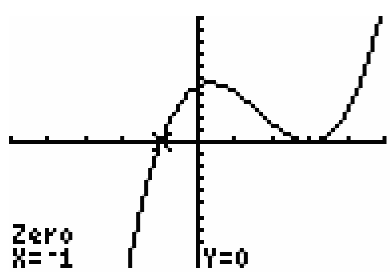
[–5, 5] by [–10, 10]



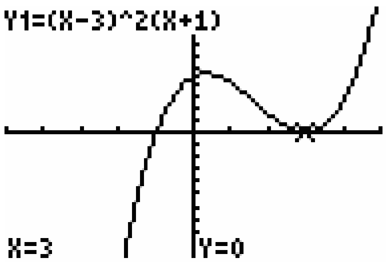
[–5, 5] by [–10, 10]

The function is greater than or equal to zero on the interval $[-1,1]\cup[3,\infty)$ or when $-1\le x\le 1$ or $x\ge 3$.

6. Applying the x-intercept method:



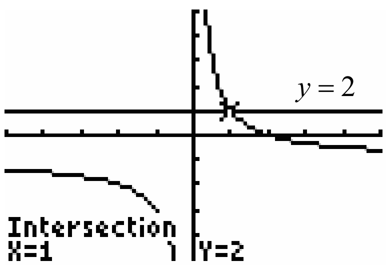
[–5, 5] by [–20, 20]



[–5, 5] by [–20, 20]

The function is less than zero on the interval $(-\infty,-1)$ or when $x<-1$.

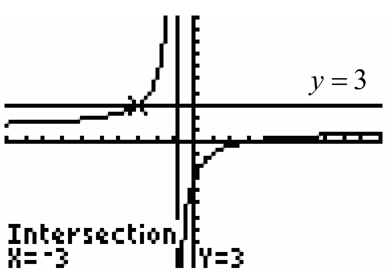
7. Applying the intersection of graphs method:



[–5, 5] by [–10, 10]

Note that the graphs intersect when $x = 1$. Also note that a vertical asymptote occurs at $x = 0$. Therefore, $\frac{4-2x}{x}>2$ on the interval $(0,1)$ or when $0<x<1$.

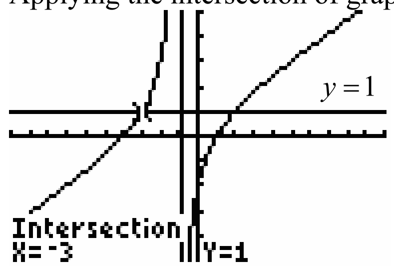
8. Applying the intersection of graphs method:



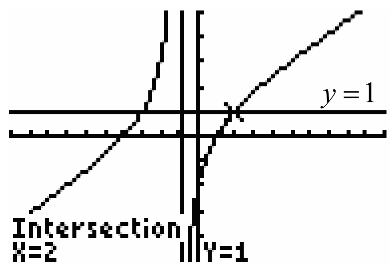
[–10, 10] by [–10, 10]

Note that the graphs intersect when $x = -3$. Also note that a vertical asymptote occurs at $x+1=0$ or $x=-1$. Therefore, $\frac{x-3}{x+1}\geq 3$ on the interval $[-3,-1)$ or when $-3\leq x<-1$.

9. Applying the intersection of graphs method:



[–10, 10] by [–5, 5]



[–10, 10] by [–5, 5]

Note that the graphs intersect when $x=-3$ and $x=2$. Also note that a vertical asymptote occurs at $x+1=0$ or $x=-1$. Therefore, $\frac{x}{2}+\frac{x-2}{x+1}\leq 1$ on the interval $(-\infty,-3]\cup(-1,2]$ or when $x\leq -3$ or $-1<x\leq 2$.

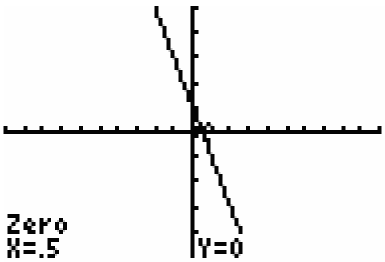
10. $\frac{x}{x-1}\leq 2x+\frac{1}{x-1}$

$\frac{x}{x-1}-2x-\frac{1}{x-1}\leq 0$

$\frac{x-1}{x-1}-2x\leq 0$

$1-2x\leq 0, \quad x\neq 1$

Applying the x-intercept method:

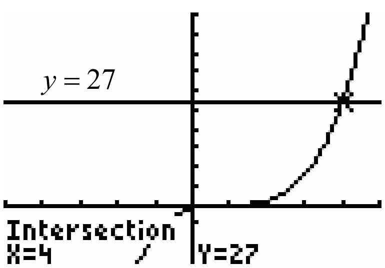


[–10, 10] by [–5, 5]

The inequality is true for all x in the domain of the function such that x is in the interval $\left[\frac{1}{2},\infty\right)$ or $x \geq \frac{1}{2}$. Recall that $x \neq 1$.

Therefore, the solution is $\left[\frac{1}{2},1\right) \cup (1,\infty)$.

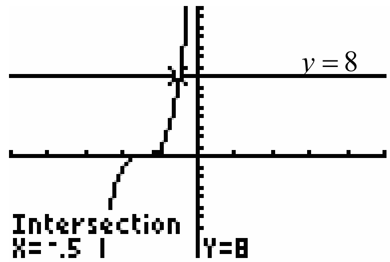
11. Applying the intersection of graphs method:



[–5, 5] by [–15, 50]

Note that the graphs intersect when $x = 4$. Therefore $(x-1)^3 > 27$ on the interval $(4,\infty)$ or when $x > 4$.

12. Applying the intersection of graphs method:

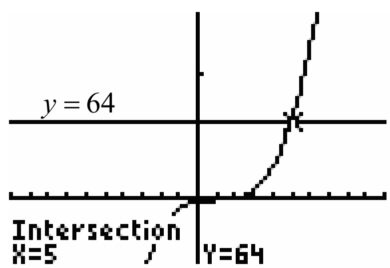


[–5, 5] by [–10, 15]

Note that the graphs intersect when $x = -\frac{1}{2}$.

Therefore, $(2x+3)^3 \leq 8$ on the interval $\left(-\infty,-\frac{1}{2}\right]$ or when $x \leq -\frac{1}{2}$.

13. Applying the intersection of graphs method:



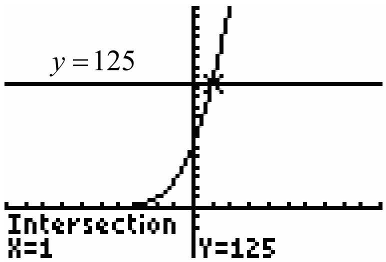
[–10, 10] by [–50, 150]

Note that the graphs intersect when $x = 5$. Therefore, $(x-1)^3 < 64$ on the interval $(-\infty,5)$ or when $x < 5$.

14. $(x+4)^3 - 125 \geq 0$

$$(x+4)^3 \geq 125$$

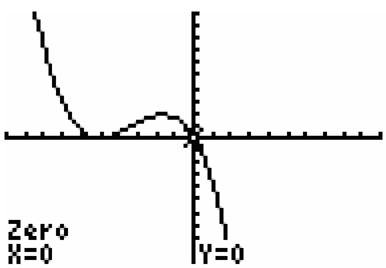
Applying the intersection of graphs method:



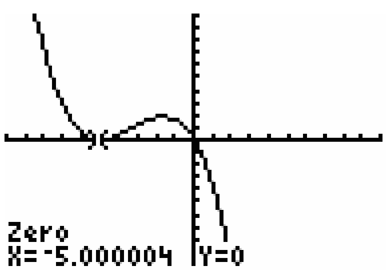
[-10, 10] by [–50, 200]

Note that the graphs intersect when $x = 1$. Therefore, $(x+4)^3 - 125 \geq 0$ on the interval $[1, \infty)$ or when $x \geq 1$.

15. Applying the intersection of graphs method:



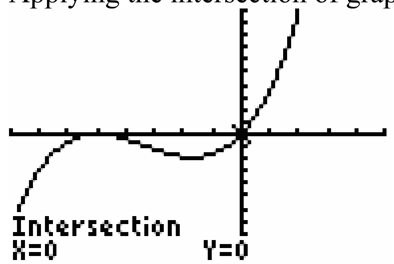
[–10, 10] by [–100, 100]



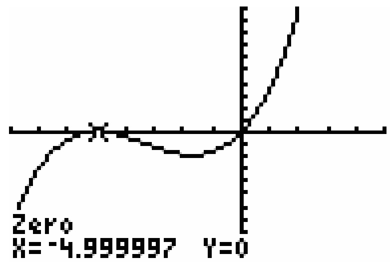
[–10, 10] by [–100, 100]

Note that the graphs intersect when $x = -5$ and $x = 0$. Therefore, $-x^3 - 10x^2 - 25x \leq 0$ on the interval $\{-5\} \cup [0, \infty)$ or when $x = -5$ or $x \geq 0$.

16. Applying the intersection of graphs method:



[–8, 5] by [–100, 100]



Note that the graphs intersect when $x = 0$ and $x = -5$. Therefore, $x^3 + 10x^2 + 25x < 0$ on the interval $(-\infty, -5) \cup (-5, 0)$ or when $x < 0$ and $x \neq -5$.

17. a. $f(x) < 0 \Rightarrow x < -3$ or $0 < x < 2$

b. $f(x) \geq 0 \Rightarrow -3 \leq x \leq 0$ or $x \geq 2$

18. a. $f(x) < 0 \Rightarrow x < 3$

b. $f(x) \geq 0 \Rightarrow x \geq 3$

19. a. $f(x) \geq 2 \Rightarrow \frac{1}{2} \leq x \leq 3$

20. a. $f(x) < 0 \Rightarrow 1 < x < 3$

b. $f(x) \geq 0 \Rightarrow x < 1$ or $x \geq 3$

Section 4.6 Exercises

21. $R = 400x - x^3$

$= x\left(400 - x^2\right)$

$= x(20 - x)(20 + x)$

To find the zeros, let $R = 0$ and solve for x.

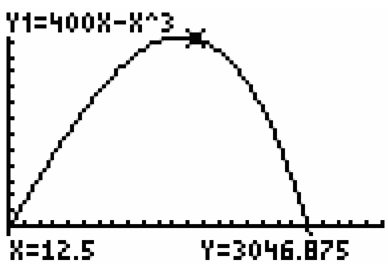
$R = 0$

$x(20 - x)(20 + x) = 0$

$x = 0,\ 20 - x = 0,\ 20 + x = 0$

$x = 0,\ x = 20,\ x = -20$

Note that since x represents product sales, only positve values of x make sense in the context of the question.



[0, 25] by [–500, 3500]

Based on the graph and the zeros calculated above, the revenue is positive, $R > 0$, in the interval $(0, 20)$ or when $0 < x < 20$. Selling between 0 and 20 units, not inclusive, generates positive revenue.

22. a. Revenue

$= (\text{Price per unit})(\text{Number of units})$

$R(x) = \left(1000 - 0.1x^2\right)(x)$

$= 1000x - 0.1x^3$

b. To find the zeros, set the revenue function equal zero and solve for x.

$1000x - 0.1x^3 = 0$

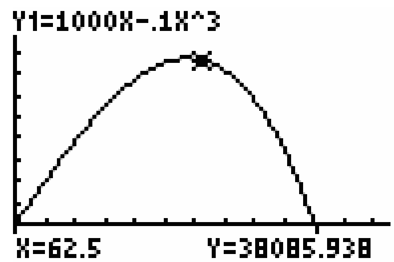
$-0.1x\left(10{,}000 - x^2\right) = 0$

$-0.1x(100 - x)(100 + x) = 0$

$-0.1x = 0,\ 100 - x = 0,\ 100 + x = 0$

$x = 0,\ x = 100,\ x = -100$

Note that since x represents product sales, only positve values of x make sense in the context of the question.



[0, 125] by [–7500, 50,000]

Based on the graph and the zeros calculated above, the revenue is positive in the interval $(0, 100)$ or when $0 < x < 100$. Selling between 0 and 100 units, not inclusive, generates positive revenue.

23. a. $V > 0$

$1296x - 144x^2 + 4x^3 > 0$

Find the zeros:

$1296x - 144x^2 + 4x^3 = 0$

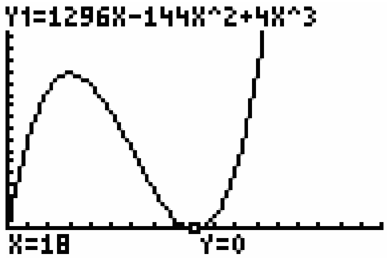
$4x^3 - 144x^2 + 1296x = 0$

$4x\left(x^2 - 36x + 324\right) = 0$

$4x(x - 18)(x - 18) = 0$

$4x = 0,\ x - 18 = 0,\ x - 18 = 0$

$x = 0,\ x = 18,\ x = 18$



[0, 36] by [–500, 5000]

Based on the graph and the zeros calculated above, the volume is positive in the interval $(0,18)\cup(18,\infty)$ or when $0<x<18$ or $x>18$.

b. In the context of the question, the largest possible cut is 18 centimeters. Therefore, to generate a positive volume, the size of the cut, *x*, must be in the interval $(0,18)$ or $0<x<18$.

24. $V>0$

$192x-56x^2+4x^3>0$

Find the zeros:

$192x-56x^2+4x^3=0$

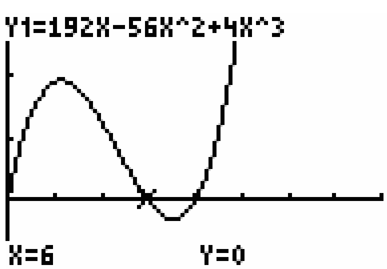
$4x^3-56x^2+192x=0$

$4x(x^2-14x+48)=0$

$4x(x-6)(x-8)=0$

$4x=0,\quad x-6=0,\quad x-8=0$

$x=0,\quad x=6,\quad x=8$



[0, 16] by [–100, 300]

Based on the graph and the zeros calculated above, the volume is positive in the interval $(0,6)\cup(8,\infty)$ or when $0<x<6$ or $x>8$.
In the physical context of the question, the largest possible cut is 6 inches. A cut of larger than 6 inches would not be possible on the side of the cardboard measuring 12 inches. Therefore, to generate a positive volume, the size of the cut, *x*, must be in the interval $(0,6)$ or $0<x<6$.

25. $C(x)\ge 1200$

$3x^3-6x^2-300x+1800\ge 1200$

$3x^3-6x^2-300x+600\ge 0$

Find the zeros:

$3x^3-6x^2-300x+600=0$

$3(x^3-2x^2-100x+200)=0$

$3\left[(x^3-2x^2)+(-100x+200)\right]=0$

$3\left[x^2(x-2)+(-100)(x-2)\right]=0$

$3(x-2)(x^2-100)=0$

$3(x-2)(x+10)(x-10)=0$

$x-2=0,\quad x+10=0,\quad x-10=0$

$x=2,\quad x=-10,\quad x=10$

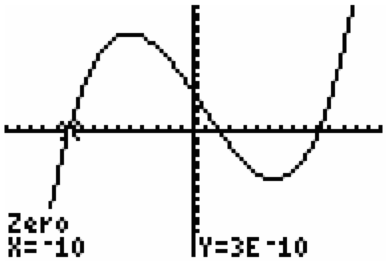
Sign chart:

Function	---	+++	---	+++
3	+++	+++	+++	+++
$(x-10)$	---	---	---	+++
$(x+10)$	---	+++	+++	+++
$(x-2)$	---	---	+++	+++
		–10	2	10

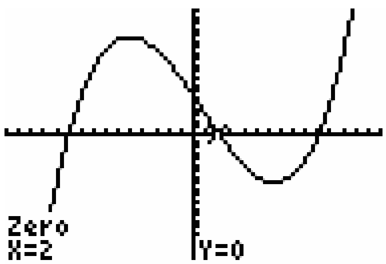
Based on the sign chart, the function is greater than zero on the intervals $(-10,2)$ and $(10,\infty)$. Considering the context of the question, the number of units can not be negative. The endpoints of the interval would be part of the solution because the question uses the phrase "at least." Therefore, total cost is at least $120,000 if

$0 \le x \le 2$ or if $x \ge 10$. In interval notation the solution is $[0,2]$ or $[10,\infty)$.

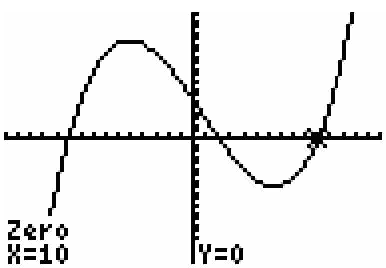
Applying the x-intercept method:



[–15, 15] by [–2000, 2000]



[–15, 15] by [–2000, 2000]



[–15, 15] by [–2000, 2000]

In context of the problem, the number of units must be greater than or equal to zero. Therefore, based on the graphs, the total cost is greater than or equal to $120,000 on the intervals $[0,2]$ or $[10,\infty)$ or when $0 \le x \le 2$ or $x \ge 10$.

26. $P(x) \ge 400$

$-x^3 + 2x^2 + 400x - 400 \ge 400$

$-x^3 + 2x^2 + 400x - 800 \ge 0$

Find the zeros:

$-x^3 + 2x^2 + 400x - 800 = 0$

$-1\left(x^3 - 2x^2 - 400x + 800\right) = 0$

$-1\left[\left(x^3 - 2x^2\right) + \left(-400x + 800\right)\right] = 0$

$-1\left[x^2(x-2) + (-400)(x-2)\right] = 0$

$-1(x-2)\left(x^2 - 400\right) = 0$

$-1(x-2)(x+20)(x-20) = 0$

$x-2=0,\quad x+20=0,\quad x-20=0$

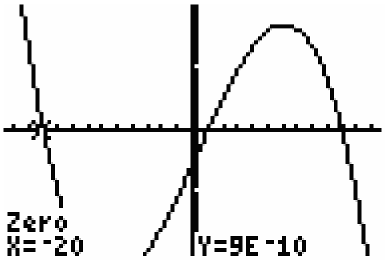
$x=2,\quad x=-20,\quad x=20$

Sign chart:

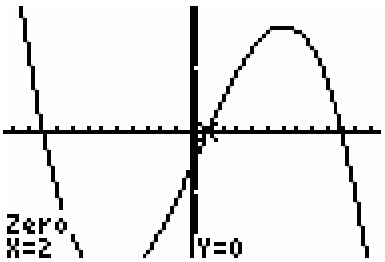
Function	+++	---	+++	---
-1	---	---	---	---
$(x-20)$	---	---	---	+++
$(x+20)$	---	+++	+++	+++
$(x-2)$	---	---	+++	+++
	–20	2	20	

Based on the sign chart, the function is greater than zero on the intervals $(-\infty,-20)$ and $(2,20)$. Considering the context of the question, the number of units can not be negative. The endpoints of the interval would be part of the solution because the question uses the phrase “at least.” Therefore, total cost is at least $40,000 if $2 \le x \le 20$. In interval notation the solution is $[2,20]$.

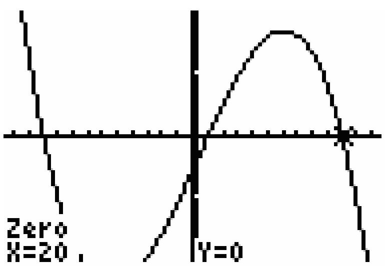
Applying the x-intercept method:



[–25, 25] by [–3000, 3000]



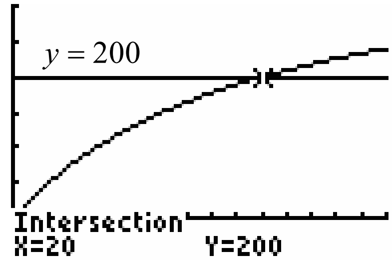
[–25, 25] by [–3000, 3000]



[–25, 25] by [–3000, 3000]

In context of the problem, the number of units must be greater than or equal to zero. Therefore, based on the graphs, the total cost is greater than or equal to $40,000 on the interval $[2,20]$ or when $2 \le x \le 20$.

27. Applying the intersection of graphs method:

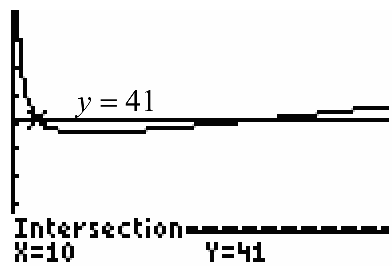


[0, 30] by [–50, 300]

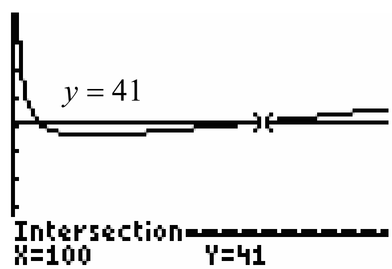
Note that the graphs intersect when $x = 20$. Therefore, $\frac{400x}{x+20} \ge 200$ on the interval $[20, \infty)$ or when $x \ge 20$.

Therefore, spending $20,000 or more on advertising creates sales of at least $200,000.

28. Applying the intersection of graphs method:



[0, 150] by [–10, 80]



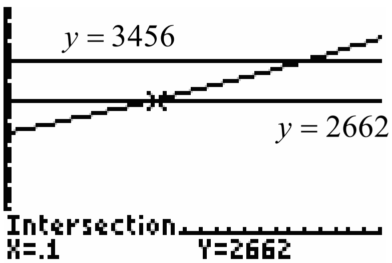
[0, 150] by [–10, 80]

Note that the graphs intersect when $x = 10$ or $x = 100$. Therefore,

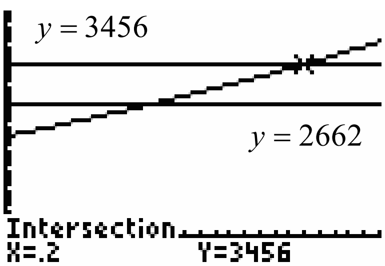
$\overline{C} = \dfrac{100 + 30x + 0.1x^2}{x} \le 41$ on the interval $[10,100]$ or when $10 \le x \le 100$.

Therefore producing between 1000 units and 10,000 units inclusive generates an average cost of at most $41 per unit.

29. Applying the intersection of graphs method



[0, 0.25] by [–500, 4500]

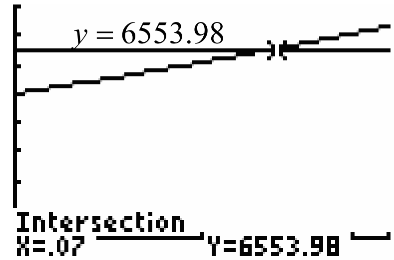


[0, 0.25] by [–500, 4500]

Note that the graphs intersect when $r = 0.10$ or $r = 0.20$. Therefore, $2662 \le 2000(1+r)^3 \le 3456$ on the interval $[0.10, 0.20]$ or when $0.10 \le r \le 0.20$.

Therefore, interest rates between 10% and 20% inclusive generate future values between $2662 and $3456 inclusive.

30. Applying the intersection of graphs method:

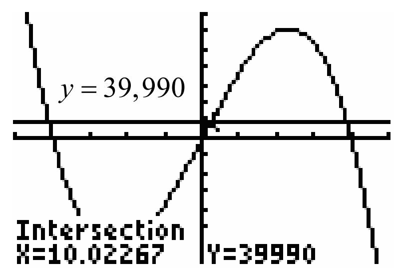


[0, 0.10] by [–500, 8000]

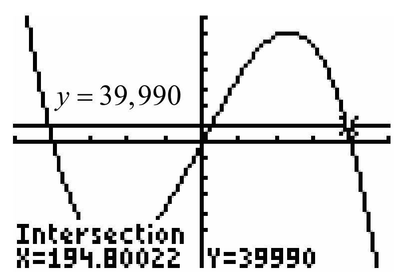
Note that the graphs intersect when $r = 0.07$. Therefore, $5000(1+r)^4 \ge 6553.98$ on the interval $[0.07, \infty)$ or when $r \ge 0.07$.

Therefore, interest rates greater than or equal to 7% generate future values of at least $6553.98.

31. Applying the intersection of graphs method:



[–250, 250] by [–350,000, 350,000]



[–250, 250] by [–350,000, 350,000]

Note that the graphs intersect when $x \approx 10.02$ and when $x \approx 194.80$. Therefore, $R = 4000x - 0.1x^3 \ge 39,990$ on the interval $[10.02, 194.80]$ or when $10.02 \le x \le 194.80$.

Therefore, producing and selling between 10 units and 195 units inclusive generates revenue of at least $39,990.

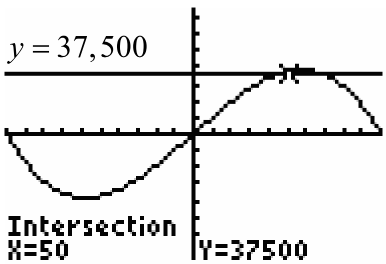
32. a. Revenue =

(Price per unit)(Number of units)

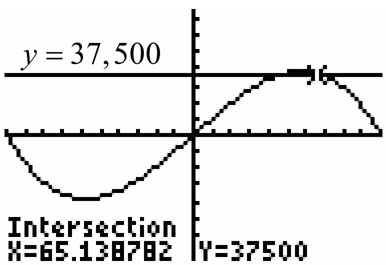
$R(x) = \left(1000 - 0.1x^2\right)(x)$

$R(x) = 1000x - 0.1x^3$

b. Applying the intersection of graphs method:



[–100, 100] by [–75,000, 75,000]



[–100, 100] by [–75,000, 75,000]

Note that the graphs intersect when $x = 50$ and when $x \approx 65.139$. Therefore,

$R(x) = 1000x - 0.1x^3 \le 37,500$ on the interval $[0, 50]$ or $[65.139, \infty)$ or when $0 \le x \le 50$ or $x \ge 65.139$.

Therefore, producing and selling between 0 units and 50 units inclusive or more than 65 units generates revenue of at most $37,500.

33. Considering the supply function and solving for q:

$6p - q = 180$

$-q = 180 - 6p$

$q = 6p - 180$

Considering the demand function and solving for q:

$(p + 20)q = 30,000$

$q = \dfrac{30,000}{p + 20}$

Supply > Demand

$6p - 180 > \dfrac{30,000}{p + 20}$ LCM: $p + 20$

$(p + 20)(6p - 180) > (p + 20)\left(\dfrac{30,000}{p + 20}\right)$

$(p + 20)(6p - 180) > 30,000$

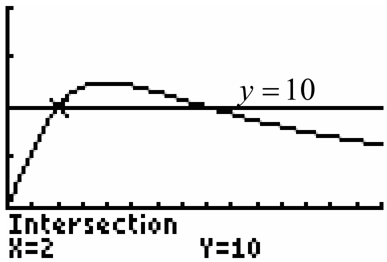
$(p + 20)(6p - 180) - 30,000 > 0$

Applying the x-intercept method:

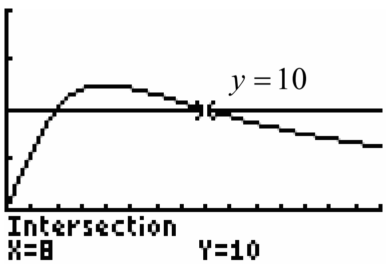
[0, 100] by [–50,000, 50,000]

When the price is at least $80, supply exceeds demand. Note that only positive values of p make sense in the context of the question.

34. Applying the intersection of graphs method:



[0, 15] by [–5, 20]



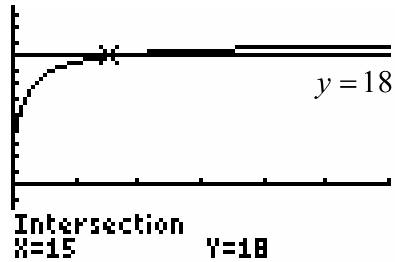
[0, 15] by [–5, 20]

Note that the graphs intersect when $t = 2$ and when $t = 8$. Therefore,

$C(t) = \frac{200t}{2t^2 + 32} \geq 10$ on the interval $[2,8]$ or when $2 \leq t \leq 8$.

Therefore, the drug concentration remains at least 10% between 2 hours and 8 hours inclusive. The results contradict the claim of the of the drug company that the drug remains at a 10% for at least 8 hours. The calculations suggest that the concentration is at least 10% for only 6 hours, $8 - 2 = 6$.

35. a. Applying the intersection of graphs method:



[0, 60] by [–10, 25]

Note that the graphs intersect when $t = 15$. Therefore, $f(t) = \frac{30 + 40t}{5 + 2t} < 18$ on the interval $[0,15)$ or when $0 \leq t < 15$.

For the first 15 months of the operation, the number of employees is at most 18.

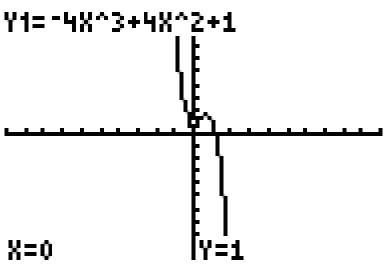
b. The number of employees is less than 18 until month 15. Therefore, the number of employees is less than 18 on the interval $[0,15)$ or when $0 \leq t < 15$. Thinking in terms of discrete months, for months 0 through 14 the number of employees is less than 18.

Chapter 4 Skills Check

1. The degree of the polynomial is the highest exponent. In this case, the degree of the polynomial is 4.

2. A fourth degree polynomial function is called a quartic function.

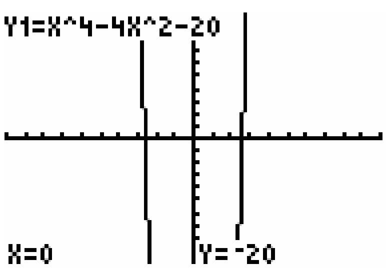
3.



[–10, 10] by [–10, 10]

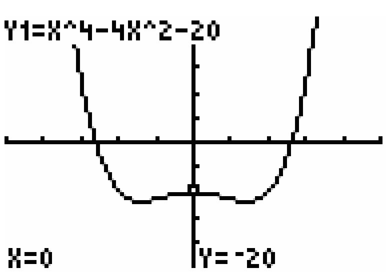
Yes. The graph is complete on the given viewing window.

4. a.



[–10, 10] by [–10, 10]

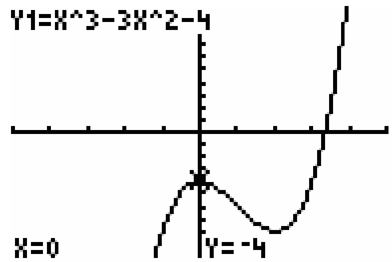
b.



[–5, 5] by [–50, 50]

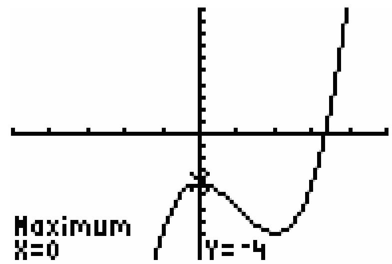
Viewing windows may vary.

5. a.



[–5, 5] by [–10, 10]

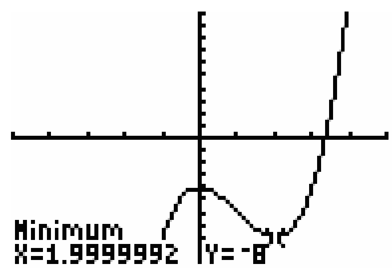
b.



[–5, 5] by [–10, 10]

The local maximum is $(0,-4)$.

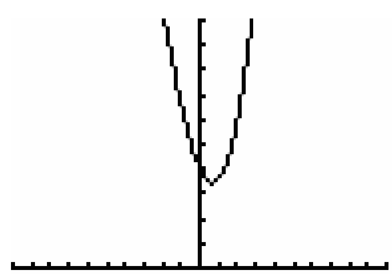
c.



[–5, 5] by [–10, 10]

The local minimum is $(2,-8)$.

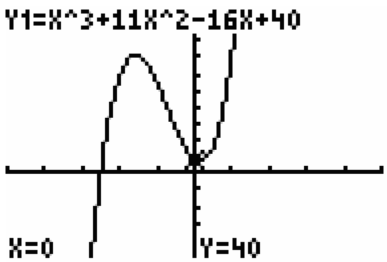
6. a. $y = x^3 + 11x^2 - 16x + 40$



[–10, 10] by [0, 100]

b. No. The graph is not complete.

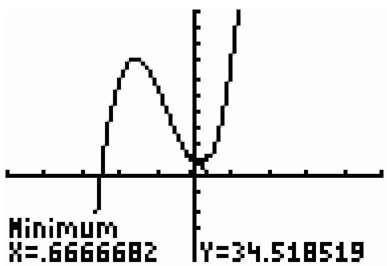
c.



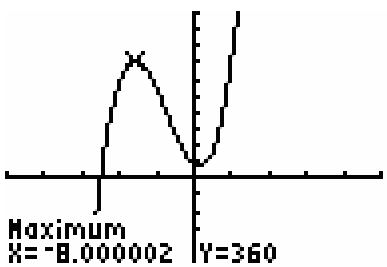
[–25, 25] by [–250, 500]

Viewing windows may vary.

d.



[–25, 25] by [–250, 500]



[–25, 25] by [–250, 500]

7. $x^3 - 16x = 0$

$x(x^2 - 16) = 0$

$x(x+4)(x-4) = 0$

$x = 0, \quad x + 4 = 0, \quad x - 4 = 0$

$x = 0, \quad x = -4, \quad x = 4$

8. $2x^4 - 8x^2 = 0$

$2x^2(x^2 - 4) = 0$

$2x^2(x+2)(x-2) = 0$

$2x^2 = 0, \quad x + 2 = 0, \quad x - 2 = 0$

$x = 0, \quad x = -2, \quad x = 2$

9. $x^4 - x^3 - 20x^2 = 0$

$x^2(x^2 - x - 20) = 0$

$x(x-5)(x+4) = 0$

$x = 0, \quad x - 5 = 0, \quad x + 4 = 0$

$x = 0, \quad x = 5, \quad x = -4$

10. $x^3 - 15x^2 + 56x = 0$

$x(x^2 - 15x + 56) = 0$

$x(x-7)(x-8) = 0$

$x = 0, \quad x - 7 = 0, \quad x - 8 = 0$

$x = 0, \quad x = 7, \quad x = 8$

11. $4x^3 - 20x^2 - 4x + 20 = 0$

$4(x^3 - 5x^2 - x + 5) = 0$

$4\left[(x^3 - 5x^2) + (-x + 5)\right] = 0$

$4\left[x^2(x-5) + -1(x-5)\right] = 0$

$4(x-5)(x^2 - 1) = 0$

$4(x-5)(x+1)(x-1) = 0$

$x - 5 = 0, \quad x + 1 = 0, \quad x - 1 = 0$

$x = 5, \quad x = -1, \quad x = 1$

12. $12x^3 - 9x^2 - 48x + 36 = 0$

$3\left(4x^3 - 3x^2 - 16x + 12\right) = 0$

$3\left[\left(4x^3 - 3x^2\right) + \left(-16x + 12\right)\right] = 0$

$3\left[x^2\left(4x - 3\right) + \left(-4\right)\left(4x - 3\right)\right] = 0$

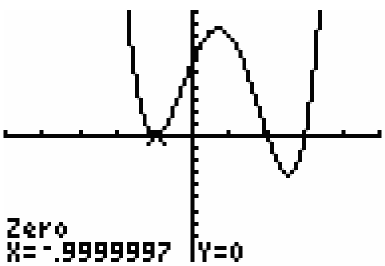
$3\left(4x - 3\right)\left(x^2 - 4\right) = 0$

$3\left(4x - 3\right)\left(x + 2\right)\left(x - 2\right) = 0$

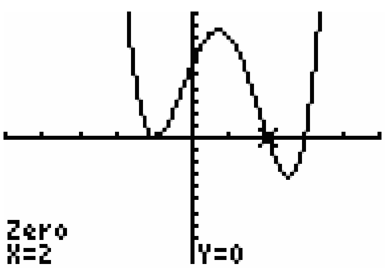
$4x - 3 = 0, \quad x + 2 = 0, \quad x - 2 = 0$

$x = \dfrac{3}{4}, \quad x = -2, \quad x = 2$

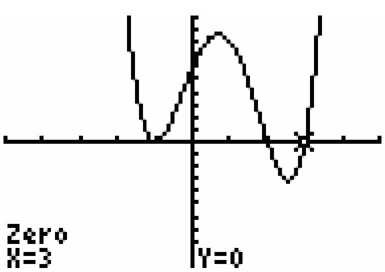
13. Applying the *x*-intercept method:



[–5, 5] by [–10, 10]



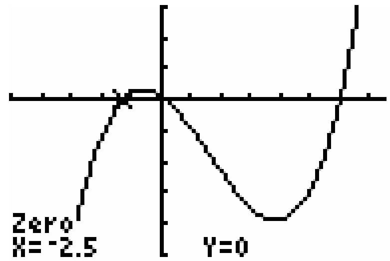
[–5, 5] by [–10, 10]



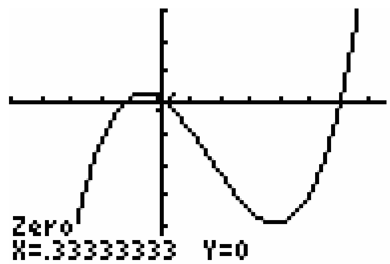
[–5, 5] by [–10, 10]

Based on the graphs, the solutions are $x = -1, x = 2,$ and $x = 3$.

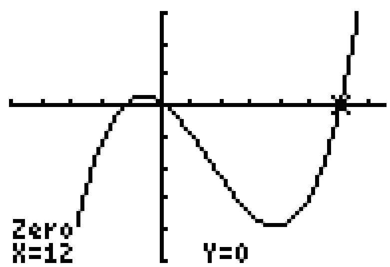
14. Applying the *x*-intercept method:



[–10, 15] by [–2500, 1500]



[–10, 15] by [–2500, 1500]



[–10, 15] by [–2500, 1500]

Based on the graphs, the solutions are $x = -2.5, x = \dfrac{1}{3},$ and $x = 12$.

15. $\left(x - 4\right)^3 = 8$

$\sqrt[3]{\left(x - 4\right)^3} = \sqrt[3]{8}$

$x - 4 = 2$

$x = 6$

16. $5(x-3)^4 = 80$

$(x-3)^4 = 16$

$\sqrt[4]{(x-3)^4} = \sqrt[4]{16}$

$x - 3 = 2$

$x = 5$

17.

2)	4	−3	0	2	−8
		8	10	20	44
	4	5	10	22	36

$4x^3 + 5x^2 + 10x + 22 + \frac{36}{x-2}$

18.

1)	2	5	−11	4
		2	7	−4
	2	7	−4	0

The remaining polynomial is $2x^2 + 7x - 4$.

Set the polynomial equal to zero and solve.

$2x^2 + 7x - 4 = 0$

$(2x-1)(x+4) = 0$

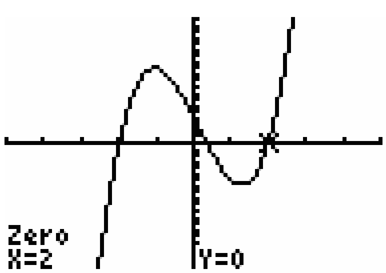
$2x - 1 = 0, x + 4 = 0$

$2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$x + 4 = 0 \Rightarrow x = -4$

The solutions are $x = 1, x = -4$, and $x = \frac{1}{2}$.

19. Applying the x-intercept method:



[–5, 5] by [–20, 20]

It appears that $x = 2$ is a zero.

2)	3	−1	−12	4
		6	10	−4
	3	5	−2	0

The remaining quadratic factor is $3x^2 + 5x - 2$.

Applying the quadratic formula:

$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{49}}{6}$

$x = \frac{-5 \pm 7}{6}$

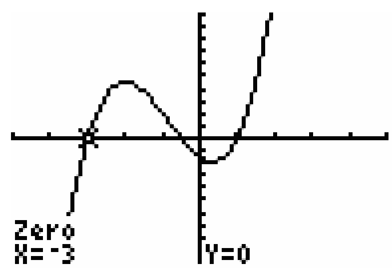
$x = \frac{-5+7}{6} = \frac{2}{6} = \frac{1}{3}$

or

$x = \frac{-5-7}{6} = \frac{-12}{6} = -2$

The solutions are $x = 2, x = -2$, and $x = \frac{1}{3}$

20. Applying the x-intercept method:



[–5, 5] by [–20, 20]

It appears that $x = -3$ is a zero.

−3)	2	5	−4	−3
		−6	3	3
	2	−1	−1	0

The remaining quadratic factor is $2x^2 - x - 1$.

Applying the quadratic formula:

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-1)}}{2(2)}$$

$$x=\frac{1\pm\sqrt{9}}{4}$$

$$x=\frac{1\pm 3}{4}$$

$$x=\frac{1+3}{4}=\frac{4}{4}=1$$

or

$$x=\frac{1-3}{4}=\frac{-2}{4}=-\frac{1}{2}$$

The solutions are $x=-3, x=-\frac{1}{2}$, and $x=1$.

21. a. To find y - intercepts, let $x=0$ and solve for y.

$$y=\frac{1-(0)^2}{0+2}=\frac{1}{2}$$

$$\left(0,\frac{1}{2}\right)$$

To find x - intercepts, let the numerator equal zero and solve for x.

$$1-x^2=0$$

$$x^2=1$$

$$\sqrt{x^2}=\pm\sqrt{1}$$

$$x=\pm 1$$

$$(-1,0),(1,0)$$

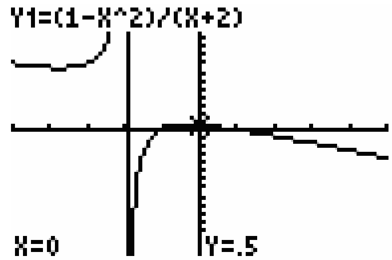
b. To find the vertical asymptote let $q(x)=0$.

$$x+2=0$$

$$x=-2$$

$x=-2$ is the vertical asymptote.

The degree of the numerator is greater than the degree of the denominator. Therefore, there is not a horizontal asymptote.

c. 

[–5, 5] by [–15, 15]

22. a. To find y - intercepts, let $x=0$ and solve for y.

$$y=\frac{3(0)-2}{0-3}=\frac{-2}{-3}=\frac{2}{3}$$

$$\left(0,\frac{2}{3}\right)$$

To find x - intercepts, let the numerator equal zero and solve for x.

$$3x-2=0$$

$$3x=2$$

$$x=\frac{2}{3}$$

$$\left(\frac{2}{3},0\right)$$

b. To find the vertical asymptote let $q(x)=0$.

$$x-3=0$$

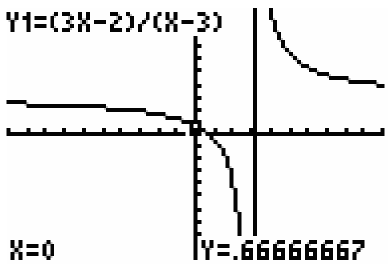
$$x=3$$

$x=3$ is the vertical asymptote.

The degree of the numerator equals the degree of the denominator. Therefore, the ratio of the leading coefficients is the horizontal asymptote.

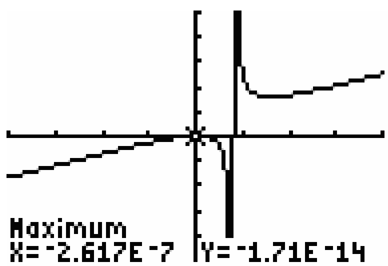
$$y=\frac{3}{1}=3$$

c.

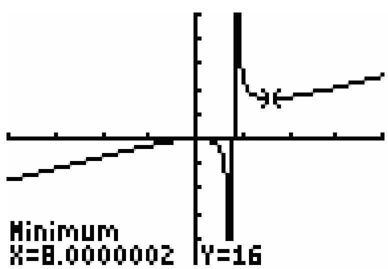


[–10, 10] by [–10, 10]

23.



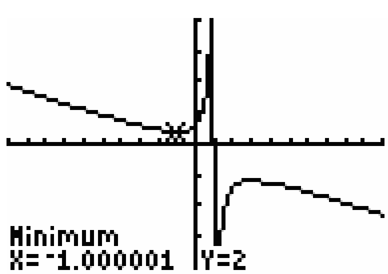
[–20, 20] by [–50, 50]



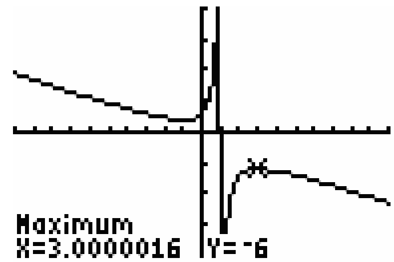
[–20, 20] by [–50, 50]

The local maximum is $(0,0)$, while the local minimum is $(8,16)$.

24.



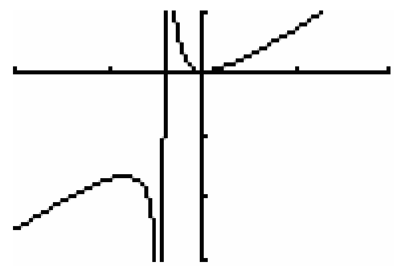
[–10, 10] by [–20, 20]



[–10, 10] by [–20, 20

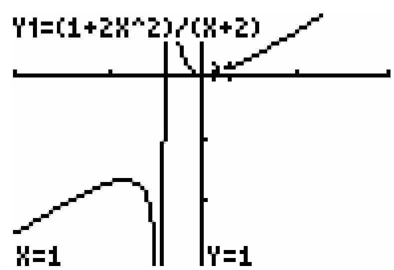
The local maximum is $(3,-6)$, while the local minimum is $(-1,2)$.

25. a.

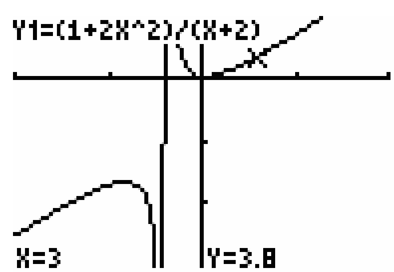


[–10, 10] by [–30, 10]

b.



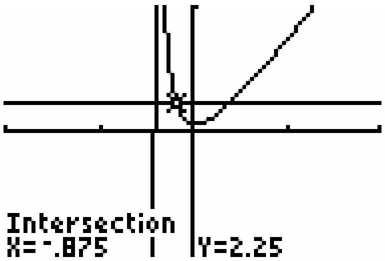
[–10, 10] by [–30, 10]



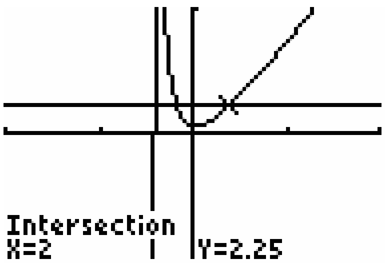
[–10, 10] by [–30, 10]

When $x = 1$, $y = 1$. When $x = 3$, $y = 3.8$.

c.



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 10]

If $y = 2.25$, then $x = -0.875$ or $x = 2$.

d. $\frac{9}{4} = \frac{1+2x^2}{x+2}$ LCM: $4(x+2)$

$4(x+2)\left(\frac{9}{4}\right) = 4(x+2)\left(\frac{1+2x^2}{x+2}\right)$

$9(x+2) = 4(1+2x^2)$

$9x + 18 = 4 + 8x^2$

$8x^2 - 9x - 14 = 0$

$(8x+7)(x-2) = 0$

$8x + 7 = 0 \Rightarrow 8x = -7 \Rightarrow x = -\frac{7}{8}$

$x - 2 = 0 \Rightarrow x = 2$

The solutions are $x = 2, x = -\frac{7}{8}$.

26. $x^4 - 13x^2 + 36 = 0$

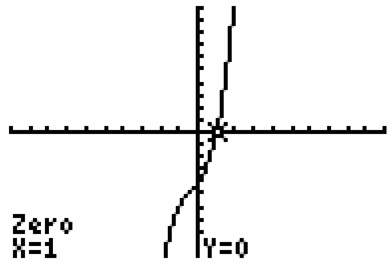
$(x^2 - 9)(x^2 - 4) = 0$

$(x+3)(x-3)(x+2)(x-2) = 0$

$x + 3 = 0, \quad x - 3 = 0, \quad x + 2 = 0, \quad x - 2 = 0$

$x = -3, \ x = 3, \ x = -2, \ x = 2$

27. Applying the x-intercept method:



[–10, 10] by [–10, 10]

It appears that $x = 1$ is a zero.

1)	1	1	2	–4
		1	2	4
	1	2	4	0

The remaining quadratic factor is $x^2 + 2x + 4$.
Set the remaining polynomial equal to zero and solve.

$x^2 + 2x + 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

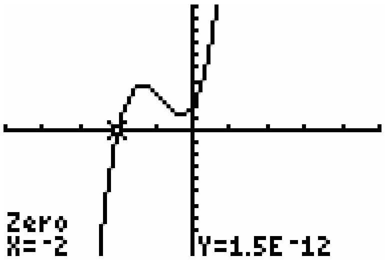
$x = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm 2i\sqrt{3}}{2}$

$x = -1 \pm i\sqrt{3}$

The solutions are $x = 1, \ x = -1 \pm i\sqrt{3}$.

28. Applying the x-intercept method:



[–5, 5] by [–10, 10]

It appears that $x = -2$ is a zero.

–2	4	10	5	2
		–8	–4	–2
	4	2	1	0

The remaining quadratic factor is $4x^2 + 2x + 1$.
Set the remaining polynomial equal to zero and solve.

$$4x^2 + 2x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{-12}}{8}$$

$$x = \frac{-2 \pm 2i\sqrt{3}}{8}$$

$$x = \frac{-1 \pm i\sqrt{3}}{4}$$

$$x = \frac{-1}{4} \pm \frac{\sqrt{3}}{4}i$$

The solutions are $x = -2,\ x = \frac{-1}{4} + \frac{\sqrt{3}}{4}i,$ and $\frac{-1}{4} - \frac{\sqrt{3}}{4}i.$

29. $x^3 - 5x^2 \geq 0$

$x^2(x-5) \geq 0$

$x^2(x-5) = 0$

$x - 5 = 0, \quad x^2 = 0$

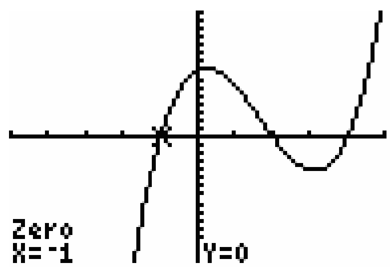
$x = 5, \quad x = 0$

Sign chart:

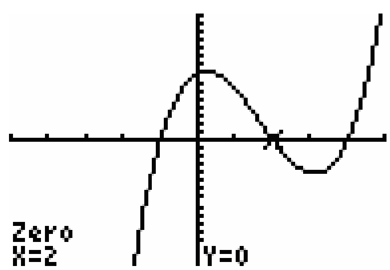
Function	---	---	+++
x^2	+++	+++	+++
$(x-5)$	---	---	+++
		0	5

Based on the sign chart, the function is greater than or equal to zero on the interval $[5, \infty)$ or when $x \geq 5$. In addition, the function is equal to zero when $x = 0$.

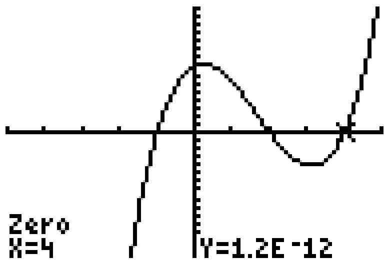
30. Applying the x-intercept method:



[–5, 5] by [–15, 15]



[–5, 5] by [–15, 15]



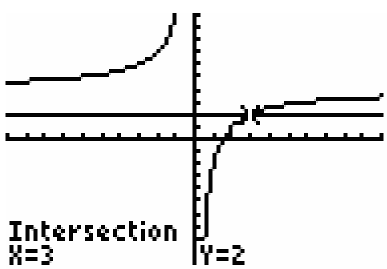
[–5, 5] by [–15, 15]

Based on the graphs, the function is greater than or equal to zero on the intervals $[-1,2]$ or $[4,\infty)$ or when $x \ge 4$ or $-1 \le x \le 2$.

31. $2 < \dfrac{4x-6}{x}$

$\dfrac{4x-6}{x} > 2$

Applying the intersection of graphs method:

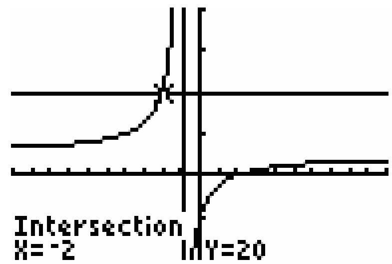


[–10, 10] by [–10, 10]

Note that the graphs intersect when $x = 3$. Also note that a vertical asymptote occurs at $x = 0$. Therefore, $\dfrac{4x-6}{x} > 2$ on the interval $(-\infty,0)\cup(3,\infty)$ or when $x < 0$ or $x > 3$.

32. $\dfrac{5x-10}{x+1} \ge 20$

Applying the intersection of graphs method:

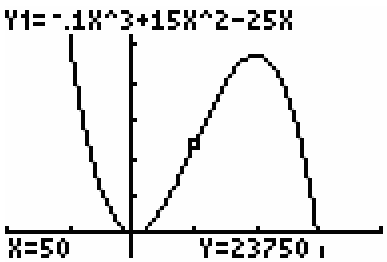


[–10, 10] by [–20, 40]

Note that the graphs intersect when $x = -2$. Also note that a vertical asymptote occurs at $x = -1$. Therefore, $\dfrac{5x-10}{x+1} \ge 20$ on the interval $[-2,-1)$ or when $-2 \le x < -1$.

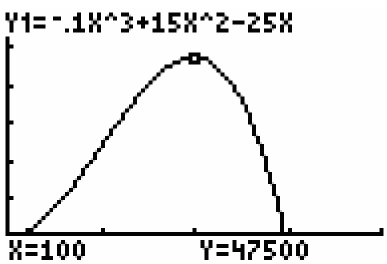
Chapter 4 Review Exercises

33. a.



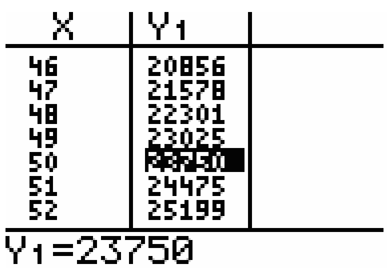
[–100, 200] by [–2000, 60,000]

b.



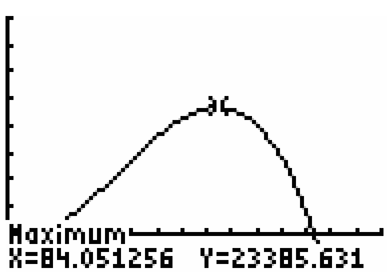
[0, 200] by [0, 60,000]

c.



When 50,000 units are produced, the revenue is $23,750.

34.



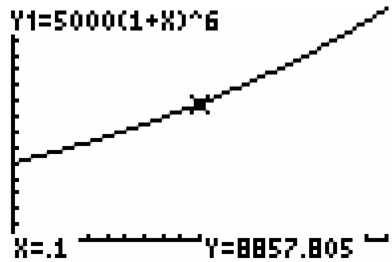
[0, 150] by [–6000, 40,000]

When 84,051 units are sold, the maximum revenue of $23,385.63 is generated.

35. a. Using the table feature of a TI-83 graphing calculator:

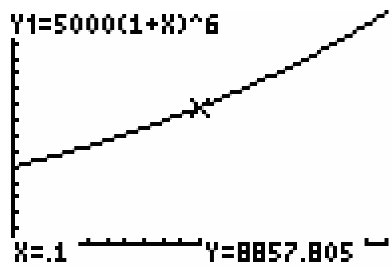
r (rate)	*S* (future value)
1%	5307.60
5%	6700.48
10%	8857.81
15%	11,565.30

b.

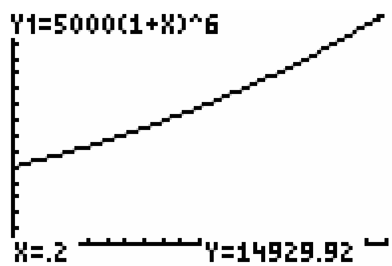


[0, 0.2] by [–1000, 15,000]

c.



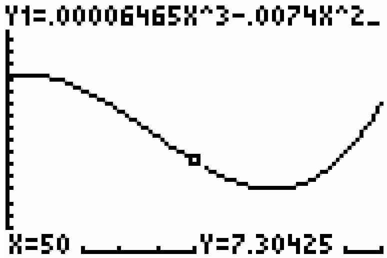
[0, 0.2] by [–1000, 15,000]



[0, 0.2] by [–1000, 15,000]

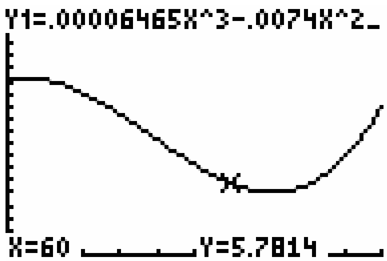
The difference in the future values is $14,929.92 - 8857.81 = \$6072.11$.

36. a.



[0, 100] by [0, 20]

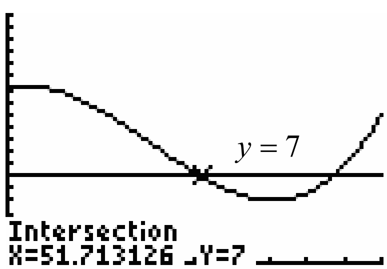
b.



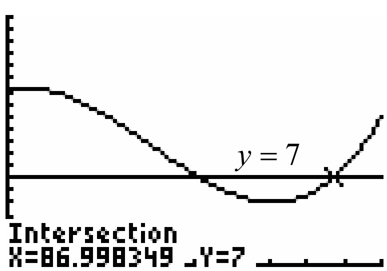
[0, 100] by [0, 20]

In 1960, the model predicts the percentage to be 5.78%. The prediction is relatively close to actual value of 5.4%.

c. Applying intersection of graphs method:



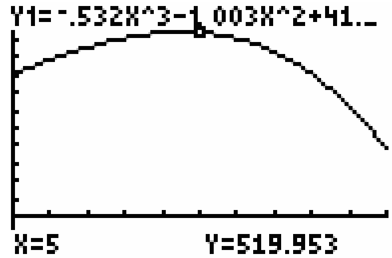
[0, 100] by [0, 20]



[0, 100] by [0, 20]

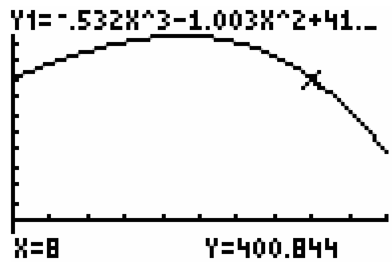
Based on the graphs, in approximately 1952 and again in 1987 the percentage is 7%.

37. a.



[0, 10] by [–100, 600]

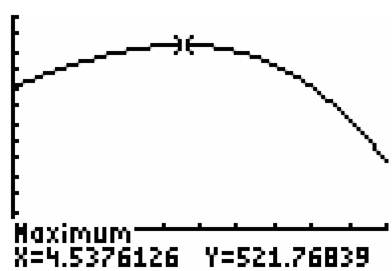
b.



[0, 10] by [–100, 600]

Based on the model, the United Nations debt is approximately $400.844 million in 1998.

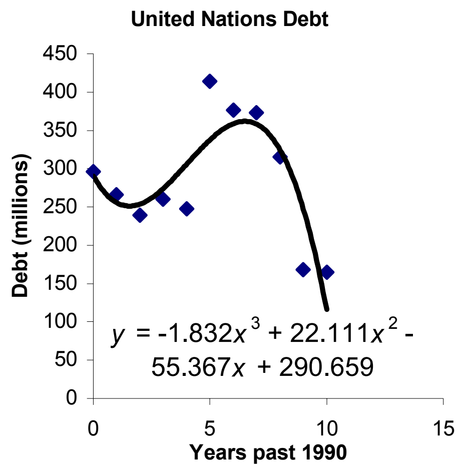
c.



[0, 10] by [–100, 600]

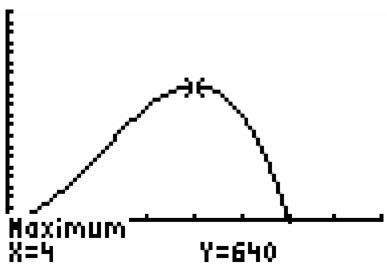
The maximum debt is approximately $521.768 million occurring between 1994 and 1995.

38. a.



b. See part a) above.

39.



[0, 8] by [0, 1000]

An intensity level of 4 allows the maximum amount of photosynthesis to take place.

40.

$$9261 = 8000(1+r)^3$$
$$(1+r)^3 = \frac{9261}{8000}$$
$$\sqrt[3]{(1+r)^3} = \sqrt[3]{\frac{9261}{8000}}$$
$$1+r = 1.05$$
$$r = 1.05 - 1$$
$$r = 0.05$$

An interest rate of 5% creates a future value of $9261 after 3 years.

41. a.

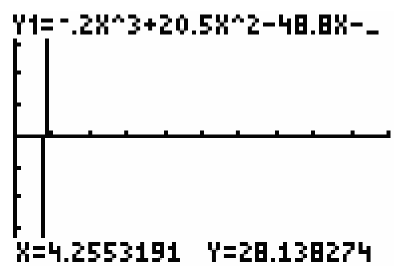
$$V = 0$$
$$324x - 72x^2 + 4x^3 = 0$$
$$4x^3 - 72x^2 + 324x = 0$$
$$4x(x^2 - 18x + 81) = 0$$
$$4x(x-9)(x-9) = 0$$
$$4x = 0, \quad x - 9 = 0, \quad x - 9 = 0$$
$$x = 0, \quad x = 9, \quad x = 9$$

b. If the values of x from part *a* are used to cut squares from corners of a piece of tin, no box can be created. Either no square is cut or the squares encompass all the tin. Therefore the volume of the box is zero.

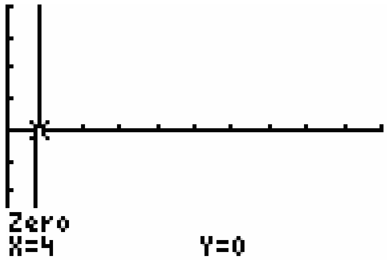
c. Reasonable values of x would allow for a box to be created. An x-value larger than zero and less than half the length of the edge of the piece of tin would allow for a box to be created. Therefore, reasonable values are

$$0 < x < \frac{18}{2} \quad \text{or} \quad 0 < x < 9.$$

42. a.



[0, 40] by [–15, 20]



[0, 40] by [–15, 20]

The x-intercept is $(4,0)$.

b.

4)	–0.2	20.5	–48.8	–120
		–0.8	78.8	120
	–0.2	19.7	30	0

The remaining quadratic factor is $-0.2x^2 + 19.7x + 30$.

c. Set the remaining polynomial equal to zero and solve.

$$-0.2x^2 + 19.7x + 30 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(19.7) \pm \sqrt{(19.7)^2 - 4(-0.2)(30)}}{2(-0.2)}$$

$$x = \frac{-19.7 \pm \sqrt{412.09}}{-0.4}$$

$$x = \frac{-19.7 \pm 20.3}{-0.4}$$

$$x = \frac{-19.7 + 20.3}{-0.4},\ x = \frac{-19.7 - 20.3}{-0.4}$$

$$x = -1.5,\ x = 100$$

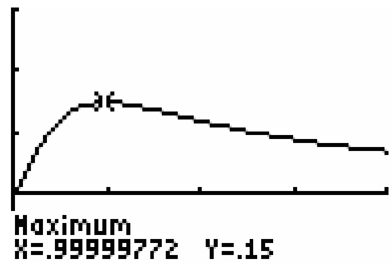
The solutions are $x = 4$, $x = -1.5$, and $x = 100$.

d. Since negative solutions do not make sense in the context of the question, break-even occurs when 400 units are produced or when 10,000 units are produced.

43. a. Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $C = 0$.

b. As the time increases, the concentration of the drug approaches zero percent.

c.



[0, 4] by [0, 0.3]

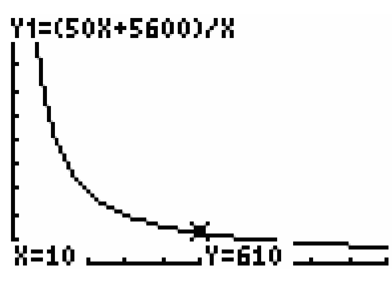
The maximum drug concentration is 15% occurring after one hour.

44. a. $\overline{C}(0)$ does not exist. If no units are produced, an average cost per unit can not be calculated.

b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $\overline{C}(x) = \frac{50}{1} = 50$.

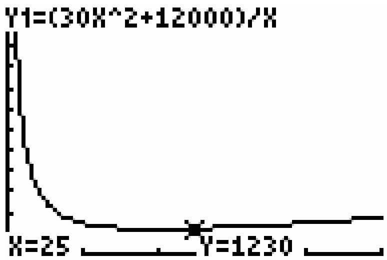
As the number of units produced increases without bound, the average cost per unit approaches $50.

c. The function decreases as x increases.



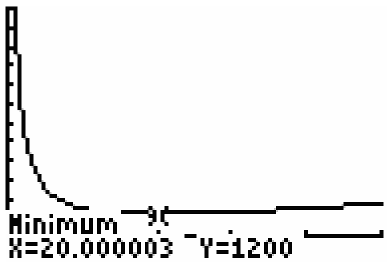
[0, 20] by [0, 5000]

45. a.



[0, 50] by [0, 12,000]

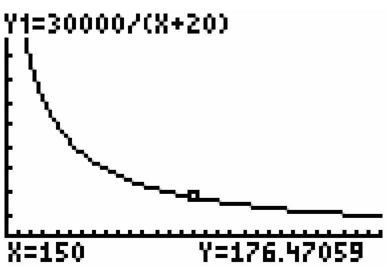
b.



[0, 50] by [0, 12,000]

The minimum average cost is \$1200 occurring when 20 units are produced.

46.



[0, 300] by [0, 1000]

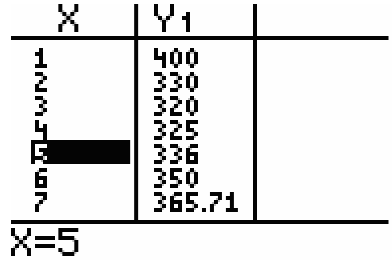
47. a.

$$C(x) = 200 + 20x + \frac{180}{x}$$

$$C(x) = \frac{200x}{x} + \frac{20x^2}{x} + \frac{180}{x}$$

$$C(x) = \frac{20x^2 + 200x + 180}{x}$$

b.

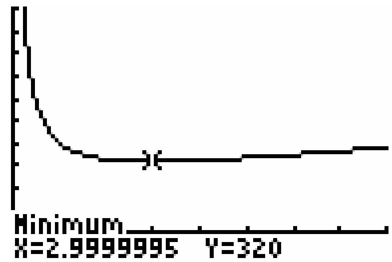


X	Y1
1	400
2	330
3	320
4	325
5	336
6	350
7	365.71

X=5

Using 5 plates creates a cost of \$336.

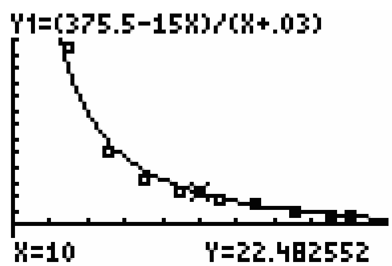
c.



[0, 8] by [–100, 1000]

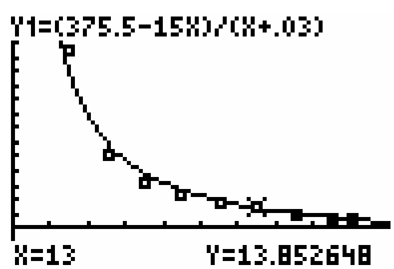
Using 3 plates creates a minimum cost of \$320.

48. a.



[0, 20] by [–25, 150]

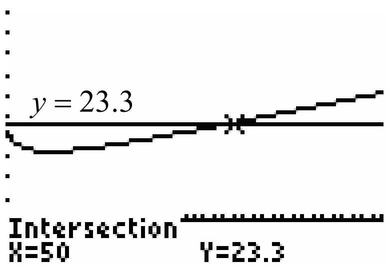
b.



[0, 20] by [–25, 150]

During the 1993-94 school year, the model predicts 13.85 students per computer. The prediction is close to the actual value of 14 students per computer, as displayed in the table.

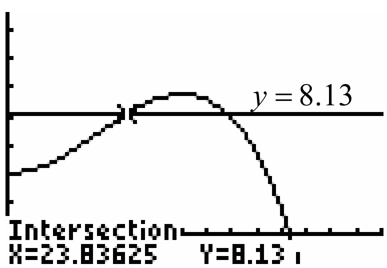
49. Applying the intersection of graphs method:



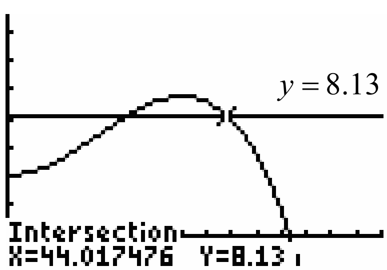
[4, 80] by [–10, 50]

Note that $S \geq 23.3$ occurs on the interval $[50, \infty)$ or when $x \geq 50$. Fifty or more hours of training results in sales greater than $23,300.

50. Applying the intersection of graphs method:



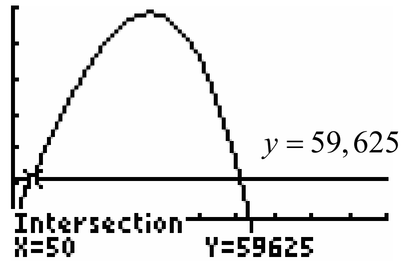
[0, 75] by [–2, 15]



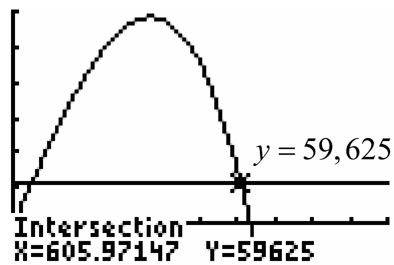
[0, 75] by [–2, 15]

Note that $y \geq 8.13$ when $23.836 \leq x \leq 44.017$. Between 1974 and 1994 inclusive, the homicide rate is at least 8.13 per 100,000 people.

51. Applying the intersection of graphs method:



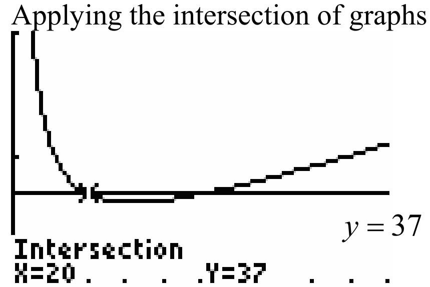
[0, 1000] by [–50,000, 300,000]



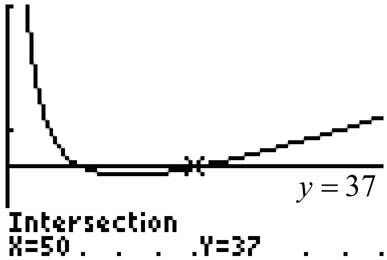
[0, 1000] by [–50,000, 300,000]

Note that $R \geq 59{,}625$ when $x \geq 50$ and $x \leq 605.97$. Selling 50 or more units but no more than 605 units creates a revenue stream of at least $59,625.

52. Applying the intersection of graphs method



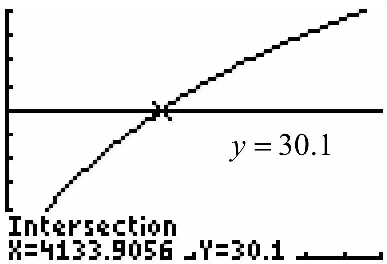
[0, 100] by [30, 50]



[0, 100] by [30, 50]

$\overline{C} \leq 37$ when $20 \leq x \leq 50$. The average cost is at most $37 when between 20 and 50 units inclusive are produced.

53. Applying the intersection of graphs method:



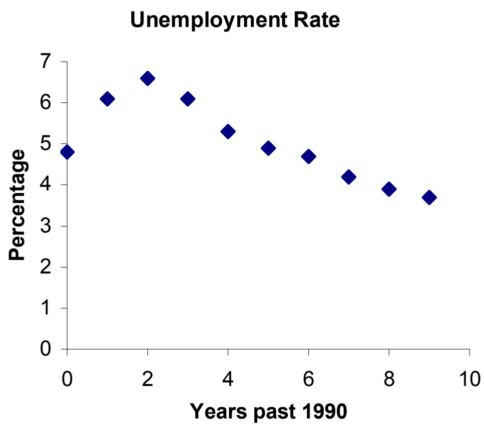
[0, 10,000] by [0, 50]

Note that $p \geq 30.1$ when $x \geq 4133.91$. To remove at least 30.1% of the particulate pollution will cost at least $4133.91.

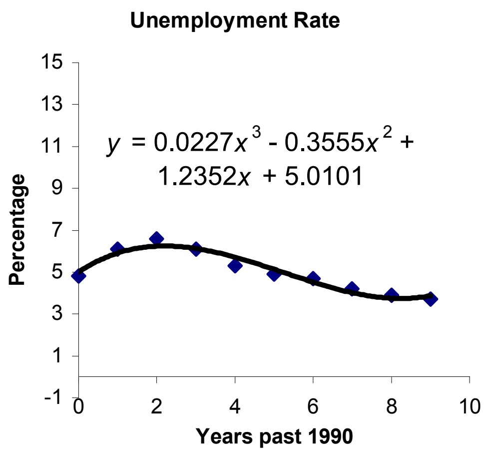
Group Activity/Extended Applications

Unemployment Rates

1.

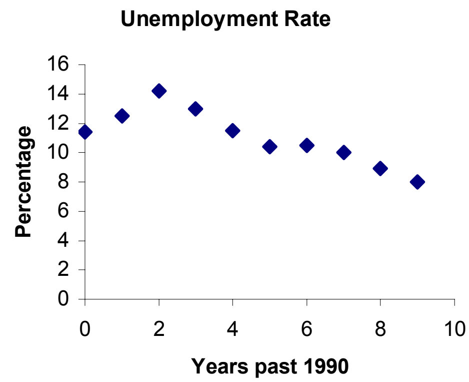


2.



3. See Exercise 2 above.

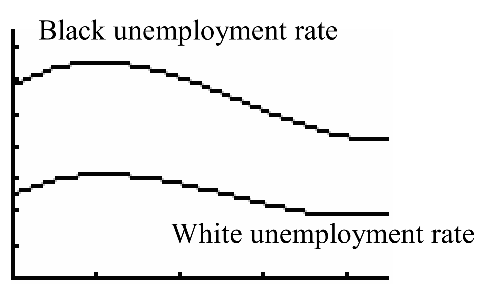
4.



5.

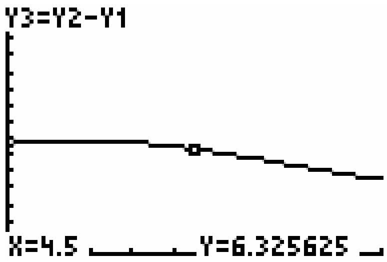
6. See Exercise 5 above.

7.



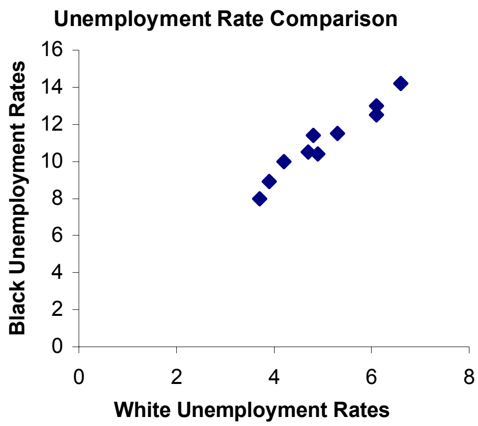
[0, 9] by [0, 15]

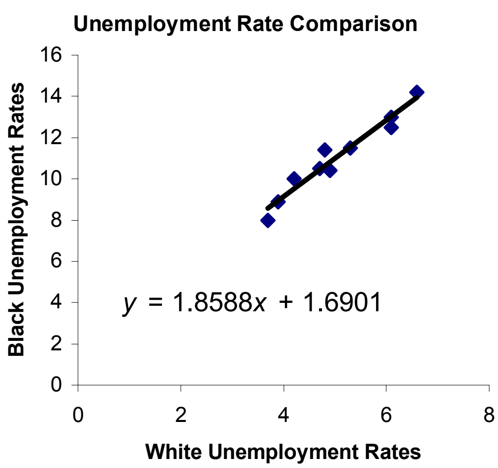
8. Black unemployment rates are much higher than white unemployment rates throughout the 1990’s.

9. 

[0, 9] by [0, 15]

The unemployment rate gap seems to be decreasing.

10. 

11. 

12. See Exercise 11 above.

13. No. The graph suggests that the unemployment rates move together in a more or less linear fashion. That is, as one rate rises, the other rate rises. Based on the data analysis, affirmative action plans might be useful in helping to narrow the unemployment rate gap.

Printing

A. 1. Assuming the printer uses 10 plates, then $1000 \cdot 10 = 10{,}000$ impressions can be made per hour. If 10,000 impressions are made per hour, it will take $\dfrac{100{,}000}{10{,}000} = 10$ hours to complete all the invitations.

2. Since it costs \$128 per hour to run the press, the cost of using 10 plates is $10 \cdot 128 = \$1280$.

3. The 10 plates cost $10 \cdot 8 = \$80$.

4. The total cost of finishing the job is $1280 + 80 = \$1360$

B. 1. Let x = number of plates. Then, the cost of the plates is $8x$.

2. Using x plates implies x invitations can be made per impression.

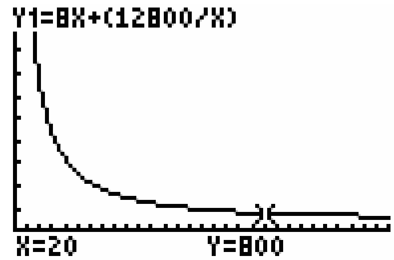
3. $1000x$ invitations per hour
Creating all 100,000 invitations would require $\dfrac{100{,}000}{1000x} = \dfrac{100}{x}$ hours.

4. $C(x) = 8x + 128\left(\dfrac{100}{x}\right)$

$C(x) = 8x + \left(\dfrac{12{,}800}{x}\right)$, where

x represents the number of plates and $C(x)$ represents the cost of the 100,000 invitations in dollars.

5.



[0, 30] by [0, 10,000]

Producing 20 plates minimizes the cost, since the number of plates, x, that can be produced is between 1 and 20 inclusive.

6. Considering a table of values yields

X	Y1
1	12808
2	6416
3	4290.7
4	3232
5	2600
6	2181.3
7	1884.6

X=1

X	Y1
8	1664
9	1494.2
10	1360
11	1251.6
12	1162.7
13	1088.6
14	1026.3

X=8

X	Y1
14	1026.3
15	973.33
16	928
17	888.94
18	855.11
19	825.68
20	800

X=20

Producing 20 plates creates a minimum cost of \$800 for printing the 100,000 invitations.