Chapter 3

Quadratic Functions and Equations; Inequalities

Exercise Set 3.1

1. $\sqrt{-3} = \sqrt{-1 \cdot 3} = \sqrt{-1} \cdot \sqrt{3} = i\sqrt{3}$, or $\sqrt{3}i$

2. $\sqrt{-21} = \sqrt{-1 \cdot 21} = i\sqrt{21}$, or $\sqrt{21}i$

3. $\sqrt{-25} = \sqrt{-1 \cdot 25} = \sqrt{-1} \cdot \sqrt{25} = i \cdot 5 = 5i$

4. $\sqrt{-100} = \sqrt{-1 \cdot 100} = i \cdot 10 = 10i$

5. $-\sqrt{-33} = -\sqrt{-1 \cdot 33} = -\sqrt{-1} \cdot \sqrt{33} = -i\sqrt{33}$, or $-\sqrt{33}i$

6. $-\sqrt{-59} = -\sqrt{-1 \cdot 59} = -i\sqrt{59}$, or $-\sqrt{59}i$

7. $-\sqrt{-81} = -\sqrt{-1 \cdot 81} = -\sqrt{-1} \cdot \sqrt{81} = -i \cdot 9 = -9i$

8. $-\sqrt{-9} = -\sqrt{-1 \cdot 9} = -\sqrt{-1} \cdot \sqrt{9} = -i \cdot 3 = -3i$

9. $\sqrt{-98} = \sqrt{-1 \cdot 98} = \sqrt{-1} \cdot \sqrt{98} = i\sqrt{49 \cdot 2} =$
$i \cdot 7\sqrt{2} = 7i\sqrt{2}$, or $7\sqrt{2}i$

10. $\sqrt{-28} = \sqrt{-1 \cdot 28} = i\sqrt{28} = i\sqrt{4 \cdot 7} = 2i\sqrt{7}$, or $2\sqrt{7}i$

11. $(-5 + 3i) + (7 + 8i)$
$= (-5 + 7) + (3i + 8i)$ Collecting the real parts and the imaginary parts
$= 2 + (3 + 8)i$
$= 2 + 11i$

12. $(-6 - 5i) + (9 + 2i) = (-6 + 9) + (-5i + 2i) = 3 - 3i$

13. $(4 - 9i) + (1 - 3i)$
$= (4 + 1) + (-9i - 3i)$ Collecting the real parts and the imaginary parts
$= 5 + (-9 - 3)i$
$= 5 - 12i$

14. $(7 - 2i) + (4 - 5i) = (7 + 4) + (-2i - 5i) = 11 - 7i$

15. $(12 + 3i) + (-8 + 5i)$
$= (12 - 8) + (3i + 5i)$
$= 4 + 8i$

16. $(-11 + 4i) + (6 + 8i) = (-11 + 6) + (4i + 8i) = -5 + 12i$

17. $(-1 - i) + (-3 - i)$
$= (-1 - 3) + (-i - i)$
$= -4 - 2i$

18. $(-5 - i) + (6 + 2i) = (-5 + 6) + (-i + 2i) = 1 + i$

19. $(3 + \sqrt{-16}) + (2 + \sqrt{-25}) = (3 + 4i) + (2 + 5i)$
$= (3 + 2) + (4i + 5i)$
$= 5 + 9i$

20. $(7 - \sqrt{-36}) + (2 + \sqrt{-9}) = (7 - 6i) + (2 + 3i) =$
$(7 + 2) + (-6i + 3i) = 9 - 3i$

21. $(10 + 7i) - (5 + 3i)$
$= (10 - 5) + (7i - 3i)$ The 5 and the $3i$ are both being subtracted.
$= 5 + 4i$

22. $(-3 - 4i) - (8 - i) = (-3 - 8) + [-4i - (-i)] =$
$-11 - 3i$

23. $(13 + 9i) - (8 + 2i)$
$= (13 - 8) + (9i - 2i)$ The 8 and the $2i$ are both being subtracted.
$= 5 + 7i$

24. $(-7 + 12i) - (3 - 6i) = (-7 - 3) + [12i - (-6i)] =$
$-10 + 18i$

25. $(6 - 4i) - (-5 + i)$
$= [6 - (-5)] + (-4i - i)$
$= (6 + 5) + (-4i - i)$
$= 11 - 5i$

26. $(8 - 3i) - (9 - i) = (8 - 9) + [-3i - (-i)] = -1 - 2i$

27. $(-5 + 2i) - (-4 - 3i)$
$= [-5 - (-4)] + [2i - (-3i)]$
$= (-5 + 4) + (2i + 3i)$
$= -1 + 5i$

28. $(-6 + 7i) - (-5 - 2i) = [-6 - (-5)] + [7i - (-2i)] =$
$-1 + 9i$

29. $(4 - 9i) - (2 + 3i)$
$= (4 - 2) + (-9i - 3i)$
$= 2 - 12i$

30. $(10 - 4i) - (8 + 2i) = (10 - 8) + (-4i - 2i) =$
$2 - 6i$

31. $\sqrt{-4} \cdot \sqrt{-36} = 2i \cdot 6i = 12i^2 = 12(-1) = -12$

32. $\sqrt{-49} \cdot \sqrt{-9} = 7i \cdot 3i = 21i^2 = -21$

33. $\sqrt{-81} \cdot \sqrt{-25} = 9i \cdot 5i = 45i^2 = 45(-1) = -45$

34. $\sqrt{-16} \cdot \sqrt{-100} = 4i \cdot 10i = 40i^2 = -40$

35. $7i(2-5i)$
$= 14i - 35i^2$ Using the distributive law
$= 14i + 35$ $i^2 = -1$
$= 35 + 14i$ Writing in the form $a + bi$

36. $3i(6+4i) = 18i + 12i^2 = 18i - 12 = -12 + 18i$

37. $-2i(-8+3i)$
$= 16i - 6i^2$ Using the distributive law
$= 16i + 6$ $i^2 = -1$
$= 6 + 16i$ Writing in the form $a + bi$

38. $-6i(-5+i) = 30i - 6i^2 = 30i + 6 = 6 + 30i$

39. $(1+3i)(1-4i)$
$= 1 - 4i + 3i - 12i^2$ Using FOIL
$= 1 - 4i + 3i - 12(-1)$ $i^2 = -1$
$= 1 - i + 12$
$= 13 - i$

40. $(1-2i)(1+3i) = 1 + 3i - 2i - 6i^2 = 1 + i + 6 = 7 + i$

41. $(2+3i)(2+5i)$
$= 4 + 10i + 6i + 15i^2$ Using FOIL
$= 4 + 10i + 6i - 15$ $i^2 = -1$
$= -11 + 16i$

42. $(3-5i)(8-2i) = 24 - 6i - 40i + 10i^2 = 24 - 6i - 40i - 10 = 14 - 46i$

43. $(-4+i)(3-2i)$
$= -12 + 8i + 3i - 2i^2$ Using FOIL
$= -12 + 8i + 3i + 2$ $i^2 = -1$
$= -10 + 11i$

44. $(5-2i)(-1+i) = -5 + 5i + 2i - 2i^2 = -5 + 5i + 2i + 2 = -3 + 7i$

45. $(8-3i)(-2-5i)$
$= -16 - 40i + 6i + 15i^2$
$= -16 - 40i + 6i - 15$ $i^2 = -1$
$= -31 - 34i$

46. $(7-4i)(-3-3i) = -21 - 21i + 12i + 12i^2 = -21 - 21i + 12i - 12 = -33 - 9i$

47. $(3+\sqrt{-16})(2+\sqrt{-25})$
$= (3+4i)(2+5i)$
$= 6 + 15i + 8i + 20i^2$
$= 6 + 15i + 8i - 20$ $i^2 = -1$
$= -14 + 23i$

48. $(7-\sqrt{-16})(2+\sqrt{-9}) = (7-4i)(2+3i) = 14 + 21i - 8i - 12i^2 = 14 + 21i - 8i + 12 = 26 + 13i$

49. $(5-4i)(5+4i) = 5^2 - (4i)^2$
$= 25 - 16i^2$
$= 25 + 16$ $i^2 = -1$
$= 41$

50. $(5+9i)(5-9i) = 25 - 81i^2 = 25 + 81 = 106$

51. $(3+2i)(3-2i)$
$= 9 - 6i + 6i - 4i^2$
$= 9 - 6i + 6i + 4$ $i^2 = -1$
$= 13$

52. $(8+i)(8-i) = 64 - 8i + 8i - i^2 = 64 - 8i + 8i + 1 = 65$

53. $(7-5i)(7+5i)$
$= 49 + 35i - 35i - 25i^2$
$= 49 + 35i - 35i + 25$ $i^2 = -1$
$= 74$

54. $(6-8i)(6+8i) = 36 + 48i - 48i - 64i^2 = 36 + 48i - 48i + 64 = 100$

55. $(4+2i)^2$
$= 16 + 2\cdot 4\cdot 2i + (2i)^2$ Recall $(A+B)^2 = A^2 + 2AB + B^2$
$= 16 + 16i + 4i^2$
$= 16 + 16i - 4$ $i^2 = -1$
$= 12 + 16i$

56. $(5-4i)^2 = 25 - 40i + 16i^2 = 25 - 40i - 16 = 9 - 40i$

57. $(-2+7i)^2$
$= (-2)^2 + 2(-2)(7i) + (7i)^2$ Recall $(A+B)^2 = A^2 + 2AB + B^2$
$= 4 - 28i + 49i^2$
$= 4 - 28i - 49$ $i^2 = -1$
$= -45 - 28i$

58. $(-3+2i)^2 = 9 - 12i + 4i^2 = 9 - 12i - 4 = 5 - 12i$

59. $(1-3i)^2$
$= 1^2 - 2\cdot 1\cdot (3i) + (3i)^2$
$= 1 - 6i + 9i^2$
$= 1 - 6i - 9$ $i^2 = -1$
$= -8 - 6i$

60. $(2-5i)^2 = 4 - 20i + 25i^2 = 4 - 20i - 25 = -21 - 20i$

61. $(-1-i)^2$
$= (-1)^2 - 2(-1)(i) + i^2$
$= 1 + 2i + i^2$
$= 1 + 2i - 1$ $i^2 = -1$
$= 2i$

62. $(-4-2i)^2 = 16+16i+4i^2 = 16+16i-4 = 12+16i$

63.
$$\begin{aligned}&(3+4i)^2\\&= 9+2\cdot 3\cdot 4i+(4i)^2\\&= 9+24i+16i^2\\&= 9+24i-16 \qquad i^2=-1\\&= -7+24i\end{aligned}$$

64. $(6+5i)^2 = 36+60i+25i^2 = 36+60i-25 = 11+60i$

65.
$$\begin{aligned}&\frac{3}{5-11i}\\&= \frac{3}{5-11i}\cdot\frac{5+11i}{5+11i} \qquad 5-11i \text{ is the conjugate of } 5+11i.\\&= \frac{3(5+11i)}{(5-11i)(5+11i)}\\&= \frac{15+33i}{25-121i^2}\\&= \frac{15+33i}{25+121} \qquad i^2=-1\\&= \frac{15+33i}{146}\\&= \frac{15}{146}+\frac{33}{146}i \qquad \text{Writing in the form } a+bi\end{aligned}$$

66.
$$\begin{aligned}\frac{i}{2+i} &= \frac{i}{2+i}\cdot\frac{2-i}{2-i}\\&= \frac{2i-i^2}{4-i^2}\\&= \frac{2i+1}{4+1}\\&= \frac{1}{5}+\frac{2}{5}i\end{aligned}$$

67.
$$\begin{aligned}&\frac{5}{2+3i}\\&= \frac{5}{2+3i}\cdot\frac{2-3i}{2-3i} \qquad 2-3i \text{ is the conjugate of } 2+3i.\\&= \frac{5(2-3i)}{(2+3i)(2-3i)}\\&= \frac{10-15i}{4-9i^2}\\&= \frac{10-15i}{4+9} \qquad i^2=-1\\&= \frac{10-15i}{13}\\&= \frac{10}{13}-\frac{15}{13}i \qquad \text{Writing in the form } a+bi\end{aligned}$$

68.
$$\begin{aligned}\frac{-3}{4-5i} &= \frac{-3}{4-5i}\cdot\frac{4+5i}{4+5i}\\&= \frac{-12-15i}{16-25i^2}\\&= \frac{-12-15i}{16+25}\\&= -\frac{12}{41}-\frac{15}{41}i\end{aligned}$$

69.
$$\begin{aligned}&\frac{4+i}{-3-2i}\\&= \frac{4+i}{-3-2i}\cdot\frac{-3+2i}{-3+2i} \qquad -3+2i \text{ is the conjugate of the divisor.}\\&= \frac{(4+i)(-3+2i)}{(-3-2i)(-3+2i)}\\&= \frac{-12+5i+2i^2}{9-4i^2}\\&= \frac{-12+5i-2}{9+4} \qquad i^2=-1\\&= \frac{-14+5i}{13}\\&= -\frac{14}{13}+\frac{5}{13}i \qquad \text{Writing in the form } a+bi\end{aligned}$$

70.
$$\begin{aligned}\frac{5-i}{-7+2i} &= \frac{5-i}{-7+2i}\cdot\frac{-7-2i}{-7-2i}\\&= \frac{-35-3i+2i^2}{49-4i^2}\\&= \frac{-35-3i-2}{49+4}\\&= -\frac{37}{53}-\frac{3}{53}i\end{aligned}$$

71.
$$\begin{aligned}&\frac{5-3i}{4+3i}\\&= \frac{5-3i}{4+3i}\cdot\frac{4-3i}{4-3i} \qquad 4-3i \text{ is the conjugate of } 4+3i.\\&= \frac{(5-3i)(4-3i)}{(4+3i)(4-3i)}\\&= \frac{20-27i+9i^2}{16-9i^2}\\&= \frac{20-27i-9}{16+9} \qquad i^2=-1\\&= \frac{11-27i}{25}\\&= \frac{11}{25}-\frac{27}{25}i \qquad \text{Writing in the form } a+bi\end{aligned}$$

72.
$$\begin{aligned}\frac{6+5i}{3-4i} &= \frac{6+5i}{3-4i}\cdot\frac{3+4i}{3+4i}\\&= \frac{18+39i+20i^2}{9-16i^2}\\&= \frac{18+39i-20}{9+16}\\&= -\frac{2}{25}+\frac{39}{25}i\end{aligned}$$

73. $\dfrac{2+\sqrt{3}i}{5-4i}$

$= \dfrac{2+\sqrt{3}i}{5-4i}\cdot\dfrac{5+4i}{5+4i}$ $\quad 5+4i$ is the conjugate of the divisor.

$= \dfrac{(2+\sqrt{3}i)(5+4i)}{(5-4i)(5+4i)}$

$= \dfrac{10+8i+5\sqrt{3}i+4\sqrt{3}i^2}{25-16i^2}$

$= \dfrac{10+8i+5\sqrt{3}i-4\sqrt{3}}{25+16}$ $\quad i^2=-1$

$= \dfrac{10-4\sqrt{3}+(8+5\sqrt{3})i}{41}$

$= \dfrac{10-4\sqrt{3}}{41}+\dfrac{8+5\sqrt{3}}{41}i$ $\quad$ Writing in the form $a+bi$

74. $\dfrac{\sqrt{5}+3i}{1-i} = \dfrac{\sqrt{5}+3i}{1-i}\cdot\dfrac{1+i}{1+i}$

$= \dfrac{\sqrt{5}+\sqrt{5}i+3i+3i^2}{1-i^2}$

$= \dfrac{\sqrt{5}+\sqrt{5}i+3i-3}{1+1}$

$= \dfrac{\sqrt{5}-3}{2}+\dfrac{\sqrt{5}+3}{2}i$

75. $\dfrac{1+i}{(1-i)^2}$

$= \dfrac{1+i}{1-2i+i^2}$

$= \dfrac{1+i}{1-2i-1}$ $\quad i^2=-1$

$= \dfrac{1+i}{-2i}$

$= \dfrac{1+i}{-2i}\cdot\dfrac{2i}{2i}$ $\quad 2i$ is the conjugate of $-2i$.

$= \dfrac{(1+i)(2i)}{(-2i)(2i)}$

$= \dfrac{2i+2i^2}{-4i^2}$

$= \dfrac{2i-2}{4}$ $\quad i^2=-1$

$= -\dfrac{2}{4}+\dfrac{2}{4}i$

$= -\dfrac{1}{2}+\dfrac{1}{2}i$

76. $\dfrac{1-i}{(1+i)^2} = \dfrac{1-i}{1+2i+i^2}$

$= \dfrac{1-i}{1+2i-1}$

$= \dfrac{1-i}{2i}$

$= \dfrac{1-i}{2i}\cdot\dfrac{-2i}{-2i}$

$= \dfrac{-2i+2i^2}{-4i^2}$

$= \dfrac{-2i-2}{4}$

$= -\dfrac{1}{2}-\dfrac{1}{2}i$

77. $\dfrac{4-2i}{1+i}+\dfrac{2-5i}{1+i}$

$= \dfrac{6-7i}{1+i}$ $\quad$ Adding

$= \dfrac{6-7i}{1+i}\cdot\dfrac{1-i}{1-i}$ $\quad 1-i$ is the conjugate of $1+i$.

$= \dfrac{(6-7i)(1-i)}{(1+i)(1-i)}$

$= \dfrac{6-13i+7i^2}{1-i^2}$

$= \dfrac{6-13i-7}{1+1}$ $\quad i^2=-1$

$= \dfrac{-1-13i}{2}$

$= -\dfrac{1}{2}-\dfrac{13}{2}i$

78. $\dfrac{3+2i}{1-i}+\dfrac{6+2i}{1-i} = \dfrac{9+4i}{1-i}$

$= \dfrac{9+4i}{1-i}\cdot\dfrac{1+i}{1+i}$

$= \dfrac{9+13i+4i^2}{1-i^2}$

$= \dfrac{9+13i-4}{1+1}$

$= \dfrac{5}{2}+\dfrac{13}{2}i$

79. $i^{11}=i^{10}\cdot i=(i^2)^5\cdot i=(-1)^5\cdot i=-1\cdot i=-i$

80. $i^7=i^6\cdot i=(i^2)^3\cdot i=(-1)^3\cdot i=-1\cdot i=-i$

81. $i^{35}=i^{34}\cdot i=(i^2)^{17}\cdot i=(-1)^{17}\cdot i=-1\cdot i=-i$

82. $i^{24}=(i^2)^{12}=(-1)^{12}=1$

83. $i^{64}=(i^2)^{32}=(-1)^{32}=1$

84. $i^{42}=(i^2)^{21}=(-1)^{21}=-1$

85. $(-i)^{71}=(-1\cdot i)^{71}=(-1)^{71}\cdot i^{71}=-i^{70}\cdot i=$
$-(i^2)^{35}\cdot i=-(-1)^{35}\cdot i=-(-1)i=i$

86. $(-i)^6=i^6=(i^2)^3=(-1)^3=-1$

87. $(5i)^4 = 5^4 \cdot i^4 = 625(i^2)^2 = 625(-1)^2 = 625 \cdot 1 = 625$

88. $(2i)^5 = 32i^5 = 32 \cdot i^4 \cdot i = 32(i^2)^2 \cdot i = 32(-1)^2 \cdot i = 32 \cdot 1 \cdot i = 32i$

89. First find the slope of the given line.

$$\begin{aligned} 3x - 6y &= 7 \\ -6y &= -3x + 7 \\ y &= \frac{1}{2}x - \frac{7}{6} \end{aligned}$$

The slope is $\frac{1}{2}$. The slope of the desired line is the opposite of the reciprocal of $\frac{1}{2}$, or -2. Write a slope-intercept equation of the line containing $(3, -5)$ with slope -2.

$$\begin{aligned} y - (-5) &= -2(x - 3) \\ y + 5 &= -2x + 6 \\ y &= -2x + 1 \end{aligned}$$

90. The domain of f is the set of all real numbers as is the domain of g. Then the domain of $(f - g)(x)$ is the set of all real numbers, or $(-\infty, \infty)$.

91. The domain of f is the set of all real numbers as is the domain of g. When $x = -\frac{5}{3}$, $g(x) = 0$, so the domain of f/g is $\left(-\infty, -\frac{5}{3}\right) \cup \left(-\frac{5}{3}, \infty\right)$.

92. $(f - g)(x) = f(x) - g(x) = x^2 + 4 - (3x + 5) = x^2 - 3x - 1$

93. $(f/g)(2) = \dfrac{f(2)}{g(2)} = \dfrac{2^2 + 4}{3 \cdot 2 + 5} = \dfrac{4 + 4}{6 + 5} = \dfrac{8}{11}$

94.

$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - 3(x+h) + 4 - (x^2 - 3x + 4)}{h} \\ &= \frac{x^2 + 2xh + h^2 - 3x - 3h + 4 - x^2 + 3x - 4}{h} \\ &= \frac{2xh + h^2 - 3h}{h} \\ &= \frac{h(2x + h - 3)}{h} \\ &= 2x + h - 3 \end{aligned}$$

95. $(a+bi)+(a-bi) = 2a$, a real number. Thus, the statement is true.

96. $(a+bi)+(c+di) = (a+c)+(b+d)i$. The conjugate of this sum is $(a+c)-(b+d)i = a+c-bi-di = (a-bi)+(c-di)$, the sum of the conjugates of the individual complex numbers. Thus, the statement is true.

97. $(a+bi)(c+di) = (ac-bd)+(ad+bc)i$. The conjugate of the product is $(ac-bd)-(ad+bc)i =$ $(a-bi)(c-di)$, the product of the conjugates of the individual complex numbers. Thus, the statement is true.

98. $\dfrac{1}{z} = \dfrac{1}{a+bi} \cdot \dfrac{a-bi}{a-bi} = \dfrac{a}{a^2+b^2} + \dfrac{-b}{a^2+b^2}i$

99. $z\overline{z} = (a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2$

100.

$$\begin{aligned} z + 6\overline{z} &= 7 \\ a + bi + 6(a - bi) &= 7 \\ a + bi + 6a - 6bi &= 7 \\ 7a - 5bi &= 7 \end{aligned}$$

Then $7a = 7$, so $a = 1$, and $-5b = 0$, so $b = 0$. Thus, $z = 1$.

101.

$$\begin{aligned} &[x - (3+4i)][x - (3-4i)] \\ &= [x - 3 - 4i][x - 3 + 4i] \\ &= [(x-3) - 4i][(x-3) + 4i] \\ &= (x-3)^2 - (4i)^2 \\ &= x^2 - 6x + 9 - 16i^2 \\ &= x^2 - 6x + 9 + 16 \qquad i^2 = -1 \\ &= x^2 - 6x + 25 \end{aligned}$$

Exercise Set 3.2

1. $(2x-3)(3x-2) = 0$

$2x - 3 = 0$ *or* $3x - 2 = 0$ Using the principle of zero products

$2x = 3$ *or* $3x = 2$

$x = \frac{3}{2}$ *or* $x = \frac{2}{3}$

The solutions are $\frac{3}{2}$ and $\frac{2}{3}$.

2. $(5x-2)(2x+3) = 0$

$x = \frac{2}{5}$ *or* $x = -\frac{3}{2}$

The solutions are $\frac{2}{5}$ and $-\frac{3}{2}$.

3. $x^2 - 8x - 20 = 0$

$(x-10)(x+2) = 0$ Factoring

$x - 10 = 0$ *or* $x + 2 = 0$ Using the principle of zero products

$x = 10$ *or* $x = -2$

The solutions are 10 and -2.

4. $x^2 + 6x + 8 = 0$

$(x+2)(x+4) = 0$

$x = -2$ *or* $x = -4$

The solutions are -2 and -4.

5. $3x^2 + x - 2 = 0$

$(3x-2)(x+1) = 0$ Factoring

$3x - 2 = 0$ *or* $x + 1 = 0$ Using the principle of zero products

$x = \frac{2}{3}$ *or* $x = -1$

The solutions are $\frac{2}{3}$ and -1.

6. $10x^2 - 16x + 6 = 0$

$2(5x - 3)(x - 1) = 0$

$x = \frac{3}{5}$ *or* $x = 1$

The solutions are $\frac{3}{5}$ and 1.

7. $4x^2 - 12 = 0$

$4x^2 = 12$

$x^2 = 3$

$x = \sqrt{3}$ *or* $x = -\sqrt{3}$ Using the principle of square roots

The solutions are $\sqrt{3}$ and $-\sqrt{3}$.

8. $6x^2 = 36$

$x^2 = 6$

$x = \sqrt{6}$ *or* $x = -\sqrt{6}$

The solutions are $\sqrt{6}$ and $-\sqrt{6}$.

9. $3x^2 = 21$

$x^2 = 7$

$x = \sqrt{7}$ *or* $x = -\sqrt{7}$ Using the principle of square roots

The solutions are $\sqrt{7}$ and $-\sqrt{7}$.

10. $2x^2 - 20 = 0$

$2x^2 = 20$

$x^2 = 10$

$x = \sqrt{10}$ *or* $x = -\sqrt{10}$

The solutions are $\sqrt{10}$ and $-\sqrt{10}$.

11. $5x^2 + 10 = 0$

$5x^2 = -10$

$x^2 = -2$

$x = \sqrt{2}i$ *or* $x = -\sqrt{2}i$

The solutions are $\sqrt{2}i$ and $-\sqrt{2}i$.

12. $4x^2 + 12 = 0$

$4x^2 = -12$

$x^2 = -3$

$x = \sqrt{3}i$ *or* $x = -\sqrt{3}i$

The solutions are $\sqrt{3}i$ and $-\sqrt{3}i$.

13. $x^2 + 16 = 0$

$x^2 = -16$

$x = \sqrt{-16}$ *or* $x = -\sqrt{-16}$

$x = 4i$ *or* $x = -4i$

The solutions are $4i$ and $-4i$.

14. $x^2 + 25 = 0$

$x^2 = -25$

$x = 5i$ *or* $x = -5i$

The solutions are $5i$ and $-5i$.

15. $2x^2 = 6x$

$2x^2 - 6x = 0$ Subtracting $6x$ on both sides

$2x(x - 3) = 0$

$2x = 0$ *or* $x - 3 = 0$

$x = 0$ *or* $x = 3$

The solutions are 0 and 3.

16. $18x + 9x^2 = 0$

$9x(2 + x) = 0$

$x = 0$ *or* $x = -2$

The solutions are -2 and 0.

17. $3y^3 - 5y^2 - 2y = 0$

$y(3y^2 - 5y - 2) = 0$

$y(3y + 1)(y - 2) = 0$

$y = 0$ *or* $3y + 1 = 0$ *or* $y - 2 = 0$

$y = 0$ *or* $y = -\frac{1}{3}$ *or* $y = 2$

The solutions are $-\frac{1}{3}$, 0 and 2.

18. $3t^3 + 2t = 5t^2$

$3t^3 - 5t^2 + 2t = 0$

$t(t - 1)(3t - 2) = 0$

$t = 0$ *or* $t = 1$ *or* $t = \frac{2}{3}$

The solutions are 0, $\frac{2}{3}$, and 1.

19. $7x^3 + x^2 - 7x - 1 = 0$

$x^2(7x + 1) - (7x + 1) = 0$

$(x^2 - 1)(7x + 1) = 0$

$(x + 1)(x - 1)(7x + 1) = 0$

$x + 1 = 0$ *or* $x - 1 = 0$ *or* $7x + 1 = 0$

$x = -1$ *or* $x = 1$ *or* $x = -\frac{1}{7}$

The solutions are -1, $-\frac{1}{7}$, and 1.

20. $3x^3 + x^2 - 12x - 4 = 0$

$x^2(3x + 1) - 4(3x + 1) = 0$

$(3x + 1)(x^2 - 4) = 0$

$(3x + 1)(x + 2)(x - 2) = 0$

$x = -\frac{1}{3}$ *or* $x = -2$ *or* $x = 2$

The solutions are -2, $-\frac{1}{3}$, and 2.

21. a) The graph crosses the x-axis at $(-4, 0)$ and at $(2, 0)$. These are the x-intercepts.

b) The zeros of the function are the first coordinates of the x-intercepts of the graph. They are -4 and 2.

22. a) $(-1, 0)$, $(2, 0)$

b) -1, 2

23. a) The graph crosses the x-axis at $(-1, 0)$ and at $(3, 0)$. These are the x-intercepts.

b) The zeros of the function are the first coordinates of the x-intercepts of the graph. They are -1 and 3.

24. a) $(-3, 0)$, $(1, 0)$

b) -3, 1

25. a) The graph crosses the x-axis at $(-2, 0)$ and at $(2, 0)$. These are the x-intercepts.

b) The zeros of the function are the first coordinates of the x-intercepts of the graph. They are -2 and 2.

26. a) $(-1, 0)$, $(1, 0)$

b) -1, 1

27. a) The graph has only one x-intercept, $(1, 0)$.

b) The zero of the function is the first coordinate of the x-intercept of the graph, 1.

28. a) $(-2, 0)$

b) -2

29.
$$x^2 + 6x = 7$$
$$x^2 + 6x + 9 = 7 + 9 \quad \text{Completing the square: } \tfrac{1}{2} \cdot 6 = 3 \text{ and } 3^2 = 9$$
$$(x+3)^2 = 16 \quad \text{Factoring}$$
$$x + 3 = \pm 4 \quad \text{Using the principle of square roots}$$
$$x = -3 \pm 4$$
$$x = -3 - 4 \quad or \quad x = -3 + 4$$
$$x = -7 \quad or \quad x = 1$$
The solutions are -7 and 1.

30.
$$x^2 + 8x = -15$$
$$x^2+8x+16 = -15+16 \quad (\tfrac{1}{2} \cdot 8 = 4 \text{ and } 4^2 = 16)$$
$$(x+4)^2 = 1$$
$$x + 4 = \pm 1$$
$$x = -4 \pm 1$$
$$x = -4 - 1 \quad or \quad x = -4 + 1$$
$$x = -5 \quad or \quad x = -3$$
The solutions are -5 and -3.

31.
$$x^2 = 8x - 9$$
$$x^2 - 8x = -9 \quad \text{Subtracting } 8x$$
$$x^2 - 8x + 16 = -9 + 16 \quad \text{Completing the square: } \tfrac{1}{2}(-8) = -4 \text{ and } (-4)^2 = 16$$
$$(x-4)^2 = 7 \quad \text{Factoring}$$
$$x - 4 = \pm\sqrt{7} \quad \text{Using the principle of square roots}$$
$$x = 4 \pm \sqrt{7}$$
The solutions are $4 - \sqrt{7}$ and $4 + \sqrt{7}$, or $4 \pm \sqrt{7}$.

32.
$$x^2 = 22 + 10x$$
$$x^2 - 10x = 22$$
$$x^2 - 10x + 25 = 22 + 25 \quad (\tfrac{1}{2}(-10) = -5 \text{ and } (-5)^2 = 25)$$
$$(x-5)^2 = 47$$
$$x - 5 = \pm\sqrt{47}$$
$$x = 5 \pm \sqrt{47}$$
The solutions are $5 - \sqrt{47}$ and $5 + \sqrt{47}$, or $5 \pm \sqrt{47}$.

33.
$$x^2 + 8x + 25 = 0$$
$$x^2 + 8x = -25 \quad \text{Subtracting 25}$$
$$x^2 + 8x + 16 = -25 + 16 \quad \text{Completing the square: } \tfrac{1}{2} \cdot 8 = 4 \text{ and } 4^2 = 16$$
$$(x+4)^2 = -9 \quad \text{Factoring}$$
$$x + 4 = \pm 3i \quad \text{Using the principle of square roots}$$
$$x = -4 \pm 3i$$
The solutions are $-4 - 3i$ and $-4 + 3i$, or $-4 \pm 3i$.

34.
$$x^2 + 6x + 13 = 0$$
$$x^2 + 6x = -13$$
$$x^2 + 6x + 9 = -13 + 9 \quad \left(\tfrac{1}{2} \cdot 6 = 3 \text{ and } 3^2 = 9\right)$$
$$(x+3)^2 = -4$$
$$x + 3 = \pm 2i$$
$$x = -3 \pm 2i$$
The solutions are $-3 - 2i$ and $-3 + 2i$, or $-3 \pm 2i$.

35.
$$3x^2 + 5x - 2 = 0$$
$$3x^2 + 5x = 2 \quad \text{Adding 2}$$
$$x^2 + \frac{5}{3}x = \frac{2}{3} \quad \text{Dividing by 3}$$
$$x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{2}{3} + \frac{25}{36} \quad \text{Completing the square: } \tfrac{1}{2} \cdot \tfrac{5}{3} = \tfrac{5}{6} \text{ and } (\tfrac{5}{6})^2 = \tfrac{25}{36}$$
$$\left(x + \frac{5}{6}\right)^2 = \frac{49}{36} \quad \text{Factoring and simplifying}$$
$$x + \frac{5}{6} = \pm\frac{7}{6} \quad \text{Using the principle of square roots}$$
$$x = -\frac{5}{6} \pm \frac{7}{6}$$
$$x = -\frac{5}{6} - \frac{7}{6} \quad or \quad x = -\frac{5}{6} + \frac{7}{6}$$
$$x = -\frac{12}{6} \quad or \quad x = \frac{2}{6}$$
$$x = -2 \quad or \quad x = \frac{1}{3}$$
The solutions are -2 and $\frac{1}{3}$.

36. $2x^2 - 5x - 3 = 0$

$2x^2 - 5x = 3$

$x^2 - \frac{5}{2}x = \frac{3}{2}$

$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$ $\left(\frac{1}{2}(-\frac{5}{2}) = -\frac{5}{4}\right.$ and $\left.(-\frac{5}{4})^2 = \frac{25}{16}\right)$

$\left(x - \frac{5}{4}\right)^2 = \frac{49}{16}$

$x - \frac{5}{4} = \pm\frac{7}{4}$

$x = \frac{5}{4} \pm \frac{7}{4}$

$x = \frac{5}{4} - \frac{7}{4}$ *or* $x = \frac{5}{4} + \frac{7}{4}$

$x = -\frac{1}{2}$ *or* $x = 3$

The solutions are $-\frac{1}{2}$ and 3.

37. $x^2 - 2x = 15$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$ Factoring

$x - 5 = 0$ *or* $x + 3 = 0$

$x = 5$ *or* $x = -3$

The solutions are 5 and -3.

38. $x^2 + 4x = 5$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x + 5 = 0$ *or* $x - 1 = 0$

$x = -5$ *or* $x = 1$

The solutions are -5 and 1.

39. $5m^2 + 3m = 2$

$5m^2 + 3m - 2 = 0$

$(5m-2)(m+1) = 0$ Factoring

$5m - 2 = 0$ *or* $m + 1 = 0$

$m = \frac{2}{5}$ *or* $m = -1$

The solutions are $\frac{2}{5}$ and -1.

40. $2y^2 - 3y - 2 = 0$

$(2y+1)(y-2) = 0$

$2y + 1 = 0$ *or* $y - 2 = 0$

$y = -\frac{1}{2}$ *or* $y = 2$

The solutions are $-\frac{1}{2}$ and 2.

41. $3x^2 + 6 = 10x$

$3x^2 - 10x + 6 = 0$

We use the quadratic formula. Here $a = 3$, $b = -10$, and $c = 6$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 3 \cdot 6}}{2 \cdot 3}$ Substituting

$= \frac{10 \pm \sqrt{28}}{6} = \frac{10 \pm 2\sqrt{7}}{6}$

$= \frac{2(5 \pm \sqrt{7})}{2 \cdot 3} = \frac{5 \pm \sqrt{7}}{3}$

The solutions are $\frac{5 - \sqrt{7}}{3}$ and $\frac{5 + \sqrt{7}}{3}$, or $\frac{5 \pm \sqrt{7}}{3}$.

42. $3t^2 + 8t + 3 = 0$

$t = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3}$

$= \frac{-8 \pm \sqrt{28}}{6} = \frac{-8 \pm 2\sqrt{7}}{6}$

$= \frac{2(-4 \pm \sqrt{7})}{2 \cdot 3} = \frac{-4 \pm \sqrt{7}}{3}$

The solutions are $\frac{-4 - \sqrt{7}}{3}$ and $\frac{-4 + \sqrt{7}}{3}$, or $\frac{-4 \pm \sqrt{7}}{3}$.

43. $x^2 + x + 2 = 0$

We use the quadratic formula. Here $a = 1$, $b = 1$, and $c = 2$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$ Substituting

$= \frac{-1 \pm \sqrt{-7}}{2}$

$= \frac{-1 \pm \sqrt{7}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$

The solutions are $-\frac{1}{2} - \frac{\sqrt{7}}{2}i$ and $-\frac{1}{2} + \frac{\sqrt{7}}{2}i$, or $-\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$.

44. $x^2 + 1 = x$

$x^2 - x + 1 = 0$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$

$= \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

The solutions are $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} + \frac{\sqrt{3}}{2}i$, or $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

45. $5t^2 - 8t = 3$

$5t^2 - 8t - 3 = 0$

We use the quadratic formula. Here $a = 5$, $b = -8$, and $c = -3$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 5(-3)}}{2 \cdot 5}$$
$$= \frac{8 \pm \sqrt{124}}{10} = \frac{8 \pm 2\sqrt{31}}{10}$$
$$= \frac{2(4 \pm \sqrt{31})}{2 \cdot 5} = \frac{4 \pm \sqrt{31}}{5}$$

The solutions are $\dfrac{4 - \sqrt{31}}{5}$ and $\dfrac{4 + \sqrt{31}}{5}$, or $\dfrac{4 \pm \sqrt{31}}{5}$.

46. $5x^2 + 2 = x$

$5x^2 - x + 2 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 5 \cdot 2}}{2 \cdot 5}$$
$$= \frac{1 \pm \sqrt{-39}}{10} = \frac{1 \pm \sqrt{39}i}{10}$$
$$= \frac{1}{10} \pm \frac{\sqrt{39}}{10}i$$

The solutions are $\dfrac{1}{10} - \dfrac{\sqrt{39}}{10}i$ and $\dfrac{1}{10} + \dfrac{\sqrt{39}}{10}i$, or $\dfrac{1}{10} \pm \dfrac{\sqrt{39}}{10}i$.

47. $3x^2 + 4 = 5x$

$3x^2 - 5x + 4 = 0$

We use the quadratic formula. Here $a = 3$, $b = -5$, and $c = 4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3}$$
$$= \frac{5 \pm \sqrt{-23}}{6} = \frac{5 \pm \sqrt{23}i}{6}$$
$$= \frac{5}{6} \pm \frac{\sqrt{23}}{6}i$$

The solutions are $\dfrac{5}{6} - \dfrac{\sqrt{23}}{6}i$ and $\dfrac{5}{6} + \dfrac{\sqrt{23}}{6}i$, or $\dfrac{5}{6} \pm \dfrac{\sqrt{23}}{6}i$.

48. $2t^2 - 5t = 1$

$2t^2 - 5t - 1 = 0$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2(-1)}}{2 \cdot 2}$$
$$= \frac{5 \pm \sqrt{33}}{4}$$

The solutions are $\dfrac{5 - \sqrt{33}}{4}$ and $\dfrac{5 + \sqrt{33}}{4}$, or $\dfrac{5 \pm \sqrt{33}}{4}$.

49. $x^2 - 8x + 5 = 0$

We use the quadratic formula. Here $a = 1$, $b = -8$, and $c = 5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$
$$= \frac{8 \pm \sqrt{44}}{2} = \frac{8 \pm 2\sqrt{11}}{2}$$
$$= \frac{2(4 \pm \sqrt{11})}{2} = 4 \pm \sqrt{11}$$

The solutions are $4 - \sqrt{11}$ and $4 + \sqrt{11}$, or $4 \pm \sqrt{11}$.

50. $x^2 - 6x + 3 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$
$$= \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm 2\sqrt{6}}{2}$$
$$= \frac{2(3 \pm \sqrt{6})}{2} = 3 \pm \sqrt{6}$$

The solutions are $3 - \sqrt{6}$ and $3 + \sqrt{6}$, or $3 \pm \sqrt{6}$.

51. $3x^2 + x = 5$

$3x^2 + x - 5 = 0$

We use the quadratic formula. We have $a = 3$, $b = 1$, and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$$
$$= \frac{-1 \pm \sqrt{61}}{6}$$

The solutions are $\dfrac{-1 - \sqrt{61}}{6}$ and $\dfrac{-1 + \sqrt{61}}{6}$, or $\dfrac{-1 \pm \sqrt{61}}{6}$.

52. $5x^2 + 3x = 1$

$5x^2 + 3x - 1 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 5 \cdot (-1)}}{2 \cdot 5}$$
$$= \frac{-3 \pm \sqrt{29}}{10}$$

The solutions are $\dfrac{-3 - \sqrt{29}}{10}$ and $\dfrac{-3 + \sqrt{29}}{10}$, or $\dfrac{-3 \pm \sqrt{29}}{10}$.

53. $2x^2+1=5x$

$2x^2-5x+1=0$

We use the quadratic formula. We have $a=2$, $b=-5$, and $c=1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 2\cdot 1}}{2\cdot 2}=\frac{5\pm\sqrt{17}}{4}$$

The solutions are $\frac{5-\sqrt{17}}{4}$ and $\frac{5+\sqrt{17}}{4}$, or $\frac{5\pm\sqrt{17}}{4}$.

54. $4x^2+3=x$

$4x^2-x+3=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 4\cdot 3}}{2\cdot 4}$$

$$=\frac{1\pm\sqrt{-47}}{8}=\frac{1\pm\sqrt{47}i}{8}=\frac{1}{8}\pm\frac{\sqrt{47}}{8}i$$

The solutions are $\frac{1}{8}-\frac{\sqrt{47}}{8}i$ and $\frac{1}{8}+\frac{\sqrt{47}}{8}i$, or $\frac{1}{8}\pm\frac{\sqrt{47}}{8}i$.

55. $5x^2+2x=-2$

$5x^2+2x+2=0$

We use the quadratic formula. We have $a=5$, $b=2$, and $c=2$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm\sqrt{2^2-4\cdot 5\cdot 2}}{2\cdot 5}$$

$$=\frac{-2\pm\sqrt{-36}}{10}=\frac{-2\pm 6i}{10}$$

$$=\frac{2(-1\pm 3i)}{2\cdot 5}=\frac{-1\pm 3i}{5}$$

$$=-\frac{1}{5}\pm\frac{3}{5}i$$

The solutions are $-\frac{1}{5}-\frac{3}{5}i$ and $-\frac{1}{5}+\frac{3}{5}i$, or $-\frac{1}{5}\pm\frac{3}{5}i$.

56. $3x^2+3x=-4$

$3x^2+3x+4=0$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot 3\cdot 4}}{2\cdot 3}$$

$$=\frac{-3\pm\sqrt{-39}}{6}=\frac{-3\pm\sqrt{39}i}{6}$$

$$=-\frac{1}{2}\pm\frac{\sqrt{39}}{6}i$$

The solutions are $-\frac{1}{2}-\frac{\sqrt{39}}{6}i$ and $-\frac{1}{2}+\frac{\sqrt{39}}{6}i$ or $-\frac{1}{2}\pm\frac{\sqrt{39}}{6}i$.

57. $4x^2=8x+5$

$4x^2-8x-5=0$

$a=4, b=-8, c=-5$

$b^2-4ac=(-8)^2-4\cdot 4(-5)=144$

Since $b^2-4ac>0$, there are two different real-number solutions.

58. $4x^2-12x+9=0$

$b^2-4ac=(-12)^2-4\cdot 4\cdot 9=0$

There is one real-number solution.

59. $x^2+3x+4=0$

$a=1,\ b=3,\ c=4$

$b^2-4ac=3^2-4\cdot 1\cdot 4=-7$

Since $b^2-4ac<0$, there are two different imaginary-number solutions.

60. $x^2-2x+4=0$

$b^2-4ac=(-2)^2-4\cdot 1\cdot 4=-12<0$

There are two different imaginary-number solutions.

61. $9x^2+6x+1=0$

$a=9,\ b=6,\ c=1$

$b^2-4ac=6^2-4\cdot 9\cdot 1=0$

Since $b^2-4ac=0$, there is one real-number solution.

62. $5t^2-4t=11$

$5t^2-4t-11=0$

$b^2-4ac=(-4)^2-4\cdot 5(-11)=236>0$

There are two different real-number solutions.

63. $x^2+6x+5=0$ Setting $f(x)=0$

$(x+5)(x+1)=0$ Factoring

$x+5=0$ *or* $x+1=0$

$x=-5$ *or* $x=-1$

The zeros of the function are -5 and -1.

64. $x^2-x-2=0$

$(x+1)(x-2)=0$

$x+1=0$ *or* $x-2=0$

$x=-1$ *or* $x=2$

The zeros of the function are -1 and 2.

65. $x^2-3x-3=0$

$a=1$, $b=-3$, $c=-3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 1\cdot(-3)}}{2\cdot 1}$$

$$=\frac{3\pm\sqrt{9+12}}{2}$$

$$=\frac{3\pm\sqrt{21}}{2}$$

The zeros of the function are $\frac{3-\sqrt{21}}{2}$ and $\frac{3+\sqrt{21}}{2}$, or $\frac{3\pm\sqrt{21}}{2}$.

We use a calculator to find decimal approximations for the zeros:

$$\frac{3+\sqrt{21}}{2}\approx 3.791 \text{ and } \frac{3-\sqrt{21}}{2}\approx -0.791.$$

66. $3x^2+8x+2=0$

$$x=\frac{-8\pm\sqrt{8^2-4\cdot 3\cdot 2}}{2\cdot 3}$$
$$=\frac{-8\pm\sqrt{40}}{6}=\frac{-8\pm 2\sqrt{10}}{6}$$
$$=\frac{-4\pm\sqrt{10}}{3}$$

The zeros of the function are $\frac{-4-\sqrt{10}}{3}$ and $\frac{-4+\sqrt{10}}{3}$, or $\frac{-4\pm\sqrt{10}}{3}$.

$\frac{-4+\sqrt{10}}{3}\approx -0.279$ and $\frac{-4-\sqrt{10}}{3}\approx -2.387$.

67. $x^2-5x+1=0$

$a=1,\ b=-5,\ c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 1\cdot 1}}{2\cdot 1}$$
$$=\frac{5\pm\sqrt{25-4}}{2}$$
$$=\frac{5\pm\sqrt{21}}{2}$$

The zeros of the function are $\frac{5-\sqrt{21}}{2}$ and $\frac{5+\sqrt{21}}{2}$, or $\frac{5\pm\sqrt{21}}{2}$.

We use a calculator to find decimal approximations for the zeros:

$\frac{5+\sqrt{21}}{2}\approx 4.791$ and $\frac{5-\sqrt{21}}{2}\approx 0.209$.

68. $x^2-3x-7=0$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 1\cdot(-7)}}{2\cdot 1}$$
$$=\frac{3\pm\sqrt{37}}{2}$$

The zeros of the function are $\frac{3-\sqrt{37}}{2}$ and $\frac{3+\sqrt{37}}{2}$, or $\frac{3\pm\sqrt{37}}{2}$.

$\frac{3+\sqrt{37}}{2}\approx 4.541$ and $\frac{3-\sqrt{37}}{2}\approx -1.541$.

69. $x^2+2x-5=0$

$a=1,\ b=2,\ c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{2^2-4\cdot 1\cdot(-5)}}{2\cdot 1}$$
$$=\frac{-2\pm\sqrt{4+20}}{2}=\frac{-2\pm\sqrt{24}}{2}$$
$$=\frac{-2\pm 2\sqrt{6}}{2}=-1\pm\sqrt{6}$$

The zeros of the function are $-1+\sqrt{6}$ and $-1-\sqrt{6}$, or $-1\pm\sqrt{6}$.

We use a calculator to find decimal approximations for the zeros:

$-1+\sqrt{6}\approx 1.449$ and $-1-\sqrt{6}\approx -3.449$

70. $x^2-x-4=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 1\cdot(-4)}}{2\cdot 1}$$
$$=\frac{1\pm\sqrt{17}}{2}$$

The zeros of the function are $\frac{1+\sqrt{17}}{2}$ or $\frac{1-\sqrt{17}}{2}$, or $\frac{1\pm\sqrt{17}}{2}$.

$\frac{1+\sqrt{17}}{2}\approx 2.562$ and $\frac{1-\sqrt{17}}{2}\approx -1.562$.

71. $2x^2-x+4=0$

$a=2,\ b=-1,\ c=4$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 2\cdot 4}}{2\cdot 2}$$
$$=\frac{1\pm\sqrt{-31}}{4}=\frac{1\pm\sqrt{31}i}{4}$$
$$=\frac{1}{4}\pm\frac{\sqrt{31}}{4}i$$

The zeros of the function are $\frac{1}{4}-\frac{\sqrt{31}}{4}i$ and $\frac{1}{4}+\frac{\sqrt{31}}{4}i$, or $\frac{1}{4}\pm\frac{\sqrt{31}}{4}i$.

72. $2x^2+3x+2=0$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot 2\cdot 2}}{2\cdot 2}$$
$$=\frac{-3\pm\sqrt{-7}}{4}=\frac{-3\pm\sqrt{7}i}{4}$$
$$=-\frac{3}{4}\pm\frac{\sqrt{7}}{4}i$$

The zeros of the function are $-\frac{3}{4}-\frac{\sqrt{7}}{4}i$ and $-\frac{3}{4}+\frac{\sqrt{7}}{4}i$, or $-\frac{3}{4}\pm\frac{\sqrt{7}}{4}i$.

73. $3x^2-x-1=0$

$a=3,\ b=-1,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 3\cdot(-1)}}{2\cdot 3}$$
$$=\frac{1\pm\sqrt{13}}{6}$$

The zeros of the function are $\frac{1-\sqrt{13}}{6}$ and $\frac{1+\sqrt{13}}{6}$, or $\frac{1\pm\sqrt{13}}{6}$.

We use a calculator to find decimal approximations for the zeros:

$\frac{1+\sqrt{13}}{6} \approx 0.768$ and $\frac{1-\sqrt{13}}{6} \approx -0.434$.

74. $3x^2+5x+1=0$

$$x = \frac{-5\pm\sqrt{5^2-4\cdot 3\cdot 1}}{2\cdot 3} = \frac{-5\pm\sqrt{13}}{6}$$

The zeros of the function are $\frac{-5-\sqrt{13}}{6}$ and $\frac{-5+\sqrt{13}}{6}$, or $\frac{-5\pm\sqrt{13}}{6}$.

$\frac{-5+\sqrt{13}}{6} \approx -0.232$ and $\frac{-5-\sqrt{13}}{6} \approx -1.434$.

75. $5x^2-2x-1=0$

$a=5,\ b=-2,\ c=-1$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-2)\pm\sqrt{(-2)^2-4\cdot 5\cdot(-1)}}{2\cdot 5} = \frac{2\pm\sqrt{24}}{10} = \frac{2\pm 2\sqrt{6}}{10} = \frac{2(1\pm\sqrt{6})}{2\cdot 5} = \frac{1\pm\sqrt{6}}{5}$$

The zeros of the function are $\frac{1-\sqrt{6}}{5}$ and $\frac{1+\sqrt{6}}{5}$, or $\frac{1\pm\sqrt{6}}{5}$.

We use a calculator to find decimal approximations for the zeros:

$\frac{1+\sqrt{6}}{5} \approx 0.690$ and $\frac{1-\sqrt{6}}{5} \approx -0.290$.

76. $4x^2-4x-5=0$

$$x = \frac{-(-4)\pm\sqrt{(-4)^2-4\cdot 4\cdot(-5)}}{2\cdot 4} = \frac{4\pm\sqrt{96}}{8} = \frac{4\pm 4\sqrt{6}}{8} = \frac{4(1\pm\sqrt{6})}{4\cdot 2} = \frac{1\pm\sqrt{6}}{2}$$

The zeros of the function are $\frac{1-\sqrt{6}}{2}$ and $\frac{1+\sqrt{6}}{2}$, or $\frac{1\pm\sqrt{6}}{2}$.

$\frac{1+\sqrt{6}}{2} \approx 1.725$ and $\frac{1-\sqrt{6}}{2} \approx -0.725$.

77. $4x^2+3x-3=0$

$a=4,\ b=3,\ c=-3$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-3\pm\sqrt{3^2-4\cdot 4\cdot(-3)}}{2\cdot 4} = \frac{-3\pm\sqrt{57}}{8}$$

The zeros of the function are $\frac{-3-\sqrt{57}}{8}$ and $\frac{-3+\sqrt{57}}{8}$, or $\frac{-3\pm\sqrt{57}}{8}$.

We use a calculator to find decimal approximations for the zeros:

$\frac{-3+\sqrt{57}}{8} \approx 0.569$ and $\frac{-3-\sqrt{57}}{8} \approx -1.319$.

78. $x^2+6x-3=0$

$$x = \frac{-6\pm\sqrt{6^2-4\cdot 1\cdot(-3)}}{2\cdot 1} = \frac{-6\pm\sqrt{48}}{2} = \frac{-6\pm 4\sqrt{3}}{2} = \frac{2(-3\pm 2\sqrt{3})}{2} = -3\pm 2\sqrt{3}$$

The zeros of the function are $-3-2\sqrt{3}$ and $-3+2\sqrt{3}$, or $-3\pm 2\sqrt{3}$.

$-3+2\sqrt{3} \approx 0.464$ and $-3-2\sqrt{3} \approx -6.464$.

79. $x^4-3x^2+2=0$

Let $u=x^2$.

$u^2-3u+2=0$ Substituting u for x^2

$(u-1)(u-2)=0$

$u-1=0$ *or* $u-2=0$

$u=1$ *or* $u=2$

Now substitute x^2 for u and solve for x.

$x^2=1$ *or* $x^2=2$

$x=\pm 1$ *or* $x=\pm\sqrt{2}$

The solutions are -1, 1, $-\sqrt{2}$, and $\sqrt{2}$.

80. $x^4+3=4x^2$

$x^4-4x^2+3=0$

Let $u=x^2$.

$u^2-4u+3=0$ Substituting u for x^2

$(u-1)(u-3)=0$

$u-1=0$ *or* $u-3=0$

$u=1$ *or* $u=3$

Substitute x^2 for u and solve for x.

$x^2=1$ *or* $x^2=3$

$x=\pm 1$ *or* $x=\pm\sqrt{3}$

The solutions are -1, 1, $-\sqrt{3}$, and $\sqrt{3}$.

81. $x^4 + 3x^2 = 10$

$x^4 + 3x^2 - 10 = 0$

Let $u = x^2$.

$u^2 + 3u - 10 = 0$ Substituting u for x^2

$(u+5)(u-2) = 0$

$u + 5 = 0$ *or* $u - 2 = 0$

$u = -5$ *or* $u = 2$

Now substitute x^2 for u and solve for x.

$x^2 = -5$ *or* $x^2 = 2$

$x = \pm\sqrt{5}i$ *or* $x = \pm\sqrt{2}$

The solutions are $-\sqrt{5}i$, $\sqrt{5}i$, $-\sqrt{2}$, and $\sqrt{2}$.

82. $x^4 - 8x^2 = 9$

$x^4 - 8x^2 - 9 = 0$

Let $u = x^2$.

$u^2 - 8u - 9 = 0$ Substituting u for x^2

$(u-9)(u+1) = 0$

$u - 9 = 0$ *or* $u + 1 = 0$

$u = 9$ *or* $u = -1$

Now substitute x^2 for u and solve for x.

$x^2 = 9$ *or* $x^2 = -1$

$x = \pm 3$ *or* $x = \pm i$

The solutions are -3, 3, i, and $-i$.

83. $y^4 + 4y^2 - 5 = 0$

Let $u = y^2$.

$u^2 + 4u - 5 = 0$ Substituting u for y^2

$(u+5)(u-1) = 0$

$u + 5 = 0$ *or* $u - 1 = 0$

$u = -5$ *or* $u = 1$

Now substitute y^2 for u and solve for y.

$y^2 = -5$ *or* $y^2 = 1$

$y = \pm\sqrt{5}i$ *or* $y = \pm 1$

The solutions are $-\sqrt{5}i$, $\sqrt{5}i$, -1, and 1.

84. $y^4 - 15y^2 - 16 = 0$

Let $u = y^2$.

$u^2 - 15u - 16 = 0$ Substituting u for y^2

$(u-16)(u+1) = 0$

$u - 16 = 0$ *or* $u + 1 = 0$

$u = 16$ *or* $u = -1$

Now substitute y^2 for u and solve for y.

$y^2 = 16$ *or* $y^2 = -1$

$y = \pm 4$ *or* $y = \pm i$

The solutions are -4, 4, $-i$, and i.

85. $x - 3\sqrt{x} - 4 = 0$

Let $u = \sqrt{x}$.

$u^2 - 3u - 4 = 0$ Substituting u for $\sqrt{x}$

$(u+1)(u-4) = 0$

$u + 1 = 0$ *or* $u - 4 = 0$

$u = -1$ *or* $u = 4$

Now substitute $\sqrt{x}$ for u and solve for x.

$\sqrt{x} = -1$ *or* $\sqrt{x} = 4$

No solution $\quad x = 16$

Note that $\sqrt{x}$ must be nonnegative, so $\sqrt{x} = -1$ has no solution. The number 16 checks and is the solution. The solution is 16.

86. $2x - 9\sqrt{x} + 4 = 0$

Let $u = \sqrt{x}$.

$2u^2 - 9u + 4 = 0$ Substituting u for $\sqrt{x}$

$(2u-1)(u-4) = 0$

$2u - 1 = 0$ *or* $u - 4 = 0$

$u = \frac{1}{2}$ *or* $u = 4$

Substitute $\sqrt{x}$ for u and solve for u.

$\sqrt{x} = \frac{1}{2}$ *or* $\sqrt{x} = 4$

$x = \frac{1}{4}$ *or* $x = 16$

Both numbers check. The solutions are $\frac{1}{4}$ and 16.

87. $m^{2/3} - 2m^{1/3} - 8 = 0$

Let $u = m^{1/3}$.

$u^2 - 2u - 8 = 0$ Substituting u for $m^{1/3}$

$(u+2)(u-4) = 0$

$u + 2 = 0$ *or* $u - 4 = 0$

$u = -2$ *or* $u = 4$

Now substitute $m^{1/3}$ for u and solve for m.

$m^{1/3} = -2$ *or* $m^{1/3} = 4$

$(m^{1/3})^3 = (-2)^3$ *or* $(m^{1/3})^3 = 4^3$ Using the principle of powers

$m = -8$ *or* $m = 64$

The solutions are -8 and 64.

88. $t^{2/3} + t^{1/3} - 6 = 0$

Let $u = t^{1/3}$.

$u^2 + u - 6 = 0$

$(u+3)(u-2) = 0$

$u + 3 = 0$ *or* $u - 2 = 0$

$u = -3$ *or* $u = 2$

Substitute $t^{1/3}$ for u and solve for t.

$t^{1/3} = -3$ *or* $t^{1/3} = 2$

$t = -27$ *or* $t = 8$

The solutions are -27 and 8.

89. $x^{1/2} - 3x^{1/4} + 2 = 0$

Let $u = x^{1/4}$.

$u^2 - 3u + 2 = 0$ Substituting u for $x^{1/4}$

$(u-1)(u-2) = 0$

$u - 1 = 0$ *or* $u - 2 = 0$

$u = 1$ *or* $u = 2$

Now substitute $x^{1/4}$ for u and solve for x.

$x^{1/4} = 1$ *or* $x^{1/4} = 2$

$(x^{1/4})^4 = 1^4$ *or* $(x^{1/4})^4 = 2^4$

$x = 1$ *or* $x = 16$

The solutions are 1 and 16.

90. $x^{1/2} - 4x^{1/4} = -3$

$x^{1/2} - 4x^{1/4} + 3 = 0$

Let $u = x^{1/4}$.

$u^2 - 4u + 3 = 0$

$(u-1)(u-3) = 0$

$u - 1 = 0$ *or* $u - 3 = 0$

$u = 1$ *or* $u = 3$

Substitute $x^{1/4}$ for u and solve for x.

$x^{1/4} = 1$ *or* $x^{1/4} = 3$

$x = 1$ *or* $x = 81$

The solutions are 1 and 81.

91. $(2x-3)^2 - 5(2x-3) + 6 = 0$

Let $u = 2x - 3$.

$u^2 - 5u + 6 = 0$ Substituting u for $2x - 3$

$(u-2)(u-3) = 0$

$u - 2 = 0$ *or* $u - 3 = 0$

$u = 2$ *or* $u = 3$

Now substitute $2x - 3$ for u and solve for x.

$2x - 3 = 2$ *or* $2x - 3 = 3$

$2x = 5$ *or* $2x = 6$

$x = \frac{5}{2}$ *or* $x = 3$

The solutions are $\frac{5}{2}$ and 3.

92. $(3x+2)^2 + 7(3x+2) - 8 = 0$

Let $u = 3x + 2$.

$u^2 + 7u - 8 = 0$ Substituting u for $3x + 2$

$(u+8)(u-1) = 0$

$u + 8 = 0$ *or* $u - 1 = 0$

$u = -8$ *or* $u = 1$

Substitute $3x + 2$ for u and solve for x.

$3x + 2 = -8$ *or* $3x + 2 = 1$

$3x = -10$ *or* $3x = -1$

$x = -\frac{10}{3}$ *or* $x = -\frac{1}{3}$

The solutions are $-\frac{10}{3}$ and $-\frac{1}{3}$.

93. $(2t^2 + t)^2 - 4(2t^2 + t) + 3 = 0$

Let $u = 2t^2 + t$.

$u^2 - 4u + 3 = 0$ Substituting u for $2t^2 + t$

$(u-1)(u-3) = 0$

$u - 1 = 0$ *or* $u - 3 = 0$

$u = 1$ *or* $u = 3$

Now substitute $2t^2 + t$ for u and solve for t.

$2t^2 + t = 1$ *or* $2t^2 + t = 3$

$2t^2 + t - 1 = 0$ *or* $2t^2 + t - 3 = 0$

$(2t-1)(t+1) = 0$ *or* $(2t+3)(t-1) = 0$

$2t-1=0$ *or* $t+1=0$ *or* $2t+3=0$ *or* $t-1 = 0$

$t=\frac{1}{2}$ *or* $t=-1$ *or* $t=-\frac{3}{2}$ *or* $t = 1$

The solutions are $\frac{1}{2}$, -1, $-\frac{3}{2}$ and 1.

94. $12 = (m^2 - 5m)^2 + (m^2 - 5m)$

$0 = (m^2 - 5m)^2 + (m^2 - 5m) - 12$

Let $u = m^2 - 5m$.

$0 = u^2 + u - 12$ Substituting u for $m^2 - 5m$

$0 = (u+4)(u-3)$

$u + 4 = 0$ *or* $u - 3 = 0$

$u = -4$ *or* $u = 3$

Substitute $m^2 - 5m$ for u and solve for m.

$m^2 - 5m = -4$ *or* $m^2 - 5m = 3$

$m^2 - 5m + 4 = 0$ *or* $m^2 - 5m - 3 = 0$

$(m-1)(m-4) = 0$ *or*

$$m = \frac{-(-5) \pm \sqrt{(-5)^2 - 4\cdot 1(-3)}}{2\cdot 1}$$

$m = 1$ *or* $m = 4$ *or* $m = \frac{5 \pm \sqrt{37}}{2}$

The solutions are 1, 4, $\frac{5 - \sqrt{37}}{2}$, and $\frac{5 + \sqrt{37}}{2}$, or 1, 4, and $\frac{5 \pm \sqrt{37}}{2}$.

95. Substitute 40 for $h(x)$ and solve for x.

$40 = 0.012x^2 - 0.583x + 35.727$

$0 = 0.012x^2 - 0.583x - 4.273$

$a = 0.012,\ b = -0.583,\ c = -4.273$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.583) \pm \sqrt{(-0.583)^2 - 4(0.012)(-4.273)}}{2(0.012)}$$

$$= \frac{0.583 \pm \sqrt{0.544993}}{0.024}$$

$x \approx -6.5$ *or* $x \approx 55.0$

Since we are looking for a year after 1940, we use the positive solution. There were 40 million multigenerational households about 55 yr after 1940, or in 1995.

96. Solve: $55 = 0.012x^2 - 0.583x + 35.727$

$x \approx -23$ *or* $x \approx 71$

Since we are looking for a year after 1940, we use the positive solution. There were 55 million multigenerational households about 71 yr after 1940, or in 2011.

97. Substitute 50 for $t(x)$ and solve for x.

$$50 = 0.16x^2 + 0.46x + 21.36$$
$$0 = 0.16x^2 + 0.46x - 28.64$$

$a = 0.16,\ b = 0.46,\ c = -28.64$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.46 \pm \sqrt{(0.46)^2 - 4(0.16)(-28.64)}}{2(0.16)} = \frac{-0.46 \pm \sqrt{18.5412}}{0.32}$$

$x \approx -15$ *or* $x \approx 12$

We use the positive solution because a negative number has no meaning in this situation. The average U.S. household received 50 TV channels about 12 yr after 1985, or in 1997.

98. Solve: $88 = 0.16x^2 + 0.46x + 21.36$

$x \approx -22$ *or* $x \approx 19$

We use the positive solution because a negative number has no meaning in this situation. The average U.S. household received 88 TV channels about 19 yr after 1985, or in 2004.

99. ***Familiarize and Translate***. We will use the formula $s = 16t^2$, substituting 1670 for s.

$$1670 = 16t^2$$

Carry out. We solve the equation.

$1670 = 16t^2$

$104.375 = t^2$ Dividing by 16 on both sides

$10.216 \approx t$ Taking the square root on both sides

Check. When $t = 10.216$, $s = 16(10.216)^2 \approx 1670$. The answer checks.

State. It would take an object about 10.216 sec to reach the ground.

100. Solve: $630 = 16t^2$

$t \approx 6.275$ sec

101. ***Familiarize***. Let $w =$ the width of the rug. Then $w+1 =$ the length.

Translate. We use the Pythagorean equation.

$$w^2 + (w+1)^2 = 5^2$$

Carry out. We solve the equation.

$$w^2 + (w+1)^2 = 5^2$$
$$w^2 + w^2 + 2w + 1 = 25$$
$$2w^2 + 2w + 1 = 25$$
$$2w^2 + 2w - 24 = 0$$
$$2(w+4)(w-3) = 0$$
$$w + 4 = 0 \quad or \quad w - 3 = 0$$
$$w = -4 \quad or \quad w = 3$$

Since the width cannot be negative, we consider only 3. When $w = 3$, $w + 1 = 3 + 1 = 4$.

Check. The length, 4 ft, is 1 ft more than the width, 3 ft. The length of a diagonal of a rectangle with width 3 ft and length 4 ft is $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. The answer checks.

State. The length is 4 ft, and the width is 3 ft.

102. Let $x =$ the length of the longer leg.

Solve: $x^2 + (x-7)^2 = 13^2$

$x = -5$ *or* $x = 12$

Only 12 has meaning in the original problem. The length of one leg is 12 cm, and the length of the other leg is $12-7$, or 5 cm.

103. ***Familiarize***. Let $n =$ the smaller number. Then $n + 5 =$ the larger number.

Translate.

$$\underbrace{\text{The product of the numbers}}_{\downarrow\; n(n+5)} \quad \underset{=}{\overset{\text{is}}{\downarrow}} \quad \underset{36}{\overset{36.}{\downarrow}}$$

Carry out.

$$n(n+5) = 36$$
$$n^2 + 5n = 36$$
$$n^2 + 5n - 36 = 0$$
$$(n+9)(n-4) = 0$$
$$n + 9 = 0 \quad or \quad n - 4 = 0$$
$$n = -9 \quad or \quad n = 4$$

If $n = -9$, then $n + 5 = -9 + 5 = -4$. If $n = 4$, then $n + 5 = 4 + 5 = 9$.

Check. The number -4 is 5 more than -9 and $(-4)(-9) = 36$, so the pair -9 and -4 check. The number 9 is 5 more than 4 and $9 \cdot 4 = 36$, so the pair 4 and 9 also check.

State. The numbers are -9 and -4 or 4 and 9.

104. Let $n =$ the larger number.

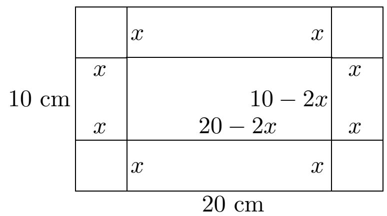
Solve: $n(n-6) = 72$

$n = -6$ *or* $n = 12$

When $n = -6$, then $n - 6 = -6 - 6 = -12$, so one pair of numbers is -6 and -12. When $n = 12$, then $n - 6 = 12 - 6 = 6$, so the other pair of numbers is 6 and 12.

105. ***Familiarize***. We add labels to the drawing in the text.

We let x represent the length of a side of the square in each corner. Then the length and width of the resulting base are represented by $20 - 2x$ and $10 - 2x$, respectively. Recall that for a rectangle, Area = length × width.



Translate.

$\underbrace{\text{The area of the base}}$ is $\underbrace{96\text{ cm}^2}$.

$(20-2x)(10-2x) = 96$

Carry out. We solve the equation.

$200-60x+4x^2 = 96$

$4x^2-60x+104 = 0$

$x^2-15x+26 = 0$

$(x-13)(x-2) = 0$

$x-13=0$ *or* $x-2=0$

$x=13$ *or* $x=2$

Check. When $x=13$, both $20-2x$ and $10-2x$ are negative numbers, so we only consider $x=2$. When $x=2$, then $20-2x = 20-2\cdot 2 = 16$ and $10-2x = 10-2\cdot 2 = 6$, and the area of the base is $16\cdot 6$, or 96 cm^2. The answer checks.

State. The length of the sides of the squares is 2 cm.

106. We have $170 = 2l+2w$, so $w = 85-l$.

Solve: $l(85-l) = 1750$

$l = 35$ *or* $l = 50$

Choosing the larger number to be the length, we find that the length of the petting area is 50 m, and the width is 35 m.

107. ***Familiarize***. We have $P = 2l+2w$, or $28 = 2l+2w$. Solving for w, we have

$28 = 2l+2w$

$14 = l+w$ Dividing by 2

$14-l = w.$

Then we have l = the length of the rug and $14-l$ = the width, in feet. Recall that the area of a rectangle is the product of the length and the width.

Translate.

$\underbrace{\text{The area}}$ is $\underbrace{48\text{ ft}^2}$.

$\downarrow \quad \downarrow \quad \downarrow$

$l(14-l) = 48$

Carry out. We solve the equation.

$l(14-l) = 48$

$14l-l^2 = 48$

$0 = l^2-14l+48$

$0 = (l-6)(l-8)$

$l-6=0$ *or* $l-8=0$

$l=6$ *or* $l=8$

If $l=6$, then $14-l = 14-6 = 8$.

If $l=8$, then $14-l = 14-8 = 6$.

In either case, the dimensions are 8 ft by 6 ft. Since we usually consider the length to be greater than the width, we let 8 ft = the length and 6 ft = the width.

Check. The perimeter is $2\cdot 8$ ft$+2\cdot 6$ ft $= 16$ ft $+12$ ft $=$ 28 ft. The answer checks.

State. The length of the rug is 8 ft, and the width is 6 ft.

108. Let w = the width of the frame.

Solve: $(10-2w)(8-2w) = 48$

$w = 1$ or $w = 8$

Only 1 has meaning in the original problem. The width of the frame is 1 in.

109. $f(x) = 4-5x = -5x+4$

The function can be written in the form $y = mx+b$, so it is a linear function.

110. $f(x) = 4-5x^2 = -5x^2+4$

The function can be written in the form $f(x) = ax^2+bx+c$, $a \neq 0$, so it is a quadratic function.

111. $f(x) = 7x^2$

The function is in the form $f(x) = ax^2+bx+c$, $a\neq 0$, so it is a quadratic function.

112. $f(x) = 23x+6$

The function is in the form $f(x) = mx+b$, so it is a linear function.

113. $f(x) = 1.2x-(3.6)^2$

The function is in the form $f(x) = mx+b$, so it is a linear function.

114. $f(x) = 2-x-x^2 = -x^2-x+2$

The function can be written in the form $f(x) = ax^2+bx+c$, $a\neq 0$, so it is a quadratic function.

115. In 2010, $x = 2010-2004 = 6$.

$a(6) = 1.24(6)+9.24 = 7.44+9.24 = 16.68$

In 2010, \$16.68 billion was spent on antipsychotic drugs.

116. Solve: $24 = 1.24x+9.24$

$x \approx 12$

Spending on antipsychotic drugs will reach \$24 billion about 12 yr after 2004, or in 2016.

117. Test for symmetry with respect to the x-axis:

$3x^2+4y^2 = 5$ Original equation

$3x^2+4(-y)^2 = 5$ Replacing y by $-y$

$3x^2+4y^2 = 5$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x^2+4y^2 = 5$ Original equation

$3(-x)^2+4y^2 = 5$ Replacing x by $-x$

$3x^2+4y^2 = 5$ Simplifying

The last equation is equivalent to the original equation, so the equation is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x^2 + 4y^2 = 5$ Original equation

$3(-x)^2 + 4(-y)^2 = 5$ Replacing x by $-x$ and y by $-y$

$3x^2 + 4y^2 = 5$ Simplifying

The last equation is equivalent to the original equation, so the equation is symmetric with respect to the origin.

118. Test for symmetry with respect to the x-axis:

$y^3 = 6x^2$ Original equation

$(-y)^3 = 6x^2$ Replacing y by $-y$

$-y^3 = 6x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y^3 = 6x^2$ Original equation

$y^3 = 6(-x)^2$ Replacing x by $-x$

$y^3 = 6x^2$ Simplifying

The last equation is equivalent to the original equation, so the equation is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^3 = 6x^2$ Original equation

$(-y)^3 = 6(-x)^2$ Replacing x by $-x$ y by $-y$

$-y^3 = 6x^2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

119. $f(x) = 2x^3 - x$

$f(-x) = 2(-x)^3 - (-x) = -2x^3 + x$

$-f(x) = -2x^3 + x$

$f(x) \neq f(-x)$ so f is not even

$f(-x) = -f(x)$, so f is odd.

120. $f(x) = 4x^2 + 2x - 3$

$f(-x) = 4(-x)^2 + 2(-x) - 3 = 4x^2 - 2x - 3$

$-f(x) = -4x^2 - 2x + 3$

$f(x) \neq f(-x)$ so f is not even

$f(-x) \neq -f(x)$, so f is not odd.

Thus $f(x) = 4x^2 + 2x - 3$ is neither even nor odd.

121. a) $kx^2 - 17x + 33 = 0$

$k(3)^2 - 17(3) + 33 = 0$ Substituting 3 for x

$9k - 51 + 33 = 0$

$9k = 18$

$k = 2$

b) $2x^2 - 17x + 33 = 0$ Substituting 2 for k

$(2x - 11)(x - 3) = 0$

$2x - 11 = 0$ *or* $x - 3 = 0$

$x = \dfrac{11}{2}$ *or* $x = 3$

The other solution is $\dfrac{11}{2}$.

122. a) $kx^2 - 2x + k = 0$

$k(-3)^2 - 2(-3) + k = 0$ Substituting -3 for x

$9k + 6 + k = 0$

$10k = -6$

$k = -\dfrac{3}{5}$

b) $-\dfrac{3}{5}x^2 - 2x - \dfrac{3}{5} = 0$ Substituting $-\dfrac{3}{5}$ for k

$3x^2 + 10x + 3 = 0$ Multiplying by -5

$(3x + 1)(x + 3) = 0$

$3x + 1 = 0$ *or* $x + 3 = 0$

$3x = -1$ *or* $x = -3$

$x = -\dfrac{1}{3}$ *or* $x = -3$

The other solution is $-\dfrac{1}{3}$.

123. a) $(1 + i)^2 - k(1 + i) + 2 = 0$ Substituting $1 + i$ for x

$1 + 2i - 1 - k - ki + 2 = 0$

$2 + 2i = k + ki$

$2(1 + i) = k(1 + i)$

$2 = k$

b) $x^2 - 2x + 2 = 0$ Substituting 2 for k

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i$$

The other solution is $1 - i$.

124. a) $x^2 - (6 + 3i)x + k = 0$

$3^2 - (6 + 3i) \cdot 3 + k = 0$ Substituting 3 for x

$9 - 18 - 9i + k = 0$

$k = 9 + 9i$

b) $x^2-(6+3i)x+9+9i=0$

$$x=\frac{-[-(6+3i)]\pm\sqrt{[-(6+3i)]^2-4(1)(9+9i)}}{2\cdot 1}$$

$$x=\frac{6+3i\pm\sqrt{36+36i-9-36-36i}}{2}$$

$$x=\frac{6+3i\pm\sqrt{-9}}{2}=\frac{6+3i\pm 3i}{2}$$

$$x=\frac{6+3i+3i}{2} \quad or \quad x=\frac{6+3i-3i}{2}$$

$$x=\frac{6+6i}{2} \quad or \quad x=\frac{6}{2}$$

$$x=3+3i \quad or \quad x=3$$

The other solution is $3+3i$.

125. $(x-2)^3=x^3-2$

$$x^3-6x^2+12x-8=x^3-2$$
$$0=6x^2-12x+6$$
$$0=6(x^2-2x+1)$$
$$0=6(x-1)(x-1)$$

$x-1=0$ *or* $x-1=0$

$x=1$ *or* $x=1$

The solution is 1.

126. $(x+1)^3=(x-1)^3+26$

$$x^3+3x^2+3x+1=x^3-3x^2+3x-1+26$$
$$x^3+3x^2+3x+1=x^3-3x^2+3x+25$$
$$6x^2-24=0$$
$$6(x^2-4)=0$$
$$6(x+2)(x-2)=0$$

$x+2=0$ *or* $x-2=0$

$x=-2$ *or* $x=2$

The solutions are -2 and 2.

127. $(6x^3+7x^2-3x)(x^2-7)=0$

$$x(6x^2+7x-3)(x^2-7)=0$$
$$x(3x-1)(2x+3)(x^2-7)=0$$

$x=0$ *or* $3x-1=0$ *or* $2x+3=0$ *or* $x^2-7=0$

$x=0$ *or* $x=\frac{1}{3}$ *or* $x=-\frac{3}{2}$ *or* $x=\sqrt{7}$ *or* $x=-\sqrt{7}$

The exact solutions are $-\sqrt{7}$, $-\frac{3}{2}$, 0, $\frac{1}{3}$, and $\sqrt{7}$.

128. $\left(x-\frac{1}{5}\right)\left(x^2-\frac{1}{4}\right)+\left(x-\frac{1}{5}\right)\left(x^2+\frac{1}{8}\right)=0$

$$\left(x-\frac{1}{5}\right)\left(2x^2-\frac{1}{8}\right)=0$$
$$\left(x-\frac{1}{5}\right)(2)\left(x+\frac{1}{4}\right)\left(x-\frac{1}{4}\right)=0$$

$x=\frac{1}{5}$ *or* $x=-\frac{1}{4}$ *or* $x=\frac{1}{4}$

The solutions are $-\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{4}$.

129. $x^2+x-\sqrt{2}=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-1\pm\sqrt{1^2-4\cdot 1(-\sqrt{2})}}{2\cdot 1}=\frac{-1\pm\sqrt{1+4\sqrt{2}}}{2}$$

The solutions are $\frac{-1\pm\sqrt{1+4\sqrt{2}}}{2}$.

130. $x^2+\sqrt{5}x-\sqrt{3}=0$

Use the quadratic formula. Here $a=1$, $b=\sqrt{5}$, and $c=-\sqrt{3}$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-\sqrt{5}\pm\sqrt{(\sqrt{5})^2-4\cdot 1(-\sqrt{3})}}{2\cdot 1}$$
$$=\frac{-\sqrt{5}\pm\sqrt{5+4\sqrt{3}}}{2}$$

The solutions are $\frac{-\sqrt{5}\pm\sqrt{5+4\sqrt{3}}}{2}$.

131. $2t^2+(t-4)^2=5t(t-4)+24$

$$2t^2+t^2-8t+16=5t^2-20t+24$$
$$0=2t^2-12t+8$$
$$0=t^2-6t+4 \qquad \text{Dividing by 2}$$

Use the quadratic formula.

$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-6)\pm\sqrt{(-6)^2-4\cdot 1\cdot 4}}{2\cdot 1}$$
$$=\frac{6\pm\sqrt{20}}{2}=\frac{6\pm 2\sqrt{5}}{2}$$
$$=\frac{2(3\pm\sqrt{5})}{2}=3\pm\sqrt{5}$$

The solutions are $3\pm\sqrt{5}$.

132. $9t(t+2)-3t(t-2)=2(t+4)(t+6)$

$$9t^2+18t-3t^2+6t=2t^2+20t+48$$
$$4t^2+4t-48=0$$
$$4(t+4)(t-3)=0$$

$t+4=0$ *or* $t-3=0$

$t=-4$ *or* $t=3$

The solutions are -4 and 3.

133. $\sqrt{x-3}-\sqrt[4]{x-3}=2$

Substitute u for $\sqrt[4]{x-3}$.

$$u^2-u-2=0$$
$$(u-2)(u+1)=0$$

$u-2=0$ *or* $u+1=0$

$u=2$ *or* $u=-1$

Substitute $\sqrt[4]{x-3}$ for u and solve for x.

$\sqrt[4]{x-3} = 2$ *or* $\sqrt[4]{x-3} = 1$

$x - 3 = 16$ No solution

$x = 19$

The value checks. The solution is 19.

134.
$$x^2+3x+1-\sqrt{x^2+3x+1}=8$$
$$x^2+3x+1-\sqrt{x^2+3x+1}-8=0$$
$$u^2-u-8=0$$
$$u=\frac{1+\sqrt{33}}{2} \text{ or } u=\frac{1-\sqrt{33}}{2}$$
$$\sqrt{x^2+3x+1}=\frac{1+\sqrt{33}}{2} \text{ or } \sqrt{x^2+3x+1}=\frac{1-\sqrt{33}}{2}$$
$$x^2+3x+1=\frac{34+2\sqrt{33}}{4} \text{ or } x^2+3x+1=\frac{34-2\sqrt{33}}{4}$$
$$x^2+3x+\frac{-15-\sqrt{33}}{2}=0 \text{ or } x^2+3x+\frac{-15+\sqrt{33}}{2}=0$$
$$x=\frac{-3\pm\sqrt{39+2\sqrt{33}}}{2} \text{ or } x=\frac{-3\pm\sqrt{39-2\sqrt{33}}}{2}$$

Only $\dfrac{-3\pm\sqrt{39+2\sqrt{33}}}{2}$ checks. The solutions are $\dfrac{-3\pm\sqrt{39+2\sqrt{33}}}{2}$.

135.
$$\left(y+\frac{2}{y}\right)^2+3y+\frac{6}{y}=4$$
$$\left(y+\frac{2}{y}\right)^2+3\left(y+\frac{2}{y}\right)-4=0$$

Substitute u for $y+\dfrac{2}{y}$.

$$u^2+3u-4=0$$
$$(u+4)(u-1)=0$$
$$u=-4 \text{ or } u=1$$

Substitute $y+\dfrac{2}{y}$ for u and solve for y.

$$y+\frac{2}{y}=-4 \text{ or } y+\frac{2}{y}=1$$
$$y^2+2=-4y \text{ or } y^2+2=y$$
$$y^2+4y+2=0 \text{ or } y^2-y+2=0$$
$$y=\frac{-4\pm\sqrt{4^2-4\cdot1\cdot2}}{2\cdot1} \text{ or } y=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot1\cdot2}}{2\cdot1}$$
$$y=\frac{-4\pm\sqrt{8}}{2} \text{ or } y=\frac{1\pm\sqrt{-7}}{2}$$
$$y=\frac{-4\pm2\sqrt{2}}{2} \text{ or } y=\frac{1\pm\sqrt{7}i}{2}$$
$$y=-2\pm\sqrt{2} \text{ or } y=\frac{1}{2}\pm\frac{\sqrt{7}}{2}i$$

The solutions are $-2\pm\sqrt{2}$ and $\dfrac{1}{2}\pm\dfrac{\sqrt{7}}{2}i$.

136. $\dfrac{1}{2}at+v_0t+x_0=0$

Use the quadratic formula. Here $a=\dfrac{1}{2}a$, $b=v_0$, and $c=x_0$.

$$t=\frac{-v_0\pm\sqrt{(v_0)^2-4\cdot\frac{1}{2}a\cdot x_0}}{2\cdot\frac{1}{2}a}$$
$$t=\frac{-v_0\pm\sqrt{v_0^2-2ax_0}}{a}$$

Exercise Set 3.3

1. a) The minimum function value occurs at the vertex, so the vertex is $\left(-\dfrac{1}{2},-\dfrac{9}{4}\right)$.

b) The axis of symmetry is a vertical line through the vertex. It is $x=-\dfrac{1}{2}$.

c) The minimum value of the function is $-\dfrac{9}{4}$.

2. a) $\left(-\dfrac{1}{2},\dfrac{25}{4}\right)$

b) Axis of symmetry: $x=-\dfrac{1}{2}$

c) Maximum: $\dfrac{25}{4}$

3. $f(x)=x^2-8x+12$ 16 completes the square for x^2-8x.

$=x^2-8x+16-16+12$ Adding $16-16$ on the right side

$=(x^2-8x+16)-16+12$

$=(x-4)^2-4$ Factoring and simplifying

$=(x-4)^2+(-4)$ Writing in the form $f(x)=a(x-h)^2+k$

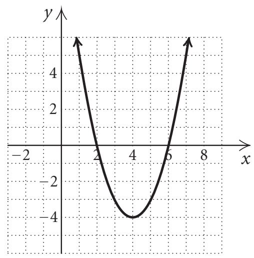
a) Vertex: $(4,-4)$

b) Axis of symmetry: $x=4$

c) Minimum value: -4

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
4	-4
2	0
1	5
5	-3
6	0



$f(x) = x^2 - 8x + 12$

4. $g(x) = x^2 + 7x - 8$

$= x^2 + 7x + \frac{49}{4} - \frac{49}{4} - 8 \quad \left(\frac{1}{2} \cdot 7 = \frac{7}{2} \text{ and } \left(\frac{7}{2}\right)^2 = \frac{49}{4}\right)$

$= \left(x + \frac{7}{2}\right)^2 - \frac{81}{4}$

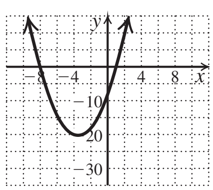
$= \left[x - \left(-\frac{7}{2}\right)\right]^2 + \left(-\frac{81}{4}\right)$

a) Vertex: $\left(-\frac{7}{2}, -\frac{81}{4}\right)$

b) Axis of symmetry: $x = -\frac{7}{2}$

c) Minimum value: $-\frac{81}{4}$

d)



$g(x) = x^2 + 7x - 8$

5. $f(x) = x^2 - 7x + 12 \qquad \frac{49}{4}$ completes the square for $x^2 - 7x$.

$= x^2 - 7x + \frac{49}{4} - \frac{49}{4} + 12$ Adding $\frac{49}{4} - \frac{49}{4}$ on the right side

$= \left(x^2 - 7x + \frac{49}{4}\right) - \frac{49}{4} + 12$

$= \left(x - \frac{7}{2}\right)^2 - \frac{1}{4}$ Factoring and simplifying

$= \left(x - \frac{7}{2}\right)^2 + \left(-\frac{1}{4}\right)$ Writing in the form $f(x) = a(x-h)^2 + k$

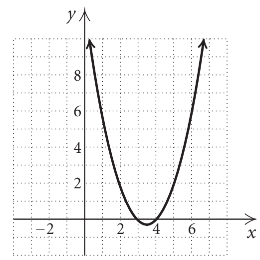
a) Vertex: $\left(\frac{7}{2}, -\frac{1}{4}\right)$

b) Axis of symmetry: $x = \frac{7}{2}$

c) Minimum value: $-\frac{1}{4}$

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
$\frac{7}{2}$	$-\frac{1}{4}$
4	0
5	2
3	0
1	6



$f(x) = x^2 - 7x + 12$

6. $g(x) = x^2 - 5x + 6$

$= x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 6 \quad \left(\frac{1}{2}(-5) = -\frac{5}{2} \text{ and } \left(-\frac{5}{2}\right)^2 = \frac{25}{4}\right)$

$= \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$

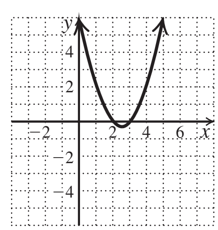
$= \left(x - \frac{5}{2}\right)^2 + \left(-\frac{1}{4}\right)$

a) Vertex: $\left(\frac{5}{2}, -\frac{1}{4}\right)$

b) Axis of symmetry: $x = \frac{5}{2}$

c) Minimum value: $-\frac{1}{4}$

d)



$g(x) = x^2 - 5x + 6$

7. $f(x) = x^2 + 4x + 5 \qquad$ 4 completes the square for $x^2 + 4x$

$= x^2 + 4x + 4 - 4 + 5$ Adding $4 - 4$ on the right side

$= (x + 2)^2 + 1$ Factoring and simplifying

$= [x - (-2)]^2 + 1$ Writing in the form $f(x) = a(x-h)^2 + k$

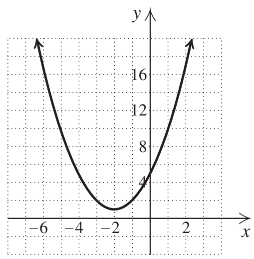
a) Vertex: $(-2, 1)$

b) Axis of symmetry: $x = -2$

c) Minimum value: 1

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
-2	1
-1	2
0	5
-3	2
-4	5



$f(x) = x^2 + 4x + 5$

8. $f(x) = x^2 + 2x + 6$

$= x^2 + 2x + 1 - 1 + 6 \quad \left(\frac{1}{2} \cdot 2 = 1 \text{ and } 1^2 = 1\right)$

$= (x+1)^2 + 5$

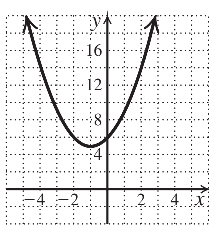
$= [x - (-1)]^2 + 5$

a) Vertex: $(-1, 5)$

b) Axis of symmetry: $x = -1$

c) Minimum value: 5

d)



$f(x) = x^2 + 2x + 6$

9. $g(x) = \frac{x^2}{2} + 4x + 6$

$= \frac{1}{2}(x^2 + 8x) + 6$ Factoring $\frac{1}{2}$ out of the first two terms

$= \frac{1}{2}(x^2 + 8x + 16 - 16) + 6$ Adding $16 - 16$ inside the parentheses

$= \frac{1}{2}(x^2 + 8x + 16) - \frac{1}{2} \cdot 16 + 6$ Removing -16 from within the parentheses

$= \frac{1}{2}(x+4)^2 - 2$ Factoring and simplifying

$= \frac{1}{2}[x - (-4)]^2 + (-2)$

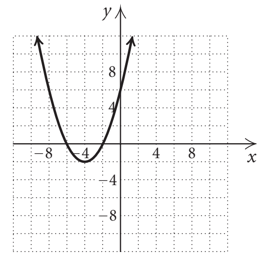
a) Vertex: $(-4, -2)$

b) Axis of symmetry: $x = -4$

c) Minimum value: -2

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$g(x)$
-4	-2
-2	0
0	6
-6	0
-8	6



$g(x) = \frac{x^2}{2} + 4x + 6$

10. $g(x) = \frac{x^2}{3} - 2x + 1$

$= \frac{1}{3}(x^2 - 6x) + 1$

$= \frac{1}{3}(x^2 - 6x + 9 - 9) + 1$

$= \frac{1}{3}(x^2 - 6x + 9) - \frac{1}{3} \cdot 9 + 1$

$= \frac{1}{3}(x-3)^2 - 2$

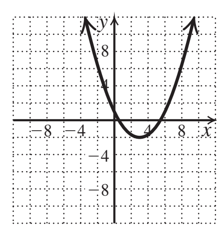
$= \frac{1}{3}(x-3)^2 + (-2)$

a) Vertex: $(3, -2)$

b) Axis of symmetry: $x = 3$

c) Minimum value: -2

d)



$g(x) = \frac{x^2}{3} - 2x + 1$

11. $g(x) = 2x^2 + 6x + 8$

$= 2(x^2 + 3x) + 8$ Factoring 2 out of the first two terms

$= 2\left(x^2 + 3x + \frac{9}{4} - \frac{9}{4}\right) + 8$ Adding $\frac{9}{4} - \frac{9}{4}$ inside the parentheses

$= 2\left(x^2 + 3x + \frac{9}{4}\right) - 2 \cdot \frac{9}{4} + 8$ Removing $-\frac{9}{4}$ from within the parentheses

$= 2\left(x + \frac{3}{2}\right)^2 + \frac{7}{2}$ Factoring and simplifying

$= 2\left[x - \left(-\frac{3}{2}\right)\right]^2 + \frac{7}{2}$

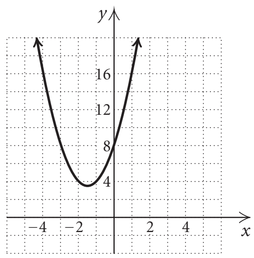
a) Vertex: $\left(-\frac{3}{2}, \frac{7}{2}\right)$

b) Axis of symmetry: $x = -\frac{3}{2}$

c) Minimum value: $\frac{7}{2}$

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
$-\frac{3}{2}$	$\frac{7}{2}$
-1	4
0	8
-2	4
-3	8



$g(x) = 2x^2 + 6x + 8$

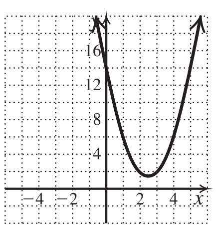
12. $f(x) = 2x^2 - 10x + 14$
$= 2(x^2 - 5x) + 14$
$= 2\left(x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right) + 14$
$= 2\left(x^2 - 5x + \frac{25}{4}\right) - 2 \cdot \frac{25}{4} + 14$
$= 2\left(x - \frac{5}{2}\right)^2 + \frac{3}{2}$

a) Vertex: $\left(\frac{5}{2}, \frac{3}{2}\right)$

b) Axis of symmetry: $x = \frac{5}{2}$

c) Minimum value: $\frac{3}{2}$

d)



$f(x) = 2x^2 - 10x + 14$

13. $f(x) = -x^2 - 6x + 3$
$= -(x^2 + 6x) + 3$ 9 completes the square for $x^2 + 6x$.
$= -(x^2 + 6x + 9 - 9) + 3$
$= -(x + 3)^2 - (-9) + 3$ Removing -9 from the parentheses
$= -(x + 3)^2 + 9 + 3$
$= -[x - (-3)]^2 + 12$

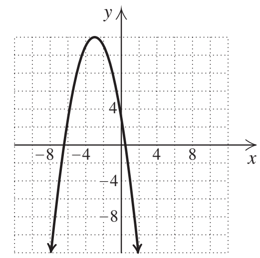
a) Vertex: $(-3, 12)$

b) Axis of symmetry: $x = -3$

c) Maximum value: 12

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
-3	12
0	3
1	-4
-6	3
-7	-4



$f(x) = -x^2 - 6x + 3$

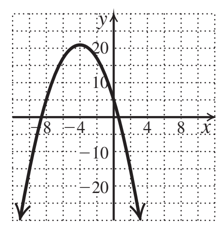
14. $f(x) = -x^2 - 8x + 5$
$= -(x^2 + 8x) + 5$
$= -(x^2 + 8x + 16 - 16) + 5$
$\left(\frac{1}{2} \cdot 8 = 4 \textit{ and } 4^2 = 16\right)$
$= -(x^2 + 8x + 16) - (-16) + 5$
$= -(x^2 + 8x + 16) + 21$
$= -[x - (-4)]^2 + 21$

a) Vertex: $(-4, 21)$

b) Axis of symmetry: $x = -4$

c) Maximum value: 21

d)



$f(x) = -x^2 - 8x + 5$

15. $g(x) = -2x^2 + 2x + 1$
$= -2(x^2 - x) + 1$ Factoring -2 out of the first two terms
$= -2\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right) + 1$ Adding $\frac{1}{4} - \frac{1}{4}$ inside the parentheses
$= -2\left(x^2 - x + \frac{1}{4}\right) - 2\left(-\frac{1}{4}\right) + 1$
Removing $-\frac{1}{4}$ from within the parentheses
$= -2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$

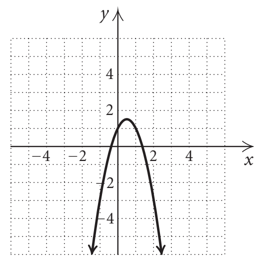
a) Vertex: $\left(\frac{1}{2}, \frac{3}{2}\right)$

b) Axis of symmetry: $x = \frac{1}{2}$

c) Maximum value: $\frac{3}{2}$

d) We plot the vertex and find several points on either side of it. Then we plot these points and connect them with a smooth curve.

x	$f(x)$
$\frac{1}{2}$	$\frac{3}{2}$
1	1
2	-3
0	1
-1	-3



$g(x) = -2x^2 + 2x + 1$

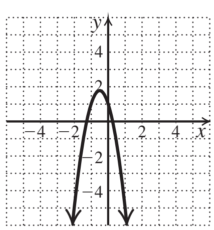
16.
$$\begin{aligned} f(x) &= -3x^2 - 3x + 1 \\ &= -3(x^2 + x) + 1 \\ &= -3\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 1 \\ &= -3\left(x^2 + x + \frac{1}{4}\right) - 3\left(-\frac{1}{4}\right) + 1 \\ &= -3\left(x + \frac{1}{2}\right)^2 + \frac{7}{4} \\ &= -3\left[x - \left(-\frac{1}{2}\right)\right]^2 + \frac{7}{4} \end{aligned}$$

a) Vertex: $\left(-\frac{1}{2}, \frac{7}{4}\right)$

b) Axis of symmetry: $x = -\frac{1}{2}$

c) Maximum value: $\frac{7}{4}$

d)



$f(x) = -3x^2 - 3x + 1$

17. The graph of $y = (x + 3)^2$ has vertex $(-3, 0)$ and opens up. It is graph (f).

18. The graph of $y = -(x-4)^2 + 3$ has vertex $(4, 3)$ and opens down. It is graph (e).

19. The graph of $y = 2(x - 4)^2 - 1$ has vertex $(4, -1)$ and opens up. It is graph (b).

20. The graph of $y = x^2 - 3$ has vertex $(0, -3)$ and opens up. It is graph (g).

21. The graph of $y = -\frac{1}{2}(x + 3)^2 + 4$ has vertex $(-3, 4)$ and opens down. It is graph (h).

22. The graph of $y = (x - 3)^2$ has vertex $(3, 0)$ and opens up. It is graph (a).

23. The graph of $y = -(x + 3)^2 + 4$ has vertex $(-3, 4)$ and opens down. It is graph (c).

24. The graph of $y = 2(x - 1)^2 - 4$ has vertex $(1, -4)$ and opens up. It is graph (d).

25. The function $f(x) = -3x^2 + 2x + 5$ is of the form $f(x) = ax^2 + bx + c$ with $a < 0$, so it is true that it has a maximum value.

26. The statement is false. While $-\frac{b}{2a}$ is the first coordinate of the vertex, the vertex is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

27. The statement is false. The graph of $h(x) = (x + 2)^2$ can be obtained by translating the graph of $h(x) = x^2$ two units to the *left*.

28. The statement is false. The vertex of the graph of the function $g(x) = 2(x - 4)^2 - 1$ is $(4, -1)$.

29. The function $f(x) = -(x + 2)^2 - 4$ can be written as $f(x) = -[x - (-2)]^2 - 4$, so it is true that the axis of symmetry is $x = -2$.

30. The function $f(x) = 3(x - 1)^2 + 5$ is of the form $f(x) = a(x - h)^2 + k$ with $a > 0$, so it is true that the function has a minimum value and that value is k, or 5.

31. $f(x) = x^2 - 6x + 5$

a) The x-coordinate of the vertex is
$$-\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3.$$
Since $f(3) = 3^2 - 6 \cdot 3 + 5 = -4$, the vertex is $(3, -4)$.

b) Since $a = 1 > 0$, the graph opens up so the second coordinate of the vertex, -4, is the minimum value of the function.

c) The range is $[-4, \infty)$.

d) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(3, \infty)$ and decreasing on $(-\infty, 3)$.

32. $f(x) = x^2 + 4x - 5$

a) $-\frac{b}{2a} = -\frac{4}{2 \cdot 1} = -2$

$f(-2) = (-2)^2 + 4(-2) - 5 = -9$

The vertex is $(-2, -9)$.

b) Since $a = 1 > 0$, the graph opens up. The minimum value of $f(x)$ is -9.

c) Range: $[-9, \infty)$

d) Increasing: $(-2, \infty)$; decreasing: $(-\infty, -2)$

33. $f(x) = 2x^2 + 4x - 16$

a) The x-coordinate of the vertex is
$$-\frac{b}{2a} = -\frac{4}{2 \cdot 2} = -1.$$
Since $f(-1) = 2(-1)^2 + 4(-1) - 16 = -18$, the vertex is $(-1, -18)$.

b) Since $a = 2 > 0$, the graph opens up so the second coordinate of the vertex, -18, is the minimum value of the function.

c) The range is $[-18, \infty)$.

d) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

34. $f(x) = \frac{1}{2}x^2 - 3x + \frac{5}{2}$

a) $-\frac{b}{2a} = -\frac{-3}{2 \cdot \frac{1}{2}} = 3$

$f(3) = \frac{1}{2} \cdot 3^2 - 3 \cdot 3 + \frac{5}{2} = -2$

The vertex is $(3, -2)$.

b) Since $a = \frac{1}{2} > 0$, the graph opens up. The minimum value of $f(x)$ is -2.

c) Range: $[-2, \infty)$

d) Increasing: $(3, \infty)$; decreasing: $(-\infty, 3)$

35. $f(x) = -\frac{1}{2}x^2 + 5x - 8$

a) The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{5}{2\left(-\frac{1}{2}\right)} = 5.$$

Since $f(5) = -\frac{1}{2} \cdot 5^2 + 5 \cdot 5 - 8 = \frac{9}{2}$, the vertex is $\left(5, \frac{9}{2}\right)$.

b) Since $a = -\frac{1}{2} < 0$, the graph opens down so the second coordinate of the vertex, $\frac{9}{2}$, is the maximum value of the function.

c) The range is $\left(-\infty, \frac{9}{2}\right]$.

d) Since the graph opens down, function values increase to the left of the vertex and decrease to the right of the vertex. Thus, $f(x)$ is increasing on $(-\infty, 5)$ and decreasing on $(5, \infty)$.

36. $f(x) = -2x^2 - 24x - 64$

a) $-\frac{b}{2a} = -\frac{-24}{2(-2)} = -6$.

$f(-6) = -2(-6)^2 - 24(-6) - 64 = 8$

The vertex is $(-6, 8)$.

b) Since $a = -2 < 0$, the graph opens down. The maximum value of $f(x)$ is 8.

c) Range: $(-\infty, 8]$

d) Increasing: $(-\infty, -6)$; decreasing: $(-6, \infty)$

37. $f(x) = 3x^2 + 6x + 5$

a) The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{6}{2 \cdot 3} = -1.$$

Since $f(-1) = 3(-1)^2 + 6(-1) + 5 = 2$, the vertex is $(-1, 2)$.

b) Since $a = 3 > 0$, the graph opens up so the second coordinate of the vertex, 2, is the minimum value of the function.

c) The range is $[2, \infty)$.

d) Since the graph opens up, function values decrease to the left of the vertex and increase to the right of the vertex. Thus, $f(x)$ is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

38. $f(x) = -3x^2 + 24x - 49$

a) $-\frac{b}{2a} = -\frac{24}{2(-3)} = 4$.

$f(4) = -3 \cdot 4^2 + 24 \cdot 4 - 49 = -1$

The vertex is $(4, -1)$.

b) Since $a = -3 < 0$, the graph opens down. The maximum value of $f(x)$ is -1.

c) Range: $(-\infty, -1]$

d) Increasing: $(-\infty, 4)$; decreasing: $(4, \infty)$

39. $g(x) = -4x^2 - 12x + 9$

a) The x-coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{-12}{2(-4)} = -\frac{3}{2}.$$

Since $g\left(-\frac{3}{2}\right) = -4\left(-\frac{3}{2}\right)^2 - 12\left(-\frac{3}{2}\right) + 9 = 18$, the vertex is $\left(-\frac{3}{2}, 18\right)$.

b) Since $a = -4 < 0$, the graph opens down so the second coordinate of the vertex, 18, is the maximum value of the function.

c) The range is $(-\infty, 18]$.

d) Since the graph opens down, function values increase to the left of the vertex and decrease to the right of the vertex. Thus, $g(x)$ is increasing on $\left(-\infty, -\frac{3}{2}\right)$ and decreasing on $\left(-\frac{3}{2}, \infty\right)$.

40. $g(x) = 2x^2 - 6x + 5$

a) $-\frac{b}{2a} = -\frac{-6}{2 \cdot 2} = \frac{3}{2}$

$g\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 5 = \frac{1}{2}$

The vertex is $\left(\frac{3}{2}, \frac{1}{2}\right)$.

b) Since $a = 2 > 0$, the graph opens up. The minimum value of $g(x)$ is $\frac{1}{2}$.

c) Range: $\left[\frac{1}{2}, \infty\right)$

d) Increasing: $\left(\frac{3}{2}, \infty\right)$; decreasing: $\left(-\infty, \frac{3}{2}\right)$

41. ***Familiarize and Translate***. The function $s(t) = -16t^2 + 20t + 6$ is given in the statement of the problem.

Carry out. The function $s(t)$ is quadratic and the coefficient of t^2 is negative, so $s(t)$ has a maximum value. It occurs at the vertex of the graph of the function. We find the first coordinate of the vertex. This is the time at which the ball reaches its maximum height.

$$t = -\frac{b}{2a} = -\frac{20}{2(-16)} = 0.625$$

The second coordinate of the vertex gives the maximum height.

$$s(0.625) = -16(0.625)^2 + 20(0.625) + 6 = 12.25$$

Check. Completing the square, we write the function in the form $s(t) = -16(t - 0.625)^2 + 12.25$. We see that the coordinates of the vertex are $(0.625, 12.25)$, so the answer checks.

State. The ball reaches its maximum height after 0.625 seconds. The maximum height is 12.25 ft.

42. Find the first coordinate of the vertex:

$$t = -\frac{60}{2(-16)} = 1.875$$

Then $s(1.875) = -16(1.875)^2 + 60(1.875) + 30 = 86.25$. Thus the maximum height is reached after 1.875 sec. The maximum height is 86.25 ft.

43. ***Familiarize and Translate***. The function $s(t) = -16t^2 + 120t + 80$ is given in the statement of the problem.

Carry out. The function $s(t)$ is quadratic and the coefficient of t^2 is negative, so $s(t)$ has a maximum value. It occurs at the vertex of the graph of the function. We find the first coordinate of the vertex. This is the time at which the rocket reaches its maximum height.

$$t = -\frac{b}{2a} = -\frac{120}{2(-16)} = 3.75$$

The second coordinate of the vertex gives the maximum height.

$$s(3.75) = -16(3.75)^2 + 120(3.75) + 80 = 305$$

Check. Completing the square, we write the function in the form $s(t) = -16(t - 3.75)^2 + 305$. We see that the coordinates of the vertex are $(3.75, 305)$, so the answer checks.

State. The rocket reaches its maximum height after 3.75 seconds. The maximum height is 305 ft.

44. Find the first coordinate of the vertex:

$$t = -\frac{150}{2(-16)} = 4.6875$$

Then $s(4.6875) = -16(4.6875)^2 + 150(4.6875) + 40 = 391.5625$. Thus the maximum height is reached after 4.6875 sec. The maximum height is 391.5625 ft.

45. ***Familiarize***. Using the label in the text, we let $x =$ the height of the file. Then the length $= 10$ and the width $= 18 - 2x$.

Translate. Since the volume of a rectangular solid is length $\times$ width $\times$ height we have

$$V(x) = 10(18 - 2x)x, \text{ or } -20x^2 + 180x.$$

Carry out. Since $V(x)$ is a quadratic function with $a = -20 < 0$, the maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{180}{2(-20)} = 4.5.$$

Check. When $x = 4.5$, then $18 - 2x = 9$ and $V(x) = 10 \cdot 9(4.5)$, or 405. As a partial check, we can find $V(x)$ for a value of x less than 4.5 and for a value of x greater than 4.5. For instance, $V(4.4) = 404.8$ and $V(4.6) = 404.8$. Since both of these values are less than 405, our result appears to be correct.

State. The file should be 4.5 in. tall in order to maximize the volume.

46. Let $w =$ the width of the garden. Then the length $= 32 - 2w$ and the area is given by $A(w) = (32 - 2w)w$, or $-2w^2 + 32w$. The maximum function value occurs at the vertex of the graph of $A(w)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{32}{2(-2)} = 8.$$

When $w = 8$, then $32 - 2w = 16$ and the area is $16 \cdot 8$, or 128 ft^2. A garden with dimensions 8 ft by 16 ft yields this area.

47. ***Familiarize***. Let $b =$ the length of the base of the triangle. Then the height $= 20 - b$.

Translate. Since the area of a triangle is $\frac{1}{2} \times$ base$\times$height, we have

$$A(b) = \frac{1}{2}b(20 - b), \text{ or } -\frac{1}{2}b^2 + 10b.$$

Carry out. Since $A(b)$ is a quadratic function with $a = -\frac{1}{2} < 0$, the maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{10}{2\left(-\frac{1}{2}\right)} = 10.$$

When $b = 10$, then $20 - b = 20 - 10 = 10$, and the area is $\frac{1}{2} \cdot 10 \cdot 10 = 50$ cm^2.

Check. As a partial check, we can find $A(b)$ for a value of b less than 10 and for a value of b greater than 10. For instance, $V(9.9) = 49.995$ and $V(10.1) = 49.995$. Since both of these values are less than 50, our result appears to be correct.

State. The area is a maximum when the base and the height are both 10 cm.

48. Let b = the length of the base. Then $69 - b$ = the height and $A(b) = b(69-b)$, or $-b^2+69b$. The maximum function value occurs at the vertex of the graph of $A(b)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{69}{2(-1)} = 34.5.$$

When $b = 34.5$, then $69-b = 34.5$. The area is a maximum when the base and height are both 34.5 cm.

49. $C(x) = 0.1x^2 - 0.7x + 1.625$

Since $C(x)$ is a quadratic function with $a = 0.1 > 0$, a minimum function value occurs at the vertex of the graph of $C(x)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{-0.7}{2(0.1)} = 3.5.$$

Thus, 3.5 hundred, or 350 chairs should be built to minimize the average cost per chair.

50. $P(x) = R(x) - C(x)$

$P(x) = 5x - (0.001x^2 + 1.2x + 60)$

$P(x) = -0.001x^2 + 3.8x - 60$

Since $P(x)$ is a quadratic function with $a = -0.001 < 0$, a maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{3.8}{2(-0.001)} = 1900.$$

$P(1900) = -0.001(1900)^2 + 3.8(1900) - 60 = 3550$

Thus, the maximum profit is \$3550. It occurs when 1900 units are sold.

51. $P(x) = R(x) - C(x)$

$P(x) = (50x - 0.5x^2) - (10x + 3)$

$P(x) = -0.5x^2 + 40x - 3$

Since $P(x)$ is a quadratic function with $a = -0.5 < 0$, a maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{40}{2(-0.5)} = 40.$$

$P(40) = -0.5(40)^2 + 40 \cdot 40 - 3 = 797$

Thus, the maximum profit is \$797. It occurs when 40 units are sold.

52. $P(x) = R(x) - C(x)$

$P(x) = 20x - 0.1x^2 - (4x + 2)$

$P(x) = -0.1x^2 + 16x - 2$

Since $P(x)$ is a quadratic function with $a = -0.1 < 0$, a maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{16}{2(-0.1)} = 80.$$

$P(80) = -0.1(80)^2 + 16(80) - 2 = 638$

Thus, the maximum profit is \$638. It occurs when 80 units are sold.

53. ***Familiarize***. Using the labels on the drawing in the text, we let x = the width of each corral and $240 - 3x$ = the total length of the corrals.

Translate. Since the area of a rectangle is length × width, we have

$$A(x) = (240 - 3x)x = -3x^2 + 240x.$$

Carry out. Since $A(x)$ is a quadratic function with $a = -3 < 0$, the maximum function value occurs at the vertex of the graph of $A(x)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{240}{2(-3)} = 40.$$

$A(40) = -3(40)^2 + 240(40) = 4800$

Check. As a partial check we can find $A(x)$ for a value of x less than 40 and for a value of x greater than 40. For instance, $A(39.9) = 4799.97$ and $A(40.1) = 4799.97$. Since both of these values are less than 4800, our result appears to be correct.

State. The largest total area that can be enclosed is 4800 yd^2.

54. $\frac{1}{2} \cdot 2\pi x + 2x + 2y = 24$, so $y = 12 - \frac{\pi x}{2} - x$.

$$A(x) = \frac{1}{2} \cdot \pi x^2 + 2x\left(12 - \frac{\pi x}{2} - x\right)$$

$$A(x) = \frac{\pi x^2}{2} + 24x - \pi x^2 - 2x^2$$

$$A(x) = 24x - \frac{\pi x^2}{2} - 2x^2, \text{ or } 24x - \left(\frac{\pi}{2} + 2\right)x^2$$

Since $A(x)$ is a quadratic function with $a = -\left(\frac{\pi}{2} + 2\right) < 0$, the maximum function value occurs at the vertex of the graph of $A(x)$. The first coordinate of the vertex is

$$\frac{-b}{2a} = -\frac{24}{2\left[-\left(\frac{\pi}{2} + 2\right)\right]} = \frac{24}{\pi + 4}.$$

When $x = \frac{24}{\pi + 4}$, then $y = \frac{24}{\pi + 4}$. Thus, the maximum amount of light will enter when the dimensions of the rectangular part of the window are $2x$ by y, or $\frac{48}{\pi + 4}$ ft by $\frac{24}{\pi + 4}$ ft, or approximately 6.72 ft by 3.36 ft.

55. ***Familiarize***. We let s = the height of the elevator shaft, t_1 = the time it takes the screwdriver to reach the bottom of the shaft, and t_2 = the time it takes the sound to reach the top of the shaft.

Translate. We know that $t_1 + t_2 = 5$. Using the information in Example 7 we also know that

$$s = 16t_1^2, \quad \textit{or} \quad t_1 = \frac{\sqrt{s}}{4} \text{ and}$$

$$s = 1100t_2, \quad \textit{or} \quad t_2 = \frac{s}{1100}.$$

Then $\frac{\sqrt{s}}{4} + \frac{s}{1100} = 5$.

Carry out. We solve the last equation above.

$$\frac{\sqrt{s}}{4}+\frac{s}{1100}=5$$
$$275\sqrt{s}+s=5500 \quad \text{Multiplying by 1100}$$
$$s+275\sqrt{s}-5500=0$$

Let $u=\sqrt{s}$ and substitute.

$$u^2+275u-5500=0$$
$$u=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{We only want the positive solution.}$$
$$=\frac{-275+\sqrt{275^2-4\cdot 1(-5500)}}{2\cdot 1}$$
$$=\frac{-275+\sqrt{97,625}}{2}\approx 18.725$$

Since $u\approx 18.725$, we have $\sqrt{s}=18.725$, so $s\approx 350.6$.

Check. If $s\approx 350.6$, then $t_1=\frac{\sqrt{s}}{4}=\frac{\sqrt{350.6}}{4}\approx$ 4.68 and $t_2=\frac{s}{1100}=\frac{350.6}{1100}\approx 0.32$, so $t_1+t_2=4.68+0.32=5$.

The result checks.

State. The elevator shaft is about 350.6 ft tall.

56. Let s = the height of the cliff, t_1 = the time it takes the balloon to hit the ground, and t_2 = the time it takes for the sound to reach the top of the cliff. Then we have

$$t_1+t_2=3,$$
$$s=16t_1^2, \quad or \quad t_1=\frac{\sqrt{s}}{4}, \text{ and}$$
$$s=1100t_2, \quad or \quad t_2=\frac{s}{1100}, \text{ so}$$
$$\frac{\sqrt{s}}{4}+\frac{s}{1100}=3.$$

Solving the last equation, we find that $s\approx 132.7$ ft.

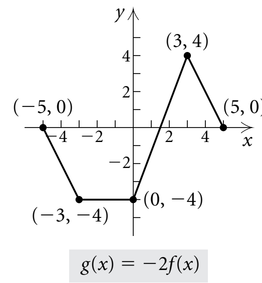
57. $f(x)=3x-7$

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)-7-(3x-7)}{h}$$
$$=\frac{3x+3h-7-3x+7}{h}$$
$$=\frac{3h}{h}=3$$

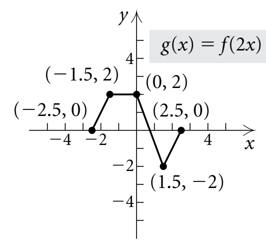
58. $f(x)=2x^2-x+4$

$$f(x+h)=2(x+h)^2-(x+h)+4$$
$$=2(x^2+2xh+h^2)-(x+h)+4$$
$$=2x^2+4xh+2h^2-x-h+4$$
$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2x^2+4xh+2h^2-x-h+4-(2x^2-x+4)}{h}$$
$$=\frac{2x^2+4xh+2h^2-x-h+4-2x^2+x-4}{h}$$
$$=\frac{4xh+2h^2-h}{h}=\frac{h(4x+2h-1)}{h}$$
$$=4x+2h-1$$

59. The graph of $f(x)$ is stretched vertically and reflected across the x-axis.



60.



61. $f(x)=-0.2x^2-3x+c$

The x-coordinate of the vertex of $f(x)$ is $-\frac{b}{2a}=-\frac{-3}{2(-0.2)}=-7.5$. Now we find c such that $f(-7.5)=-225$.

$$-0.2(-7.5)^2-3(-7.5)+c=-225$$
$$-11.25+22.5+c=-225$$
$$c=-236.25$$

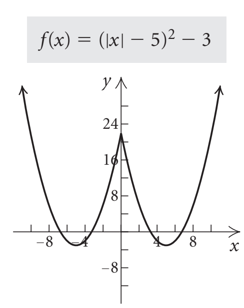
62. $f(x)=-4x^2+bx+3$

The x-coordinate of the vertex of $f(x)$ is $-\frac{b}{2(-4)}$, or $\frac{b}{8}$.

Now we find b such that $f\left(\frac{b}{8}\right)=50$.

$$-4\left(\frac{b}{8}\right)^2+b\cdot\frac{b}{8}+3=50$$
$$-\frac{b^2}{16}+\frac{b^2}{8}+3=50$$
$$\frac{b^2}{16}=47$$
$$b^2=16\cdot 47$$
$$b=\pm\sqrt{16\cdot 47}$$
$$b=\pm 4\sqrt{47}$$

63.



64. $f(x) = a(x-h)^2 + k$

$1 = a(-3-4)^2 - 5$, so $a = \frac{6}{49}$. Then

$f(x) = \frac{6}{49}(x-4)^2 - 5$.

65. First we find the radius r of a circle with circumference x:

$$2\pi r = x$$
$$r = \frac{x}{2\pi}$$

Then we find the length s of a side of a square with perimeter $24 - x$:

$$4s = 24 - x$$
$$s = \frac{24-x}{4}$$

Then S = area of circle + area of square

$$S = \pi r^2 + s^2$$
$$S(x) = \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{24-x}{4}\right)^2$$
$$S(x) = \left(\frac{1}{4\pi} + \frac{1}{16}\right)x^2 - 3x + 36$$

Since $S(x)$ is a quadratic function with $a = \frac{1}{4\pi} + \frac{1}{16} > 0$, the minimum function value occurs at the vertex of the graph of $S(x)$. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{-3}{2\left(\frac{1}{4\pi} + \frac{1}{16}\right)} = \frac{24\pi}{4+\pi}.$$

Then the string should be cut so that one piece is $\frac{24\pi}{4+\pi}$ in., or about 10.56 in. The other piece will be $24 - \frac{24\pi}{4+\pi}$, or $\frac{96}{4+\pi}$ in., or about 13.44 in.

Chapter 3 Mid-Chapter Mixed Review

1. The statement is true. See page 238 in the text.

2. The statement is false. See page 244 in the text.

3. The statement is true. See page 248 in the text.

4. For $f(x) = 3(x+4)^2+5 = 3[x-(-4)]^2+5$, we have $h = -4$ and $k = 5$, so the vertex is $(-4, 5)$. The given statement is false.

5. $\sqrt{-36} = \sqrt{-1 \cdot 36} = \sqrt{-1} \cdot \sqrt{36} = i \cdot 6 = 6i$

6. $\sqrt{-5} = \sqrt{-1 \cdot 5} = \sqrt{-1} \cdot \sqrt{5} = i\sqrt{5}$, or $\sqrt{5}i$

7. $-\sqrt{-16} = -\sqrt{-1 \cdot 16} = -\sqrt{-1} \cdot \sqrt{16} = -i \cdot 4 = -4i$

8. $\sqrt{-32} = \sqrt{-1 \cdot 32} = i\sqrt{32} = i\sqrt{16 \cdot 2} = 4i\sqrt{2}$, or $4\sqrt{2}i$

9. $(3-2i) + (-4+3i) = (3-4) + (-2i+3i) = -1+i$

10.
$$\begin{aligned}(-5+i) - (2-4i) &= (-5-2) + [i - (-4i)]\\ &= -7 + (i+4i)\\ &= -7+5i\end{aligned}$$

11.
$$\begin{aligned}(2+3i)(4-5i) &= 8 - 10i + 12i - 15i^2\\ &= 8 + 2i + 15\\ &= 23 + 2i\end{aligned}$$

12.
$$\begin{aligned}\frac{3+i}{-2+5i} &= \frac{3+i}{-2+5i} \cdot \frac{-2-5i}{-2-5i}\\ &= \frac{-6-17i-5i^2}{4-25i^2}\\ &= \frac{-6-17i+5}{4+25}\\ &= \frac{-1-17i}{29} = -\frac{1}{29} - \frac{17}{29}i\end{aligned}$$

13. $i^{13} = i^{12} \cdot i = (i^2)^6 \cdot i = (-1)^6 \cdot i = i$

14. $i^{44} = (i^2)^{22} = (-1)^{22} = 1$

15. $(-i)^5 = (-1 \cdot i)^5 = (-1)^5 i^5 = -i^4 \cdot i = -(i^2)^2 \cdot i = -(-1)^2 \cdot i = -i$

16. $(2i)^6 = 2^6 \cdot i^6 = 64(i^2)^3 = 64(-1)^3 = -64$

17.
$$x^2 + 3x - 4 = 0$$
$$(x+4)(x-1) = 0$$
$$x + 4 = 0 \quad or \quad x - 1 = 0$$
$$x = -4 \quad or \quad x = 1$$

The solutions are -4 and 1.

18.
$$2x^2 + 6 = -7x$$
$$2x^2 + 7x + 6 = 0$$
$$(2x+3)(x+2) = 0$$
$$2x + 3 = 0 \quad or \quad x + 2 = 0$$
$$2x = -3 \quad or \quad x = -2$$
$$x = -\frac{3}{2} \quad or \quad x = -2$$

The solutions are $-\frac{3}{2}$ and -2.

19.
$$4x^2 = 24$$
$$x^2 = 6$$
$$x = \sqrt{6} \; or \; x = -\sqrt{6}$$

The solutions are $\sqrt{6}$ and $-\sqrt{6}$, or $\pm\sqrt{6}$.

20.
$$x^2 + 100 = 0$$
$$x^2 = -100$$
$$x = 10i \; or \; x = -10i$$

The solutions are $10i$ and $-10i$, or $\pm 10i$.

21. $4x^2 - 8x - 3 = 0$

$$4x^2 - 8x = 3$$
$$x^2 - 2x = \frac{3}{4}$$
$$x^2 - 2x + 1 = \frac{3}{4} + 1 \quad \text{Completing the square: } \frac{1}{2}(-2) = -1 \text{ and } (-1)^2 = 1$$
$$(x-1)^2 = \frac{7}{4}$$
$$x - 1 = \pm\frac{\sqrt{7}}{2}$$
$$x = 1 + \frac{\sqrt{7}}{2}$$
$$x = \frac{2 \pm \sqrt{7}}{2}$$

The zeros are $\frac{2+\sqrt{7}}{2}$ and $\frac{2-\sqrt{7}}{4}$, or $\frac{2 \pm \sqrt{7}}{2}$.

22. $x^2 - 3x - 5 = 0$

a) $b^2 - 4ac = (-3)^2 - 4 \cdot 1 \cdot (-5) = 9 + 20 = 29 > 0$

There are two real-number solutions.

b) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-5)}}{2 \cdot 1}$$
$$= \frac{3 \pm \sqrt{29}}{2}$$

The solutions are $\frac{3 \pm \sqrt{29}}{2}$.

We can use a calculator to find decimal approximations of the solutions.

$$\frac{3+\sqrt{29}}{2} \approx 4.193 \text{ and } \frac{3-\sqrt{29}}{2} \approx -1.193$$

23. $4x^2 - 12x + 9 = 0$

a) $b^2 - 4ac = (-12)^2 - 4 \cdot 4 \cdot 9 = 144 - 144 = 0$

There is one real-number solution.

b) $$4x^2 - 12x + 9 = 0$$
$$(2x - 3)^2 = 0$$
$$2x - 3 = 0$$
$$2x = 3$$
$$x = \frac{3}{2}$$

The solution is $\frac{3}{2}$.

24. $3x^2 + 2x = -1$

$$3x^2 + 2x + 1 = 0$$

a) $b^2 - 4ac = 2^2 - 4 \cdot 3 \cdot 1 = 4 - 12 = -8 < 0$

There are two nonreal solutions.

b) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3}$$
$$= \frac{-2 \pm \sqrt{-8}}{2 \cdot 3} = \frac{-2 \pm \sqrt{-4 \cdot 2}}{2 \cdot 3}$$
$$= \frac{-2 \pm 2i\sqrt{2}}{2 \cdot 3} = \frac{2(-1 \pm i\sqrt{2})}{2 \cdot 3}$$
$$= \frac{-1 \pm \sqrt{2}i}{3} = -\frac{1}{3} \pm \frac{\sqrt{2}}{3}i$$

The solutions are $-\frac{1}{3} \pm \frac{\sqrt{2}}{3}i$.

25. $x^4 + 5x^2 - 6 = 0$

Let $u = x^2$.

$$u^2 + 5u - 6 = 0 \quad \text{Substituting}$$
$$(u+6)(u-1) = 0$$
$$u + 6 = 0 \quad or \quad u - 1 = 0$$
$$u = -6 \quad or \quad u = 1$$
$$x^2 = -6 \quad or \quad x^2 = 1$$
$$x = \pm\sqrt{6}i \quad or \quad x = \pm 1$$

The solutions are $\pm\sqrt{6}i$ and ± 1.

26. $2x - 5\sqrt{x} + 2 = 0$

Let $u = \sqrt{x}$.

$$2u^2 - 5u + 2 = 0 \quad \text{Substituting}$$
$$(2u - 1)(u - 2) = 0$$
$$2u - 1 = 0 \quad or \quad u - 2 = 0$$
$$u = \frac{1}{2} \quad or \quad u = 2$$
$$\sqrt{x} = \frac{1}{2} \quad or \quad \sqrt{x} = 2$$
$$x = \frac{1}{4} \quad or \quad x = 4 \quad \text{Squaring both sides}$$

Both numbers check. The solutions are $\frac{1}{4}$ and 4.

27. ***Familiarize***. Let x = the smaller number. Then $x + 2$ = the larger number.

Translate.

$$\underbrace{\text{The product of the numbers}}_{\downarrow} \;\; \underset{\downarrow}{\text{is}} \;\; \underset{\downarrow}{35.}$$
$$x(x+2) \quad = \quad 35$$

Carry out.

$$x(x+2) = 35$$
$$x^2 + 2x = 35$$
$$x^2 + 2x - 35 = 0$$
$$(x+7)(x-5) = 0$$
$$x + 7 = 0 \quad or \quad x - 5 = 0$$
$$x = -7 \quad or \quad x = 5$$

If $x = -7$, then $x + 2 = -7 + 2 = -5$; if $x = 5$, then $x + 2 = 5 + 2 = 7$.

Check. -5 is 2 more than -7, and $(-7)(-5) = 35$. Also, 7 is 2 more than 5, and $5 \cdot 7 = 35$. The numbers check.

State. The numbers are 5 and 7 or -7 and -5.

28.
$$\begin{aligned} f(x) &= x^2 - 6x + 7 \\ &= x^2 - 6x + 9 - 9 + 7 \\ &= (x^2 - 6x + 9) - 9 + 7 \\ &= (x-3)^2 - 2 \\ &= (x-3)^2 + (-2) \end{aligned}$$

a) Vertex: $(3, -2)$

b) Axis of symmetry: $x = 3$

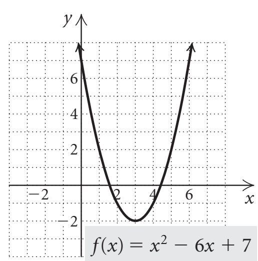
c) Minimum value: -2

d) Range: $[-2, \infty)$

e) Increasing: $(-2, \infty)$; decreasing: $(-\infty, -2)$

f)

x	$f(x)$
3	-2
1	2
2	-1
4	-1
5	2



29.
$$\begin{aligned} f(x) &= -2x^2 - 4x - 5 \\ &= -2(x^2 + 2x) - 5 \\ &= -2(x^2 + 2x + 1 - 1) - 5 \\ &= -2(x^2 + 2x + 1) - 2(-1) - 5 \\ &= -2(x+1)^2 - 3 \\ &= -2[x - (-1)]^2 + (-3) \end{aligned}$$

a) Vertex: $(-1, -3)$

b) Axis of symmetry: $x = -1$

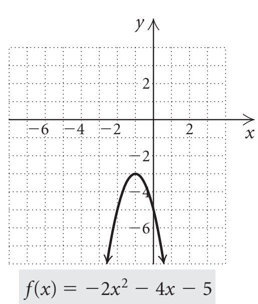
c) Maximum value: -3

d) Range: $(-\infty, -3]$

e) Increasing: $(-\infty, -3)$; decreasing: $(-3, \infty)$

f)

x	$f(x)$
-1	-3
-3	-11
-2	-5
0	-5
1	-11



30. The sum of two imaginary numbers is not always an imaginary number. For example, $(2 + i) + (3 - i) = 5$, a real number.

31. Use the discriminant. If $b^2 - 4ac < 0$, there are no x-intercepts. If $b^2 - 4ac = 0$, there is one x-intercept. If $b^2 - 4ac > 0$, there are two x-intercepts.

32. Completing the square was used in Section 3.2 to solve quadratic equations. It was used again in this section to write quadratic functions in the form $f(x) = a(x-h)^2 + k$.

33. The x-intercepts of $g(x)$ are also $(x_1, 0)$ and $(x_2, 0)$. This is true because $f(x)$ and $g(x)$ have the same zeros. Consider $g(x) = 0$, or $-ax^2 - bx - c = 0$. Multiplying by -1 on both sides, we get an equivalent equation $ax^2 + bx + c = 0$, or $f(x) = 0$.

Exercise Set 3.4

1.
$$\begin{aligned} \frac{1}{4} + \frac{1}{5} &= \frac{1}{t}, \text{ LCD is } 20t \\ 20t\left(\frac{1}{4} + \frac{1}{5}\right) &= 20t \cdot \frac{1}{t} \\ 20t \cdot \frac{1}{4} + 20t \cdot \frac{1}{5} &= 20t \cdot \frac{1}{t} \\ 5t + 4t &= 20 \\ 9t &= 20 \\ t &= \frac{20}{9} \end{aligned}$$

Check:

$$\begin{array}{c|c} \frac{1}{4} + \frac{1}{5} = \frac{1}{t} & \\ \hline \frac{1}{4} + \frac{1}{5} \ ? & \frac{1}{\frac{20}{9}} \\ \frac{5}{20} + \frac{4}{20} & 1 \cdot \frac{9}{20} \\ \frac{9}{20} & \frac{9}{20} \end{array} \quad \text{TRUE}$$

The solution is $\frac{20}{9}$.

2.
$$\begin{aligned} \frac{1}{3} - \frac{5}{6} &= \frac{1}{x}, && \text{LCD is } 6x \\ 2x - 5x &= 6 && \text{Multiplying by } 6x \\ -3x &= 6 \\ x &= -2 \end{aligned}$$

-2 checks. The solution is -2.

3.
$$\begin{aligned} \frac{x+2}{4} - \frac{x-1}{5} &= 15, \text{ LCD is } 20 \\ 20\left(\frac{x+2}{4} - \frac{x-1}{5}\right) &= 20 \cdot 15 \\ 5(x+2) - 4(x-1) &= 300 \\ 5x + 10 - 4x + 4 &= 300 \\ x + 14 &= 300 \\ x &= 286 \end{aligned}$$

The solution is 286.

4. $\dfrac{t+1}{3} - \dfrac{t-1}{2} = 1$, LCD is 6

$2t+2-3t+3 = 6$ Multiplying by 6

$-t = 1$

$t = -1$

The solution is -1.

5. $\dfrac{1}{2} + \dfrac{2}{x} = \dfrac{1}{3} + \dfrac{3}{x}$, LCD is $6x$

$6x\left(\dfrac{1}{2} + \dfrac{2}{x}\right) = 6x\left(\dfrac{1}{3} + \dfrac{3}{x}\right)$

$3x + 12 = 2x + 18$

$3x - 2x = 18 - 12$

$x = 6$

Check:

$\dfrac{1}{2} + \dfrac{2}{x}$	$=$	$\dfrac{1}{3} + \dfrac{3}{x}$	
$\dfrac{1}{2} + \dfrac{2}{6}$	?	$\dfrac{1}{3} + \dfrac{3}{6}$	
$\dfrac{1}{2} + \dfrac{1}{3}$		$\dfrac{1}{3} + \dfrac{1}{2}$	TRUE

The solution is 6.

6. $\dfrac{1}{t} + \dfrac{1}{2t} + \dfrac{1}{3t} = 5$, LCD is $6t$

$6+3+2 = 30t$ Multiplying by $6t$

$11 = 30t$

$\dfrac{11}{30} = t$

$\dfrac{11}{30}$ checks. The solution is $\dfrac{11}{30}$.

7. $\dfrac{5}{3x+2} = \dfrac{3}{2x}$, LCD is $2x(3x+2)$

$2x(3x+2)\cdot\dfrac{5}{3x+2} = 2x(3x+2)\cdot\dfrac{3}{2x}$

$2x\cdot 5 = 3(3x+2)$

$10x = 9x + 6$

$x = 6$

6 checks, so the solution is 6.

8. $\dfrac{2}{x-1} = \dfrac{3}{x+2}$, LCD is $(x-1)(x+2)$

$2(x+2) = 3(x-1)$

$2x + 4 = 3x - 3$

$7 = x$

The answer checks. The solution is 7.

9. $x + \dfrac{6}{x} = 5$, LCD is x

$x\left(x + \dfrac{6}{x}\right) = x\cdot 5$

$x^2 + 6 = 5x$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x - 2 = 0$ *or* $x - 3 = 0$

$x = 2$ *or* $x = 3$

Both numbers check. The solutions are 2 and 3.

10. $x - \dfrac{12}{x} = 1$, LCD is x

$x^2 - 12 = x$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ *or* $x = -3$

Both numbers check. The solutions are 4 and -3.

11. $\dfrac{6}{y+3} + \dfrac{2}{y} = \dfrac{5y-3}{y^2-9}$

$\dfrac{6}{y+3} + \dfrac{2}{y} = \dfrac{5y-3}{(y+3)(y-3)}$,

LCD is $y(y+3)(y-3)$

$y(y+3)(y-3)\left(\dfrac{6}{y+3} + \dfrac{2}{y}\right) = y(y+3)(y-3)\cdot\dfrac{5y-3}{(y+3)(y-3)}$

$6y(y-3) + 2(y+3)(y-3) = y(5y-3)$

$6y^2 - 18y + 2(y^2-9) = 5y^2 - 3y$

$6y^2 - 18y + 2y^2 - 18 = 5y^2 - 3y$

$8y^2 - 18y - 18 = 5y^2 - 3y$

$3y^2 - 15y - 18 = 0$

$y^2 - 5y - 6 = 0$

$(y-6)(y+1) = 0$

$y - 6 = 0$ *or* $y + 1 = 0$

$y = 6$ *or* $y = -1$

Both numbers check. The solutions are 6 and -1.

12. $\dfrac{3}{m+2} + \dfrac{2}{m} = \dfrac{4m-4}{m^2-4}$

$\dfrac{3}{m+2} + \dfrac{2}{m} = \dfrac{4m-4}{(m+2)(m-2)}$,

LCD is $m(m+2)(m-2)$

$3m^2 - 6m + 2m^2 - 8 = 4m^2 - 4m$

Multiplying by $m(m+2)(m-2)$

$m^2 - 2m - 8 = 0$

$(m-4)(m+2) = 0$

$m = 4$ or $m = -2$

Only 4 checks. The solution is 4.

13. $\frac{2x}{x-1} = \frac{5}{x-3}$, LCD is $(x-1)(x-3)$

$$(x-1)(x-3)\cdot\frac{2x}{x-1} = (x-1)(x-3)\cdot\frac{5}{x-3}$$
$$2x(x-3) = 5(x-1)$$
$$2x^2-6x = 5x-5$$
$$2x^2-11x+5 = 0$$
$$(2x-1)(x-5) = 0$$
$$2x-1=0 \quad or \quad x-5=0$$
$$2x=1 \quad or \quad x=5$$
$$x=\frac{1}{2} \quad or \quad x=5$$

Both numbers check. The solutions are $\frac{1}{2}$ and 5.

14. $\frac{2x}{x+7} = \frac{5}{x+1}$, LCD is $(x+7)(x+1)$

$$2x(x+1) = 5(x+7)$$
$$2x^2+2x = 5x+35$$
$$2x^2-3x-35 = 0$$
$$(2x+7)(x-5) = 0$$
$$x=-\frac{7}{2} \quad or \quad x=5$$

Both numbers check. The solutions are $-\frac{7}{2}$ and 5.

15. $\frac{2}{x+5}+\frac{1}{x-5} = \frac{16}{x^2-25}$

$\frac{2}{x+5}+\frac{1}{x-5} = \frac{16}{(x+5)(x-5)}$,

LCD is $(x+5)(x-5)$

$$(x+5)(x-5)\left(\frac{2}{x+5}+\frac{1}{x-5}\right) = (x+5)(x-5)\cdot\frac{16}{(x+5)(x-5)}$$
$$2(x-5)+x+5 = 16$$
$$2x-10+x+5 = 16$$
$$3x-5 = 16$$
$$3x = 21$$
$$x = 7$$

7 checks, so the solution is 7.

16. $\frac{2}{x^2-9}+\frac{5}{x-3} = \frac{3}{x+3}$

$\frac{2}{(x+3)(x-3)}+\frac{5}{x-3} = \frac{3}{x+3}$, LCD is $(x+3)(x-3)$

$$2+5(x+3) = 3(x-3)$$
$$2+5x+15 = 3x-9$$
$$5x+17 = 3x-9$$
$$2x = -26$$
$$x = -13$$

The answer checks. The solution is -13.

17. $\frac{3x}{x+2}+\frac{6}{x} = \frac{12}{x^2+2x}$

$\frac{3x}{x+2}+\frac{6}{x} = \frac{12}{x(x+2)}$, LCD is $x(x+2)$

$$x(x+2)\left(\frac{3x}{x+2}+\frac{6}{x}\right) = x(x+2)\cdot\frac{12}{x(x+2)}$$
$$3x\cdot x+6(x+2) = 12$$
$$3x^2+6x+12 = 12$$
$$3x^2+6x = 0$$
$$3x(x+2) = 0$$
$$3x=0 \quad or \quad x+2=0$$
$$x=0 \quad or \quad x=-2$$

Neither 0 nor -2 checks, so the equation has no solution.

18. $\frac{3y+5}{y^2+5y}+\frac{y+4}{y+5} = \frac{y+1}{y}$

$\frac{3y+5}{y(y+5)}+\frac{y+4}{y+5} = \frac{y+1}{y}$, LCD is $y(y+5)$

$$3y+5+y^2+4y = y^2+6y+5$$

Multiplying by $y(y+5)$

$$y = 0$$

0 does not check. There is no solution.

19. $\frac{1}{5x+20}-\frac{1}{x^2-16} = \frac{3}{x-4}$

$\frac{1}{5(x+4)}-\frac{1}{(x+4)(x-4)} = \frac{3}{x-4}$,

LCD is $5(x+4)(x-4)$

$$5(x+4)(x-4)\left(\frac{1}{5(x+4)}-\frac{1}{(x+4)(x-4)}\right) = 5(x+4)(x-4)\cdot\frac{3}{x-4}$$
$$x-4-5 = 15(x+4)$$
$$x-9 = 15x+60$$
$$-14x-9 = 60$$
$$-14x = 69$$
$$x = -\frac{69}{14}$$

$-\frac{69}{14}$ checks, so the solution is $-\frac{69}{14}$.

20. $\frac{1}{4x+12}-\frac{1}{x^2-9} = \frac{5}{x-3}$

$\frac{1}{4(x+3)}-\frac{1}{(x+3)(x-3)} = \frac{5}{x-3}$,

LCD is $4(x+3)(x-3)$

$$x-3-4 = 20x+60$$
$$-19x = 67$$
$$x = -\frac{67}{19}$$

$-\frac{67}{19}$ checks. The solution is $-\frac{67}{19}$.

21. $$\frac{2}{5x+5} - \frac{3}{x^2-1} = \frac{4}{x-1}$$

$$\frac{2}{5(x+1)} - \frac{3}{(x+1)(x-1)} = \frac{4}{x-1},$$

LCD is $5(x+1)(x-1)$

$$5(x{+}1)(x{-}1)\left(\frac{2}{5(x{+}1)} - \frac{3}{(x{+}1)(x{-}1)}\right) = 5(x{+}1)(x{-}1)\cdot\frac{4}{x{-}1}$$

$$2(x-1) - 5\cdot 3 = 20(x+1)$$
$$2x-2-15 = 20x+20$$
$$2x-17 = 20x+20$$
$$-18x-17 = 20$$
$$-18x = 37$$
$$x = -\frac{37}{18}$$

$-\frac{37}{18}$ checks, so the solution is $-\frac{37}{18}$.

22. $$\frac{1}{3x+6} - \frac{1}{x^2-4} = \frac{3}{x-2}$$

$$\frac{1}{3(x+2)} - \frac{1}{(x+2)(x-2)} = \frac{3}{x-2},$$

LCD is $3(x+2)(x-2)$

$$x-2-3 = 9x+18$$
$$x-5 = 9x+18$$
$$-8x = 23$$
$$x = -\frac{23}{8}$$

$-\frac{23}{8}$ checks. The solution is $-\frac{23}{8}$.

23. $$\frac{8}{x^2-2x+4} = \frac{x}{x+2} + \frac{24}{x^3+8},$$

LCD is $(x+2)(x^2-2x+4)$

$$(x{+}2)(x^2{-}2x{+}4)\cdot\frac{8}{x^2{-}2x{+}4} =$$
$$(x{+}2)(x^2{-}\,2x{+}4)\left(\frac{x}{x{+}2} + \frac{24}{(x{+}2)(x^2{-}2x{+}4)}\right)$$

$$8(x+2) = x(x^2-2x+4)+24$$
$$8x+16 = x^3-2x^2+4x+24$$
$$0 = x^3-2x^2-4x+8$$
$$0 = x^2(x-2)-4(x-2)$$
$$0 = (x-2)(x^2-4)$$
$$0 = (x-2)(x+2)(x-2)$$

$x-2=0$ or $x+2=0$ or $x-2=0$

$x=2$ or $x=-2$ or $x=2$

Only 2 checks. The solution is 2.

24. $$\frac{18}{x^2-3x+9} - \frac{x}{x+3} = \frac{81}{x^3+27},$$

LCD is $(x+3)(x^2-3x+9)$

$18x+54-x^3+3x^2-9x = 81$ Multiplying by $(x+3)(x^2-3x+9)$

$$-x^3+3x^2+9x-27 = 0$$
$$-x^2(x-3)+9(x-3) = 0$$
$$(x-3)(9-x^2) = 0$$
$$(x-3)(3+x)(3-x) = 0$$

$x=3$ or $x=-3$

Only 3 checks. The solution is 3.

25. $$\frac{x}{x-4} - \frac{4}{x+4} = \frac{32}{x^2-16}$$

$$\frac{x}{x-4} - \frac{4}{x+4} = \frac{32}{(x{+}4)(x{-}4)},$$

LCD is $(x{+}4)(x{-}4)$

$$(x{+}4)(x{-}4)\left(\frac{x}{x{-}4} - \frac{4}{x{+}4}\right) = (x{+}4)(x{-}4)\cdot\frac{32}{(x{+}4)(x{-}4)}$$

$$x(x+4)-4(x-4) = 32$$
$$x^2+4x-4x+16 = 32$$
$$x^2+16 = 32$$
$$x^2 = 16$$
$$x = \pm 4$$

Neither 4 nor -4 checks, so the equation has no solution.

26. $$\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$$

$$\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{(x+1)(x-1)},$$

LCD is $(x+1)(x-1)$

$$x^2+x-x+1 = 2$$
$$x^2 = 1$$
$$x = \pm 1$$

Neither 1 nor -1 checks. There is no solution.

27. $$\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x^2-6x}$$

$$\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x(x-6)},\ \text{LCD is } x(x-6)$$

$$x(x-6)\left(\frac{1}{x-6} - \frac{1}{x}\right) = x(x-6)\cdot\frac{6}{x(x-6)}$$

$$x-(x-6) = 6$$
$$x-x+6 = 6$$
$$6 = 6$$

We get an equation that is true for all real numbers. Note, however, that when $x=6$ or $x=0$, division by 0 occurs in the original equation. Thus, the solution set is $\{x|x$ is a real number *and* $x \neq 6$ *and* $x \neq 0\}$, or $(-\infty,0)\cup(0,6)\cup(6,\infty)$.

28. $\frac{1}{x-15} - \frac{1}{x} = \frac{15}{x^2-15x}$

$\frac{1}{x-15} - \frac{1}{x} = \frac{15}{x(x-15)}$, LCD is $x(x-15)$

$x-(x-15) = 15$

$x-x+15 = 15$

$15 = 15$

We get an equation that is true for all real numbers. Note, however, that when $x = 0$ or $x = 15$, division by 0 occurs in the original equation. Thus, the solution set is $\{x|x \text{ is a real number } and\ x \neq 0\ and\ x \neq 15\}$, or $(-\infty, 0) \cup (0, 15) \cup (15, \infty)$.

29. $\sqrt{3x-4} = 1$

$(\sqrt{3x-4})^2 = 1^2$

$3x-4 = 1$

$3x = 5$

$x = \frac{5}{3}$

Check:

$\sqrt{3x-4} = 1$	
$\sqrt{3\cdot\frac{5}{3}-4}$	? 1
$\sqrt{5-4}$	
$\sqrt{1}$	
1	1 TRUE

The solution is $\frac{5}{3}$.

30. $\sqrt{4x+1} = 3$

$4x+1 = 9$

$4x = 8$

$x = 2$

The answer checks. The solution is 2.

31. $\sqrt{2x-5} = 2$

$(\sqrt{2x-5})^2 = 2^2$

$2x-5 = 4$

$2x = 9$

$x = \frac{9}{2}$

Check:

$\sqrt{2x-5} = 2$	
$\sqrt{2\cdot\frac{9}{2}-5}$	? 2
$\sqrt{9-5}$	
$\sqrt{4}$	
2	2 TRUE

The solution is $\frac{9}{2}$.

32. $\sqrt{3x+2} = 6$

$3x+2 = 36$

$3x = 34$

$x = \frac{34}{3}$

The answer checks. The solution is $\frac{34}{3}$.

33. $\sqrt{7-x} = 2$

$(\sqrt{7-x})^2 = 2^2$

$7-x = 4$

$-x = -3$

$x = 3$

Check:

$\sqrt{7-x} = 2$	
$\sqrt{7-3}$	? 2
$\sqrt{4}$	
2	2 TRUE

The solution is 3.

34. $\sqrt{5-x} = 1$

$5-x = 1$

$4 = x$

The answer checks. The solution is 4.

35. $\sqrt{1-2x} = 3$

$(\sqrt{1-2x})^2 = 3^2$

$1-2x = 9$

$-2x = 8$

$x = -4$

Check:

$\sqrt{1-2x} = 3$	
$\sqrt{1-2(-4)}$	? 3
$\sqrt{1+8}$	
$\sqrt{9}$	
3	3 TRUE

The solution is -4.

36. $\sqrt{2-7x} = 2$

$2-7x = 4$

$-7x = 2$

$x = -\frac{2}{7}$

The answer checks. The solution is $-\frac{2}{7}$.

37. $\sqrt[3]{5x-2} = -3$

$(\sqrt[3]{5x-2})^3 = (-3)^3$

$5x-2 = -27$

$5x = -25$

$x = -5$

Check:

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt[3]{5x-2} = -3} \\ \hline
\sqrt[3]{5(-5)-2} \ ? & -3 \\
\sqrt[3]{-25-2} & \\
\sqrt[3]{-27} & \\
-3 & -3 \quad \text{TRUE}
\end{array}$$

The solution is -5.

38. $\sqrt[3]{2x+1} = -5$

$$\begin{aligned}
2x+1 &= -125 \\
2x &= -126 \\
x &= -63
\end{aligned}$$

The answer checks. The solution is -63.

39.

$$\begin{aligned}
\sqrt[4]{x^2-1} &= 1 \\
(\sqrt[4]{x^2-1})^4 &= 1^4 \\
x^2-1 &= 1 \\
x^2 &= 2 \\
x &= \pm\sqrt{2}
\end{aligned}$$

Check:

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt[4]{x^2-1} = 1} \\ \hline
\sqrt[4]{(\pm\sqrt{2})^2-1} \ ? & 1 \\
\sqrt[4]{2-1} & \\
\sqrt[4]{1} & \\
1 & 1 \quad \text{TRUE}
\end{array}$$

The solutions are $\pm\sqrt{2}$.

40. $\sqrt[5]{3x+4} = 2$

$$\begin{aligned}
3x+4 &= 32 \\
3x &= 28 \\
x &= \frac{28}{3}
\end{aligned}$$

The answer checks. The solution is $\dfrac{28}{3}$.

41. $\sqrt{y-1}+4 = 0$

$$\sqrt{y-1} = -4$$

The principal square root is never negative. Thus, there is no solution.

If we do not observe the above fact, we can continue and reach the same answer.

$$\begin{aligned}
(\sqrt{y-1})^2 &= (-4)^2 \\
y-1 &= 16 \\
y &= 17
\end{aligned}$$

Check:

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt{y-1}+4 = 0} \\ \hline
\sqrt{17-1}+4 \ ? & 0 \\
\sqrt{16}+4 & \\
4+4 & \\
8 & 0 \quad \text{FALSE}
\end{array}$$

Since 17 does not check, there is no solution.

42.

$$\begin{aligned}
\sqrt{m+1}-5 &= 8 \\
\sqrt{m+1} &= 13 \\
m+1 &= 169 \\
m &= 168
\end{aligned}$$

The answer checks. The solution is 168.

43.

$$\begin{aligned}
\sqrt{b+3}-2 &= 1 \\
\sqrt{b+3} &= 3 \\
(\sqrt{b+3})^2 &= 3^2 \\
b+3 &= 9 \\
b &= 6
\end{aligned}$$

Check:

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt{b+3}-2 = 1} \\ \hline
\sqrt{6+3}-2 \ ? & 1 \\
\sqrt{9}-2 & \\
3-2 & \\
1 & 1 \quad \text{TRUE}
\end{array}$$

The solution is 6.

44.

$$\begin{aligned}
\sqrt{x-4}+1 &= 5 \\
\sqrt{x-4} &= 4 \\
x-4 &= 16 \\
x &= 20
\end{aligned}$$

The answer checks. The solution is 20.

45.

$$\begin{aligned}
\sqrt{z+2}+3 &= 4 \\
\sqrt{z+2} &= 1 \\
(\sqrt{z+2})^2 &= 1^2 \\
z+2 &= 1 \\
z &= -1
\end{aligned}$$

Check:

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt{z+2}+3 = 4} \\ \hline
\sqrt{-1+2}+3 \ ? & 4 \\
\sqrt{1}+3 & \\
1+3 & \\
4 & 4 \quad \text{TRUE}
\end{array}$$

The solution is -1.

46.

$$\begin{aligned}
\sqrt{y-5}-2 &= 3 \\
\sqrt{y-5} &= 5 \\
y-5 &= 25 \\
y &= 30
\end{aligned}$$

The answer checks. The solution is 30.

47.

$$\begin{aligned}
\sqrt{2x+1}-3 &= 3 \\
\sqrt{2x+1} &= 6 \\
(\sqrt{2x+1})^2 &= 6^2 \\
2x+1 &= 36 \\
2x &= 35 \\
x &= \frac{35}{2}
\end{aligned}$$

Check:

$$\begin{array}{r|l} \sqrt{2x+1}-3 = 3 & \\ \hline \sqrt{2\cdot\frac{35}{2}+1}-3 \ ? & 3 \\ \sqrt{35+1}-3 & \\ \sqrt{36}-3 & \\ 6-3 & \\ 3 & 3 \quad \text{TRUE} \end{array}$$

The solution is $\frac{35}{2}$.

48. $\sqrt{3x-1}+2 = 7$

$$\begin{aligned} \sqrt{3x-1} &= 5 \\ 3x-1 &= 25 \\ 3x &= 26 \\ x &= \frac{26}{3} \end{aligned}$$

The answer checks. The solution is $\frac{26}{3}$.

49. $\sqrt{2-x}-4 = 6$

$$\begin{aligned} \sqrt{2-x} &= 10 \\ (\sqrt{2-x})^2 &= 10^2 \\ 2-x &= 100 \\ -x &= 98 \\ x &= -98 \end{aligned}$$

Check:

$$\begin{array}{r|l} \sqrt{2-x}-4 = 6 & \\ \hline \sqrt{2-(-98)}-4 \ ? & 6 \\ \sqrt{100}-4 & \\ 10-4 & \\ 6 & 6 \quad \text{TRUE} \end{array}$$

The solution is -98.

50. $\sqrt{5-x}+2 = 8$

$$\begin{aligned} \sqrt{5-x} &= 6 \\ 5-x &= 36 \\ -x &= 31 \\ x &= -31 \end{aligned}$$

The answer checks. The solution is -31.

51. $\sqrt[3]{6x+9}+8 = 5$

$$\begin{aligned} \sqrt[3]{6x+9} &= -3 \\ (\sqrt[3]{6x+9})^3 &= (-3)^3 \\ 6x+9 &= -27 \\ 6x &= -36 \\ x &= -6 \end{aligned}$$

Check:

$$\begin{array}{r|l} \sqrt[3]{6x+9}+8 = 5 & \\ \hline \sqrt[3]{6(-6)+9}+8 \ ? & 5 \\ \sqrt[3]{-27}+8 & \\ -3+8 & \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The solution is -6.

52. $\sqrt[5]{2x-3}-1 = 1$

$$\begin{aligned} \sqrt[5]{2x-3} &= 2 \\ 2x-3 &= 32 \\ 2x &= 35 \\ x &= \frac{35}{2} \end{aligned}$$

The answer checks. The solution is $\frac{35}{2}$.

53. $\sqrt{x+4}+2 = x$

$$\begin{aligned} \sqrt{x+4} &= x-2 \\ (\sqrt{x+4})^2 &= (x-2)^2 \\ x+4 &= x^2-4x+4 \\ 0 &= x^2-5x \\ 0 &= x(x-5) \end{aligned}$$

$x = 0$ *or* $x-5 = 0$

$x = 0$ *or* $x = 5$

Check:

For 0:

$$\begin{array}{r|l} \sqrt{x+4}+2 = x & \\ \hline \sqrt{0+4}+2 \ ? & 0 \\ 2+2 & \\ 4 & 0 \quad \text{FALSE} \end{array}$$

For 5:

$$\begin{array}{r|l} \sqrt{x+4}+2 = x & \\ \hline \sqrt{5+4}+2 \ ? & 5 \\ \sqrt{9}+2 & \\ 3+2 & \\ 5 & 5 \quad \text{TRUE} \end{array}$$

The number 5 checks but 0 does not. The solution is 5.

54. $\sqrt{x+1}+1 = x$

$$\begin{aligned} \sqrt{x+1} &= x-1 \\ x+1 &= x^2-2x+1 \\ 0 &= x^2-3x \\ 0 &= x(x-3) \end{aligned}$$

$x = 0$ *or* $x = 3$

Only 3 checks. The solution is 3.

55. $\sqrt{x-3}+5=x$

$$\begin{aligned}\sqrt{x-3} &= x-5\\ (\sqrt{x-3})^2 &= (x-5)^2\\ x-3 &= x^2-10x+25\\ 0 &= x^2-11x+28\\ 0 &= (x-4)(x-7)\end{aligned}$$

$x-4=0$ *or* $x-7=0$

$x=4$ *or* $x=7$

Check:

For 4:

$\sqrt{x-3}+5=x$		
$\sqrt{4-3}+5$?	4	
$\sqrt{1}+5$		
$1+5$		
6	4	FALSE

For 7:

$\sqrt{x-3}+5=x$		
$\sqrt{7-3}+5$?	7	
$\sqrt{4}+5$		
$2+5$		
7	7	TRUE

The number 7 checks but 4 does not. The solution is 7.

56. $\sqrt{x+3}-1=x$

$$\begin{aligned}\sqrt{x+3} &= x+1\\ x+3 &= x^2+2x+1\\ 0 &= x^2+x-2\\ 0 &= (x+2)(x-1)\end{aligned}$$

$x=-2$ *or* $x=1$

Only 1 checks. The solution is 1.

57. $\sqrt{x+7}=x+1$

$$\begin{aligned}(\sqrt{x+7})^2 &= (x+1)^2\\ x+7 &= x^2+2x+1\\ 0 &= x^2+x-6\\ 0 &= (x+3)(x-2)\end{aligned}$$

$x+3=0$ *or* $x-2=0$

$x=-3$ *or* $x=2$

Check:

For -3:

$\sqrt{x+7}=$	$x+1$	
$\sqrt{-3+7}$?	$-3+1$	
$\sqrt{4}$	-2	
2	-2	FALSE

For 2:

$\sqrt{x+7}=$	$x+1$	
$\sqrt{2+7}$?	$2+1$	
$\sqrt{9}$	3	
3	3	TRUE

The number 2 checks but -3 does not. The solution is 2.

58. $\sqrt{6x+7}=x+2$

$$\begin{aligned}6x+7 &= x^2+4x+4\\ 0 &= x^2-2x-3\\ 0 &= (x-3)(x+1)\end{aligned}$$

$x=3$ *or* $x=-1$

Both values check. The solutions are 3 and -1.

59. $\sqrt{3x+3}=x+1$

$$\begin{aligned}(\sqrt{3x+3})^2 &= (x+1)^2\\ 3x+3 &= x^2+2x+1\\ 0 &= x^2-x-2\\ 0 &= (x-2)(x+1)\end{aligned}$$

$x-2=0$ *or* $x+1=0$

$x=2$ *or* $x=-1$

Check:

For 2:

$\sqrt{3x+3}=$	$x+1$	
$\sqrt{3\cdot 2+3}$?	$2+1$	
$\sqrt{9}$	3	
3	3	TRUE

For -1:

$\sqrt{3x+3}=$	$x+1$	
$\sqrt{3(-1)+3}$?	$-1+1$	
$\sqrt{0}$	0	
0	0	TRUE

Both numbers check. The solutions are 2 and -1.

60. $\sqrt{2x+5}=x-5$

$$\begin{aligned}2x+5 &= x^2-10x+25\\ 0 &= x^2-12x+20\\ 0 &= (x-2)(x-10)\end{aligned}$$

$x=2$ *or* $x=10$

Only 10 checks. The solution is 10.

61. $\sqrt{5x+1}=x-1$

$$\begin{aligned}(\sqrt{5x+1})^2 &= (x-1)^2\\ 5x+1 &= x^2-2x+1\\ 0 &= x^2-7x\\ 0 &= x(x-7)\end{aligned}$$

$x=0$ *or* $x-7=0$

$x=0$ *or* $x=7$

Check:

For 0:

$$\begin{array}{r|l} \sqrt{5x+1} = & x-1 \\ \hline \sqrt{5\cdot 0+1} \ ? & 0-1 \\ \sqrt{1} & -1 \\ 1 & -1 \qquad \text{FALSE} \end{array}$$

For 7:

$$\begin{array}{r|l} \sqrt{5x+1} = & x-1 \\ \hline \sqrt{5\cdot 7+1} \ ? & 7-1 \\ \sqrt{36} & 6 \\ 6 & 6 \qquad \text{TRUE} \end{array}$$

The number 7 checks but 0 does not. The solution is 7.

62. $\sqrt{7x+4} = x+2$

$$\begin{aligned} 7x+4 &= x^2+4x+4 \\ 0 &= x^2-3x \\ 0 &= x(x-3) \end{aligned}$$

$x=0$ *or* $x=3$

Both numbers check. The solutions are 0 and 3.

63. $\sqrt{x-3}+\sqrt{x+2} = 5$

$$\begin{aligned} \sqrt{x+2} &= 5-\sqrt{x-3} \\ (\sqrt{x+2})^2 &= (5-\sqrt{x-3})^2 \\ x+2 &= 25-10\sqrt{x-3}+(x-3) \\ x+2 &= 22-10\sqrt{x-3}+x \\ 10\sqrt{x-3} &= 20 \\ \sqrt{x-3} &= 2 \\ (\sqrt{x-3})^2 &= 2^2 \\ x-3 &= 4 \\ x &= 7 \end{aligned}$$

Check:

$$\begin{array}{r|l} \sqrt{x-3}+\sqrt{x+2} = & 5 \\ \hline \sqrt{7-3}+\sqrt{7+2} \ ? & 5 \\ \sqrt{4}+\sqrt{9} & \\ 2+3 & \\ 5 & 5 \qquad \text{TRUE} \end{array}$$

The solution is 7.

64. $\sqrt{x}-\sqrt{x-5} = 1$

$$\begin{aligned} \sqrt{x} &= \sqrt{x-5}+1 \\ x &= x-5+2\sqrt{x-5}+1 \\ 4 &= 2\sqrt{x-5} \\ 2 &= \sqrt{x-5} \\ 4 &= x-5 \\ 9 &= x \end{aligned}$$

The answer checks. The solution is 9.

65. $\sqrt{3x-5}+\sqrt{2x+3}+1 = 0$

$$\sqrt{3x-5}+\sqrt{2x+3} = -1$$

The principal square root is never negative. Thus the sum of two principal square roots cannot equal -1. There is no solution.

66.

$$\begin{aligned} \sqrt{2m-3} &= \sqrt{m+7}-2 \\ 2m-3 &= m+7-4\sqrt{m+7}+4 \\ m-14 &= -4\sqrt{m+7} \\ m^2-28m+196 &= 16m+112 \\ m^2-44m+84 &= 0 \\ (m-2)(m-42) &= 0 \end{aligned}$$

$m=2$ *or* $m=42$

Only 2 checks. The solution is 2.

67. $\sqrt{x}-\sqrt{3x-3} = 1$

$$\begin{aligned} \sqrt{x} &= \sqrt{3x-3}+1 \\ (\sqrt{x})^2 &= (\sqrt{3x-3}+1)^2 \\ x &= (3x-3)+2\sqrt{3x-3}+1 \\ 2-2x &= 2\sqrt{3x-3} \\ 1-x &= \sqrt{3x-3} \\ (1-x)^2 &= (\sqrt{3x-3})^2 \\ 1-2x+x^2 &= 3x-3 \\ x^2-5x+4 &= 0 \\ (x-4)(x-1) &= 0 \end{aligned}$$

$x=4$ *or* $x=1$

The number 4 does not check, but 1 does. The solution is 1.

68. $\sqrt{2x+1}-\sqrt{x} = 1$

$$\begin{aligned} \sqrt{2x+1} &= \sqrt{x}+1 \\ 2x+1 &= x+2\sqrt{x}+1 \\ x &= 2\sqrt{x} \\ x^2 &= 4x \\ x^2-4x &= 0 \\ x(x-4) &= 0 \end{aligned}$$

$x=0$ *or* $x=4$

Both values check. The solutions are 0 and 4.

69. $\sqrt{2y-5}-\sqrt{y-3} = 1$

$$\begin{aligned} \sqrt{2y-5} &= \sqrt{y-3}+1 \\ (\sqrt{2y-5})^2 &= (\sqrt{y-3}+1)^2 \\ 2y-5 &= (y-3)+2\sqrt{y-3}+1 \\ y-3 &= 2\sqrt{y-3} \\ (y-3)^2 &= (2\sqrt{y-3})^2 \\ y^2-6y+9 &= 4(y-3) \\ y^2-6y+9 &= 4y-12 \\ y^2-10y+21 &= 0 \\ (y-7)(y-3) &= 0 \end{aligned}$$

$y=7$ *or* $y=3$

Both numbers check. The solutions are 7 and 3.

70.
$$\begin{aligned}\sqrt{4p+5}+\sqrt{p+5}&=3\\ \sqrt{4p+5}&=3-\sqrt{p+5}\\ 4p+5&=9-6\sqrt{p+5}+p+5\\ 3p-9&=-6\sqrt{p+5}\\ p-3&=-2\sqrt{p+5}\\ p^2-6p+9&=4p+20\\ p^2-10p-11&=0\\ (p-11)(p+1)&=0\end{aligned}$$
$p=11$ *or* $p=-1$

Only -1 checks. The solution is -1.

71.
$$\begin{aligned}\sqrt{y+4}-\sqrt{y-1}&=1\\ \sqrt{y+4}&=\sqrt{y-1}+1\\ (\sqrt{y+4})^2&=(\sqrt{y-1}+1)^2\\ y+4&=y-1+2\sqrt{y-1}+1\\ 4&=2\sqrt{y-1}\\ 2&=\sqrt{y-1} \qquad \text{Dividing by 2}\\ 2^2&=(\sqrt{y-1})^2\\ 4&=y-1\\ 5&=y\end{aligned}$$

The answer checks. The solution is 5.

72.
$$\begin{aligned}\sqrt{y+7}+\sqrt{y+16}&=9\\ \sqrt{y+7}&=9-\sqrt{y+16}\\ y+7&=81-18\sqrt{y+16}+y+16\\ -90&=-18\sqrt{y+16}\\ 5&=\sqrt{y+16}\\ 25&=y+16\\ 9&=y\end{aligned}$$

The answer checks. The solution is 9.

73.
$$\begin{aligned}\sqrt{x+5}+\sqrt{x+2}&=3\\ \sqrt{x+5}&=3-\sqrt{x+2}\\ (\sqrt{x+5})^2&=(3-\sqrt{x+2})^2\\ x+5&=9-6\sqrt{x+2}+x+2\\ -6&=-6\sqrt{x+2}\\ 1&=\sqrt{x+2} \qquad \text{Dividing by } -6\\ 1^2&=(\sqrt{x+2})^2\\ 1&=x+2\\ -1&=x\end{aligned}$$

The answer checks. The solution is -1.

74.
$$\begin{aligned}\sqrt{6x+6}&=5+\sqrt{21-4x}\\ 6x+6&=25+10\sqrt{21-4x}+21-4x\\ 10x-40&=10\sqrt{21-4x}\\ x-4&=\sqrt{21-4x}\\ x^2-8x+16&=21-4x\\ x^2-4x-5&=0\\ (x-5)(x+1)&=0\end{aligned}$$
$x=5$ *or* $x=-1$

Only 5 checks. The solution is 5.

75.
$$\begin{aligned}x^{1/3}&=-2\\ (x^{1/3})^3&=(-2)^3 \qquad (x^{1/3}=\sqrt[3]{x})\\ x&=-8\end{aligned}$$

The value checks. The solution is -8.

76.
$$\begin{aligned}t^{1/5}&=2\\ t&=32\end{aligned}$$

The value checks. The solution is 32.

77.
$$\begin{aligned}t^{1/4}&=3\\ (t^{1/4})^4&=3^4 \qquad (t^{1/4}=\sqrt[4]{t})\\ t&=81\end{aligned}$$

The value checks. The solution is 81.

78. $m^{1/2}=-7$

The principal square root is never negative. There is no solution.

79.
$$\begin{aligned}\frac{P_1V_1}{T_1}&=\frac{P_2V_2}{T_2}\\ P_1V_1T_2&=P_2V_2T_1 \qquad \text{Multiplying by } T_1T_2 \text{ on both sides}\\ \frac{P_1V_1T_2}{P_2V_2}&=T_1 \qquad \text{Dividing by } P_2V_2 \text{ on both sides}\end{aligned}$$

80.
$$\begin{aligned}\frac{1}{F}&=\frac{1}{m}+\frac{1}{p}\\ mp&=Fp+Fm\\ mp&=F(p+m)\\ \frac{mp}{p+m}&=F\end{aligned}$$

81.
$$\begin{aligned}W&=\sqrt{\frac{1}{LC}}\\ W^2&=\left(\sqrt{\frac{1}{LC}}\right)^2 \qquad \text{Squaring both sides}\\ W^2&=\frac{1}{LC}\\ CW^2&=\frac{1}{L} \qquad \text{Multiplying by } C\\ C&=\frac{1}{LW^2} \qquad \text{Dividing by } W^2\end{aligned}$$

82.
$$\begin{aligned}s&=\sqrt{\frac{A}{6}}\\ s^2&=\frac{A}{6}\\ 6s^2&=A\end{aligned}$$

83. $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$RR_1R_2 \cdot \frac{1}{R} = RR_1R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Multiplying by RR_1R_2 on both sides

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 - RR_2 = RR_1$$ Subtracting RR_2 on both sides

$$R_2(R_1 - R) = RR_1$$ Factoring

$$R_2 = \frac{RR_1}{R_1 - R}$$ Dividing by $R_1 - R$ on both sides

84. $$\frac{1}{t} = \frac{1}{a} + \frac{1}{b}$$

$$ab = bt + at$$ Multiplying by abt

$$ab = t(b + a)$$

$$\frac{ab}{b + a} = t$$

85. $$I = \sqrt{\frac{A}{P}} - 1$$

$$I + 1 = \sqrt{\frac{A}{P}}$$ Adding 1

$$(I + 1)^2 = \left(\sqrt{\frac{A}{P}}\right)^2$$

$$I^2 + 2I + 1 = \frac{A}{P}$$

$$P(I^2 + 2I + 1) = A$$ Multplying by P

$$P = \frac{A}{I^2 + 2I + 1}$$ Dividing by $I^2 + 2I + 1$

We could also express this result as $P = \dfrac{A}{(I + 1)^2}$.

86. $$T = 2\pi\sqrt{\frac{1}{g}}$$

$$T^2 = 4\pi^2 \cdot \frac{1}{g}$$

$$gT^2 = 4\pi^2$$

$$g = \frac{4\pi^2}{T^2}$$

87. $$\frac{1}{F} = \frac{1}{m} + \frac{1}{p}$$

$$Fmp \cdot \frac{1}{F} = Fmp\left(\frac{1}{m} + \frac{1}{p}\right)$$ Multiplying by Fmp on both sides

$$mp = Fp + Fm$$

$$mp - Fp = Fm$$ Subtracting Fp on both sides

$$p(m - F) = Fm$$ Factoring

$$p = \frac{Fm}{m - F}$$ Dividing by $m - F$ on both sides

88. $$\frac{V^2}{R^2} = \frac{2g}{R + h}$$

$$V^2(R + h) = 2gR^2$$ Multiplying by $R^2(R + h)$

$$V^2R + V^2h = 2gR^2$$

$$V^2h = 2gR^2 - V^2R$$

$$h = \frac{2gR^2 - V^2R}{V^2}, \text{ or}$$

$$\frac{2gR^2}{V^2} - R$$

89. $15 - 2x = 0$ Setting $f(x) = 0$

$$15 = 2x$$

$$\frac{15}{2} = x, \text{ or}$$

$$7.5 = x$$

The zero of the function is $\dfrac{15}{2}$, or 7.5.

90. $$-3x + 9 = 0$$

$$-3x = -9$$

$$x = 3$$

The zero of the function is 3.

91. ***Familiarize***. Let f = the number of highway fatalities involving distracted driving in 2004.

Translate.

Fatalities in 2004	plus	18% more	is	fatalities in 2008
↓	↓	↓	↓	↓
f	$+$	$0.18f$	$=$	5870

Carry out. We solve the equation.

$$f + 0.18f = 5870$$

$$1.18f = 5870$$

$$f \approx 4975$$

Check. 18% of 4975 is $0.18(4975) \approx 896$ and $4975 + 896 = 5871 \approx 5870$. The answer checks. (Remember that we rounded the value of f.)

State. About 4975 highway fatalities involved distracted driving in 2004.

92. Let d = the number of acres Disneyland occupies.

Solve: $(d + 11) + d = 181$

$d = 85$, so Disneyland occupies 85 acres and the Mall of America occupies $85 + 11$, or 96 acres.

93. $$(x - 3)^{2/3} = 2$$

$$[(x - 3)^{2/3}]^3 = 2^3$$

$$(x - 3)^2 = 8$$

$$x^2 - 6x + 9 = 8$$

$$x^2 - 6x + 1 = 0$$

$$a = 1,\ b = -6,\ c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$
$$= \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2}$$
$$= \frac{2(3 \pm 2\sqrt{2})}{2} = 3 \pm 2\sqrt{2}$$

Both values check. The solutions are $3 \pm 2\sqrt{2}$.

94. $\frac{x+3}{x+2} - \frac{x+4}{x+3} = \frac{x+5}{x+4} - \frac{x+6}{x+5}$,

LCD is $(x+2)(x+3)(x+4)(x+5)$

$x^4 + 15x^3 + 83x^2 + 201x + 180 - x^4 - 15x^3 -$
$82x^2 - 192x - 160 = x^4 + 15x^3 + 81x^2 + 185x +$
$150 - x^4 - 15x^3 - 80x^2 - 180x - 144$

$$x^2 + 9x + 20 = x^2 + 5x + 6$$
$$4x = -14$$
$$x = -\frac{7}{2}$$

The number $-\frac{7}{2}$ checks. The solution is $-\frac{7}{2}$.

95. $\sqrt{x+5} + 1 = \frac{6}{\sqrt{x+5}}$, LCD is $\sqrt{x+5}$

$x + 5 + \sqrt{x+5} = 6$ Multiplying by $\sqrt{x+5}$

$$\sqrt{x+5} = 1 - x$$
$$x + 5 = 1 - 2x + x^2$$
$$0 = x^2 - 3x - 4$$
$$0 = (x-4)(x+1)$$

$x = 4$ *or* $x = -1$

Only -1 checks. The solution set is -1.

96. $\sqrt{15 + \sqrt{2x+80}} = 5$

$$\left(\sqrt{15 + \sqrt{2x+80}}\right)^2 = 5^2$$
$$15 + \sqrt{2x+80} = 25$$
$$\sqrt{2x+80} = 10$$
$$(\sqrt{2x+80})^2 = 10^2$$
$$2x + 80 = 100$$
$$2x = 20$$
$$x = 10$$

This number checks. The solution is 10.

97. $x^{2/3} = x$

$$(x^{2/3})^3 = x^3$$
$$x^2 = x^3$$
$$0 = x^3 - x^2$$
$$0 = x^2(x-1)$$

$x^2 = 0$ *or* $x - 1 = 0$

$x = 0$ *or* $x = 1$

Both numbers check. The solutions are 0 and 1.

Exercise Set 3.5

1. $|x| = 7$

The solutions are those numbers whose distance from 0 on a number line is 7. They are -7 and 7. That is,

$x = -7$ *or* $x = 7$.

The solutions are -7 and 7.

2. $|x| = 4.5$

$x = -4.5$ *or* $x = 4.5$

The solutions are -4.5 and 4.5.

3. $|x| = 0$

The distance of 0 from 0 on a number line is 0. That is,

$x = 0$.

The solution is 0.

4. $|x| = \frac{3}{2}$

$x = -\frac{3}{2}$ *or* $x = \frac{3}{2}$

The solutions are $-\frac{3}{2}$ and $\frac{3}{2}$.

5. $|x| = \frac{5}{6}$

$x = -\frac{5}{6}$ *or* $x = \frac{5}{6}$

The solutions are $-\frac{5}{6}$ and $\frac{5}{6}$.

6. $|x| = -\frac{3}{5}$

The absolute value of a number is nonnegative. Thus, there is no solution.

7. $|x| = -10.7$

The absolute value of a number is nonnegative. Thus, the equation has no solution.

8. $|x| = 12$

$x = -12$ *or* $x = 12$

The solutions are -12 and 12.

9. $|3x| = 1$

$3x = -1$ *or* $3x = 1$

$x = -\frac{1}{3}$ *or* $x = \frac{1}{3}$

The solutions are $-\frac{1}{3}$ and $\frac{1}{3}$.

10. $|5x| = 4$

$5x = -4$ *or* $5x = 4$

$x = -\frac{4}{5}$ *or* $x = \frac{4}{5}$

The solutions are $-\frac{4}{5}$ and $\frac{4}{5}$.

11. $|8x| = 24$

$8x = -24$ *or* $8x = 24$

$x = -3$ *or* $x = 3$

The solutions are -3 and 3.

12. $|6x| = 0$

$6x = 0$

$x = 0$

The solution is 0.

13. $|x-1| = 4$

$x - 1 = -4$ *or* $x - 1 = 4$

$x = -3$ *or* $x = 5$

The solutions are -3 and 5.

14. $|x-7| = 5$

$x - 7 = -5$ *or* $x - 7 = 5$

$x = 2$ *or* $x = 12$

The solutions are 2 and 12.

15. $|x+2| = 6$

$x + 2 = -6$ *or* $x + 2 = 6$

$x = -8$ *or* $x = 4$

The solutions are -8 and 4.

16. $|x+5| = 1$

$x + 5 = -1$ *or* $x + 5 = 1$

$x = -6$ *or* $x = -4$

The solutions are -6 and -4.

17. $|3x+2| = 1$

$3x + 2 = -1$ *or* $3x + 2 = 1$

$3x = -3$ *or* $3x = -1$

$x = -1$ *or* $x = -\frac{1}{3}$

The solutions are -1 and $-\frac{1}{3}$.

18. $|7x-4| = 8$

$7x - 4 = -8$ *or* $7x - 4 = 8$

$7x = -4$ *or* $7x = 12$

$x = -\frac{4}{7}$ *or* $x = \frac{12}{7}$

The solutions are $-\frac{4}{7}$ and $\frac{12}{7}$.

19. $\left|\frac{1}{2}x - 5\right| = 17$

$\frac{1}{2}x - 5 = -17$ *or* $\frac{1}{2}x - 5 = 17$

$\frac{1}{2}x = -12$ *or* $\frac{1}{2}x = 22$

$x = -24$ *or* $x = 44$

The solutions are -24 and 44.

20. $\left|\frac{1}{3}x - 4\right| = 13$

$\frac{1}{3}x - 4 = -13$ *or* $\frac{1}{3}x - 4 = 13$

$\frac{1}{3}x = -9$ *or* $\frac{1}{3}x = 17$

$x = -27$ *or* $x = 51$

The solutions are -27 and 51.

21. $|x-1| + 3 = 6$

$|x-1| = 3$

$x - 1 = -3$ *or* $x - 1 = 3$

$x = -2$ *or* $x = 4$

The solutions are -2 and 4.

22. $|x+2| - 5 = 9$

$|x+2| = 14$

$x + 2 = -14$ *or* $x + 2 = 14$

$x = -16$ *or* $x = 12$

The solutions are -16 and 12.

23. $|x+3| - 2 = 8$

$|x+3| = 10$

$x + 3 = -10$ *or* $x + 3 = 10$

$x = -13$ *or* $x = 7$

The solutions are -13 and 7.

24. $|x-4| + 3 = 9$

$|x-4| = 6$

$x - 4 = -6$ *or* $x - 4 = 6$

$x = -2$ *or* $x = 10$

The solutions are -2 and 10.

25. $|3x+1| - 4 = -1$

$|3x+1| = 3$

$3x + 1 = -3$ *or* $3x + 1 = 3$

$3x = -4$ *or* $3x = 2$

$x = -\frac{4}{3}$ *or* $x = \frac{2}{3}$

The solutions are $-\frac{4}{3}$ and $\frac{2}{3}$.

26. $|2x-1| - 5 = -3$

$|2x-1| = 2$

$2x - 1 = -2$ *or* $2x - 1 = 2$

$2x = -1$ *or* $2x = 3$

$x = -\frac{1}{2}$ *or* $x = \frac{3}{2}$

The solutions are $-\frac{1}{2}$ and $\frac{3}{2}$.

27. $|4x-3|+1=7$

$$|4x-3|=6$$

$$4x-3=-6 \quad or \quad 4x-3=6$$

$$4x=-3 \quad or \quad 4x=9$$

$$x=-\frac{3}{4} \quad or \quad x=\frac{9}{4}$$

The solutions are $-\frac{3}{4}$ and $\frac{9}{4}$.

28. $|5x+4|+2=5$

$$|5x+4|=3$$

$$5x+4=-3 \quad or \quad 5x+4=3$$

$$5x=-7 \quad or \quad 5x=-1$$

$$x=-\frac{7}{5} \quad or \quad x=-\frac{1}{5}$$

The solutions are $-\frac{7}{5}$ and $-\frac{1}{5}$.

29. $12-|x+6|=5$

$$-|x+6|=-7$$

$$|x+6|=7 \quad \text{Multiplying by } -1$$

$$x+6=-7 \quad or \quad x+6=7$$

$$x=-13 \quad or \quad x=1$$

The solutions are -13 and 1.

30. $9-|x-2|=7$

$$2=|x-2|$$

$$x-2=-2 \quad or \quad x-2=2$$

$$x=0 \quad or \quad x=4$$

The solutions are 0 and 4.

31. $7-|2x-1|=6$

$$-|2x-1|=-1$$

$$|2x-1|=1 \quad \text{Multiplying by } -1$$

$$2x-1=-1 \quad or \quad 2x-1=1$$

$$2x=0 \quad or \quad 2x=2$$

$$x=0 \quad or \quad x=1$$

The solutions are 0 and 1.

32. $5-|4x+3|=2$

$$-|4x+3|=-3$$

$$|4x+3|=3$$

$$4x+3=-3 \quad or \quad 4x+3=3$$

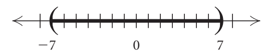
$$4x=-6 \quad or \quad 4x=0$$

$$x=-\frac{3}{2} \quad or \quad x=0$$

The solutions are $-\frac{3}{2}$ and 0.

33. $|x|<7$

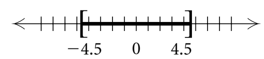
To solve we look for all numbers x whose distance from 0 is less than 7. These are the numbers between -7 and 7. That is, $-7<x<7$. The solution set is $(-7,7)$. The graph is shown below.



34. $|x|\le 4.5$

$-4.5\le x\le 4.5$

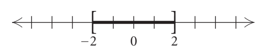
The solution set is $[-4.5,4.5]$.



35. $|x|\le 2$

$-2\le x\le 2$

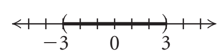
The solution set is $[-2,2]$. The graph is shown below.



36. $|x|<3$

$-3<x<3$

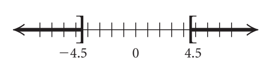
The solution set is $(-3,3)$.



37. $|x|\ge 4.5$

To solve we look for all numbers x whose distance from 0 is greater than or equal to 4.5. That is, $x\le -4.5$ or $x\ge 4.5$. The solution set and its graph are as follows.

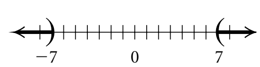
$\{x|x\le -4.5 \text{ or } x\ge 4.5\}$, or $(-\infty,-4.5]\cup[4.5,\infty)$



38. $|x|>7$

$x<-7$ or $x>7$

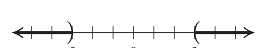
The solution set is $(-\infty,-7)\cup(7,\infty)$.



39. $|x|>3$

$x<-3 \quad or \quad x>3$

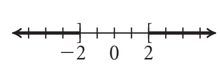
The solution set is $(-\infty,-3)\cup(3,\infty)$. The graph is shown below.



40. $|x|\ge 2$

$x\le -2 \quad or \quad x\ge 2$

The solution set is $(-\infty,-2]\cup[2,\infty)$.

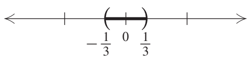


41. $|3x| < 1$

$-1 < 3x < 1$

$-\frac{1}{3} < x < \frac{1}{3}$ Dividing by 3

The solution set is $\left(-\frac{1}{3}, \frac{1}{3}\right)$. The graph is shown below.

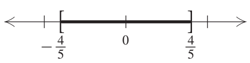


42. $|5x| \le 4$

$-4 \le 5x \le 4$

$-\frac{4}{5} \le x \le \frac{4}{5}$

The solution set is $\left[-\frac{4}{5}, \frac{4}{5}\right]$.

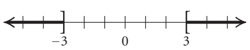


43. $|2x| \ge 6$

$2x \le -6$ *or* $2x \ge 6$

$x \le -3$ *or* $x \ge 3$

The solution set is $(-\infty, -3] \cup [3, \infty)$. The graph is shown below.

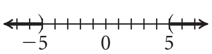


44. $|4x| > 20$

$4x < -20$ *or* $4x > 20$

$x < -5$ *or* $x > 5$

The solution set is $(-\infty, -5) \cup (5, \infty)$.



45. $|x + 8| < 9$

$-9 < x + 8 < 9$

$-17 < x < 1$ Subtracting 8

The solution set is $(-17, 1)$. The graph is shown below.

46. $|x + 6| \le 10$

$-10 \le x + 6 \le 10$

$-16 \le x \le 4$

The solution set is $[-16, 4]$.

47. $|x + 8| \ge 9$

$x + 8 \le -9$ *or* $x + 8 \ge 9$

$x \le -17$ *or* $x \ge 1$ Subtracting 8

The solution set is $(-\infty, -17] \cup [1, \infty)$. The graph is shown below.

48. $|x + 6| > 10$

$x + 6 < -10$ *or* $x + 6 > 10$

$x < -16$ *or* $x > 4$

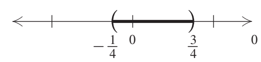
The solution set is $(-\infty, -16) \cup (4, \infty)$.

49. $\left|x - \frac{1}{4}\right| < \frac{1}{2}$

$-\frac{1}{2} < x - \frac{1}{4} < \frac{1}{2}$

$-\frac{1}{4} < x < \frac{3}{4}$ Adding $\frac{1}{4}$

The solution set is $\left(-\frac{1}{4}, \frac{3}{4}\right)$. The graph is shown below.

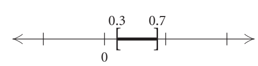


50. $|x - 0.5| \le 0.2$

$-0.2 \le x - 0.5 \le 0.2$

$0.3 \le x \le 0.7$

The solution set is [0.3,0.7].



51. $|2x + 3| \le 9$

$-9 \le 2x + 3 \le 9$

$-12 \le 2x \le 6$ Subtracting 3

$-6 \le x \le 3$ Dividing by 2

The solution set is $[-6, 3]$. The graph is shown below.

52. $|3x + 4| < 13$

$-13 < 3x + 4 < 13$

$-17 < 3x < 9$

$-\frac{17}{3} < x < 3$

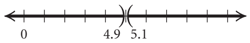
The solution set is $\left(-\frac{17}{3}, 3\right)$.

53. $|x-5|>0.1$

$x-5<-0.1$ *or* $x-5>0.1$

$x<4.9$ *or* $x>5.1$ Adding 5

The solution set is $(-\infty,4.9)\cup(5.1,\infty)$. The graph is shown below.

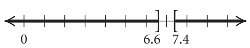


54. $|x-7|\geq 0.4$

$x-7\leq -0.4$ *or* $x-7\geq 0.4$

$x\leq 6.6$ *or* $x\geq 7.4$

The solution set is $(-\infty,6.6]\cup[7.4,\infty)$.



55. $|6-4x|\geq 8$

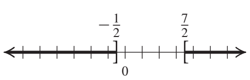
$6-4x\leq -8$ *or* $6-4x\geq 8$

$-4x\leq -14$ *or* $-4x\geq 2$ Subtracting 6

$x\geq \frac{14}{4}$ *or* $x\leq -\frac{2}{4}$ Dividing by -4 and reversing the inequality symbols

$x\geq \frac{7}{2}$ *or* $x\leq -\frac{1}{2}$ Simplifying

The solution set is $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{7}{2},\infty\right)$. The graph is shown below.



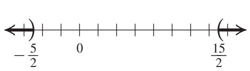
56. $|5-2x|>10$

$5-2x<-10$ *or* $5-2x>10$

$-2x<-15$ *or* $-2x>5$

$x>\frac{15}{2}$ *or* $x<-\frac{5}{2}$

The solution set is $\left(-\infty,-\frac{5}{2}\right)\cup\left(\frac{15}{2},\infty\right)$.



57. $\left|x+\frac{2}{3}\right|\leq\frac{5}{3}$

$-\frac{5}{3}\leq x+\frac{2}{3}\leq\frac{5}{3}$

$-\frac{7}{3}\leq x\leq 1$ Subtracting $\frac{2}{3}$

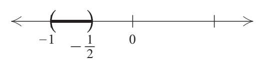
The solution set is $\left[-\frac{7}{3},1\right]$. The graph is shown below.

58. $\left|x+\frac{3}{4}\right|<\frac{1}{4}$

$-\frac{1}{4}<x+\frac{3}{4}<\frac{1}{4}$

$-1<x<-\frac{1}{2}$

The solution set is $\left(-1,-\frac{1}{2}\right)$.



59. $\left|\frac{2x+1}{3}\right|>5$

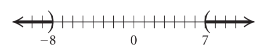
$\frac{2x+1}{3}<-5$ *or* $\frac{2x+1}{3}>5$

$2x+1<-15$ *or* $2x+1>15$ Multiplying by 3

$2x<-16$ *or* $2x>14$ Subtracting 1

$x<-8$ *or* $x>7$ Dividing by 2

The solution set is $\{x|x<-8 \text{ or } x>7\}$, or $(-\infty,-8)\cup(7,\infty)$. The graph is shown below.



60. $\left|\frac{2x-1}{3}\right|\geq\frac{5}{6}$

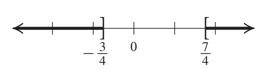
$\frac{2x-1}{3}\leq-\frac{5}{6}$ *or* $\frac{2x-1}{3}\geq\frac{5}{6}$

$2x-1\leq-\frac{5}{2}$ *or* $2x-1\geq\frac{5}{2}$

$2x\leq-\frac{3}{2}$ *or* $2x\geq\frac{7}{2}$

$x\leq-\frac{3}{4}$ *or* $x\geq\frac{7}{4}$

The solution set is $\left(-\infty,-\frac{3}{4}\right]\cup\left[\frac{7}{4},\infty\right)$.



61. $|2x-4|<-5$

Since $|2x-4|\geq 0$ for all x, there is no x such that $|2x-4|$ would be less than -5. There is no solution.

62. $|3x+5|<0$

$|3x+5|\geq 0$ for all x, so there is no solution.

63. y-intercept

64. distance formula

65. relation

66. function

67. horizontal lines

68. parallel

69. decreasing

70. symmetric with respect to the y-axis

71. $|3x-1| > 5x-2$

$3x-1 < -(5x-2)$ *or* $3x-1 > 5x-2$

$3x-1 < -5x+2$ *or* $1 > 2x$

$8x < 3$ *or* $\frac{1}{2} > x$

$x < \frac{3}{8}$ *or* $\frac{1}{2} > x$

The solution set is $\left(-\infty, \frac{3}{8}\right) \cup \left(-\infty, \frac{1}{2}\right)$. This is equivalent to $\left(-\infty, \frac{1}{2}\right)$.

72. $|x+2| \le |x-5|$

Divide the set of real numbers into three intervals:

$(-\infty,-2)$, $[-2,5)$, and $[5,\infty,)$.

Find the solution set of $|x+2| \le |x-5|$ in each interval. Then find the union of the three solution sets.

If $x < -2$, then $|x+2| = -(x+2)$ and $|x-5| = -(x-5)$.

Solve: $x < -2$ *and* $-(x+2) \le -(x-5)$

$x < -2$ *and* $-x-2 \le -x+5$

$x < -2$ *and* $-2 \le 5$

The solution set for this interval is $(-\infty,-2)$.

If $-2 \le x < 5$, then $|x+2| = x+2$ and $|x-5| = -(x-5)$.

Solve: $-2 \le x < 5$ *and* $x+2 \le -(x-5)$

$-2 \le x < 5$ *and* $x+2 \le -x+5$

$-2 \le x < 5$ *and* $2x \le 3$

$-2 \le x < 5$ *and* $x \le \frac{3}{2}$

The solution set for this interval is $\left[-2, \frac{3}{2}\right]$.

If $x \ge 5$, then $|x+2| = x+2$ and $|x-5| = x-5$.

Solve: $x \ge 5$ *and* $x+2 \le x-5$

$x \ge 5$ *and* $2 \le -5$

The solution set for this interval is $\emptyset$.

The union of the above three solution set is $\left(-\infty, \frac{3}{2}\right]$. This is the solution set of $|x+2| \le |x-5|$.

73. $|p-4| + |p+4| < 8$

If $p < -4$, then $|p-4| = -(p-4)$ and $|p+4| = -(p+4)$.

Solve: $-(p-4) + [-(p+4)] < 8$

$-p+4-p-4 < 8$

$-2p < 8$

$p > -4$

Since this is false for all values of p in the interval $(-\infty,-4)$ there is no solution in this interval.

If $p \ge -4$, then $|p+4| = p+4$.

Solve: $|p-4| + p+4 < 8$

$|p-4| < 4-p$

$p-4 > -(4-p)$ *and* $p-4 < 4-p$

$p-4 > p-4$ *and* $2p < 8$

$-4 > -4$ *and* $p < 4$

Since $-4 > -4$ is false for all values of p, there is no solution in the interval $[-4,\infty)$.

Thus, $|p-4| + |p+4| < 8$ has no solution.

74. $|x| + |x+1| < 10$

If $x < -1$, then $|x| = -x$ and $|x+1| = -(x+1)$ and we have:

$x < -1$ *and* $-x + [-(x+1)] < 10$

$x < -1$ *and* $-x-x-1 < 10$

$x < -1$ *and* $-2x-1 < 10$

$x < -1$ *and* $-2x < 11$

$x < -1$ *and* $x > -\frac{11}{2}$

The solution set for this interval is $\left(-\frac{11}{2}, -1\right)$.

If $-1 \le x < 0$, then $|x| = -x$ and $|x+1| = x+1$ and we have:

$-1 \le x$ *and* $-x+x+1 < 10$

$-1 \le x$ *and* $1 < 10$

The solution set for this interval is $[-1,0]$.

If $x \ge 0$, then $|x| = x$ and $|x+1| = x+1$ and we have:

$x \ge 0$ *and* $x+x+1 < 10$

$x \ge 0$ *and* $2x+1 < 10$

$x \ge 0$ *and* $2x < 9$

$x \ge 0$ *and* $x < \frac{9}{2}$

The solution set for this interval is $\left[0, \frac{9}{2}\right)$.

The union of the three solution sets above is $\left(-\frac{11}{2}, \frac{9}{2}\right)$. This is the solution set of $|x| + |x+1| < 10$.

75. $|x-3| + |2x+5| > 6$

Divide the set of real numbers into three intervals: $\left(-\infty, -\frac{5}{2}\right)$, $\left[-\frac{5}{2}, 3\right)$, and $[3,\infty)$.

Find the solution set of $|x-3| + |2x+5| > 6$ in each interval. Then find the union of the three solution sets.

If $x < -\frac{5}{2}$, then $|x-3| = -(x-3)$ and $|2x+5| = -(2x+5)$.

Solve: $x < -\frac{5}{2}$ *and* $-(x-3) + [-(2x+5)] > 6$

$x < -\frac{5}{2}$ *and* $-x+3-2x-5 > 6$

$x < -\frac{5}{2}$ *and* $-3x > 8$

$x < -\frac{5}{2}$ *and* $x < -\frac{8}{3}$

The solution set in this interval is $\left(-\infty, -\dfrac{8}{3}\right)$.

If $-\dfrac{5}{2} \le x < 3$, then $|x-3| = -(x-3)$ and $|2x+5| = 2x+5$.

Solve: $-\dfrac{5}{2} \le x < 3$ *and* $-(x-3)+2x+5 > 6$

$-\dfrac{5}{2} \le x < 3$ *and* $-x+3+2x+5 > 6$

$-\dfrac{5}{2} \le x < 3$ *and* $x > -2$

The solution set in this interval is $(-2, 3)$.

If $x \ge 3$, then $|x-3| = x-3$ and $|2x+5| = 2x+5$.

Solve: $x \ge 3$ *and* $x-3+2x+5 > 6$

$x \ge 3$ *and* $3x > 4$

$x \ge 3$ *and* $x > \dfrac{4}{3}$

The solution set in this interval is $[3, \infty)$.

The union of the above solution sets is $\left(-\infty, -\dfrac{8}{3}\right) \cup (-2, \infty)$. This is the solution set of $|x-3| + |2x+5| > 6$.

Chapter 3 Review Exercises

1. The statement is true. See page 246 in the text.

2. The statement is true. See page 258 in the text.

3. The statement is false. For example, $3^2 = (-3)^2$, but $3 \ne -3$.

4. The statement is false. See Exercise 17 in Exercise Set 3.5, for example.

5. $(2y+5)(3y-1) = 0$

$2y+5 = 0$ *or* $3y-1 = 0$

$2y = -5$ *or* $3y = 1$

$y = -\dfrac{5}{2}$ *or* $y = \dfrac{1}{3}$

The solutions are $-\dfrac{5}{2}$ and $\dfrac{1}{3}$.

6. $x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5$ *or* $x = 1$

7. $3x^2 + 2x = 8$

$3x^2 + 2x - 8 = 0$

$(x+2)(3x-4) = 0$

$x+2 = 0$ *or* $3x-4 = 0$

$x = -2$ *or* $3x = 4$

$x = -2$ *or* $x = \dfrac{4}{3}$

The solutions are -2 and $\dfrac{4}{3}$.

8. $5x^2 = 15$

$x^2 = 3$

$x = -\sqrt{3}$ *or* $x = \sqrt{3}$

9. $x^2 + 10 = 0$

$x^2 = -10$

$x = -\sqrt{-10}$ *or* $x = \sqrt{-10}$

$x = -\sqrt{10}i$ *or* $x = \sqrt{10}i$

The solutions are $-\sqrt{10}i$ and $\sqrt{10}i$.

10. $x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

11. $x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x+5 = 0$ *or* $x-3 = 0$

$x = -5$ *or* $x = 3$

The zeros of the function are -5 and 3.

12. $2x^2 - x - 5 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\cdot 2\cdot(-5)}}{2\cdot 2}$$

$$= \frac{1 \pm \sqrt{41}}{4}$$

13. $3x^2 + 2x + 3 = 0$

$a = 3$, $b = 2$, $c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 3\cdot 3}}{2\cdot 3}$$

$$= \frac{-2 \pm \sqrt{-32}}{2\cdot 3} = \frac{-2 \pm \sqrt{-16\cdot 2}}{2\cdot 3} = \frac{-2 \pm 4i\sqrt{2}}{2\cdot 3}$$

$$= \frac{2(-1 \pm 2i\sqrt{2})}{2\cdot 3} = \frac{-1 \pm 2i\sqrt{2}}{3}$$

The zeros of the function are $\dfrac{-1 \pm 2i\sqrt{2}}{3}$.

14. $\dfrac{5}{2x+3} + \dfrac{1}{x-6} = 0$, LCD is $(2x+3)(x-6)$

$5(x-6) + 2x + 3 = 0$

$5x - 30 + 2x + 3 = 0$

$7x - 27 = 0$

$7x = 27$

$x = \dfrac{27}{7}$

This number checks.

15.

$$\frac{3}{8x+1}+\frac{8}{2x+5}=1$$

LCD is $(8x+1)(2x+5)$

$$\begin{aligned}(8x+1)(2x+5)\left(\frac{3}{8x+1}+\frac{8}{2x+5}\right) &= (8x+1)(2x+5)\cdot 1\\ 3(2x+5)+8(8x+1) &= (8x+1)(2x+5)\\ 6x+15+64x+8 &= 16x^2+42x+5\\ 70x+23 &= 16x^2+42x+5\\ 0 &= 16x^2-28x-18\\ 0 &= 2(8x^2-14x-9)\\ 0 &= 2(2x+1)(4x-9)\end{aligned}$$

$$\begin{aligned}2x+1=0 \quad &or \quad 4x-9=0\\ 2x=-1 \quad &or \quad 4x=9\\ x=-\frac{1}{2} \quad &or \quad x=\frac{9}{4}\end{aligned}$$

Both numbers check. The solutions are $-\frac{1}{2}$ and $\frac{9}{4}$.

16.

$$\begin{aligned}\sqrt{5x+1}-1 &= \sqrt{3x}\\ 5x+1-2\sqrt{5x+1}+1 &= 3x\\ -2\sqrt{5x+1} &= -2x-2\\ \sqrt{5x+1} &= x+1\\ 5x+1 &= x^2+2x+1\\ 0 &= x^2-3x\\ 0 &= x(x-3)\end{aligned}$$

$x=0$ *or* $x=3$

Both numbers check.

17. $\sqrt{x-1}-\sqrt{x-4}=1$

$$\begin{aligned}\sqrt{x-1} &= \sqrt{x-4}+1\\ (\sqrt{x-1})^2 &= (\sqrt{x-4}+1)^2\\ x-1 &= x-4+2\sqrt{x-4}+1\\ x-1 &= x-3+2\sqrt{x-4}\\ 2 &= 2\sqrt{x-4}\\ 1 &= \sqrt{x-4} \qquad \text{Dividing by 2}\\ 1^2 &= (\sqrt{x-4})^2\\ 1 &= x-4\\ 5 &= x\end{aligned}$$

This number checks. The solution is 5.

18. $|x-4|=3$

$$\begin{aligned}x-4=-3 \quad &or \quad x-4=3\\ x=1 \quad &or \quad x=7\end{aligned}$$

The solutions are 1 and 7.

19. $|2y+7|=9$

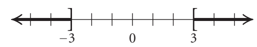
$$\begin{aligned}2y+7=-9 \quad &or \quad 2y+7=9\\ 2y=-16 \quad &or \quad 2y=2\\ y=-8 \quad &or \quad y=1\end{aligned}$$

The solutions are -8 and 1.

20. $|5x|\geq 15$

$$\begin{aligned}5x\leq -15 \quad &or \quad 5x\geq 15\\ x\leq -3 \quad &or \quad x\geq 3\end{aligned}$$

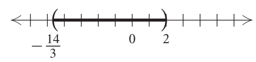
The solution set is $(-\infty,-3]\cup[3,\infty)$. The graph is shown below.



21. $|3x+4|<10$

$$\begin{aligned}&-10<3x+4<10\\ &-14<3x<6\\ &-\frac{14}{3}<x<2\end{aligned}$$

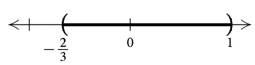
The solution set is $\left(-\frac{14}{3},2\right)$. The graph is shown below.



22.

$$\begin{aligned}|6x-1| &< 5\\ -5<6x-1 &< 5\\ -4<6x &< 6\\ -\frac{2}{3}<x &< 1\end{aligned}$$

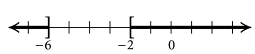
$\left(-\frac{2}{3},1\right)$



23. $|x+4|\geq 2$

$$\begin{aligned}x+4\leq -2 \quad &or \quad x+4\geq 2\\ x\leq -6 \quad &or \quad x\geq -2\end{aligned}$$

The solution is $(-\infty,-6]\cup[-2,\infty)$.



24.

$$\begin{aligned}\frac{1}{M}+\frac{1}{N} &= \frac{1}{P}\\ NP+MP &= MN \qquad \text{Multiplying by MNP}\\ P(N+M) &= MN\\ P &= \frac{MN}{N+M}\end{aligned}$$

25. $-\sqrt{-40}=-\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{10}=-2\sqrt{10}i$

26.

$$\begin{aligned}\sqrt{-12}\cdot\sqrt{-20} &= \sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{3}\cdot\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{5}\\ &= 2i\sqrt{3}\cdot 2i\sqrt{5}\\ &= 4i^2\sqrt{3\cdot 5}\\ &= -4\sqrt{15}\end{aligned}$$

27. $\dfrac{\sqrt{-49}}{-\sqrt{-64}}=\dfrac{7i}{-8i}=-\dfrac{7}{8}$

28.

$$\begin{aligned}(6+2i)+(-4-3i) &= (6-4)+(2i-3i)\\ &= 2-i\end{aligned}$$

29. $(3-5i)-(2-i) = (3-2)+[-5i-(-i)]$
$= 1-4i$

30. $(6+2i)(-4-3i) = -24-18i-8i-6i^2$
$= -24-26i+6$
$= -18-26i$

31. $\dfrac{2-3i}{1-3i} = \dfrac{2-3i}{1-3i}\cdot\dfrac{1+3i}{1+3i}$
$= \dfrac{2+3i-9i^2}{1-9i^2}$
$= \dfrac{2+3i+9}{1+9}$
$= \dfrac{11+3i}{10}$
$= \dfrac{11}{10}+\dfrac{3}{10}i$

32. $i^{23} = (i^2)^{11}\cdot i = (-1)^{11}\cdot i = -1\cdot i = -i$

33. $x^2-3x = 18$
$x^2-3x+\dfrac{9}{4} = 18+\dfrac{9}{4}$ $\left(\dfrac{1}{2}(-3)=-\dfrac{3}{2} \text{ and } \left(-\dfrac{3}{2}\right)^2=\dfrac{9}{4}\right)$
$\left(x-\dfrac{3}{2}\right)^2 = \dfrac{81}{4}$
$x-\dfrac{3}{2} = \pm\dfrac{9}{2}$
$x = \dfrac{3}{2}\pm\dfrac{9}{2}$
$x = \dfrac{3}{2}-\dfrac{9}{2}$ *or* $x = \dfrac{3}{2}+\dfrac{9}{2}$
$x = -3$ *or* $x = 6$
The solutions are -3 and 6.

34. $3x^2-12x-6 = 0$
$3x^2-12x = 6$
$x^2-4x = 2$
$x^2-4x+4 = 2+4$ $\left(\dfrac{1}{2}(-4)=-2 \text{ and } (-2)^2 = 4\right)$
$(x-2)^2 = 6$
$x-2 = \pm\sqrt{6}$
$x = 2\pm\sqrt{6}$

35. $3x^2+10x = 8$
$3x^2+10x-8 = 0$
$(x+4)(3x-2) = 0$
$x+4 = 0$ *or* $3x-2 = 0$
$x = -4$ *or* $3x = 2$
$x = -4$ *or* $x = \dfrac{2}{3}$
The solutions are -4 and $\dfrac{2}{3}$.

36. $r^2-2r+10 = 0$
$r = \dfrac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot10}}{2\cdot1}$
$= \dfrac{2\pm\sqrt{-36}}{2} = \dfrac{2\pm6i}{2}$
$= 1\pm3i$

37. $x^2 = 10+3x$
$x^2-3x-10 = 0$
$(x+2)(x-5) = 0$
$x+2 = 0$ *or* $x-5 = 0$
$x = -2$ *or* $x = 5$
The solutions are -2 and 5.

38. $x = 2\sqrt{x}-1$
$x-2\sqrt{x}+1 = 0$
Let $u = \sqrt{x}$.
$u^2-2u+1 = 0$
$(u-1)^2 = 0$
$u-1 = 0$
$u = 1$
Substitute $\sqrt{x}$ for u and solve for x.
$\sqrt{x} = 1$
$x = 1$

39. $y^4-3y^2+1 = 0$
Let $u = y^2$.
$u^2-3u+1 = 0$
$u = \dfrac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot1}}{2\cdot1} = \dfrac{3\pm\sqrt{5}}{2}$
Substitute y^2 for u and solve for y.
$y^2 = \dfrac{3\pm\sqrt{5}}{2}$
$y = \pm\sqrt{\dfrac{3\pm\sqrt{5}}{2}}$
The solutions are $\pm\sqrt{\dfrac{3\pm\sqrt{5}}{2}}$.

40. $(x^2-1)^2-(x^2-1)-2 = 0$
Let $u = x^2-1$.
$u^2-u-2 = 0$
$(u+1)(u-2) = 0$
$u+1 = 0$ *or* $u-2 = 0$
$u = -1$ *or* $u = 2$
Substitute x^2-1 for u and solve for x.
$x^2-1 = -1$ *or* $x^2-1 = 2$
$x^2 = 0$ *or* $x^2 = 3$
$x = 0$ *or* $x = \pm\sqrt{3}$

41. $(p-3)(3p+2)(p+2)=0$

$$p-3=0 \quad or \quad 3p+2=0 \quad or \quad p+2=0$$
$$p=3 \quad or \quad 3p=-2 \quad or \quad p=-2$$
$$p=3 \quad or \quad p=-\frac{2}{3} \quad or \quad p=-2$$

The solutions are -2, $-\frac{2}{3}$ and 3.

42.
$$x^3+5x^2-4x-20=0$$
$$x^2(x+5)-4(x+5)=0$$
$$(x+5)(x^2-4)=0$$
$$(x+5)(x+2)(x-2)=0$$
$$x+5=0 \quad or \quad x+2=0 \quad or \quad x-2=0$$
$$x=-5 \quad or \quad x=-2 \quad or \quad x=2$$

43.
$$f(x)=-4x^2+3x-1$$
$$=-4\left(x^2-\frac{3}{4}x\right)-1$$
$$=-4\left(x^2-\frac{3}{4}x+\frac{9}{64}-\frac{9}{64}\right)-1$$
$$=-4\left(x^2-\frac{3}{4}x+\frac{9}{64}\right)-4\left(-\frac{9}{64}\right)-1$$
$$=-4\left(x^2-\frac{3}{4}x+\frac{9}{64}\right)+\frac{9}{16}-1$$
$$=-4\left(x-\frac{3}{8}\right)^2-\frac{7}{16}$$

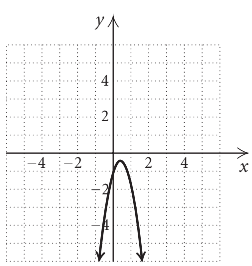
a) Vertex: $\left(\frac{3}{8},-\frac{7}{16}\right)$

b) Axis of symmetry: $x=\frac{3}{8}$

c) Maximum value: $-\frac{7}{16}$

d) Range: $\left(-\infty,-\frac{7}{16}\right]$

e)



$f(x)=-4x^2+3x-1$

44.
$$f(x)=5x^2-10x+3$$
$$=5(x^2-2x)+3$$
$$=5(x^2-2x+1-1)+3$$
$$=5(x^2-2x+1)-5\cdot 1+3$$
$$=5(x-1)^2-2$$

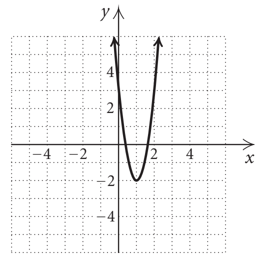
a) Vertex: $(1,-2)$

b) Axis of symmetry: $x=1$

c) Minimum value: -2

d) Range: $[-2,\infty)$

e)



$f(x)=5x^2-10x+3$

45. The graph of $y=(x-2)^2$ has vertex $(2,0)$ and opens up. It is graph (d).

46. The graph of $y=(x+3)^2-4$ has vertex $(-3,-4)$ and opens up. It is graph (c).

47. The graph of $y=-2(x+3)^2+4$ has vertex $(-3,4)$ and opens down. It is graph (b).

48. The graph of $y=-\frac{1}{2}(x-2)^2+5$ has vertex $(2,5)$ and opens down. It is graph (a).

49. ***Familiarize***. Using the labels in the textbook, the legs of the right triangle are represented by x and $x+10$.

Translate. We use the Pythagorean theorem.

$$x^2+(x+10)^2=50^2$$

Carry out. We solve the equation.

$$x^2+(x+10)^2=50^2$$
$$x^2+x^2+20x+100=2500$$
$$2x^2+20x-2400=0$$
$$2(x^2+10x-1200)=0$$
$$2(x+40)(x-30)=0$$
$$x+40=0 \quad or \quad x-30=0$$
$$x=-40 \quad or \quad x=30$$

Check. Since the length cannot be negative, we need to check only 30. If $x=30$, then $x+10=30+10=40$. Since $30^2+40^2=900+1600=2500=50^2$, the answer checks.

State. The lengths of the legs are 30 ft and 40 ft.

50. Let $s=$ Cassidy's speed, in km/h. Then $s-7=$ Logan's speed. After 4 hr, Cassidy has traveled $4s$ km and Logan has traveled $4(s-7)$ km.

Solve: $(4s)^2+[4(s-7)]^2=68^2$

$s=15$, so Cassidy's speed is 15 km/h, and Logan's speed is $15-7$, or 8 km/h.

51. ***Familiarize***. Using the drawing in the textbook, let $w=$ the width of the sidewalk, in ft. Then the length of the new parking lot is $80-2w$, and its width is $60-2w$.

Translate. We use the formula for the area of a rectangle, $A = lw$.

$$\underbrace{\text{New area}}_{\downarrow} \quad \underset{\downarrow}{\text{is}} \quad \underset{\downarrow}{\frac{2}{3}} \quad \underset{\downarrow}{\text{of}} \quad \underbrace{\text{old area}}_{\downarrow}$$

$$(80-2w)(60-2w) = \frac{2}{3} \cdot 80 \cdot 60$$

Carry out. We solve the equation.

$$(80-2w)(60-2w) = \frac{2}{3} \cdot 80 \cdot 60$$

$$4800 - 280w + 4w^2 = \frac{2}{3} \cdot 80 \cdot 3 \cdot 20$$

$$4w^2 - 280w + 4800 = 3200$$

$$4w^2 - 280w + 1600 = 0$$

$$w^2 - 70w + 400 = 0 \qquad \text{Dividing by 4}$$

We use the quadratic formula.

$$w = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-70) \pm \sqrt{(-70)^2 - 4 \cdot 1 \cdot 400}}{2 \cdot 1}$$

$$= \frac{70 \pm \sqrt{3300}}{2}$$

$$= \frac{70 \pm \sqrt{33 \cdot 100}}{2} = \frac{70 \pm 10\sqrt{33}}{2}$$

$$= 35 \pm 5\sqrt{33}$$

$35 + 5\sqrt{33} \approx 63.7$ and $35 - 5\sqrt{33} \approx 6.3$

Check. The width of the sidewalk cannot be 63.7 ft because this width exceeds the width of the original parking lot, 60 ft. We check $35 - 5\sqrt{33} \approx 6.3$. If the width of the sidewalk in about 6.3 ft, then the length of the new parking lot is $80 - 2(6.3)$, or 67.4, and the width is $60 - 2(6.3)$, or 47.4. The area of a parking lot with these dimensions is $(67.4)(47.4) = 3194.76$. Two-thirds of the area of the original parking lot is $\frac{2}{3} \cdot 80 \cdot 60 = 3200$. Since $3194.76 \approx 3200$, this answer checks.

State. The width of the sidewalk is $35 - 5\sqrt{33}$ ft, or about 6.3 ft.

52. ***Familiarize***. Let l = the length of the toy corral, in ft. Then the width is $\frac{24-2l}{2}$, or $12 - l$. The height of the corral is 2 ft.

Translate. We use the formula for the volume of a rectangular solid, $V = lwh$.

$$V(l) = l(12-l)(2)$$

$$= 24l - 2l^2$$

$$= -2l^2 + 24l$$

Carry out. Since $V(l)$ is a quadratic function with $a = -2 < 0$, the maximum function value occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$$-\frac{b}{2a} = -\frac{24}{2(-2)} = 6$$

When $l = 6$, then $12 - l = 12 - 6 = 6$.

Check. The volume of a corral with length 6 ft, width 6 ft, and height 2 ft is $6 \cdot 6 \cdot 2$, or 72 ft^3. As a partial check, we can find $V(l)$ for a value of l less than 6 and for a value of l greater than 6. For instance, $V(5.9) = 71.98$ and $V(6.1) = 71.98$. Since both of these values are less than 72, our result appears to be correct.

State. The dimensions of the corral should be 6 ft by 6 ft.

53. ***Familiarize***. Using the labels in the textbook, let x = the length of the sides of the squares, in cm. Then the length of the base of the box is $20 - 2x$ and the width of the base is $10 - 2x$.

Translate. We use the formula for the area of a rectangle, $A = lw$.

$$90 = (20-2x)(10-2x)$$

$$90 = 200 - 60x + 4x^2$$

$$0 = 4x^2 - 60x + 110$$

$$0 = 2x^2 - 30x + 55 \qquad \text{Dividing by 2}$$

We use the quadratic formula.

Carry out.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-30) \pm \sqrt{(-30)^2 - 4 \cdot 2 \cdot 55}}{2 \cdot 2}$$

$$= \frac{30 \pm \sqrt{460}}{4}$$

$$= \frac{30 \pm \sqrt{4 \cdot 115}}{4} = \frac{30 \pm 2\sqrt{115}}{4}$$

$$= \frac{15 \pm \sqrt{115}}{2}$$

$\frac{15 + \sqrt{115}}{2} \approx 12.9$ and $\frac{15 - \sqrt{115}}{2} \approx 2.1$.

Check. The length of the sides of the squares cannot be 12.9 cm because this length exceeds the width of the piece of aluminum. We check 2.1 cm. If the sides of the squares are 2.1 cm, then the length of the base of the box is $20 - 2(2.1) = 15.8$, and the width is $10 - 2(2.1) = 5.8$. The area of the base is $15.8(5.8) = 91.64 \approx 90$. This answer checks.

State. The length of the sides of the squares is $\frac{15 - \sqrt{115}}{2}$ cm, or about 2.1 cm.

54. $2x^2 - 5x + 1 = 0$

$a = 2,\ b = -5,\ c = 1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{5 \pm \sqrt{25-8}}{4}$$

$$= \frac{5 \pm \sqrt{17}}{4}$$

Answer B is correct.

55.
$$\begin{aligned}\sqrt{4x+1}+\sqrt{2x}&=1\\ \sqrt{4x+1}&=1-\sqrt{2x}\\ (\sqrt{4x+1})^2&=(1-\sqrt{2x})^2\\ 4x+1&=1-2\sqrt{2x}+2x\\ 2x&=-2\sqrt{2x}\\ x&=-\sqrt{2x}\\ x^2&=(-\sqrt{2x})^2\\ x^2&=2x\\ x^2-2x&=0\\ x(x-2)&=0\end{aligned}$$
$x=0$ *or* $x=2$

Only 0 checks, so answer B is correct.

56. The graph of $f(x)=(x-2)^2-3$ has vertex $(2,-3)$. Thus the correct graph is A.

57.
$$\begin{aligned}\sqrt{\sqrt{\sqrt{x}}}&=2\\ \left(\sqrt{\sqrt{\sqrt{x}}}\right)^2&=2^2\\ \sqrt{\sqrt{x}}&=4\\ \left(\sqrt{\sqrt{x}}\right)^2&=4^2\\ \sqrt{x}&=16\\ (\sqrt{x})^2&=16^2\\ x&=256\end{aligned}$$
The answer checks. The solution is 256.

58.
$$\begin{aligned}(t-4)^{4/5}&=3\\ \left[(t-4)^{4/5}\right]^5&=3^5\\ (t-4)^4&=243\\ t-4&=\pm\sqrt[4]{243}\\ t&=4\pm\sqrt[4]{243}\end{aligned}$$
The exact solutions are $4+\sqrt[4]{243}$ and $4-\sqrt[4]{243}$. The approximate solutions are 7.948 and 0.052.

59.
$$\begin{aligned}(x-1)^{2/3}&=4\\ (x-1)^2&=4^3\\ x-1&=\pm\sqrt{64}\\ x-1&=\pm 8\end{aligned}$$
$x-1=-8$ *or* $x-1=8$

$x=-7$ *or* $x=9$

Both numbers check. The solutions are -7 and 9.

60.
$$\begin{aligned}(2y-2)^2+y-1&=5\\ 4y^2-8y+4+y-1&=5\\ 4y^2-7y+3&=5\\ 4y^2-7y-2&=0\\ (4y+1)(y-2)&=0\end{aligned}$$
$4y+1=0$ *or* $y-2=0$

$4y=-1$ *or* $y=2$

$y=-\frac{1}{4}$ *or* $y=2$

The solutions are $-\frac{1}{4}$ and 2.

61. $\sqrt{x+2}+\sqrt[4]{x+2}-2=0$

Let $u=\sqrt[4]{x+2}$, so $u^2=(\sqrt[4]{x+2})^2=\sqrt{x+2}$.

$$\begin{aligned}u^2+u-2&=0\\ (u+2)(u-1)&=0\end{aligned}$$
$u=-2$ *or* $u=1$

Substitute $\sqrt[4]{x+2}$ for u and solve for x.

$\sqrt[4]{x+2}=-2$ *or* $\sqrt[4]{x+2}=1$

No real solution $\quad x+2=1$

$x=-1$

This number checks. The solution is -1.

62. ***Familiarize***. When principal P is deposited in an account at interest rate r, compounded annually, the amount A to which it grows in t years is given by $A=P(1+r)^t$. In 2 years the \$3500 deposit had grown to $\$3500(1+r)^2$. In one year the \$4000 deposit had grown to $\$4000(1+r)$.

Translate. The amount in the account at the end of 2 years was \$8518.35, so we have

$$3500(1+r)^2+4000(1+r)=8518.35.$$

Carry out. We solve the equation. Let $u=1+r$.

$$\begin{aligned}3500u^2+4000u&=8518.35\\ 3500u^2+4000u-8518.35&=0\end{aligned}$$

Using the quadratic formula, we find that $u=1.09$ or $u\approx-2.23$. Substitute $1+r$ for u and solve for r.

$1+r=1.09$ *or* $1+r=-2.23$

$r=0.09$ *or* $r=-3.23$

Check. Since the interest rate cannot be negative, we need to check only 0.09. At 9%, the \$3500 deposit would grow to $\$3500(1+0.09)^2$, or \$4158.35. The \$4000 deposit would grow to $\$4000(1+0.09)$, or \$4360. Since \$4158.35+\$4360 = \$8518.35, the answer checks.

State. The interest rate was 9%.

63. The maximum value occurs at the vertex. The first coordinate of the vertex is $-\frac{b}{2a}=-\frac{b}{2(-3)}=\frac{b}{6}$ and $f\left(\frac{b}{6}\right)=2$.

$$\begin{aligned}-3\left(\frac{b}{6}\right)^2+b\left(\frac{b}{6}\right)-1&=2\\ -\frac{b^2}{12}+\frac{b^2}{6}-1&=2\\ -b^2+2b^2-12&=24\\ b^2&=36\\ b&=\pm 6\end{aligned}$$

64. The product of two imaginary numbers is not always an imaginary number. For example, $i\cdot i=i^2=-1$, a real number.

65. No; consider the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. If $b^2 - 4ac = 0$, then $x = \dfrac{-b}{2a}$, so there is one real zero. If $b^2 - 4ac > 0$, then $\sqrt{b^2 - 4ac}$ is a real number and there are two real zeros. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is an imaginary number and there are two imaginary zeros. Thus, a quadratic function cannot have one real zero and one imaginary zero.

66. You can conclude that $|a_1| = |a_2|$ since these constants determine how wide the parabolas are. Nothing can be concluded about the h's and the k's.

67. When both sides of an equation are multiplied by the LCD, the resulting equation might not be equivalent to the original equation. One or more of the possible solutions of the resulting equation might make a denominator of the original equation 0.

68. When both sides of an equation are raised to an even power, the resulting equation might not be equivalent to the original equation. For example, the solution set of $x = -2$ is $\{-2\}$, but the solution set of $x^2 = (-2)^2$, or $x^2 = 4$, is $\{-2, 2\}$.

69. Absolute value is nonnegative.

70. $|x| \geq 0 > p$ for any real number x.

Chapter 3 Test

1. $(2x-1)(x+5) = 0$

$2x - 1 = 0 \quad or \quad x + 5 = 0$

$2x = 1 \quad or \quad x = -5$

$x = \dfrac{1}{2} \quad or \quad x = -5$

The solutions are $\dfrac{1}{2}$ and -5.

2. $6x^2 - 36 = 0$

$6x^2 = 36$

$x^2 = 6$

$x = -\sqrt{6} \quad or \quad x = \sqrt{6}$

The solutions are $-\sqrt{6}$ and $\sqrt{6}$.

3. $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = -2i \quad or \quad x = 2i$

The solutions are $-2i$ and $2i$.

4. $x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x + 1 = 0 \quad or \quad x - 3 = 0$

$x = -1 \quad or \quad x = 3$

The solutions are -1 and 3.

5. $x^2 - 5x + 3 = 0$

$a = 1,\ b = -5,\ c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$= \frac{5 \pm \sqrt{13}}{2}$$

The solutions are $\dfrac{5 + \sqrt{13}}{2}$ and $\dfrac{5 - \sqrt{13}}{2}$.

6. $2t^2 - 3t + 4 = 0$

$a = 2,\ b = -3,\ c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$

$$= \frac{3 \pm \sqrt{-23}}{4} = \frac{3 \pm i\sqrt{23}}{4}$$

$$= \frac{3}{4} \pm \frac{\sqrt{23}}{4}i$$

The solutions are $\dfrac{3}{4} + \dfrac{\sqrt{23}}{4}i$ and $\dfrac{3}{4} - \dfrac{\sqrt{23}}{4}i$.

7. $x + 5\sqrt{x} - 36 = 0$

Let $u = \sqrt{x}$.

$u^2 + 5u - 36 = 0$

$(u+9)(u-4) = 0$

$u + 9 = 0 \quad or \quad u - 4 = 0$

$u = -9 \quad or \quad u = 4$

Substitute $\sqrt{x}$ for u and solve for x.

$\sqrt{x} = -9 \quad or \quad \sqrt{x} = 4$

No solution $\qquad x = 16$

The number 16 checks. It is the solution.

8. $\dfrac{3}{3x+4} + \dfrac{2}{x-1} = 2$, LCD is $(3x+4)(x-1)$

$$(3x+4)(x-1)\left(\frac{3}{3x+4} + \frac{2}{x-1}\right) = (3x+4)(x-1)(2)$$

$$3(x-1) + 2(3x+4) = 2(3x^2 + x - 4)$$

$$3x - 3 + 6x + 8 = 6x^2 + 2x - 8$$

$$9x + 5 = 6x^2 + 2x - 8$$

$$0 = 6x^2 - 7x - 13$$

$$0 = (x+1)(6x-13)$$

$x + 1 = 0 \quad or \quad 6x - 13 = 0$

$x = -1 \quad or \quad 6x = 13$

$x = -1 \quad or \quad x = \dfrac{13}{6}$

Both numbers check. The solutions are -1 and $\dfrac{13}{6}$.

9. $\sqrt{x+4}-2=1$

$\sqrt{x+4}=3$

$(\sqrt{x+4})^2=3^2$

$x+4=9$

$x=5$

This number checks. The solution is 5.

10. $\sqrt{x+4}-\sqrt{x-4}=2$

$$\begin{aligned}\sqrt{x+4}&=\sqrt{x-4}+2\\(\sqrt{x+4})^2&=(\sqrt{x-4}+2)^2\\x+4&=x-4+4\sqrt{x-4}+4\\4&=4\sqrt{x-4}\\1&=\sqrt{x-4}\\1^2&=(\sqrt{x-4})^2\\1&=x-4\\5&=x\end{aligned}$$

This number checks. The solution is 5.

11. $|x+4|=7$

$x+4=-7$ *or* $x+4=7$

$x=-11$ *or* $x=3$

The solutions are -11 and 3.

12. $|4y-3|=5$

$4y-3=-5$ *or* $4y-3=5$

$4y=-2$ *or* $4y=8$

$y=-\frac{1}{2}$ *or* $y=2$

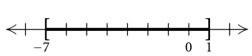
The solutions are $-\frac{1}{2}$ and 2.

13. $|x+3|\le 4$

$-4\le x+3\le 4$

$-7\le x\le 1$

The solution set is $[-7,1]$.



14. $|2x-1|<5$

$-5<2x-1<5$

$-4<2x<6$

$-2<x<3$

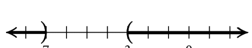
The solution set is $(-2,3)$.

15. $|x+5|>2$

$x+5<-2$ *or* $x+5>2$

$x<-7$ *or* $x>-3$

The solution set is $(-\infty,-7)\cup(-3,\infty)$.



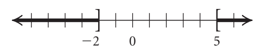
16. $|3-2x|\ge 7$

$3-2x\le -7$ *or* $3-2x\ge 7$

$-2x\le -10$ *or* $-2x\ge 4$

$x\ge 5$ *or* $x\le -2$

The solution set is $(-\infty,-2]\cup[5,\infty)$.



17. $\frac{1}{A}+\frac{1}{B}=\frac{1}{C}$

$$\begin{aligned}ABC\left(\frac{1}{A}+\frac{1}{B}\right)&=ABC\cdot\frac{1}{C}\\BC+AC&=AB\\AC&=AB-BC\\AC&=B(A-C)\\\frac{AC}{A-C}&=B\end{aligned}$$

18. $R=\sqrt{3np}$

$R^2=(\sqrt{3np})^2$

$R^2=3np$

$\frac{R^2}{3p}=n$

19. $x^2+4x=1$

$x^2+4x+4=1+4 \quad \left(\frac{1}{2}(4)=2 \text{ and } 2^2=4\right)$

$(x+2)^2=5$

$x+2=\pm\sqrt{5}$

$x=-2\pm\sqrt{5}$

The solutions are $-2+\sqrt{5}$ and $-2-\sqrt{5}$.

20. ***Familiarize and Translate***. We will use the formula $s=16t^2$, substituting 2063 for s.

$2063=16t^2$

Carry out. We solve the equation.

$2063=16t^2$

$\frac{2063}{16}=t^2$

$11.4\approx t$

Check. When $t=11.4$, $s=16(11.4)^2=2079.36\approx 2063$. The answer checks.

State. It would take an object about 11.4 sec to reach the ground.

21. $\sqrt{-43}=\sqrt{-1}\cdot\sqrt{43}=i\sqrt{43}$, or $\sqrt{43}i$

22. $-\sqrt{-25}=-\sqrt{-1}\cdot\sqrt{25}=-5i$

23. $(5-2i)-(2+3i)=(5-2)+(-2i-3i)$

$=3-5i$

24. $(3+4i)(2-i)=6-3i+8i-4i^2$

$=6+5i+4 \quad (i^2=-1)$

$=10+5i$

25. $\frac{1-i}{6+2i} = \frac{1-i}{6+2i} \cdot \frac{6-2i}{6-2i}$

$= \frac{6-2i-6i+2i^2}{36-4i^2}$

$= \frac{6-8i-2}{36+4}$

$= \frac{4-8i}{40}$

$= \frac{4}{40} - \frac{8}{40}i$

$= \frac{1}{10} - \frac{1}{5}i$

26. $i^{33} = (i^2)^{16} \cdot i = (-1)^{16} \cdot i = 1 \cdot i = i$

27. $4x^2 - 11x - 3 = 0$

$(4x+1)(x-3) = 0$

$4x+1 = 0$ *or* $x - 3 = 0$

$4x = -1$ *or* $x = 3$

$x = -\frac{1}{4}$ *or* $x = 3$

The zeros of the functions are $-\frac{1}{4}$ and 3.

28. $2x^2 - x - 7 = 0$

$a = 2,\ b = -1,\ c = -7$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-7)}}{2 \cdot 2}$

$= \frac{1 \pm \sqrt{57}}{4}$

The solutions are $\frac{1+\sqrt{57}}{4}$ and $\frac{1-\sqrt{57}}{4}$.

29. $f(x) = -x^2 + 2x + 8$

$= -(x^2 - 2x) + 8$

$= -(x^2 - 2x + 1 - 1) + 8$

$= -(x^2 - 2x + 1) - (-1) + 8$

$= -(x^2 - 2x + 1) + 1 + 8$

$= -(x-1)^2 + 9$

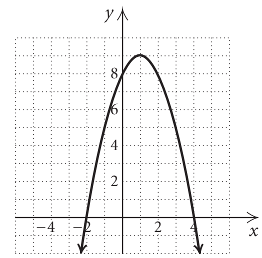
a) Vertex: $(1, 9)$

b) Axis of symmetry: $x = 1$

c) Maximum value: 9

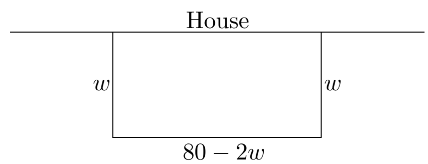
d) Range: $(-\infty, 9]$

e)



$f(x) = -x^2 + 2x + 8$

30. ***Familiarize***. We make a drawing, letting w = the width of the rectangle, in ft. This leaves $80 - w - w$, or $80 - 2w$ ft of fencing for the length.



Translating. The area of a rectangle is given by length times width.

$A(w) = (80 - 2w)w$

$= 80w - 2w^2$, or $-2w^2 + 80w$

Carry out. This is a quadratic function with $a < 0$, so it has a maximum value that occurs at the vertex of the graph of the function. The first coordinate of the vertex is

$w = -\frac{b}{2a} = -\frac{80}{2(-2)} = 20.$

If $w = 20$, then $80 - 2w = 80 - 2 \cdot 20 = 40$.

Check. The area of a rectangle with length 40 ft and width 20 ft is $40 \cdot 20$, or 800 ft^2. As a partial check, we can find $A(w)$ for a value of w less than 20 and for a value of w greater than 20. For instance, $A(19.9) = 799.98$ and $A(20.1) = 799.98$. Since both of these values are less than 800, the result appears to be correct.

State. The dimensions for which the area is a maximum are 20 ft by 40 ft.

31. $f(x) = x^2 - 2x - 1$

$= (x^2 - 2x + 1 - 1) - 1$ Completing the square

$= (x^2 - 2x + 1) - 1 - 1$

$= (x-1)^2 - 2$

The graph of this function opens up and has vertex $(1, -2)$. Thus the correct graph is C.

32. The maximum value occurs at the vertex. The first coordinate of the vertex is $-\frac{b}{2a} = -\frac{(-4)}{2a} = \frac{2}{a}$ and $f\left(\frac{2}{a}\right) = 12$. Then we have:

$$\begin{aligned}
a\left(\frac{2}{a}\right)^2 - 4\left(\frac{2}{a}\right) + 3 &= 12 \\
a \cdot \frac{4}{a^2} - \frac{8}{a} + 3 &= 12 \\
\frac{4}{a} - \frac{8}{a} + 3 &= 12 \\
-\frac{4}{a} + 3 &= 12 \\
-\frac{4}{a} &= 9 \\
-4 &= 9a \\
-\frac{4}{9} &= a
\end{aligned}$$