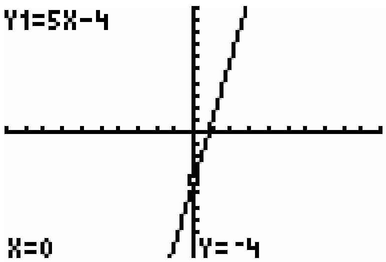
Chapter 6
Special Topics

Section 6.1 Skills Check

1. $y \le 5x - 4$



[–10, 10] by [–10, 10]

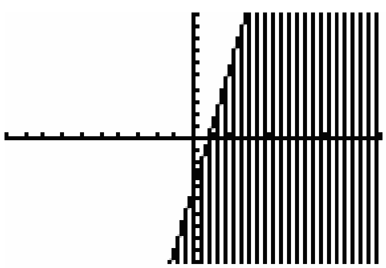
Note that the line is solid because of the "equal to" in the given inequality.

Test (0, 0).

$0 \le 5(0) - 4$

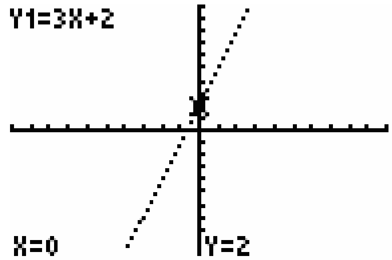
$0 \le -4$

Since the statement is false, the region not containing (0, 0) is the solution to the inequality.



[–10, 10] by [–10, 10]

2. $y > 3x + 2$



[–10, 10] by [–10, 10]

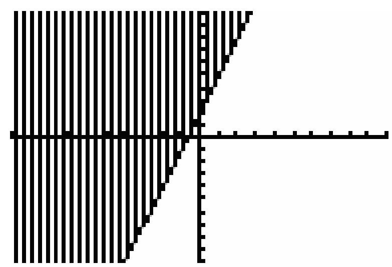
Note that the line is dashed because "equal to" is not part of the inequality.

Test (0, 0).

$0 > 3(0) + 2$

$0 > 2$

Since the statement is false, the region not containing (0, 0) is the solution to the inequality.



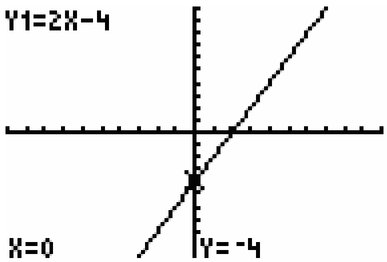
[–10, 10] by [–10, 10]

3. $6x - 3y \ge 12$

$$-3y \ge -6x + 12$$

$$\frac{-3y}{-3} \le \frac{-6x + 12}{-3}$$

$$y \le 2x - 4$$



[–10, 10] by [–10, 10]

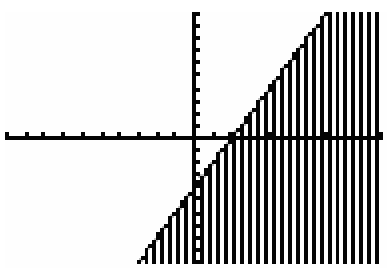
Note that the line is solid because of the "equal to" in the given inequality.

Test (0, 0).

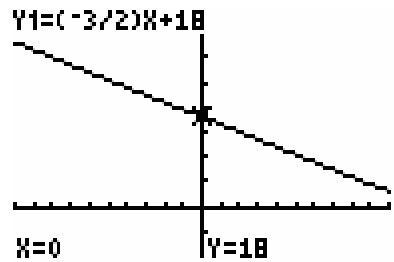
$0 \le 2(0) - 4$

$0 \le -4$

Since the statement is false, the region not containing (0, 0) is the solution to the inequality.



[–10, 10] by [–10, 10]



[–10, 10] by [–10, 40]

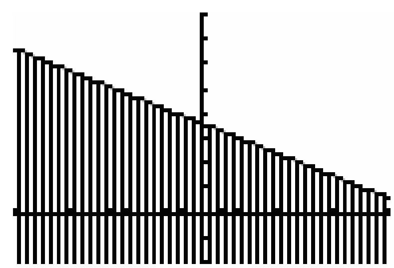
Note that the line is solid because of the "equal to" in the given inequality.

Test (0, 0).

$(0) \le -\frac{3}{2}(0) + 18$

$0 \le 18$

Since the statement is true, the region containing (0, 0) is the solution to the inequality.



[–10, 10] by [–10, 40]

4. $\frac{x}{2} + \frac{y}{3} \le 6$

LCM: 6

$6\left(\frac{x}{2} + \frac{y}{3}\right) \le 6(6)$

$3x + 2y \le 36$

$2y \le -3x + 36$

$y \le \frac{-3x + 36}{2}$

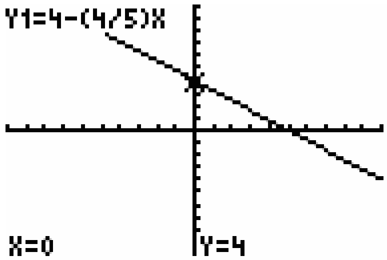
$y \le -\frac{3}{2}x + 18$

5. $4x + 5y \le 20$

$5y \le 20 - 4x$

$y \le \frac{20 - 4x}{5}$

$y \le 4 - \frac{4}{5}x$



[–10, 10] by [–10, 40]

Note that the line is solid because of the "equal to" in the given inequality.

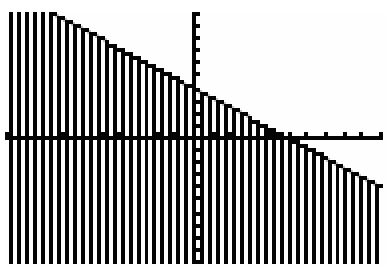
Test (0, 0).

$$y \le 4 - \frac{4}{5}x$$

$$0 \le 4 - \frac{4}{5}(0)$$

$$0 \le 4$$

Since the statement is true, the region containing (0, 0) is the solution to the inequality.



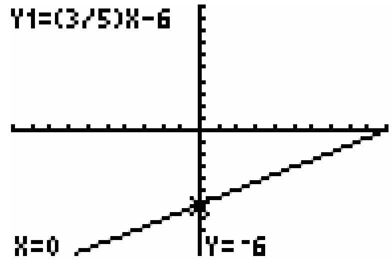
[–10, 10] by [–10, 40]

6. $3x - 5y \ge 30$

$$-5y \ge 30 - 3x$$

$$y \le \frac{30 - 3x}{-5}$$

$$y \le \frac{3}{5}x - 6$$



[–10, 10] by [–10, 40]

Note that the line is solid because of the "equal to" in the given inequality.

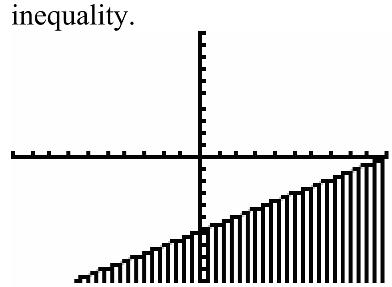
Test (0, 0).

$$y \le \frac{3}{5}x - 6$$

$$0 \le \frac{3}{5}(0) - 6$$

$$0 \le -6$$

Since the statement is false, the region not containing (0, 0) is the solution to the inequality.



[–10, 10] by [–10, 40]

7. To determine the solution region and the corners of the solution region, pick a point in a potential solution region and test it. For example, pick $(2,2)$.

$x + y \le 5$

$2 + 2 \le 5$

$4 \le 5$

True statement

$2x + y \le 8$

$2(2) + 2 \le 8$

$4 + 2 \le 8$

$6 \le 8$

True statement

$x \ge 0$

$2 \ge 0$

True statement

$y \ge 0$

$2 \ge 0$

True statement

Since all the inequalities are true at the point $(2,2)$, the region that contains $(2,2)$ is the solution region. The corners of the region are $(0,0),(4,0),(3,2),$ and $(0,5)$.

8. To determine the solution region and the corners of the solution region, pick a point in a potential solution region and test it. Pick $(2,2)$.

$2x + y \le 12$

$2(2) + 2 \le 12$

$6 \le 12$

True statement

$x + 5y \le 15$

$2 + 5(2) \le 15$

$12 \le 15$

True statement

$x \ge 0$

$2 \ge 0$

True statement

$y \ge 0$

$2 \ge 0$

True statement

Since all the inequalities are true at the point $(2,2)$, the region that contains $(2,2)$ is the solution region. The corners of the region are $(0,3),(0,0),(5,2),$ and $(6,0)$.

9. To determine the solution region and the corners of the solution region, pick a point in a potential solution region and test it. Pick $(2,4)$.

$4x + 2y > 8$

$4(2) + 2(8) > 8$

$24 > 8$

True statement

$3x + y > 5$

$3(2) + 4 > 5$

$10 > 5$

True statement

$x \ge 0$

$2 \ge 0$

True statement

$y \ge 0$

$4 \ge 0$

True statement

Since all the inequalities are true at the point $(2,4)$, the region that contains $(2,4)$ is the solution region. The corners of the region are $(2,0),(0,5),$ and $(1,2)$.

10. To determine the solution region and the corners of the solution region, pick a point in a potential solution region and test it. Pick $(3,3)$.

$2x+4y \geq 12$

$2(3)+4(3) \geq 12$

$18 \geq 12$

True statement

$x+3y \geq 8$

$3+3(3) \geq 8$

$12 \geq 8$

True statement

$x \geq 0$

$3 \geq 0$

True statement

$y \geq 0$

$3 \geq 0$

True statement

Since all the inequalities are true at the point $(3,3)$, the region that contains $(3,3)$ is the solution region. The corners of the region are $(8,0),(0,3)$, and $(2,2)$.

11. To determine the solution region and the corners of the solution region, pick a point in a potential solution region and test it. Pick $(3,3)$.

$2x+6y \geq 12$

$2(3)+6(3) \geq 12$

$6+18 \geq 12$

$24 \geq 12$

True statement

$3x+y \geq 5$

$3(3)+3 \geq 5$

$12 \geq 5$

True statement

$x+2y \geq 5$

$3+2(3) \geq 5$

$9 \geq 5$

True statement

$x \geq 0$

$3 \geq 0$

True statement

$y \geq 0$

$3 \geq 0$

True statement

Since all the inequalities are true at the point $(3,3)$, the region that contains $(3,3)$ is the solution region. The corners of the region are $(0,5),(1,2),(3,1)$, and $(6,0)$.

12. To determine the solution region and the corners of the solution region, pick a point in a potential solution region and test it. Choose $(6,1)$.

$2x+y \geq 12$

$2(6)+1 \geq 12$

$13 \geq 12$

True statement

$x+y \leq 8$

$6+1 \leq 8$

$7 \leq 8$

True statement

$2x + y \le 14$

$2(6) + 1 \le 14$

$13 \le 14$

True statement

$x \ge 0$

$6 \ge 0$

True statement

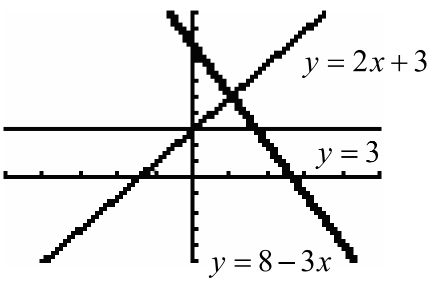
$y \ge 0$

$1 \ge 0$

True statement

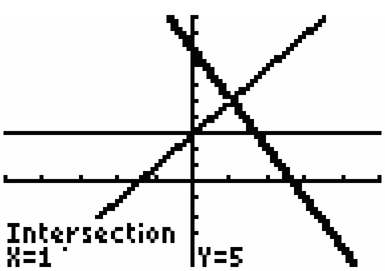
Since all the inequalities are true at the point $(6,1)$, the region that contains $(6,1)$ is the solution region. The corners of the region are $(6,0),(6,2),(4,4)$, and $(7,0)$.

13. The graph of the system is

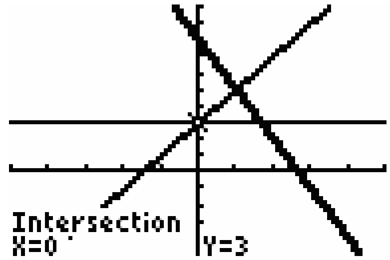


[–5, 5] by [–5, 10]

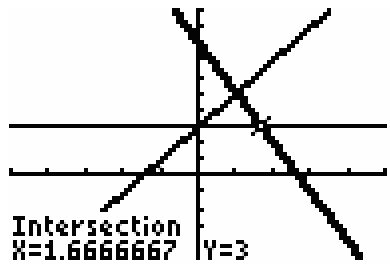
Note that $y = 3$ is a dashed line. The other two lines are solid. The intersection point between $y = 8 - 3x$ and $y = 2x + 3$ is $(1,5)$.



The intersection point between $y = 3$ and $y = 2x + 3$ is $(0,3)$.



The intersection point between $y = 3$ and $y = 8 - 3x$ is $\left(\frac{5}{3}, 3\right)$.



To determine the solution region, pick a point to test. Pick $(1,4)$.

$y \le 8 - 3x$

$4 \le 8 - 3(1)$

$4 \le 5$

True statement

$y \le 2x + 3$

$4 \le 2(1) + 3$

$4 \le 5$

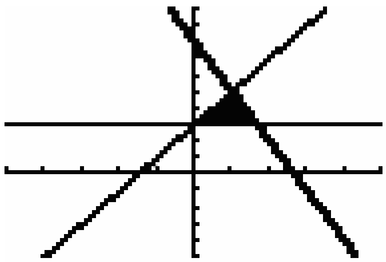
True statement

$y > 3$

$4 > 3$

True statement

Since all the inequalities that form the system are true at the point $(1,4)$, the region that contains $(1,4)$ is the solution region. The corners of the region are $(0,3),(1,5)$, and $\left(\frac{5}{3}, 3\right)$.



[–10, 10] by [–10, 10]

Recall that $y = 3$ is dashed. The other two lines are solid.

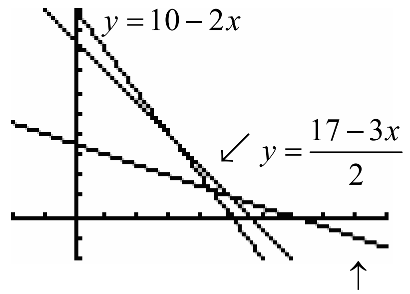
14. Rewriting the system:

$$2x + y \ge 10$$
$$y \ge 10 - 2x$$
and
$$3x + 2y \ge 17$$
$$2y \ge 17 - 3x$$
$$y \ge \frac{17 - 3x}{2}$$
and
$$x + 2y \ge 7$$
$$2y \ge 7 - x$$
$$y \ge \frac{7 - x}{2}$$

The new system is

$$\begin{cases} y \ge 10 - 2x \\ y \ge \dfrac{17 - 3x}{2} \\ y \ge \dfrac{7 - x}{2} \end{cases}$$

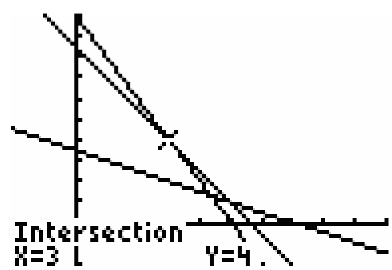
The graph of the system is



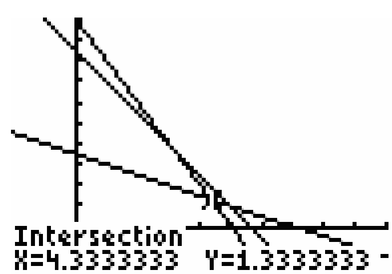
[–2, 10] by [–2, 10]

Note that all the lines are solid.

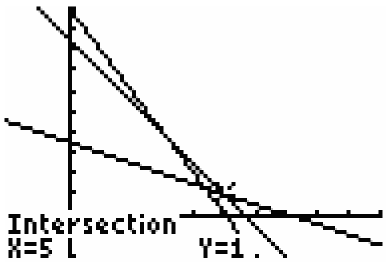
Determining the intersection point between $y = 10 - 2x$ and $y = \frac{17 - 3x}{2}$ yields $(3, 4)$.



Determining the intersection point between $y = 10 - 2x$ and $y = \frac{7 - x}{2}$ yields $\left(4\frac{1}{3}, 1\frac{1}{3}\right)$.



Determining the intersection point between $y = \frac{7 - x}{2}$ and $y = \frac{17 - 3x}{2}$ yields $(5, 1)$.



To determine the solution region, pick a point to test. Pick $(5,4)$.

$y \geq 10 - 2x$

$4 \geq 10 - 2(5)$

$4 \geq 0$

True statement

$y \geq \dfrac{17 - 3x}{2}$

$4 \geq \dfrac{17 - 3(5)}{2}$

$4 \geq \dfrac{2}{2}$

$4 \geq 1$

True statement

$y \geq \dfrac{7 - x}{2}$

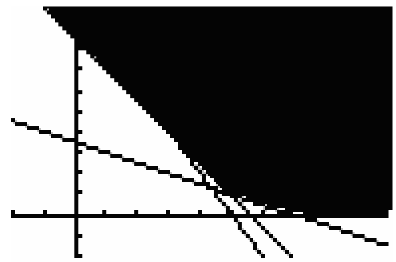
$4 \geq \dfrac{7 - 5}{2}$

$4 \geq \dfrac{2}{2}$

$4 \geq 1$

True statement

Since all the inequalities that form the system are true at the point $(5,4)$, the region that contains $(5,4)$ is the solution region. Considering the graph of the system, the corners of the region are $(0,10), (3,4), (5,1)$, and $(7,0)$.



[2, 10] by [–2, 10]

Recall that all the lines are solid.

15. Rewriting the inequalities:

$2x + y < 5$

$y < -2x + 5$

and

$2x - y > -1$

$-y > -1 - 2x$

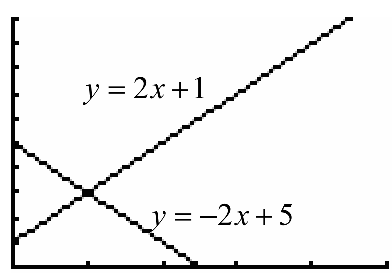
$\dfrac{-y}{-1} < \dfrac{-1 - 2x}{-1}$

$y < 2x + 1$

The new system is

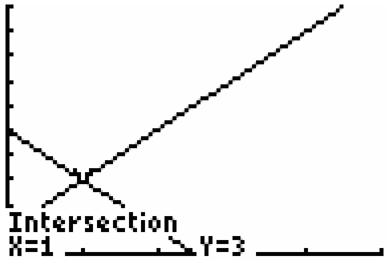
$$\begin{cases} y < -2x + 5 \\ y < 2x + 1 \\ x \geq 0, y \geq 0 \end{cases}$$

The graph of the system is



[0, 5] by [0, 10]

Note that both lines are dashed. The intersection point between the two lines is $(1,3)$.



To determine the solution region, pick a point to test. Pick $(1,2)$.

$y < -2x+5$

$2 < -2(1)+5$

$2 < 3$

True statement

$y < 2x+1$

$2 < 2(1)+1$

$2 < 3$

True statement

$x \geq 0$

$1 \geq 0$

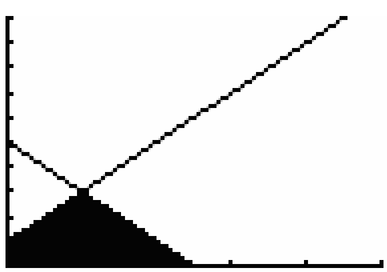
True statement

$y \geq 0$

$2 \geq 0$

True statement

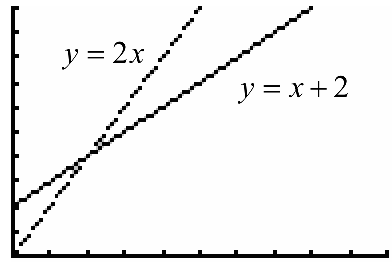
Since all the inequalities that form the system are true at the point $(1,2)$, the region that contains $(1,2)$ is the solution region. Considering the graph, the corners of the region are $(0,0)$, $(0,1)$, $(2.5,0)$ and $(1,3)$.



[0, 5] by [0, 10]

Recall that both lines are dashed.

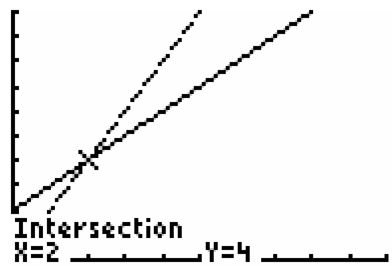
16. The graph of the system is



[0, 10] by [0, 10]

Note that both lines are dashed because there is no "equal to" in either inequality statement.

The intersection point between the two lines is $(2,4)$.



To determine the solution region, pick a point to test. Pick $(4,7)$.

$y < 2x$

$7 < 2(4)$

$7 < 8$

True statement

$y > x+2$

$7 > 4+2$

$7 > 6$

True statement

$x \ge 0$

$4 \ge 0$

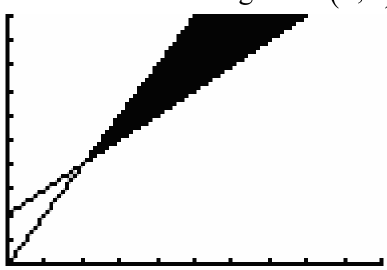
True statement

$y \ge 0$

$7 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(4,7)$, the region that contains $(4,7)$ is the solution region. The corner of the region is $(2,4)$.



[0, 10] by [0, 10]

Both lines are dashed.

17. Rewriting the system:

$$x + 2y \ge 4$$
$$2y \ge 4 - x$$
$$y \ge \frac{4-x}{2}$$

and

$$x + y \le 5$$
$$y \le 5 - x$$

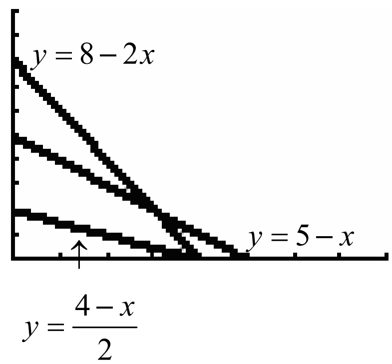
and

$$2x + y \le 8$$
$$y \le 8 - 2x$$

The new system is

$$\begin{cases} y \ge \dfrac{4-x}{2} \\ y \le 5 - x \\ y \le 8 - 2x \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is



[0, 8] by [0, 10]

Note that all the lines are solid.

Determining the intersection point between $y = 8 - 2x$ and $y = 5 - x$:

$8 - 2x = 5 - x$

$-x = -3$

$x = 3$

Substituting to find y

$y = 5 - x$

$y = 5 - 3$

$y = 2$

The intersection point is $(3,2)$.

Determining the intersection point between $y = 8 - 2x$ and $y = \frac{4-x}{2}$:

$8-2x=\dfrac{4-x}{2}$

$2(8-2x)=2\left(\dfrac{4-x}{2}\right)$

$16-4x=4-x$

$-3x=-12$

$x=4$

Substituting to find y

$y=8-2x$

$y=8-2(4)$

$y=0$

The intersection point is $(4,0)$.

To determine the solution region, pick a point to test. Pick $(1,3)$.

$y\ge\dfrac{4-x}{2}$

$3\ge\dfrac{4-1}{2}$

$3\ge\dfrac{3}{2}$

True statement

$y\le 5-x$

$3\le 5-1$

$3\le 4$

True statement

$y\le 8-2x$

$3\le 8-2(1)$

$3\le 6$

True statement

$x\ge 0$

$1\ge 0$

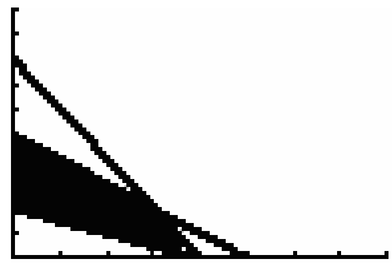
True statement

$y\ge 0$

$3\ge 0$

True statement

Since all the inequalities that form the system are true at the point $(1,3)$, the region that contains $(1,3)$ is the solution region. Considering the graph of the system, the corners of the region are $(0,2),(0,5),(3,2)$, and $(4,0)$.



[0, 8] by [0, 10]

Recall that all the lines are solid.

18. Rewriting the system:

$x+y<4$

$y<4-x$

and

$x+2y<6$

$2y<6-x$

$y<\dfrac{6-x}{2}$

and

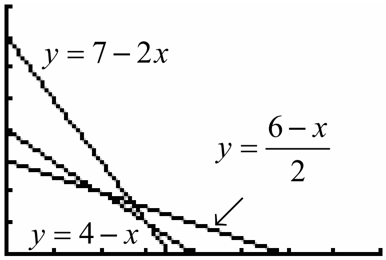
$2x+y<7$

$y<7-2x$

The new system is

$$\begin{cases} y<4-x \\ y<\dfrac{6-x}{2} \\ y<7-2x \\ x\ge 0, y\ge 0 \end{cases}$$

The graph of the system is



[0, 8] by [0, 8]

Note that all the lines are dashed.

Determining the intersection point between $y = 7 - 2x$ and $y = 4 - x$:

$7 - 2x = 4 - x$

$-x = -3$

$x = 3$

Substituting to find y

$y = 4 - x$

$y = 4 - 3$

$y = 1$

The intersection point is $(3,1)$.

Determining the intersection point between $y = 4 - x$ and $y = \frac{6-x}{2}$:

$4 - x = \frac{6-x}{2}$

$2(4 - x) = 2\left(\frac{6-x}{2}\right)$

$8 - 2x = 6 - x$

$-x = -2$

$x = 2$

Substituting to find y

$y = 4 - x$

$y = 4 - 2$

$y = 2$

The intersection point is $(2,2)$.

To determine the solution region, pick a point to test. Pick $(1,2)$.

$y < 4 - x$

$2 < 4 - 1$

$2 < 3$

True statement

$y < \frac{6-x}{2}$

$2 < \frac{6-1}{2}$

$2 < \frac{5}{2}$

True statement

$y < 7 - 2x$

$2 < 7 - 2(1)$

$2 < 5$

True statement

$x \geq 0$

$1 \geq 0$

True statement

$y \geq 0$

$2 \geq 0$

True statement

Since all the inequalities that form the system are true at the point $(1,2)$, the region that contains $(1,2)$ is the solution region. Considering the graph of the system, the corners of the region are $(0,0), (0,3), (3.5,0), (2,2)$, and $(3,1)$.

[0, 8] by [0, 10]

Recall that all the lines are dashed.

19. Rewriting the system:

$$x^2 - 3y < 4$$
$$-3y < 4 - x^2$$
$$y > \frac{4 - x^2}{-3}$$
$$y > \frac{x^2 - 4}{3}$$

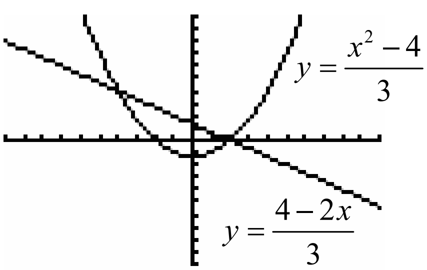
and

$$2x + 3y < 4$$
$$3y < 4 - 2x$$
$$y < \frac{4 - 2x}{3}$$

The new system is

$$\begin{cases} y > \dfrac{x^2 - 4}{3} \\ y < \dfrac{4 - 2x}{3} \end{cases}$$

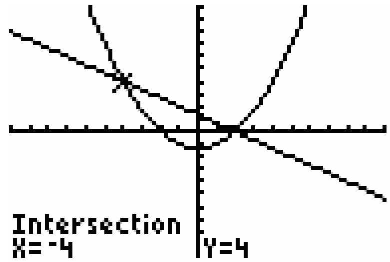
The graph of the system is



[–10, 10] by [–10, 10]

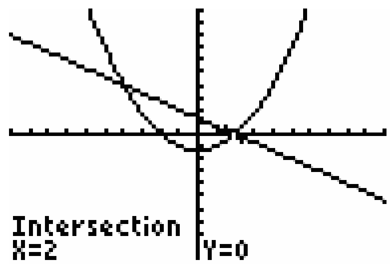
Note that all the lines are dashed.

Determining the intersection points between the two functions graphically yields



[–10, 10] by [–10, 10]

and



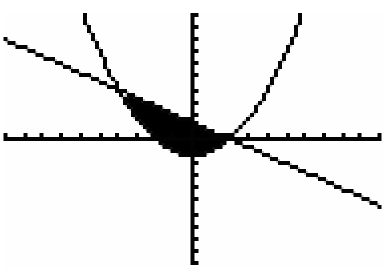
[–10, 10] by [–10, 10]

The intersection points are $(-4, 4)$ and $(2, 0)$.

To determine the solution region, pick a point to test. Pick $(0, 0)$.

$$y > \frac{x^2 - 4}{3}$$
$$0 > \frac{(0)^2 - 4}{3}$$
$$0 > -\frac{4}{3}$$
True statement
$$y < \frac{4 - 2x}{3}$$
$$0 < \frac{4 - 2(0)}{3}$$
$$0 < \frac{4}{3}$$
True statement

Since all the inequalities that form the system are true at the point $(0,0)$, the region that contains $(0,0)$ is the solution region. Considering the graph of the system, the corners of the region are $(-4,4)$ and $(2,0)$.



[–10, 10] by [–10, 10]

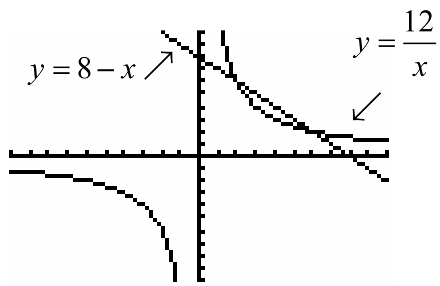
Recall that all the lines are dashed.

20. Rewriting the system:

$$x + y \le 8$$
$$y \le 8 - x$$
and
$$xy \ge 12$$
$$y \ge \frac{12}{x}$$

The new system is
$$\begin{cases} y \le 8 - x \\ y \ge \dfrac{12}{x} \end{cases}$$

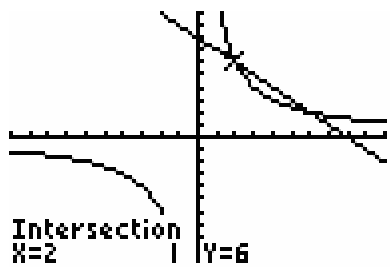
The graph of the system is



[–10, 10] by [–10, 10]

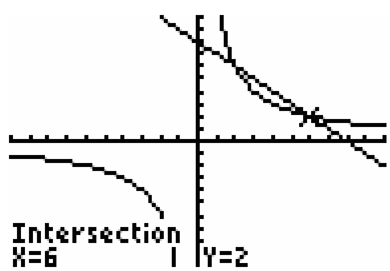
Note that all the lines are solid.

Determining the intersection points between the two functions graphically yields



[–10, 10] by [–10, 10]

and



[–10, 10] by [–10, 10]

The intersection points are $(2,6)$ and $(6,2)$.

To determine the solution region, pick a point to test. Pick $(4,3.5)$.

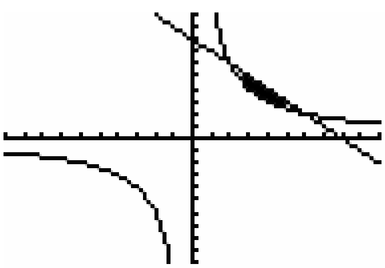
$$\begin{aligned}&y \le 8-x\\&3.5 \le 8-4\\&3.5 \le 4\end{aligned}$$

True statement

$$\begin{aligned}&y \ge \frac{12}{x}\\&3.5 \ge \frac{12}{4}\\&3.5 \ge 3\end{aligned}$$

True statement

Since all the inequalities that form the system are true at the point $(4,3.5)$, the region that contains $(4,3.5)$ is the solution region. Considering the graph of the system, the corners of the region are $(2,6)$ and $(6,2)$.



[–10, 10] by [–10, 10]

Recall that all the lines are solid.

21. Rewriting the system:

$$\begin{aligned}x^2 - y - 8x &\le -6\\ -y &\le -x^2 + 8x - 6\\ y &\ge x^2 - 8x + 6\end{aligned}$$

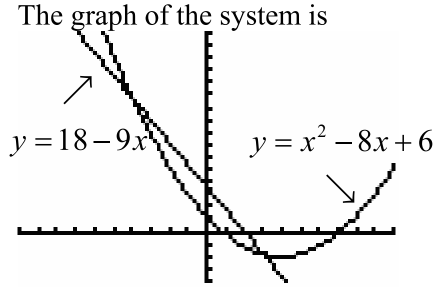
and

$$\begin{aligned}y + 9x &\le 18\\ y &\le 18 - 9x\end{aligned}$$

The new system is

$$\begin{cases} y \ge x^2 - 8x + 6\\ y \le 18 - 9x\end{cases}$$

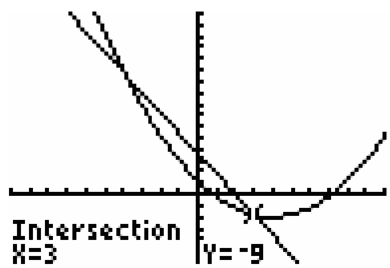
The graph of the system is



[–10, 10] by [–20, 80]

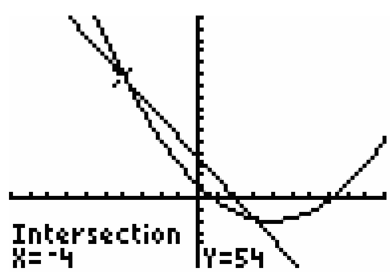
Note that all the lines are solid.

Determining the intersection points between the two functions graphically yields



[–10, 10] by [–20, 80]

and



[–10, 10] by [–20, 80]

The intersection points are $(3,-9)$ and $(-4,54)$.

To determine the solution region, pick a point to test. Pick $(0,10)$.

$y \geq x^2 - 8x + 6$

$10 \geq (0)^2 - 8(0) + 6$

$10 \geq 6$

True statement

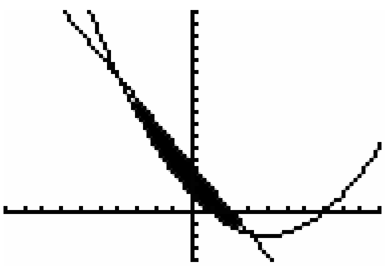
$y \leq 18 - 9x$

$10 \leq 18 - 9(0)$

$10 \leq 18$

True statement

Since all the inequalities that form the system are true at the point $(0,10)$, the region that contains $(0,10)$ is the solution region. Considering the graph of the system, the corners of the region are $(3,-9)$ and $(-4,54)$.



[–10, 10] by [–20, 80]

Recall that all the lines are solid.

22. Rewriting the system:

$x^2 - y > 0$

$-y > -x^2$

$y < x^2$

and

$2x + \sqrt{y} > 4$

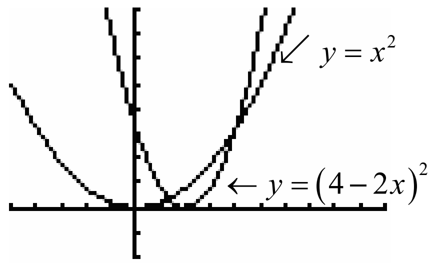
$\sqrt{y} > 4 - 2x$

$y > (4 - 2x)^2$

The new system is

$$\begin{cases} y < x^2 \\ y > (4-2x)^2 \end{cases}$$

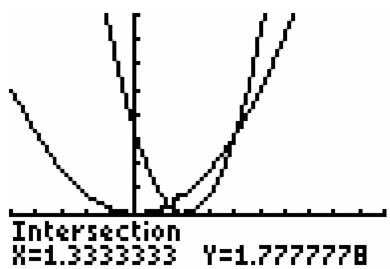
The graph of the system is



[–5, 10] by [–10, 40]

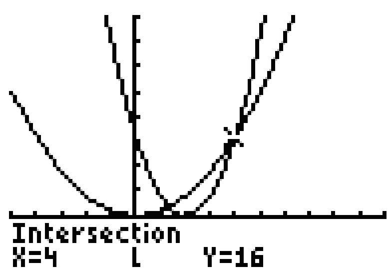
Note that all the lines are dashed.

Determining the intersection points between the two functions graphically yields



[–5, 10] by [–10, 40]

and



[–5, 10] by [–10, 40]

The intersection points are $(1.\overline{3}, 1.\overline{7})$ and $(4,16)$.

To determine the solution region, pick a point to test. Pick $(2,2)$.

$y < x^2$

$2 < (2)^2$

$2 < 4$

True statement

$y > (4-2x)^2$

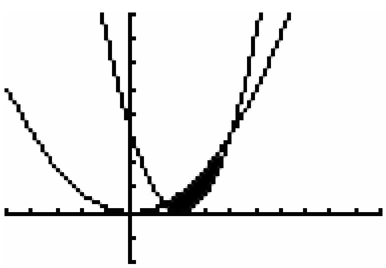
$2 > (4-2(2))^2$

$2 > (0)^2$

$2 > 0$

True statement

Since all the inequalities that form the system are true at the point $(2,2)$, the region that contains $(2,2)$ is the solution region. Considering the graph of the system, the corners of the region are $(1.\overline{3}, 1.\overline{7})$ and $(4,16)$.



[–5, 10] by [–10, 40]

Recall that all the lines are dashed.

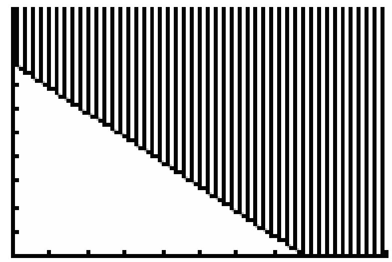
Section 6.1 Exercises

23. a. Let x = the number of Turbo blowers, and y = the number of Tornado blowers.

$x + y \geq 780$

b. $x + y \geq 780$

$y \geq 780 - x$



[0, 1000] by [0, 1000]

Note that the line is solid since an "equal to" is part of the inequality.

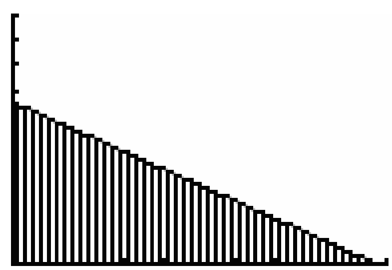
24. a. Let x = the number of electric blowers, and y = the number of gas blowers.

$78x + 117y \leq 76{,}050$

b. $78x + 117y \leq 76{,}050$

$117y \leq 76{,}050 - 78x$

$y \leq \dfrac{76{,}050 - 78x}{117}$



[0, 1000] by [0, 1000]

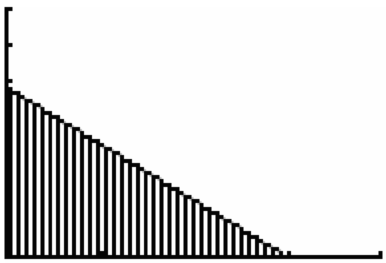
Note that the line is solid since the "equal to" is part of the inequality.

25. a. Let x = minutes of cable television time, and y = minutes of radio time.

$240x+150y\le 36{,}000$

b.

$$240x+150y\le 36{,}000$$
$$150y\le 36{,}000-240x$$
$$y\le \frac{36{,}000-240x}{150}$$



[0, 200] by [0, 350]

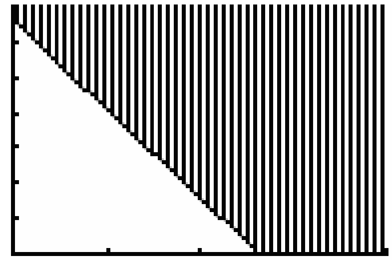
Note that the line is solid since an "equal to" is part of the inequality.

26. a. Let x = minutes of cable television time, and y = minutes of radio time. Then, $\frac{1}{4}x$ represents the number of cars sold based on cable advertising, while $\frac{1}{10}y$ represents the number of cars sold based on radio advertising. The corresponding inequality is

$\frac{1}{4}x+\frac{1}{10}y\ge 33.$

b.

$$\frac{1}{4}x+\frac{1}{10}y\ge 33$$
$$\text{LCM: } 20$$
$$20\left(\frac{1}{4}x+\frac{1}{10}y\right)\ge 20(33)$$
$$5x+2y\ge 660$$
$$2y\ge 660-5x$$
$$y\ge \frac{660-5x}{2}$$



[0, 200] by [0, 350]

Note that the line is solid since an "equal to" is part of the inequality.

27. a. Let x = minutes of television time, and y = minutes of radio time. Then, $0.12x$ represents the number, in millions, of registered voters reached by television advertising, and $0.009y$ represents the number, in millions, of registered votersreached by radio advertising. The corresponding inequalities are

$$\begin{cases} x+y\ge 100 \\ 0.12x+0.009y\ge 7.56 \\ x\ge 0, y\ge 0 \end{cases}$$

b. Rewriting the system:

$$x+y\ge 100$$
$$y\ge 100-x$$

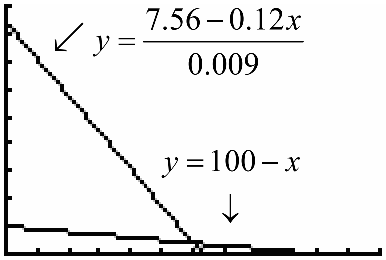
and

$$0.12x+0.009y\ge 7.56$$
$$0.009y\ge 7.56-0.12x$$
$$y\ge \frac{7.56-0.12x}{0.009}$$

The new system is

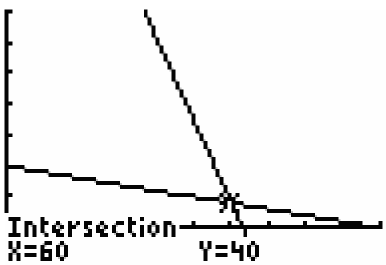
$$\begin{cases} y\ge 100-x \\ y\ge \dfrac{7.56-0.12x}{0.009} \\ x\ge 0, y\ge 0 \end{cases}$$

The graph of the system is



[0, 120] by [0, 900]

The intersection point between the two lines is $(60, 40)$. Both lines are solid since there is an "equal to" in both inequalities.



To determine the solution region, pick a point to test. Pick $(60,100)$.

$x + y \geq 100$

$60 + 100 \geq 100$

$160 \geq 100$

True statement

$0.12x + 0.009y \geq 7.56$

$0.12(60) + 0.009(100) \geq 7.56$

$7.2 + 0.9 \geq 7.56$

$8.1 \geq 7.56$

True statement

$x \geq 0$

$60 \geq 0$

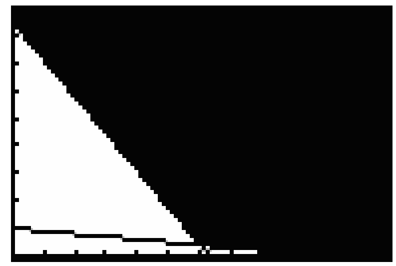
True statement

$y \geq 0$

$100 \geq 0$

True statement

Since all the inequalities that form the system are true at the point $(60,100)$, the region that contains $(60,100)$ is the solution region. The graph of the solution is



[0, 120] by [0, 900]

Recall that both lines are solid.

One of the corner points is the intersection point of the two lines, $(60, 40)$. Another corner point occurs where $y = \frac{7.56 - 0.12x}{0.009}$ crosses the y-axis. A third corner point occurs where $y = 100 - x$ crosses the x-axis.

To find the y-intercept, let $x = 0$.

$y = \frac{7.56 - 0.12(0)}{0.009}$

$y = \frac{7.56}{0.009}$

$y = 840$

$(0,840)$

To find the x-intercept, let $y = 0$.

$0 = 100 - x$

$x = 100$

$(100, 0)$

Therefore, the corner points of the solution region are $(60, 40), (100, 0)$, and $(0,840)$.

28. The system of inequalities is

$$\begin{cases} 200x + 500y \le 5000 \\ 240,000x + 300,000y \le 3,600,000 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting the system:

$$200x + 500y \le 5000$$
$$500y \le 5000 - 200x$$
$$y \le \frac{5000 - 200x}{500}$$

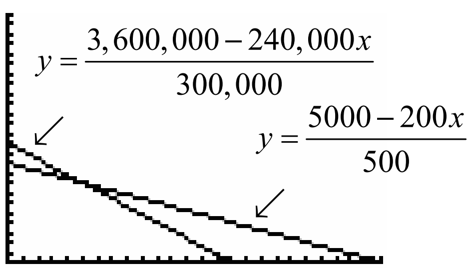
and

$$240,000x + 300,000y \le 3,600,000$$
$$300,000y \le 3,600,000 - 240,000x$$
$$y \le \frac{3,600,000 - 240,000x}{300,000}$$

The new system is

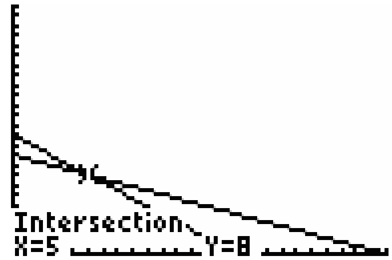
$$\begin{cases} y \le \dfrac{5000 - 200x}{500} \\ y \le \dfrac{3,600,000 - 240,000x}{300,000} \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is



[0, 25] by [0, 25]

The intersection point between the two lines is $(5, 8)$. Both lines are solid since there is an "equal to" in both inequalities.



To determine the solution region, pick a point to test. Pick $(1, 2)$.

$$200x + 500y \le 5000$$
$$200(1) + 500(2) \le 5000$$
$$200 + 500 \le 5000$$
$$700 \le 5000$$

True statement

$$240,000x + 300,000y \le 3,600,000$$
$$240,000(1) + 300,000(2) \le 3,600,000$$
$$240,000 + 600,000 \le 3,600,000$$
$$840,000 \le 3,600,000$$

True statement

$$x \ge 0$$
$$1 \ge 0$$

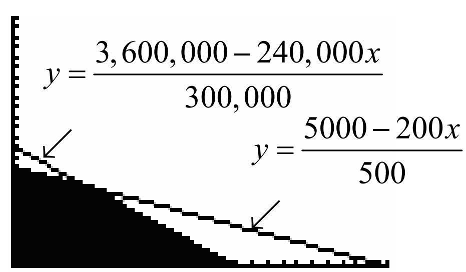
True statement

$$y \ge 0$$
$$2 \ge 0$$

True statement

Since all the inequalities that form the system are true at the point $(1, 2)$, the region that contains $(1, 2)$ is the solution region.

The graph of the solution is



[0, 25] by [0, 25]

Recall that both lines are solid.

One of the corner points is the intersection point of the two lines, $(5,8)$. Another corner point occurs at $(0,0)$. A third corner point occurs where $y=\dfrac{3,600,000-240,000x}{300,000}$ crosses the x-axis. A fourth corner point occurs where $y=\dfrac{5000-200x}{500}$ crosses the y-axis.

To find the y-intercept, let $x=0$.

$$y=\frac{5000-200(0)}{500}$$
$$y=\frac{5000}{500}$$
$$y=10$$
$$(0,10)$$

To find the x-intercept, let $y=0$.

$$0=\frac{3,600,000-240,000x}{300,000}$$
$$0=3,600,000-240,000x$$
$$240,000x=3,600,000$$
$$x=\frac{3,600,000}{240,000}=15$$
$$(15,0)$$

Therefore, the corner points of the solution region are $(5,8),(0,0),(0,10)$ and $(15,0)$.

29. The system of inequalities is

$$\begin{cases} x+y\ge 780 \\ 78x+117y\le 76,050 \\ x\ge 0, y\ge 0 \end{cases}$$

Rewriting the system:

$$x+y\ge 780$$
$$y\ge 780-x$$

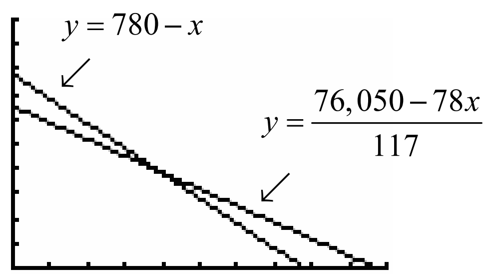
and

$$78x+117y\le 76,050$$
$$117y\le 76,050-78x$$
$$y\le\frac{76,050-78x}{117}$$

The new system is

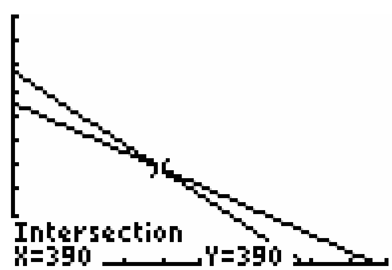
$$\begin{cases} y\ge 780-x \\ y\le\dfrac{76,050-78x}{117} \\ x\ge 0, y\ge 0 \end{cases}$$

The graph of the system is



[0, 1000] by [0, 1000]

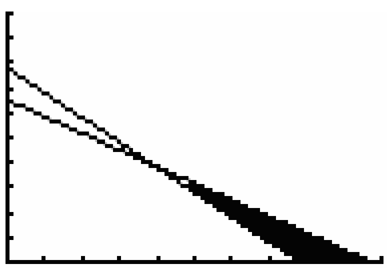
The intersection point between the two lines is $(390,390)$. Both lines are solid since there is an "equal to" in both inequalities.



To determine the solution region, pick a point to test. Pick $(700,100)$.

$x + y \geq 780$
$700 + 100 \geq 780$
$800 \geq 780$
True statement
$78x + 117y \leq 76{,}050$
$78(700) + 117(100) \leq 76{,}050$
$54{,}600 + 11{,}700 \leq 76{,}050$
$66{,}300 \leq 76{,}050$
True statement
$x \geq 0$
$700 \geq 0$
True statement
$y \geq 0$
$100 \geq 0$
True statement

Since all the inequalities that form the system are true at the point $(700,100)$, the region that contains $(700,100)$ is the solution region. The graph of the solution is



[0, 1000] by [0, 1000]

Recall that both lines are solid.

One of the corner points is the intersection point of the two lines, $(390,390)$. A second corner point occurs where $y = 780 - x$ crosses the x-axis. A third corner point occurs where $y = \frac{76{,}050 - 78x}{117}$ crosses the x-axis.

To find the x-intercept, let $y = 0$.
$y = 780 - x$
$0 = 780 - x$
$x = 780$
$(780,0)$
To find the x-intercept, let $y = 0$.
$0 = \frac{76{,}050 - 78x}{117}$
$0 = 76{,}050 - 78x$
$78x = 76{,}050$
$x = \frac{76{,}050}{78} = 975$
$(975,0)$

Therefore, the corner points of the solution region are $(390,390)$, $(780,0)$ and $(975,0)$.

30. The system of inequalities is

$$\begin{cases} 240x + 150y \leq 36{,}000 \\ \frac{1}{4}x + \frac{1}{10}y \geq 33 \\ x \geq 0, y \geq 0 \end{cases}$$

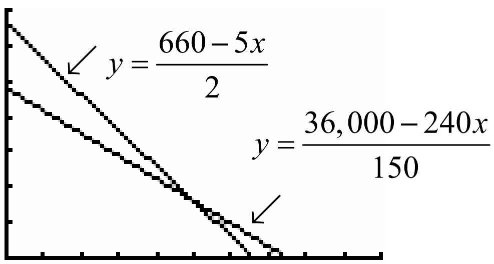
Rewriting the system:

$240x + 150y \leq 36{,}000$
$150y \leq 36{,}000 - 240x$
$y \leq \frac{36{,}000 - 240x}{150}$
and
$\frac{1}{4}x + \frac{1}{10}y \geq 33$
LCM: 20
$20\left(\frac{1}{4}x + \frac{1}{10}y\right) \geq 20(33)$
$5x + 2y \geq 660$
$2y \geq 660 - 5x$
$y \geq \frac{660 - 5x}{2}$

The new system is

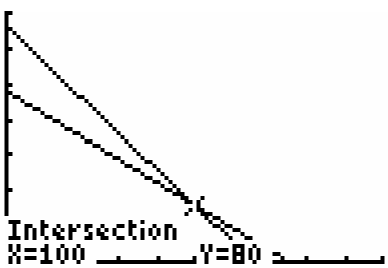
$$\begin{cases} y \le \dfrac{36{,}000 - 240x}{150} \\ y \ge \dfrac{660 - 5x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is



[0, 200] by [0, 350]

The intersection point between the two lines is $(100, 80)$. Both lines are solid since there is an "equal to" in both inequalities.



To determine the solution region, pick a point to test. Pick $(125, 30)$.

$$y \le \frac{36{,}000 - 240x}{150}$$

$$30 \le \frac{36{,}000 - 240(125)}{150}$$

$$30 \le \frac{6000}{150}$$

$$30 \le 40$$

True statement

$$y \ge \frac{660 - 5x}{2}$$

$$30 \ge \frac{660 - 5(125)}{2}$$

$$30 \ge \frac{35}{2}$$

$$30 \ge 17.5$$

True statement

$$x \ge 0$$

$$125 \ge 0$$

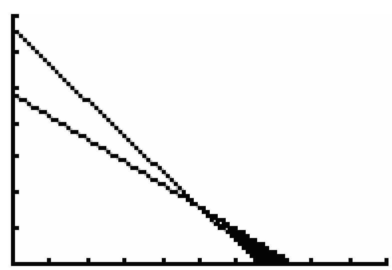
True statement

$$y \ge 0$$

$$30 \ge 0$$

True statement

Since all the inequalities that form the system are true at the point $(125, 30)$, the region that contains $(125, 30)$ is the solution region. The graph of the solution is



[0, 200] by [0, 350]

Recall that both lines are solid.

One of the corner points is the intersection point of the two lines, $(100, 80)$. A second corner point occurs where $y = \frac{36,000 - 240x}{150}$ crosses the x-axis. A third corner point occurs where $y = \frac{660 - 5x}{2}$ crosses the x-axis.

To find the x-intercept, let $y = 0$.

$0 = \frac{36,000 - 240x}{150}$

$0 = 36,000 - 240x$

$240x = 36,000$

$x = \frac{36,000}{240} = 150$

$(150, 0)$

To find the x-intercept, let $y = 0$.

$0 = \frac{660 - 5x}{2}$

$0 = 660 - 5x$

$5x = 660$

$x = \frac{660}{5} = 132$

$(132, 0)$

Therefore, the corner points of the solution region are $(100, 80), (150, 0)$ and $(132, 0)$.

31. a. Let x = number of manufacturing days on assembly line 1, and y = number of manufacturing days on assembly line 2.

Then, the system of inequalities is

$$\begin{cases} 80x + 40y \ge 3200 \\ 20x + 20y \ge 1000 \\ 100x + 40y \ge 3400 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting the system:

$80x + 40y \ge 3200$

$40y \ge 3200 - 80x$

$y \ge \frac{3200 - 80x}{40}$

$y \ge 80 - 2x$

and

$20x + 20y \ge 1000$

$20y \ge 1000 - 20x$

$y \ge \frac{1000 - 20x}{20}$

$y \ge 50 - x$

and

$100x + 40y \ge 3400$

$40y \ge 3400 - 100x$

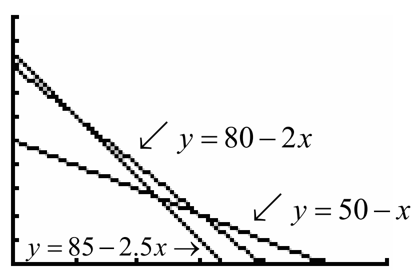
$y \ge \frac{3400 - 100x}{40}$

$y \ge 85 - 2.5x$

The new system is

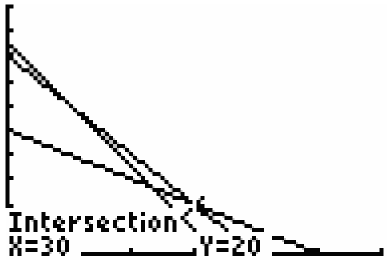
$$\begin{cases} y \ge 80 - 2x \\ y \ge 50 - x \\ y \ge 85 - 2.5x \\ x \ge 0, y \ge 0 \end{cases}$$

b. To solve the system, first consider the graph of the system

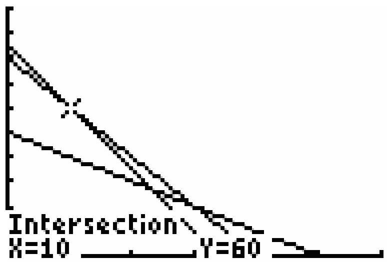


[0, 60] by [0, 100]

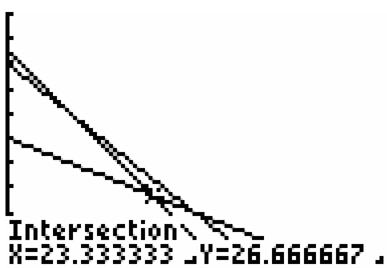
The intersection point between $y = 80 - 2x$ and $y = 50 - x$ is $(30, 20)$.



The intersection point between $y = 80 - 2x$ and $y = 85 - 2.5x$ is $(10, 60)$.



The intersection point between $y = 50 - x$ and $y = 85 - 2.5x$ is $\left(23.\bar{3}, 26.\bar{6}\right)$.

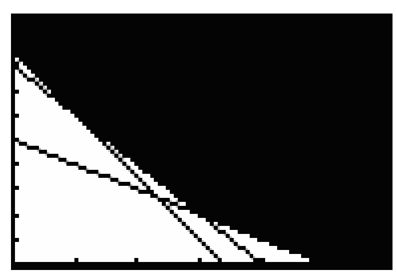


Note that all the lines are solid because there is an "equal to" as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(25, 40)$.

$80x + 40y \geq 3200$
$80(25) + 40(40) \geq 3200$
$3600 \geq 3200$
True statement
$20x + 20y \geq 1000$
$20(25) + 20(40) \geq 1000$
$500 + 800 \geq 1000$
$1300 \geq 1000$
True statement
$100x + 40y \geq 3400$
$100(25) + 40(40) \geq 3400$
$4100 \geq 3400$
True statement
$x \geq 0$
$25 \geq 0$
True statement
$y \geq 0$
$40 \geq 0$
True statement

Since all the inequalities that form the system are true at the point $(25, 40)$, the region that contains $(25, 40)$ is the solution region. The graph of the solution is



[0, 60] by [0, 100]

Recall that all lines are solid.

One of the corner points is the intersection point between $y = 80 - 2x$ and $y = 85 - 2.5x$, which is $(10, 60)$. A second corner point is the intersection point between $y = 80 - 2x$ and $y = 50 - x$, which is $(30, 20)$. A third corner point occurs where $y = 50 - x$ crosses the x-axis. A fourth corner point occurs where $y = 85 - 2.5x$ crosses the y-axis. Therefore, to find the x-intercept, let $y = 0$.

$0 = 50 - x$

$x = 50$

$(50, 0)$

To find the y-intercept, let $x = 0$.

$y = 85 - 2.5(0)$

$y = 85$

$(0, 85)$

Therefore, the corner points of the solution region are $(10, 60), (30, 20), (50, 0)$ and $(0, 85)$.

32. a. Let x = number of newspaper ad packages, and y = number of radio ad packages.

The system of inequalities is

$$\begin{cases} 1000x + 3000y \le 18{,}000 \\ 18x + 36y \ge 252 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting the system:

$$1000x + 3000y \le 18{,}000$$
$$3000y \le 18{,}000 - 1000x$$
$$y \le \frac{18{,}000 - 1000x}{3000}$$

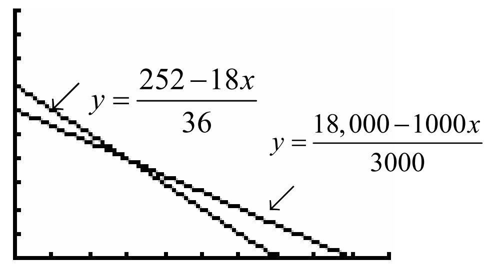
and

$$18x + 36y \ge 252$$
$$36y \ge 252 - 18x$$
$$y \ge \frac{252 - 18x}{36}$$

The new system is

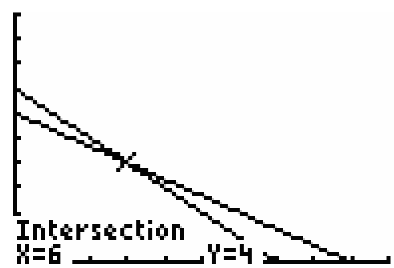
$$\begin{cases} y \le \dfrac{18{,}000 - 1000x}{3000} \\ y \ge \dfrac{252 - 18x}{36} \\ x \ge 0, y \ge 0 \end{cases}$$

b. To solve the system, first consider the graph of the system



[0, 20] by [0, 10]

The intersection point between the two lines is $(6, 4)$.



Note that the lines are solid because there is an “equal to” as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(11, 2)$.

$y \le \frac{18,000-1000x}{3000}$

$2 \le \frac{18,000-1000(11)}{3000}$

$2 \le \frac{7000}{3000}$

$2 \le 2.\overline{3}$

True statement

$y \ge \frac{252-18x}{36}$

$2 \ge \frac{252-18(11)}{36}$

$2 \ge \frac{54}{36}$

$2 \ge 1.5$

True statement

$x \ge 0$

$11 \ge 0$

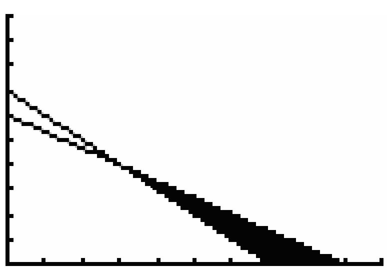
True statement

$y \ge 0$

$2 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(11,2)$, the region that contains $(11,2)$ is the solution region. The graph of the solution is



[0, 20] by [0, 10]

Recall that all lines are solid.

One of the corner points is the intersection point between the two lines, which is $(6,4)$. A second corner point occurs where $y = \frac{18,000-1000x}{3000}$ crosses the x-axis. A third corner point occurs where $y = \frac{252-18x}{36}$ crosses the x-axis.

To find the x-intercept, let $y = 0$.

$0 = \frac{18,000-1000x}{3000}$

$0 = 18,000-1000x$

$1000x = 18,000$

$x = \frac{18,000}{1000} = 18$

$(18,0)$

To find the x-intercept, let $y = 0$.

$0 = \frac{252-18x}{36}$

$0 = 252-18x$

$18x = 252$

$x = \frac{252}{18} = 14$

$(14,0)$

Therefore the corner points of the solution region are $(6,4)$, $(14,0)$ and $(18,0)$.

33. Let x = the number of bass, and y = the number of trout.

Then, the system of inequalities is

$$\begin{cases} 4x+10y \le 1600 \\ 6x+7y \le 1600 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting the system:

$$4x + 10y \le 1600$$
$$10y \le 1600 - 4x$$
$$y \le \frac{1600 - 4x}{10}$$

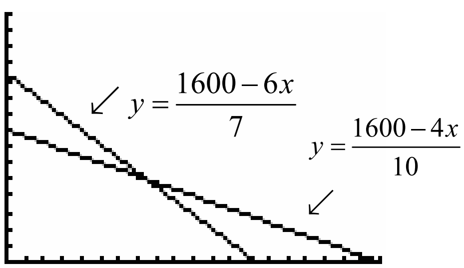
and

$$6x + 7y \le 1600$$
$$7y \le 1600 - 6x$$
$$y \le \frac{1600 - 6x}{7}$$

The new system is

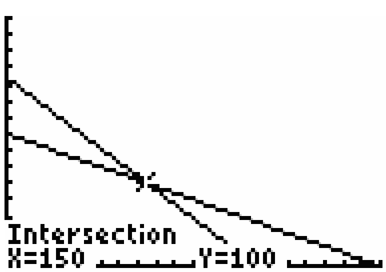
$$\begin{cases} y \le \dfrac{1600 - 4x}{10} \\ y \le \dfrac{1600 - 6x}{7} \\ x \ge 0, y \ge 0 \end{cases}$$

To solve the system, first consider the graph of the system



[0, 400] by [0, 300]

The intersection point between the two lines is $(150,100)$.



Note that the lines are solid because there is an "equal to" as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(100,75)$.

$$y \le \frac{1600 - 4x}{10}$$
$$75 \le \frac{1600 - 4(100)}{10}$$
$$75 \le \frac{1200}{10}$$
$$75 \le 120$$

True statement

$$y \le \frac{1600 - 6x}{7}$$
$$75 \le \frac{1600 - 6(100)}{7}$$
$$75 \le \frac{1000}{7}$$
$$75 \le \sim 142$$

True statement

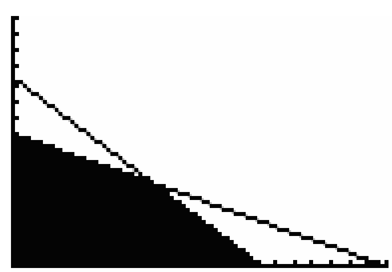
$$x \ge 0$$
$$100 \ge 0$$

True statement

$$y \ge 0$$
$$75 \ge 0$$

True statement

Since all the inequalities that form the system are true at the point $(100,75)$, the region that contains $(100,75)$ is the solution region. The graph of the solution is



[0, 400] by [0, 300]

Recall that all lines are solid.

One of the corner points is the intersection point between the two lines, which is $(150,100)$. A second corner point occurs where $y=\frac{1600-4x}{10}$ crosses the y-axis. A third corner point occurs where $y=\frac{1600-6x}{7}$ crosses the x-axis.

To find the y-intercept, let $x=0$.

$$y=\frac{1600-4(0)}{10}$$
$$y=\frac{1600}{10}$$
$$y=160$$
$$(0,160)$$

To find the x-intercept, let $y=0$.

$$0=\frac{1600-6x}{7}$$
$$0=1600-6x$$
$$6x=1600$$
$$x=\frac{1600}{6}=266.\overline{6}$$
$$(266.\overline{6},0)$$

A fourth corner point occurs at the origin, $(0,0)$. Therefore, the corner points of the solution region are $(0,0),(150,100),(0,160)$ and $(266.\overline{6},0)$.

34. a. Let x = number of inkjet printers, and let y = number of laserjet printers.

Then, the system of inequalities is

$$\begin{cases} x+y\le 120 \\ 2x+6y\le 400 \\ x\ge 0, y\ge 0 \end{cases}$$

Rewriting the system:

$$x+y\le 120$$
$$y\le 120-x$$

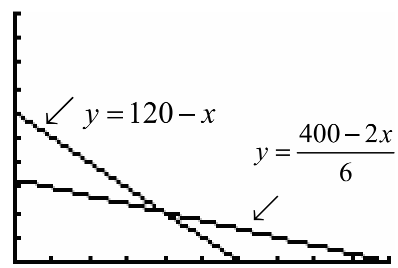
and

$$2x+6y\le 400$$
$$6y\le 400-2x$$
$$y\le\frac{400-2x}{6}$$

The new system is

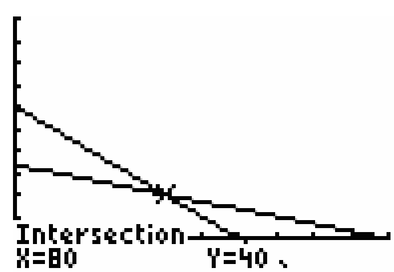
$$\begin{cases} y\le 120-x \\ y\le\frac{400-2x}{6} \\ x\ge 0, y\ge 0 \end{cases}$$

b. To solve the system, first consider the graph of the system



[0, 200] by [0, 200]

The intersection point between the two lines is $(80,40)$.



Note that the lines are solid because there is an "equal to" as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(5,10)$.

$y \le 120 - x$
$10 \le 120 - 5$
$10 \le 115$
True statement
$y \le \dfrac{400 - 2x}{6}$
$10 \le \dfrac{400 - 2(5)}{6}$
$10 \le \dfrac{390}{6}$
$10 \le 65$
True statement
$x \ge 0$
$5 \ge 0$
True statement
$y \ge 0$
$10 \ge 0$
True statement

Since all the inequalities that form the system are true at the point $(5,10)$, the region that contains $(5,10)$ is the solution region. The graph of the solution is

[0, 200] by [0, 200]

Recall that all lines are solid.

One of the corner points is the intersection point between the two lines, which is $(80,40)$. A second corner point occurs where $y = 120 - x$ crosses the x-axis. A third corner point occurs where $y = \dfrac{400 - 2x}{6}$ crosses the y-axis.

To find the x-intercept, let $y = 0$.
$0 = 120 - x$
$x = 120$
$(120,0)$
To find the y-intercept, let $x = 0$.
$y = \dfrac{400 - 2(0)}{6}$
$y = \dfrac{400}{6}$
$y = 66.\overline{6}$
$(0, 66.\overline{6})$

A fourth corner point occurs at the origin, $(0,0)$. Therefore, the corner points of the solution region are $(0,0), (80,40), (120,0)$ and $(0, 66.\overline{6})$.

35. a. Let x = number of Standard chairs, and let y = number of Deluxe chairs.

Then, the system of inequalities is

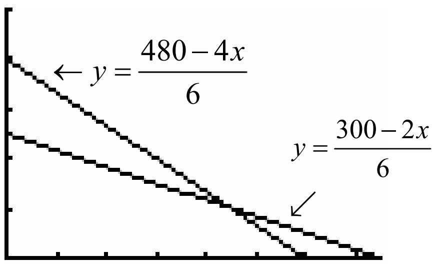
$$\begin{cases} 4x + 6y \le 480 \\ 2x + 6y \le 300 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting the system:
$4x + 6y \le 480$
$6y \le 480 - 4x$
$y \le \dfrac{480 - 4x}{6}$
and
$2x + 6y \le 300$
$6y \le 300 - 2x$
$y \le \dfrac{300 - 2x}{6}$

The new system is

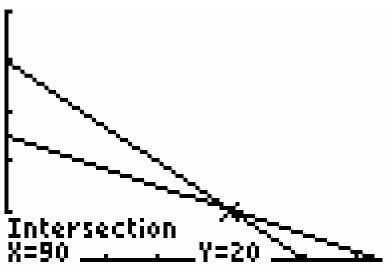
$$\begin{cases} y \le \dfrac{480-4x}{6} \\ y \le \dfrac{300-2x}{6} \\ x \ge 0, y \ge 0 \end{cases}$$

b. To solve the system, first consider the graph of the system



[0, 150] by [0, 100]

The intersection point between the two lines is $(90, 20)$.



Note that the lines are solid because there is an “equal to” as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(5,10)$.

$4x + 6y \le 480$

$4(5) + 6(10) \le 480$

$80 \le 480$

True statement

$2x + 6y \le 300$

$2(5) + 6(10) \le 300$

$70 \le 300$

True statement

$x \ge 0$

$5 \ge 0$

True statement

$y \ge 0$

$10 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(5,10)$, the region that contains $(5,10)$ is the solution region. The graph of the solution is

[0, 150] by [0, 100]

Recall that all lines are solid.

One of the corner points is the intersection point between the two lines, which is $(90, 20)$. A second corner point occurs where $y = \dfrac{480-4x}{6}$ crosses the *x*-axis. A third corner point occurs where $y = \dfrac{300-2x}{6}$ crosses the *y*-axis.

To find the x-intercept, let $y = 0$.

$$0 = \frac{480 - 4x}{6}$$
$$0 = 480 - 4x$$
$$4x = 480$$
$$x = \frac{480}{4}$$
$$(120, 0)$$

To find the y-intercept, let $x = 0$.

$$y = \frac{300 - 2(0)}{6}$$
$$y = \frac{300}{6}$$
$$y = 50$$
$$(0, 50)$$

A fourth corner point occurs at the origin, $(0,0)$. Therefore, the corner points of the solution region are $(0,0), (90,20), (120,0)$ and $(0,50)$.

36. a. Let x = number of Safecut chainsaws, and y = number of Deluxe Safecut chainsaws.

Then, the system of inequalities is

$$\begin{cases} 2x + 3y \le 36 \\ 1x + \frac{1}{2}y \le 12 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting the system:

$$2x + 3y \le 36$$
$$3y \le 36 - 2x$$
$$y \le \frac{36 - 2x}{3}$$

and

$$1x + \frac{1}{2}y \le 12$$

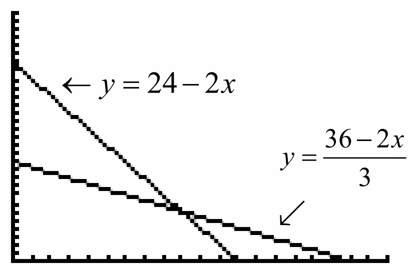
LCM: 2

$$2\left(1x + \frac{1}{2}y\right) \le 2(12)$$
$$2x + y \le 24$$
$$y \le 24 - 2x$$

The new system is

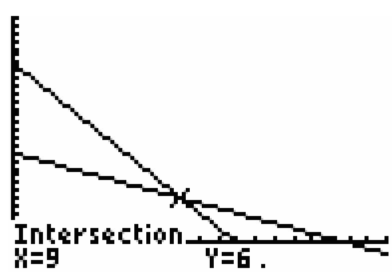
$$\begin{cases} y \le \frac{36 - 2x}{3} \\ y \le 24 - 2x \\ x \ge 0, y \ge 0 \end{cases}$$

b. To solve the system, first consider the graph of the system



[0, 20] by [0, 30]

The intersection point between the two lines is $(9, 6)$.



Note that the lines are solid because there is an "equal to" as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(5,5)$.

$$y \le \frac{36-2x}{3}$$
$$5 \le \frac{36-2(5)}{3}$$
$$5 \le \frac{26}{3}$$
$$5 \le 8.\overline{6}$$

True statement

$$y \le 24-2x$$
$$5 \le 24-2(5)$$
$$5 \le 14$$

True statement

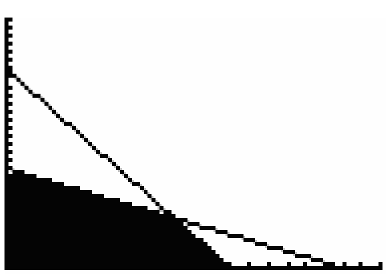
$$x \ge 0$$
$$5 \ge 0$$

True statement

$$y \ge 0$$
$$5 \ge 0$$

True statement

Since all the inequalities that form the system are true at the point $(5,5)$, the region that contains $(5,5)$ is the solution region. The graph of the solution is



[0, 20] by [0, 30]

Recall that all lines are solid.

One of the corner points is the intersection point between the two lines, $(9,6)$. A second corner point occurs where $y = 24-2x$ crosses the x-axis. A third corner point occurs where $y = \frac{36-2x}{3}$ crosses the y-axis.

To find the x-intercept, let $y = 0$.

$$0 = 24-2x$$
$$2x = 24$$
$$x = 12$$
$$(12,0)$$

To find the y-intercept, let $x = 0$.

$$y = \frac{36-2(0)}{3}$$
$$y = \frac{36}{3}$$
$$y = 12$$
$$(0,12)$$

A fourth corner point occurs at the origin, $(0,0)$. Therefore, the corner points of the solution region are $(0,0),(9,6),(12,0)$ and $(0,12)$.

37. a. Let x = number of commercial heating systems, and y = number of domestic heating systems.

Then, the system of inequalities is

$$\begin{cases} x+y \le 1400 \\ x \ge 500 \\ y \ge 750 \end{cases}$$

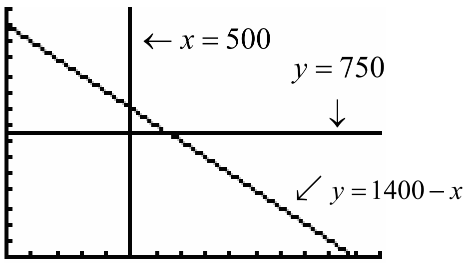
Rewriting the system:

$$x+y \le 1400$$
$$y \le 1400-x$$

The new system is

$$\begin{cases} y \le 1400 - x \\ x \ge 500 \\ y \ge 750 \end{cases}$$

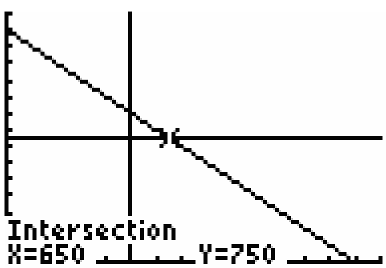
b. To solve the system, first consider the graph of the system



[0, 1500] by [0, 1500]

The intersection point between the $x = 500$ and $y = 1400 - x$ is $(500, 900)$. Note that solving by the substitution method yields $y = 1400 - 500 = 900$.

The intersection point between $y = 750$ and $y = 1400 - x$ is $(650, 750)$.



The intersection point between $x = 500$ and $y = 750$ is $(500, 750)$.

Note that the lines are solid because there is an "equal to" as part of all the inequalities.

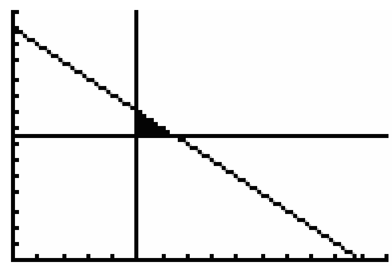
To determine the solution region, pick a point to test. Pick $(600, 775)$.

$y \le 1400 - x$
$775 \le 1400 - 600$
$775 \le 800$
True statement

$x \ge 500$
$600 \ge 500$
True statement

$y \ge 750$
$775 \ge 750$
True statement

Since all the inequalities that form the system are true at the point $(600, 775)$, the region that contains $(600, 775)$ is the solution region. The graph of the solution is



[0, 1500] by [0, 1500]

Recall that all lines are solid.

One of the corner points is the intersection point between $x = 500$ and $y = 1400 - x$, which is $(500, 900)$.

Another corner point is the intersection point between $y = 750$ and $y = 1400 - x$, which is $(650, 750)$.

A third corner point occurs at the intersection between $x = 500$ and $y = 750$, which is $(500, 750)$.

Therefore, the corner points of the solution region are $(500, 750), (500, 900),$ and $(650, 750)$.

38. Let x = length and width of the box, and let y = height of the box.

The system of inequalities is

$$\begin{cases} x^2 y \geq 500 \\ 4xy + x^2 \leq 500 \\ x \geq 0, y \geq 0 \end{cases}$$

Rewriting the system yields

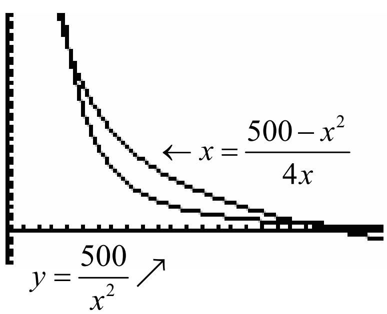
$$y \geq \frac{500}{x^2}$$

$$y \leq \frac{500 - x^2}{4x}$$

The new system is

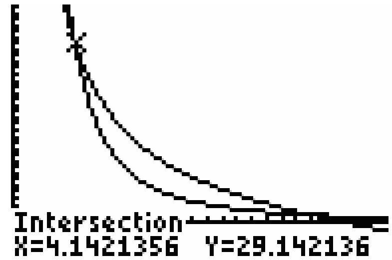
$$\begin{cases} y \geq \dfrac{500}{x^2} \\ y \leq \dfrac{500 - x^2}{4x} \\ x \geq 0, y \geq 0 \end{cases}$$

To solve the system, first consider the graph of the system



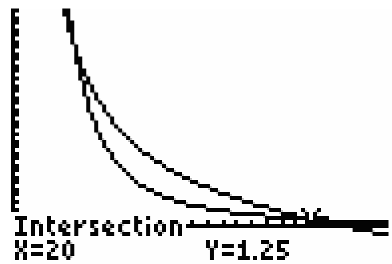
[0, 25] by [–5, 35]

Solving for the intersection points of the two functions graphically,



[0, 25] by [–5, 35]

and



[0, 25] by [–5, 35]

The intersection points are approximately $(4.14, 29)$ and $(20, 1.25)$.

Note that the lines are solid because there is an "equal to" as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(10, 8)$.

$$y \geq \frac{500}{x^2}$$

$$8 \geq \frac{500}{(10)^2}$$

$$8 \geq 5$$

True statement

$$y \leq \frac{500 - x^2}{4x}$$

$$8 \leq \frac{500 - (10)^2}{4(10)}$$

$$8 \leq \frac{400}{40}$$

$$8 \leq 10$$

True statement

$x \geq 0$

$10 \geq 0$

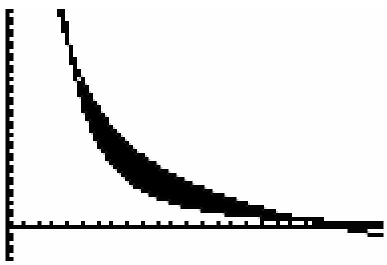
True statement

$y \geq 0$

$8 \geq 0$

True statement

Since all the inequalities that form the system are true at the point $(10,8)$, the region that contains $(10,8)$ is the solution region. The graph of the solution is



[0, 25] by [–5, 35]

Recall that all lines are solid.

If $4.14 \leq x \leq 20$, the box will have a volume of at least 500 cubic centimeters.

Section 6.2 Skills Check

1. Test the corner points of the feasible region in the objective function.

At $(0,0)$:	$f=4(0)+9(0)=0$
At $(0,40)$:	$f=4(0)+9(40)=360$
At $(67,0)$:	$f=4(67)+9(0)=268$
At $(10,38)$:	$f=4(10)+9(38)=382$

The maximum value is 382 occurring at $(10,38)$, and the minimum value is 0 occurring at $(0,0)$.

2. Test the corner points of the feasible region in the objective function.

At $(0,0)$:	$f=2(0)+3(0)=0$
At $(0,20)$:	$f=2(0)+3(20)=60$
At $\left(28\frac{1}{3},0\right)$:	$f=2\left(28\frac{1}{3}\right)+3(0)=56.\overline{6}$
At $(14,17.2)$:	$f=2(14)+3(17.2)=79.6$

The maximum value is 79.6 occurring at $(14,17.2)$, and the minimum value is 0 occurring at $(0,0)$.

3. Test the corner points of the feasible region in the objective function.

At $(0,0)$:	$f=4(0)+2(0)=0$
At $(0,2)$:	$f=4(0)+2(2)=4$
At $(2,4)$:	$f=4(2)+2(4)=16$
At $(4,3)$:	$f=4(4)+2(3)=22$
At $(5,0)$:	$f=4(5)+2(0)=20$

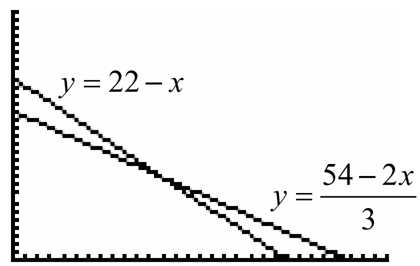
The maximum value is 22 occurring at $(4,3)$, and the minimum value is 0 occurring at $(0,0)$.

4. Test the corner points of the feasible region in the objective function.

At $(0,0)$:	$f=3(0)+9(0)=0$
At $(0,5)$:	$f=3(0)+9(5)=45$
At $(3,4)$:	$f=3(3)+9(4)=45$
At $(5,2)$:	$f=3(5)+9(2)=33$
At $(6,0)$:	$f=3(6)+9(0)=18$

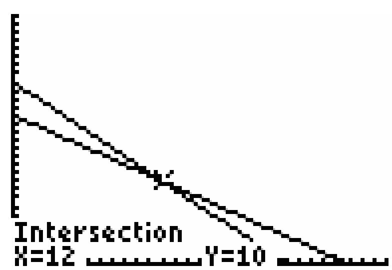
The maximum value is 45 occurring at $(0,5)$ and $(3,4)$. Therefore, the maximum value occurs at any point along the boundary line connecting $(0,5)$ and $(3,4)$. The minimum value is 0 occurring at $(0,0)$.

5. **a.** The graph of the system is



[0, 30] by [0, 30]

The intersection point between the two lines is $(12,10)$.



To determine the solution region, pick a point to test. Pick $(1,1)$.

$y \le \frac{54-2x}{3}$

$1 \le \frac{54-2(1)}{3}$

$1 \le \frac{52}{3}$

$1 \le 17.\overline{3}$

True statement

$y \le 22 - x$

$1 \le 22 - 1$

$1 \le 21$

True statement

$x \ge 0$

$1 \ge 0$

True statement

$y \ge 0$

$1 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The graph of the solution is

[0, 30] by [0, 30]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The graph above represents the feasible region. The corners of the region are $(0,0),(12,10),(22,0)$, and $(0,18)$.

b. Testing the corner points of the feasible region in the objective function yields

At $(0,0)$: $f = 3(0)+5(0)=0$

At $(0,18)$: $f = 3(0)+5(18)=90$

At $(22,0)$: $f = 3(22)+5(0)=66$

At $(12,10)$: $f = 3(12)+5(10)=86$

The maximum value is 90 occurring at $(0,18)$.

6. a. To solve the system and determine the feasible region, first graph the system. To graph the system, rewrite the system by solving for y in the inequalities.

$x + 2y \le 16$

$2y \le 16 - x$

$y \le \frac{16-x}{2}$

and

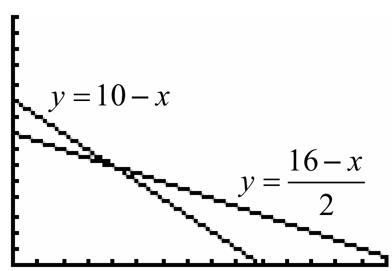
$x + y \le 10$

$y \le 10 - x$

The new system is

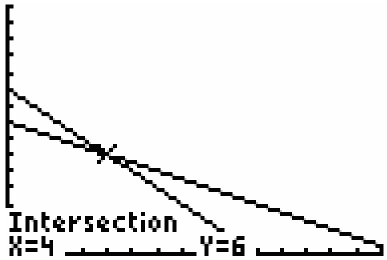
$$\begin{cases} y \le \frac{16-x}{2} \\ y \le 10 - x \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is



[0, 15] by [0, 15]

The intersection point between the two lines is $(4,6)$.



To determine the solution region, pick a point to test. Pick $(1,1)$.

$$y \le \frac{16-x}{2}$$
$$1 \le \frac{16-1}{2}$$
$$1 \le \frac{15}{2}$$
$$1 \le 7.5$$
True statement
$$y \le 10-x$$
$$1 \le 10-1$$
$$1 \le 9$$
True statement
$$x \ge 0$$
$$1 \ge 0$$
True statement
$$y \ge 0$$
$$1 \ge 0$$
True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The graph of the solution is

[0, 15] by [0, 15]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The graph above represents the feasible region. The corners of the region are $(0,0),(4,6),(10,0)$, and $(0,8)$.

b. Testing the corner points of the feasible region in the objective function yields

At $(0,0)$:	$f=3(0)+4(0)=0$
At $(0,8)$:	$f=3(0)+4(8)=32$
At $(10,0)$:	$f=3(10)+4(0)=30$
At $(4,6)$:	$f=3(4)+4(6)=36$

The maximum value is 36 occurring at $(4,6)$.

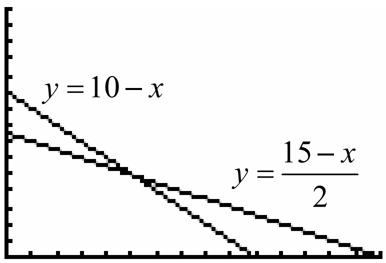
7. a. To solve the system and determine the feasible region, first graph the system. To graph the system, rewrite the system by solving for y in the inequalities.

$$x+2y \ge 15$$
$$2y \ge 15-x$$
$$y \ge \frac{15-x}{2}$$
and
$$x+y \ge 10$$
$$y \ge 10-x$$

The new system is

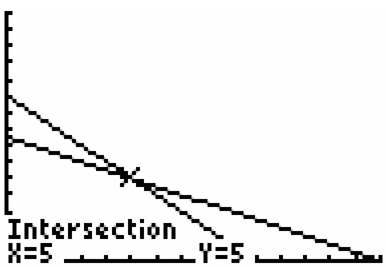
$$\begin{cases} y \geq \dfrac{15-x}{2} \\ y \geq 10-x \\ x \geq 0, y \geq 0 \end{cases}$$

The graph of the system is



[0, 15] by [0, 15]

The intersection point between the two lines is $(5,5)$.



To determine the solution region, pick a point to test. Pick $(5,6)$.

$y \geq \dfrac{15-x}{2}$

$6 \geq \dfrac{15-5}{2}$

$6 \geq \dfrac{10}{2}$

$6 \geq 5$

True statement

$y \geq 10-x$

$6 \geq 10-5$

$6 \geq 5$

True statement

$x \geq 0$

$5 \geq 0$

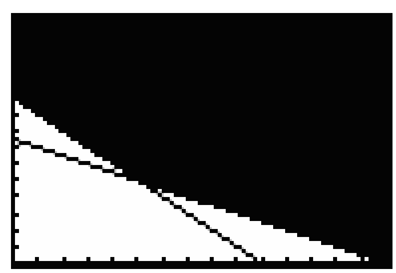
True statement

$y \geq 0$

$6 \geq 0$

True statement

Since all the inequalities that form the system are true at the point $(5,6)$, the region that contains $(5,6)$ is the solution region. The graph of the solution is



[0, 15] by [0, 15]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The graph above represents the feasible region. The corners of the region are $(5,5), (0,10)$, and $(15,0)$.

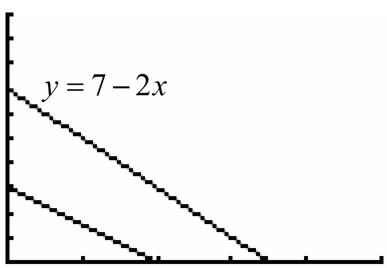
b. Testing the corner points of the feasible region in the objective function yields

At $(5,5)$: $g = 4(5) + 2(5) = 30$
At $(0,10)$: $g = 4(0) + 2(10) = 20$
At $(15,0)$: $g = 4(15) + 2(0) = 60$

The minimum value is 20 occurring at $(0,10)$.

8. Rewriting the system of constraints and graphing the system yields

$$\begin{cases} y \geq \dfrac{6-3x}{2} \\ y \leq 7 - 2x \\ x \geq 0, y \geq 0 \end{cases}$$



[0, 5] by [0, 10]

To determine the solution region, pick a point to test. Pick $(1,4)$.

$y \geq \dfrac{6-3x}{2}$

$4 \geq \dfrac{6-3(1)}{2}$

$4 \geq 1.5$

True statement

$y \leq 7 - 2x$

$4 \leq 7 - 2(1)$

$4 \leq 5$

True statement

$x \geq 0$

$1 \geq 0$

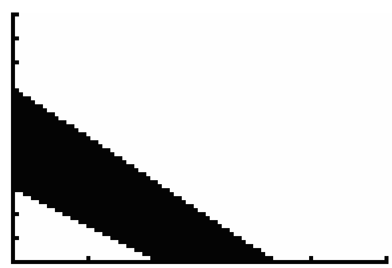
True statement

$y \geq 0$

$4 \geq 0$

True statement

Since all the inequalities that form the system are true at the point $(1,4)$, the region that contains $(1,4)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 5] by [0, 10]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,3),(0,7),(2,0),$ and $(3.5,0)$.

Testing the corner points of the feasible region to maximize the objective function, $f = 400x + 300y$, yields

At $(0,3)$: $f = 400(0) + 300(3) = 900$

At $(0,7)$: $f = 400(0) + 300(7) = 2100$

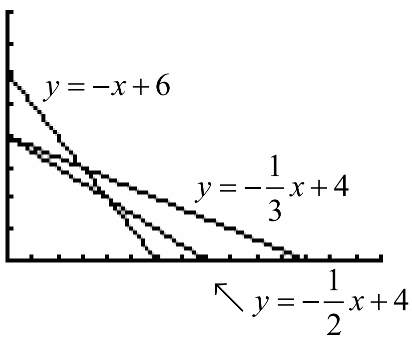
At $(2,0)$: $f = 400(2) + 300(0) = 800$

At $(3.5,0)$: $f = 400(3.5) + 300(0) = 1400$

The maximum value is 2100 occurring at $(0,7)$.

9. Graphing the system yields

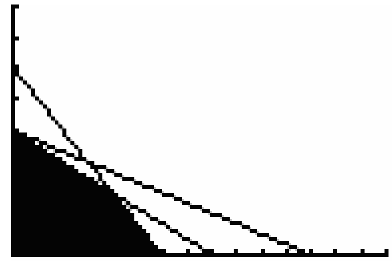
$$\begin{cases} y \le -\frac{1}{2}x + 4 \\ y \le -x + 6 \\ y \le -\frac{1}{3}x + 4 \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 15] by [0, 8]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

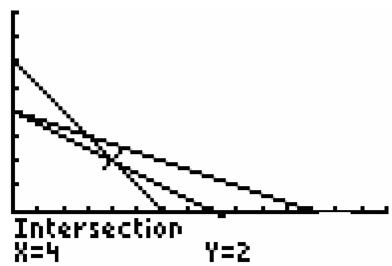
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 15] by [0, 8]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(0,4),(6,0),$ and $(4,2)$.



Testing the corner points of the feasible region to maximize the objective function, $f = 20x + 30y$, yields

At $(0,0)$: $f = 20(0) + 30(0) = 0$

At $(0,4)$: $f = 20(0) + 30(4) = 120$

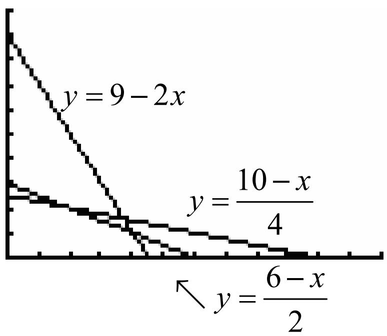
At $(6,0)$: $f = 20(6) + 30(0) = 120$

At $(4,2)$: $f = 20(4) + 30(2) = 140$

The maximum value is 140 occurring at $(4,2)$.

10. Rewriting the system of constraints and graphing the system yields

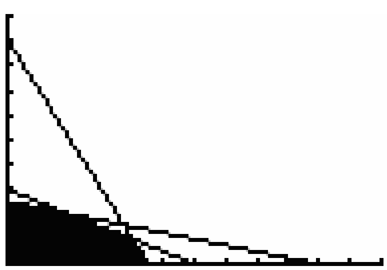
$$\begin{cases} y \le \dfrac{6-x}{2} \\ y \le \dfrac{10-x}{4} \\ y \le 9-2x \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 12] by [0, 10]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

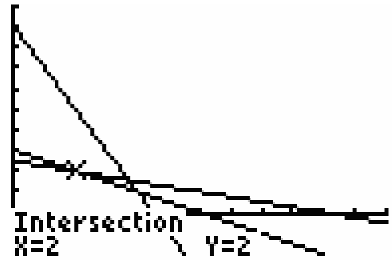
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

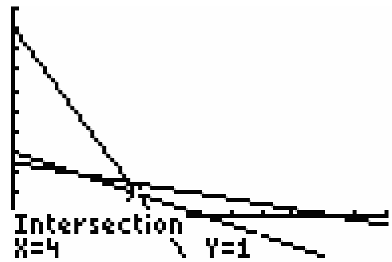


[0, 12] by [0, 10]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(0,2.5),(4.5,0),(2,2)$,and $(4,1)$.





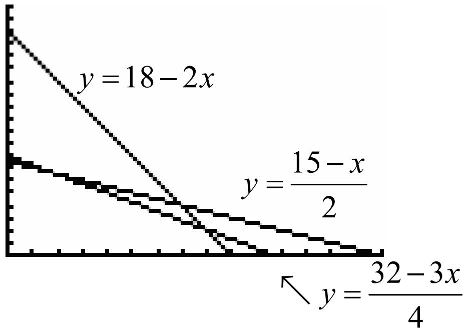
Testing the corner points of the feasible region to maximize the objective function, $f = 100x + 100y$, yields

At $(0,0)$:	$f = 100(0) + 100(0) = 0$
At $(0,2.5)$:	$f = 100(0) + 100(2.5) = 250$
At $(4.5,0)$:	$f = 100(4.5) + 100(0) = 450$
At $(2,2)$:	$f = 100(2) + 100(2) = 400$
At $(4,1)$:	$f = 100(4) + 100(1) = 500$

The maximum value is 500 occurring at $(4,1)$.

11. Rewriting the system of constraints and graphing the system yields

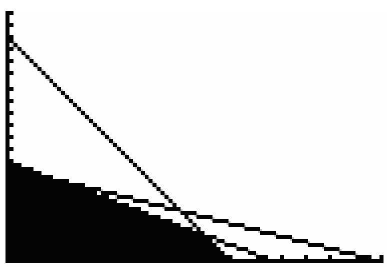
$$\begin{cases} y \le \dfrac{32-3x}{4} \\ y \le \dfrac{15-x}{2} \\ y \le 18-2x \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 15] by [0, 20]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

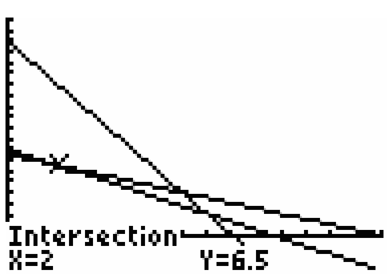
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

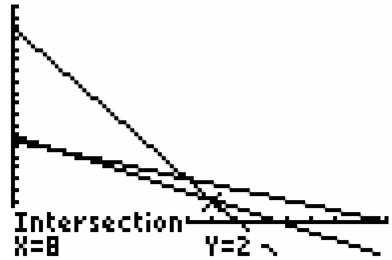


[0, 15] by [0, 20]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(0,7.5),(9,0),(2,6.5)$,and $(8,2)$.





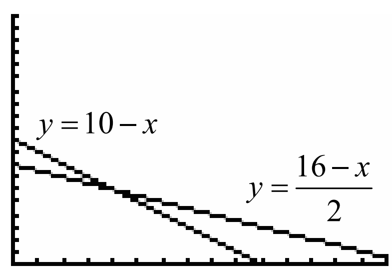
Testing the corner points of the feasible region to maximize the objective function, $f = 80x + 160y$, yields

At $(0,0)$: $f = 80(0) + 160(0) = 0$
At $(0,7.5)$: $f = 80(0) + 160(7.5) = 1200$
At $(9,0)$: $f = 80(9) + 160(0) = 720$
At $(2,6.5)$: $f = 80(2) + 160(6.5) = 1200$
At $(8,2)$: $f = 80(8) + 160(2) = 960$

The maximum value is 1200 occurring at $(0,7.5)$ and $(2,6.5)$. Since the maximum occurs at two corner points, the maximum also occurs at all points along the line segment connecting $(0,7.5)$ and $(2,6.5)$.

12. Rewriting the system of constraints and graphing the system yields

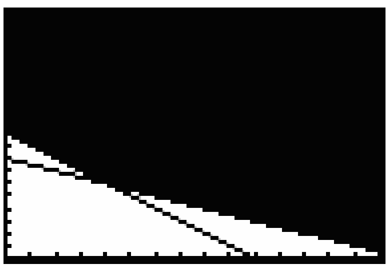
$$\begin{cases} y \ge \dfrac{16 - x}{2} \\ y \ge 10 - x \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 15] by [0, 20]

To determine the solution region, pick a point to test. Pick $(5,10)$. When substituted into the inequalities that form the system, the point $(5,10)$ creates true statements in all cases.

Since all the inequalities that form the system are true at the point $(5,10)$, the region that contains $(5,10)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 15] by [0, 20]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,10), (16,0)$, and $(4,6)$.

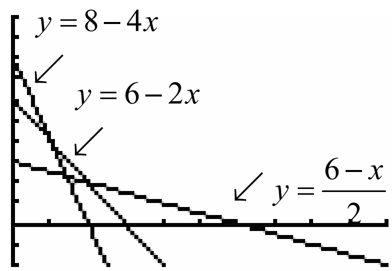
Testing the corner points of the feasible region to minimize the objective function, $g = 30x + 40y$, yields

At $(0,10)$:	$g = 30(0) + 40(10) = 400$
At $(16,0)$:	$g = 30(16) + 40(0) = 480$
At $(4,6)$:	$g = 30(4) + 40(6) = 360$

The minimum value is 360 occurring at $(4,6)$.

13. Rewriting the system of constraints and graphing the system yields

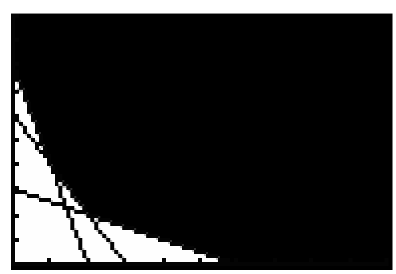
$$\begin{cases} y \ge 6 - 2x \\ y \ge 8 - 4x \\ y \ge \dfrac{6-x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 10] by [–2, 10]

To determine the solution region, pick a point to test. Pick $(2,4)$. When substituted into the inequalities that form the system, the point $(2,4)$ creates true statements in all cases.

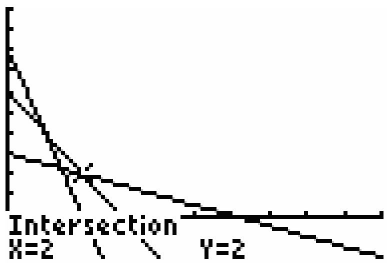
Since all the inequalities that form the system are true at the point $(2,4)$, the region that contains $(2,4)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

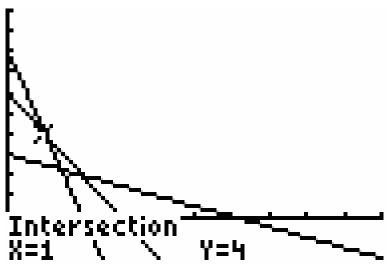


[0, 10] by [0, 10]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,8),(6,0),(1,4)$ and $(2,2)$.





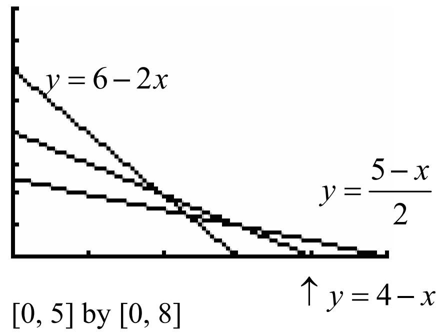
Testing the corner points of the feasible region to minimize the objective function, $g = 40x + 30y$, yields

At $(0,8)$: $g = 40(0) + 30(8) = 240$
At $(6,0)$: $g = 40(6) + 30(0) = 240$
At $(1,4)$: $g = 40(1) + 30(4) = 160$
At $(2,2)$: $g = 40(2) + 30(2) = 140$

The minimum value is 140 occurring at $(2,2)$.

14. Rewriting the system of constraints and graphing the system yields

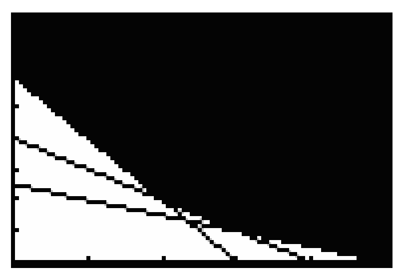
$$\begin{cases} y \geq \dfrac{5-x}{2} \\ y \geq 4 - x \\ y \geq 6 - 2x \\ x \geq 0, y \geq 0 \end{cases}$$



[0, 5] by [0, 8]

To determine the solution region, pick a point to test. Pick $(2,4)$. When substituted into the inequalities that form the system, the point $(2,4)$ creates true statements in all cases.

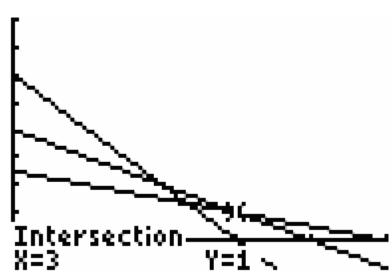
Since all the inequalities that form the system are true at the point $(2,4)$, the region that contains $(2,4)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

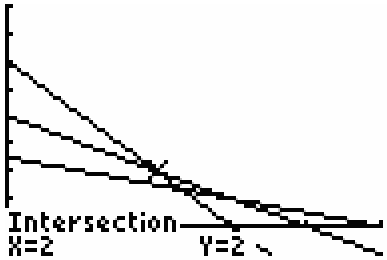


[0, 5] by [0, 8]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,6),(5,0),(3,1)$ and $(2,2)$.





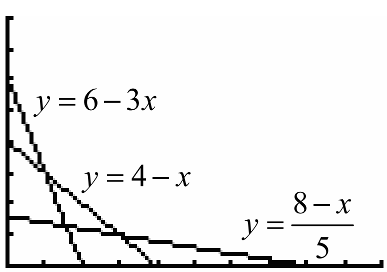
Testing the corner points of the feasible region to minimize the objective function, $g = 30x + 40y$, yields

At $(0,6)$: $g = 30(0) + 40(6) = 240$
At $(5,0)$: $g = 30(5) + 40(0) = 150$
At $(3,1)$: $g = 30(3) + 40(1) = 130$
At $(2,2)$: $g = 30(2) + 40(2) = 140$

The minimum value is 130 occurring at $(3,1)$.

15. Rewriting the system of constraints and graphing the system yields

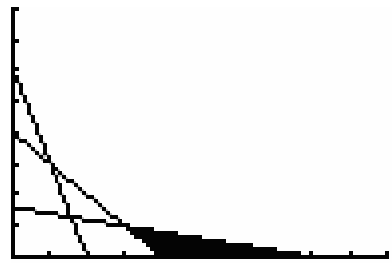
$$\begin{cases} y \geq 6 - 3x \\ y \geq 4 - x \\ y \leq \dfrac{8-x}{5} \\ x \geq 0, y \geq 0 \end{cases}$$



[0, 5] by [0, 8]

To determine the solution region, pick a point to test. Pick $\left(4, \frac{1}{2}\right)$. When substituted into the inequalities that form the system, the point $\left(4, \frac{1}{2}\right)$ creates true statements in all cases.

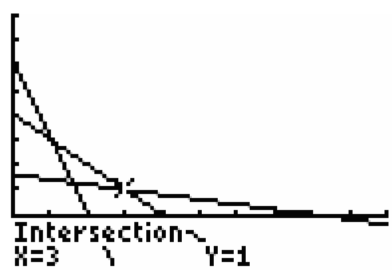
Since all the inequalities that form the system are true at the point $\left(4, \frac{1}{2}\right)$, the region that contains $\left(4, \frac{1}{2}\right)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 5] by [0, 8]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(4,0), (8,0)$, and $(3,1)$.



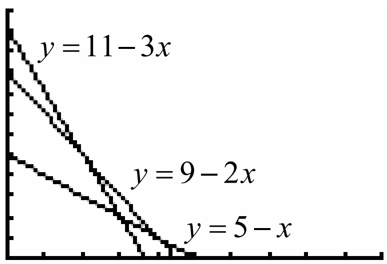
Testing the corner points of the feasible region to minimize the objective function, $g = 46x + 23y$, yields

At $(4,0)$: $g = 46(4) + 23(0) = 184$
At $(8,0)$: $g = 46(8) + 23(0) = 368$
At $(3,1)$: $g = 46(3) + 23(1) = 161$

The minimum value is 161 occurring at $(3,1)$.

16. Rewriting the system of constraints and graphing the system yields

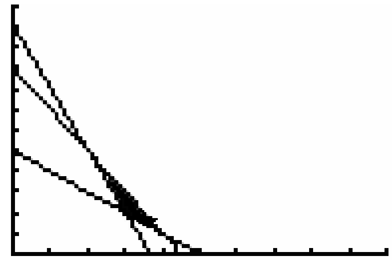
$$\begin{cases} y \le 9-2x \\ y \ge 11-3x \\ y \ge 5-x \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 10] by [0, 12]

To determine the solution region, pick a point to test. Pick $\left(3, 2\frac{1}{2}\right)$. When substituted into the inequalities that form the system, the point $\left(3, 2\frac{1}{2}\right)$ creates true statements in all cases.

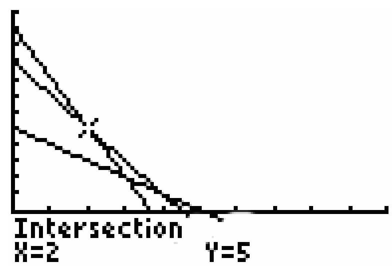
Since all the inequalities that form the system are true at the point $\left(3, 2\frac{1}{2}\right)$, the region that contains $\left(3, 2\frac{1}{2}\right)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

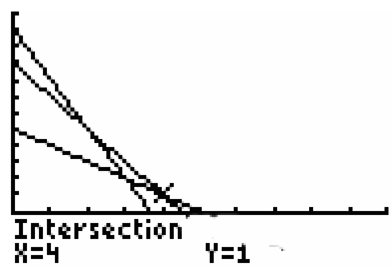


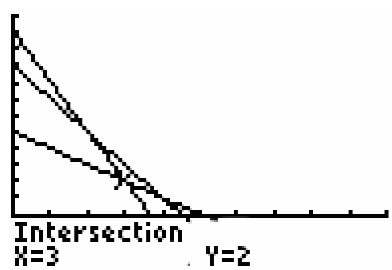
[0, 10] by [0, 12]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(2,5),(4,1),$ and $(3,2)$.







a. Testing the corner points of the feasible region to maximize the objective function, $f = 4x + 5y$, yields

At $(2,5)$: $f = 4(2) + 5(5) = 33$
At $(4,1)$: $f = 4(4) + 5(1) = 21$
At $(3,2)$: $f = 4(3) + 5(2) = 22$

The maximum value is 33 occurring at $(2,5)$.

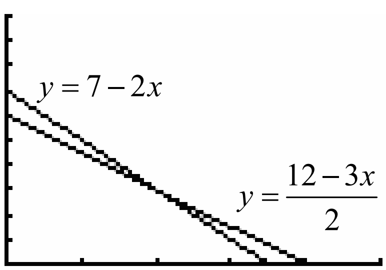
b. Testing the corner points of the feasible region to minimize the objective function, $g = 3x + 2y$, yields

At $(2,5)$: $g = 3(2) + 2(5) = 16$
At $(4,1)$: $g = 3(4) + 2(1) = 14$
At $(3,2)$: $g = 3(3) + 2(2) = 13$

The minimum value is 13 occurring at $(3,2)$.

17. Rewriting the system of constraints and graphing the system yields

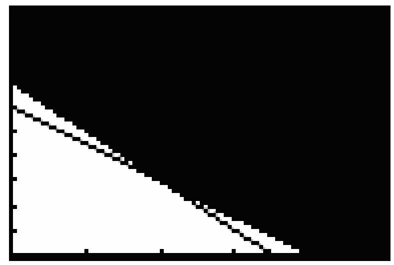
$$\begin{cases} y \geq \dfrac{12-3x}{2} \\ y \geq 7-2x \\ x \geq 0, y \geq 0 \end{cases}$$



[0, 5] by [0, 10]

To determine the solution region, pick a point to test. Pick $(1,10)$. When substituted into the inequalities that form the system, the point $(1,10)$ creates true statements in all cases.

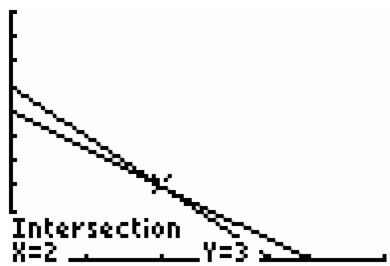
Since all the inequalities that form the system are true at the point $(1,10)$, the region that contains $(1,10)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 5] by [0, 10]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,7), (4,0)$, and $(2,3)$.



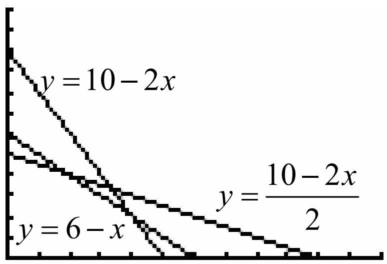
Testing the corner points of the feasible region to minimize the objective function, $g = 60x + 10y$, yields

At $(0,7)$: $g = 60(0) + 10(7) = 70$
At $(4,0)$: $g = 60(4) + 10(0) = 240$
At $(2,3)$: $g = 60(2) + 10(3) = 150$

The minimum value is 70 occurring at $(0,7)$.

18. Rewriting the system of constraints and graphing the system yields

$$\begin{cases} y \le \dfrac{10-x}{2} \\ y \le 6-x \\ y \le 10-2x \\ x \ge 0, y \ge 0 \end{cases}$$



[0, 12] by [0, 12]

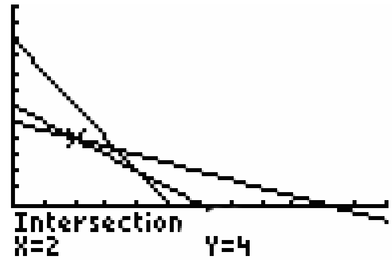
To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

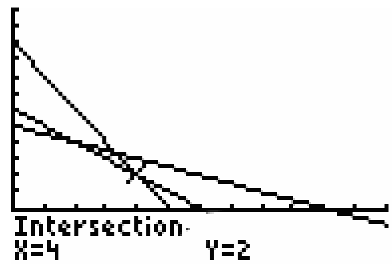
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

[0, 12] by [0, 12]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,5),(5,0),(4,2),$and $(2,4)$.





Testing the corner points of the feasible region to maximize the objective function, $g = 10x + 10y$, yields

At $(0,5)$:	$g = 10(0) + 10(5) = 50$
At $(5,0)$:	$g = 10(5) + 10(0) = 50$
At $(4,2)$:	$g = 10(4) + 10(2) = 60$
At $(2,4)$:	$g = 10(2) + 10(4) = 60$

The maximum value is 60 occurring at $(4,2)$ and $(2,4)$. Since there are two corner points that maximize the objective function, any point along the line segment connecting $(4,2)$ and $(2,4)$ also maximizes the objective function. There infinitely many solutions to the question all lying along the line segment connecting $(4,2)$ and $(2,4)$.

Section 6.2 Exercises

19. See the solution to Section 6.1, question 29. The corner points of the feasible region are $(390,390),(780,0)$ and $(975,0)$.

Testing the corner points of the feasible region to maximize the objective function, $f=32x+45y$, yields

At $(390,390)$: $f=32(390)+45(390)$
$=\$30,030$

At $(780,0)$: $f=32(780)+45(0)$
$=\$24,960$

At $(975,0)$: $f=32(975)+45(0)$
$=\$31,200$

The maximum is \$31,200 occurring at $(975,0)$. To maximize profit, the company needs to manufacture 975 Turbo models and 0 Tornado models.

20. See the solution to Section 6.1, question 34. The corner points of the feasible region are $(0,0),(80,40),(120,0)$ and $\left(0,66.\overline{6}\right)$.

Testing the corner points of the feasible region to maximize the objective function, $f=80x+120y$, yields

At $(0,0)$: $f=80(0)+120(0)$
$=\$0$

At $(80,40)$: $f=80(80)+120(40)$
$=\$11,200$

At $(120,0)$: $f=80(120)+120(0)$
$=\$9600$

At $\left(0,66\frac{2}{3}\right)$: $f=80(0)+120\left(66\frac{2}{3}\right)$
$=\$8000$

The maximum is \$11,200 occurring at $(80,40)$. To maximize profit, the company needs to manufacture 80 Inkjet printers and 40 Laserjet printers.

21. See the solution to Section 6.1, question 36. The corner points of the feasible region are $(0,0),(9,6),(12,0)$ and $(0,12)$.

Testing the corner points of the feasible region to maximize the objective function, $f=24x+30y$, yields

At $(0,0)$: $f=24(0)+30(0)=\$0$

At $(9,6)$: $f=24(9)+30(6)=\$396$

At $(12,0)$: $f=24(12)+30(0)=\$288$

At $(0,12)$: $f=24(0)+30(12)=\$360$

The maximum is \$396 occurring at $(9,6)$. To maximize profit, the company needs to sell 9 Safecut chainsaws and 6 Deluxe chainsaws.

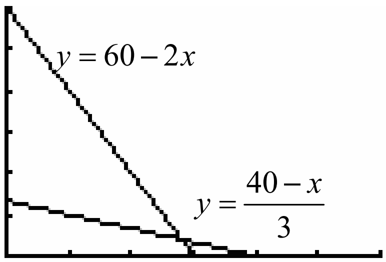
22. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of Lawn King mowers, and let y represent the number of Lawn Master mowers.

$$\begin{cases} 2x+1y\le 60 \\ 1x+3y\le 40 \\ x\ge 0, y\ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y\le 60-2x \\ y\le \dfrac{40-x}{3} \\ x\ge 0, y\ge 0 \end{cases}$$

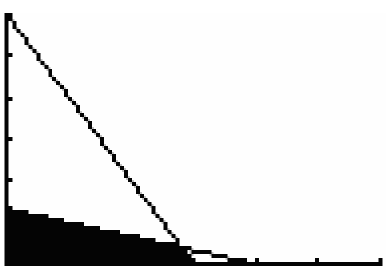
Graphing the system yields



[0, 60] by [0, 60]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

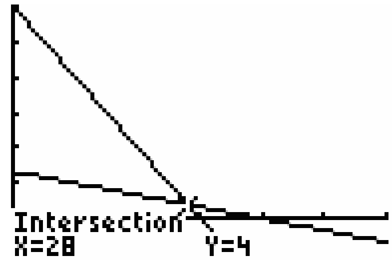
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 60] by [0, 60]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(30,0),\left(0,13\frac{1}{3}\right)$ and $(28,4)$.



Testing the corner points of the feasible region to maximize the objective function, $f = 150x + 200y$, yields

At $(0,0)$: $f = 150(0) + 200(0) = \$0$

At $(30,0)$: $f = 150(30) + 200(0) = \$4500$

At $\left(0,13\frac{1}{3}\right)$: $f = 150(0) + 200\left(13\frac{1}{3}\right) = \$2666.67$

At $(28,4)$: $f = 150(28) + 200(4) = \$5000$

The maximum value is \$5000 occurring at $(28,4)$. To produce a maximum profit the company needs to produce and sell 28 Lawn King mowers and 4 Lawn Master mowers.

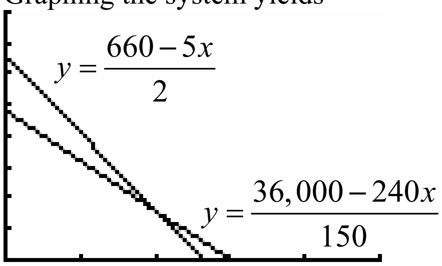
23. a. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of cable television commercials, and let y represent the number of radio commercials.

$$\begin{cases} 240x + 150y \le 36,000 \\ \frac{1}{4}x + \frac{1}{10}y \ge 33 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le \dfrac{36,000-240x}{150} \\ y \ge \dfrac{660-5x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

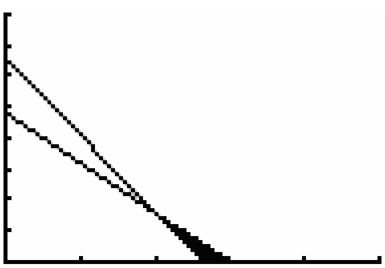
Graphing the system yields



[0, 250] by [0, 400]

To determine the solution region, pick a point to test. Pick $(125,30)$. When substituted into the inequalities that form the system, the point $(125,30)$ creates true statements in all cases.

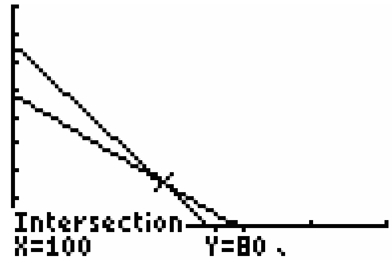
Since all the inequalities that form the system are true at the point $(125,30)$, the region that contains $(125,30)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 250] by [0, 400]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(150,0),(132,0),$ and $(100,80)$.



Testing the corner points of the feasible region to maximize the objective function, $f=500x+550y$, yields

$$\begin{aligned} \text{At } (132,0):\quad f &= 500(132)+550(0) \\ &= \$66,000 \\ \text{At } (150,0):\quad f &= 500(150)+550(0) \\ &= \$75,000 \\ \text{At } (100,80):\quad f &= 500(100)+550(80) \\ &= \$94,000 \end{aligned}$$

The maximum value is \$94,000 occurring at $(100,80)$. To produce a maximum profit the company needs to buy 100 minutes on cable television and 80 minutes on radio.

b. See part a) above. The maximum value is \$94,000.

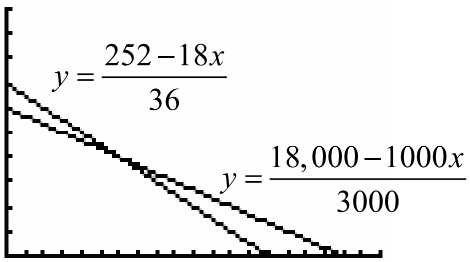
24. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of newspaper ad packages, and let y represent the number of radio ad packages.

$$\begin{cases} 1000x+3000y \le 18,000 \\ 18x+36y \ge 252 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le \dfrac{18,000 - 1000x}{3000} \\ y \ge \dfrac{252 - 18x}{36} \\ x \ge 0, y \ge 0 \end{cases}$$

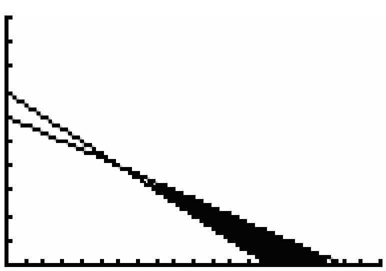
Graphing the system yields



[0, 20] by [0, 10]

To determine the solution region, pick a point to test. Pick $(12,1.5)$. When substituted into the inequalities that form the system, the point $(12,1.5)$ creates true statements in all cases.

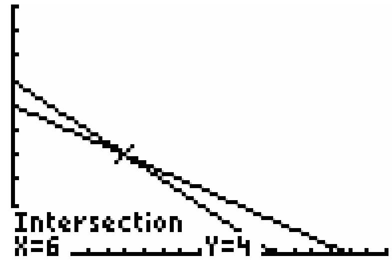
Since all the inequalities that form the system are true at the point $(12,1.5)$, the region that contains $(12,1.5)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 20] by [0, 10]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(14,0), (18,0)$, and $(6,4)$.



Testing the corner points of the feasible region to maximize the objective function, $f = 6000x + 8000y$, yields

At $(14,0)$: $f = 6000(14) + 8000(0)$
$= 84,000$
At $(18,0)$: $f = 6000(18) + 8000(0)$
$= 108,000$
At $(6,4)$: $f = 6000(6) + 8000(4)$
$= 68,000$

The maximum value is 108,000 occurring at $(18,0)$. To reach the maximum number of people, the company needs to buy 18 newspaper ad packages and 0 radio ad packages.

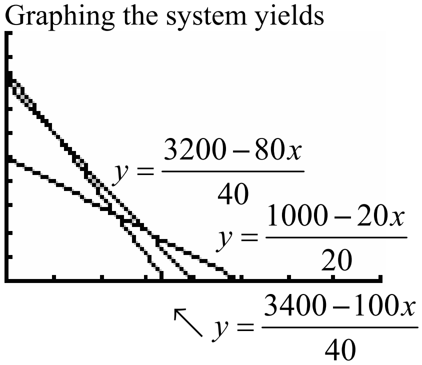
25. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let *x* represent the number of assembly line 1 days, and let *y* represent the number assembly line 2 days.

$$\begin{cases} 80x + 40y \ge 3200 \\ 20x + 20y \ge 1000 \\ 100x + 40y \ge 3400 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \ge \dfrac{3200-80x}{40} \\ y \ge \dfrac{1000-20x}{20} \\ y \ge \dfrac{3400-100x}{40} \\ x \ge 0, y \ge 0 \end{cases}$$

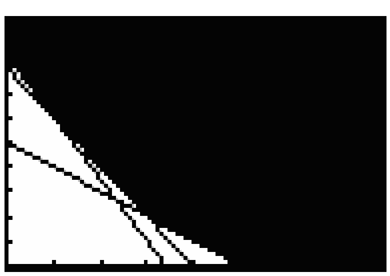
Graphing the system yields



[0, 80] by [0, 100]

To determine the solution region, pick a point to test. Pick $(25,40)$. When substituted into the inequalities that form the system, the point $(25,40)$ creates true statements in all cases.

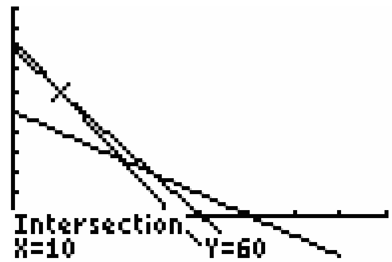
Since all the inequalities that form the system are true at the point $(25,40)$, the region that contains $(25,40)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

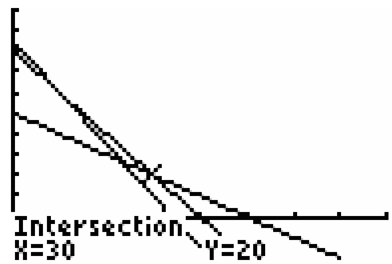


[0, 80] by [0, 100]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,85),(50,0),(30,20)$ and $(10,60)$.





Testing the corner points of the feasible region to minimize the objective function, $g = 20{,}000x + 40{,}000y$, yields

At $(0,85)$: $g = 20{,}000(0) + 40{,}000(85) = \$3{,}400{,}000$

At $(50,0)$: $g = 20{,}000(50) + 40{,}000(0) = \$1{,}000{,}000$

At $(10,60)$: $g = 20{,}000(10) + 40{,}000(60) = \$2{,}600{,}000$

At $(30,20)$: $g = 20{,}000(30) + 40{,}000(20) = \$1{,}400{,}000$

The minimum value is \$1,000,000 occurring at $(50,0)$. To minimize the cost, the company needs to manufacture the television sets on assembly line 1 for 50 days and assembly line 2 for zero days.

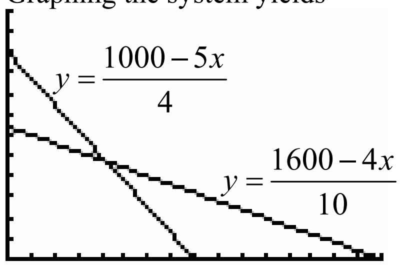
26. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of trout, and let y represent the number of bass.

$$\begin{cases} 4x + 10y \le 1600 \\ 5x + 4y \le 1000 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le \dfrac{1600 - 4x}{10} \\ y \le \dfrac{1000 - 5x}{4} \\ x \ge 0, y \ge 0 \end{cases}$$

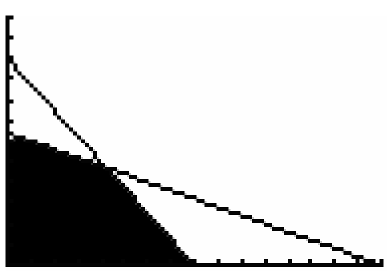
Graphing the system yields



[0, 400] by [0, 300]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

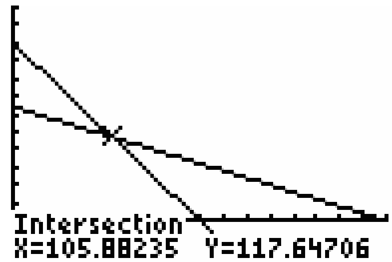
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 400] by [0, 300]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0), (200,0), (0,160)$, and approximately $(105.9, 117.6)$.



Testing the corner points of the feasible region to maximize the objective function, $f = x + y$, yields

At $(0,0)$: $f = 0 + 0 = 0$
At $(200,0)$: $f = 200 + 0 = 200$
At $(0,160)$: $f = 0 + 160 = 160$
At $(105.9, 117.6)$: $f = 105.9 + 117.6$
$= 223.5$

The maximum value is approximately 224 occurring at $(105.9, 117.6)$. The maximum number of fish that can be supported in the pond is 224, with 106 trout and 118 bass.

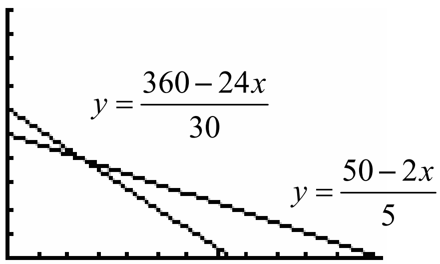
27. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of Van Buren models, and let y represent the number of Jefferson models.

$$\begin{cases} 200x + 500y \le 5000 \\ 240{,}000x + 300{,}000y \le 3{,}600{,}000 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le \dfrac{50-2x}{5} \\ y \le \dfrac{360-24x}{30} \\ x \ge 0, y \ge 0 \end{cases}$$

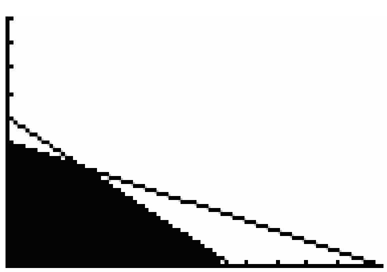
Graphing the system yields



[0, 25] by [0, 20]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

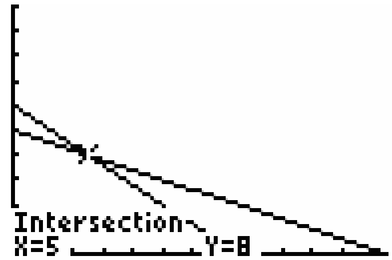
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 25] by [0, 20]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(15,0),(0,10),$ and $(5,8)$.



Testing the corner points of the feasible region to maximize the objective function, $f=60{,}000x+75{,}000y$, yields

At $(0,0)$: $f=60{,}000(0)+75{,}000(0)$
$=\$0$

At $(15,0)$: $f=60{,}000(15)+75{,}000(0)$
$=\$900{,}000$

At $(0,10)$: $f=60{,}000(0)+75{,}000(10)$
$=\$750{,}000$

At $(5,8)$: $f=60{,}000(5)+75{,}000(8)$
$=\$900{,}000$

The maximum profit is \$900,000 occurring at $(15,0)$ and $(5,8)$. Since there are two corner points that maximize the objective function, any point along the line segment connecting $(15,0)$ and $(5,8)$ also maximizes the objective function. Therefore, the contractor can build 15 Van Buren models and 0 Jefferson models, 5 Van Buren models and 8 Jefferson models, or any other whole number combination that lies along the line segment connecting $(15,0)$ and $(5,8)$. In particular the possible solutions are $(10,4),(15,0)$ and $(5,8)$.

28. a. See the solution to Section 6.1, question 35. The corner points of the feasible region are $(0,0),(90,20),(120,0)$ and $(0,50)$.

Testing the corner points of the feasible region to maximize the objective function, $f = 178x + 267y$, yields

At $(0,0)$: $f = 178(0) + 267(0) = \$0$

At $(90,20)$: $f = 178(90) + 267(20) = \$21,360$

At $(120,0)$: $f = 178(120) + 267(0) = \$21,360$

At $(0,50)$: $f = 178(0) + 267(50) = \$13,350$

The maximum is \$21,360 occurring at $(90,20)$ and $(120,0)$. Since two corner points maximize the objective function, any point along the line segment connecting the two points $(90,20)$ and $(120,0)$ also maximizes the objective function. Therefore manufacturing 90 Standard chairs and 20 Deluxe chairs or 120 Standard chairs and 0 Deluxe chairs or any other whole number pair along the line segment connecting $(90,20)$ and $(120,0)$ will yield a maximum revenue of \$21,360.

b. There are multiple answers all lying along the line segment connecting $(90,20)$ and $(120,0)$.

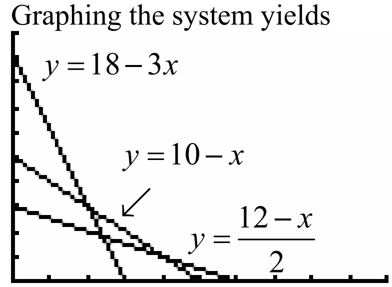
29. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of weeks operating facility 1, and let y represent the number of weeks operating facility 2.

$$\begin{cases} 400x + 400y \ge 4000 \\ 300x + 100y \ge 1800 \\ 200x + 400y \ge 2400 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \ge 10 - x \\ y \ge 18 - 3x \\ y \ge \dfrac{12 - x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

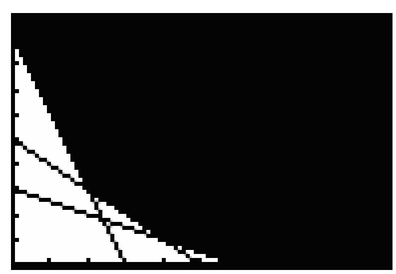
Graphing the system yields



[0, 20] by [0, 20]

To determine the solution region, pick a point to test. Pick $(10,5)$. When substituted into the inequalities that form the system, the point $(10,5)$ creates true statements in all cases.

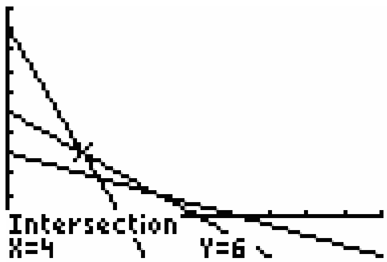
Since all the inequalities that form the system are true at the point $(10,5)$, the region that contains $(10,5)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

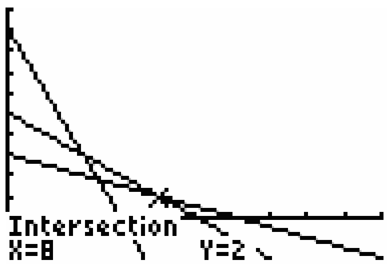


[0, 20] by [0, 20]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,18), (12,0), (4,6)$ and $(8,2)$.





Testing the corner points of the feasible region to minimize the objective function, $g = 15{,}000x + 20{,}000y$, yields

$$\begin{aligned}
\text{At } (0,18)&: g = 15{,}000(0) + 20{,}000(18) \\
&= \$360{,}000 \\
\text{At } (12,0)&: g = 15{,}000(12) + 20{,}000(0) \\
&= \$180{,}000 \\
\text{At } (4,6)&: g = 15{,}000(4) + 20{,}000(6) \\
&= \$180{,}000 \\
\text{At } (8,2)&: g = 15{,}000(8) + 20{,}000(2) \\
&= \$160{,}000
\end{aligned}$$

The minimum value is \$160,000 occurring at $(8,2)$. Operating facility 1 for 8 weeks and facility 2 for 2 weeks yields a minimum cost of \$160,000 for filling the orders.

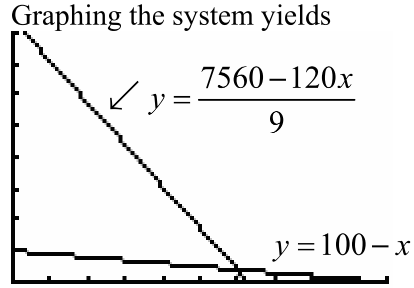
30. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of minutes of television ads, and let y represent the number of minutes of radio ads.

$$\begin{cases} 0.12x + 0.009y \geq 7.56 \\ x + y \geq 100 \\ x \geq 0, y \geq 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \geq \dfrac{7560 - 120x}{9} \\ y \geq 100 - x \\ x \geq 0, y \geq 0 \end{cases}$$

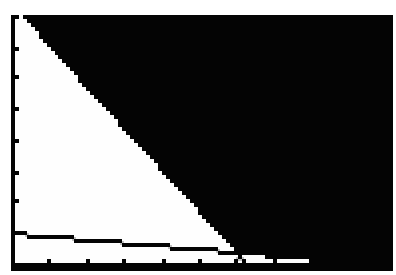
Graphing the system yields



[0, 100] by [0, 800]

To determine the solution region, pick a point to test. Pick $(50,400)$. When substituted into the inequalities that form the system, the point $(50,400)$ creates true statements in all cases.

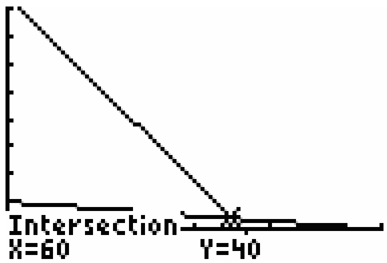
Since all the inequalities that form the system are true at the point $(50,400)$, the region that contains $(50,400)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 100] by [0, 800]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,840)$, $(100,0)$, and $(60,40)$.



Testing the corner points of the feasible region to minimize the objective function, $g = 1000x + 200y$, yields

At $(0,840)$: $g = 1000(0) + 200(840)$
$= \$168,000$
At $(100,0)$: $g = 1000(100) + 200(0)$
$= \$100,000$
At $(60,40)$: $g = 1000(60) + 200(40)$
$= \$68,000$

The minimum value is \$68,000 occurring at $(60,40)$. Buying 60 minutes of television time and 40 minutes of radio time minimizes the cost.

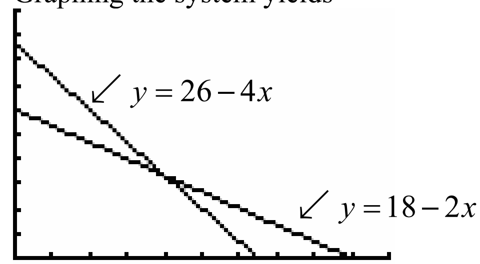
31. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let *x* represent the number of servings of Diet A, and let *y* represent the number of servings of Diet B.

$$\begin{cases} 2x + 1y \geq 18 \\ 4x + 1y \geq 26 \\ x \geq 0, y \geq 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \geq 18 - 2x \\ y \geq 26 - 4x \\ x \geq 0, y \geq 0 \end{cases}$$

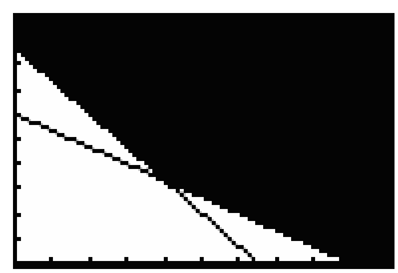
Graphing the system yields



[0, 10] by [0, 30]

To determine the solution region, pick a point to test. Pick $(5,10)$. When substituted into the inequalities that form the system, the point $(5,10)$ creates true statements in all cases.

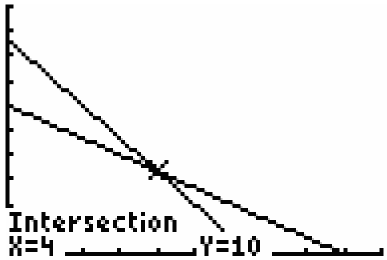
Since all the inequalities that form the system are true at the point $(5,10)$, the region that contains $(5,10)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 10] by [0, 30]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,26), (9,0),$ and $(4,10)$.



Testing the corner points of the feasible region to minimize the objective function, $g = 0.09x + 0.035y$, yields

$$\text{At } (0,26): g = 0.09(0) + 0.035(26) = 0.91$$
$$\text{At } (9,0): g = 0.09(9) + 0.035(0) = 0.81$$
$$\text{At } (4,10): g = 0.09(4) + 0.035(10) = 0.71$$

The minimum value is 0.71 occurring at $(4,10)$. Four servings of Diet A and ten servings of Diet B yields the minimum amount of detrimental substance, 0.71 oz.

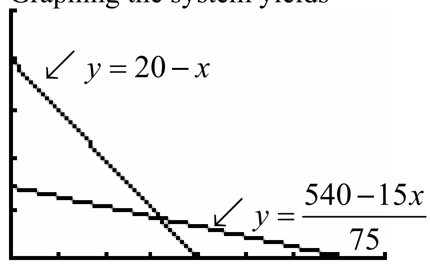
32. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let *x* represent the number of hours running assembly line 1, and let *y* represent the number of hours running assembly line 2.

$$\begin{cases} 15x + 75y \geq 540 \\ 20x + 20y \geq 400 \\ x \geq 0, y \geq 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \geq \dfrac{540 - 15x}{75} \\ y \geq 20 - x \\ x \geq 0, y \geq 0 \end{cases}$$

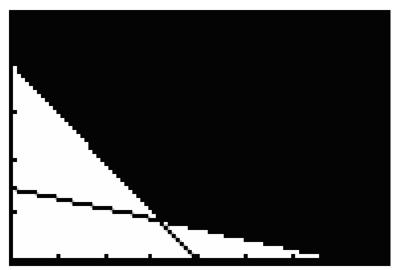
Graphing the system yields



[0, 40] by [0, 25]

To determine the solution region, pick a point to test. Pick $(20,10)$. When substituted into the inequalities that form the system, the point $(20,10)$ creates true statements in all cases.

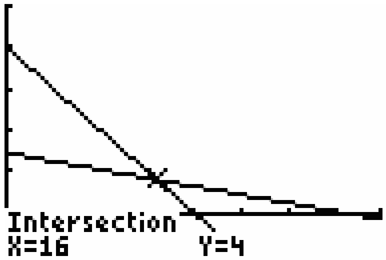
Since all the inequalities that form the system are true at the point $(20,10)$, the region that contains $(20,10)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 40] by [0, 25]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,20), (36,0),$ and $(16,4)$.



Testing the corner points of the feasible region to minimize the objective function, $g = 300x + 600y$, yields

$$\text{At } (0,20): g = 300(0) + 600(20) = \$12,000$$

$$\text{At } (36,0): g = 300(36) + 600(0) = \$10,800$$

$$\text{At } (16,4): g = 300(16) + 600(4) = \$7200$$

The minimum value is \$7200 occurring at $(16,4)$. The minimum manufacturing cost occurs when running assembly line 1 for 16 hours and assembly line 2 for 4 hours.

Section 6.3 Skills Check

1. $f(1)=2(1)+3=5$
$f(2)=2(2)+3=7$
$f(3)=2(3)+3=9$
$f(4)=2(4)+3=11$
$f(5)=2(5)+3=13$
$f(6)=2(6)+3=15$
The first six terms are 5, 7, 9, 11, 13, and 15.

2. $f(1)=\frac{1}{2(1)}+(1)=\frac{3}{2}$
$f(2)=\frac{1}{2(2)}+(2)=\frac{9}{4}$
$f(3)=\frac{1}{2(3)}+(3)=\frac{19}{6}$
$f(4)=\frac{1}{2(4)}+(4)=\frac{33}{8}$
The first four terms are $\frac{3}{2},\frac{9}{4},\frac{19}{6},$ and $\frac{33}{8}$.

3. $a_1=\frac{10}{1}=10$
$a_2=\frac{10}{2}=5$
$a_3=\frac{10}{3}$
$a_4=\frac{10}{4}=\frac{5}{2}$
$a_5=\frac{10}{5}=2$
The first five terms are 10, 5, $\frac{10}{3},\frac{5}{2},$ and 2.

4. $a_1=(-1)^{(1)}2(1)=-1(2)=-2$
$a_2=(-1)^{(2)}2(2)=1(4)=4$
$a_3=(-1)^{(3)}2(3)=-1(6)=-6$
$a_4=(-1)^{(4)}2(4)=1(8)=8$
$a_5=(-1)^{(5)}2(5)=-1(10)=-10$
The first five terms are
$-2, 4, -6, 8,$ and -10.

5. To move from one term in the sequence to the next term, add two. Therefore the next three terms are 9, 11, and 13.

6. To move from one term in the sequence to the next term, add three. Therefore the next three terms are 11, 14, and 17.

7. $a_n=a_1+(n-1)d$
$a_n=-3+(n-1)(4)$
$a_n=-3+4n-4$
$a_n=4n-7$
$a_8=4(8)-7=25$

8. $a_n=a_1+(n-1)d$
$a_n=5+(n-1)(15)$
$a_n=5+15n-15$
$a_n=15n-10$
$a_{40}=15(40)-10=590$

9. To move from one term in the sequence to the next term, multiply by 2. Therefore the next four terms are 24, 48, 96, and 192.

10. $a_n = a_1 r^{n-1}$

$a_2 = a_1 r^{2-1}$

$20 = 8r$

$r = \frac{20}{8} = \frac{5}{2}$

$a_n = 8\left(\frac{5}{2}\right)^{n-1}$

$a_4 = 8\left(\frac{5}{2}\right)^{4-1} = 8\left(\frac{5}{2}\right)^{3} = 125$

$a_5 = 8\left(\frac{5}{2}\right)^{5-1} = 8\left(\frac{5}{2}\right)^{4} = 312.5$

$a_6 = 8\left(\frac{5}{2}\right)^{6-1} = 8\left(\frac{5}{2}\right)^{5} = 781.25$

$a_7 = 8\left(\frac{5}{2}\right)^{7-1} = 8\left(\frac{5}{2}\right)^{6} = 1953.125$

The next four terms are $125, 312.5, 781.25,$ and 1953.125.

11. $a_n = a_1 r^{n-1}$

$a_n = 10(3)^{n-1}$

$a_6 = 10(3)^{6-1} = 10(3)^5 = 2430$

12. $a_n = a_1 r^{n-1}$

$a_n = 48\left(-\frac{1}{2}\right)^{n-1}$

$a_{10} = 48\left(-\frac{1}{2}\right)^{10-1} = 48\left(-\frac{1}{2}\right)^{9} = -\frac{3}{32}$

13. $a_n = a_{n-1} - 2$ implies that the next term in the sequence is the previous term in the sequence minus 2. For example, $a_2 = a_{2-1} - 2 = a_1 - 2$. The second term is the first term minus 2. Therefore, if the first term is 5, then the first four terms will be 5, 3, 1, and –1.

14. $a_n = a_{n-1} + 3$ implies that the next term in the sequence is the previous term in the sequence plus 3. For example, $a_2 = a_{2-1} + 3 = a_1 + 3$. The second term is the first term plus 3. Therefore, if the first term is 8, then the first six terms will be 8, 11, 14, 17, 20, and 23.

15. $a_n = 2a_{n-1} + 3$

$a_1 = 2$

$a_2 = 2a_{2-1} + 3 = 2a_1 + 3 = 2(2) + 3 = 7$

$a_3 = 2a_{3-1} + 3 = 2a_2 + 3 = 2(7) + 3 = 17$

$a_4 = 2a_{4-1} + 3 = 2a_3 + 3 = 2(17) + 3 = 37$

The first four terms are 2, 7, 17, and 37.

16. $a_n = \frac{a_{n-1} + 4}{2}$

$a_1 = 26$

$a_2 = \frac{a_{2-1} + 4}{2} = \frac{a_1 + 4}{2} = \frac{26 + 4}{2} = \frac{30}{2} = 15$

$a_3 = \frac{a_{3-1} + 4}{2} = \frac{a_2 + 4}{2} = \frac{15 + 4}{2} = \frac{19}{2} = 9.5$

$a_4 = \frac{a_{4-1} + 4}{2} = \frac{a_3 + 4}{2} = \frac{9.5 + 4}{2}$

$= \frac{13.5}{2}$

$= 6.75$

$a_5 = \frac{a_{5-1} + 4}{2} = \frac{a_4 + 4}{2} = \frac{6.75 + 4}{2}$

$= \frac{10.75}{2}$

$= 5.375$

The first five terms are 26, 15, 9.5, 6.75, and 5.375.

Section 6.3 Exercises

17. Let a_1 represent the starting salary

$a_n = a_1 + (n-1)d$

$a_n = a_1 + (n-1)1500$

$a_8 = a_1 + (8-1)1500$

$a_8 = a_1 + (7)1500 = a_1 + 10,500$

The salary would increase by \$10,500.

18. Using the straight-line depreciation method implies that after 5 years the car will be completely depreciated. So, each year the car would be depreciated $\frac{35,000}{5} = \$7000$.

Therefore, the sequence that represents the value of the car each year after depreciation is 28,000, 21,000, 14,000, 7000, and 0.

19. a. $f(n) = 300 + 60n$

b. $f(1) = 300 + 60(1) = 360$

$f(2) = 300 + 60(2) = 420$

$f(3) = 300 + 60(3) = 480$

$f(4) = 300 + 60(4) = 540$

$f(5) = 300 + 60(5) = 600$

$f(6) = 300 + 60(6) = 660$

The first six terms are 360, 420, 480, 540, 600, and 660.

20. $a_n = a_1 + (n-1)d$

$a_n = -2000 + (n-1)400$

$a_n = -2000 + 400n - 400$

$a_n = 400n - 2400$

$a_{12} = 400(12) - 2400 = 2400$

The profit in the twelfth month is \$2400.

21. a. Job 1

$a_n = a_1 + (n-1)d$

$a_n = 40,000 + (n-1)2000$

$a_n = 2000n + 38,000$

Job 2

$a_n = a_1 + (n-1)d$

$a_n = 36,000 + (n-1)2400$

$a_n = 2400n + 33,600$

After 5 years, $n = 6$.

Job 1:

$a_6 = 2000(6) + 38,000 = 50,000$

Job 2:

$a_6 = 2400(6) + 33,600 = 48,000$

After 5 years, Job 1 pays \$2000 more than Job 2.

b. After 10 years, $n = 11$.

Job 1:

$a_{11} = 2000(11) + 38,000 = 60,000$

Job 2:

$a_{11} = 2400(11) + 33,600 = 60,000$

The salaries are the same.

c. To decide between the jobs on the basis of salary requires an analysis of the length of time you plan to stay in the job. If you plan to stay in the job less than 10 years, Job 1 is clearly better.

22. a. $a_n = a_1 + (n-1)d$

$a_n = 0.99 + (n-1)(0.25)$

$a_n = 0.25n + 0.74$

b. The first six terms of the sequence are 0.99, 1.24, 1.49, 1.74, 1.99, and 2.24.

23. a. Year 1: $S = P + Prt$

$S = 1000 + 1000(5\%)(1)$

$S = 1000 + 1000(0.05)(1)$

$S = 1000 + 50$

$S = 1050$

Year 2: $S = P + Prt$

$S = 1050 + 1050(5\%)(1)$

$S = 1050 + 1050(0.05)(1)$

$S = 1050 + 52.5$

$S = 1102.50$

Year 3: $S = P + Prt$

$S = 1102.50 + 1102.50(5\%)(1)$

$S = 1102.50 + 1102.50(0.05)(1)$

$S = 1102.50 + 55.13$

$S = 1157.63$

Year 4: $S = P + Prt$

$S = 1157.63 + 1157.63(5\%)(1)$

$S = 1157.63 + 1157.63(0.05)(1)$

$S = 1157.63 + 57.88$

$S = 1215.51$

The sequence is 1050, 1102.50, 1157.63, and 1215.51.

24. $a_n = a_1 r^{n-1}$

$a_n = 10{,}000(1.06)^{n-1}$

$a_1 = 10{,}000$

$a_2 = 10{,}000(1.06)^{2-1}$

$= 10{,}000(1.06) = 10{,}600$

$a_3 = 10{,}000(1.06)^{3-1}$

$= 10{,}000(1.06)^2 = 11{,}236$

$a_4 = 10{,}000(1.06)^{4-1}$

$= 10{,}000(1.06)^3 = 11{,}910.16$

25. a. $20\%(50{,}000) = (0.20)(50{,}000)$

$= 10{,}000$

$50{,}000 - 3(10{,}000) = \$20{,}000$

The value after 3 years is \$20,000.

b. $a_n = a_1 + (n-1)d$

$a_n = 40{,}000 + (n-1)(-10{,}000)$

$a_n = 40{,}000 - 10{,}000n + 10{,}000$

$a_n = 50{,}000 - 10{,}000n$

Note that n represents the number of years of depreciation.

c. The value of the car depreciates as follows: 50,000, 40,000, 30,000, 20,000, 10,000, and 0.

26. $a_n = a_1 r^{n-1}$

$a_n = 32{,}000(1.08)^{n-1}$

$n = 5$

$a_5 = 32{,}000(1.08)^{5-1}$

$a_5 = 32{,}000(1.08)^4 = \$43{,}535.65$

27. $a_n = a_1 r^{n-1}$

$a_n = 5000(2)^{n-1}$

Since $n = 1$ represents zero hours having passed, then after six hours $n = 7$.

$a_7 = 5000(2)^{7-1}$

$a_7 = 5000(2)^6 = 320{,}000$

After 6 hours the number of bacteria in the culture is 320,000.

28. $a_n = a_1 r^{n-1}$

$a_n = 2000(1.20)^{n-1}$

After 10 hours, $n = 11$.

$a_{11} = 2000(1.20)^{11-1}$

$a_{11} = 2000(1.20)^{10} = 12{,}383.47384$

After 10 hours the number of bacteria in the culture is approximately 12,383.

29. $s(n) = 128\left(\frac{1}{4}\right)^n$, where n = the number of bounces, and s = the height of the bounce.

$s(4) = 128\left(\frac{1}{4}\right)^4 = \frac{1}{2}$

The height after the fourth bounce is $\frac{1}{2}$ foot.

30. $s(n) = 64\left(\frac{3}{4}\right)^n$, where n = the number of bounces, and s = the height of the bounce.

$s(4) = 64\left(\frac{3}{4}\right)^4 = 20.25$

The height after the fourth bounce is 20.25 feet.

31. If a company loses 2% of it's profit, then 98% of it's profit remains. Therefore, $f(n) = 8{,}000{,}000(0.98)^n$, where n = the number of years and $f(n)$ = the company's profit.

$f(5) = 8{,}000{,}000(0.98)^5$

$f(5) = 7{,}231{,}366.37$

After five years the company's projected profit is $7,231,366.37.

32. $f(n) = 81\left(\frac{1}{3}\right)^n$, where n represents the number of stokes and $f(n)$ is the amount removed by the pump on that stroke.

$f(5) = 81\left(\frac{1}{3}\right)^5 = \frac{1}{3}$

The pump removes $\frac{1}{3}$ cm^3 of water on the fifth stroke.

33. $a_n = a_1 + (n-1)d$

$a_n = 54{,}000 + (n-1)(3600)$

$a_n = 54{,}000 + 3600n - 3600$

$a_n = 50{,}400 + 3600n$

X	Y1
11	90000
12	93600
13	97200
14	100800
15	104400
16	108000
17	111600

X=16

When n = 16 or after 15 years, the salary is twice the original amount of $54,000. Therefore the salary doubles after 15 years.

34. $f(n) = 2500(1.08)^n$

X	Y1
9	4997.5
10	5397.3
11	5829.1
12	6295.4
13	6799.1
14	7343
15	7930.4

X=11

The account reaches a value of $5829.10 after 11 years.

35. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 10{,}000\left(1 + \frac{0.08}{365}\right)^{365(10)}$

$A = 10{,}000(1.000219178)^{3650}$

$A = 22{,}253.46$

The future value of $10,000 compounded daily at 8% is $22,253.46.

36. $10,000(1.08)^{10} = 21,589.25$

The future value of $10,000 compounded annually at 8% is $21,589.25.

37. The next number in the sequence is the sum of the previous two numbers in the sequence. Therefore, the next four numbers in the sequence are 13, 21, 34, and 55.

38. **a.** 21, 34, and 55

b. Mother, grandparents, and great-grandparents.

39. Making a payment of 1% interest plus 10% of the balance at the beginning of the month implies that she will make a payment of 11% of balance each month. However, the balance will decrease by 10%, since 1% of the payment is interest.

$$\begin{cases} \text{Month 1} & \\ \text{Payment:} & 10{,}000(0.11) = 1100 \\ \text{Balance after} & \\ \text{Payment:} & 10{,}000(1.01) - 1100 = 9000 \end{cases}$$

$$\begin{cases} \text{Month 2} & \\ \text{Payment:} & 9000(0.11) = 990 \\ \text{Balance after} & \\ \text{Payment:} & 9000(1.01) - 990 = 8100 \end{cases}$$

$$\begin{cases} \text{Month 3} & \\ \text{Payment:} & 8100(0.11) = 891 \\ \text{Balance after} & \\ \text{Payment:} & 8100(1.01) - 891 = 7290 \end{cases}$$

$$\begin{cases} \text{Month 4} & \\ \text{Payment:} & 7290(0.11) = 801.90 \end{cases}$$

The sequence of payments is $1100, $990, $891, and $801.90.

Section 6.4 Skills Check

1. Note that $r = \frac{3}{9} = \frac{1}{3}$.

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_6 = \frac{9\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\left(\frac{1}{3}\right)} = \frac{364}{27}$$

2. Note that $d = 3-1 = 5-3 = 2$.

$$a_n = a_1 + (n-1)d$$
$$a_n = 1 + (n-1)(2)$$
$$a_n = 2n-1$$
$$a_7 = 2(7)-1 = 13$$
$$s_n = \frac{n(a_1+a_n)}{2}$$
$$s_7 = \frac{7(a_1+a_7)}{2} = \frac{7(1+13)}{2} = 49$$

3. Note that $d = 10-7 = 13-10 = 3$.

$$a_n = a_1 + (n-1)d$$
$$a_n = 7 + (n-1)(3)$$
$$a_n = 3n+4$$
$$a_{10} = 3(10)+4 = 34$$
$$s_n = \frac{n(a_1+a_n)}{2}$$
$$s_{10} = \frac{10(a_1+a_{10})}{2} = \frac{10(7+34)}{2} = 205$$

4. Note that $d = 12-5 = 19-12 = 7$.

$$a_n = a_1 + (n-1)d$$
$$a_n = 5 + (n-1)(7)$$
$$a_n = 7n-2$$
$$a_{20} = 7(20)-2 = 138$$
$$s_n = \frac{n(a_1+a_n)}{2}$$
$$s_{20} = \frac{20(a_1+a_{20})}{2} = \frac{20(5+138)}{2} = 1430$$

5. $a_n = a_1 + (n-1)d$

$$a_n = -4 + (n-1)(2)$$
$$a_n = 2n-6$$
$$a_{15} = 2(15)-6 = 24$$
$$s_n = \frac{n(a_1+a_n)}{2}$$
$$s_{15} = \frac{15(a_1+a_{15})}{2} = \frac{15(-4+24)}{2} = 150$$

6. $a_n = a_1 + (n-1)d$

$$a_n = 50 + (n-1)(3)$$
$$a_n = 3n+47$$
$$a_{10} = 3(10)+47 = 77$$
$$s_n = \frac{n(a_1+a_n)}{2}$$
$$s_{10} = \frac{10(a_1+a_{15})}{2} = \frac{10(50+77)}{2} = 635$$

7. $s_n = \frac{a_1\left(1-r^n\right)}{1-r}$

$$s_{15} = \frac{3\left(1-(2)^{15}\right)}{1-(2)} = 98,301$$

8. $s_n = \frac{a_1\left(1-r^n\right)}{1-r}$

$$s_{12} = \frac{48\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-\left(\frac{1}{2}\right)} = \frac{12,285}{128}$$

9. Note that $r = \frac{10}{5} = \frac{20}{10} = 2$.

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_{10} = \frac{5\left(1-(2)^{10}\right)}{1-(2)} = 5115$$

10. Note that $r = \frac{50}{100} = \frac{25}{50} = \frac{1}{2}$.

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_{15} = \frac{100\left(1-\left(\frac{1}{2}\right)^{15}\right)}{1-\left(\frac{1}{2}\right)} = \frac{819{,}175}{4096}$$

11. Note that $r = \frac{9}{81} = \frac{1}{9} = \frac{1}{9}$.

$$S = \frac{a_1}{1-r}$$

$$S = \frac{81}{1-\left(\frac{1}{9}\right)} = \frac{729}{8}$$

12. Note that $r = \frac{32}{64} = \frac{16}{32} = \frac{1}{2}$.

$$S = \frac{a_1}{1-r}$$

$$S = \frac{64}{1-\left(\frac{1}{2}\right)} = 128$$

13. $\sum_{i=1}^{6} 2^i = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$

$= 2 + 4 + 8 + 16 + 32 + 64 = 126$

14. $\sum_{i=1}^{5} 4^i = 4^1 + 4^2 + 4^3 + 4^4 + 4^5$

$= 4 + 16 + 64 + 256 + 1024 = 1364$

15. $\sum_{i=1}^{4}\left(\frac{1+i}{i}\right) = 2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} = \frac{73}{12}$

16. $\sum_{i=1}^{5}\left(\frac{1}{2}\right)^i = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{31}{32}$

17. $S = \frac{a_1}{1-r} = \frac{\frac{3}{4}}{1-\frac{3}{4}} = 3$

18. $S = \frac{a_1}{1-r} = \frac{\frac{5}{6}}{1-\frac{5}{6}} = 5$

19. Since $r = \frac{4}{3} \geq 1$, the infinite sum does not exist. No solution.

20. Note that $r = \frac{600}{800} = \frac{3}{4}$.

$$S = \frac{a_1}{1-r}$$

$$S = \frac{800}{1-\left(\frac{3}{4}\right)} = 3200$$

Section 6.4 Exercises

21. Note that $d = 400$ and $a_1 = -2000$.

$$a_n = a_1 + (n-1)d$$
$$a_n = -2000 + (n-1)(400)$$
$$a_n = 400n - 2400$$
$$a_{12} = 400(12) - 2400 = 2400$$
$$s_n = \frac{n(a_1 + a_n)}{2}$$
$$s_{12} = \frac{12(a_1 + a_{12})}{2}$$
$$= \frac{12(-2000 + 2400)}{2} = 2400$$

The profit for the year is \$2400.

22. $(1500)(7) = 10{,}500$. The raises over the next seven years total \$10,500.

23. $1 + 2 + 3 + 5 = 11$. A male bee has 11 ancestors through four generations.

24. a. Job 1

$$a_n = a_1 + (n-1)d$$
$$a_n = 40{,}000 + (n-1)(2000)$$
$$a_n = 2000n + 38{,}000$$
$$a_5 = 2000(5) + 38{,}000 = 48{,}000$$
$$s_n = \frac{n(a_1 + a_n)}{2}$$
$$s_5 = \frac{5(a_1 + a_5)}{2}$$
$$= \frac{5(40{,}000 + 48{,}000)}{2}$$
$$= 220{,}000$$

Job 2

$$a_n = a_1 + (n-1)d$$
$$a_n = 36{,}000 + (n-1)(2400)$$
$$a_n = 2400n + 33{,}600$$
$$a_5 = 2400(5) + 33{,}600 = 45{,}600$$
$$s_n = \frac{n(a_1 + a_n)}{2}$$
$$s_5 = \frac{5(a_1 + a_5)}{2}$$
$$= \frac{5(36{,}000 + 45{,}600)}{2}$$
$$= 204{,}000$$

Job 1 produces \$16,000 more in income over a 5-year period.

b. Job 1

$a_n = a_1 + (n-1)d$

$a_n = 40{,}000 + (n-1)(2000)$

$a_n = 2000n + 38{,}000$

$a_{12} = 2000(12) + 38{,}000 = 62{,}000$

$s_n = \dfrac{n(a_1 + a_n)}{2}$

$s_{12} = \dfrac{12(a_1 + a_{12})}{2}$

$= \dfrac{12(40{,}000 + 62{,}000)}{2}$

$= 612{,}000$

Job 2

$a_n = a_1 + (n-1)d$

$a_n = 36{,}000 + (n-1)(2400)$

$a_n = 2400n + 33{,}600$

$a_{12} = 2400(12) + 33{,}600 = 62{,}400$

$s_n = \dfrac{n(a_1 + a_n)}{2}$

$s_{12} = \dfrac{12(a_1 + a_{12})}{2}$

$= \dfrac{12(36{,}000 + 62{,}400)}{2}$

$= 590{,}400$

Job 1 produces $21,600 more income over a 12-year period.

c. The length of time a person stays in the job determines which job is best from a salary point of view. For the first 5 or 12 years, Job 1 produces more income. However, since the raises for Job 1 are greater, eventually the income generated from Job 2 will exceed the income generated from Job 1.

25. a. $a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)(1)$

$a_n = n$

$a_{12} = 12$

$s_n = \dfrac{n(a_1 + a_n)}{2}$

$s_{12} = \dfrac{12(1+12)}{2} = 78$

The clock chimes 78 times in a 12-hour period.

b. In a 24-hour period, the clock chimes twice as many times as in a 12-hour period. Therefore the clock chimes 156 times every 24 hours.

26. a. $1+2+3+4=10$

The fractions are $\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$.

The sequence of depreciation is

$(36{,}000)\left(\frac{4}{10}\right), (36{,}000)\left(\frac{3}{10}\right),$

$(36{,}000)\left(\frac{2}{10}\right), (36{,}000)\left(\frac{1}{10}\right)$

or

$14{,}400,\ 10{,}800,\ 7200,\ 3600$

b. Sum of the sequence =

$(36{,}000)\left(\frac{4}{10}\right) + (36{,}000)\left(\frac{3}{10}\right)$

$+ (36{,}000)\left(\frac{2}{10}\right) + (36{,}000)\left(\frac{1}{10}\right)$

Sum of the sequence =

$14{,}400 + 10{,}800 + 7200 + 3600$

Sum of the sequence = $36{,}000$

27. Note that the sequence is geometric with a common ratio of 1.10. Therefore,

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_{12} = \frac{2000\left(1-(1.10)^{12}\right)}{1-(1.10)} \approx 42,768.57$$

The total profit over the first 12 months is $42,768.57.

28. If the company projects a 2% loss in profit, then 98% of the profit remains. Therefore, the series is geometric with a common ratio of 98% or 0.98.

$$a_1 = 8,000,000(0.98) = 7,840,000$$

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_5 = \frac{7,840,000\left(1-(0.98)^5\right)}{1-(0.98)}$$

$$s_5 \approx 37,663,047.65$$

The total profit over the next five years will be approximately $37,663,047.65.

29. If the pump removes $\frac{1}{3}$ of the water with each stroke, then $\frac{2}{3}$ of the water remains after each stroke. Therefore, a geometric sequence can be created with a common ratio of $\frac{2}{3}$.

$$a_1 = 81\left(\frac{2}{3}\right) = 54$$

$$a_n = a_1 r^{n-1}$$

$$a_n = 54\left(\frac{2}{3}\right)^{n-1}$$

$$a_4 = 54\left(\frac{2}{3}\right)^{4-1} = 54\left(\frac{2}{3}\right)^3 = 16\,\text{cm}^3$$

The amount of water in the container after four strokes is 16 cm^3.

30. Note that the sequence is geometric with a common ratio of 1.08. Therefore,

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_5 = \frac{32,000\left(1-(1.08)^5\right)}{1-(1.08)}$$

$$s_5 \approx 187,731.23$$

The total income over the first 5 years is $187,731.23.

31. a. 5

b. $5 \bullet 5 = 25$

c. $5^3 = 125$

$5^4 = 625$

d. The sequence is geometric with a common ratio of 5.

32. a. Recall that the sequence is geometric with a common ratio of 5.

$$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

$$s_{12} = \frac{5\left(1-(5)^{12}\right)}{1-(5)}$$

$$s_5 = 305,175,780$$

b. The number is larger than the population of the United States!

33. Since 10% of the balance is paid each month, then 90% of the balance remains. Note that the situation is modeled by a geometric series with a common ratio of 0.90.

$$a_n = 10,000(0.90)^n$$

$$a_{12} = 10,000(0.90)^{12} = 2824.30$$

34. $a_n = 10,000(0.90)^n$

Using the table feature in the graphing calculator yields

X	Y1
2	8100
3	7290
4	6561
5	5904.9
6	5314.4
7	4783
8	4304.7

X=7

After about 7 months, half the debt is retired.

35. Note that the situation is modeled by a geometric function having a common ratio of $\frac{1}{4}$.

$a_1 = 128\left(\frac{1}{4}\right) = 32$

$a_n = 32\left(\frac{1}{4}\right)^{n-1}$, where n represents the number of bounces and a_n represents the height after that bounce. Note that when the ball hits the ground for the fifth time, it has bounced four times.

$$s_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

$$s_4 = \frac{32\left(1 - \left(\frac{1}{4}\right)^4\right)}{1 - \left(\frac{1}{4}\right)}$$

$$s_4 = 42.5$$

Note that s_4 is the sum of the rebound heights. To calculate the distance the ball actually travels, add in the initial distance before the first rebound of 128 feet and double s_4 to take into consideration that the ball rises and falls with each bounce. Therefore the total distance traveled is $128 + 2(42.5) = 213$ feet.

36. Note that the situation is modeled by a geometric function having a common ratio of $\frac{3}{4}$.

$a_1 = 64\left(\frac{3}{4}\right) = 48$

$a_n = 48\left(\frac{3}{4}\right)^{n-1}$, where n represents the number of bounces and a_n represents the height after that bounce. Note that when the ball hits the ground for the 4 time, it has bounced 3 times.

$$s_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

$$s_3 = \frac{48\left(1 - \left(\frac{3}{4}\right)^3\right)}{1 - \left(\frac{3}{4}\right)}$$

$$s_3 = 111$$

Note that s_3 is the sum of the rebound heights. To calculate the distance the ball actually travels, add in the initial distance before the first rebound of 64 feet and double s_3 to take into consideration that the ball rises and falls with each bounce. Therefore, the total distance traveled is $64 + 2(111) = 286$ feet.

37. a. Note that if the car loses 16% of its value each year, then it retains 84% of its value. The value of the car is modeled by a geometric sequence with a common ratio of 0.84.
The value of the car after n years is given by $a_n = 35{,}000(0.84)^n$. The amount of depreciation after n years is given by the original value of the car less the depreciated value. Therefore,

$$s_n = 35{,}000 - 35{,}000(0.84)^n$$

$$s_n = 35{,}000\left(1 - (0.84)^n\right)$$

b. $a_n = 35{,}000(0.84)^n$

38. a. If 8% is paid each month, then 92% remains. The situation is modeled by a geometric sequence having a common ratio of 0.92.

$a_n = 15{,}000(0.92)^n$

$a_{12} = 15{,}000(0.92)^{12} = \$5515.00$

b. $a_n = 15{,}000(0.92)^n$

$a_{24} = 15{,}000(0.92)^{24} = \$2027.68$

39. $s_n = \dfrac{a_1(1-r^n)}{1-r}$

$$s_{96} = \frac{100\left(1-\left(1+\frac{0.12}{12}\right)^{96}\right)}{1-\left(1+\frac{0.12}{12}\right)}$$

$$s_{96} = \frac{100\left(1-(1.01)^{96}\right)}{1-(1.01)}$$

$s_{96} \approx 15{,}992.73$

The future value of the annuity is \$15,992.73.

40. $s_n = \dfrac{a_1(1-r^n)}{1-r}$

$$s_{40} = \frac{300\left(1-\left(1+\frac{0.08}{4}\right)^{40}\right)}{1-\left(1+\frac{0.08}{4}\right)}$$

$$s_{40} = \frac{300\left(1-(1.02)^{40}\right)}{1-(1.02)}$$

$s_{96} \approx 18{,}120.59$

The future value of the annuity is \$18,120.59.

41. $s_n = \dfrac{a_1(1-r^n)}{1-r}$

$$s_n = \frac{\left(R(1+i)^{-n}\right)\left(1-(1+i)^n\right)}{1-(1+i)}$$

$$s_n = \frac{R(1+i)^{-n}\left(1-(1+i)^n\right)}{1-1-i}$$

$$s_n = \frac{R\left[(1+i)^{-n}(1)-(1+i)^{-n}(1+i)^n\right]}{-i}$$

$$s_n = \frac{R\left[(1+i)^{-n}-(1+i)^{-n+n}\right]}{-i}$$

$$s_n = \frac{R\left[(1+i)^{-n}-(1+i)^0\right]}{-i}$$

$$s_n = \frac{R\left[(1+i)^{-n}-1\right]}{-i}$$

$$s_n = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Section 6.5 Skills Check

1. $f'(x) = (4x^4 + 6x^2 + 2x) + (6x^4 + 3x^2 + 24x^2 + 12)$

$f'(x) = 10x^4 + 33x^2 + 2x + 12$

2. $f'(x) = (3x^4 - 9x^3) + (2x^4 - 3x^3 - 8x + 12)$

$f'(x) = 5x^4 - 12x^3 - 8x + 12$

3. $f'(x) = \dfrac{3x^4 - (2x^4 - 6x)}{x^4}$

$f'(x) = \dfrac{x^4 + 6x}{x^4} = \dfrac{x(x^3 + 6)}{x^4} = \dfrac{x^3 + 6}{x^3}$

4. $f'(x) = (3x^4 - 6x^3 - 12x^2 + 24x) - (2x^4 - 6x^3 + 4x)$

$f'(x) = x^4 - 12x^2 + 20x$

5. $f'(2) = 4(2)^3 - 3(2)^2 + 4(2) - 2 = 26$

6. $f'(-1) = ((-1)^2 - 1)(6(-1)) + (3(-1)^2 + 1)(2(-1))$

$= (0)(-6) + (4)(-2)$

$= -8$

7. $f'(3) = \dfrac{((3)^2 - 1)(2) + (2(3))(2(3))}{((3)^2 - 1)^2}$

$= \dfrac{(8)(2) + (6)(6)}{(8)^2}$

$= \dfrac{16 + 36}{64}$

$= \dfrac{52}{64} = \dfrac{13}{16}$

8. $f'(2) = 8(2)^3 + 6(2)^2 - 5(2) - 10$

$= 64 + 24 - 10 - 10$

$= 68$

The slope of the tangent line to $f(x)$ at $x = 2$ is 68.

9. $y - y_1 = m(x - x_1)$

$y - 8 = 12(x - 2)$

$y - 8 = 12x - 24$

$y = 12x - 16$

10. a. $f(x + h) = 3(x + h) - 2$

b. $f(x + h) - f(x)$

$= [3(x + h) - 2] - [3x - 2]$

$= 3x + 3h - 2 - 3x + 2$

$= 3h$

c. $\dfrac{f(x + h) - f(x)}{h}$

$= \dfrac{3h}{h}$

$= 3$

11. a. $f(x + h) = 4(x + h) + 5$

b. $f(x+h)-f(x)$

$=[4(x+h)+5]-[4x+5]$

$=4x+4h+5-4x-5$

$=4h$

c. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{4h}{h}$

$=4$

12. a. $f(x+h)=12-2(x+h)$

b. $f(x+h)-f(x)$

$=[12-2(x+h)]-[12-2x]$

$=12-2x-2h-12+2x$

$=-2h$

c. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{-2h}{h}$

$=-2$

13. $8x^2+4y=12$

$4y=12-8x^2$

$y=\dfrac{12-8x^2}{4}$

$y=\dfrac{12}{4}-\dfrac{8x^2}{4}$

$y=3-2x^2$

14. $3x^2-2y=6$

$-2y=6-3x^2$

$y=\dfrac{6-3x^2}{-2}$

$y=\dfrac{6}{-2}+\dfrac{3}{2}x^2$

$y=\dfrac{3}{2}x^2-3$

or

$y=\dfrac{3x^2-6}{2}$

15. $9x^3+5y=18$

$5y=18-9x^3$

$y=\dfrac{18-9x^3}{5}$

16. $12x^3=6y-24$

$6y-24=12x^3$

$6y=12x^3+24$

$y=\dfrac{12x^3+24}{6}$

$y=\dfrac{12x^3}{6}+\dfrac{24}{6}$

$y=2x^3+4$

17. a. $0=2x-2$

$2x=2$

$x=1$

b. Recall that the x-coordinate of the vertex is given by $\dfrac{-b}{2a}$.

$\dfrac{-b}{2a}=\dfrac{-(-2)}{2(1)}=\dfrac{2}{2}=1$

The solution to part a) and the x-coordinate of the vertex are equal.

c. $y = (1)^2 - 2(1) + 5 = 4$

$(1, 4)$

18. a. $0 = 1 - 2x$

$2x = 1$

$x = \frac{1}{2}$

b. Recall that the x-coordinate of the vertex is given by $\frac{-b}{2a}$.

$\frac{-b}{2a} = \frac{-(1)}{2(-1)} = \frac{-1}{-2} = \frac{1}{2}$

The solution to part a) and the x-coordinate of the vertex are equal.

c. $y = 5 + \left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2$

$y = 5 + \frac{1}{2} - \frac{1}{4} = \frac{21}{4}$

$\left(\frac{1}{2}, \frac{21}{4}\right)$

19. a. $0 = 12x - 24$

$12x = 24$

$x = 2$

b. Recall that the x-coordinate of the vertex is given by $\frac{-b}{2a}$.

$\frac{-b}{2a} = \frac{-(-24)}{2(6)} = \frac{24}{12} = 2$

The solution to part a) and the x-coordinate of the vertex are equal.

c. $y = 6(2)^2 - 24(2) + 15$

$y = 24 - 48 + 15 = -9$

$(2, -9)$

20. a. $0 = -3 - 2x$

$2x = -3$

$x = \frac{-3}{2}$

b. $y = -4 - 3x - x^2$

Recall that the x-coordinate of the vertex is given by $\frac{-b}{2a}$.

$\frac{-b}{2a} = \frac{-(-3)}{2(-1)} = \frac{3}{-2} = -\frac{3}{2}$

The solution to part a) and the x-coordinate of the vertex are equal.

c. $y = -\left(4 + 3\left(-\frac{3}{2}\right) + \left(-\frac{3}{2}\right)^2\right)$

$y = -\left(4 - \frac{9}{2} + \frac{9}{4}\right) = -\left(\frac{7}{4}\right) = -\frac{7}{4}$

$\left(-\frac{3}{2}, -\frac{7}{4}\right)$

21. a. $3x^2 - 18x - 48 = 0$

$3(x^2 - 6x - 16) = 0$

$3(x - 8)(x + 2) = 0$

$x = 8,\ x = -2$

b. $x = 8$

$y = (8)^3 - 9(8)^2 - 48(8) + 15$

$y = -433$

$(8, -433)$

$x = -2$

$y = (-2)^3 - 9(-2)^2 - 48(-2) + 15$

$y = 67$

$(-2, 67)$

c. $x = -2$ produces the maximum.

22. a. $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3,\ x = -1$

b. $x = 3$

$y = (3)^3 - 3(3)^2 - 9(3) + 5$

$y = -22$

$(3, -22)$

$x = -1$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 5$

$y = 10$

$(-1, 10)$

c. $x = -1$ produces the maximum.

23. $y = 3\sqrt{x} = 3\sqrt[2]{x^1} = 3x^{\frac{1}{2}}$

24. $y = 6\sqrt[3]{x^1} = 6x^{\frac{1}{3}}$

25. $y = 2\sqrt[3]{x^2} = 2x^{\frac{2}{3}}$

26. $y = 4\sqrt{x^3} = 4\sqrt[2]{x^3} = 4x^{\frac{3}{2}}$

27. $y = \sqrt[3]{x^2 + 1} = (x^2 + 1)^{\frac{1}{3}}$

28. $y = \sqrt[4]{x^3 - 2} = (x^3 - 2)^{\frac{1}{4}}$

29. $y = \sqrt[3]{(x^3 - 2)^2} = (x^3 - 2)^{\frac{2}{3}}$

30. $y = \sqrt{(5x-3)^3} = \sqrt[2]{(5x-3)^3} = (5x-3)^{\frac{3}{2}}$

31. $y = \sqrt{x} + \sqrt[3]{2x}$

$y = x^{\frac{1}{2}} + (2x)^{\frac{1}{3}}$

32. $y = \sqrt[3]{(2x)^2} + \sqrt[2]{(4x)^3}$

$y = (2x)^{\frac{2}{3}} + (4x)^{\frac{3}{2}}$

33. $y^2 + 4x - 3 = 0$

$y^2 = 3 - 4x$

$\sqrt{y^2} = \pm\sqrt{3 - 4x}$

$y = \pm\sqrt{3 - 4x}$

34. $5x - y^2 = 12$

$y^2 + 12 = 5x$

$y^2 = 5x - 12$

$\sqrt{y^2} = \pm\sqrt{5x - 12}$

$y = \pm\sqrt{5x - 12}$

35. $y^2 + y - 6x = 0$

$a = 1,\ b = 1,\ c = -6x$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-6x)}}{2(1)}$$

$$y = \frac{-1 \pm \sqrt{1 + 24x}}{2}$$

36. $-2y^2 + y + (2x+4) = 0$

$a = -2,\ b = 1,\ c = 2x+4$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-1 \pm \sqrt{(1)^2 - 4(-2)(2x+4)}}{2(-2)}$$

$$y = \frac{-1 \pm \sqrt{1 + 8(2x+4)}}{-4}$$

$$y = \frac{-1 \pm \sqrt{1 + 16x + 32}}{-4}$$

$$y = \frac{-1 \pm \sqrt{16x+33}}{-4} \text{ or } \frac{1 \pm \sqrt{16x+33}}{4}$$

37. $y = u^2$

$u = 4x^3 + 5$

38. $(5x^2 - 4x)^7$

39. $u = x^3 + x$

$y = \sqrt{x^3 + x}$

$y = \sqrt{u} = u^{\frac{1}{2}}$

40. $6u^5 \bullet 4x^3$

$= 6(x^4+5)^5 \bullet 4x^3$

$= 24x^3(x^4+5)^5$

41. $\frac{3}{2}u^{\frac{1}{2}} \bullet 2x$

$= \frac{3}{2}(x^2-1)^{\frac{1}{2}} \bullet 2x$

$= 3x\sqrt{x^2-1}$

42. $y = 3x + x^{-1} - 2x^{-2}$

43. $y = 3x^{-1} - 4x^{-2} - 6$

44. $y = 5 + x^{-3} - \frac{2}{x^{\frac{1}{2}}} + \frac{3}{x^{\frac{2}{3}}}$

$y = 5 + x^{-3} - 2x^{-\frac{1}{2}} + 3x^{-\frac{2}{3}}$

45. $y = (x^2 - 3)^{-3}$

46. $f'(x) = 8\left(\frac{1}{x^4}\right) + 5\left(\frac{1}{x^2}\right) + x$

$f'(x) = \frac{8}{x^4} + \frac{5}{x^2} + x$

47. $f'(x) = -6\left(\frac{1}{x^4}\right) + 4\left(\frac{1}{x^2}\right) + \frac{1}{x}$

$f'(x) = \frac{-6}{x^4} + \frac{4}{x^2} + \frac{1}{x}$

48. $f'(x) = -3(3x-2)^{-3} = \frac{-3}{(3x-2)^3}$

49. $f'(x) = (4x^2-3)^{-\frac{1}{2}}(8x)$

$f'(x) = \frac{8x}{(4x^2-3)^{\frac{1}{2}}}$

$f'(x) = \frac{8x}{\sqrt{4x^2-3}}$

50. $\ln\left(\frac{3x-2}{x+1}\right) = \ln(3x-2) - \ln(x+1)$

51. $\log\left[x^3(3x-4)^5\right]$

$\log\left(x^3\right)+\log\left[(3x-4)^5\right]$

$3\log x+5\log(3x-4)$

52. $\ln\left(\dfrac{\sqrt[4]{4x+1}}{4x^2}\right)$

$\ln\left(\sqrt[4]{4x+1}\right)-\ln\left(4x^2\right)$

$\ln(4x+1)^{\frac{1}{4}}-\left[\ln 4+\ln x^2\right]$

$\dfrac{1}{4}\ln(4x+1)-\ln 4-2\ln x$

53. $f'(x)=0$

$3x^2-3x=0$

$3x(x-1)=0$

$3x=0,\ x-1=0$

$x=0,\ x=1$

54. $f'(x)=0$

$3x^3-14x^2+8x=0$

$x\left(3x^2-14x+8\right)=0$

$x(3x-2)(x-4)=0$

$x=0,\ 3x-2=0,\ x-4=0$

$x=0,\ x=\dfrac{2}{3},\ x=4$

55. $f'(x)=0$

$\left(x^2-4\right)3(x-3)^2+(x-3)^3(2x)=0$

$(x-3)^2\left[3\left(x^2-4\right)+(x-3)(2x)\right]=0$

$(x-3)^2\left[3x^2-12+2x^2-6x\right]=0$

$(x-3)^2\left[5x^2-6x-12\right]=0$

$(x-3)^2=0$ implies $x=3$

$5x^2-6x-12=0$

$a=5,\ b=-6,\ c=-12$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(5)(-12)}}{2(5)}$

$x=\dfrac{6\pm\sqrt{36+240}}{10}$

$x=\dfrac{6\pm\sqrt{276}}{10}$

$x=\dfrac{6\pm\sqrt{4\bullet 69}}{10}$

$x=\dfrac{6\pm 2\sqrt{69}}{10}$

$x=\dfrac{3\pm\sqrt{69}}{5}$

$x=3,\ x=\dfrac{3+\sqrt{69}}{5},\ x=\dfrac{3-\sqrt{69}}{5}$

56. $f'(x)=0$

$$\frac{(x+1)\left[3(2x-3)^2 2\right]-(2x-3)^3}{(x+1)^2}=0$$

Only consider the numerator equal zero.
If the numerator is zero, then the expression is zero.

$$(x+1)\left[3(2x-3)^2 2\right]-(2x-3)^3=0$$

$$(2x-3)^2\left[6(x+1)-(2x-3)\right]=0$$

$$(2x-3)^2\left[6x+6-2x+3\right]=0$$

$$(2x-3)^2(4x+9)=0$$

$$2x-3=0, 4x+9=0$$

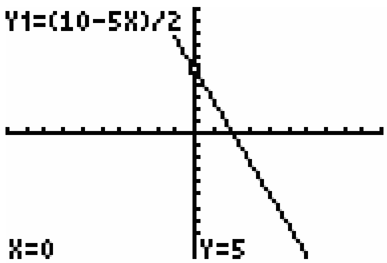
$$x=\frac{3}{2}, x=-\frac{9}{4}$$

Chapter 6 Skills Check

1. $5x + 2y \le 10$

$$2y \le 10 - 5x$$

$$y \le \frac{10 - 5x}{2}$$



[–10, 10] by [–10, 10]

Note that the line is solid because of the "equal to" in the given inequality.

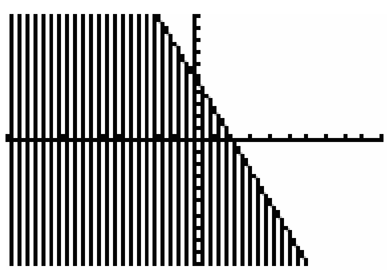
Test (0, 0).

$$y \le \frac{10 - 5x}{2}$$

$$0 \le \frac{10 - 5(0)}{2}$$

$$0 \le 5$$

Since the statement is true, the region containing (0, 0) is the solution to the inequality.



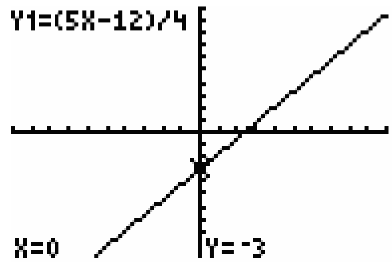
[–10, 10] by [–10, 10]

2. $5x - 4y > 12$

$$-4y > 12 - 5x$$

$$y < \frac{12 - 5x}{-4}$$

$$y < \frac{5x - 12}{4}$$



[–10, 10] by [–10, 10]

Note that the line is dashed because there is no "equal to" in the given inequality.

Test (0, 0).

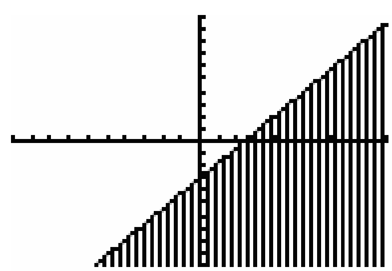
$$y < \frac{5x - 12}{4}$$

$$0 < \frac{5(0) - 12}{4}$$

$$0 < \frac{-12}{4}$$

$$0 < -3$$

Since the statement is false, the region not containing (0, 0) is the solution to the inequality.



[–10, 10] by [–10, 10]

3. Rewriting the system yields

$2x + y \le 3$

$y \le 3 - 2x$

and

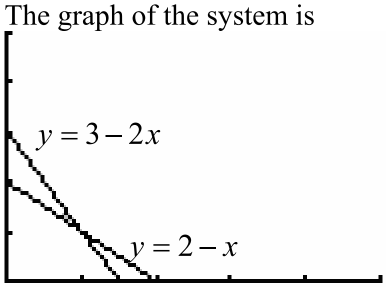
$x + y \le 2$

$y \le 2 - x$

The new system is

$$\begin{cases} y \le 3 - 2x \\ y \le 2 - x \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is



[0, 5] by [0, 5]

Note that all the lines are solid.
Determining the intersection point between $y = 3 - 2x$ and $y = 2 - x$:

$3 - 2x = 2 - x$

$-x = -1$

$x = 1$

Substituting to find y

$y = 2 - x$

$y = 2 - 1$

$y = 1$

The intersection point is $(1,1)$.

To determine the solution region, pick a point to test. Pick $\left(\frac{1}{2}, 1\right)$.

$y \le 3 - 2x$

$1 \le 3 - 2\left(\frac{1}{2}\right)$

$1 \le 2$

True statement

$y \le 2 - x$

$1 \le 2 - \frac{1}{2}$

$1 \le \frac{3}{2}$

True statement

$x \ge 0$

$\frac{1}{2} \ge 0$

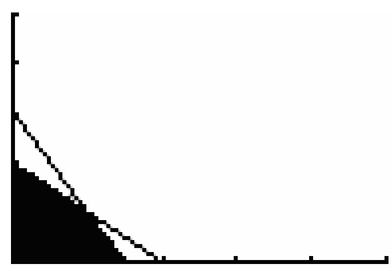
True statement

$y \ge 0$

$1 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $\left(\frac{1}{2}, 1\right)$, the region that contains $\left(\frac{1}{2}, 1\right)$ is the solution region. Considering the graph of the system, the corners of the region are $(0,0), (0,2), (1,1)$, and $\left(\frac{3}{2}, 0\right)$.



[0, 5] by [0, 5]

Recall that all the lines are solid.

4. Rewriting the system:

$3x+2y\le 6$

$2y\le 6-3x$

$y\le\dfrac{6-3x}{2}$

and

$3x+6y\le 12$

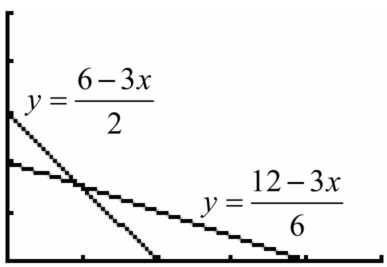
$6y\le 12-3x$

$y\le\dfrac{12-3x}{6}$

The new system is

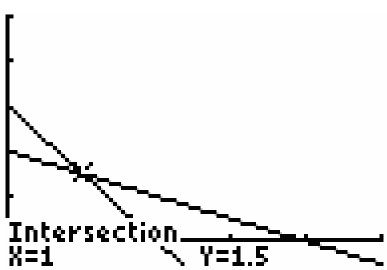
$$\begin{cases} y\le\dfrac{6-3x}{2} \\ y\le\dfrac{12-3x}{6} \\ x\ge 0, y\ge 0 \end{cases}$$

The graph of the system is



[0, 5] by [0, 5]

Note that all the lines are solid. The intersection point between the two lines is $(1,1.5)$.



To determine the solution region, pick a point to test. Pick $(1,1)$.

$y\le\dfrac{6-3x}{2}$

$1\le\dfrac{6-3(1)}{2}$

$1\le\dfrac{3}{2}$

True statement

$y\le\dfrac{12-3x}{6}$

$1\le\dfrac{12-3(1)}{6}$

$1\le\dfrac{3}{2}$

True statement

$x\ge 0$

$1\ge 0$

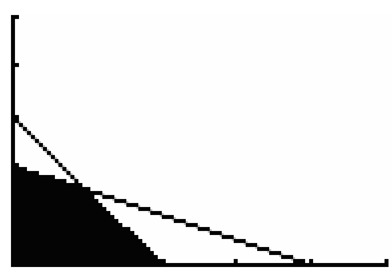
True statement

$y\ge 0$

$1\ge 0$

True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. Considering the graph of the system, the corners of the region are $(0,0),(0,2),(1,1.5)$, and $(2,0)$.



[0, 5] by [0, 5]

Recall that all the lines are solid.

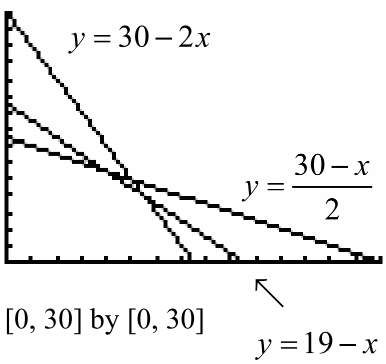
5. Rewriting the system yields

$$2x + y \le 30$$
$$y \le 30 - 2x$$
and
$$x + y \le 19$$
$$y \le 19 - x$$
and
$$x + 2y \le 30$$
$$2y \le 30 - x$$
$$y \le \frac{30 - x}{2}$$

The new system is

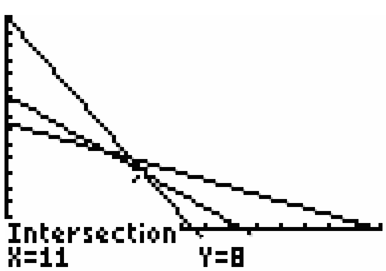
$$\begin{cases} y \le 30 - 2x \\ y \le 19 - x \\ y \le \dfrac{30 - x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is

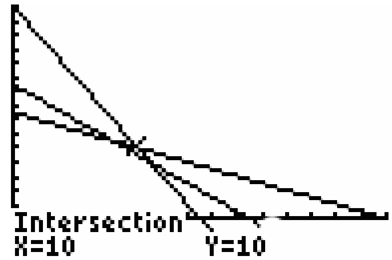


[0, 30] by [0, 30]

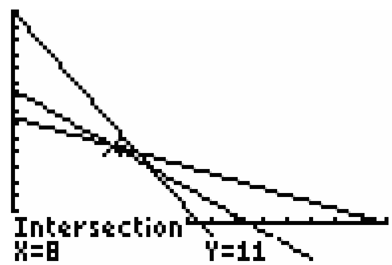
The intersection point between $y = 30 - 2x$ and $y = 19 - x$ is $(11, 8)$.



The intersection point between $y = 30 - 2x$ and $y = \frac{30 - x}{2}$ is $(10, 10)$.



The intersection point between $y = 19 - x$ and $y = \frac{30 - x}{2}$ is $(8, 11)$.



Note that all the lines are solid because there is an “equal to” as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(1, 1)$.

$$2x + y \le 30$$
$$2(1) + (1) \le 30$$
$$3 \le 30$$
True statement
$$x + y \le 19$$
$$(1) + (1) \le 19$$
$$2 \le 19$$
True statement

$x + 2y \le 30$

$(1) + 2(1) \le 30$

$3 \le 30$

True statement

$x \ge 0$

$1 \ge 0$

True statement

$y \ge 0$

$1 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The graph of the solution is

[0, 30] by [0, 30]

Recall that all lines are solid.

One of the corner points is the intersection point between $y = 30 - 2x$ and $y = 19 - x$, which is $(11,8)$. A second corner point is the intersection point between $y = 19 - x$ and $y = \frac{30 - x}{2}$, which is $(8,11)$. A third corner point occurs where $y = 30 - 2x$ crosses the x-axis. A fourth corner point occurs where $y = \frac{30 - x}{2}$ crosses the y-axis.

To find the x-intercept, let $y = 0$.

$0 = 30 - 2x$

$2x = 30$

$x = 15$

$(15,0)$

To find the y-intercept, let $x = 0$.

$y = \frac{30 - x}{2}$

$y = \frac{30 - (0)}{2}$

$y = 15$

$(0,15)$

Therefore, the corner points of the solution region are $(8,11),(11,8),(15,0),(0,15)$, and $(0,0)$.

6. Rewriting the system:

$2x + y \le 10$

$y \le 10 - 2x$

and

$x + 2y \le 11$

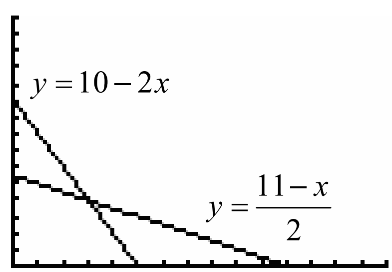
$2y \le 11 - x$

$y \le \frac{11 - x}{2}$

The new system is

$$\begin{cases} y \le 10 - 2x \\ y \le \frac{11 - x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

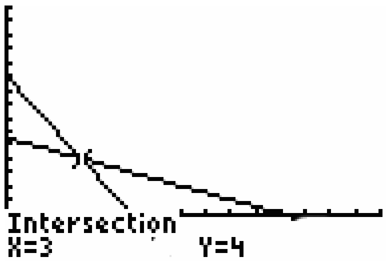
The graph of the system is



[0, 15] by [0, 15]

Note that all the lines are solid.

The intersection point between the two lines is $(3,4)$.



To determine the solution region, pick a point to test. Pick $(1,1)$.

$y \le 10-2x$

$1 \le 10-2(1)$

$1 \le 8$

True statement

$y \le \dfrac{11-x}{2}$

$y \le \dfrac{11-(1)}{2}$

$1 \le 5$

True statement

$x \ge 0$

$1 \ge 0$

True statement

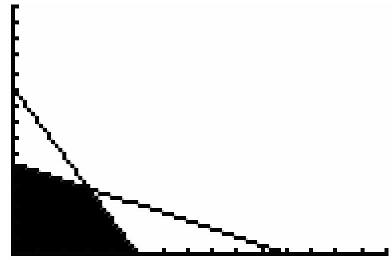
$y \ge 0$

$1 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. Considering the graph of the system, the corners of the region are

$(0,0), \left(0,\dfrac{11}{2}\right), (3,4)$, and $(5,0)$.



[0, 15] by [0, 15]

Recall that all the lines are solid.

7. Rewriting the system:

$$15x - x^2 - y \ge 0$$
$$-y \ge x^2 - 15x$$
$$y \le 15x - x^2$$

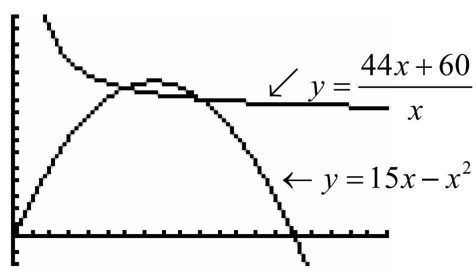
and

$$y - \frac{44x+60}{x} \ge 0$$
$$y \le \frac{44x+60}{x}$$

The new system is

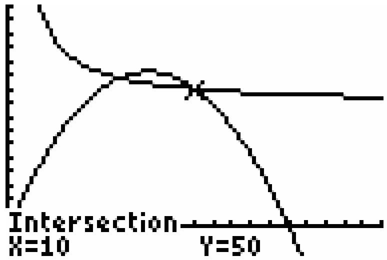
$$\begin{cases} y \le 15x - x^2 \\ y \ge \dfrac{44x+60}{x} \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is



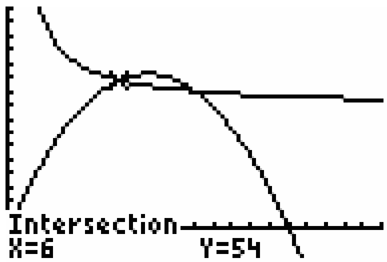
[0, 20] by [–10, 80]

Note that all the lines are solid. Determining the intersection points between the two functions graphically yields



[0, 20] by [–10, 80]

and



[0, 20] by [–10, 80]

The intersection points are $(10,50)$ and $(6,54)$.

To determine the solution region, pick a point to test. Pick $(8,54)$.

$y \le 15x - x^2$

$54 \le 15(8) - (8)^2$

$54 \le 56$

True statement

$y \ge \dfrac{44x + 60}{x}$

$54 \ge \dfrac{44(8) + 60}{(8)}$

$54 \ge 51.5$

True statement

$x \ge 0$

$8 \ge 0$

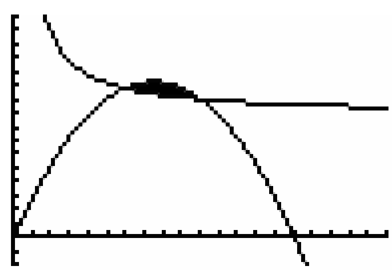
True statement

$y \ge 0$

$54 \ge 0$

True statement

Since all the inequalities that form the system are true at the point $(8,54)$, the region that contains $(8,54)$ is the solution region. Considering the graph of the system, the corners of the region are $(10,50)$ and $(6,54)$.



[0, 20] by [–10, 80]

Recall that all the lines are solid.

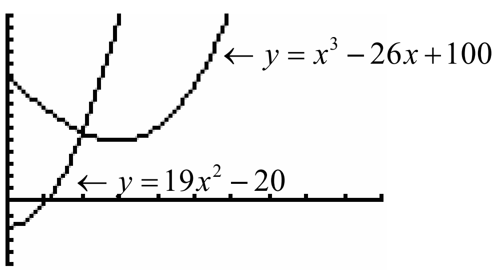
8. Rewriting the system yields

$$x^3 - 26x + 100 - y \geq 0$$
$$-y \geq -x^3 + 26x - 100$$
$$y \leq x^3 - 26x + 100$$
and
$$19x^2 - 20 - y \leq 0$$
$$-y \leq 20 - 19x^2$$
$$y \geq 19x^2 - 20$$

The new system is

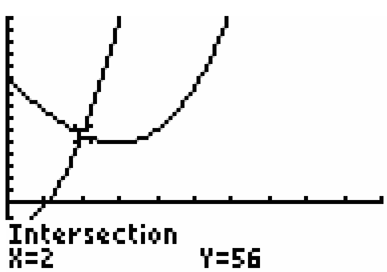
$$\begin{cases} y \leq x^3 - 26x + 100 \\ y \geq 19x^2 - 20 \\ x \geq 0, y \geq 0 \end{cases}$$

The graph of the system is



[0, 10] by [–50, 150]

Note that all the lines are solid.
Determining the intersection point between the two functions graphically yields



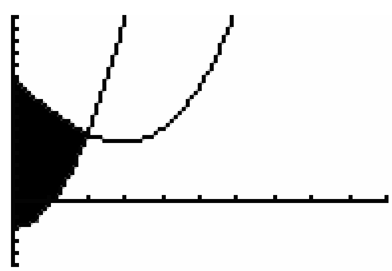
[0, 10] by [–50, 150]

The intersection point is $(2,56)$.

To determine the solution region, pick a point to test. Pick $(1,1)$.

$$y \leq x^3 - 26x + 100$$
$$1 \leq (1)^3 - 26(1) + 100$$
$$1 \leq 75$$
True statement
$$y \geq 19x^2 - 20$$
$$1 \geq 19(1)^2 - 20$$
$$1 \geq -1$$
True statement
$$x \geq 0$$
$$1 \geq 0$$
True statement
$$y \geq 0$$
$$1 \geq 0$$
True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. Considering the graph of the system, the corners of the region are $(2,56), (0,100)$, and $(0,-20)$.

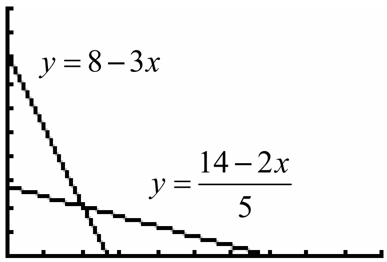


[0, 10] by [–50, 150]

Recall that all the lines are solid.

9. Rewriting the system of constraints and graphing the system yields

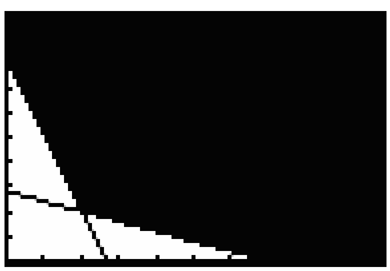
$$\begin{cases} y \geq 8 - 3x \\ y \geq \dfrac{14 - 2x}{5} \\ x \geq 0, y \geq 0 \end{cases}$$



[0, 10] by [0, 10]

To determine the solution region, pick a point to test. Pick $(2,3)$. When substituted into the inequalities that form the system, the point $(2,3)$ creates true statements in all cases.

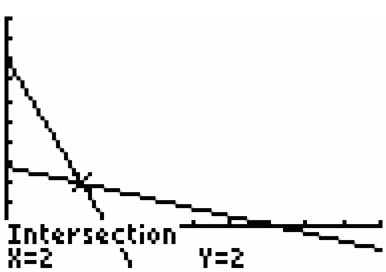
Since all the inequalities that form the system are true at the point $(2,3)$, the region that contains $(2,3)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 10] by [0, 10]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,8),(7,0),$ and $(2,2)$.



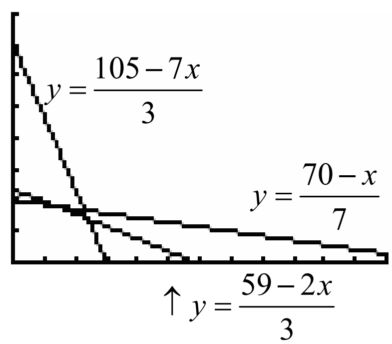
Testing the corner points of the feasible region to minimize the objective function, $g = 3x + 4y$, yields

At $(0,8)$: $g = 3(0) + 4(8) = 32$
At $(7,0)$: $g = 3(7) + 4(0) = 21$
At $(2,2)$: $g = 3(2) + 4(2) = 14$

The minimum value is 14 occurring at $(2,2)$.

10. Rewriting the system of constraints and graphing the system yields

$$\begin{cases} y \le \dfrac{105 - 7x}{3} \\ y \le \dfrac{59 - 2x}{5} \\ y \le \dfrac{70 - x}{7} \\ x \ge 0, y \ge 0 \end{cases}$$

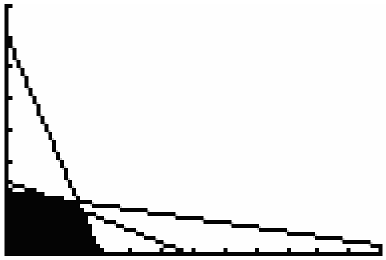


[0, 60] by [0, 40]

To determine the solution region, pick a point to test. Pick $(2,3)$. When substituted into the inequalities that form the system, the point $(2,3)$ creates true statements in all cases.

Since all the inequalities that form the system are true at the point $(2,3)$, the region that contains $(2,3)$ is the solution region.

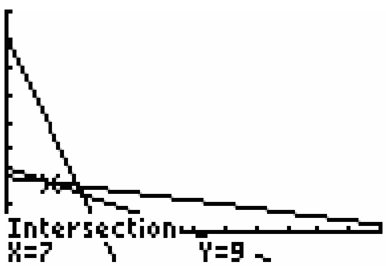
The solution represents the feasible region. The graph of the feasible region is

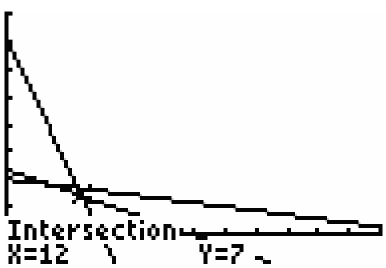


[0, 60] by [0, 40]

Note that since all the inequalities contain an “equal to”, all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(0,10),(15,0),(12,7)$,and $(7,9)$.





Testing the corner points of the feasible region to maximize the objective function, $f = 7x + 12y$, yields

At $(0,0)$: $f = 7(0) + 12(0) = 0$

At $(0,10)$: $f = 7(0) + 12(10) = 120$

At $(15,0)$: $f = 7(15) + 12(0) = 105$

At $(12,7)$: $f = 7(12) + 12(7) = 168$

At $(7,9)$: $f = 7(7) + 12(9) = 157$

The maximum value is 168 occurring at $(12,7)$.

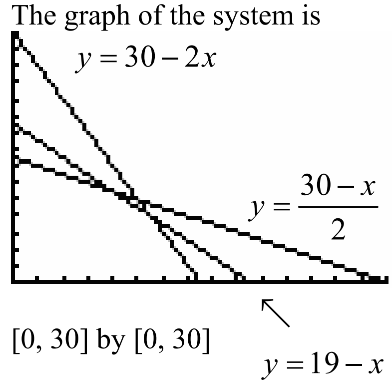
11. Rewriting the system:

$$2x + y \le 30$$
$$y \le 30 - 2x$$
and
$$x + y \le 19$$
$$y \le 19 - x$$
and
$$x + 2y \le 30$$
$$2y \le 30 - x$$
$$y \le \frac{30 - x}{2}$$

The new system is

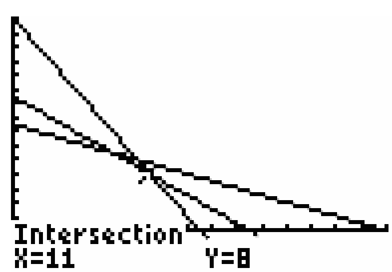
$$\begin{cases} y \le 30 - 2x \\ y \le 19 - x \\ y \le \dfrac{30 - x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

The graph of the system is

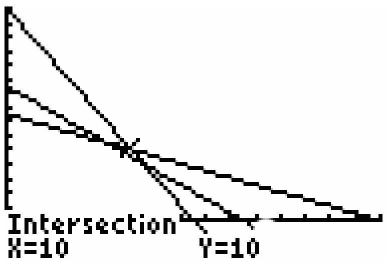


[0, 30] by [0, 30]

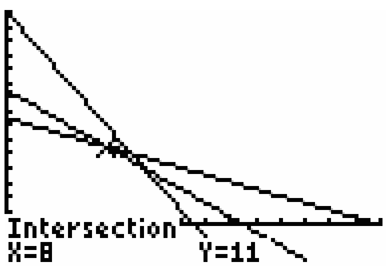
The intersection point between $y = 30 - 2x$ and $y = 19 - x$ is $(11,8)$.



The intersection point between $y = 30 - 2x$ and $y = \frac{30 - x}{2}$ is $(10,10)$.



The intersection point between $y = 19 - x$ and $y = \frac{30 - x}{2}$ is $(8,11)$.



Note that all the lines are solid because there is an "equal to" as part of all the inequalities.

To determine the solution region, pick a point to test. Pick $(1,1)$.

$2x + y \le 30$
$2(1) + (1) \le 30$
$3 \le 30$
True statement
$x + y \le 19$
$(1) + (1) \le 19$
$2 \le 19$
True statement
$x + 2y \le 30$
$(1) + 2(1) \le 30$
$3 \le 30$
True statement
$x \ge 0$
$1 \ge 0$
True statement
$y \ge 0$
$1 \ge 0$
True statement

Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The graph of the solution is

[0, 30] by [0, 30]

Recall that all lines are solid.

One of the corner points is the intersection point between $y = 30 - 2x$ and $y = 19 - x$, which is $(11,8)$. A second corner point is the intersection point between $y = 19 - x$ and $y = \frac{30 - x}{2}$, which is $(8,11)$. A third corner point occurs where $y = 30 - 2x$ crosses the x-axis. A fourth corner point occurs where $y = \frac{30 - x}{2}$ crosses the y-axis.

To find the x-intercept, let $y = 0$.

$0 = 30 - 2x$
$2x = 30$
$x = 15$
$(15, 0)$

To find the y-intercept, let $x = 0$.

$y = \frac{30 - x}{2}$
$y = \frac{30 - (0)}{2}$
$y = 15$
$(0, 15)$

Therefore, the corner points of the solution region are $(8,11), (11,8), (15,0), (0,15)$, and $(0,0)$.

Testing the corner points of the feasible region to maximize the objective function, $f = 3x + 5y$, yields

At $(0,0)$: $f = 3(0) + 5(0) = 0$
At $(0,15)$: $f = 3(0) + 5(15) = 75$
At $(15,0)$: $f = 3(15) + 5(0) = 45$
At $(8,11)$: $f = 3(8) + 5(11) = 79$
At $(11,8)$: $f = 3(11) + 5(8) = 73$

The maximum value is 79 occurring at $(8,11)$.

12. Geometric, with a common ratio of 6.
$\frac{\frac{2}{3}}{\frac{1}{9}} = 6, \frac{4}{\frac{2}{3}} = 6, \frac{24}{4} = 6$, etc.

13. Arithmetic, with a common difference of 12.
$16 - 4 = 12, 28 - 16 = 12$, etc.

14. Geometric, with a common ratio of $-\frac{3}{4}$.

$\frac{-12}{16} = -\frac{3}{4}, \frac{9}{-12} = -\frac{3}{4}$, etc.

15. $a_n = a_1 r^{n-1}$
$a_n = 64r^{n-1}$
$a_8 = 64r^{8-1} = \frac{1}{2}$
$2(64r^7) = 2\left(\frac{1}{2}\right)$
$128r^7 = 1$
$r^7 = \frac{1}{128}$
$r = \sqrt[7]{\frac{1}{128}} = \frac{1}{2}$
$a_n = 64\left(\frac{1}{2}\right)^{n-1}$
$a_5 = 64\left(\frac{1}{2}\right)^{5-1} = 64\left(\frac{1}{2}\right)^4 = 4$

16. Note that the common ratio is 3 and that the first term is $\frac{1}{9}$.
$a_n = a_1 r^{n-1}$
$a_n = \frac{1}{9}(3)^{n-1}$
$a_6 = \frac{1}{9}(3)^{6-1} = \frac{1}{9}(3)^5 = 27$

17. $s_n = \frac{a_1(1 - r^n)}{1 - r}$
$s_{10} = \frac{5(1 - (-2)^{10})}{1 - (-2)}$
$s_{10} = \frac{5(-1023)}{3} = -1705$

18. Note that the sequence is arithmetic with a common difference of 3.

$a_n = a_1 + (n-1)d$

$a_n = 3 + (n-1)(3)$

$a_n = 3n$

$a_{12} = 3(12) = 36$

$s_n = \dfrac{n(a_1 + a_n)}{2}$

$s_{12} = \dfrac{12(a_1 + a_{12})}{2} = \dfrac{12(3+36)}{2} = 234$

19. $a_1 = \left(\dfrac{4}{5}\right)^1 = \dfrac{4}{5}$

$r = \dfrac{4}{5}$

$S = \dfrac{a_1}{1-r} = \dfrac{\frac{4}{5}}{1-\frac{4}{5}} = \dfrac{\frac{4}{5}}{\frac{1}{5}} = 4$

20. $(x^3 + 2x + 3)(2x) + (x^2 - 5)(3x^2 + 2)$

$= 2x^4 + 4x^2 + 6x + (3x^4 + 2x^2 - 15x^2 - 10)$

$= 5x^4 - 9x^2 + 6x - 10$

21. $x^3 - 8x^2 - 9x = 0$

$x(x^2 - 8x - 9) = 0$

$x(x-9)(x+1) = 0$

$x = 0,\ x - 9 = 0,\ x + 1 = 0$

$x = 0,\ x = 9,\ x = -1$

22. $y = x + x^{-2} - \dfrac{2}{x^{\frac{3}{2}}} + \dfrac{3}{x^{\frac{1}{3}}}$

$y = x + x^{-2} - 2x^{-\frac{3}{2}} + 3x^{-\frac{1}{3}}$

23. $f'(x) = \dfrac{8}{x^3} + \dfrac{5}{x} + 4x$

24. $f'(x) = 0$

$2x^3 + x^2 - x = 0$

$x(2x^2 + x - 1) = 0$

$x(2x-1)(x+1) = 0$

$x = 0,\ 2x - 1 = 0,\ x + 1 = 0$

$x = 0,\ x = \dfrac{1}{2},\ x = -1$

Chapter 6 Review

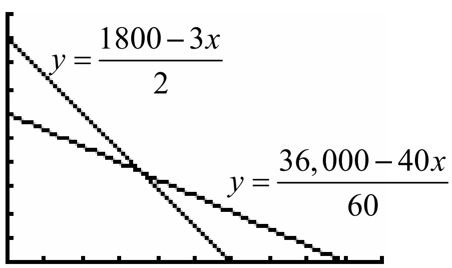
25. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of Deluxe model DVD players, and let y represent the number of Superior model DVD players.

$$\begin{cases} 3x + 2y \le 1800 \\ 40x + 60y \le 36{,}000 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le \dfrac{1800 - 3x}{2} \\ y \le \dfrac{36{,}000 - 40x}{60} \\ x \ge 0, y \ge 0 \end{cases}$$

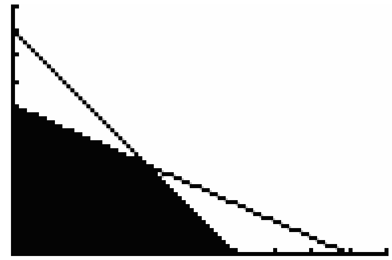
Graphing the system yields



[0, 1000] by [0, 1000]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

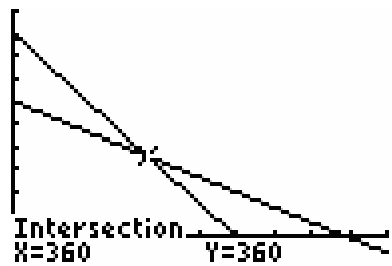
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 1000] by [0, 1000]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0), (0,600), (600,0)$, and $(360,360)$.



Testing the corner points of the feasible region to maximize the objective function, $f = 30x + 40y$, yields

At $(0,0)$:
$f = 30(0) + 40(0) = \$0$
At $(600,0)$:
$f = 30(600) + 40(0) = \$18{,}000$
At $(0,600)$:
$f = 30(0) + 40(600) = \$24{,}000$
At $(360,360)$:
$f = 30(360) + 40(360) = \$25{,}200$

The maximum value is \$25,200 occurring at $(360,360)$. To produce a maximum profit the company needs to produce 360 Deluxe models and 360 Superior models.

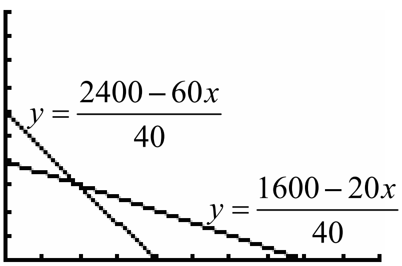
26. a. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of days of production at the Pottstown plant, and let y represent the number of days of production at the Ethica plant.

$$\begin{cases} 20x + 40y \ge 1600 \\ 60x + 40y \ge 2400 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \ge \dfrac{1600 - 20x}{40} \\ y \ge \dfrac{2400 - 60x}{40} \\ x \ge 0, y \ge 0 \end{cases}$$

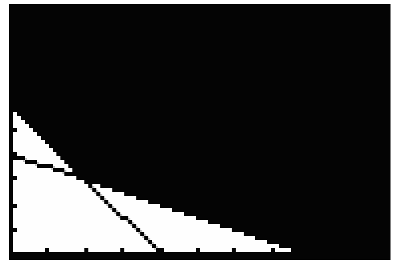
Graphing the system yields



[0, 100] by [0, 100]

To determine the solution region, pick a point to test. Pick $(20,40)$. When substituted into the inequalities that form the system, the point $(20,40)$ creates true statements in all cases.

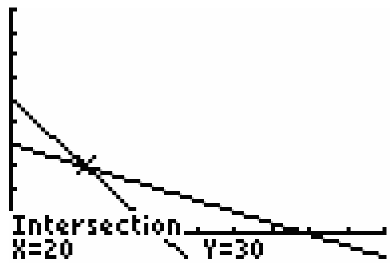
Since all the inequalities that form the system are true at the point $(20,40)$, the region that contains $(20,40)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 100] by [0, 100]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(80,0)$, $(0,60)$, and $(20,30)$.



Testing the corner points of the feasible region to minimize the objective function, $g = 20{,}000x + 24{,}000y$, yields

At $(80,0)$:
$$g = 20{,}000(80) + 24{,}000(0) = \$1{,}600{,}000$$
At $(0,60)$:
$$g = 20{,}000(0) + 24{,}000(60) = \$1{,}440{,}000$$
At $(20,30)$:
$$g = 20{,}000(20) + 24{,}000(30) = \$1{,}120{,}000$$

The minimum cost is \$1,120,000 occurring at $(20,30)$. Therefore, operating the Pottstown plant for 20 days and the Ethica plant for 30 days produces the minimum manufacturing cost of \$1,120,000.

b. Refer to part a). The minimum cost is \$1,200,000.

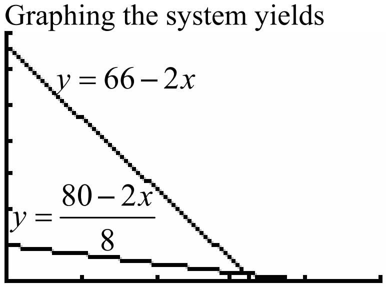
27. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of pounds of Feed A, and let y represent the number of pounds of Feed B.

$$\begin{cases} 2x+8y\ge 80 \\ 4x+2y\ge 132 \\ x\ge 0, y\ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y\ge \dfrac{80-2x}{8} \\ y\ge 66-2x \\ x\ge 0, y\ge 0 \end{cases}$$

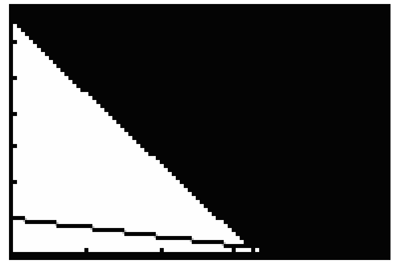
Graphing the system yields



[0, 50] by [0, 70]

To determine the solution region, pick a point to test. Pick $(25,50)$. When substituted into the inequalities that form the system, the point $(25,50)$ creates true statements in all cases.

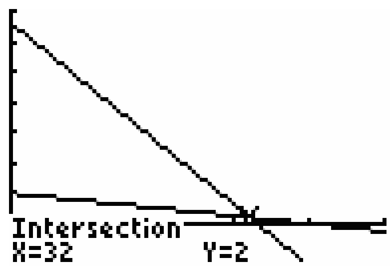
Since all the inequalities that form the system are true at the point $(25,50)$, the region that contains $(25,50)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 50] by [0, 70]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,66)$, $(32,2)$ and $(40,0)$.



Testing the corner points of the feasible region to minimize the objective function, $g=1.40x+1.60y$, yields

At $(0,66)$:
$g=1.40(0)+1.60(66)=\$105.60$
At $(32,2)$:
$g=1.40(32)+1.60(2)=\$48$
At $(40,0)$:
$g=1.40(40)+1.60(0)=\$56$

The minimum value is \$48 occurring at $(32,2)$. To minimize the cost of the feed, the laboratory needs to use 32 pounds of Feed A and 2 pounds of Feed B.

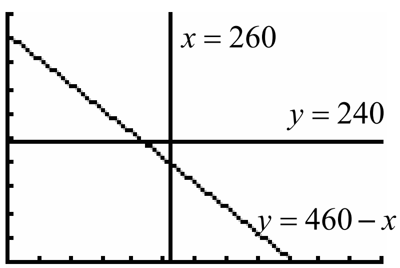
28. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the number of leaf blowers, and let y represent the number of weed wackers.

$$\begin{cases} x + y \leq 460 \\ x \leq 260 \\ y \leq 240 \\ x \geq 0, y \geq 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \leq 460 - x \\ x \leq 260 \\ y \leq 240 \\ x \geq 0, y \geq 0 \end{cases}$$

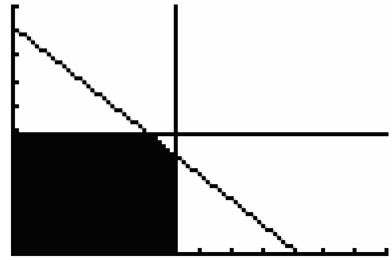
Graphing the system yields



[0, 600] by [0, 500]

To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

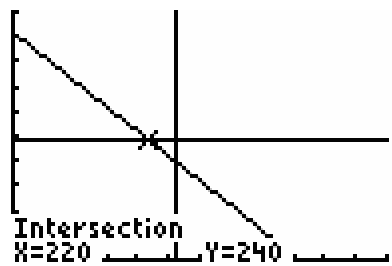
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

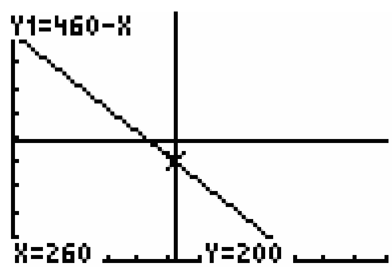


[0, 600] by [0, 500]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(0,240),(260,0),(220,240)$, and $(260,200)$.





Testing the corner points of the feasible region to maximize the objective function, $f = 5x + 10y$, yields

At $(0,0)$:

$f = 5(0) + 10(0) = \$0$

At $(0,240)$:

$f = 5(0) + 10(240) = \$2400$

At $(260,0)$:

$f = 5(260) + 10(0) = \$1300$

At $(220,240)$:

$f = 5(220) + 10(240) = \$3500$

At $(260,200)$:

$f = 5(260) + 10(200) = \$3300$

The maximum value is \$3500 occurring at $(220,240)$. To produce a maximum profit the company needs to produce 220 leaf blowers and 240 weed wackers.

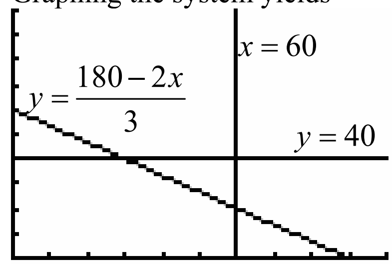
29. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let x represent the two-bedroom apartments rented, and let y represent the number of three bedroom apartments rented.

$$\begin{cases} 2x + 3y \le 180 \\ x \le 60 \\ y \le 40 \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le \dfrac{180 - 2x}{3} \\ x \le 60 \\ y \le 40 \\ x \ge 0, y \ge 0 \end{cases}$$

Graphing the system yields



[0, 100] by [0, 100]

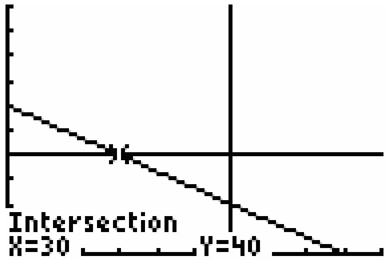
To determine the solution region, pick a point to test. Pick $(1,1)$. When substituted into the inequalities that form the system, the point $(1,1)$ creates true statements in all cases.

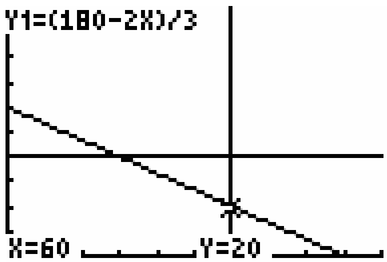
Since all the inequalities that form the system are true at the point $(1,1)$, the region that contains $(1,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is

[0, 100] by [0, 100]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0), (0,40), (60,0), (30,40)$, and $(60,20)$.





Testing the corner points of the feasible region to maximize the objective function, $f = 800x + 1150y$, yields

At $(0,0)$:
$f = 800(0) + 1150(0) = \$0$
At $(0,40)$:
$f = 800(0) + 1150(40) = \$46,000$
At $(60,0)$:
$f = 800(60) + 1150(0) = \$48,000$
At $(30,40)$:
$f = 800(30) + 1150(40) = \$70,000$
At $(60,20)$:
$f = 800(60) + 1150(20) = \$71,000$

The maximum value is \$71,000 occurring at $(60,20)$. To produce a maximum profit the woman needs to rent sixty two-bedroom apartments and twenty 3-bedroom apartments.

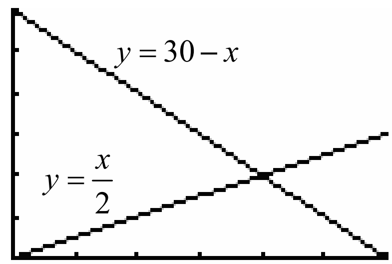
30. Determine the feasible region by solving and graphing the system of inequalities that represent the constraints. Let *x* represent the amount of auto loans in millions of dollars, and let *y* represent the amount of home equity loans in millions of dollars.

$$\begin{cases} x + y \le 30 \\ x \ge 2y \\ x \ge 0, y \ge 0 \end{cases}$$

Rewriting yields

$$\begin{cases} y \le 30 - x \\ y \le \dfrac{x}{2} \\ x \ge 0, y \ge 0 \end{cases}$$

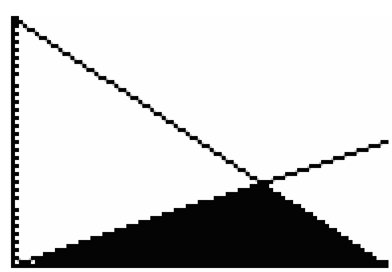
Graphing the system yields



[0, 30] by [0, 30]

To determine the solution region, pick a point to test. Pick $(15,1)$. When substituted into the inequalities that form the system, the point $(15,1)$ creates true statements in all cases.

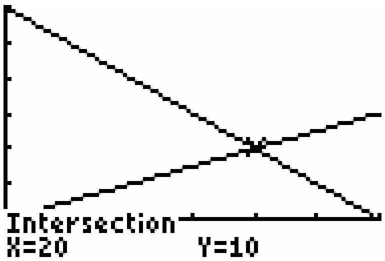
Since all the inequalities that form the system are true at the point $(15,1)$, the region that contains $(15,1)$ is the solution region. The solution represents the feasible region. The graph of the feasible region is



[0, 30] by [0, 30]

Note that since all the inequalities contain an "equal to", all the boundary lines are solid.

The corner points of the feasible region are $(0,0),(30,0)$, and $(20,10)$.



Testing the corner points of the feasible region to maximize the objective function, $f=0.08x+0.07y$, yields

At $(0,0)$: $f=0.08(0)+0.07(0)=\$0$
At $(30,0)$: $f=0.08(30)+0.07(0)=\$2.4$
At $(20,10)$: $f=0.08(20)+0.07(10)=\$2.3$

The maximum value is \$2.4 million occurring at $(30,0)$. To produce a maximum profit the finance company should make \$30 million in auto loans and no home equity loans.

31. a. Job 1: $a_n=a_1+(n-1)d$

$$a_n=20,000+(n-1)(1000)$$
$$a_n=1000n+19,000$$
$$a_5=1000(5)+19,000=24,000$$

Job 2: $a_n=a_1+(n-1)d$

$$a_n=18,000+(n-1)(1600)$$
$$a_n=1600n+16,400$$
$$a_5=1600(5)+16,400=24,400$$

During the fifth year of employment, job two produces a higher salary.

b. Job 1: $s_n=\dfrac{n(a_1+a_n)}{2}$

$$s_5=\frac{5(a_1+a_5)}{2}$$
$$s_5=\frac{5(20,000+24,000)}{2}$$
$$s_5=110,000$$

Job 2: $s_n=\dfrac{n(a_1+a_n)}{2}$

$$s_5=\frac{5(a_1+a_5)}{2}$$
$$s_5=\frac{5(18,000+24,400)}{2}$$
$$s_5=106,000$$

Over the first five years of employment, the total salary earned from job one is higher than the total salary earned for job two.

32. Note that the given series is geometric with a common ratio of 0.6.

$$s_n=\frac{a_1(1-r^n)}{1-r}$$
$$s_{21}=\frac{400\left(1-(0.6)^{21}\right)}{1-(0.6)}$$
$$s_{21}=999.978063$$

The level of the drug in the bloodstream after 21 doses over 21 days is approximately 999.98 mg.

33. a. Note that the question is modeled by a geometric series with a common ratio of 2 and an initial value of 1.

$$a_1=1, r=2$$
$$a_n=a_1r^{n-1}$$
$$a_n=1(2)^{n-1}$$
$$a_{64}=1(2)^{64-1}=(2)^{63}\approx 9.22337\times 10^{18}$$

b. $a_1 = 1,\ r = 2$

$$s_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

$$s_{64} = \frac{1\left(1 - (2)^{64}\right)}{1 - (2)}$$

$$s_{64} \approx 1.84467 \times 10^{19}$$

34. $a_1 = 20{,}000,\ r = 1.06$

$a_n = a_1 r^{n-1}$

Note that after 5 years, $n = 6$.

$a_6 = 20{,}000(1.06)^{6-1}$

$a_6 = 20{,}000(1.06)^5$

$a_6 = 26{,}764.51$

After earning interest for five years, the value of the investment is $26,764.51.

35. $$s_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

$$s_{60} = \frac{300\left(1 - \left(1 + \frac{0.12}{12}\right)^{60}\right)}{1 - \left(1 + \frac{0.12}{12}\right)}$$

$$s_{60} = \frac{300\left(1 - (1.01)^{60}\right)}{1 - (1.01)}$$

$$s_{60} \approx 24{,}500.90$$

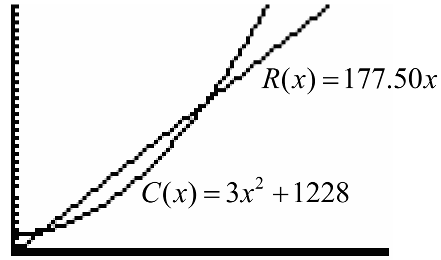
The future value of the annuity is $24,500.90.

36. Recall that to earn a profit, revenue must exceed cost.

$R(x) > C(x)$

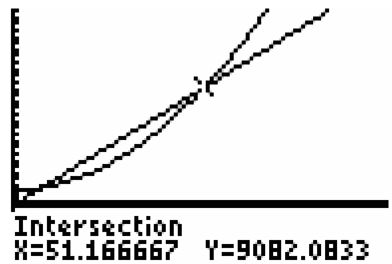
$177.50x > 3x^2 + 1228$

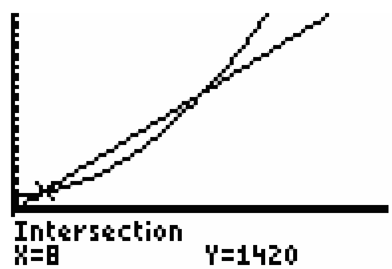
Graphing the functions yields



[0, 100] by [0, 15,000]

The two intersection points are approximately $(51.18, 9082.08)$ and $(8, 1420)$





If $8 < x < 52$, $R(x) > C(x)$ and profit is achieved.

Chapter 6 Extended Application

Year	Period (months)	Salary, Plan I	Salary, Plan II
1	0–6 6–12	20,000 20,000	20,000 20,300
2	12–18 18–24	20,500 20,500	20,600 20,900
3	24–30 30–36	21,000 21,000	21,200 21,500
Total for 3 years		\$123,000	\$124,500

1. The raises in Plan I total \$3000.

2. The raises in Plan II total \$4,500.

3. Plan II is clearly better for the employee. It yields an extra \$1500 over the 3-year period.

4. See complete table above.

Year	Period (months)	Salary, Plan I	Salary, Plan II
Total for 3 years		\$123,000	\$124,500
Year 3	36–42 42–48	21,500 21,500	21,800 22,100
Total for 4 years		\$166,000	\$168,400

Plan II continues to be better. It yields \$8,400 in raises, whereas Plan I yields \$6000 in raises.

5. For at least the first four years, the school board will not save money by awarding \$300 raises every six months instead of \$1000 annual raises. The school board did not take into consideration that the semi-annual raises would be awarded more frequently and therefore would compound faster than annual raises.

6. The difference in the raises over four years is $8400 - 6000 = \$2400$.
200 teachers $\times$ \$2400 per teacher
$= \$480,000$

The school district will spend an additional \$480,000 over the first four years.