Chapter 4

Polynomial and Rational Functions

Exercise Set 4.1

1. $g(x) = \frac{1}{2}x^3 - 10x + 8$

The leading term is $\frac{1}{2}x^3$ and the leading coefficient is $\frac{1}{2}$. The degree of the polynomial is 3, so the polynomial is cubic.

2. $f(x) = 15x^2 - 10 + 0.11x^4 - 7x^3 =$
$0.11x^4 - 7x^3 + 15x^2 - 10$

The leading term is $0.11x^4$ and the leading coefficient is 0.11. The degree of the polynomial is 4, so the polynomial is quartic.

3. $h(x) = 0.9x - 0.13$

The leading term is $0.9x$ and the leading coefficient is 0.9. The degree of the polynomial is 1, so the polynomial is linear.

4. $f(x) = -6 = -6x^0$

The leading term and leading coefficient are both -6. The degree of the polynomial is 0, so the polynomial is constant.

5. $g(x) = 305x^4 + 4021$

The leading term is $305x^4$ and the leading coefficient is 305. The degree of the polynomial is 4, so the polynomial is quartic.

6. $h(x) = 2.4x^3 + 5x^2 - x + \frac{7}{8}$

The leading term is $2.4x^3$ and the leading coefficient is 2.4. The degree of the polynomial is 3, so the polynomial is cubic.

7. $h(x) = -5x^2 + 7x^3 + x^4 = x^4 + 7x^3 - 5x^2$

The leading term is x^4 and the leading coefficient is 1 ($x^4 = 1 \cdot x^4$). The degree of the polynomial is 4, so the polynomial is quartic.

8. $f(x) = 2 - x^2 = -x^2 + 2$

The leading term is $-x^2$ and the leading coefficient is -1. The degree of the polynomial is 2, so the polynomial is quadratic.

9. $g(x) = 4x^3 - \frac{1}{2}x^2 + 8$

The leading term is $4x^3$ and the leading coefficient is 4. The degree of the polynomial is 3, so the polynomial is cubic.

10. $f(x) = 12 + x = x + 12$

The leading term is x and the leading coefficient is 1 ($x = 1 \cdot x$). The degree of the polynomial is 1, so the polynomial is linear.

11. $f(x) = -3x^3 - x + 4$

The leading term is $-3x^3$. The degree, 3, is odd and the leading coefficient, -3, is negative. Thus the end behavior of the graph is like that of (d).

12. $f(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 6x^2 + x - 5$

The leading term is $\frac{1}{4}x^4$. The degree, 4, is even and the leading coefficient, $\frac{1}{4}$, is positive. Thus the end behavior of the graph is like that of (a).

13. $f(x) = -x^6 + \frac{3}{4}x^4$

The leading term is $-x^6$. The degree, 6, is even and the leading coefficient, -1, is negative. Thus the end behavior of the graph is like that of (b).

14. $f(x) = \frac{2}{5}x^5 - 2x^4 + x^3 - \frac{1}{2}x + 3$

The leading term is $\frac{2}{5}x^5$. The degree, 5, is odd and the leading coefficient, $\frac{2}{5}$, is positive. Thus the end behavior of the graph is like that of (c).

15. $f(x) = -3.5x^4 + x^6 + 0.1x^7 = 0.1x^7 + x^6 - 3.5x^4$

The leading term is $0.1x^7$. The degree, 7, is odd and the leading coefficient, 0.1, is positive. Thus the end behavior of the graph is like that of (c).

16. $f(x) = -x^3 + x^5 - 0.5x^6 = -0.5x^6 + x^5 - x^3$

The leading term is $-0.5x^6$. The degree, 6, is even and the leading coefficient, -0.5, is negative. Thus the end behavior of the graph is like that of (b).

17. $f(x) = 10 + \frac{1}{10}x^4 - \frac{2}{5}x^3 = \frac{1}{10}x^4 - \frac{2}{5}x^3 + 10$

The leading term is $\frac{1}{10}x^4$. The degree, 4, is even and the leading coefficient, $\frac{1}{10}$, is positive. Thus the end behavior of the graph is like that of (a).

18. $f(x) = 2x + x^3 - x^5 = -x^5 + x^3 + 2x$

The leading term is $-x^5$. The degree, 5, is odd and the leading coefficient, -1, is negative. Thus the end behavior of the graph is like that of (d).

19. $f(x) = -x^6 + 2x^5 - 7x^2$

The leading term is $-x^6$. The degree, 6, is even and the leading coefficient, -1, is negative. Thus, (c) is the correct graph.

20. $f(x) = 2x^4 - x^2 + 1$

The leading term is $2x^4$. The degree, 4, is even and the leading coefficient, 2, is positive. Thus, (b) is the correct graph.

21. $f(x) = x^5 + \frac{1}{10}x - 3$

The leading term is x^5. The degree, 5, is odd and the leading coefficient, 1, is positive. Thus, (d) is the correct graph.

22. $f(x) = -x^3 + x^2 - 2x + 4$

The leading term is $-x^3$. The degree, 3, is odd and the leading coefficient, -1, is negative. Thus, (a) is the correct graph.

23. $f(x) = x^3 - 9x^2 + 14x + 24$

$f(4) = 4^3 - 9 \cdot 4^2 + 14 \cdot 4 + 24 = 0$

Since $f(4) = 0$, 4 is a zero of $f(x)$.

$f(5) = 5^3 - 9 \cdot 5^2 + 14 \cdot 5 + 24 = -6$

Since $f(5) \neq 0$, 5 is not a zero of $f(x)$.

$f(-2) = (-2)^3 - 9(-2)^2 + 14(-2) + 24 = -48$

Since $f(-2) \neq 0$, -2 is not a zero of $f(x)$.

24. $f(x) = 2x^3 - 3x^2 + x + 6$

$f(2) = 2 \cdot 2^3 - 3 \cdot 2^2 + 2 + 6 = 12$

$f(2) \neq 0$, so 2 is not a zero of $f(x)$.

$f(3) = 2 \cdot 3^3 - 3 \cdot 3^2 + 3 + 6 = 36$

$f(3) \neq 0$, so 3 is not a zero of $f(x)$.

$f(-1) = 2(-1)^3 - 3(-1)^2 + (-1) + 6 = 0$

$f(-1) = 0$, so -1 is a zero of $f(x)$.

25. $g(x) = x^4 - 6x^3 + 8x^2 + 6x - 9$

$g(2) = 2^4 - 6 \cdot 2^3 + 8 \cdot 2^2 + 6 \cdot 2 - 9 = 3$

Since $g(2) \neq 0$, 2 is not a zero of $g(x)$.

$g(3) = 3^4 - 6 \cdot 3^3 + 8 \cdot 3^2 + 6 \cdot 3 - 9 = 0$

Since $g(3) = 0$, 3 is a zero of $g(x)$.

$g(-1) = (-1)^4 - 6(-1)^3 + 8(-1)^2 + 6(-1) - 9 = 0$

Since $g(-1) = 0$, -1 is a zero of $g(x)$.

26. $g(x) = x^4 - x^3 - 3x^2 + 5x - 2$

$g(1) = 1^4 - 1^3 - 3 \cdot 1^2 + 5 \cdot 1 - 2 = 0$

Since $g(1) = 0$, 1 is a zero of $g(x)$.

$g(-2) = (-2)^4 - (-2)^3 - 3(-2)^2 + 5(-2) - 2 = 0$

Since $g(-2) = 0$, -2 is a zero of $g(x)$.

$g(3) = 3^4 - 3^3 - 3 \cdot 3^2 + 5 \cdot 3 - 2 = 40$

Since $g(3) \neq 0$, 3 is not a zero of $g(x)$.

27. $f(x) = (x+3)^2(x-1) = (x+3)(x+3)(x-1)$

To solve $f(x) = 0$ we use the principle of zero products, solving $x + 3 = 0$ and $x - 1 = 0$. The zeros of $f(x)$ are -3 and 1.

The factor $x + 3$ occurs twice. Thus the zero -3 has a multiplicity of two.

The factor $x - 1$ occurs only one time. Thus the zero 1 has a multiplicity of one.

28. $f(x) = (x+5)^3(x-4)(x+1)^2$

-5, multiplicity 3; 4, multiplicity 1;

-1, multiplicity 2

29. $f(x) = -2(x-4)(x-4)(x-4)(x+6) = -2(x-4)^3(x+6)$

To solve $f(x) = 0$ we use the principle of zero products, solving $x - 4 = 0$ and $x + 6 = 0$. The zeros of $f(x)$ are 4 and -6.

The factor $x - 4$ occurs three times. Thus the zero 4 has a multiplicity of 3.

The factor $x + 6$ occurs only one time. Thus the zero -6 has a multiplicity of 1.

30. $f(x) = \left(x+\frac{1}{2}\right)(x+7)(x+7)(x+5) = \left(x+\frac{1}{2}\right)(x+7)^2(x+5)$

$-\frac{1}{2}$, multiplicity 1; -7, multiplicity 2;

-5, multiplicity 1

31. $f(x) = (x^2 - 9)^3 = [(x+3)(x-3)]^3 = (x+3)^3(x-3)^3$

To solve $f(x) = 0$ we use the principle of zero products, solving $x + 3 = 0$ and $x - 3 = 0$. The zeros of $f(x)$ are -3 and 3.

The factors $x+3$ and $x-3$ each occur three times so each zero has a multiplicity of 3.

32. $f(x) = (x^2 - 4)^2 = [(x+2)(x-2)]^2 = (x+2)^2(x-2)^2$

-2, multiplicity 2; 2, multiplicity 2

33. $f(x) = x^3(x-1)^2(x+4)$

To solve $f(x) = 0$ we use the principle of zero products, solving $x = 0$, $x - 1 = 0$, and $x + 4 = 0$. The zeros of $f(x)$ are 0, 1, and -4.

The factor x occurs three times. Thus the zero 0 has a multiplicity of three.

The factor $x - 1$ occurs twice. Thus the zero 1 has a multiplicity of two.

The factor $x + 4$ occurs only one time. Thus the zero -4 has a multiplicity of one.

34. $f(x) = x^2(x+3)^2(x-4)(x+1)^4$

0, multiplicity 2; -3, multiplicity 2;

4, multiplicity 1; -1, multiplicity 4

35. $f(x) = -8(x-3)^2(x+4)^3x^4$

To solve $f(x) = 0$ we use the principle of zero products, solving $x - 3 = 0$, $x + 4 = 0$, and $x = 0$. The zeros of $f(x)$ are 3, -4, and 0.

The factor $x - 3$ occurs twice. Thus the zero 3 has a multiplicity of 2.

The factor $x + 4$ occurs three times. Thus the zero -4 has a multiplicity of 3.

The factor x occurs four times. Thus the zero 0 has a multiplicity of 4.

36. $f(x) = (x^2 - 5x + 6)^2$
$= [(x-3)(x-2)]^2$
$= (x-3)^2(x-2)^2$

3, multiplicity 2; 2, multiplicity 2

37. $f(x) = x^4 - 4x^2 + 3$

We factor as follows:

$f(x) = (x^2 - 3)(x^2 - 1)$
$= (x - \sqrt{3})(x + \sqrt{3})(x-1)(x+1)$

The zeros of the function are $\sqrt{3}$, $-\sqrt{3}$, 1, and -1. Each has a multiplicity of 1.

38. $f(x) = x^4 - 10x^2 + 9$
$= (x^2 - 9)(x^2 - 1)$
$= (x+3)(x-3)(x+1)(x-1)$

± 3, ± 1; each has a multiplicity of 1.

39. $f(x) = x^3 + 3x^2 - x - 3$

We factor by grouping:

$f(x) = x^2(x+3) - (x+3)$
$= (x^2 - 1)(x+3)$
$= (x-1)(x+1)(x+3)$

The zeros of the function are 1, -1, and -3. Each has a multiplicity of 1.

40. $f(x) = x^3 - x^2 - 2x + 2$
$= x^2(x-1) - 2(x-1)$
$= (x^2 - 2)(x-1)$
$= (x - \sqrt{2})(x + \sqrt{2})(x-1)$

$\sqrt{2}$, $-\sqrt{2}$, 1; each has a multiplicity of 1.

41. $f(x) = 2x^3 - x^2 - 8x + 4$
$= x^2(2x-1) - 4(2x-1)$
$= (2x-1)(x^2-4)$
$= (2x-1)(x+2)(x-2)$

The zeros of the function are $\frac{1}{2}$, -2, and 2. Each has a multiplicity of 1.

42. $f(x) = 3x^3 + x^2 - 48x - 16$
$= x^2(3x+1) - 16(3x+1)$
$= (3x+1)(x^2-16)$
$= (3x+1)(x+4)(x-4)$

$-\frac{1}{3}$, -4, 4; each has a multiplicity of 1

43. Graphing the function, we see that the graph touches the x-axis at $(3,0)$ but does not cross it, so the statement is false.

44. Graphing the function, we see that the statement is true.

45. Graphing the function, we see that the statement is true.

46. The number 4 is not a zero of the polynomial, so the statement is false.

47. For 1995, $x = 1995 - 1990 = 5$.

$f(5) = -0.056316(5)^4 - 19.500154(5)^3 + 584.892054(5)^2 - 1518.5717(5) + 94,299.1990 \approx 98,856$ twin births

For 2005, $x = 2005 - 1990 = 15$.

$f(15) = -0.056316(15)^4 - 19.500154(15)^3 + 584.892054(15)^2 - 1518.5717(15) + 94,299.1990 \approx 134,457$ twin births

48. For 1916, $x = 1916 - 1900 = 16$.

$f(16) = -0.004091(16)^4 + 1.275179(16)^3 - 142.58929(16)^2 + 5069.1067(16) + 197,909.1675 \approx 247,467$ mi

For 1960, $x = 1960 - 1900 = 60$.

$f(60) = -0.004091(60)^4 + 1.275179(60)^3 - 142.58929(60)^2 + 5069.1067(60) + 197,909.1675 \approx 211,153$ mi

For 1985, $x = 1985 - 1900 = 85$.

$f(85) = -0.004091(85)^4 + 1.275179(85)^3 - 142.58929(85)^2 + 5069.1067(85) + 197,909.1675 \approx 168,142$ mi

For 2010, $x = 2010 - 1900 = 110$.

$f(110) = -0.004091(110)^4 + 1.275179(110)^3 - 142.58929(110)^2 + 5069.1067(110) + 197,909.1675 \approx$ 128,480 mi

49. $d(3) = 0.010255(3)^3 - 0.340119(3)^2 + 7.397499(3) + 6.618361 \approx 26$ yr

$d(12) = 0.010255(12)^3 - 0.340119(12)^2 + 7.397499(12) + 6.618361 \approx 64$ yr

$d(16) = 0.010255(16)^3 - 0.340119(16)^2 + 7.397499(16) + 6.618361 \approx 80$ yr

50. 5 ft 7 in. = 67 in.

$W(67) = \left(\frac{67}{12.3}\right)^3 \approx 162$ lb

51. We substitute 294 for $s(t)$ and solve for t.

$$294 = 4.9t^2 + 34.3t$$
$$0 = 4.9t^2 + 34.3t - 294$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-34.3 \pm \sqrt{(34.3)^2 - 4(4.9)(-294)}}{2(4.9)}$$
$$= \frac{-34.3 \pm \sqrt{6938.89}}{9.8}$$

$t = 5$ *or* $t = -12$

Only the positive number has meaning in the situation. It will take the stone 5 sec to reach the ground.

52. First find the number of games played.

$N(x) = x^2 - x$

$N(9) = 9^2 - 9 = 72$

Now multiply the number of games by the cost per game to find the total cost.

$72 \cdot 110 = 7920$

It will cost \$7920 to play the entire schedule.

53. For 2002, $x = 2002 - 2000 = 2$.

$h(2) = 56.8328(2)^4 - 1554.7494(2)^3 + 10,451.8211(2)^2 - 5655.7692(2) + 140,589.1608 \approx \$159,556$

For 2005, $x = 2005 - 2000 = 5$.

$h(5) = 56.8328(5)^4 - 1554.7494(5)^3 + 10,451.8211(5)^2 - 5655.7692(5) + 140,589.1608 \approx \$214,783$

For 2008, $x = 2008 - 2000 = 8$.

$h(8) = 56.8328(8)^4 - 1554.7494(8)^3 + 10,451.8211(8)^2 - 5655.7692(8) + 140,589.1608 \approx \$201,015$

For 2009, $x = 2009 - 2000 = 9$.

$h(9) = 56.8328(9)^4 - 1554.7494(9)^3 + 10,451.8211(9)^2 - 5655.7692(9) + 140,589.1608 \approx \$175,752$

54. For 1945, $x = 1945 - 1940 = 5$.

$f(5) = -0.006093(5)^4 + 0.849362(5)^3 - 51.892087(5)^2 + 1627.3581(5) + 41,334.7289 \approx 48,276.579$ thousand, or 48,276,579

For 1985, $x = 1985 - 1940 = 45$.

$f(45) = -0.006093(45)^4 + 0.849362(45)^3 - 51.892087(45)^2 + 1627.3581(45) + 41,334.7289 \approx 61,897.371$ thousand, or 61,897,371

For 2008, $x = 2008 - 1940 = 68$.

$f(68) = -0.006093(68)^4 + 0.849362(68)^3 - 51.892087(68)^2 + 1627.3581(68) + 41,334.7289 \approx 48,835.938$ thousand, or 48,835,938

55.

$$\begin{aligned} A &= P(1+i)^t \\ 9039.75 &= 8000(1+i)^2 \quad \text{Substituting} \\ \frac{9039.75}{8000} &= (1+i)^2 \\ \pm 1.063 &\approx 1+i \quad \text{Taking the square root on both sides} \\ -1 \pm 1.063 &\approx i \end{aligned}$$

$$-1 + 1.063 \approx i \quad or \quad -1 - 1.063 \approx i$$
$$0.063 \approx i \quad or \quad -2.063 \approx i$$

Only the positive result has meaning in this application. The interest rate is about 0.063, or 6.3%.

56. Solve: $11,193.64 = 10,000(1+i)^2$

$i = 0.058 \quad or \quad i = -2.058$

Only the positive result has meaning in this application. The interest rate is 0.058, or 5.8%.

57.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1-(-5)]^2 + (0-3)^2} \\ &= \sqrt{4^2 + (-3)^2} = \sqrt{16+9} \\ &= \sqrt{25} = 5 \end{aligned}$$

58. $d = \sqrt{(-2-4)^2 + (-4-2)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

59.

$$\begin{aligned} (x-3)^2 + (y+5)^2 &= 49 \\ (x-3)^2 + [y-(-5)]^2 &= 7^2 \end{aligned}$$

Center: $(3, -5)$; radius: 7

60. The center of the circle is the midpoint of a segment that is a diameter:

$$\left(\frac{-6+(-2)}{2}, \frac{5+1}{2}\right) = (-4, 3).$$

The length of a radius is the distance from the center to one of the endpoints of the diameter:

$$\begin{aligned} r &= \sqrt{[-6-(-4)]^2 + (5-3)^2} \\ &= \sqrt{4+4} = \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

61.

$$\begin{aligned} 2y - 3 &\ge 1 - y + 5 \\ 2y - 3 &\ge 6 - y && \text{Collecting like terms} \\ 3y - 3 &\ge 6 && \text{Adding } y \\ 3y &\ge 9 && \text{Adding 3} \\ y &\ge 3 && \text{Dividing by 3} \end{aligned}$$

The solution set is $\{y|y \ge 3\}$, or $[3, \infty)$.

62.

$$\begin{aligned} (x-2)(x+5) &> x(x-3) \\ x^2 + 3x - 10 &> x^2 - 3x \\ 6x - 10 &> 0 \\ 6x &> 10 \\ x &> \frac{5}{3} \end{aligned}$$

The solution set is $\left\{x \middle| x > \frac{5}{3}\right\}$, or $\left(\frac{5}{3}, \infty\right)$.

63. $|x+6| \ge 7$

$$x + 6 \le -7 \quad or \quad x + 6 \ge 7$$
$$x \le -13 \quad or \quad x \ge 1$$

The solution set is $\{x|x \le -13 \ or \ x \ge 1\}$, or $(-\infty, -13] \cup [1, \infty)$.

64. $\left|x + \frac{1}{4}\right| \le \frac{2}{3}$

$$-\frac{2}{3} \le x + \frac{1}{4} \le \frac{2}{3}$$
$$-\frac{11}{12} \le x \le \frac{5}{12}$$

The solution set is $\left\{x \middle| -\frac{11}{12} \le x \le \frac{5}{12}\right\}$, or $\left[-\frac{11}{12}, \frac{5}{12}\right]$.

65. $f(x) = (x^5 - 1)^2(x^2 + 2)^3$

The leading term of $(x^5 - 1)^2$ is $(x^5)^2$, or x^{10}. The leading term of $(x^2 + 2)^3$ is $(x^2)^3$, or x^6. Then the leading term of $f(x)$ is $x^{10} \cdot x^6$, or x^{16}, and the degree of $f(x)$ is 16.

66. $f(x) = (10 - 3x^5)^2(5 - x^4)^3(x + 4)$

The leading term of $(10 - 3x^5)^2$ is $(3x^5)^2$, or $9x^{10}$. The leading term of $(5 - x^4)^3$ is $(-x^4)^3$, or $-x^{12}$. The leading term of $x + 4$ is x. Then the leading term of $f(x)$ is $9x^{10}(-x^{12})(x) = -9x^{23}$, and the degree of $f(x)$ is 23.

Exercise Set 4.2

1. $f(x) = x^5 - x^2 + 6$

a) This function has degree 5, so its graph can have at most 5 real zeros.

b) This function has degree 5, so its graph can have at most 5 x-intercepts.

c) This function has degree 5, so its graph can have at most $5 - 1$, or 4, turning points.

2. $f(x) = -x^2 + x^4 - x^6 + 3 = -x^6 + x^4 - x^2 + 3$

a) This function has degree 6, so its graph can have at most 6 real zeros.

b) This function has degree 6, so its graph can have at most 6 x-intercepts.

c) This function has degree 6, so its graph can have at most $6 - 1$, or 5, turning points.

3. $f(x) = x^{10} - 2x^5 + 4x - 2$

a) This function has degree 10, so its graph can have at most 10 real zeros.

b) This function has degree 10, so its graph can have at most 10 x-intercepts.

c) This function has degree 10, so its graph can have at most $10 - 1$, or 9, turning points.

4. $f(x) = \frac{1}{4}x^3 + 2x^2$

a) This function has degree 3, so its graph can have at most 3 real zeros.

b) This function has degree 3, so its graph can have at most 3 x-intercepts.

c) This function has degree 3, so its graph can have at most $3 - 1$, or 2, turning points.

5. $f(x) = -x - x^3 = -x^3 - x$

a) This function has degree 3, so its graph can have at most 3 real zeros.

b) This function has degree 3, so its graph can have at most 3 x-intercepts.

c) This function has degree 3, so its graph can have at most $3 - 1$, or 2, turning points.

6. $f(x) = -3x^4 + 2x^3 - x - 4$

a) This function has degree 4, so its graph can have at most 4 real zeros.

b) This function has degree 4, so its graph can have at most 4 x-intercepts.

c) This function has degree 4, so its graph can have at most $4 - 1$, or 3, turning points.

7. $f(x) = \frac{1}{4}x^2 - 5$

The leading term is $\frac{1}{4}x^2$. The sign of the leading coefficient, $\frac{1}{4}$, is positive and the degree, 2, is even, so we would choose either graph (b) or graph (d). Note also that $f(0) = -5$, so the y-intercept is $(0, -5)$. Thus, graph (d) is the graph of this function.

8. $f(x) = -0.5x^6 - x^5 + 4x^4 - 5x^3 - 7x^2 + x - 3$

The leading term is $-0.5x^6$. The sign of the leading coefficient, -0.5, is negative and the degree, 6, is even. Thus, graph (a) is the graph of this function.

9. $f(x) = x^5 - x^4 + x^2 + 4$

The leading term is x^5. The sign of the leading coefficient, 1, is positive and the degree, 5, is odd. Thus, graph (f) is the graph of this function.

10. $f(x) = -\frac{1}{3}x^3 - 4x^2 + 6x + 42$

The leading term is $-\frac{1}{3}x^3$. The sign of the leading coefficient, $-\frac{1}{3}$, is negative and the degree, 3, is odd, so we would choose either graph (c) or graph (e). Note also that $f(0) = 42$, so the y-intercept is $(0, 42)$. Thus, graph (c) is the graph of this function.

11. $f(x) = x^4 - 2x^3 + 12x^2 + x - 20$

The leading term is x^4. The sign of the leading coefficient, 1, is positive and the degree, 4, is even, so we would choose either graph (b) or graph (d). Note also that $f(0) = -20$, so the y-intercept is $(0, -20)$. Thus, graph (b) is the graph of this function.

12. $f(x) = -0.3x^7 + 0.11x^6 - 0.25x^5 + x^4 + x^3 - 6x - 5$

The leading term is $-0.3x^7$. The sign of the leading coefficient, -0.3, is negative and the degree, 7, is odd, so we would choose either graph (c) or graph (e). Note also that $f(0) = -5$, so the y-intercept is $(0, -5)$. Thus, graph (e) is the graph of this function.

13. $f(x) = -x^3 - 2x^2$

1. The leading term is $-x^3$. The degree, 3, is odd and the leading coefficient, -1, is negative so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$.
2. We solve $f(x) = 0$.

$$\begin{aligned} -x^3 - 2x^2 &= 0 \\ -x^2(x+2) &= 0 \\ -x^2 = 0 \quad &or \quad x + 2 = 0 \\ x^2 = 0 \quad &or \quad\quad x = -2 \\ x = 0 \quad &or \quad\quad x = -2 \end{aligned}$$

   The zeros of the function are 0 and -2, so the x-intercepts of the graph are $(0, 0)$ and $(-2, 0)$.
3. The zeros divide the x-axis into 3 intervals, $(-\infty, -2)$, $(-2, 0)$, and $(0, \infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

In $(-\infty, -2)$, test -3:

$f(-3) = -(-3)^3 - 2(-3)^2 = 9 > 0$

In $(-2, 0)$, test -1:

$f(-1) = -(-1)^3 - 2(-1)^2 = -1 < 0$

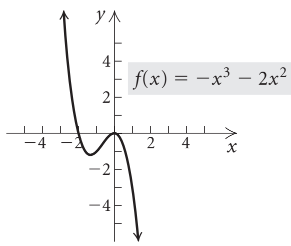
In $(0, \infty)$, test 1:

$f(1) = -1^3 - 2 \cdot 1^2 = -3 < 0$

Thus the graph lies above the x-axis on $(-\infty, -2)$ and below the x-axis on $(-2, 0)$ and $(0, \infty)$. We also know the points $(-3, 9)$, $(-1, -1)$, and $(1, -3)$ are on the graph.

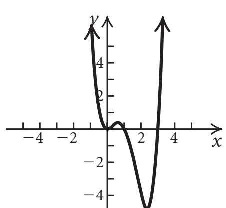
4. From Step 2 we see that the y-intercept is $(0, 0)$.
5. We find additional points on the graph and then draw the graph.

x	$f(x)$
-2.5	3.125
-1.5	-1.125
1.5	-7.875



6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

14.



$g(x) = x^4 - 4x^3 + 3x^2$

15. $h(x) = x^2 + 2x - 3$

1. The leading term is x^2. The degree, 2, is even and leading coefficient, 1, is positive so as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to \infty$.
2. We solve $h(x) = 0$.

$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x + 3 = 0 \quad \textit{or} \quad x - 1 = 0$$
$$x = -3 \quad \textit{or} \quad x = 1$$

The zeros of the function are -3 and 1, so the x-intercepts of the graph are $(-3, 0)$ and $(1, 0)$.

3. The zeros divide the x-axis into 3 intervals, $(-\infty, -3)$, $(-3, 1)$, and $(1, \infty)$. We choose a value for x from each interval and find $h(x)$. This tells us the sign of $h(x)$ for all values of x in that interval.

In $(-\infty, -3)$, test -4:

$h(-4) = (-4)^2 + 2(-4) - 3 = 5 > 0$

In $(-3, 1)$, test 0:

$h(0) = 0^2 + 2 \cdot 0 - 3 = -3 < 0$

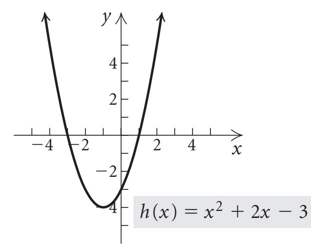
In $(1, \infty)$, test 2:

$h(2) = 2^2 + 2 \cdot 2 - 3 = 5 > 0$

Thus the graph lies above the x-axis on $(-\infty, -3)$ and on $(1, \infty)$. It lies below the x-axis on $(-3, 1)$. We also know the points $(-4, 5)$, $(0, -3)$, and $(2, 5)$ are on the graph.

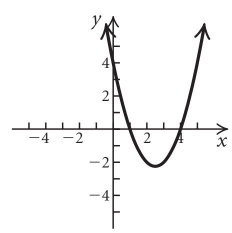
4. From Step 3 we see that the y-intercept is $(0, -3)$.
5. We find additional points on the graph and then draw the graph.

x	$h(x)$
-2	-3
-1	-4
3	12



6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

16.



$f(x) = x^2 - 5x + 4$

17. $h(x) = x^5 - 4x^3$

1. The leading term is x^5. The degree, 5, is odd and the leading coefficient, 1, is positive so as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to -\infty$.
2. We solve $h(x) = 0$.

$$x^5 - 4x^3 = 0$$
$$x^3(x^2 - 4) = 0$$
$$x^3(x+2)(x-2) = 0$$
$$x^3 = 0 \quad \textit{or} \quad x + 2 = 0 \quad \textit{or} \quad x - 2 = 0$$
$$x = 0 \quad \textit{or} \quad x = -2 \quad \textit{or} \quad x = 2$$

The zeros of the function are 0, -2, and 2 so the x-intercepts of the graph are $(0, 0)$, $(-2, 0)$, and $(2, 0)$.

3. The zeros divide the x-axis into 4 intervals, $(-\infty, -2)$, $(-2, 0)$, $(0, 2)$, and $(2, \infty)$. We choose a value for x from each interval and find $h(x)$. This tells us the sign of $h(x)$ for all values of x in that interval.

In $(-\infty, -2)$, test -3:

$h(-3) = (-3)^5 - 4(-3)^3 = -135 < 0$

In $(-2, 0)$, test -1:

$h(-1) = (-1)^5 - 4(-1)^3 = 3 > 0$

In $(0, 2)$, test 1:

$h(1) = 1^5 - 4 \cdot 1^3 = -3 < 0$

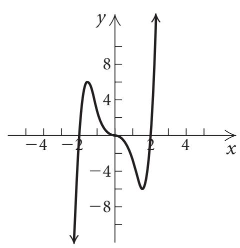
In $(2, \infty)$, test 3:

$h(3) = 3^5 - 4 \cdot 3^3 = 135 > 0$

Thus the graph lies below the x-axis on $(-\infty, -2)$ and on $(0, 2)$. It lies above the x-axis on $(-2, 0)$ and on $(2, \infty)$. We also know the points $(-3, -135)$, $(-1, 3)$, $(1, -3)$, and $(3, 135)$ are on the graph.

4. From Step 2 we see that the y-intercept is $(0, 0)$.
5. We find additional points on the graph and then draw the graph.

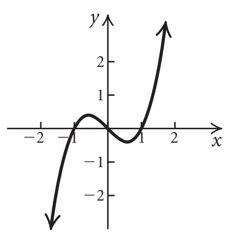
x	$h(x)$
-2.5	-35.2
-1.5	5.9
1.5	-5.9
2.5	35.2



$h(x) = x^5 - 4x^3$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

18.



$f(x) = x^3 - x$

19. $h(x) = x(x - 4)(x + 1)(x - 2)$

1. The leading term is $x \cdot x \cdot x \cdot x$, or x^4. The degree, 4, is even and the leading coefficient, 1, is positive so as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to \infty$.
2. We see that the zeros of the function are 0, 4, -1, and 2 so the x-intercepts of the graph are $(0, 0)$, $(4, 0)$, $(-1, 0)$, and $(2, 0)$.
3. The zeros divide the x-axis into 5 intervals, $(-\infty, -1)$, $(-1, 0)$, $(0, 2)$, $(2, 4)$, and $(4, \infty)$. We choose a value for x from each interval and find $h(x)$. This tells us the sign of $h(x)$ for all values of x in that interval.

In $(-\infty, -1)$, test -2:

$h(-2) = -2(-2 - 4)(-2 + 1)(-2 - 2) = 48 > 0$

In $(-1, 0)$, test -0.5:

$h(-0.5) = (-0.5)(-0.5 - 4)(-0.5 + 1)(-0.5 - 2) = -2.8125 < 0$

In $(0, 2)$, test 1:

$h(1) = 1(1 - 4)(1 + 1)(1 - 2) = 6 > 0$

In $(2, 4)$, test 3:

$h(3) = 3(3 - 4)(3 + 1)(3 - 2) = -12 < 0$

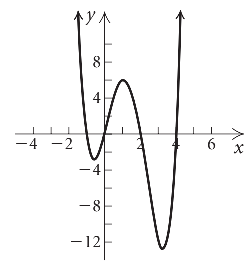
In $(4, \infty)$, test 5:

$h(5) = 5(5 - 4)(5 + 1)(5 - 2) = 90 > 0$

Thus the graph lies above the x-axis on $(-\infty, -1)$, $(0, 2)$, and $(4, \infty)$. It lies below the x-axis on $(-1, 0)$ and on $(2, 4)$. We also know the points $(-2, 48)$, $(-0.5, -2.8125)$, $(1, 6)$, $(3, -12)$, and $(5, 90)$ are on the graph.

4. From Step 2 we see that the y-intercept is $(0, 0)$.
5. We find additional points on the graph and then draw the graph.

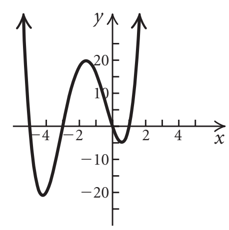
x	$h(x)$
-1.5	14.4
1.5	4.7
2.5	-6.6
4.5	30.9



$h(x) = x(x - 4)(x + 1)(x - 2)$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

20.



$f(x) = x(x-1)(x+3)(x+5)$

21. $g(x) = -\dfrac{1}{4}x^3 - \dfrac{3}{4}x^2$

1. The leading term is $-\dfrac{1}{4}x^3$. The degree, 3, is odd and the leading coefficient, $-\dfrac{1}{4}$, is negative so as $x \to \infty$, $g(x) \to -\infty$ and as $x \to -\infty$, $g(x) \to \infty$.
2. We solve $g(x) = 0$.

$$-\frac{1}{4}x^3 - \frac{3}{4}x^2 = 0$$

$$-\frac{1}{4}x^2(x + 3) = 0$$

$$-\frac{1}{4}x^2 = 0 \quad or \quad x + 3 = 0$$

$$x^2 = 0 \quad or \quad x = -3$$

$$x = 0 \quad or \quad x = -3$$

The zeros of the function are 0 and -3, so the x-intercepts of the graph are $(0, 0)$ and $(-3, 0)$.

3. The zeros divide the x-axis into 3 intervals, $(-\infty, -3)$, $(-3, 0)$, and $(0, \infty)$. We choose a value for x from each interval and find $g(x)$. This tells us the sign of $g(x)$ for all values of x in that interval.

In $(-\infty, -3)$, test -4:

$g(-4) = -\frac{1}{4}(-4)^3 - \frac{3}{4}(-4)^2 = 4 > 0$

In $(-3, 0)$, test -1:

$g(-1) = -\frac{1}{4}(-1)^3 - \frac{3}{4}(-1)^2 = -\frac{1}{2} < 0$

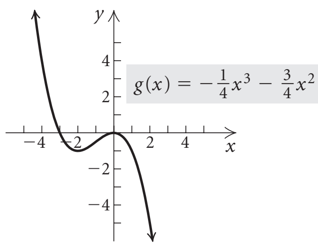
In $(0, \infty)$, test 1:

$g(1) = -\frac{1}{4} \cdot 1^3 - \frac{3}{4} \cdot 1^2 = -1 < 0$

Thus the graph lies above the x-axis on $(-\infty, -3)$ and below the x-axis on $(-3, 0)$ and on $(0, \infty)$. We also know the points $(-4, 4)$, $\left(-1, -\frac{1}{2}\right)$, and $(1, -1)$ are on the graph.

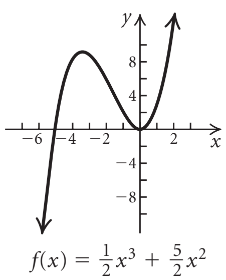
4. From Step 2 we see that the y-intercept is $(0, 0)$.
5. We find additional points on the graph and then draw the graph.

x	$g(x)$
-2	-1
2	-5
2.5	-8.6



6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

22.



23. $g(x) = -x^4 - 2x^3$

1. The leading term is $-x^4$. The degree, 4, is even and the leading coefficient, -1, is negative so as $x \to \infty$, $g(x) \to -\infty$ and as $x \to -\infty$, $g(x) \to -\infty$.
2. We solve $f(x) = 0$.

   $-x^4 - 2x^3 = 0$

   $-x^3(x + 2) = 0$

   $-x^3 = 0$ *or* $x + 2 = 0$

   $x = 0$ *or* $x = -2$

   The zeros of the function are 0 and -2, so the x-intercepts of the graph are $(0, 0)$ and $(-2, 0)$.
3. The zeros divide the x-axis into 3 intervals, $(-\infty, -2)$, $(-2, 0)$, and $(0, \infty)$. We choose a value for x from each interval and find $g(x)$. This tells us the sign of $g(x)$ for all values of x in that interval.

   In $(-\infty, -2)$, test -3:

   $g(-3) = -(-3)^4 - 2(-3)^3 = -27 < 0$

   In $(-2, 0)$, test -1:

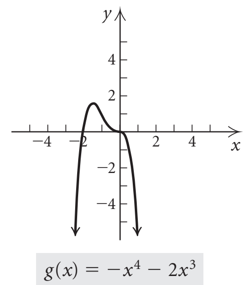
   $g(-1) = -(-1)^4 - 2(-1)^3 = 1 > 0$

   In $(0, \infty)$, test 1:

   $g(1) = -(1)^4 - 2(1)^3 = -3 < 0$

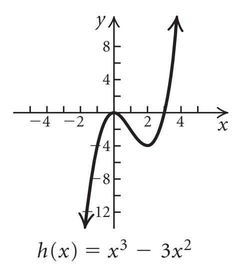
   Thus the graph lies below the x-axis on $(-\infty, -2)$ and $(0, \infty)$ and above the x-axis on $(-2, 0)$. We also know the points $(-3, -27)$, $(-1, 1)$, and $(1, -3)$ are on the graph.
4. From Step 2 we see that the y-intercept is $(0, 0)$.
5. We find additional points on the graph and then draw the graph.

x	$g(x)$
-2.5	-7.8
-1.5	1.7
0.5	-0.3
1.5	-11.8
2	-32



6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

24.



25. $f(x) = -\frac{1}{2}(x - 2)(x + 1)^2(x - 1)$

1. The leading term is $-\frac{1}{2} \cdot x \cdot x \cdot x \cdot x$, or $-\frac{1}{2}x^4$. The degree, 4, is even and the leading coefficient, $-\frac{1}{2}$, is negative so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
2. We solve $f(x) = 0$.

   $-\frac{1}{2}(x - 2)(x + 1)^2(x - 1) = 0$

   $x - 2 = 0$ *or* $(x + 1)^2 = 0$ *or* $x - 1 = 0$

   $x = 2$ *or* $x + 1 = 0$ *or* $x = 1$

   $x = 2$ *or* $x = -1$ *or* $x = 1$

   The zeros of the function are 2, -1, and 1, so the x-intercepts of the graph are $(2, 0)$, $(-1, 0)$, and $(1, 0)$.
3. The zeros divide the x-axis into 4 intervals, $(-\infty, -1)$, $(-1, 1)$, $(1, 2)$, and $(2, \infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

In $(-\infty, -1)$, test -2:

$f(-2) = -\frac{1}{2}(-2-2)(-2+1)^2(-2-1) = -6 < 0$

In $(-1, 1)$, test 0:

$f(0) = -\frac{1}{2}(0-2)(0+1)^2(0-1) = -1 < 0$

In $(1, 2)$, test 1.5:

$f(1.5) = -\frac{1}{2}(1.5-2)(1.5+1)^2(1.5-1) = 0.78125 > 0$

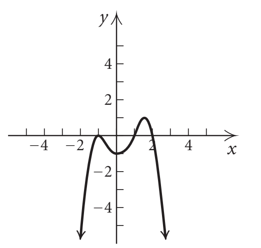
In $(2, \infty)$, test 3:

$f(3) = -\frac{1}{2}(3-2)(3+1)^2(3-1) = -16 < 0$

Thus the graph lies below the x-axis on $(-\infty, -1)$, $(-1, 1)$, and $(2, \infty)$ and above the x-axis on $(1, 2)$. We also know the points $(-2, -6)$, $(0, -1)$, $(1.5, 0.78125)$, and $(3, -16)$ are on the graph.

4. From Step 2 we know that $f(0) = -1$ so the y-intercept is $(0, -1)$.
5. We find additional points on the graph and then draw the graph.

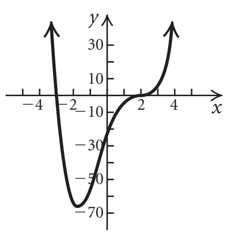
x	$f(x)$
-3	-40
-0.5	-0.5
0.5	-0.8
1.5	0.8



$f(x) = -\frac{1}{2}(x-2)(x+1)^2(x-1)$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

26.



$g(x) = (x-2)^3(x+3)$

27. $g(x) = -x(x-1)^2(x+4)^2$

1. The leading term is $-x \cdot x \cdot x \cdot x \cdot x$, or $-x^5$. The degree, 5, is odd and the leading coefficient, -1, is negative so as $x \to \infty$, $g(x) \to -\infty$ and as $x \to -\infty$, $g(x) \to \infty$.
2. We solve $g(x) = 0$.

$-x(x-1)^2(x+4)^2 = 0$

$-x = 0 \quad or \quad (x-1)^2 = 0 \quad or \quad (x+4)^2 = 0$

$x = 0 \quad or \quad x - 1 = 0 \quad or \quad x + 4 = 0$

$x = 0 \quad or \quad x = 1 \quad or \quad x = -4$

The zeros of the function are 0, 1, and -4, so the x-intercepts are $(0, 0)$, $(1, 0)$, and $(-4, 0)$.

3. The zeros divide the x-axis into 4 intervals, $(-\infty, -4)$, $(-4, 0)$, $(0, 1)$, and $(1, \infty)$. We choose a value for x from each interval and find $g(x)$. This tells us the sign of $g(x)$ for all values of x in that interval.

In $(-\infty, -4)$, test -5:

$g(-5) = -(-5)(-5-1)^2(-5+4)^2 = 180 > 0$

In $(-4, 0)$, test -1:

$g(-1) = -(-1)(-1-1)^2(-1+4)^2 = 36 > 0$

In $(0, 1)$, test 0.5:

$g(0.5) = -0.5(0.5-1)^2(-0.5+4)^2 = -2.53125 < 0$

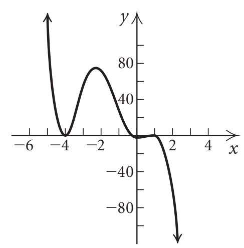
In $(1, \infty)$, test 2:

$g(2) = -2(2-1)^2(2+4)^2 = -72 < 0$

Thus the graph lies above the x-axis on $(-\infty, -4)$ and on $(-4, 0)$ and below the x-axis on $(0, 1)$ and $(1, \infty)$. We also know the points $(-5, 180)$, $(-1, 36)$, $(0.5, -2.53125)$, and $(2, -72)$ are on the graph.

4. From Step 2 we see that the y-intercept is (0,0).
5. We find additional points on the graph and then draw the graph.

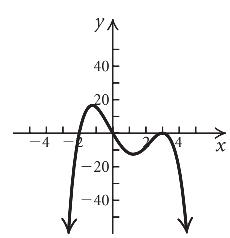
x	$g(x)$
-3	4.8
-2	72
1.5	-11.3



$g(x) = -x(x-1)^2(x+4)^2$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

28.



$h(x) = -x(x-3)(x-3)(x+2)$

29. $f(x) = (x-2)^2(x+1)^4$

1. The leading term is $x \cdot x \cdot x \cdot x \cdot x \cdot x$, or x^6. The degree, 6, is even and the leading coefficient, 1, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.
2. We see that the zeros of the function are 2 and -1 so the x-intercepts of the graph are $(2, 0)$ and $(-1, 0)$.
3. The zeros divide the x-axis into 3 intervals, $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

In $(-\infty, -1)$, test -2:

$f(-2) = (-2-2)^2(-2+1)^4 = 16 > 0$

In $(-1, 2)$, test 0:

$f(0) = (0-2)^2(0+1)^4 = 4 > 0$

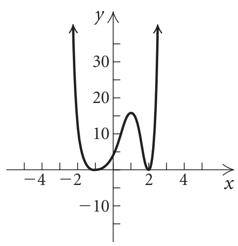
In $(2, \infty)$, test 3:

$f(3) = (3-2)^2(3+1)^4 = 256 > 0$

Thus the graph lies above the x-axis on all 3 intervals. We also know the points $(-2, 16)$, $(0, 4)$, and $(3, 256)$ are on the graph.

4. From Step 3 we know that $f(0) = 4$ so the y-intercept is $(0, 4)$.
5. We find additional points on the graph and then draw the graph.

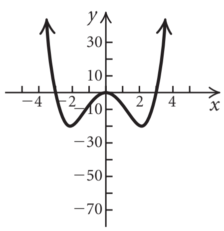
x	$f(x)$
-1.5	0.8
-0.5	0.4
1	16
1.5	9.8



$f(x) = (x-2)^2(x+1)^4$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

30.



$g(x) = x^4 - 9x^2$

31. $g(x) = -(x-1)^4$

1. The leading term is $-1 \cdot x \cdot x \cdot x \cdot x$, or $-x^4$. The degree, 4, is even and the leading coefficient, -1, is negative so as $x \to \infty$, $g(x) \to -\infty$ and as $x \to -\infty$, $g(x) \to -\infty$.
2. We see that the zero of the function is 1, so the x-intercept is $(1, 0)$.
3. The zero divides the x-axis into 2 intervals, $(-\infty, 1)$ and $(1, \infty)$. We choose a value for x from each interval and find $g(x)$. This tells us the sign of $g(x)$ for all values of x in that interval.

   In $(-\infty, 1)$, test 0:

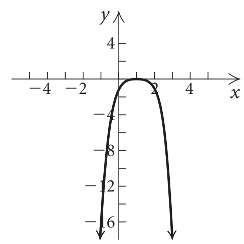
   $g(0) = -(0-1)^4 = -1 < 0$

   In $(1, \infty)$, test 2:

   $g(2) = -(2-1)^4 = -1 < 0$

   Thus the graph lies below the x-axis on both intervals. We also know the points $(0, -1)$ and $(2, -1)$ are on the graph.
4. From Step 3 we know that $g(0) = -1$ so the y-intercept is $(0, -1)$.
5. We find additional points on the graph and then draw the graph.

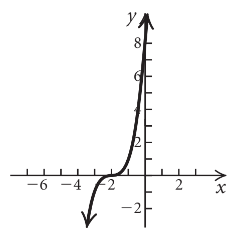
x	$g(x)$
-1	-16
-0.5	-5.1
1.5	0.1
3	-16



$g(x) = -(x-1)^4$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

32.



$h(x) = (x+2)^3$

33. $h(x) = x^3 + 3x^2 - x - 3$

1. The leading term is x^3. The degree, 3, is odd and the leading coefficient, 1, is positive so as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to -\infty$.
2. We solve $h(x) = 0$.

$$x^3 + 3x^2 - x - 3 = 0$$
$$x^2(x+3) - (x+3) = 0$$
$$(x+3)(x^2-1) = 0$$
$$(x+3)(x+1)(x-1) = 0$$
$$x+3 = 0 \quad \text{or} \quad x+1 = 0 \quad \text{or} \quad x-1 = 0$$
$$x = -3 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 1$$

   The zeros of the function are -3, -1, and 1 so the x-intercepts of the graph are $(-3, 0)$, $(-1, 0)$, and $(1, 0)$.
3. The zeros divide the x-axis into 4 intervals, $(-\infty, -3)$, $(-3, -1)$, $(-1, 1)$, and $(1, \infty)$. We choose a value for x from each interval and find $h(x)$. This tells us the sign of $h(x)$ for all values of x in that interval.

   In $(-\infty, -3)$, test -4:

   $h(-4) = (-4)^3 + 3(-4)^2 - (-4) - 3 = -15 < 0$

   In $(-3, -1)$, test -2:

   $h(-2) = (-2)^3 + 3(-2)^2 - (-2) - 3 = 3 > 0$

   In $(-1, 1)$, test 0:

   $h(0) = 0^3 + 3 \cdot 0^2 - 0 - 3 = -3 < 0$

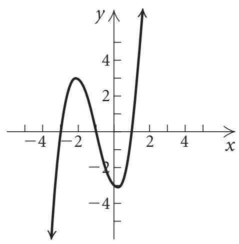
   In $(1, \infty)$, test 2:

   $h(2) = 2^3 + 3 \cdot 2^2 - 2 - 3 = 15 > 0$

Thus the graph lies below the x-axis on $(-\infty, -3)$ and on $(-1, 1)$ and above the x-axis on $(-3, -1)$ and on $(1, \infty)$. We also know the points $(-4, -15)$, $(-2, 3)$, $(0, -3)$, and $(2, 15)$ are on the graph.

4. From Step 3 we know that $h(0) = -3$ so the y-intercept is $(0, -3)$.
5. We find additional points on the graph and then draw the graph.

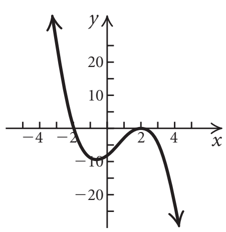
x	$h(x)$
-4.5	-28.9
-2.5	2.6
0.5	-2.6
2.5	28.9



$h(x) = x^3 + 3x^2 - x - 3$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

34.



$g(x) = -x^3 + 2x^2 + 4x - 8$

35. $f(x) = 6x^3 - 8x^2 - 54x + 72$

1. The leading term is $6x^3$. The degree, 3, is odd and the leading coefficient, 6, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
2. We solve $f(x) = 0$.

$$6x^3 - 8x^2 - 54x + 72 = 0$$
$$2(3x^3 - 4x^2 - 27x + 36) = 0$$
$$2[x^2(3x - 4) - 9(3x - 4)] = 0$$
$$2(3x - 4)(x^2 - 9) = 0$$
$$2(3x - 4)(x + 3)(x - 3) = 0$$
$$3x - 4 = 0 \quad or \quad x + 3 = 0 \quad or \quad x - 3 = 0$$
$$x = \frac{4}{3} \quad or \quad x = -3 \quad or \quad x = 3$$

The zeros of the function are $\frac{4}{3}$, -3, and 3, so the x-intercepts of the graph are $\left(\frac{4}{3}, 0\right)$, $(-3, 0)$, and $(3, 0)$.

3. The zeros divide the x-axis into 4 intervals, $(-\infty, -3)$, $\left(-3, \frac{4}{3}\right)$, $\left(\frac{4}{3}, 3\right)$, and $(3, \infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

In $(-\infty, -3)$, test -4:

$f(-4) = 6(-4)^3 - 8(-4)^2 - 54(-4) + 72 = -224 < 0$

In $\left(-3, \frac{4}{3}\right)$, test 0:

$f(0) = 6 \cdot 0^3 - 8 \cdot 0^2 - 54 \cdot 0 + 72 = 72 > 0$

In $\left(\frac{4}{3}, 3\right)$, test 2:

$f(2) = 6 \cdot 2^3 - 8 \cdot 2^2 - 54 \cdot 2 + 72 = -20 < 0$

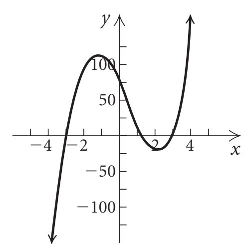
In $(3, \infty)$, test 4:

$f(4) = 6 \cdot 4^3 - 8 \cdot 4^2 - 54 \cdot 4 + 72 = 112 > 0$

Thus the graph lies below the x-axis on $(-\infty, -3)$ and on $\left(\frac{4}{3}, 3\right)$ and above the x-axis on $\left(-3, \frac{4}{3}\right)$ and on $(3, \infty)$. We also know the points $(-4, -224)$, $(0, 72)$, $(2, -20)$, and $(4, 112)$ are on the graph.

4. From Step 3 we know that $f(0) = 72$ so the y-intercept is $(0, 72)$.
5. We find additional points on the graph and then draw the graph.

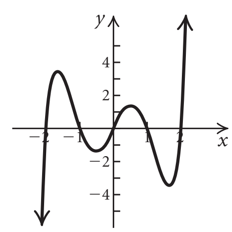
x	$f(x)$
-1	112
1	16
3.5	42.25



$f(x) = 6x^3 - 8x^2 - 54x + 72$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

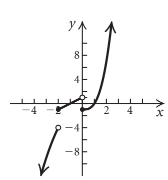
36.



$h(x) = x^5 - 5x^3 + 4x$

37. We graph $g(x) = -x+3$ for $x \leq -2$, $g(x)=4$ for $2 < x < 1$, and $g(x) = \frac{1}{2}x^3$ for $x \geq 1$.

38.



$$h(x) = \begin{cases} -x^2, & \text{for } x < -2, \\ x + 1, & \text{for } -2 \leq x < 0, \\ x^3 - 1, & \text{for } x \geq 0 \end{cases}$$

39. $f(-5) = (-5)^3 + 3(-5)^2 - 9(-5) - 13 = -18$

$f(-4) = (-4)^3 + 3(-4)^2 - 9(-4) - 13 = 7$

By the intermediate value theorem, since $f(-5)$ and $f(-4)$ have opposite signs then $f(x)$ has a zero between -5 and -4.

40. $f(1) = 1^3 + 3 \cdot 1^2 - 9 \cdot 1 - 13 = -18$

$f(2) = 2^3 + 3 \cdot 2^2 - 9 \cdot 2 - 13 = -11$

Since both $f(1)$ and $f(2)$ are negative, we cannot use the intermediate value theorem to determine if there is a zero between 1 and 2.

41. $f(-3) = 3(-3)^2 - 2(-3) - 11 = 22$

$f(-2) = 3(-2)^2 - 2(-2) - 11 = 5$

Since both $f(-3)$ and $f(-2)$ are positive, we cannot use the intermediate value theorem to determine if there is a zero between -3 and -2.

42. $f(2) = 3 \cdot 2^2 - 2 \cdot 2 - 11 = -3$

$f(3) = 3 \cdot 3^2 - 2 \cdot 3 - 11 = 10$

By the intermediate value theorem, since $f(2)$ and $f(3)$ have opposite signs then $f(x)$ has a zero between 2 and 3.

43. $f(2) = 2^4 - 2 \cdot 2^2 - 6 = 2$

$f(3) = 3^4 - 2 \cdot 3^2 - 6 = 57$

Since both $f(2)$ and $f(3)$ are positive, we cannot use the intermediate value theorem to determine if there is a zero between 2 and 3.

44. $f(1) = 2 \cdot 1^5 - 7 \cdot 1 + 1 = -4$

$f(2) = 2 \cdot 2^5 - 7 \cdot 2 + 1 = 51$

By the intermediate value theorem, since $f(1)$ and $f(2)$ have opposite signs then $f(x)$ has a zero between 1 and 2.

45. $f(4) = 4^3 - 5 \cdot 4^2 + 4 = -12$

$f(5) = 5^3 - 5 \cdot 5^2 + 4 = 4$

By the intermediate value theorem, since $f(4)$ and $f(5)$ have opposite signs then $f(x)$ has a zero between 4 and 5.

46. $f(-3) = (-3)^4 - 3(-3)^2 + (-3) - 1 = 50$

$f(-2) = (-2)^4 - 3(-2)^2 + (-2) - 1 = 1$

Since both $f(-3)$ and $f(-2)$ are positive, we cannot use the intermediate value theorem to determine if there is a zero between -3 and -2.

47. The graph of $y = x$, or $y = x + 0$, has y-intercept $(0, 0)$, so (d) is the correct answer.

48. The graph of $x = -4$ is a vertical line 4 units to the left of the y-axis, so (f) is the correct answer.

49. The graph of $y - 2x = 6$, or $y = 2x + 6$, has y-intercept $(0, 6)$, so (e) is the correct answer.

50. When $y = 0$, $x = -2$ and when $x = 0$, $y = -3$ so the intercepts of the graph are $(-2, 0)$ and $(0, -3)$. Thus (a) is the correct answer.

51. The graph of $y = 1 - x$, or $y = -x + 1$, has y-intercept $(0, 1)$, so (b) is the correct answer.

52. The graph of $y = 2$ is a horizontal line 2 units above the x-axis, so (c) is the correct answer.

53.
$$\begin{aligned}
2x - \frac{1}{2} &= 4 - 3x \\
5x - \frac{1}{2} &= 4 && \text{Adding } 3x \\
5x &= \frac{9}{2} && \text{Adding } \frac{1}{2} \\
x &= \frac{1}{5} \cdot \frac{9}{2} && \text{Multiplying by } \frac{1}{5} \\
x &= \frac{9}{10}
\end{aligned}$$

The solution is $\frac{9}{10}$.

54.
$$\begin{aligned}
x^3 - x^2 - 12x &= 0 \\
x(x^2 - x - 12) &= 0 \\
x(x-4)(x+3) &= 0 \\
x = 0 \quad \text{or} \quad x - 4 = 0 \quad &\text{or} \quad x + 3 = 0 \\
x = 0 \quad \text{or} \quad x = 4 \quad &\text{or} \quad x = -3
\end{aligned}$$

55.
$$\begin{aligned}
6x^2 - 23x - 55 &= 0 \\
(3x+5)(2x-11) &= 0 \\
3x + 5 = 0 \quad &\text{or} \quad 2x - 11 = 0 \\
3x = -5 \quad &\text{or} \quad 2x = 11 \\
x = -\frac{5}{3} \quad &\text{or} \quad x = \frac{11}{2}
\end{aligned}$$

The solutions are $-\frac{5}{3}$ and $\frac{11}{2}$.

56.
$$\begin{aligned}
\frac{3}{4}x + 10 &= \frac{1}{5} + 2x \\
15x + 200 &= 4 + 40x && \text{Multiplying by 20} \\
196 &= 25x \\
\frac{196}{25} &= x
\end{aligned}$$

Exercise Set 4.3

1. a)
$$\begin{array}{r|l}
 & x^3 - 7x^2 + 8x + 16 \\
\hline
x+1 & x^4 - 6x^3 + x^2 + 24x - 20 \\
 & \underline{x^4 + x^3} \\
 & -7x^3 + x^2 \\
 & \underline{-7x^3 - 7x^2} \\
 & 8x^2 + 24x \\
 & \underline{8x^2 + 8x} \\
 & 16x - 20 \\
 & \underline{16x + 16} \\
 & -4
\end{array}$$

Since the remainder is not 0, $x + 1$ is not a factor of $f(x)$.

b)
$$\begin{array}{rl}
 & x^3-4x^2-7x+10 \\
x-2 & \overline{)\,x^4-6x^3+x^2+24x-20} \\
 & \underline{x^4-2x^3} \\
 & -4x^3+x^2 \\
 & \underline{-4x^3+8x^2} \\
 & -7x^2+24x \\
 & \underline{-7x^2+14x} \\
 & 10x-20 \\
 & \underline{10x-20} \\
 & 0
\end{array}$$

Since the remainder is 0, $x-2$ is a factor of $f(x)$.

c)
$$\begin{array}{rl}
 & x^3-11x^2+56x-256 \\
x+5 & \overline{)\,x^4-6x^3+x^2+24x-20} \\
 & \underline{x^4+5x^3} \\
 & -11x^3+x^2 \\
 & \underline{-11x^3-55x^2} \\
 & 56x^2+24x \\
 & \underline{56x^2+280x} \\
 & -256x-20 \\
 & \underline{-256x-1280} \\
 & 1260
\end{array}$$

Since the remainder is not 0, $x+5$ is not a factor of $f(x)$.

2. a)
$$\begin{array}{rl}
 & x^2-6x+13 \\
x+5 & \overline{)\,x^3-x^2-17x-15} \\
 & \underline{x^3+5x^2} \\
 & -6x^2-17x \\
 & \underline{-6x^2-30x} \\
 & 13x-15 \\
 & \underline{13x+65} \\
 & -80
\end{array}$$

Since the remainder is not 0, $x+5$ is not a factor of $h(x)$.

b)
$$\begin{array}{rl}
 & x^2-2x-15 \\
x+1 & \overline{)\,x^3-x^2-17x-15} \\
 & \underline{x^3+x^2} \\
 & -2x^2-17x \\
 & \underline{-2x^2-2x} \\
 & -15x-15 \\
 & \underline{-15x-15} \\
 & 0
\end{array}$$

Since the remainder is 0, $x+1$ is a factor of $h(x)$.

c)
$$\begin{array}{rl}
 & x^2-4x-5 \\
x+3 & \overline{)\,x^3-x^2-17x-15} \\
 & \underline{x^3+3x^2} \\
 & -4x^2-17x \\
 & \underline{-4x^2-12x} \\
 & -5x-15 \\
 & \underline{-5x-15} \\
 & 0
\end{array}$$

Since the remainder is 0, $x+3$ is a factor of $h(x)$.

3. a)
$$\begin{array}{rl}
 & x^2+2x-3 \\
x-4 & \overline{)\,x^3-2x^2-11x+12} \\
 & \underline{x^3-4x^2} \\
 & 2x^2-11x \\
 & \underline{2x^2-8x} \\
 & -3x+12 \\
 & \underline{-3x+12} \\
 & 0
\end{array}$$

Since the remainder is 0, $x-4$ is a factor of $g(x)$.

b)
$$\begin{array}{rl}
 & x^2+x-8 \\
x-3 & \overline{)\,x^3-2x^2-11x+12} \\
 & \underline{x^3-3x^2} \\
 & x^2-11x \\
 & \underline{x^2-3x} \\
 & -8x+12 \\
 & \underline{-8x+24} \\
 & -12
\end{array}$$

Since the remainder is not 0, $x-3$ is not a factor of $g(x)$.

c)
$$\begin{array}{rl}
 & x^2-x-12 \\
x-1 & \overline{)\,x^3-2x^2-11x+12} \\
 & \underline{x^3-x^2} \\
 & -x^2-11x \\
 & \underline{-x^2+x} \\
 & -12x+12 \\
 & \underline{-12x+12} \\
 & 0
\end{array}$$

Since the remainder is 0, $x-1$ is a factor of $g(x)$.

4. a)
$$\begin{array}{rl}
 & x^3+2x^2-7x+4 \\
x+6 & \overline{)\,x^4+8x^3+5x^2-38x+24} \\
 & \underline{x^4+6x^3} \\
 & 2x^3+5x^2 \\
 & \underline{2x^3+12x^2} \\
 & -7x^2-38x \\
 & \underline{-7x^2-42x} \\
 & 4x+24 \\
 & \underline{4x+24} \\
 & 0
\end{array}$$

Since the remainder is 0, $x+6$ is a factor of $f(x)$.

b)
$$\begin{array}{rl}
 & x^3+7x^2-2x-36 \\
x+1 & \overline{)\,x^4+8x^3+5x^2-38x+24} \\
 & \underline{x^4+x^3} \\
 & 7x^3+5x^2 \\
 & \underline{7x^3+7x^2} \\
 & -2x^2-38x \\
 & \underline{-2x^2-2x} \\
 & -36x+24 \\
 & \underline{-36x-36} \\
 & 60
\end{array}$$

Since the remainder is not 0, $x+1$ is not a factor of $f(x)$.

c)
$$\begin{array}{r|l} & x^3+12x^2+53x+174 \\ \hline x-4 & x^4+8x^3+5x^2-38x+24 \\ & \underline{x^4-4x^3} \\ & \quad 12x^3+5x^2 \\ & \quad \underline{12x^3-48x^2} \\ & \qquad 53x^2-38x \\ & \qquad \underline{53x^2-212x} \\ & \qquad\quad 174x+24 \\ & \qquad\quad \underline{174x-696} \\ & \qquad\qquad 720 \end{array}$$

Since the remainder is not 0, $x-4$ is not a factor of $f(x)$.

5.
$$\begin{array}{r|l} & x^2-2x+4 \\ \hline x+2 & x^3+0x^2+0x-8 \\ & \underline{x^3+2x^2} \\ & \quad -2x^2+0x \\ & \quad \underline{-2x^2-4x} \\ & \qquad 4x-8 \\ & \qquad \underline{4x+8} \\ & \qquad\quad -16 \end{array}$$

$x^3-8=(x+2)(x^2-2x+4)-16$

6.
$$\begin{array}{r|l} & 2x^2+3x+10 \\ \hline x-3 & 2x^3-3x^2+x-1 \\ & \underline{2x^3-6x^2} \\ & \quad 3x^2+x \\ & \quad \underline{3x^2-9x} \\ & \qquad 10x-1 \\ & \qquad \underline{10x-30} \\ & \qquad\quad 29 \end{array}$$

$2x^3-3x^2+x-1=(x-3)(2x^2+3x+10)+29$

7.
$$\begin{array}{r|l} & x^2-3x+2 \\ \hline x+9 & x^3+6x^2-25x+18 \\ & \underline{x^3+9x^2} \\ & \quad -3x^2-25x \\ & \quad \underline{-3x^2-27x} \\ & \qquad 2x+18 \\ & \qquad \underline{2x+18} \\ & \qquad\quad 0 \end{array}$$

$x^3+6x^2-25x+18=(x+9)(x^2-3x+2)+0$

8.
$$\begin{array}{r|l} & x^2-4x-5 \\ \hline x-5 & x^3-9x^2+15x+25 \\ & \underline{x^3-5x^2} \\ & \quad -4x^2+15x \\ & \quad \underline{-4x^2+20x} \\ & \qquad -5x+25 \\ & \qquad \underline{-5x+25} \\ & \qquad\quad 0 \end{array}$$

$x^3-9x^2+15x+25=(x-5)(x^2-4x-5)+0$

9.
$$\begin{array}{r|l} & x^3-2x^2+2x-4 \\ \hline x+2 & x^4+0x^3-2x^2+0x+3 \\ & \underline{x^4+2x^3} \\ & \quad -2x^3-2x^2 \\ & \quad \underline{-2x^3-4x^2} \\ & \qquad 2x^2+0x \\ & \qquad \underline{2x^2+4x} \\ & \qquad\quad -4x+3 \\ & \qquad\quad \underline{-4x-8} \\ & \qquad\qquad 11 \end{array}$$

$x^4-2x^2+3=(x+2)(x^3-2x^2+2x-4)+11$

10.
$$\begin{array}{r|l} & x^3+7x^2+7x+7 \\ \hline x-1 & x^4+6x^3+0x^2+0x+0 \\ & \underline{x^4-x^3} \\ & \quad 7x^3+0x^2 \\ & \quad \underline{7x^3-7x^2} \\ & \qquad 7x^2+0x \\ & \qquad \underline{7x^2-7x} \\ & \qquad\quad 7x+0 \\ & \qquad\quad \underline{7x-7} \\ & \qquad\qquad 7 \end{array}$$

$x^4+6x^3=(x-1)(x^3+7x^2+7x+7)+7$

11. $(2x^4+7x^3+x-12)\div(x+3)$

$=(2x^4+7x^3+0x^2+x-12)\div[x-(-3)]$

$$\begin{array}{r|rrrrr} -3 & 2 & 7 & 0 & 1 & -12 \\ & & -6 & -3 & 9 & -30 \\ \hline & 2 & 1 & -3 & 10 & -42 \end{array}$$

The quotient is $2x^3+x^2-3x+10$. The remainder is -42.

12.
$$\begin{array}{r|rrrr} 2 & 1 & -7 & 13 & 3 \\ & & 2 & -10 & 6 \\ \hline & 1 & -5 & 3 & 9 \end{array}$$

$Q(x)=x^2-5x+3,\ R(x)=9$

13. $(x^3-2x^2-8)\div(x+2)$

$=(x^3-2x^2+0x-8)\div[x-(-2)]$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 0 & -8 \\ & & -2 & 8 & -16 \\ \hline & 1 & -4 & 8 & -24 \end{array}$$

The quotient is x^2-4x+8. The remainder is -24.

14.
$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & 10 \\ & & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 12 \end{array}$$

$Q(x)=x^2+2x+1,\ R(x)=12$

15. $(3x^3-x^2+4x-10)\div(x+1)$

$=(3x^3-x^2+4x-10)\div[x-(-1)]$

$$\begin{array}{r|rrrr} -1 & 3 & -1 & 4 & -10 \\ & & -3 & 4 & -8 \\ \hline & 3 & -4 & 8 & -18 \end{array}$$

The quotient is $3x^2-4x+8$. The remainder is -18.

16.
$$\begin{array}{r|rrrrr} -3 & 4 & 0 & 0 & -2 & 5 \\ & & -12 & 36 & -108 & 330 \\ \hline & 4 & -12 & 36 & -110 & 335 \end{array}$$

$Q(x)=4x^3-12x^2+36x-110,\ R(x)=335$

17. $(x^5 + x^3 - x) \div (x - 3)$

$= (x^5 + 0x^4 + x^3 + 0x^2 - x + 0) \div (x - 3)$

$$\begin{array}{r|rrrrr|r} 3 & 1 & 0 & 1 & 0 & -1 & 0 \\ & & 3 & 9 & 30 & 90 & 267 \\ \hline & 1 & 3 & 10 & 30 & 89 & 267 \end{array}$$

The quotient is $x^4 + 3x^3 + 10x^2 + 30x + 89$.

The remainder is 267.

18.
$$\begin{array}{r|rrrrrrr|r} -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 2 \\ & & -1 & 2 & -3 & 4 & -4 & 4 & -4 \\ \hline & 1 & -2 & 3 & -4 & 4 & -4 & 4 & -2 \end{array}$$

$Q(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 4x^2 - 4x + 4,\ R(x) = -2$

19. $(x^4 - 1) \div (x - 1)$

$= (x^4 + 0x^3 + 0x^2 + 0x - 1) \div (x - 1)$

$$\begin{array}{r|rrrr|r} 1 & 1 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

The quotient is $x^3 + x^2 + x + 1$. The remainder is 0.

20.
$$\begin{array}{r|rrrrr|r} -2 & 1 & 0 & 0 & 0 & 0 & 32 \\ & & -2 & 4 & -8 & 16 & -32 \\ \hline & 1 & -2 & 4 & -8 & 16 & 0 \end{array}$$

$Q(x) = x^4 - 2x^3 + 4x^2 - 8x + 16,\ R(x) = 0$

21. $(2x^4 + 3x^2 - 1) \div \left(x - \frac{1}{2}\right)$

$(2x^4 + 0x^3 + 3x^2 + 0x - 1) \div \left(x - \frac{1}{2}\right)$

$$\begin{array}{r|rrrr|r} \frac{1}{2} & 2 & 0 & 3 & 0 & -1 \\ & & 1 & \frac{1}{2} & \frac{7}{4} & \frac{7}{8} \\ \hline & 2 & 1 & \frac{7}{2} & \frac{7}{4} & -\frac{1}{8} \end{array}$$

The quotient is $2x^3 + x^2 + \frac{7}{2}x + \frac{7}{4}$. The remainder is $-\frac{1}{8}$.

22.
$$\begin{array}{r|rrrr|r} \frac{1}{4} & 3 & 0 & -2 & 0 & 2 \\ & & \frac{3}{4} & \frac{3}{16} & -\frac{29}{64} & -\frac{29}{256} \\ \hline & 3 & \frac{3}{4} & -\frac{29}{16} & -\frac{29}{64} & \frac{483}{256} \end{array}$$

$Q(x) = 3x^3 + \frac{3}{4}x^2 - \frac{29}{16}x - \frac{29}{64},\ R(x) = \frac{483}{256}$

23. $f(x) = x^3 - 6x^2 + 11x - 6$

Find $f(1)$.

$$\begin{array}{r|rrr|r} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$f(1) = 0$

Find $f(-2)$.

$$\begin{array}{r|rrr|r} -2 & 1 & -6 & 11 & -6 \\ & & -2 & 16 & -54 \\ \hline & 1 & -8 & 27 & -60 \end{array}$$

$f(-2) = -60$

Find $f(3)$.

$$\begin{array}{r|rrr|r} 3 & 1 & -6 & 11 & -6 \\ & & 3 & -9 & 6 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

$f(3) = 0$

24.
$$\begin{array}{r|rrr|r} -3 & 1 & 7 & -12 & -3 \\ & & -3 & -12 & 72 \\ \hline & 1 & 4 & -24 & 69 \end{array}$$

$f(-3) = 69$

$$\begin{array}{r|rrr|r} -2 & 1 & 7 & -12 & -3 \\ & & -2 & -10 & 44 \\ \hline & 1 & 5 & -22 & 41 \end{array}$$

$f(-2) = 41$

$$\begin{array}{r|rrr|r} 1 & 1 & 7 & -12 & -3 \\ & & 1 & 8 & -4 \\ \hline & 1 & 8 & -4 & -7 \end{array}$$

$f(1) = -7$

25. $f(x) = x^4 - 3x^3 + 2x + 8$

Find $f(-1)$.

$$\begin{array}{r|rrrr|r} -1 & 1 & -3 & 0 & 2 & 8 \\ & & -1 & 4 & -4 & 2 \\ \hline & 1 & -4 & 4 & -2 & 10 \end{array}$$

$f(-1) = 10$

Find $f(4)$.

$$\begin{array}{r|rrrr|r} 4 & 1 & -3 & 0 & 2 & 8 \\ & & 4 & 4 & 16 & 72 \\ \hline & 1 & 1 & 4 & 18 & 80 \end{array}$$

$f(4) = 80$

Find $f(-5)$.

$$\begin{array}{r|rrrr|r} -5 & 1 & -3 & 0 & 2 & 8 \\ & & -5 & 40 & -200 & 990 \\ \hline & 1 & -8 & 40 & -198 & 998 \end{array}$$

$f(-5) = 998$

26.
$$\begin{array}{r|rrrr|r} -10 & 2 & 0 & 1 & -10 & 1 \\ & & -20 & 200 & -2010 & 20,200 \\ \hline & 2 & -20 & 201 & -2020 & 20,201 \end{array}$$

$f(-10) = 20,201$

$$\begin{array}{r|rrrr|r} 2 & 2 & 0 & 1 & -10 & 1 \\ & & 4 & 8 & 18 & 16 \\ \hline & 2 & 4 & 9 & 8 & 17 \end{array}$$

$f(2) = 17$

$$\begin{array}{r|rrrr|r} 3 & 2 & 0 & 1 & -10 & 1 \\ & & 6 & 18 & 57 & 141 \\ \hline & 2 & 6 & 19 & 47 & 142 \end{array}$$

$f(3) = 142$

27. $f(x) = 2x^5 - 3x^4 + 2x^3 - x + 8$

Find $f(20)$.

$$\begin{array}{r|rrrrr|r} 20 & 2 & -3 & 2 & 0 & -1 & 8 \\ & & 40 & 740 & 14,840 & 296,800 & 5,935,980 \\ \hline & 2 & 37 & 742 & 14,840 & 296,799 & 5,935,988 \end{array}$$

$f(20) = 5,935,988$

Find $f(-3)$.

$$\begin{array}{r|rrrrrr} -3 & 2 & -3 & 2 & 0 & -1 & 8 \\ & & -6 & 27 & -87 & 261 & -780 \\ \hline & 2 & -9 & 29 & -87 & 260 & -772 \end{array}$$

$f(-3) = -772$

28.
$$\begin{array}{r|rrrrrr} -10 & 1 & -10 & 20 & 0 & -5 & -100 \\ & & -10 & 200 & -2200 & 22,000 & -219,950 \\ \hline & 1 & -20 & 220 & -2200 & 21,995 & -220,050 \end{array}$$

$f(-10) = -220,050$

$$\begin{array}{r|rrrrrr} 5 & 1 & -10 & 20 & 0 & -5 & -100 \\ & & 5 & -25 & -25 & -125 & -650 \\ \hline & 1 & -5 & -5 & -25 & -130 & -750 \end{array}$$

$f(5) = -750$

29. $f(x) = x^4 - 16$

Find $f(2)$.

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -16 \\ & & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$$

$f(2) = 0$

Find $f(-2)$.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 0 & 0 & -16 \\ & & -2 & 4 & -8 & 16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$f(-2) = 0$

Find $f(3)$.

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & 0 & -16 \\ & & 3 & 9 & 27 & 81 \\ \hline & 1 & 3 & 9 & 27 & 65 \end{array}$$

$f(3) = 65$

Find $f(1-\sqrt{2})$.

$$\begin{array}{r|rrrrr} 1-\sqrt{2} & 1 & 0 & 0 & 0 & -16 \\ & & 1-\sqrt{2} & 3-2\sqrt{2} & 7-5\sqrt{2} & 17-12\sqrt{2} \\ \hline & 1 & 1-\sqrt{2} & 3-2\sqrt{2} & 7-5\sqrt{2} & 1-12\sqrt{2} \end{array}$$

$f(1-\sqrt{2}) = 1 - 12\sqrt{2}$

30.
$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & 32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 64 \end{array}$$

$f(2) = 64$

$f(-2) = 0$ (See Exercise 20.)

$$\begin{array}{r|rrrrrr} 3 & 1 & 0 & 0 & 0 & 0 & 32 \\ & & 3 & 9 & 27 & 81 & 243 \\ \hline & 1 & 3 & 9 & 27 & 81 & 275 \end{array}$$

$f(3) = 275$

$$\begin{array}{l} \underline{2+3i\,|} \\ \begin{array}{rrrrrr} 1 & 0 & 0 & 0 & 0 & 32 \\ & 2+3i & -5+12i & -46+9i & -119-120i & 122-597i \\ \hline 1 & 2+3i & -5+12i & -46+9i & -119-120i & 154-597i \end{array} \end{array}$$

$f(2+3i) = 154 - 597i$

31. $f(x) = 3x^3 + 5x^2 - 6x + 18$

If -3 is a zero of $f(x)$, then $f(-3) = 0$. Find $f(-3)$ using synthetic division.

$$\begin{array}{r|rrrr} -3 & 3 & 5 & -6 & 18 \\ & & -9 & 12 & -18 \\ \hline & 3 & -4 & 6 & 0 \end{array}$$

Since $f(-3) = 0$, -3 is a zero of $f(x)$.

If 2 is a zero of $f(x)$, then $f(2) = 0$. Find $f(2)$ using synthetic division.

$$\begin{array}{r|rrrr} 2 & 3 & 5 & -6 & 18 \\ & & 6 & 22 & 32 \\ \hline & 3 & 11 & 16 & 50 \end{array}$$

Since $f(2) \neq 0$, 2 is not a zero of $f(x)$.

32.
$$\begin{array}{r|rrrr} -4 & 3 & 11 & -2 & 8 \\ & & -12 & 4 & -8 \\ \hline & 3 & -1 & 2 & 0 \end{array}$$

$f(-4) = 0$, so -4 is a zero of $f(x)$.

$$\begin{array}{r|rrrr} 2 & 3 & 11 & -2 & 8 \\ & & 6 & 34 & 64 \\ \hline & 3 & 17 & 32 & 72 \end{array}$$

$f(2) \neq 0$, so 2 is not a zero of $f(x)$.

33. $h(x) = x^4 + 4x^3 + 2x^2 - 4x - 3$

If -3 is a zero of $h(x)$, then $h(-3) = 0$. Find $h(-3)$ using synthetic division.

$$\begin{array}{r|rrrrr} -3 & 1 & 4 & 2 & -4 & -3 \\ & & -3 & -3 & 3 & 3 \\ \hline & 1 & 1 & -1 & -1 & 0 \end{array}$$

Since $h(-3) = 0$, -3 is a zero of $h(x)$.

If 1 is a zero of $h(x)$, then $h(1) = 0$. Find $h(1)$ using synthetic division.

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & 2 & -4 & -3 \\ & & 1 & 5 & 7 & 3 \\ \hline & 1 & 5 & 7 & 3 & 0 \end{array}$$

Since $h(1) = 0$, 1 is a zero of $h(x)$.

34.
$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 1 & 24 & -20 \\ & & 2 & -8 & -14 & 20 \\ \hline & 1 & -4 & -7 & 10 & 0 \end{array}$$

$g(2) = 0$, so 2 is a zero of $g(x)$.

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 1 & 24 & -20 \\ & & -1 & 7 & -8 & -16 \\ \hline & 1 & -7 & 8 & 16 & -36 \end{array}$$

$g(-1) \neq 0$, so -1 is not a zero of $g(x)$.

35. $g(x) = x^3 - 4x^2 + 4x - 16$

If i is a zero of $g(x)$, then $g(i) = 0$. Find $g(i)$ using synthetic division. Keep in mind that $i^2 = -1$.

$$\begin{array}{r|rrrr} i & 1 & -4 & 4 & -16 \\ & & i & -4i-1 & 3i+4 \\ \hline & 1 & -4+i & 3-4i & -12+3i \end{array}$$

Since $g(i) \neq 0$, i is not a zero of $g(x)$.

If $-2i$ is a zero of $g(x)$, then $g(-2i) = 0$. Find $g(-2i)$ using synthetic division. Keep in mind that $i^2 = -1$.

$$\begin{array}{r|rrrr} -2i & 1 & -4 & 4 & -16 \\ & & -2i & 8i-4 & 16 \\ \hline & 1 & -4-2i & 8i & 0 \end{array}$$

Since $g(-2i) = 0$, $-2i$ is a zero of $g(x)$.

36.
$$\begin{array}{r|rrrr} \frac{1}{3} & 1 & -1 & -\frac{1}{9} & \frac{1}{9} \\ & & \frac{1}{3} & -\frac{2}{9} & -\frac{1}{9} \\ \hline & 1 & -\frac{2}{3} & -\frac{1}{3} & 0 \end{array}$$

$h\left(\frac{1}{3}\right) = 0$, so $\frac{1}{3}$ is a zero of $h(x)$.

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -\frac{1}{9} & \frac{1}{9} \\ & & 2 & 2 & \frac{34}{9} \\ \hline & 1 & 1 & \frac{17}{9} & \frac{35}{9} \end{array}$$

$h(2) \neq 0$, so 2 is not a zero of $h(x)$.

37. $f(x) = x^3 - \frac{7}{2}x^2 + x - \frac{3}{2}$

If -3 is a zero of $f(x)$, then $f(-3) = 0$. Find $f(-3)$ using synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & -\frac{7}{2} & 1 & -\frac{3}{2} \\ & & -3 & \frac{39}{2} & -\frac{123}{2} \\ \hline & 1 & -\frac{13}{2} & \frac{41}{2} & -63 \end{array}$$

Since $f(-3) \neq 0$, -3 is not a zero of $f(x)$.

If $\frac{1}{2}$ is a zero of $f(x)$, then $f\left(\frac{1}{2}\right) = 0$.

Find $f\left(\frac{1}{2}\right)$ using synthetic division.

$$\begin{array}{r|rrrr} \frac{1}{2} & 1 & -\frac{7}{2} & 1 & -\frac{3}{2} \\ & & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{4} \\ \hline & 1 & -3 & -\frac{1}{2} & -\frac{7}{4} \end{array}$$

Since $f\left(\frac{1}{2}\right) \neq 0$, $\frac{1}{2}$ is not a zero of $f(x)$.

38.
$$\begin{array}{r|rrrr} i & 1 & 2 & 1 & 2 \\ & & i & -1+2i & -2 \\ \hline & 1 & 2+i & 2i & 0 \end{array}$$

$f(i) = 0$, so i is a zero of $f(x)$.

$$\begin{array}{r|rrrr} -i & 1 & 2 & 1 & 2 \\ & & -i & -1-2i & -2 \\ \hline & 1 & 2-i & -2i & 0 \end{array}$$

$f(-i) = 0$, so $-i$ is a zero of $f(x)$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$f(-2) = 0$, so -2 is a zero of $f(x)$.

39. $f(x) = x^3 + 4x^2 + x - 6$

Try $x - 1$. Use synthetic division to see whether $f(1) = 0$.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

Since $f(1) = 0$, $x - 1$ is a factor of $f(x)$. Thus $f(x) = (x-1)(x^2 + 5x + 6)$.

Factoring the trinomial we get

$f(x) = (x-1)(x+2)(x+3)$.

To solve the equation $f(x) = 0$, use the principle of zero products.

$(x-1)(x+2)(x+3) = 0$

$x - 1 = 0$ *or* $x + 2 = 0$ *or* $x + 3 = 0$

$x = 1$ *or* $x = -2$ *or* $x = -3$

The solutions are 1, -2, and -3.

40.
$$\begin{array}{r|rrrr} 2 & 1 & 5 & -2 & -24 \\ & & 2 & 14 & 24 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-2)(x^2 + 7x + 12) \\ &= (x-2)(x+3)(x+4) \end{aligned}$$

The solutions of $f(x) = 0$ are 2, -3, and -4.

41. $f(x) = x^3 - 6x^2 + 3x + 10$

Try $x - 1$. Use synthetic division to see whether $f(1) = 0$.

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 3 & 10 \\ & & 1 & -5 & -2 \\ \hline & 1 & -5 & -2 & 8 \end{array}$$

Since $f(1) \neq 0$, $x - 1$ is not a factor of $P(x)$.

Try $x+1$. Use synthetic division to see whether $f(-1) = 0$.

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 3 & 10 \\ & & -1 & 7 & -10 \\ \hline & 1 & -7 & 10 & 0 \end{array}$$

Since $f(-1) = 0$, $x + 1$ is a factor of $f(x)$.

Thus $f(x) = (x+1)(x^2 - 7x + 10)$.

Factoring the trinomial we get

$f(x) = (x+1)(x-2)(x-5)$.

To solve the equation $f(x) = 0$, use the principle of zero products.

$(x+1)(x-2)(x-5) = 0$

$x + 1 = 0$ *or* $x - 2 = 0$ *or* $x - 5 = 0$

$x = -1$ *or* $x = 2$ *or* $x = 5$

The solutions are -1, 2, and 5.

42.
$$\begin{array}{r|rrrr} 1 & 1 & 2 & -13 & 10 \\ & & 1 & 3 & -10 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-1)(x^2 + 3x - 10) \\ &= (x-1)(x-2)(x+5) \end{aligned}$$

The solutions of $f(x) = 0$ are 1, 2, and -5.

43. $f(x) = x^3 - x^2 - 14x + 24$

Try $x + 1$, $x - 1$, and $x + 2$. Using synthetic division we find that $f(-1) \neq 0$, $f(1) \neq 0$ and $f(-2) \neq 0$. Thus $x + 1$, $x - 1$, and $x + 2$, are not factors of $f(x)$.

Try $x - 2$. Use synthetic division to see whether $f(2) = 0$.

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -14 & 24 \\ & & 2 & 2 & -24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

Since $f(2) = 0$, $x - 2$ is a factor of $f(x)$. Thus $f(x) = (x-2)(x^2+x-12)$.

Factoring the trinomial we get

$f(x) = (x-2)(x+4)(x-3)$

To solve the equation $f(x) = 0$, use the principle of zero products.

$(x-2)(x+4)(x-3) = 0$

$x - 2 = 0$ *or* $x + 4 = 0$ *or* $x - 3 = 0$

$x = 2$ *or* $x = -4$ *or* $x = 3$

The solutions are 2, −4, and 3.

44.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$f(x) = (x-2)(x^2-x-12)$

$\quad = (x-2)(x-4)(x+3)$

The solutions of $f(x) = 0$ are 2, 4, and −3.

45. $f(x) = x^4 - 7x^3 + 9x^2 + 27x - 54$

Try $x+1$ and $x-1$. Using synthetic division we find that $f(-1) \neq 0$ and $f(1) \neq 0$. Thus $x+1$ and $x-1$ are not factors of $f(x)$. Try $x+2$. Use synthetic division to see whether $f(-2) = 0$.

$$\begin{array}{r|rrrrr} -2 & 1 & -7 & 9 & 27 & -54 \\ & & -2 & 18 & -54 & 54 \\ \hline & 1 & -9 & 27 & -27 & 0 \end{array}$$

Since $f(-2) = 0$, $x+2$ is a factor of $f(x)$. Thus $f(x) = (x+2)(x^3-9x^2+27x-27)$.

We continue to use synthetic division to factor $g(x) = x^3 - 9x^2 + 27x - 27$. Trying $x+2$ again and $x-2$ we find that $g(-2) \neq 0$ and $g(2) \neq 0$. Thus $x+2$ and $x-2$ are not factors of $g(x)$. Try $x-3$.

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

Since $g(3) = 0$, $x-3$ is a factor of $x^3 - 9x^2 + 27x - 27$.

Thus $f(x) = (x+2)(x-3)(x^2-6x+9)$.

Factoring the trinomial we get

$f(x) = (x+2)(x-3)(x-3)^2$, or $f(x) = (x+2)(x-3)^3$.

To solve the equation $f(x) = 0$, use the principle of zero products.

$(x+2)(x-3)(x-3)(x-3) = 0$

$x + 2 = 0$ *or* $x - 3 = 0$ *or* $x - 3 = 0$ *or* $x - 3 = 0$

$x = -2$ *or* $x = 3$ *or* $x = 3$ *or* $x = 3$

The solutions are −2 and 3.

46.

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -7 & 34 & -24 \\ & & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$f(x) = (x-1)(x-2)(x^2-x-12)$

$\quad = (x-1)(x-2)(x-4)(x+3)$

The solutions of $f(x) = 0$ are 1, 2, 4, and −3.

47. $f(x) = x^4 - x^3 - 19x^2 + 49x - 30$

Try $x-1$. Use synthetic division to see whether $f(1) = 0$.

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -19 & 49 & -30 \\ & & 1 & 0 & -19 & 30 \\ \hline & 1 & 0 & -19 & 30 & 0 \end{array}$$

Since $f(1) = 0$, $x-1$ is a factor of $f(x)$. Thus $f(x) = (x-1)(x^3-19x+30)$.

We continue to use synthetic division to factor $g(x) = x^3 - 19x + 30$. Trying $x-1$, $x+1$, and $x+2$ we find that $g(1) \neq 0$, $g(-1) \neq 0$, and $g(-2) \neq 0$. Thus $x-1$, $x+1$, and $x+2$ are not factors of $x^3 - 19x + 30$. Try $x-2$.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -19 & 30 \\ & & 2 & 4 & -30 \\ \hline & 1 & 2 & -15 & 0 \end{array}$$

Since $g(2) = 0$, $x-2$ is a factor of $x^3 - 19x + 30$.

Thus $f(x) = (x-1)(x-2)(x^2+2x-15)$.

Factoring the trinomial we get

$f(x) = (x-1)(x-2)(x-3)(x+5)$.

To solve the equation $f(x) = 0$, use the principle of zero products.

$(x-1)(x-2)(x-3)(x+5) = 0$

$x-1 = 0$ *or* $x-2 = 0$ *or* $x-3 = 0$ *or* $x+5 = 0$

$x = 1$ *or* $x = 2$ *or* $x = 3$ *or* $x = -5$

The solutions are 1, 2, 3, and −5.

48.

$$\begin{array}{r|rrrrr} -1 & 1 & 11 & 41 & 61 & 30 \\ & & -1 & -10 & -31 & -30 \\ \hline & 1 & 10 & 31 & 30 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 10 & 31 & 30 \\ & & -2 & -16 & -30 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

$f(x) = (x+1)(x+2)(x^2+8x+15)$

$\quad = (x+1)(x+2)(x+3)(x+5)$

The solutions of $f(x) = 0$ are −1, −2, −3, and −5.

49. $f(x) = x^4 - x^3 - 7x^2 + x + 6$

1. The leading term is x^4. The degree, 4, is even and the leading coefficient, 1, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.
2. Find the zeros of the function. We first use synthetic division to determine if $f(1) = 0$.

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -7 & 1 & 6 \\ & & 1 & 0 & -7 & -6 \\ \hline & 1 & 0 & -7 & -6 & 0 \end{array}$$

1 is a zero of the function and we have
$f(x) = (x-1)(x^3-7x-6)$.

Synthetic division shows that -1 is a zero of $g(x) = x^3 - 7x - 6$.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

Then we have $f(x) = (x-1)(x+1)(x^2 - x - 6)$.

To find the other zeros we solve the following equation:

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0 \quad or \quad x + 2 = 0$$
$$x = 3 \quad or \qquad x = -2$$

The zeros of the function are 1, -1, 3, and -2 so the x-intercepts of the graph are $(1,0)$, $(-1,0)$, $(3,0)$, and $(-2,0)$.

3. The zeros divide the x-axis into five intervals, $(-\infty,-2)$, $(-2,-1)$, $(-1,1)$, $(1,3)$, and $(3,\infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in the interval.

   In $(-\infty,-2)$, test -3:

   $f(-3) = (-3)^4 - (-3)^3 - 7(-3)^2 + (-3) + 6 = 48 > 0$

   In $(-2,-1)$, test -1.5:

   $f(-1.5) = (-1.5)^4 - (-1.5)^3 - 7(-1.5)^2 + (-1.5) + 6 = -2.8125 < 0$

   In $(-1,1)$, test 0:

   $f(0) = 0^4 - 0^3 - 7 \cdot 0^2 + 0 + 6 = 6 > 0$

   In $(1,3)$, test 2:

   $f(2) = 2^4 - 2^3 - 7 \cdot 2^2 + 2 + 6 = -12 < 0$

   In $(3,\infty)$, test 4:

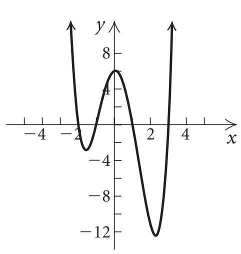
   $f(4) = 4^4 - 4^3 - 7 \cdot 4^2 + 4 + 6 = 90 > 0$

   Thus the graph lies above the x-axis on $(-\infty,-2)$, on $(-1,1)$, and on $(3,\infty)$. It lies below the x-axis on $(-2,-1)$ and on $(1,3)$. We also know the points $(-3,48)$, $(-1.5,-2.8125)$, $(0,6)$, $(2,-12)$, and $(4,90)$ are on the graph.

4. From Step 3 we see that $f(0) = 6$ so the y-intercept is $(0,6)$.

5. We find additional points on the graph and draw the graph.

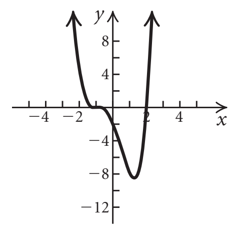
x	$f(x)$
-2.5	14.3
-0.5	3.9
0.5	4.7
2.5	-11.8



$f(x) = x^4 - x^3 - 7x^2 + x + 6$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

50.



$f(x) = x^4 + x^3 - 3x^2 - 5x - 2$

51. $f(x) = x^3 - 7x + 6$

1. The leading term is x^3. The degree, 3, is odd and the leading coefficient, 1, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.

2. Find the zeros of the function. We first use synthetic division to determine if $f(1) = 0$.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

   1 is a zero of the function and we have $f(x) = (x-1)(x^2 + x - 6)$. To find the other zeros we solve the following equation.

$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \quad or \quad x - 2 = 0$$
$$x = -3 \quad or \qquad x = 2$$

   The zeros of the function are 1, -3, and 2 so the x-intercepts of the graph are $(1,0)$, $(-3,0)$, and $(2,0)$.

3. The zeros divide the x-axis into four intervals, $(-\infty,-3)$, $(-3,1)$, $(1,2)$, and $(2,\infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in the interval.

   In $(-\infty,-3)$, test -4:

   $f(-4) = (-4)^3 - 7(-4) + 6 = -30 < 0$

   In $(-3,1)$, test 0:

   $f(0) = 0^3 - 7 \cdot 0 + 6 = 6 > 0$

   In $(1,2)$, test 1.5:

   $f(1.5) = (1.5)^3 - 7(1.5) + 6 = -1.125 < 0$

   In $(2,\infty)$, test 3:

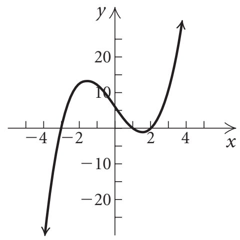
   $f(3) = 3^3 - 7 \cdot 3 + 6 = 12 > 0$

   Thus the graph lies below the x-axis on $(-\infty,-3)$ and on $(1,2)$. It lies above the x-axis on $(-3,1)$ and on $(2,\infty)$. We also know the points $(-4,-30)$, $(0,6)$, $(1.5,-1.125)$, and $(3,12)$ are on the graph.

4. From Step 3 we see that $f(0) = 6$ so the y-intercept is $(0,6)$.

5. We find additional points on the graph and draw the graph.

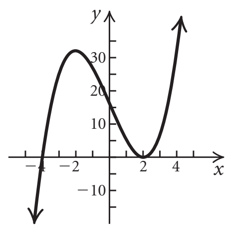
x	$f(x)$
-3.5	-12.4
-2	12
2.5	4.1
4	42



$f(x) = x^3 - 7x + 6$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

52.



$f(x) = x^3 - 12x + 16$

53. $f(x) = -x^3 + 3x^2 + 6x - 8$

1. The leading term is $-x^3$. The degree, 3, is odd and the leading coefficient, -1, is negative so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$.
2. Find the zeros of the function. We first use synthetic division to determine if $f(1) = 0$.

$$\begin{array}{r|rrrr} 1 & -1 & 3 & 6 & -8 \\ & & -1 & 2 & 8 \\ \hline & -1 & 2 & 8 & 0 \end{array}$$

1 is a zero of the function and we have $f(x) = (x-1)(-x^2+2x+8)$. To find the other zeros we solve the following equation.

$$-x^2 + 2x + 8 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x - 4 = 0 \quad or \quad x + 2 = 0$$
$$x = 4 \quad or \quad x = -2$$

The zeros of the function are 1, 4, and -2 so the x-intercepts of the graph are $(1, 0)$, $(4, 0)$, and $(-2, 0)$.

3. The zeros divide the x-axis into four intervals, $(-\infty, -2)$, $(-2, 1)$, $(1, 4)$, and $(4, \infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in the interval.

In $(-\infty, -2)$, test -3:

$f(-3) = -(-3)^3 + 3(-3)^2 + 6(-3) - 8 = 28 > 0$

In $(-2, 1)$, test 0:

$f(0) = -0^3 + 3 \cdot 0^2 + 6 \cdot 0 - 8 = -8 < 0$

In $(1, 4)$, test 2:

$f(2) = -2^3 + 3 \cdot 2^2 + 6 \cdot 2 - 8 = 8 > 0$

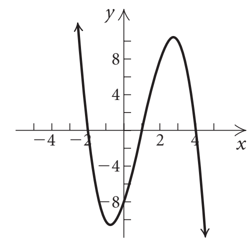
In $(4, \infty)$, test 5:

$f(5) = -5^3 + 3 \cdot 5^2 + 6 \cdot 5 - 8 = -28 < 0$

Thus the graph lies above the x-axis on $(-\infty, -2)$ and on $(1, 4)$. It lies below the x-axis on $(-2, 1)$ and on $(4, \infty)$. We also know the points $(-3, 28)$, $(0, -8)$, $(2, 8)$, and $(5, -28)$ are on the graph.

4. From Step 3 we see that $f(0) = -8$ so the y-intercept is $(0, -8)$.
5. We find additional points on the graph and draw the graph.

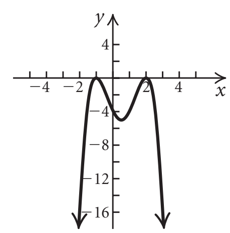
x	$f(x)$
-2.5	11.4
-1	-10
3	10
4.5	-11.4



$f(x) = -x^3 + 3x^2 + 6x - 8$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

54.



$f(x) = -x^4 + 2x^3 + 3x^2 - 4x - 4$

55.
$$2x^2 + 12 = 5x$$
$$2x^2 - 5x + 12 = 0$$
$$a = 2,\ b = -5,\ c = 12$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 12}}{2 \cdot 2}$$
$$= \frac{5 \pm \sqrt{-71}}{4}$$
$$= \frac{5 \pm i\sqrt{71}}{4} = \frac{5}{4} \pm \frac{\sqrt{71}}{4}i$$

The solutions are $\frac{5}{4} + \frac{\sqrt{71}}{4}i$ and $\frac{5}{4} - \frac{\sqrt{71}}{4}i$, or $\frac{5}{4} \pm \frac{\sqrt{71}}{4}i$.

56.
$$7x^2 + 4x = 3$$
$$7x^2 + 4x - 3 = 0$$
$$(7x - 3)(x + 1) = 0$$
$$7x - 3 = 0 \quad or \quad x + 1 = 0$$
$$7x = 3 \quad or \quad x = -1$$
$$x = \frac{3}{7} \quad or \quad x = -1$$

The solutions are $\frac{3}{7}$ and -1.

57. We substitute -14 for $g(x)$ and solve for x.

$$\begin{aligned} -14 &= x^2+5x-14 \\ 0 &= x^2+5x \\ 0 &= x(x+5) \\ x=0 \quad &or \quad x+5=0 \\ x=0 \quad &or \quad x=-5 \end{aligned}$$

When the output is -14, the input is 0 or -5.

58. $g(3) = 3^2+5\cdot 3-14 = 10$

59. We substitute -20 for $g(x)$ and solve for x.

$$\begin{aligned} -20 &= x^2+5x-14 \\ 0 &= x^2+5x+6 \\ 0 &= (x+3)(x+2) \\ x+3=0 \quad &or \quad x+2=0 \\ x=-3 \quad &or \quad x=-2 \end{aligned}$$

When the output is -20, the input is -3 or -2.

60. We have the data points $(0, 2.69)$ and $(28, 7.18)$.

$$m = \frac{7.18-2.69}{28-0} \approx 0.1604$$

We know that the y-intercept is $(0, 2.69)$, so we have $f(x) = 0.1604x + 2.69$.

In 1998, $x = 1998 - 1980 = 18$.

$f(18) = 0.1604(18) + 2.69 \approx \$5.58$

In 2012, $x = 2012 - 1980 = 32$.

$f(32) = 0.1604(32) + 2.69 \approx \$7.82$

61. Let b and h represent the length of the base and the height of the triangle, respectively.

$b + h = 30$, so $b = 30 - h$.

$$A = \frac{1}{2}bh = \frac{1}{2}(30-h)h = -\frac{1}{2}h^2 + 15h$$

Find the value of h for which A is a maximum:

$$h = \frac{-15}{2(-1/2)} = 15$$

When $h = 15$, $b = 30 - 15 = 15$.

The area is a maximum when the base and the height are each 15 in.

62. a) -5, -3, 4, 6, and 7 are zeros of the function, so $x+5$, $x+3$, $x-4$, $x-6$, and $x-7$ are factors.

b) We first write the product of the factors:

$F(x) = (x+5)(x+3)(x-4)(x-6)(x-7)$

Note that $F(0) = 5\cdot 3(-4)(-6)(-7) < 0$, but that the y-intercept of the graph is positive. Thus we must reflect $F(x)$ across the x-axis to obtain a function $P(x)$ with the given graph. We have:

$$\begin{aligned} P(x) &= -F(x) \\ P(x) &= -(x+5)(x+3)(x-4)(x-6)(x-7) \end{aligned}$$

c) Yes; two examples are $f(x) = c\cdot P(x)$, for any nonzero constant c, and $g(x) = (x-a)P(x)$.

d) No; only the function in part (b) has the given graph.

63. a) -4, -3, 2, and 5 are zeros of the function, so $x+4$, $x+3$, $x-2$, and $x-5$ are factors.

b) We first write the product of the factors:

$P(x) = (x+4)(x+3)(x-2)(x-5)$

Note that $P(0) = 4\cdot 3(-2)(-5) > 0$ and the graph shows a positive y-intercept, so this function is a correct one.

c) Yes; two examples are $f(x) = c\cdot P(x)$ for any nonzero constant c and $g(x) = (x-a)P(x)$.

d) No; only the function in part (b) has the given graph.

64. Divide $x^2 + kx + 4$ by $x - 1$.

$$\begin{array}{r|rrr} 1 & 1 & k & 4 \\ & & 1 & k+1 \\ \hline & 1 & k+1 & k+5 \end{array}$$

The remainder is $k+5$.

Divide $x^2 + kx + 4$ by $x + 1$.

$$\begin{array}{r|rrr} -1 & 1 & k & 4 \\ & & -1 & -k+1 \\ \hline & 1 & k-1 & -k+5 \end{array}$$

The remainder is $-k+5$.

Let $k+5 = -k+5$ and solve for k.

$$\begin{aligned} k+5 &= -k+5 \\ 2k &= 0 \\ k &= 0 \end{aligned}$$

65. Divide $x^3 - kx^2 + 3x + 7k$ by $x + 2$.

$$\begin{array}{r|rrrr} -2 & 1 & -k & 3 & 7k \\ & & -2 & 2k+4 & -4k-14 \\ \hline & 1 & -k-2 & 2k+7 & 3k-14 \end{array}$$

Thus $P(-2) = 3k - 14$.

We know that if $x+2$ is a factor of $f(x)$, then $f(-2) = 0$.

We solve $0 = 3k - 14$ for k.

$$\begin{aligned} 0 &= 3k-14 \\ \frac{14}{3} &= k \end{aligned}$$

66.

$$\begin{aligned} y &= \frac{1}{13}x^3 - \frac{1}{14}x \\ y &= x\left(\frac{1}{13}x^2 - \frac{1}{14}\right) \end{aligned}$$

We use the principle of zero products to find the zeros of the polynomial.

$$\begin{aligned} x=0 \quad or \quad \frac{1}{13}x^2 - \frac{1}{14} &= 0 \\ x=0 \quad or \quad \frac{1}{13}x^2 &= \frac{1}{14} \\ x=0 \quad or \quad x^2 &= \frac{13}{14} \\ x=0 \quad or \quad x &= \pm\sqrt{\frac{13}{14}} \\ x=0 \quad or \quad x &\approx \pm 0.9636 \end{aligned}$$

Only 0 and 0.9636 are in the interval $[0, 2]$.

67. $\dfrac{2x^2}{x^2-1}+\dfrac{4}{x+3}=\dfrac{12x-4}{x^3+3x^2-x-3}$,

LCM is $(x+1)(x-1)(x+3)$

$$(x+1)(x-1)(x+3)\left[\frac{2x^2}{(x+1)(x-1)}+\frac{4}{x+3}\right]=$$
$$(x+1)(x-1)(x+3)\cdot\frac{12x-4}{(x+1)(x-1)(x+3)}$$
$$2x^2(x+3)+4(x+1)(x-1)=12x-4$$
$$2x^3+6x^2+4x^2-4=12x-4$$
$$2x^3+10x^2-12x=0$$
$$x^3+5x^2-6x=0$$
$$x(x^2+5x-6)=0$$
$$x(x+6)(x-1)=0$$
$x=0$ *or* $x+6=0$ $\quad x-1=0$

$x=0$ *or* $\quad x=-6$ $\quad x=1$

Only 0 and -6 check. They are the solutions.

68. $\dfrac{6x^2}{x^2+11}+\dfrac{60}{x^3-7x^2+11x-77}=\dfrac{1}{x-7}$,

LCM is $(x^2+11)(x-7)$

$$(x^2+11)(x-7)\left[\frac{6x^2}{x^2+11}+\frac{60}{(x^2+11)(x-7)}\right]=$$
$$(x^2+11)(x-7)\cdot\frac{1}{x-7}$$
$$6x^2(x-7)+60=x^2+11$$
$$6x^3-42x^2+60=x^2+11$$
$$6x^3-43x^2+49=0$$

Use synthetic division to find factors of $f(x)=6x^3-43x^2+49$.

$$\begin{array}{r|rrrr} -1 & 6 & -43 & 0 & 49 \\ & & -6 & 49 & -49 \\ \hline & 6 & -49 & 49 & 0 \end{array}$$

Then we have:

$$(x+1)(6x^2-49x+49)=0$$
$$(x+1)(6x-7)(x-7)=0$$
$x=-1$ or $x=\dfrac{7}{6}$ or $x=7$

Only -1 and $\dfrac{7}{6}$ check.

69. Answers may vary. One possibility is $P(x)=x^{15}-x^{14}$.

70. $(x^4-y^4)\div(x-y)$

$=(x^4+0x^3+0x^2+0x-y^4)\div(x-y)$

$$\begin{array}{r|rrrrr} y & 1 & 0 & 0 & 0 & -y^4 \\ & & y & y^2 & y^3 & y^4 \\ \hline & 1 & y & y^2 & y^3 & 0 \end{array}$$

$Q(x)=x^3+x^2y+xy^2+y^3$, $R(x)=0$

71.
$$\begin{array}{r|rrrr} -i & 1 & 3i & -4i & -2 \\ & & -i & 2 & -4-2i \\ \hline & 1 & 2i & 2-4i & -6-2i \end{array}$$

$Q(x)=x^2+2ix+(2-4i)$, $R(x)=-6-2i$

72.
$$\begin{array}{r|rrr} 3+2i & 1 & -4 & -2 \\ & & 3+2i & -7+4i \\ \hline & 1 & -1+2i & -9+4i \end{array}$$

The answer is $x-1+2i$, R $-9+4i$.

73.
$$\begin{array}{r|rrr} i & 1 & -3 & 7 \\ & & i & -3i-1 \\ \hline & 1 & -3+i & 6-3i \end{array} \qquad (i^2=-1)$$

The answer is $x-3+i$, R $6-3i$.

Chapter 4 Mid-Chapter Mixed Review

1. $P(0)=5-2\cdot 0^3=5$, so the y-intercept is $(0,5)$. The given statement is false.

2. $-\dfrac{1}{2}x^4-3x^6+x^5=-3x^6+x^5-\dfrac{1}{2}x^4$

   The degree of the leading term, $-3x^6$, is 6, so the given statement is true.

3. $f(8)=(8+7)(8-8)=15\cdot 0=0$

   The given statement is true.

4. If $f(12)=0$ then, by the factor theorem, $x-12$ is a factor of $f(x)$. The given statement is false.

5. $f(x)=(x^2-10x+25)^3=[(x-5)^2]^3=(x-5)^6$

   Solving $(x-5)^6=0$, we get $x=5$.

   The factor $x-5$ occurs 6 times, so the zero has a multiplicity of 6.

6. $h(x)=2x^3+x^2-50x-25=x^2(2x+1)-25(2x+1)=(2x+1)(x^2-25)=(2x+1)(x+5)(x-5)$.

   Solving $(2x+1)(x+5)(x-5)=0$, we get

   $x=-\dfrac{1}{2}$ *or* $x=-5$ *or* $x=5$.

   Each factor occurs 1 time, so the multiplicity of each zero is 1.

7. $g(x)=x^4-3x^2+2=(x^2-1)(x^2-2)=(x+1)(x-1)(x^2-2)$

   Solving $(x+1)(x-1)(x^2-2)=0$, we get $x=-1$ *or* $x=1$ *or* $x=\pm\sqrt{2}$.

   Each factor occurs 1 time, so the multiplicity of each zero is 1.

8. $f(x)=-6(x-3)^2(x+4)$

   Solving $-6(x-3)^2(x+4)$, we get $x=3$ *or* $x=-4$.

   The factor $x-3$ occurs 2 times, so the multiplicity of the zero 3 is 2.

   The factor $x+4$ occurs 1 time, so the multiplicity of the zero -4 is 1.

9. $f(x)=x^4-x^3-6x^2$

   The sign of the leading coefficient, 1, is positive and the degree, 4, is even. Thus, graph (d) is the graph of the function.

10. $f(x) = -(x-1)^3(x+2)^2$

The sign of the leading coefficient, -1, is negative and the degree, 5, is odd. Thus, graph (a) is the graph of the function.

11. $f(x) = 6x^3 + 8x^2 - 6x - 8$

The sign of the leading coefficient, 6, is positive and the degree, 3, is odd. Thus, graph (b) is the graph of the function.

12. $f(x) = -(x-1)^3(x+1)$

The leading coefficient, -1, is negative and the degree, 4, is even. Thus, graph (c) is the graph of the function.

13. $f(-2) = (-2)^3 - 2(-2)^2 + 3 = -13$

$f(0) = 0^3 - 2\cdot 0^2 + 3 = 3$

By the intermediate value theorem, since $f(-2)$ and $f(0)$ have opposite signs, $f(x)$ has a zero between -2 and 0.

14. $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 2\left(-\frac{1}{2}\right)^2 + 3 = \frac{19}{8}$

$f(1) = 1^3 - 2\cdot 1^2 + 3 = 2$

Since both $f\left(-\frac{1}{2}\right)$ and $f(1)$ are positive, we cannot use the intermediate value theorem to determine if there is a zero between $-\frac{1}{2}$ and 1.

15.

$$\begin{array}{r|l}
 & x^3 - 5x^2 - 5x - 4 \\
x-1 & x^4 - 6x^3 + 0x^2 + x - 2 \\
 & \underline{x^4 - x^3} \\
 & -5x^3 + 0x^2 \\
 & \underline{-5x^3 + 5x^2} \\
 & -5x^2 + x \\
 & \underline{-5x^2 + 5x} \\
 & -4x - 2 \\
 & \underline{-4x + 4} \\
 & -6
\end{array}$$

$P(x) = (x-1)(x^3 - 5x^2 - 5x - 4) - \frac{6}{x-1}$

16.

$$\begin{array}{r|rrrrr}
2 & 3 & -1 & 2 & -6 & 6 \\
 & & 6 & 10 & 24 & 36 \\
\hline
 & 3 & 5 & 12 & 18 & 42
\end{array}$$

$Q(x) = 3x^3 + 5x^2 + 12x + 18$, $R(x) = 42$

17. $(x^5 - 5) \div (x+1) = (x^5 - 5) \div [x - (-1)]$

$$\begin{array}{r|rrrrrr}
-1 & 1 & 0 & 0 & 0 & 0 & -5 \\
 & & -1 & 1 & -1 & 1 & -1 \\
\hline
 & 1 & -1 & 1 & -1 & 1 & -6
\end{array}$$

$Q(x) = x^4 - x^3 + x^2 - x + 1$, $R(x) = -6$

18.

$$\begin{array}{r|rrrr}
-5 & 1 & -9 & 4 & -10 \\
 & & -5 & 70 & -370 \\
\hline
 & 1 & -14 & 74 & -380
\end{array}$$

$g(-5) = -380$

19.

$$\begin{array}{r|rrr}
\frac{1}{2} & 20 & -40 & 0 \\
 & & 10 & -15 \\
\hline
 & 20 & -30 & -15
\end{array}$$

$f\left(\frac{1}{2}\right) = -15$

20.

$$\begin{array}{r|rrrrr}
-\sqrt{2} & 5 & 1 & 0 & -1 & 0 \\
 & & -5\sqrt{2} & -\sqrt{2}+10 & 2-10\sqrt{2} & -\sqrt{2}+20 \\
\hline
 & 5 & 1-5\sqrt{2} & -\sqrt{2}+10 & 1-10\sqrt{2} & -\sqrt{2}+20
\end{array}$$

$f(-\sqrt{2}) = -\sqrt{2} + 20$, or $20 - \sqrt{2}$

21. $f(x) = x^3 - 4x^2 + 9x - 36$

If $-3i$ is a zero of $f(x)$, then $f(-3i) = 0$. We find $f(-3i)$.

$$\begin{array}{r|rrrr}
-3i & 1 & -4 & 9 & -36 \\
 & & -3i & -9+12i & 36 \\
\hline
 & 1 & -4-3i & 12i & 0
\end{array}$$

Since $f(-3i) = 0$, $-3i$ is a zero of $f(x)$.

If 3 is a zero of $f(x)$, then $f(3) = 0$. We find $f(3)$.

$$\begin{array}{r|rrrr}
3 & 1 & -4 & 9 & -36 \\
 & & 3 & -3 & 18 \\
\hline
 & 1 & -1 & 6 & -18
\end{array}$$

Since $f(3) \neq 0$, 3 is not a zero of $f(x)$.

22. $f(x) = x^6 - 35x^4 + 259x^2 - 225$

If -1 is a zero of $f(x)$, then $f(-1) = 0$. We find $f(-1)$.

$$\begin{array}{r|rrrrrrr}
-1 & 1 & 0 & -35 & 0 & 259 & 0 & -225 \\
 & & -1 & 1 & 34 & -34 & -225 & 225 \\
\hline
 & 1 & -1 & -34 & 34 & 225 & -225 & 0
\end{array}$$

Since $f(-1) = 0$, -1 is a zero of $f(x)$.

If 5 is a zero of $f(x)$, then $f(5) = 0$. We find $f(5)$.

$$\begin{array}{r|rrrrrrr}
5 & 1 & 0 & -35 & 0 & 259 & 0 & -225 \\
 & & 5 & 25 & -50 & -250 & 45 & 225 \\
\hline
 & 1 & 5 & -10 & -50 & 9 & 45 & 0
\end{array}$$

Since $f(5) = 0$, 5 is a zero of $f(x)$.

23. $h(x) = x^3 - 2x^2 - 55x + 56$

Try $x - 1$.

$$\begin{array}{r|rrrr}
1 & 1 & -2 & -55 & 56 \\
 & & 1 & -1 & -56 \\
\hline
 & 1 & -1 & -56 & 0
\end{array}$$

Since $h(1) = 0$, $x - 1$ is a factor of $h(x)$. Then $h(x) = (x-1)(x^2 - x - 56)$. Factoring the trinomial, we get $h(x) = (x-1)(x-8)(x+7)$.

Now we solve $h(x) = 0$.

$(x-1)(x-8)(x+7) = 0$

$x - 1 = 0$ *or* $x - 8 = 0$ *or* $x + 7 = 0$

$x = 1$ *or* $x = 8$ *or* $x = -7$

The solutions are 1, 8, and -7.

24. $g(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

Try $x + 1$, or $x - (-1)$.

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & -13 & 14 & 24 \\ & & -1 & 3 & 10 & -24 \\ \hline & 1 & -3 & -10 & 24 & 0 \end{array}$$

Since $g(-1) = 0$, $x - (-1)$, or $x + 1$, is a factor of $g(x)$. Then $g(x) = (x + 1)(x^3 - 3x^2 - 10x + 24)$.

Now try $x - 2$.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

Since $g(2) = 0$, $x - 2$ is a factor of $g(x)$. Then $g(x) = (x+1)(x-2)(x^2-x-12)$. Factoring the trinomial, we have $g(x) = (x + 1)(x - 2)(x - 4)(x + 3)$.

Now we solve $g(x) = 0$.

$(x + 1)(x - 2)(x - 4)(x + 3) = 0$

$x + 1 = 0$ *or* $x - 2 = 0$ *or* $x - 4 = 0$ *or* $x + 3 = 0$

$x = -1$ *or* $x = 2$ *or* $x = 4$ *or* $x = -3$

The solutions are -1, 2, 4, and -3.

25. The range of a polynomial function with an odd degree is $(-\infty, \infty)$. The range of a polynomial function with an even degree is $[s, \infty)$ for some real number s if $a_n > 0$ and is $(-\infty, s]$ for some real number s if $a_n < 0$.

26. Since we can find $f(0)$ for any polynomial function $f(x)$, it is not possible for the graph of a polynomial function to have no y-intercept. It is possible for a polynomial function to have no x-intercepts. For instance, a function of the form $f(x) = x^2 + a$, $a > 0$, has no x-intercepts. There are other examples as well.

27. If function values change from positive to negative or from negative to positive in an interval, there would have to be a zero in the interval. Thus, between a pair of consecutive zeros, all the function values must have the same sign.

28. For a polynomial $P(x)$ of degree n, when we have $P(x) = d(x) \cdot Q(x) + R(x)$, where the degree of $d(x)$ is 1, then the degree of $Q(x)$ must be $n - 1$.

Exercise Set 4.4

1. Find a polynomial function of degree 3 with -2, 3, and 5 as zeros.

Such a function has factors $x + 2$, $x - 3$, and $x - 5$, so we have $f(x) = a_n(x + 2)(x - 3)(x - 5)$.

The number a_n can be any nonzero number. The simplest polynomial will be obtained if we let it be 1. Multiplying the factors, we obtain

$$\begin{aligned} f(x) &= (x + 2)(x - 3)(x - 5) \\ &= (x^2 - x - 6)(x - 5) \\ &= x^3 - 6x^2 - x + 30. \end{aligned}$$

2. $$\begin{aligned} f(x) &= (x + 1)(x)(x - 4) \\ &= (x^2 + x)(x - 4) \\ &= x^3 - 3x^2 - 4x \end{aligned}$$

3. Find a polynomial function of degree 3 with -3, $2i$, and $-2i$ as zeros.

Such a function has factors $x + 3$, $x - 2i$, and $x + 2i$, so we have $f(x) = a_n(x + 3)(x - 2i)(x + 2i)$.

The number a_n can be any nonzero number. The simplest polynomial will be obtained if we let it be 1. Multiplying the factors, we obtain

$$\begin{aligned} f(x) &= (x + 3)(x - 2i)(x + 2i) \\ &= (x + 3)(x^2 + 4) \\ &= x^3 + 3x^2 + 4x + 12. \end{aligned}$$

4. $$\begin{aligned} f(x) &= (x - 2)(x - i)(x + i) \\ &= (x - 2)(x^2 + 1) \\ &= x^3 - 2x^2 + x - 2 \end{aligned}$$

5. Find a polynomial function of degree 3 with $\sqrt{2}$, $-\sqrt{2}$, and 3 as zeros.

Such a function has factors $x - \sqrt{2}$, $x + \sqrt{2}$, and $x - 3$, so we have $f(x) = a_n(x - \sqrt{2})(x + \sqrt{2})(x - 3)$.

The number a_n can be any nonzero number. The simplest polynomial will be obtained if we let it be 1. Multiplying the factors, we obtain

$$\begin{aligned} f(x) &= (x - \sqrt{2})(x + \sqrt{2})(x - 3) \\ &= (x^2 - 2)(x - 3) \\ &= x^3 - 3x^2 - 2x + 6. \end{aligned}$$

6. $$\begin{aligned} f(x) &= (x + 5)(x - \sqrt{3})(x + \sqrt{3}) \\ &= (x + 5)(x^2 - 3) \\ &= x^3 + 5x^2 - 3x - 15 \end{aligned}$$

7. Find a polynomial function of degree 3 with $1-\sqrt{3}$, $1+\sqrt{3}$, and -2 as zeros.

Such a function has factors $x - (1 - \sqrt{3})$, $x - (1 + \sqrt{3})$, and $x + 2$, so we have

$f(x) = a_n[x - (1 - \sqrt{3})][x - (1 + \sqrt{3})](x + 2)$.

The number a_n can be any nonzero number. The simplest polynomial will be obtained if we let it be 1. Multiplying the factors, we obtain

$$\begin{aligned} f(x) &= [x - (1 - \sqrt{3})][x - (1 + \sqrt{3})](x + 2) \\ &= [(x - 1) + \sqrt{3}][(x - 1) - \sqrt{3}](x + 2) \\ &= [(x - 1)^2 - (\sqrt{3})^2](x + 2) \\ &= (x^2 - 2x + 1 - 3)(x + 2) \\ &= (x^2 - 2x - 2)(x + 2) \\ &= x^3 - 2x^2 - 2x + 2x^2 - 4x - 4 \\ &= x^3 - 6x - 4. \end{aligned}$$

8. $$\begin{aligned} f(x) &= (x + 4)[x - (1 - \sqrt{5})][x - (1 + \sqrt{5})] \\ &= (x + 4)[(x - 1) + \sqrt{5}][(x - 1) - \sqrt{5}] \\ &= (x + 4)(x^2 - 2x + 1 - 5) \\ &= (x + 4)(x^2 - 2x - 4) \\ &= x^3 - 2x^2 - 4x + 4x^2 - 8x - 16 \\ &= x^3 + 2x^2 - 12x - 16 \end{aligned}$$

9. Find a polynomial function of degree 3 with $1+6i$, $1-6i$, and -4 as zeros.

Such a function has factors $x-(1+6i)$, $x-(1-6i)$, and $x+4$, so we have

$f(x) = a_n[x-(1+6i)][x-(1-6i)](x+4)$.

The number a_n can be any nonzero number. The simplest polynomial will be obtained if we let it be 1. Multiplying the factors, we obtain

$$\begin{aligned} f(x) &= [x-(1+6i)][x-(1-6i)](x+4) \\ &= [(x-1)-6i][(x-1)+6i](x+4) \\ &= [(x-1)^2-(6i)^2](x+4) \\ &= (x^2-2x+1+36)(x+4) \\ &= (x^2-2x+37)(x+4) \\ &= x^3-2x^2+37x+4x^2-8x+148 \\ &= x^3+2x^2+29x+148. \end{aligned}$$

10. $$\begin{aligned} f(x) &= [x-(1+4i)][x-(1-4i)](x+1) \\ &= (x^2-2x+17)(x+1) \\ &= x^3-x^2+15x+17 \end{aligned}$$

11. Find a polynomial function of degree 3 with $-\dfrac{1}{3}$, 0, and 2 as zeros.

Such a function has factors $x+\dfrac{1}{3}$, $x-0$ (or x), and $x-2$ so we have

$f(x) = a_n\left(x+\dfrac{1}{3}\right)(x)(x-2)$.

The number a_n can be any nonzero number. The simplest polynomial will be obtained if we let it be 1. Multiplying the factors, we obtain

$$\begin{aligned} f(x) &= \left(x+\frac{1}{3}\right)(x)(x-2) \\ &= \left(x^2+\frac{1}{3}x\right)(x-2) \\ &= x^3-\frac{5}{3}x^2-\frac{2}{3}x. \end{aligned}$$

12. $$\begin{aligned} f(x) &= (x+3)(x)\left(x-\frac{1}{2}\right) \\ &= (x^2+3x)(\left(x-\frac{1}{2}\right) \\ &= x^3+\frac{5}{2}x^2-\frac{3}{2}x \end{aligned}$$

13. A polynomial function of degree 5 has at most 5 real zeros. Since 5 zeros are given, these are all of the zeros of the desired function. We proceed as in Exercises 1-11, letting $a_n = 1$.

$$\begin{aligned} f(x) &= (x+1)^3(x-0)(x-1) \\ &= (x^3+3x^2+3x+1)(x^2-x) \\ &= x^5+2x^4-2x^2-x \end{aligned}$$

14. $$\begin{aligned} f(x) &= (x+2)(x-3)^2(x+1) \\ &= x^4-3x^3-7x^2+15x+18 \end{aligned}$$

15. A polynomial function of degree 4 has at most 4 real zeros. Since 4 zeros are given, these are all of the zeros of the desired function. We proceed as in Exercises 1-11, letting $a_n = 1$.

$$\begin{aligned} f(x) &= (x+1)^3(x-0) \\ &= (x^3+3x^2+3x+1)(x) \\ &= x^4+3x^3+3x^2+x \end{aligned}$$

16. $$\begin{aligned} f(x) &= \left(x+\frac{1}{2}\right)^2(x-0)(x-1)^2 \\ &= x^5-x^4-\frac{3}{4}x^3+\frac{1}{2}x^2+\frac{1}{4}x \end{aligned}$$

17. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, in addition to the three zeros given, the other zero is the conjugate of $\sqrt{3}$, or $-\sqrt{3}$.

18. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, in addition to the three zeros given, the other zero is the conjugate of $-\sqrt{2}$, or $\sqrt{2}$.

19. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, the other zeros are the conjugates of the given zeros. They are i and $2+\sqrt{5}$.

20. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, the other zeros are the conjugates of the given zeros. They are $-i$ and $-3-\sqrt{3}$.

21. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, in addition to the three zeros given, the other zero is the conjugate of $3i$, or $-3i$.

22. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, in addition to the three zeros given, the other zero is the conjugate of $-2i$, or $2i$.

23. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, the other zeros are the conjugates of the given zeros. They are $-4+3i$ and $2+\sqrt{3}$.

24. A polynomial function of degree 4 can have at most 4 zeros. Since $f(x)$ has rational coefficients, the other zeros are the conjugates of the given zeros. They are $6+5i$ and $-1-\sqrt{7}$.

25. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $\sqrt{5}$ and $-4i$, or $-\sqrt{5}$ and $4i$.

26. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $-\sqrt{3}$ and $2i$, or $\sqrt{3}$ and $-2i$.

27. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, the other zero is the conjugate of $2-i$, or $2+i$.

28. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, the other zero is the conjugate of $1-i$, or $1+i$.

29. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $-3+4i$ and $4-\sqrt{5}$, or $-3-4i$ and $4+\sqrt{5}$.

30. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $-3-3i$ and $2+\sqrt{13}$, or $-3+3i$ and $2-\sqrt{13}$.

31. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, the other zero is the conjugate of $4-i$, or $4+i$.

32. A polynomial function $f(x)$ of degree 5 has at most 5 zeros. Since $f(x)$ has rational coefficients, the other zero is the conjugate of $-3+\sqrt{2}$, or $-3-\sqrt{2}$.

33. Find a polynomial function of lowest degree with rational coefficients that has $1+i$ and 2 as some of its zeros. $1-i$ is also a zero.

Thus the polynomial function is

$f(x)=a_n(x-2)[x-(1+i)][x-(1-i)]$.

If we let $a_n=1$, we obtain

$$\begin{aligned} f(x)&=(x-2)[(x-1)-i][(x-1)+i]\\ &=(x-2)[(x-1)^2-i^2]\\ &=(x-2)(x^2-2x+1+1)\\ &=(x-2)(x^2-2x+2)\\ &=x^3-4x^2+6x-4. \end{aligned}$$

34. $$\begin{aligned} f(x)&=[x-(2-i)][x-(2+i)](x+1)\\ &=x^3-3x^2+x+5 \end{aligned}$$

35. Find a polynomial function of lowest degree with rational coefficients that has $4i$ as one of its zeros. $-4i$ is also a zero.

Thus the polynomial function is

$f(x)=a_n(x-4i)(x+4i)$.

If we let $a_n=1$, we obtain

$f(x)=(x-4i)(x+4i)=x^2+16$.

36. $f(x)=(x+5i)(x-5i)=x^2+25$

37. Find a polynomial function of lowest degree with rational coefficients that has $-4i$ and 5 as some of its zeros.

$4i$ is also a zero.

Thus the polynomial function is

$f(x)=a_n(x-5)(x+4i)(x-4i)$.

If we let $a_n=1$, we obtain

$$\begin{aligned} f(x)&=(x-5)[x^2-(4i)^2]\\ &=(x-5)(x^2+16)\\ &=x^3-5x^2+16x-80 \end{aligned}$$

38. $f(x)=(x-3)(x+i)(x-i)=x^3-3x^2+x-3$

39. Find a polynomial function of lowest degree with rational coefficients that has $1-i$ and $-\sqrt{5}$ as some of its zeros. $1+i$ and $\sqrt{5}$ are also zeros.

Thus the polynomial function is

$f(x)=a_n[x-(1-i)][x-(1+i)](x+\sqrt{5})(x-\sqrt{5})$.

If we let $a_n=1$, we obtain

$$\begin{aligned} f(x)&=[x-(1-i)][x-(1+i)](x+\sqrt{5})(x-\sqrt{5})\\ &=[(x-1)+i][(x-1)-i](x+\sqrt{5})(x-\sqrt{5})\\ &=(x^2-2x+1+1)(x^2-5)\\ &=(x^2-2x+2)(x^2-5)\\ &=x^4-2x^3+2x^2-5x^2+10x-10\\ &=x^4-2x^3-3x^2+10x-10 \end{aligned}$$

40. $$\begin{aligned} f(x)&=[x-(2-\sqrt{3})][x-(2+\sqrt{3})][x-(1+i)][x-(1-i)]\\ &=x^4-6x^3+11x^2-10x+2 \end{aligned}$$

41. Find a polynomial function of lowest degree with rational coefficients that has $\sqrt{5}$ and $-3i$ as some of its zeros.

$-\sqrt{5}$ and $3i$ are also zeros.

Thus the polynomial function is

$f(x)=a_n(x-\sqrt{5})(x+\sqrt{5})(x+3i)(x-3i)$.

If we let $a_n=1$, we obtain

$$\begin{aligned} f(x)&=(x^2-5)(x^2+9)\\ &=x^4+4x^2-45 \end{aligned}$$

42. $$\begin{aligned} f(x)&=(x+\sqrt{2})(x-\sqrt{2})(x-4i)(x+4i)\\ &=x^4+14x^2-32 \end{aligned}$$

43. $f(x)=x^3+5x^2-2x-10$

Since -5 is a zero of $f(x)$, we have $f(x)=(x+5)\cdot Q(x)$. We use synthetic division to find $Q(x)$.

$$\begin{array}{r|rrrr} -5 & 1 & 5 & -2 & -10\\ & & -5 & 0 & 10\\ \hline & 1 & 0 & -2 & 0 \end{array}$$

Then $f(x)=(x+5)(x^2-2)$. To find the other zeros we solve $x^2-2=0$.

$$\begin{aligned} x^2-2&=0\\ x^2&=2\\ x&=\pm\sqrt{2} \end{aligned}$$

The other zeros are $-\sqrt{2}$ and $\sqrt{2}$.

44. $$\begin{array}{r|rrrr} 1 & 1 & -1 & 1 & -1\\ & & 1 & 0 & 1\\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$f(x)=(x-1)(x^2+1)$

Solving $x^2+1=0$, we find that the other zeros are $-i$ and i.

45. If $-i$ is a zero of $f(x)=x^4-5x^3+7x^2-5x+6$, i is also a zero. Thus $x+i$ and $x-i$ are factors of the polynomial. Since $(x+i)(x-i)=x^2+1$, we know that $f(x)=(x^2+1)\cdot Q(x)$. Divide $x^4-5x^3+7x^2-5x+6$ by x^2+1.

$$\begin{array}{r|l} & x^2 - 5x + 6 \\ \hline x^2+1 & x^4 - 5x^3 + 7x^2 - 5x + 6 \\ & \underline{x^4 \qquad\quad + x^2} \\ & -5x^3 + 6x^2 - 5x \\ & \underline{-5x^3 \qquad\quad - 5x} \\ & 6x^2 \quad + 6 \\ & \underline{6x^2 \quad + 6} \\ & 0 \end{array}$$

Thus

$$\begin{aligned} x^4-5x^3+7x^2-5x+6 &= (x+i)(x-i)(x^2-5x+6) \\ &= (x+i)(x-i)(x-2)(x-3) \end{aligned}$$

Using the principle of zero products we find the other zeros to be i, 2, and 3.

46. $(x-2i)$ and $(x+2i)$ are both factors of $P(x) = x^4 - 16$.

$(x-2i)(x+2i) = x^2+4$

$$\begin{array}{r|l} & x^2 - 4 \\ \hline x^2+4 & x^4 + 0x^2 - 16 \\ & \underline{x^4 + 4x^2} \\ & -4x^2 - 16 \\ & \underline{-4x^2 - 16} \\ & 0 \end{array}$$

$$(x-2i)(x+2i)(x^2-4) = 0$$
$$(x-2i)(x+2i)(x+2)(x-2) = 0$$

The other zeros are $-2i$, -2, and 2.

47. $x^3 - 6x^2 + 13x - 20 = 0$

If 4 is a zero, then $x-4$ is a factor. Use synthetic division to find another factor.

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 13 & -20 \\ & & 4 & -8 & 20 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$(x-4)(x^2-2x+5) = 0$

$x-4 = 0$ *or* $x^2-2x+5 = 0$ Principle of zero products

$x = 4$ *or* $x = \dfrac{2\pm\sqrt{4-20}}{2}$ Quadratic formula

$x = 4$ *or* $x = \dfrac{2\pm 4i}{2} = 1\pm 2i$

The other zeros are $1+2i$ and $1-2i$.

48.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$(x-2)(x^2+2x+4) = 0$

$x = 2$ *or* $x = -1\pm\sqrt{3}i$

The other zeros are $-1+\sqrt{3}i$ and $-1-\sqrt{3}i$.

49. $f(x) = x^5 - 3x^2 + 1$

According to the rational zeros theorem, any rational zero of f must be of the form p/q, where p is a factor of the constant term, 1, and q is a factor of the coefficient of x^5, 1.

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1}{\pm 1}$

Possibilities for p/q: 1, -1

50. $f(x) = x^7 + 37x^5 - 6x^2 + 12$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 3, -3, 4, -4, 6, -6, 12, -12$

51. $f(x) = 2x^4 - 3x^3 - x + 8$

According to the rational zeros theorem, any rational zero of f must be of the form p/q, where p is a factor of the constant term, 8, and q is a factor of the coefficient of x^4, 2.

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2}$

Possibilities for p/q: $1,\ -1,\ 2,\ -2,\ 4,\ -4,\ 8,\ -8,\ \dfrac{1}{2},\ -\dfrac{1}{2}$

52. $f(x) = 3x^3 - x^2 + 6x - 9$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 3}$

Possibilities for p/q: $1,\ -1,\ 3,\ -3,\ 9,\ -9,\ \dfrac{1}{3},\ -\dfrac{1}{3}$

53. $f(x) = 15x^6 + 47x^2 + 2$

According to the rational zeros theorem, any rational zero of f must be of the form p/q, where p is a factor of 2 and q is a factor of 15.

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2}{\pm 1, \pm 3, \pm 5, \pm 15}$

Possibilities for p/q: $1, -1, 2, -2, \dfrac{1}{3}, -\dfrac{1}{3}, \dfrac{2}{3}, -\dfrac{2}{3}, \dfrac{1}{5}, -\dfrac{1}{5}, \dfrac{2}{5}, -\dfrac{2}{5}, \dfrac{1}{15}, -\dfrac{1}{15}, \dfrac{2}{15}, -\dfrac{2}{15}$

54. $f(x) = 10x^{25} + 3x^{17} - 35x + 6$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 5, \pm 10}$

Possibilities for p/q: $1, -1, 2, -2, 3, -3, 6, -6, \dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{3}{2}, -\dfrac{3}{2}, \dfrac{1}{5}, -\dfrac{1}{5}, \dfrac{2}{5}, -\dfrac{2}{5}, \dfrac{3}{5}, -\dfrac{3}{5}, \dfrac{6}{5}, -\dfrac{6}{5}, \dfrac{1}{10}, -\dfrac{1}{10}, \dfrac{3}{10}, -\dfrac{3}{10}$

55. $f(x) = x^3 + 3x^2 - 2x - 6$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 3, -3, 6, -6$

We use synthetic division to find a zero. We find that one zero is -3 as shown below.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -2 & -6 \\ & & -3 & 0 & 6 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

Then we have $f(x) = (x+3)(x^2-2)$.

We find the other zeros:

$$x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}.$$

There is only one rational zero, -3. The other zeros are $\pm\sqrt{2}$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x) = (x+3)(x-\sqrt{2})(x+\sqrt{2})$

56. $f(x) = x^3 - x^2 - 3x + 3$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm 1, \pm 3}{\pm 1}$

Possibilities for p/q: $1, -1, 3, -3$

We try 1.

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -3 & 3 \\ & & 1 & 0 & -3 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$f(x) = (x-1)(x^2-3)$

Now $x^2 - 3 = 0$ for $x = \pm\sqrt{3}$. Thus, there is only one rational zero, 1. The other zeros are $\pm\sqrt{3}$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x) = (x-1)(x-\sqrt{3})(x+\sqrt{3})$

57. $f(x) = 3x^3 - x^2 - 15x + 5$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$

Possibilities for p/q: $1, -1, 5, -5, \dfrac{1}{3}, -\dfrac{1}{3}, \dfrac{5}{3}, -\dfrac{5}{3}$

We use synthetic division to find a zero. We find that one zero is $\dfrac{1}{3}$ as shown below.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -15 & 5 \\ & & 1 & 0 & -5 \\ \hline & 3 & 0 & -15 & 0 \end{array}$$

Then we have $f(x) = \left(x - \dfrac{1}{3}\right)(3x^2 - 15)$, or $3\left(x - \dfrac{1}{3}\right)(x^2 - 5)$.

Now $x^2 - 5 = 0$ for $x = \pm\sqrt{5}$. Thus, there is only one rational zero, $\dfrac{1}{3}$. The other zeros are $\pm\sqrt{5}$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x) = 3\left(x - \dfrac{1}{3}\right)(x+\sqrt{5})(x-\sqrt{5})$

58. $f(x) = 4x^3 - 4x^2 - 3x + 3$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$

Possibilities for p/q: $1, -1, 3, -3, \dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{3}{2}, -\dfrac{3}{2}, \dfrac{1}{4}, -\dfrac{1}{4}, \dfrac{3}{4}, -\dfrac{3}{4}$

We try 1.

$$\begin{array}{r|rrrr} 1 & 4 & -4 & -3 & 3 \\ & & 4 & 0 & -3 \\ \hline & 4 & 0 & -3 & 0 \end{array}$$

$f(x) = (x-1)(4x^2-3)$.

Now $4x^2 - 3 = 0$ for $x = \pm\dfrac{\sqrt{3}}{2}$. Thus, there is only one rational zero, 1. The other zeros are $\pm\dfrac{\sqrt{3}}{2}$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x) = (x-1)\left(x + \dfrac{\sqrt{3}}{2}\right)\left(x - \dfrac{\sqrt{3}}{2}\right)$

59. $f(x) = x^3 - 3x + 2$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm 1, \pm 2}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2$

We use synthetic division to find a zero. We find that -2 is a zero as shown below.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

Then we have $f(x) = (x+2)(x^2 - 2x + 1) = (x+2)(x-1)^2$.

Now $(x-1)^2 = 0$ for $x = 1$. Thus, the rational zeros are -2 and 1. (The zero 1 has a multiplicity of 2.) These are the only zeros.

b) $f(x) = (x+2)(x-1)^2$

60. $f(x) = x^3 - 2x + 4$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm 1, \pm 2, \pm 4}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4$

We use synthetic division to find a zero. We find that -2 is a zero as shown below.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ & & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$f(x) = (x+2)(x^2 - 2x + 2)$

Using the quadratic formula, we find that the other zeros are $1 \pm i$. The only rational zero is -2. The other zeros are $1 \pm i$.

b) $$\begin{aligned} f(x) &= (x+2)[x-(1+i)][x-(1-i)] \\ &= (x+2)(x-1-i)(x-1+i) \end{aligned}$$

61. $f(x) = 2x^3 + 3x^2 + 18x + 27$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm 1, \pm 3, \pm 9, \pm 27}{\pm 1, \pm 2}$

Possibilities for p/q: $1, -1, 3, -3, 9, -9, 27, -27, \dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{3}{2}, -\dfrac{3}{2}, \dfrac{9}{2}, -\dfrac{9}{2}, \dfrac{27}{2}, -\dfrac{27}{2}$

We use synthetic division to find a zero. We find that $-\dfrac{3}{2}$ is a zero as shown below.

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 3 & 18 & 27 \\ & & -3 & 0 & -27 \\ \hline & 2 & 0 & 18 & 0 \end{array}$$

Then we have $f(x) = \left(x + \frac{3}{2}\right)(2x^2 + 18)$, or $2\left(x + \frac{3}{2}\right)(x^2 + 9)$.

Now $x^2 + 9 = 0$ for $x = \pm 3i$. Thus, the only rational zero is $-\frac{3}{2}$. The other zeros are $\pm 3i$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x) = 2\left(x + \frac{3}{2}\right)(x + 3i)(x - 3i)$

62. $f(x) = 2x^3 + 7x^2 + 2x - 8$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4, 8, -8, \frac{1}{2}, -\frac{1}{2}$

We try -2.

$$\begin{array}{r|rrrr} -2 & 2 & 7 & 2 & -8 \\ & & -4 & -6 & 8 \\ \hline & 2 & 3 & -4 & 0 \end{array}$$

$f(x) = (x + 2)(2x^2 + 3x - 4)$

Using the quadratic formula, we find that the other zeros are $\dfrac{-3 \pm \sqrt{41}}{4}$. The only rational zero is -2. The other zeros are $\dfrac{-3 \pm \sqrt{41}}{4}$.

b) $f(x) = (x+2)\left(x - \dfrac{-3+\sqrt{41}}{4}\right)\left(x - \dfrac{-3-\sqrt{41}}{4}\right)$

63. $f(x) = 5x^4 - 4x^3 + 19x^2 - 16x - 4$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 5}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4, \frac{1}{5}, -\frac{1}{5}, \frac{2}{5}, -\frac{2}{5}, \frac{4}{5}, -\frac{4}{5}$

We use synthetic division to find a zero. We find that 1 is a zero as shown below.

$$\begin{array}{r|rrrrr} 1 & 5 & -4 & 19 & -16 & -4 \\ & & 5 & 1 & 20 & 4 \\ \hline & 5 & 1 & 20 & 4 & 0 \end{array}$$

Then we have

$$\begin{aligned} f(x) &= (x - 1)(5x^3 + x^2 + 20x + 4) \\ &= (x - 1)[x^2(5x + 1) + 4(5x + 1)] \\ &= (x - 1)(5x + 1)(x^2 + 4). \end{aligned}$$

We find the other zeros:

$$\begin{aligned} 5x + 1 = 0 \quad &or \quad x^2 + 4 = 0 \\ 5x = -1 \quad &or \quad x^2 = -4 \\ x = -\frac{1}{5} \quad &or \quad x = \pm 2i \end{aligned}$$

The rational zeros are $-\frac{1}{5}$ and 1. The other zeros are $\pm 2i$.

b) From part (a) we see that $f(x) = (5x + 1)(x - 1)(x + 2i)(x - 2i)$, or $5\left(x + \frac{1}{5}\right)(x - 1)(x + 2i)(x - 2i)$.

64. $f(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2}{\pm 1, \pm 3}$

Possibilities for p/q: $1, -1, 2, -2, \frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

$$\begin{array}{r|rrrrr} -1 & 3 & -4 & 1 & 6 & -2 \\ & & -3 & 7 & -8 & 2 \\ \hline & 3 & -7 & 8 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -7 & 8 & -2 \\ & & 1 & -2 & 2 \\ \hline & 3 & -6 & 6 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x + 1)\left(x - \frac{1}{3}\right)(3x^2 - 6x + 6) \\ &= (x + 1)\left(x - \frac{1}{3}\right)(3)(x^2 - 2x + 2) \end{aligned}$$

Using the quadratic formula, we find that the other zeros are $1 \pm i$.

The rational zeros are -1 and $\frac{1}{3}$. The other zeros are $1 \pm i$.

b)

$$\begin{aligned} f(x) &= 3(x+1)\left(x - \frac{1}{3}\right)[x - (1+i)][x - (1-i)] \\ &= (x + 1)(3x - 1)(x - 1 - i)(x - 1 + i), \text{ or} \\ &\quad 3(x + 1)\left(x - \frac{1}{3}\right)(x - 1 - i)(x - 1 + i) \end{aligned}$$

65. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4, 8, -8$

We use synthetic division to find a zero. We find that -2 is a zero as shown below.

$$\begin{array}{r|rrrrr} -2 & 1 & -3 & -20 & -24 & -8 \\ & & -2 & 10 & 20 & 8 \\ \hline & 1 & -5 & -10 & -4 & 0 \end{array}$$

Now we determine whether -1 is a zero.

$$\begin{array}{r|rrrr} -1 & 1 & -5 & -10 & -4 \\ & & -1 & 6 & 4 \\ \hline & 1 & -6 & -4 & 0 \end{array}$$

Then we have $f(x) = (x+2)(x+1)(x^2-6x-4)$.

Use the quadratic formula to find the other zeros.

$x^2 - 6x - 4 = 0$

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} \\ &= \frac{6 \pm \sqrt{52}}{2} = \frac{6 \pm 2\sqrt{13}}{2} \\ &= 3 \pm \sqrt{13} \end{aligned}$$

The rational zeros are -2 and -1. The other zeros are $3 \pm \sqrt{13}$.

b) $f(x) = (x+2)(x+1)[x-(3+\sqrt{13})][x-(3-\sqrt{13})]$
$= (x+2)(x+1)(x-3-\sqrt{13})(x-3+\sqrt{13})$

66. $f(x) = x^4 + 5x^3 - 27x^2 + 31x - 10$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 5, -5, 10, -10$

$$\begin{array}{r|rrrrr} 1 & 1 & 5 & -27 & 31 & -10 \\ & & 1 & 6 & -21 & 10 \\ \hline & 1 & 6 & -21 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 6 & -21 & 10 \\ & & 2 & 16 & -10 \\ \hline & 1 & 8 & -5 & 0 \end{array}$$

$f(x) = (x-1)(x-2)(x^2+8x-5)$

Using the quadratic formula, we find that the other zeros are $-4 \pm \sqrt{21}$.

The rational zeros are 1 and 2. The other zeros are $-4 \pm \sqrt{21}$.

b) $f(x) = (x-1)(x-2)[x-(-4+\sqrt{21})][x-(-4-\sqrt{21})]$
$= (x-1)(x-2)(x+4-\sqrt{21})(x+4+\sqrt{21})$

67. $f(x) = x^3 - 4x^2 + 2x + 4$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4$

Synthetic division shows that neither -1 nor 1 is a zero. Try 2.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 2 & 4 \\ & & 2 & -4 & -4 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

Then we have $f(x) = (x-2)(x^2-2x-2)$. Use the quadratic formula to find the other zeros.

$x^2 - 2x - 2 = 0$

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \\ &= \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} \\ &= 1 \pm \sqrt{3} \end{aligned}$$

The only rational zero is 2. The other zeros are $1 \pm \sqrt{3}$.

b) $f(x) = (x-2)[x-(1+\sqrt{3})][x-(1-\sqrt{3})]$
$= (x-2)(x-1-\sqrt{3})(x-1+\sqrt{3})$

68. $f(x) = x^3 - 8x^2 + 17x - 4$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4$

$$\begin{array}{r|rrrr} 4 & 1 & -8 & 17 & -4 \\ & & 4 & -16 & 4 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$f(x) = (x-4)(x^2-4x+1)$

Using the quadratic formula, we find that the other zeros are $2 \pm \sqrt{3}$.

The only rational zero is 4. The other zeros are $2 \pm \sqrt{3}$.

b) $f(x) = (x-4)[x-(2+\sqrt{3})][x-(2-\sqrt{3})]$
$= (x-4)(x-2-\sqrt{3})(x-2+\sqrt{3})$

69. $f(x) = x^3 + 8$

a) $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1}$

Possibilities for p/q: $1, -1, 2, -2, 4, -4, 8, -8$

We use synthetic division to find a zero. We find that -2 is a zero as shown below.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

We have $f(x) = (x+2)(x^2-2x+4)$. Use the quadratic formula to find the other zeros.

$x^2 - 2x + 4 = 0$

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} \\ &= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2\sqrt{3}i}{2} \\ &= 1 \pm \sqrt{3}i \end{aligned}$$

The only rational zero is -2. The other zeros are $1 \pm \sqrt{3}i$.

b) $f(x) = (x+2)[x-(1+\sqrt{3}i)][x-(1-\sqrt{3}i)]$
$= (x+2)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)$

70. $f(x) = x^3 - 8$

a) As in Exercise 69, the possibilities for p/q are 1, -1, 2, -2, 4, -4, 8, and -8.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$f(x) = (x-2)(x^2+2x+4)$

Using the quadratic formula, we find that the other zeros are $-1 \pm \sqrt{3}i$.

The only rational zero is 2. The other zeros are $-1 \pm \sqrt{3}i$.

b) $f(x) = (x-2)[x-(-1+\sqrt{3}i)][x-(-1-\sqrt{3}i)]$
$= (x-2)(x+1-\sqrt{3}i)(x+1+\sqrt{3}i)$

71. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - \frac{1}{6}x + \frac{1}{6}$

$= \frac{1}{6}(2x^3 - 3x^2 - x + 1)$

a) The second form of the equation is equivalent to the first and has the advantage of having integer coefficients. Thus, we can use the rational zeros theorem for $g(x) = 2x^3 - 3x^2 - x + 1$. The zeros of $g(x)$ are the same as the zeros of $f(x)$. We find the zeros of $g(x)$.

$\frac{\text{Possibilities for } p}{\text{Possibilities for } q} : \quad \frac{\pm 1}{\pm 1, \pm 2}$

Possibilities for p/q: $1, -1, \frac{1}{2}, -\frac{1}{2}$

Synthetic division shows that $-\frac{1}{2}$ is not a zero. Try $\frac{1}{2}$.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -3 & -1 & 1 \\ & & 1 & -1 & -1 \\ \hline & 2 & -2 & -2 & 0 \end{array}$$

We have $g(x) = \left(x - \frac{1}{2}\right)(2x^2 - 2x - 2) = \left(x - \frac{1}{2}\right)(2)(x^2 - x - 1)$. Use the quadratic formula to find the other zeros.

$x^2 - x - 1 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

The only rational zero is $\frac{1}{2}$. The other zeros are $\frac{1 \pm \sqrt{5}}{2}$.

b) $f(x) = \frac{1}{6}g(x)$

$= \frac{1}{6}\left(x - \frac{1}{2}\right)(2)\left[x - \frac{1+\sqrt{5}}{2}\right]\left[x - \frac{1-\sqrt{5}}{2}\right]$

$= \frac{1}{3}\left(x - \frac{1}{2}\right)\left(x - \frac{1+\sqrt{5}}{2}\right)\left(x - \frac{1-\sqrt{5}}{2}\right)$

72. $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - \frac{1}{2}$

$= \frac{1}{6}(4x^3 - 3x^2 + 4x - 3)$

a) Find the zeros of $g(x) = 4x^3 - 3x^2 + 4x - 3$.

$\frac{\text{Possibilities for } p}{\text{Possibilities for } q} : \quad \frac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$

Possibilities for p/q: $1, -1, 3, -3, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -\frac{3}{2}, \frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, -\frac{3}{4}$

We try the possibilities until we find a zero.

$$\begin{array}{r|rrrr} \frac{3}{4} & 4 & -3 & 4 & -3 \\ & & 3 & 0 & 3 \\ \hline & 4 & 0 & 4 & 0 \end{array}$$

$g(x) = \left(x - \frac{3}{4}\right)(4x^2 + 4) = \left(x - \frac{3}{4}\right)(4)(x^2 + 1)$

Now $x^2 + 1 = 0$ when $x = \pm i$. Thus, the only rational zero is $\frac{3}{4}$. The other zeros are $\pm i$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x) = \frac{1}{6}g(x)$

$= \frac{1}{6}\left(x - \frac{3}{4}\right)(4)(x + i)(x - i)$

$= \frac{2}{3}\left(x - \frac{3}{4}\right)(x + i)(x - i)$

73. $f(x) = x^4 + 2x^3 - 5x^2 - 4x + 6$

According to the rational zeros theorem, the possible rational zeros are ± 1, ± 2 , ± 3, and ± 6. Synthetic division shows that only 1 and -3 are zeros.

74. $f(x) = x^4 - 3x^3 - 9x^2 - 3x - 10$

Possible rational zeros: ± 1, ± 2, ± 5, ± 10

Synthetic division shows that only -2 and 5 are zeros.

75. $f(x) = x^3 - x^2 - 4x + 3$

According to the rational zeros theorem, the possible rational zeros are ± 1 and ± 3. Synthetic division shows that none of these is a zero. Thus, there are no rational zeros.

76. $f(x) = 2x^3 + 3x^2 + 2x + 3$

Possible rational zeros: ± 1, ± 3, $\pm \frac{1}{2}$, $\pm \frac{3}{2}$

We try the possibilities until we find a zero.

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 3 & 2 & 3 \\ & & -3 & 0 & -3 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

$f(x) = \left(x + \frac{3}{2}\right)(2x^2 + 2) = \left(x + \frac{3}{2}\right)(2)(x^2 + 1)$

Since $g(x) = x^2 + 1$ has no real-number zeros, the only rational zero is $-\frac{3}{2}$. (We could have used factoring by grouping to find this result.)

77. $f(x) = x^4 + 2x^3 + 2x^2 - 4x - 8$

According to the rational zeros theorem, the possible rational zeros are ± 1, ± 2, ± 4, and ± 8. Synthetic division shows that none of the possibilities is a zero. Thus, there are no rational zeros.

78. $f(x) = x^4 + 6x^3 + 17x^2 + 36x + 66$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 11, \pm 22, \pm 33, \pm 66$

Synthetic division shows that there are no rational zeros.

79. $f(x) = x^5 - 5x^4 + 5x^3 + 15x^2 - 36x + 20$

According to the rational zeros theorem, the possible rational zeros are ±1, ±2, ±4, ±5, ±10, and ±20. We try -2.

$$\begin{array}{r|rrrrrr} -2 & 1 & -5 & 5 & 15 & -36 & 20 \\ & & -2 & 14 & -38 & 46 & -20 \\ \hline & 1 & -7 & 19 & -23 & 10 & 0 \end{array}$$

Thus, -2 is a zero. Now try 1.

$$\begin{array}{r|rrrrr} 1 & 1 & -7 & 19 & -23 & 10 \\ & & 1 & -6 & 13 & -10 \\ \hline & 1 & -6 & 13 & -10 & 0 \end{array}$$

1 is also a zero. Try 2.

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 13 & -10 \\ & & 2 & -8 & 10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

2 is also a zero.

We have $f(x) = (x + 2)(x - 1)(x - 2)(x^2 - 4x + 5)$. The discriminant of $x^2 - 4x + 5$ is $(-4)^2 - 4 \cdot 1 \cdot 5$, or $4 < 0$, so $x^2 - 4x + 5$ has two nonreal zeros. Thus, the rational zeros are -2, 1, and 2.

80. $f(x) = x^5 - 3x^4 - 3x^3 + 9x^2 - 4x + 12$

Possible rational zeros: ±1, ±2, ±3, ±4, ±6, ±12.

$$\begin{array}{r|rrrrrr} -2 & 1 & -3 & -3 & 9 & -4 & 12 \\ & & -2 & 10 & -14 & 10 & -12 \\ \hline & 1 & -5 & 7 & -5 & 6 & 0 \end{array}$$

-2 is a zero.

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 7 & -5 & 6 \\ & & 2 & -6 & 2 & -6 \\ \hline & 1 & -3 & 1 & -3 & 0 \end{array}$$

2 is a zero.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

3 is also a zero.

$f(x) = (x + 2)(x - 2)(x - 3)(x^2 + 1)$

Since $g(x) = x^2 + 1$ has no real-number zeros, the rational zeros are -2, 2, and 3.

81. $f(x) = 3x^5 - 2x^2 + x - 1$

The number of variations in sign in $f(x)$ is 3. Then the number of positive real zeros is either 3 or less than 3 by 2, 4, 6, and so on. Thus, the number of positive real zeros is 3 or 1.

$$\begin{aligned} f(-x) &= 3(-x)^5 - 2(-x)^2 + (-x) - 1 \\ &= -3x^5 - 2x^2 - x - 1 \end{aligned}$$

There are no variations in sign in $f(-x)$, so there are 0 negative real zeros.

82. $g(x) = 5x^6 - 3x^3 + x^2 - x$

The number of variations in sign in $g(x)$ is 3. Then the number of positive real zeros is either 3 or less than 3 by 2, 4, 6, and so on. Thus, the number of positive real zeros is 3 or 1.

$$\begin{aligned} g(-x) &= 5(-x)^6 - 3(-x)^3 + (-x)^2 - (-x) \\ &= 5x^6 + 3x^3 + x^2 + x \end{aligned}$$

There are no variations in sign in $g(-x)$, so there are 0 negative real zeros.

83. $h(x) = 6x^7 + 2x^2 + 5x + 4$

There are no variations in sign in $h(x)$, so there are 0 positive real zeros.

$$\begin{aligned} h(-x) &= 6(-x)^7 + 2(-x)^2 + 5(-x) + 4 \\ &= -6x^7 + 2x^2 - 5x + 4 \end{aligned}$$

The number of variations in sign in $h(-x)$ is 3. Thus, there are 3 or 1 negative real zeros.

84. $P(x) = -3x^5 - 7x^3 - 4x - 5$

There are no variations in sign in $P(x)$, so there are 0 positive real zeros.

$$\begin{aligned} P(-x) &= -3(-x)^5 - 7(-x)^3 - 4(-x) - 5 \\ &= 3x^5 + 7x^3 + 4x - 5 \end{aligned}$$

There is 1 variation in sign in $P(-x)$, so there is 1 negative real zero.

85. $F(p) = 3p^{18} + 2p^4 - 5p^2 + p + 3$

There are 2 variations in sign in $F(p)$, so there are 2 or 0 positive real zeros.

$$\begin{aligned} F(-p) &= 3(-p)^{18} + 2(-p)^4 - 5(-p)^2 + (-p) + 3 \\ &= 3p^{18} + 2p^4 - 5p^2 - p + 3 \end{aligned}$$

There are 2 variations in sign in $F(-p)$, so there are 2 or 0 negative real zeros.

86. $H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$

There are 2 variations in sign in $H(t)$, so there are 2 or 0 positive real zeros.

$$\begin{aligned} H(-t) &= 5(-t)^{12} - 7(-t)^4 + 3(-t)^2 + (-t) + 1 \\ &= 5t^{12} - 7t^4 + 3t^2 - t + 1 \end{aligned}$$

There are 4 variations in sign in $H(-t)$, so there are 4, 2, or 0 negative real zeros.

87. $C(x) = 7x^6 + 3x^4 - x - 10$

There is 1 variation in sign in $C(x)$, so there is 1 positive real zero.

$$\begin{aligned} C(-x) &= 7(-x)^6 + 3(-x)^4 - (-x) - 10 \\ &= 7x^6 + 3x^4 + x - 10 \end{aligned}$$

There is 1 variation in sign in $C(-x)$, so there is 1 negative real zero.

88. $g(z) = -z^{10} + 8z^7 + z^3 + 6z - 1$

There are 2 variations in sign in $g(z)$, so there are 2 or 0 positive real zeros.

$$\begin{aligned} g(-z) &= -(-z)^{10} + 8(-z)^7 + (-z)^3 + 6(-z) - 1 \\ &= -z^{10} - 8z^7 - z^3 - 6z - 1 \end{aligned}$$

There are no variations in sign in $g(-z)$, so there are 0 negative real zeros.

89. $h(t) = -4t^5 - t^3 + 2t^2 + 1$

There is 1 variation in sign in $h(t)$, so there is 1 positive real zero.

$$\begin{aligned} h(-t) &= -4(-t)^5 - (-t)^3 + 2(-t)^2 + 1 \\ &= 4t^5 + t^3 + 2t^2 + 1 \end{aligned}$$

There are no variations in sign in $h(-t)$, so there are 0 negative real zeros.

90. $P(x) = x^6 + 2x^4 - 9x^3 - 4$

There is 1 variation in sign in $P(x)$, so there is 1 positive real zero.

$$P(-x) = (-x)^6 + 2(-x)^4 - 9(-x)^3 - 4$$
$$= x^6 + +2x^4 + 9x^3 - 4$$

There is 1 variation in sign in $P(-x)$, so there is 1 negative real zero.

91. $f(y) = y^4 + 13y^3 - y + 5$

There are 2 variations in sign in $f(y)$, so there are 2 or 0 positive real zeros.

$$f(-y) = (-y)^4 + 13(-y)^3 - (-y) + 5$$
$$= y^4 - 13y^3 + y + 5$$

There are 2 variations in sign in $f(-y)$, so there are 2 or 0 negative real zeros.

92. $Q(x) = x^4 - 2x^2 + 12x - 8$

There are 3 variations in sign in $Q(x)$, so there are 3 or 1 positive real zeros.

$$Q(-x) = (-x)^4 - 2(-x)^2 + 12(-x) - 8$$
$$= x^4 - 2x^2 - 12x - 8$$

There is 1 variation in sign in $Q(-x)$, so there is 1 negative real zero.

93. $r(x) = x^4 - 6x^2 + 20x - 24$

There are 3 variations in sign in $r(x)$, so there are 3 or 1 positive real zeros.

$$r(-x) = (-x)^4 - 6(-x)^2 + 20(-x) - 24$$
$$= x^4 - 6x^2 - 20x - 24$$

There is 1 variation in sign in $r(-x)$, so there is 1 negative real zero.

94. $f(x) = x^5 - 2x^3 - 8x$

There is 1 variation in sign in $f(x)$, so there is 1 positive real zero.

$$f(-x) = (-x)^5 - 2(-x)^3 - 8(-x)$$
$$= -x^5 + 2x^3 + 8x$$

There is 1 variation in sign in $f(-x)$, so there is 1 negative real zero.

95. $R(x) = 3x^5 - 5x^3 - 4x$

There is 1 variation in sign in $R(x)$, so there is 1 positive real zero.

$$R(-x) = 3(-x)^5 - 5(-x)^3 - 4(-x)$$
$$= -3x^5 + 5x^3 + 4x$$

There is 1 variation in sign in $R(-x)$, so there is 1 negative real zero.

96. $f(x) = x^4 - 9x^2 - 6x + 4$

There are 2 variations in sign in $f(x)$, so there are 2 or 0 positive real zeros.

$$f(-x) = (-x)^4 - 9(-x)^2 - 6(-x) + 4$$
$$= x^4 - 9x^2 + 6x + 4$$

There are 2 variations in sign in $f(-x)$, so there are 2 or 0 negative real zeros.

97. $f(x) = 4x^3 + x^2 - 8x - 2$

1. The leading term is $4x^3$. The degree, 3, is odd and the leading coefficient, 4, is positive so as $x \to \infty$, $f(x) \to \infty$ and $x \to -\infty$, $f(x) \to -\infty$.
2. We find the rational zeros p/q of $f(x)$.

   $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

   Possibilities for p/q: $1, -1, 2, -2, \frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}$

   Synthetic division shows that $-\frac{1}{4}$ is a zero.

$-\frac{1}{4}$	4	1	−8	−2
		−1	0	2
	4	0	−8	0

   We have $f(x) = \left(x + \frac{1}{4}\right)(4x^2 - 8) = 4\left(x + \frac{1}{4}\right)(x^2 - 2)$. Solving $x^2 - 2 = 0$ we get $x = \pm\sqrt{2}$. Thus the zeros of the function are $-\frac{1}{4}$, $-\sqrt{2}$, and $\sqrt{2}$ so the x-intercepts of the graph are $\left(-\frac{1}{4}, 0\right)$, $(-\sqrt{2}, 0)$, and $(\sqrt{2}, 0)$.
3. The zeros divide the x-axis into 4 intervals, $(-\infty, -\sqrt{2})$, $\left(-\sqrt{2}, -\frac{1}{4}\right)$, $\left(-\frac{1}{4}, \sqrt{2}\right)$, and $(\sqrt{2}, \infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

   In $(-\infty, -\sqrt{2})$, test -2:

   $f(-2) = 4(-2)^3 + (-2)^2 - 8(-2) - 2 = -14 < 0$

   In $\left(-\sqrt{2}, -\frac{1}{4}\right)$, test -1:

   $f(-1) = 4(-1)^3 + (-1)^2 - 8(-1) - 2 = 3 > 0$

   In $\left(-\frac{1}{4}, \sqrt{2}\right)$, test 0:

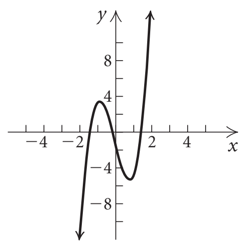
   $f(0) = 4 \cdot 0^3 + 0^2 - 8 \cdot 0 - 2 = -2 < 0$

   In $(\sqrt{2}, \infty)$, test 2:

   $f(2) = 4 \cdot 2^3 + 2^2 - 8 \cdot 2 - 2 = 18 > 0$

   Thus the graph lies below the x-axis on $(-\infty, -\sqrt{2})$ and on $\left(-\frac{1}{4}, \sqrt{2}\right)$. It lies above the x-axis on $\left(-\sqrt{2}, -\frac{1}{4}\right)$ and on $(\sqrt{2}, \infty)$. We also know the points $(-2, -14)$, $(-1, 3)$, $(0, -2)$, and $(2, 18)$ are on the graph.
4. From Step 3 we see that $f(0) = -2$ so the y-intercept is $(0, -2)$.
5. We find additional points on the graph and then draw the graph.

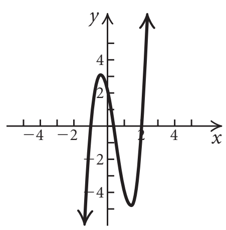
x	$f(x)$
-1.5	-1.25
-0.5	1.75
1	-5
1.5	1.75



$f(x) = 4x^3 + x^2 - 8x - 2$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

98.



$f(x) = 3x^3 - 4x^2 - 5x + 2$

99. $f(x) = 2x^4 - 3x^3 - 2x^2 + 3x$

1. The leading term is $2x^4$. The degree, 4, is even and the leading coefficient, 2, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.
2. We find the rational zeros p/q of $f(x)$. First note that $f(x) = x(2x^3 - 3x^2 - 2x + 3)$, so 0 is a zero. Now consider $g(x) = 2x^3 - 3x^2 - 2x + 3$.

   $\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q} : \dfrac{\pm 1, \pm 3}{\pm 1, \pm 2}$

   Possibilities for p/q: $1, -1, 3, -3, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -\frac{3}{2}$

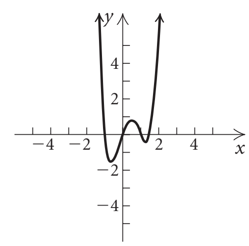
   We try 1.

   $$\begin{array}{r|rrrr} 1 & 2 & -3 & -2 & 3 \\ & & 2 & -1 & -3 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

   Then $f(x) = x(x-1)(2x^2 - x - 3)$. Using the principle of zero products to solve $2x^2 - x - 3 = 0$, we get $x = \frac{3}{2}$ or $x = -1$.

   Thus the zeros of the function are 0, 1, $\frac{3}{2}$, and -1 so the x-intercepts of the graph are $(0,0)$, $(1,0)$, $\left(\frac{3}{2}, 0\right)$, and $(-1, 0)$.
3. The zeros divide the x-axis into 5 intervals, $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$, $\left(1, \frac{3}{2}\right)$, and $\left(\frac{3}{2}, \infty\right)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.
   In $(-\infty, -1)$, test -2:
   $f(-2) = 2(-2)^4 - 3(-2)^3 - 2(-2)^2 + 3(-2) = 42 > 0$
   In $(-1, 0)$, test -0.5:
   $f(-0.5) = 2(-0.5)^4 - 3(-0.5)^3 - 2(-0.5)^2 + 3(-0.5) = -1.5 < 0$
   In $(0, 1)$, test 0.5:
   $f(0.5) = 2(0.5)^4 - 3(0.5)^3 - 2(0.5)^2 + 3(0.5) = 0.75 > 0$
   In $\left(1, \frac{3}{2}\right)$, test 1.25:
   $f(1.25) = 2(1.25)^4 - 3(1.25)^3 - 2(1.25)^2 + 3(1.25) = -0.3515625 < 0$
   In $\left(\frac{3}{2}, \infty\right)$, test 2:
   $f(2) = 2 \cdot 2^4 - 3 \cdot 2^3 - 2 \cdot 2^2 + 3 \cdot 2 = 6 > 0$
   Thus the graph lies above the x-axis on $(-\infty, -1)$, on $(0, 1)$, and on $\left(\frac{3}{2}, \infty\right)$. It lies below the x-axis on $(-1, 0)$ and on $\left(1, \frac{3}{2}\right)$. We also know the points $(-2, 42)$, $(-0.5, -1.5)$, $(0.5, 0.75)$, $(1.25, -0.3515625)$, and $(2, 6)$ are on the graph.
4. From Step 2 we know that $f(0) = 0$ so the y-intercept is $(0, 0)$.
5. We find additional points on the graph and then draw the graph.

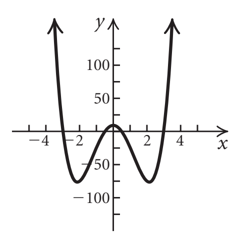
x	$f(x)$
-1.5	11.25
2.5	26.25
3	72



$f(x) = 2x^4 - 3x^3 - 2x^2 + 3x$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

100.



$f(x) = 4x^4 - 37x^2 + 9$

101. $f(x) = x^2 - 8x + 10$

a) $-\dfrac{b}{2a} = -\dfrac{-8}{2 \cdot 1} = -(-4) = 4$
   $f(4) = 4^2 - 8 \cdot 4 + 10 = -6$
   The vertex is $(4, -6)$.

b) The axis of symmetry is $x = 4$.

c) Since the coefficient of x^2 is positive, there is a minimum function value. It is the second coordinate of the vertex, -6. It occurs when $x = 4$.

102. $f(x) = 3x^2 - 6x - 1$

a) $-\frac{b}{2a} = -\frac{-6}{2 \cdot 3} = 1$

$f(1) = 3 \cdot 1^2 - 6 \cdot 1 - 1 = -4$

The vertex is $(1, -4)$.

b) $x = 1$

c) Minimum: -4 at $x = 1$

103.

$$-\frac{4}{5}x + 8 = 0$$

$$-\frac{4}{5}x = -8 \quad \text{Subtracting 8}$$

$$-\frac{5}{4}\left(-\frac{4}{5}x\right) = -\frac{5}{4}(-8) \quad \text{Multiplying by } -\frac{5}{4}$$

$$x = 10$$

The zero is 10.

104.

$$x^2 - 8x - 33 = 0$$

$$(x - 11)(x + 3) = 0$$

$x = 11$ *or* $x = -3$

The zeros are -3 and 11.

105. $g(x) = -x^3 - 2x^2$

Leading term: $-x^3$; leading coefficient: -1

The degree is 3, so the function is cubic.

Since the degree is odd and the leading coefficient is negative, as $x \to \infty$, $g(x) \to -\infty$ and as $x \to -\infty$, $g(x) \to \infty$.

106. $f(x) = -x^2 - 3x + 6$

Leading term: $-x^2$; leading coefficient: -1

The degree is 2, so the function is quadratic.

Since the degree is even and the leading coefficient is negative, as $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

107. $f(x) = -\frac{4}{9}$

Leading term: $-\frac{4}{9}$; leading coefficient: $-\frac{4}{9}$;

for all x, $f(x) = -\frac{4}{9}$

The degree is 0, so this is a constant function.

108. $h(x) = x - 2$

Leading term: x; leading coefficient: 1

The degree is 1, so the function is linear.

Since the degree is odd and the leading coefficient is positive, as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to -\infty$.

109. $g(x) = x^4 - 2x^3 + x^2 - x + 2$

Leading term: x^4; leading coefficient: 1

The degree is 4, so the function is quartic.

Since the degree is even and the leading coefficient is positive, as $x \to \infty$, $g(x) \to \infty$ and as $x \to -\infty$, $g(x) \to \infty$.

110. $h(x) = x^3 + \frac{1}{2}x^2 - 4x - 3$

Leading term: x^3; leading coefficient: 1

The degree is 3, so the function is cubic.

Since the degree is odd and the leading coefficient is positive, as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to -\infty$.

111. $f(x) = 2x^3 - 5x^2 - 4x + 3$

a) $2x^3 - 5x^2 - 4x + 3 = 0$

$\frac{\text{Possibilities for } p}{\text{Possibilities for } q}$: $\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

Possibilities for p/q: $1, -1, 3, -3, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -\frac{3}{2}$

The first possibility that is a solution of $f(x) = 0$ is -1:

-1	2	-5	-4	3
		-2	7	-3
	2	-7	3	0

Thus, -1 is a solution.

Then we have:

$$(x + 1)(2x^2 - 7x + 3) = 0$$

$$(x + 1)(2x - 1)(x - 3) = 0$$

The other solutions are $\frac{1}{2}$ and 3.

b) The graph of $y = f(x - 1)$ is the graph of $y = f(x)$ shifted 1 unit right. Thus, we add 1 to each solution of $f(x) = 0$ to find the solutions of $f(x - 1) = 0$. The solutions are $-1 + 1$, or 0; $\frac{1}{2} + 1$, or $\frac{3}{2}$; and $3 + 1$, or 4.

c) The graph of $y = f(x + 2)$ is the graph of $y = f(x)$ shifted 2 units left. Thus, we subtract 2 from each solution of $f(x) = 0$ to find the solutions of $f(x + 2) = 0$. The solutions are $-1 - 2$, or -3; $\frac{1}{2} - 2$, or $-\frac{3}{2}$; and $3 - 2$, or 1.

d) The graph of $y = f(2x)$ is a horizontal shrinking of the graph of $y = f(x)$ by a factor of 2. We divide each solution of $f(x) = 0$ by 2 to find the solutions of $f(2x) = 0$. The solutions are $\frac{-1}{2}$ or $-\frac{1}{2}$; $\frac{1/2}{2}$, or $\frac{1}{4}$; and $\frac{3}{2}$.

112. By the rational zeros theorem, only ± 1, ± 2, ± 3, ± 4, ± 6, and ± 12 can be rational solutions of $x^4 - 12 = 0$. Since none of them is a solution, the equation has no rational solutions. But $\sqrt[4]{12}$ is a solution of the equation, so $\sqrt[4]{12}$ must be irrational.

113. $P(x) = 2x^5 - 33x^4 - 84x^3 + 2203x^2 - 3348x - 10,080$

a) $2x^5 - 33x^4 - 84x^3 + 2203x^2 - 3348x - 10,080 = 0$

Trying some of the many possibilities for p/q, we find that 4 is a zero.

4	2	-33	-84	2203	-3348	$-10,080$
		8	-100	-736	5868	10,080
	2	-25	-184	1467	2520	0

Then we have:

$$(x - 4)(2x^4 - 25x^3 - 184x^2 + 1467x + 2520) = 0$$

We now use the fourth degree polynomial above to find another zero. Synthetic division shows that 4 is not a double zero, but 7 is a zero.

$$\begin{array}{r|rrrrr} 7 & 2 & -25 & -184 & 1467 & 2520 \\ & & 14 & -77 & -1827 & -2520 \\ \hline & 2 & -11 & -261 & -360 & 0 \end{array}$$

Now we have:

$(x-4)(x-7)(2x^3-11x^2-261x-360)=0$

Use the third degree polynomial above to find a third zero. Synthetic division shows that 7 is not a double zero, but 15 is a zero.

$$\begin{array}{r|rrrr} 15 & 2 & -11 & -261 & -360 \\ & & 30 & 285 & 360 \\ \hline & 2 & 19 & 24 & 0 \end{array}$$

We have:

$$\begin{aligned} P(x) &= (x-4)(x-7)(x-15)(2x^2+19x+24) \\ &= (x-4)(x-7)(x-15)(2x+3)(x+8) \end{aligned}$$

The rational zeros are 4, 7, 15, $-\frac{3}{2}$, and -8.

114. $P(x)=x^6-x^5-72x^4-81x^2+486x+5832$

a) $x^6-6x^5-72x^4-81x^2+486x+5832=0$

Synthetic division shows that we can factor as follows:

$$\begin{aligned} P(x) &= (x-3)(x+3)(x+6)(x^3-12x^2+9x-108) \\ &= (x-3)(x+3)(x+6)[x^2(x-12)+9(x-12)] \\ &= (x-3)(x+3)(x+6)(x-12)(x^2+9) \end{aligned}$$

The rational zeros are 3, -3, -6, and 12.

Exercise Set 4.5

1. $f(x)=\frac{x^2}{2-x}$

We find the value(s) of x for which the denominator is 0.

$$\begin{aligned} 2-x &= 0 \\ 2 &= x \end{aligned}$$

The domain is $\{x|x\neq 2\}$, or $(-\infty,2)\cup(2,\infty)$.

2. $f(x)=\frac{1}{x^3}$

The demominator is 0 when $x=0$, so the domain is $\{x|x\neq 0\}$, or $(-\infty,0)\cup(0,\infty)$.

3. $f(x)=\frac{x+1}{x^2-6x+5}$

We find the value(s) of x for which the denominator is 0.

$$\begin{aligned} x^2-6x+5 &= 0 \\ (x-1)(x-5) &= 0 \\ x-1=0 \quad &or \quad x-5=0 \\ x=1 \quad &or \quad \quad x=5 \end{aligned}$$

The domain is $\{x|x\neq 1 \textit{ and } x\neq 5\}$, or $(-\infty,1)\cup(1,5)\cup(5,\infty)$.

4. $f(x)=\frac{(x+4)^2}{4x-3}$

We find the value(s) of x for which the denominator is 0.

$$\begin{aligned} 4x-3 &= 0 \\ 4x &= 3 \\ x &= \frac{3}{4} \end{aligned}$$

The domain is $\left\{x\middle|x\neq\frac{3}{4}\right\}$, or $\left(-\infty,\frac{3}{4}\right)\cup\left(\frac{3}{4},\infty\right)$.

5. $f(x)=\frac{3x-4}{3x+15}$

We find the value(s) of x for which the denominator is 0.

$$\begin{aligned} 3x+15 &= 0 \\ 3x &= -15 \\ x &= -5 \end{aligned}$$

The domain is $\{x|x\neq -5\}$, or $(-\infty,-5)\cup(-5,\infty)$.

6. $f(x)=\frac{x^2+3x-10}{x^2+2x}$

We find the value(s) of x for which the denominator is 0.

$$\begin{aligned} x^2+2x &= 0 \\ x(x+2) &= 0 \\ x=0 \quad &or \quad x+2=0 \\ x=0 \quad &or \quad \quad x=-2 \end{aligned}$$

The domain is $\{x|x\neq -2 \textit{ and } x\neq 0\}$, or $(-\infty,-2)\cup(-2,0)\cup(0,\infty)$.

7. Graph (d) is the graph of $f(x)=\frac{8}{x^2-4}$.

$x^2-4=0$ when $x=\pm 2$, so $x=-2$ and $x=2$ are vertical asymptotes.

The x-axis, $y=0$, is the horizontal asymptote because the degree of the numerator is less than the degree of the denominator.

There is no oblique asymptote.

8. Graph (f) is the graph of $f(x)=\frac{8}{x^2+4}$.

$x^2+4=0$ has no real solutions, so there is no vertical asymptote.

The x-axis, $y=0$, is the horizontal asymptote because the degree of the numerator is less than the degree of the denominator.

There is no oblique asymptote.

9. Graph (e) is the graph of $f(x)=\frac{8x}{x^2-4}$.

As in Exercise 7, $x=-2$ and $x=2$ are vertical asymptotes.

The x-axis, $y=0$, is the horizontal asymptote because the degree of the numerator is less than the degree of the denominator.

There is no oblique asymptote.

10. Graph (a) is the graph of $f(x) = \dfrac{8x^2}{x^2-4}$.

As in Exercise 7, $x = 2$ and $x = -2$ are vertical asymptotes.

The numerator and denominator have the same degree, so $y = 8/1$, or $y = 8$, is the horizontal asymptote.

There is no oblique asymptote.

11. Graph (c) is the graph of $f(x) = \dfrac{8x^3}{x^2-4}$.

As in Exercise 7, $x = -2$ and $x = 2$ are vertical asymptotes.

The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote but there is an oblique asymptote. To find it we first divide to find an equivalent expression.

$$\begin{array}{r} 8x \\ x^2-4\overline{\big)\,8x^3} \\ \underline{8x^3-32x} \\ 32x \end{array}$$

$$\frac{8x^3}{x^2-4} = 8x + \frac{32x}{x^2-4}$$

Now we multiply by 1, using $(1/x^2)/(1/x^2)$.

$$\frac{32x}{x^2-4}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{\frac{32}{x}}{1-\frac{4}{x^2}}$$

As $|x|$ becomes very large, each expression with x in the denominator tends toward zero.

Then, as $|x| \to \infty$, we have

$$\frac{\frac{32}{x}}{1-\frac{4}{x^2}} \to \frac{0}{1-0}, \text{ or } 0.$$

Thus, as $|x|$ becomes very large, the graph of $f(x)$ gets very close to the graph of $y = 8x$, so $y = 8x$ is the oblique asymptote.

12. Graph (b) is the graph of $f(x) = \dfrac{8x^3}{x^2+4}$.

As in Exercise 8, there is no vertical asymptote.

The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote but there is an oblique asymptote. To find it we first divide to find an equivalent expression.

$$\frac{8x^3}{x^2+4} = 8x - \frac{32x}{x^2+4}$$

Now $\dfrac{32x}{x^2+4} = \dfrac{\frac{32}{x}}{1+\frac{4}{x^2}}$ and, as $|x| \to \infty$,

$$\frac{\frac{32}{x}}{1+\frac{4}{x^2}} \to \frac{0}{1+0}, \text{ or } 0.$$

Thus, as $y = 8x$ is the oblique asymptote.

13. $g(x) = \dfrac{1}{x^2}$

The numerator and the denominator have no common factors. The zero of the denominator is 0, so the vertical asymptote is $x = 0$.

14. $f(x) = \dfrac{4}{x+10}$

The numerator and the denominator have no common factors. $x + 10 = 0$ when $x = -10$, so the vertical asymptote is $x = -10$.

15. $h(x) = \dfrac{x+7}{2-x}$

The numerator and the denominator have no common factors. $2 - x = 0$ when $x = 2$, so the vertical asymptote is $x = 2$.

16. $g(x) = \dfrac{x^4+2}{x}$

The numerator and the denominator have no common factors. The zero of the denominator is 0, so the vertical asymptote is $x = 0$.

17. $f(x) = \dfrac{3-x}{(x-4)(x+6)}$

The numerator and the denominator have no common factors. The zeros of the denominator are 4 and -6, so the vertical asymptotes are $x = 4$ and $x = -6$.

18. $h(x) = \dfrac{x^2-4}{x(x+5)(x-2)} = \dfrac{(x+2)(x-2)}{x(x+5)(x-2)}$

The numerator and the denominator have a common factor, $x - 2$. The zeros of the denominator that are not also zeros of the numerator are 0 and -5, so the vertical asymptotes are $x = 0$ and $x = -5$.

19. $g(x) = \dfrac{x^2}{2x^2-x-3} = \dfrac{x^2}{(2x-3)(x+1)}$

The numerator and the denominator have no common factors. The zeros of the denominator are $\dfrac{3}{2}$ and -1, so the vertical asymptotes are $x = \dfrac{3}{2}$ and $x = -1$.

20. $f(x) = \dfrac{x+5}{x^2+4x-32} = \dfrac{x+5}{(x+8)(x-4)}$

The numerator and the denominator have no common factors. The zeros of the denominator are -8 and 4, so the vertical asymptotes are $x = -8$ and $x = 4$.

21. $f(x) = \dfrac{3x^2+5}{4x^2-3}$

The numerator and the denominator have the same degree and the ratio of the leading coefficients is $\dfrac{3}{4}$, so $y = \dfrac{3}{4}$ is the horizontal asymptote.

22. $g(x) = \dfrac{x+6}{x^3+2x^2}$

The degree of the numerator is less than the degree of the denominator, so $y = 0$ is the horizontal asymptote.

23. $h(x) = \dfrac{x^2 - 4}{2x^4 + 3}$

The degree of the numerator is less than the degree of the denominator, so $y = 0$ is the horizontal asymptote.

24. $f(x) = \dfrac{x^5}{x^5 + x}$

The numerator and the denominator have the same degree and the ratio of the leading coefficients is 1, so $y = 1$ is the horizontal asymptote.

25. $g(x) = \dfrac{x^3 - 2x^2 + x - 1}{x^2 - 16}$

The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote.

26. $h(x) = \dfrac{8x^4 + x - 2}{2x^4 - 10}$

The numerator and the denominator have the same degree and the ratio of the leading coefficients is 4, so $y = 4$ is the horizontal asymptote.

27. $g(x) = \dfrac{x^2 + 4x - 1}{x + 3}$

$$\begin{array}{r|l} & x + 1 \\ \hline x+3 & x^2 + 4x - 1 \\ & \underline{x^2 + 3x} \\ & \quad x - 1 \\ & \quad \underline{x + 3} \\ & \qquad -4 \end{array}$$

Then $g(x) = x + 1 + \dfrac{-4}{x + 3}$. The oblique asymptote is $y = x + 1$.

28. $f(x) = \dfrac{x^2 - 6x}{x - 5}$

$$\begin{array}{r|l} & x - 1 \\ \hline x-5 & x^2 - 6x + 0 \\ & \underline{x^2 - 5x} \\ & \quad -x + 0 \\ & \quad \underline{-x + 5} \\ & \qquad -5 \end{array}$$

Then $f(x) = x - 1 + \dfrac{-5}{x - 5}$. The oblique asymptote is $y = x - 1$.

29. $h(x) = \dfrac{x^4 - 2}{x^3 + 1}$

$$\begin{array}{r|l} & x \\ \hline x^3+1 & x^4 + 0x^3 + 0x^2 + 0x \\ & \underline{x^4 \qquad\qquad\quad + x} \\ & \qquad\qquad\qquad -x \end{array}$$

Then $h(x) = x + \dfrac{-x}{x^3 + 1}$. The oblique asymptote is $y = x$.

30. $g(x) = \dfrac{12x^3 - x}{6x^2 + 4}$

$$\begin{array}{r|l} & 2x \\ \hline 6x^2+4 & 12x^3 - x \\ & \underline{12x^3 + 8x} \\ & \qquad -9x \end{array}$$

Then $g(x) = 2x + \dfrac{-9x}{6x^2 + 4}$. The oblique asymptote is $y = 2x$.

31. $f(x) = \dfrac{x^3 - x^2 + x - 4}{x^2 + 2x - 1}$

$$\begin{array}{r|l} & x - 3 \\ \hline x^2+2x-1 & x^3 - x^2 + x - 4 \\ & \underline{x^3 + 2x^2 - x} \\ & \quad -3x^2 + 2x - 4 \\ & \quad \underline{-3x^2 - 6x + 3} \\ & \qquad\qquad 8x - 7 \end{array}$$

Then $f(x) = x - 3 + \dfrac{8x - 7}{x^2 + 2x - 1}$. The oblique asymptote is $y = x - 3$.

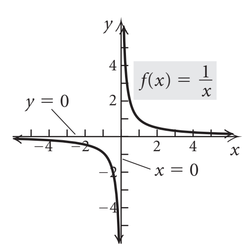
32. $h(x) = \dfrac{5x^3 - x^2 + x - 1}{x^2 - x + 2}$

$$\begin{array}{r|l} & 5x + 4 \\ \hline x^2-x+2 & 5x^3 - x^2 + x - 1 \\ & \underline{5x^3 - 5x^2 + 10x} \\ & \qquad 4x^2 - 9x - 1 \\ & \qquad \underline{4x^2 - 4x + 8} \\ & \qquad\quad -5x - 9 \end{array}$$

Then $h(x) = 5x + 4 + \dfrac{-5x - 9}{x^2 - x + 2}$. The oblique asymptote is $y = 5x + 4$.

33. $f(x) = \dfrac{1}{x}$

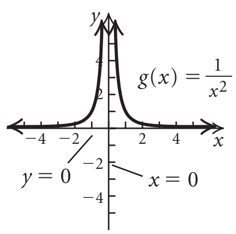
1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There are no oblique asymptotes.
3. The numerator has no zeros, so there is no x-intercept.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



34. $g(x) = \dfrac{1}{x^2}$

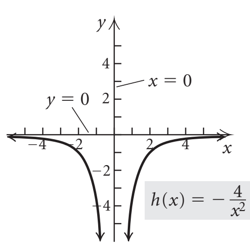
1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.

2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



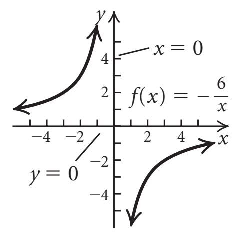
35. $h(x) = -\dfrac{4}{x^2}$

1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



36. $f(x) = -\dfrac{6}{x}$

1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.

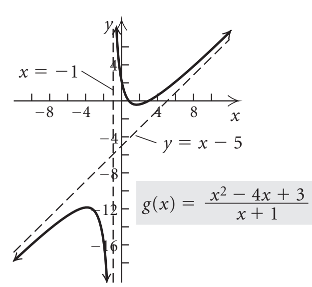


37. $g(x) = \dfrac{x^2 - 4x + 3}{x + 1} = \dfrac{(x-1)(x-3)}{x+1}$

1. The numerator and the denominator have no common factors. The denominator, $x+1$, is 0 when $x = -1$, so the domain excludes -1. It is $(-\infty, -1) \cup (-1, \infty)$. The line $x = -1$ is the vertical asymptote.
2. The degree of the numerator is 1 greater than the degree of the denominator, so we divide to find the oblique asymptote.

$$\begin{array}{r l} & x - 5 \\ x + 1 \,\big) & x^2 - 4x + 3 \\ & \underline{x^2 + x} \\ & -5x + 3 \\ & \underline{-5x - 5} \\ & 8 \end{array}$$

   The oblique asymptote is $y = x - 5$. There is no horizontal asymptote.
3. The zeros of the numerator are 1 and 3. Thus the x-intercepts are $(1, 0)$ and $(3, 0)$.
4. $g(0) = \dfrac{0^2 - 4 \cdot 0 + 3}{0 + 1} = 3$, so the y-intercept is $(0, 3)$.
5. Find other function values to determine the shape of the graph and then draw it.



38. $h(x) = \dfrac{2x^2 - x - 3}{x - 1} = \dfrac{(2x-3)(x+1)}{x-1}$

1. The numerator and the denominator have no common factors. The denominator is 0 when $x = 1$, so the domain excludes 1. It is $(-\infty, 1) \cup (1, \infty)$. The line $x = 1$ is the vertical asymptote.

2. The degree of the numerator is 1 greater than the degree of the denominator, so we divide to find the oblique asymptote.

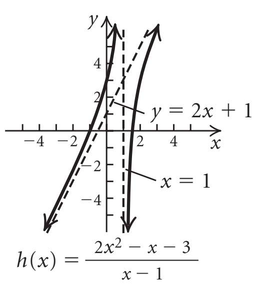
$$\begin{array}{r|l} & 2x + 1 \\ \hline x - 1 & 2x^2 - x - 3 \\ & \underline{2x^2 - 2x} \\ & \quad\;\; x - 3 \\ & \quad\;\; \underline{x - 1} \\ & \quad\;\; -2 \end{array}$$

The oblique asymptote is $y = 2x + 1$. There is no horizontal asymptote.

3. The zeros of the numerator are $\frac{3}{2}$ and -1. Thus the x-intercepts are $\left(\frac{3}{2}, 0\right)$ and $(-1, 0)$.

4. $h(0) = \frac{2 \cdot 0^2 - 0 - 3}{0 - 1} = 3$, so the y-intercept is $(0, 3)$.

5. Find other function values to determine the shape of the graph and then draw it.



39. $f(x) = \frac{-2}{x - 5}$

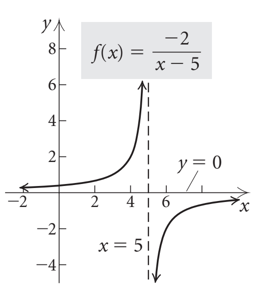
1. The numerator and the denominator have no common factors. 5 is the zero of the denominator, so the domain excludes 5. It is $(-\infty, 5) \cup (5, \infty)$. The line $x = 5$ is the vertical asymptote.

2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.

3. The numerator has no zeros, so there is no x-intercept.

4. $f(0) = \frac{-2}{0 - 5} = \frac{2}{5}$, so $\left(0, \frac{2}{5}\right)$ is the y-intercept.

5. Find other function values to determine the shape of the graph and then draw it.



40. $f(x) = \frac{1}{x - 5}$

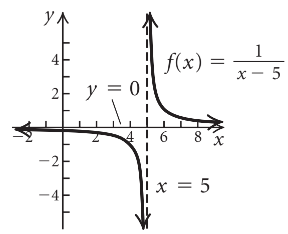
1. The numerator and the denominator have no common factors. 5 is the zero of the denominator, so the domain is $(-\infty, 5) \cup (5, \infty)$ and $x = 5$ is the vertical asymptote.

2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.

3. The numerator has no zeros, so there is no x-intercept.

4. $f(0) = \frac{1}{0 - 5} = -\frac{1}{5}$, so $\left(0, -\frac{1}{5}\right)$ is the y-intercept.

5. Find other function values to determine the shape of the graph and then draw it.



41. $f(x) = \frac{2x + 1}{x}$

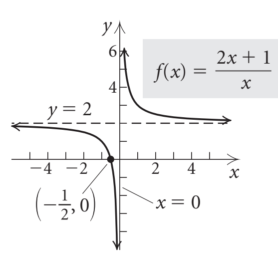
1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.

2. The numerator and denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 2/1, or 2. Thus, $y = 2$ is the horizontal asymptote. There is no oblique asymptote.

3. The zero of the numerator is the solution of $2x + 1 = 0$, or $-\frac{1}{2}$. The x-intercept is $\left(-\frac{1}{2}, 0\right)$.

4. Since 0 is not in the domain of the function, there is no y-intercept.

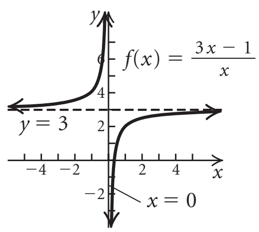
5. Find other function values to determine the shape of the graph and then draw it.



42. $f(x) = \frac{3x - 1}{x}$

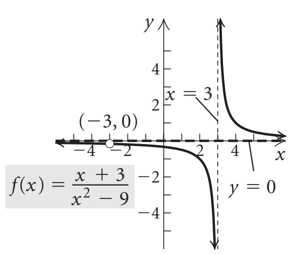
1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.

2. The numerator and denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 3/1, or 3. Thus, $y = 3$ is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is $\frac{1}{3}$, so the x-intercept is $\left(\frac{1}{3}, 0\right)$.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



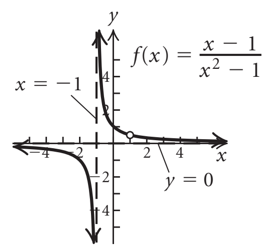
43. $f(x) = \frac{x+3}{x^2-9} = \frac{x+3}{(x+3)(x-3)}$

1. The domain of the function is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$. The numerator and denominator have the common factor $x+3$. The zeros of the denominator are -3 and 3, and the zero of the numerator is -3. Since 3 is the only zero of the denominator that is not a zero of the numerator, the only vertical asymptote is $x = 3$.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There are no oblique asymptotes.
3. The zero of the numerator, -3, is not in the domain of the function, so there is no x-intercept.
4. $f(0) = \frac{0+3}{0^2-9} = -\frac{1}{3}$, so the y-intercept is $\left(0, -\frac{1}{3}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



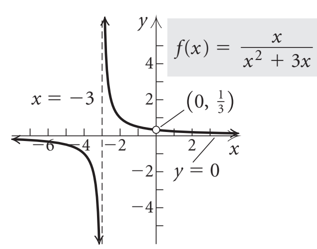
44. $f(x) = \frac{x-1}{x^2-1} = \frac{x-1}{(x+1)(x-1)}$

1. The domain of the function is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$. The numerator and denominator have the common factor $x-1$. The zeros of the denominator are -1 and 1, and the zero of the numerator is 1. Since -1 is the only zero of the denominator that is not a zero of the numerator, the only vertical asymptote is $x = -1$.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There are no oblique asymptotes.
3. The zero of the numerator, 1, is not in the domain of the function, so there is no x-intercept.
4. $f(0) = \frac{0-1}{0^2-1} = 1$, so the y-intercept is $(0, 1)$.
5. Find other function values to determine the shape of the graph and then draw it.



45. $f(x) = \frac{x}{x^2+3x} = \frac{x}{x(x+3)}$

1. The zeros of the denominator are 0 and -3, so the domain is $(-\infty, -3) \cup (-3, 0) \cup (0, \infty)$. The zero of the numerator is 0. Since -3 is the only zero of the denominator that is not also a zero of the numerator, the only vertical asymptote is $x = -3$.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is 0, but 0 is not in the domain of the function, so there is no x-intercept.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it, indicating the "hole" when $x = 0$ with an open circle.



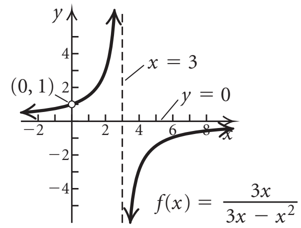
46. $f(x) = \frac{3x}{3x-x^2} = \frac{3x}{x(3-x)}$

1. The zeros of the denominator are 0 and 3, so the domain is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$. The zero of the numerator is 0. Since 3 is the only zero of the denominator that is not also a zero of the numerator, the only vertical asymptote is $x = 3$.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.

3. The zero of the numerator is 0, but 0 is not in the domain of the function, so there is no x-intercept.

4. Since 0 is not in the domain of the function, there is no y-intercept.

5. Find other function values to determine the shape of the graph and then draw it, indicating the "hole" when $x = 0$ with an open circle.



47. $f(x) = \dfrac{1}{(x-2)^2}$

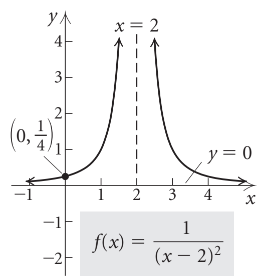
1. The numerator and the denominator have no commnon factors. 2 is the zero of the denominator, so the domain excludes 2. It is $(-\infty, 2) \cup (2, \infty)$. The line $x = 2$ is the vertical asymptote.

2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.

3. The numerator has no zeros, so there is no x-intercept.

4. $f(0) = \dfrac{1}{(0-2)^2} = \dfrac{1}{4}$, so $\left(0, \dfrac{1}{4}\right)$ is the y-intercept.

5. Find other function values to determine the shape of the graph and then draw it.



48. $f(x) = \dfrac{-2}{(x-3)^2}$

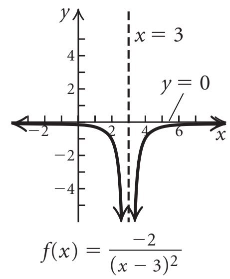
1. The numerator and the denominator have no commnon factors. 3 is the zero of the denominator, so the domain is $(-\infty, 3) \cup (3, \infty)$ and $x = 3$ is the vertical asymptote.

2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.

3. The numerator has no zeros, so there is no x-intercept.

4. $f(0) = \dfrac{-2}{(0-3)^2} = -\dfrac{2}{9}$, so $\left(0, -\dfrac{2}{9}\right)$ is the y-intercept.

5. Find other function values to determine the shape of the graph and then draw it.



49. $f(x) = \dfrac{x^2 + 2x - 3}{x^2 + 4x + 3} = \dfrac{(x+3)(x-1)}{(x+3)(x+1)}$

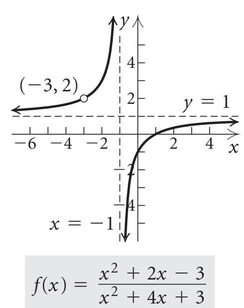
1. The zeros of the denominator are -3 and -1, so the domain is $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$. The zeros of the numerator are -3 and 1. Since -1 is the only zero of the denominator that is not also a zero of the numerator, the only vertical asymptote is $x = -1$.

2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 1/1, or 1. Thus, $y = 1$ is the horizontal asymptote. There is no oblique asymptote.

3. The only zero of the numerator that is in the domain of the function is 1, so the only x-intercept is $(1, 0)$.

4. $f(0) = \dfrac{0^2 + 2 \cdot 0 - 3}{0^2 + 4 \cdot 0 + 3} = \dfrac{-3}{3} = -1$, so the y-intercept is $(0, -1)$.

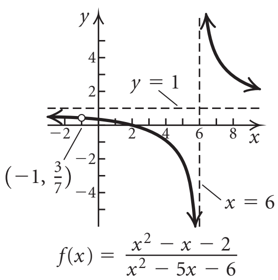
5. Find other function values to determine the shape of the graph and then draw it, indicating the "hole" when $x = -3$ with an open circle.



50. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 5x - 6} = \dfrac{(x+1)(x-2)}{(x+1)(x-6)}$

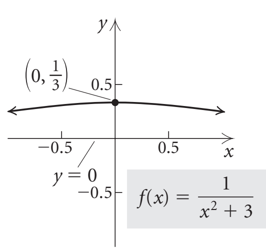
1. The zeros of the denominator are -1 and 6, so the domain is $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$. The zeros of the numerator are -1 and 2. Since 6 is the only zero of the denominator that is not also a zero of the numerator, the only vertical asymptote is $x = 6$.

2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 1/1, or 1. Thus, $y = 1$ is the horizontal asymptote. There is no oblique asymptote.
3. The only zero of the numerator that is in the domain of the function is 2, so the only x-intercept is $(2, 0)$.
4. $f(0) = \dfrac{0^2 - 0 - 2}{0^2 - 5 \cdot 0 - 6} = \dfrac{-2}{-6} = \dfrac{1}{3}$, so the y-intercept is $\left(0, \dfrac{1}{3}\right)$.
5. Find other function values to determine the shape of the graph and then draw it, indicating the "hole" when $x = -1$ with an open circle.



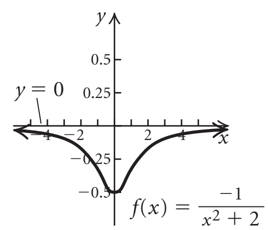
51. $f(x) = \dfrac{1}{x^2 + 3}$

1. The numerator and the denominator have no common factors. The denominator has no real-number zeros, so the domain is $(-\infty, \infty)$ and there is no vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. $f(0) = \dfrac{1}{0^2 + 3} = \dfrac{1}{3}$, so $\left(0, \dfrac{1}{3}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



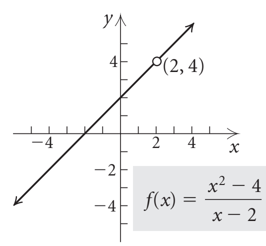
52. $f(x) = \dfrac{-1}{x^2 + 2}$

1. The numerator and the denominator have no common factors. The denominator has no real-number zeros, so the domain is $(-\infty, \infty)$ and there is no vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. $f(0) = \dfrac{-1}{0^2 + 2} = -\dfrac{1}{2}$, so $\left(0, -\dfrac{1}{2}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



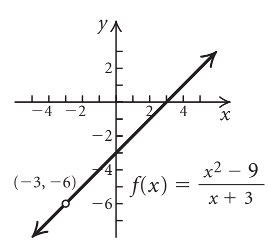
53. $f(x) = \dfrac{x^2 - 4}{x - 2} = \dfrac{(x + 2)(x - 2)}{x - 2} = x + 2,\ x \neq 2$

The graph is the same as the graph of $f(x) = x + 2$ except at $x = 2$, where there is a hole. Thus the domain is $(-\infty, 2) \cup (2, \infty)$. The zero of $f(x) = x + 2$ is -2, so the x-intercept is $(-2, 0)$; $f(0) = 2$, so the y-intercept is $(0, 2)$.



54. $f(x) = \dfrac{x^2 - 9}{x + 3} = \dfrac{(x + 3)(x - 3)}{x + 3} = x - 3,\ x \neq -3$

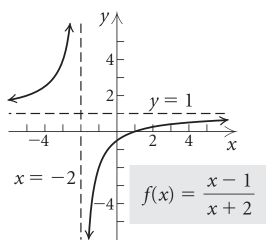
The domain is $(-\infty, -3) \cup (-3, \infty)$. The zero of $f(x) = x - 3$ is 3, so the x-intercept is $(3, 0)$; $f(0) = -3$, so the y-intercept is $(0, -3)$.



55. $f(x) = \dfrac{x - 1}{x + 2}$

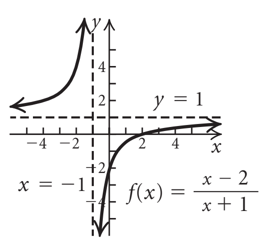
1. The numerator and the denominator have no common factors. -2 is the zero of the denominator, so the domain excludes -2. It is $(-\infty, -2) \cup (-2, \infty)$. The line $x = -2$ is the vertical asymptote.

2. The numerator and denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, $1/1$, or 1. Thus, $y = 1$ is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is 1, so the x-intercept is $(1, 0)$.
4. $f(0) = \dfrac{0-1}{0+2} = -\dfrac{1}{2}$, so $\left(0, -\dfrac{1}{2}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



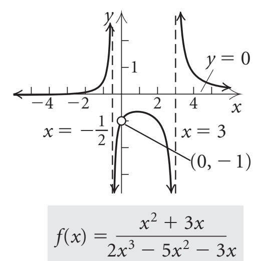
56. $f(x) = \dfrac{x-2}{x+1}$

1. The numerator and the denominator have no common factors. -1 is the zero of the denominator, so the domain is $(-\infty, -1) \cup (-1, \infty)$ and $x = -1$ is the vertical asymptote.
2. The numerator and denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, $1/1$, or 1. Thus, $y = 1$ is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is 2, so the x-intercept is $(2, 0)$.
4. $f(0) = \dfrac{0-2}{0+1} = -2$, so $(0, -2)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



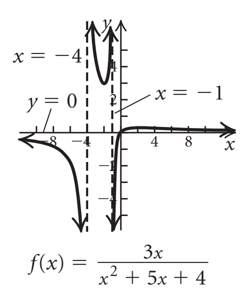
57. $f(x) = \dfrac{x^2+3x}{2x^3-5x^2-3x} = \dfrac{x(x+3)}{x(2x^2-5x-3)} = \dfrac{x(x+3)}{x(2x+1)(x-3)}$

1. The zeros of the denominator are 0, $-\dfrac{1}{2}$, and 3, so the domain is $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, 0\right) \cup (0, 3) \cup (3, \infty)$. The zeros of the numerator are 0 and -3. Since $-\dfrac{1}{2}$ and 3 are the only zeros of the denominator that are not also zeros of the numerator, the vertical asymptotes are $x = -\dfrac{1}{2}$ and $x = 3$.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The only zero of the numerator that is in the domain of the function is -3 so the only x-intercept is $(-3, 0)$.
4. 0 is not in the domain of the function, so there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it, indicating the "hole" when $x = 0$ with an open circle.



58. $f(x) = \dfrac{3x}{x^2+5x+4} = \dfrac{3x}{(x+1)(x+4)}$

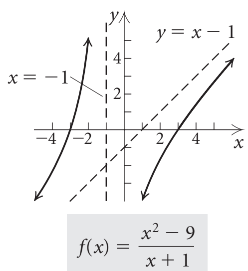
1. The numerator and the denominator have no common factors. The zeros of the denominator are -1 and -4, so the domain is $(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$ and $x = -4$ and $x = -1$ are the vertical asymptotes.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. 0 is the zero of the numerator, so $(0, 0)$ is the x-intercept.
4. From part (3) we see that $(0, 0)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



59. $f(x) = \dfrac{x^2-9}{x+1} = \dfrac{(x+3)(x-3)}{x+1}$

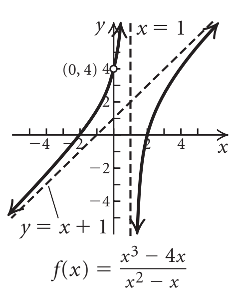
1. The numerator and the denominator have no common factors. -1 is the zero of the denominator, so the domain is $(-\infty, -1) \cup (-1, \infty)$. The line $x = -1$ is the vertical asymptote.

2. Because the degree of the numerator is one greater than the degree of the denominator, there is an oblique asymptote. Using division, we find that $\dfrac{x^2-9}{x+1} = x-1+\dfrac{-8}{x+1}$. As $|x|$ becomes very large, the graph of $f(x)$ gets close to the graph of $y = x-1$. Thus, the line $y = x-1$ is the oblique asymptote.
3. The zeros of the numerator are -3 and 3. Thus, the x-intercepts are $(-3,0)$ and $(3,0)$.
4. $f(0) = \dfrac{0^2-9}{0+1} = -9$, so $(0,-9)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



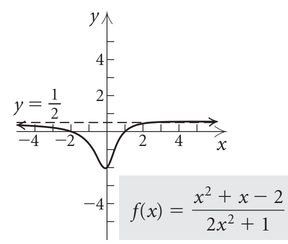
60. $f(x) = \dfrac{x^3-4x}{x^2-x} = \dfrac{x(x^2-4)}{x(x-1)} = \dfrac{x(x+2)(x-2)}{x(x-1)}$

1. The zeros of the denominator are 0 and 1, so the domain is $(-\infty,0)\cup(0,1)\cup(1,\infty)$. The zeros of the numerator are 0, -2, and 2. Since 1 is the only zero of the denominator that is not also a zero of the numerator, the only vertical asymptote is $x = 1$.
2. $\dfrac{x^3-4x}{x^2-x} = x+1+\dfrac{-3x}{x^2-x}$, so $y = x+1$ is the oblique asymptote.
3. The zeros of the numerator that are in the domain of the function are -2 and 2, so the x-intercepts are $(-2,0)$ and $(2,0)$.
4. 0 is not in the domain of the function, so there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



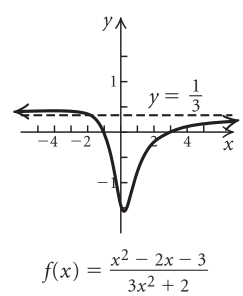
61. $f(x) = \dfrac{x^2+x-2}{2x^2+1} = \dfrac{(x+2)(x-1)}{2x^2+1}$

1. The numerator and the denominator have no common factors. The denominator has no real-number zeros, so the domain is $(-\infty,\infty)$ and there is no vertical asymptote.
2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, $1/2$. Thus, $y = 1/2$ is the horizontal asymptote. There is no oblique asymptote.
3. The zeros of the numerator are -2 and 1. Thus, the x-intercepts are $(-2,0)$ and $(1,0)$.
4. $f(0) = \dfrac{0^2+0-2}{2\cdot 0^2+1} = -2$, so $(0,-2)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



62. $f(x) = \dfrac{x^2-2x-3}{3x^2+2} = \dfrac{(x+1)(x-3)}{3x^2+2}$

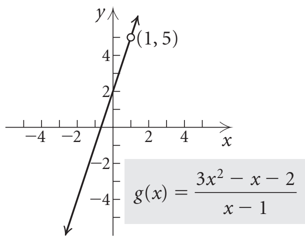
1. The numerator and the denominator have no common factors. The denominator has no real-number zeros, so the domain is $(-\infty,\infty)$ and there is no vertical asymptote.
2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, $1/3$. Thus, $y = 1/3$ is the horizontal asymptote. There is no oblique asymptote.
3. The zeros of the numerator are -1 and 3, so the x-intercepts are $(-1,0)$ and $(3,0)$.
4. $f(0) = \dfrac{0^2-2\cdot 0-3}{3\cdot 0^2+2} = -\dfrac{3}{2}$, so $\left(0,-\dfrac{3}{2}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



63. $g(x) = \dfrac{3x^2-x-2}{x-1} = \dfrac{(3x+2)(x-1)}{x-1} = 3x+2,\ x\neq 1$

The graph is the same as the graph of $g(x) = 3x+2$ except at $x = 1$, where there is a hole. Thus the domain is $(-\infty,1)\cup(1,\infty)$.

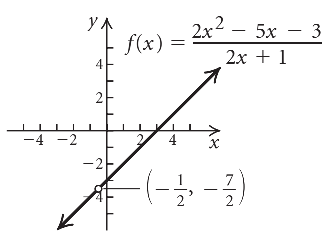
The zero of $g(x) = 3x + 2$ is $-\dfrac{2}{3}$, so the x-intercept is $\left(-\dfrac{2}{3}, 0\right)$; $g(0) = 2$, so the y-intercept is $(0, 2)$.



64. $f(x) = \dfrac{2x^2 - 5x - 3}{2x + 1} = \dfrac{(2x + 1)(x - 3)}{2x + 1} = x - 3,\ x \neq -\dfrac{1}{2}$

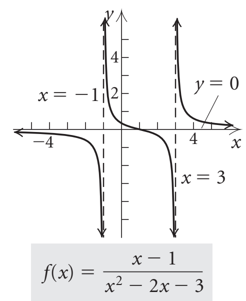
The graph is the same as the graph of $f(x) = x - 3$ except at $x = -\dfrac{1}{2}$, where there is a hole. Thus the domain is $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, \infty\right)$.

The zero of $f(x) = x - 3$ is 3, so the x-intercept is $(3, 0)$; $f(0) = \dfrac{2 \cdot 0^2 - 5 \cdot 0 - 3}{2 \cdot 0 + 1} = -3$, so the y-intercept is $(0, -3)$.



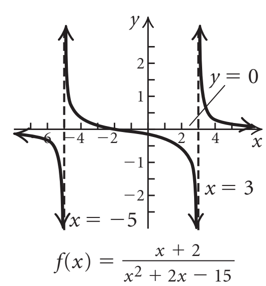
65. $f(x) = \dfrac{x - 1}{x^2 - 2x - 3} = \dfrac{x - 1}{(x + 1)(x - 3)}$

1. The numerator and the denominator have no common factors. The zeros of the denominator are -1 and 3. Thus, the domain is $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$ and the lines $x = -1$ and $x = 3$ are the vertical asymptotes.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. 1 is the zero of the numerator, so $(1, 0)$ is the x-intercept.
4. $f(0) = \dfrac{0 - 1}{0^2 - 2 \cdot 0 - 3} = \dfrac{1}{3}$, so $\left(0, \dfrac{1}{3}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



66. $f(x) = \dfrac{x + 2}{x^2 + 2x - 15} = \dfrac{x + 2}{(x + 5)(x - 3)}$

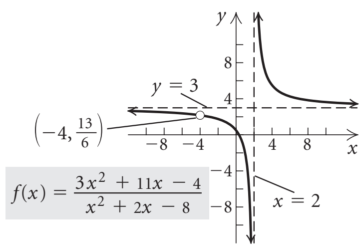
1. The numerator and the denominator have no common factors. $x^2 + 2x - 15 = (x + 5)(x - 3)$, so the domain excludes -5 and 3. It is $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$ and the lines $x = -5$ and $x = 3$ are vertical asymptotes.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. -2 is the zero of the numerator, so $(-2, 0)$ is the x-intercept.
4. $f(0) = \dfrac{0 + 2}{0^2 + 2 \cdot 0 - 15} = -\dfrac{2}{15}$, so $\left(0, -\dfrac{2}{15}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



67. $f(x) = \dfrac{3x^2 + 11x - 4}{x^2 + 2x - 8} = \dfrac{(3x - 1)(x + 4)}{(x + 4)(x - 2)}$

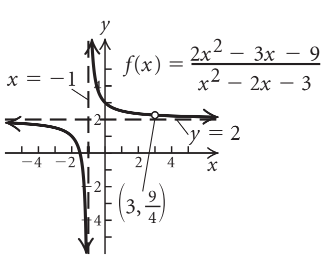
1. The domain of the function is $(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$. The numerator and the denominator have the common factor $x + 4$. The zeros of the denominator are -4 and 2, and the zeros of the numerator are $\dfrac{1}{3}$ and -4. Since 2 is the only zero of the denominator that is not a zero of the numerator, the only vertical asymptote is $x = 2$.
2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 3/1, or 3. Thus, $y = 3$ is the horizontal asymptote. There is no oblique asymptote.

3. The only zero of the numerator that is in the domain of the function is $\frac{1}{3}$, so the x-intercept is $\left(\frac{1}{3}, 0\right)$.
4. $f(0) = \dfrac{3 \cdot 0^2 + 11 \cdot 0 - 4}{0^2 + 2 \cdot 0 - 8} = \dfrac{-4}{-8} = \dfrac{1}{2}$, so the y-intercept is $\left(0, \frac{1}{2}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



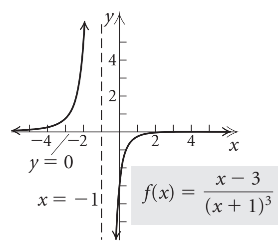
68. $f(x) = \dfrac{2x^2 - 3x - 9}{x^2 - 2x - 3} = \dfrac{(2x+3)(x-3)}{(x-3)(x+1)}$

1. The domain of the function is $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$. The numerator and the denominator have the common factor $x - 3$. The zeros of the denominator are 3 and -1, and the zeros of the numerator are $-\frac{3}{2}$ and 3. Since -1 is the only zero of the denominator that is not a zero of the numerator, the only vertical asymptote is $x = -1$.
2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, $2/1$, or 2. Thus, $y = 2$ is the horizontal asymptote. There is no oblique asymptote.
3. The only zero of the numerator that is in the domain of the function is $-\frac{3}{2}$, so the x-intercept is $\left(-\frac{3}{2}, 0\right)$.
4. $f(0) = \dfrac{2 \cdot 0^2 - 3 \cdot 0 - 9}{0^2 - 2 \cdot 0 - 3} = 3$, so the y-intercept is $(0, 3)$.
5. Find other function values to determine the shape of the graph and then draw it.



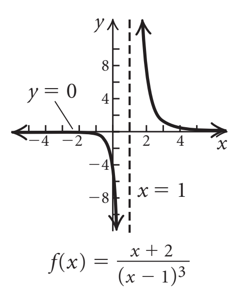
69. $f(x) = \dfrac{x-3}{(x+1)^3}$

1. The numerator and the denominator have no common factors. -1 is the zero of the denominator, so the domain excludes -1. It is $(-\infty, -1) \cup (-1, \infty)$. The line $x = -1$ is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. 3 is the zero of the numerator, so $(3, 0)$ is the x-intercept.
4. $f(0) = \dfrac{0-3}{(0+1)^3} = -3$, so $(0, -3)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



70. $f(x) = \dfrac{x+2}{(x-1)^3}$

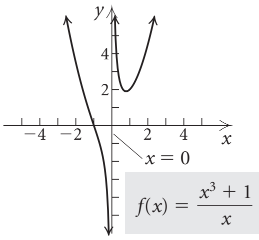
1. The numerator and the denominator have no common factors. 1 is the zero of the denominator, so the domain is $(-\infty, 1) \cup (1, \infty)$ and $x = 1$ is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. -2 is the zero of the numerator, so $(-2, 0)$ is the x-intercept.
4. $f(0) = \dfrac{0+2}{(0-1)^3} = -2$, so $(0, -2)$, is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



71. $f(x) = \dfrac{x^3 + 1}{x}$

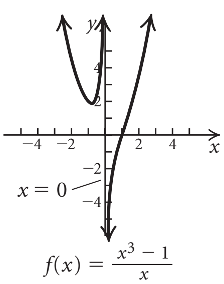
1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.
2. Because the degree of the numerator is more than one greater than the degree of the denominator, there is no horizontal or oblique asymptote.

3. The real-number zero of the numerator is -1, so the x-intercept is $(-1, 0)$.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



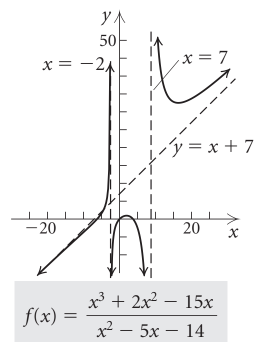
72. $f(x) = \dfrac{x^3 - 1}{x} = \dfrac{(x-1)(x^2+x+1)}{x}$

1. The numerator and the denominator have no common factors. 0 is the zero of the denominator, so the domain excludes 0. It is $(-\infty, 0) \cup (0, \infty)$. The line $x = 0$, or the y-axis, is the vertical asymptote.
2. Because the degree of the numerator is more than one greater than the degree of the denominator, there is no horizontal or oblique asymptote.
3. The real-number zero of the numerator is 1, so the x-intercept is $(1, 0)$.
4. Since 0 is not in the domain of the function, there is no y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



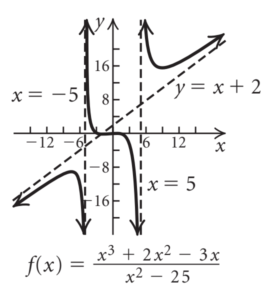
73. $f(x) = \dfrac{x^3 + 2x^2 - 15x}{x^2 - 5x - 14} = \dfrac{x(x+5)(x-3)}{(x+2)(x-7)}$

1. The numerator and the denominator have no common factors. The zeros of the denominator are -2 and 7. Thus, the domain is $(-\infty, -2) \cup (-2, 7) \cup (7, \infty)$ and the lines $x = -2$ and $x = 7$ are the vertical asymptotes.
2. Because the degree of the numerator is one greater than the degree of the denominator, there is an oblique asymptote. Using division, we find that $\dfrac{x^3 + 2x^2 - 15x}{x^2 - 5x - 14} = x + 7 + \dfrac{34x + 98}{x^2 - 5x - 14}$. As $|x|$ becomes very large, the graph of $f(x)$ gets close to the graph of $y = x + 7$. Thus, the line $y = x + 7$ is the oblique asymptote.
3. The zeros of the numerator are 0, -5, and 3. Thus, the x-intercepts are $(-5, 0)$, $(0, 0)$, and $(3, 0)$.
4. From part (3) we see that $(0, 0)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



74. $f(x) = \dfrac{x^3 + 2x^2 - 3x}{x^2 - 25} = \dfrac{x(x+3)(x-1)}{(x+5)(x-5)}$

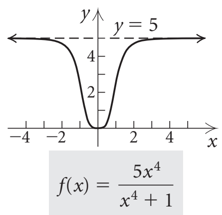
1. The numerator and the denominator have no common factors. The zeros of the denominator are -5 and 5, so the domain is $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$ and the lines $x = -5$ and $x = 5$ are the vertical asymptotes.
2. $\dfrac{x^3 + 2x^2 - 3x}{x^2 - 25} = x + 2 + \dfrac{-28x + 50}{x^2 - 25}$, so $y = x + 2$ is the oblique asymptote.
3. The zeros of the numerator are 0, -3, and 1, so the x-intercepts are $(0, 0)$, $(-3, 0)$, and $(1, 0)$.
4. From part (3) we see that $(0, 0)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



75. $f(x) = \dfrac{5x^4}{x^4 + 1}$

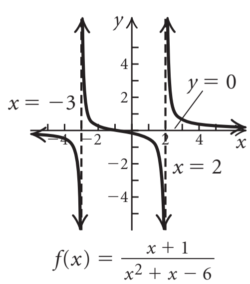
1. The numerator and the denominator have no common factors. The denominator has no real-number zeros, so the domain is $(-\infty, \infty)$ and there is no vertical asymptote.
2. The numerator and denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 5/1, or 5. Thus, $y = 5$ is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is 0, so $(0, 0)$ is the x-intercept.

4. From part (3) we see that $(0,0)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



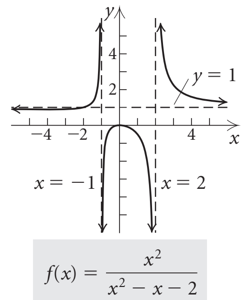
76. $f(x) = \dfrac{x+1}{x^2+x-6} = \dfrac{x+1}{(x+3)(x-2)}$

1. The numerator and the denominator have no common factors. The zeros of the denominator are -3 and 2, so the domain is $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$ and the lines $x=-3$ and $x=2$ are vertical asymptotes.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y=0$, is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is -1, so the x-intercept is $(-1,0)$.
4. $f(0) = \dfrac{0+1}{0^2+0-6} = -\dfrac{1}{6}$, so $\left(0,-\dfrac{1}{6}\right)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



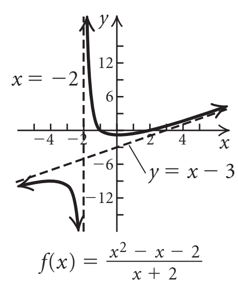
77. $f(x) = \dfrac{x^2}{x^2-x-2} = \dfrac{x^2}{(x+1)(x-2)}$

1. The numerator and the denominator have no common factors. The zeros of the denominator are -1 and 2. Thus, the domain is $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$ and the lines $x=-1$ and $x=2$ are the vertical asymptotes.
2. The numerator and denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 1/1, or 1. Thus, $y=1$ is the horizontal asymptote. There is no oblique asymptote.
3. The zero of the numerator is 0, so the x-intercept is $(0,0)$.
4. From part (3) we see that $(0,0)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



78. $f(x) = \dfrac{x^2-x-2}{x+2} = \dfrac{(x+1)(x-2)}{x+2}$

1. The numerator and the denominator have no common factors. -2 is the zero of the denominator, so the domain is $(-\infty,-2)\cup(-2,\infty)$ and the line $x=-2$ is the vertical asymptote.
2. $\dfrac{x^2-x-2}{x+2} = x-3+\dfrac{4}{x+2}$, so $y=x-3$ is the oblique asymptote.
3. The zeros of the numerator are -1 and 2, so the x-intercepts are $(-1,0)$ and $(2,0)$.
4. $f(0) = \dfrac{0^2-0-2}{0+2} = -1$, so $(0,-1)$ is the y-intercept.
5. Find other function values to determine the shape of the graph and then draw it.



79. Answers may vary. The numbers -4 and 5 must be zeros of the denominator. A function that satisfies these conditions is

$$f(x) = \frac{1}{(x+4)(x-5)}, \text{ or } f(x) = \frac{1}{x^2-x-20}.$$

80. Answers may vary. The numbers -4 and 5 must be zeros of the denominator and -2 must be a zero of the numerator.

$$f(x) = \frac{x+2}{(x+4)(x-5)}, \text{ or } f(x) = \frac{x+2}{x^2-x-20}.$$

81. Answers may vary. The numbers -4 and 5 must be zeros of the denominator and -2 must be a zero of the numerator. In addition, the numerator and denominator must have the same degree and the ratio of their leading coefficients must be 3/2. A function that satisfies these conditions is

$$f(x) = \frac{3x(x+2)}{2(x+4)(x-5)}, \text{ or } f(x) = \frac{3x^2+6x}{2x^2-2x-40}.$$

Another function that satisfies these conditions is
$g(x) = \dfrac{3(x+2)^2}{2(x+4)(x-5)}$, or $g(x) = \dfrac{3x^2+12x+12}{2x^2-2x-40}$.

82. Answers may vary. The degree of the numerator must be 1 greater than the degree of the denominator and the quotient, when long division is performed, must be $x-1$. If we let the remainder be 1, a function that satisfies these conditions is $f(x) = x - 1 + \dfrac{1}{x}$, or $f(x) = \dfrac{x^2-x+1}{x}$.

83. a) The horizontal asymptote of $N(t)$ is the ratio of the leading coefficients of the numerator and denominator, 0.8/5, or 0.16. Thus, $N(t) \to 0.16$ as $t \to \infty$.

b) The medication never completely disappears from the body; a trace amount remains.

84. a) $A(x) \to 2/1$, or 2 as $x \to \infty$.

b) As more DVDs are produced, the average cost approaches \$2.

85. a) $P(0) = 0$; $P(1) = 45.455$ thousand, or 45,455;
$P(3) = 55.556$ thousand, or 55,556;
$P(8) = 29.197$ thousand, or 29,197

b) The degree of the numerator is less than the degree of the denominator, so the x-axis is the horizontal asymptote. Thus, $P(t) \to 0$ as $t \to \infty$.

c) Eventually, no one will live in this community.

86. domain, range, domain, range

87. slope

88. slope-intercept equation

89. point-slope equation

90. x-intercept

91. $f(-x) = -f(x)$

92. vertical lines

93. midpoint formula

94. y-intercept

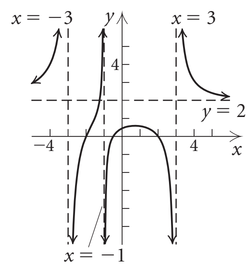
95. $f(x) = \dfrac{x^5+2x^3+4x^2}{x^2+2} = x^3 + 4 + \dfrac{-8}{x^2+2}$

As $|x| \to \infty$, $\dfrac{-8}{x^2+2} \to 0$ and the value of $f(x) \to x^3+4$. Thus, the nonlinear asymptote is $y = x^3+4$.

96. $f(x) = \dfrac{x^4+3x^2}{x^2+1} = x^2 + 2 + \dfrac{-2}{x^2+1}$

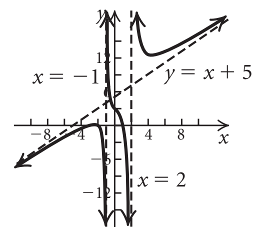
As $|x| \to \infty$, $\dfrac{-2}{x^2+1} \to 0$ and the value of $f(x) \to x^2+2$. Thus, the nonlinear asymptote is $y = x^2+2$.

97.



$f(x) = \dfrac{2x^3+x^2-8x-4}{x^3+x^2-9x-9}$

98.



$f(x) = \dfrac{x^3+4x^2+x-6}{x^2-x-2}$

Exercise Set 4.6

1.
$$x^2+2x-15=0$$
$$(x+5)(x-3)=0$$
$$x+5=0 \quad or \quad x-3=0$$
$$x=-5 \quad or \quad x=3$$

The solution set is $\{-5, 3\}$.

2. Solve $x^2+2x-15<0$.

From Exercise 1 we know the solutions of the related equation are -5 and 3. These numbers divide the x-axis into the intervals $(-\infty,-5)$, $(-5,3)$, and $(3,\infty)$. We test a value in each interval.

$(-\infty,-5)$: $f(-6) = 9 > 0$

$(-5,3)$: $f(0) = -15 < 0$

$(3,\infty)$: $f(4) = 9 > 0$

Function values are negative only in the interval $(-5,3)$. The solution set is $(-5,3)$.

3. Solve $x^2+2x-15 \le 0$.

From Exercise 2 we know the solution set of $x^2+2x-15<0$ is $(-5,3)$. The solution set of $x^2+2x-15 \le 0$ includes the endpoints of this interval. Thus the solution set is $[-5,3]$.

4. Solve $x^2+2x-15>0$.

From our work in Exercise 2 we see that the solution set is $(-\infty,-5)\cup(3,\infty)$.

5. Solve $x^2+2x-15 \ge 0$.

From Exercise 4 we know the solution set of $x^2+2x-15>0$ is $(-\infty,-5)\cup(3,\infty)$. The solution set of $x^2+2x-15 \ge 0$ includes the endpoints -5 and 3. Thus the solution set is $(-\infty,-5]\cup[3,\infty)$.

6. $\dfrac{x-2}{x+4} = 0$

$x-2 = 0$ Multiplying by $x+4$

$x = 2$

The solution set is $\{2\}$.

7. Solve $\dfrac{x-2}{x+4} > 0$.

The denominator tells us that $g(x)$ is not defined when $x = -4$. From Exercise 6 we know that $g(2) = 0$. The critical values of -4 and 2 divide the x-axis into three intervals $(-\infty, -4)$, $(-4, 2)$, and $(2, \infty)$. We test a value in each interval.

$(-\infty, -4)$: $g(-5) = 7 > 0$

$(-4, 2)$: $g(0) = -\dfrac{1}{2} < 0$

$(2, \infty)$: $g(3) = \dfrac{1}{7} > 0$

Function values are positive on $(-\infty, -4)$ and on $(2, \infty)$. The solution set is $(-\infty, -4) \cup (2, \infty)$.

8. Solve $\dfrac{x-2}{x+4} \leq 0$.

From Exercise 7 we see that the solution set of $\dfrac{x-2}{x+4} < 0$ is $(-4, 2)$. We include the zero of the function, 2, since the inequality symbol is $\leq$. The critical value -4 is not in the solution set because it is not in the domain of the function. The solution set is $(-4, 2]$.

9. Solve $\dfrac{x-2}{x+4} \geq 0$.

From Exercise 7 we know that the solution set of $\dfrac{x-2}{x+4} > 0$ is $(-\infty, -4) \cup (2, \infty)$. We include the zero of the function, 2, since the inequality symbol is $\geq$. The critical value -4 is not included because it is not in the domain of the function. The solution set is $(-\infty, -4) \cup [2, \infty)$.

10. Solve $g(x) < 0$.

From Exercise 7 we see that the solution set is $(-4, 2)$.

11. $\dfrac{7x}{(x-1)(x+5)} = 0$

$7x = 0$ Multiplying by $(x-1)(x+5)$

$x = 0$

The solution set is $\{0\}$.

12. Solve $\dfrac{7x}{(x-1)(x+5)} \leq 0$

The denominator tells us the function is not defined when $x = 1$ or $x = -5$. From Exercise 11 we know $f(0) = 0$. Test a value in each interval determined by the critical values, -5, 0, and 1.

$(-\infty, -5)$: $h(-6) = -6 < 0$

$(-5, 0)$: $h(-1) = 0.875 > 0$

$(0, 1)$: $h(0.5) \approx -1.273 < 0$

$(1, \infty)$: $h(2) = 2 > 0$

Function values are negative on $(-\infty, -5)$ and on $(0, 1)$. We also include the zero of the function, 0, in the solution set because the inequality symbol is $\leq$. The critical values -5 and 1 are not included because they are not in the domain of the function. The solution set is $(-\infty, -5) \cup [0, 1)$.

13. Solve $\dfrac{7x}{(x-1)(x+5)} \geq 0$.

From our work in Exercise 12 we see that function values are positive on $(-5, 0)$ and on $(1, \infty)$. We also include the zero of the function, 0, in the solution set because the inequality symbol is $\geq$. The critical values -5 and 1 are not included because they are not in the domain of the function. The solution set is $(-5, 0] \cup (1, \infty)$.

14. Solve $\dfrac{7x}{(x-1)(x+5)} > 0$.

From our work in Exercise 12 we see that the solution set is $(-5, 0) \cup (1, \infty)$.

15. Solve $\dfrac{7x}{(x-1)(x+5)} < 0$.

From our work in Exercise 12 we see that the solution set is $(-\infty, -5) \cup (0, 1)$.

16.

$$x^5 - 9x^3 = 0$$
$$x^3(x^2 - 9) = 0$$
$$x^3(x+3)(x-3) = 0$$

The solution set is $\{0, -3, 3\}$.

17. Solve $x^5 - 9x^3 < 0$.

From Exercise 16 we know the solutions of the related equation are -3, 0, and 3. These numbers divide the x-axis into the intervals $(-\infty, -3)$, $(-3, 0)$, $(0, 3)$, and $(3, \infty)$. We test a value in each interval.

$(-\infty, -3)$: $g(-4) = -448 < 0$

$(-3, 0)$: $g(-1) = 8 > 0$

$(0, 3)$: $g(1) = -8 < 0$

$(3, \infty)$: $g(4) = 448 > 0$

Function values are negative on $(-\infty, -3)$ and on $(0, 3)$. The solution set is $(-\infty, -3) \cup (0, 3)$.

18. Solve $x^5 - 9x^3 \leq 0$.

From Exercise 17 we know the solution set of $x^5 - 9x^3 < 0$ is $(-\infty, -3) \cup (0, 3)$. The solution set of $x^5 - 9x^3 \leq 0$ includes the endpoints -3, 0, and 3. Thus the solution set is $(-\infty, -3] \cup [0, 3]$.

19. Solve $x^5 - 9x^3 > 0$.

From our work in Exercise 17 we see that the solution set is $(-3, 0) \cup (3, \infty)$.

20. Solve $x^5 - 9x^3 \geq 0$.

From Exercise 19 we know the solution set of $x^5 - 9x^3 > 0$ is $(-3, 0) \cup (3, \infty)$. The solution set of $x^5 - 9x^3 \geq 0$ includes the endpoints -3, 0, and 3. Thus the solution set is $[-3, 0] \cup [3, \infty)$.

21. First we find an equivalent inequality with 0 on one side.

$$x^3 + 6x^2 < x + 30$$
$$x^3 + 6x^2 - x - 30 < 0$$

From the graph we see that the x-intercepts of the related function occur at $x = -5$, $x = -3$, and $x = 2$. They divide the x-axis into the intervals $(-\infty, -5)$, $(-5, -3)$, $(-3, 2)$, and $(2, \infty)$. From the graph we see that the function has negative values only on $(-\infty, -5)$ and $(-3, 2)$. Thus, the solution set is $(-\infty, -5) \cup (-3, 2)$.

22. From the graph we see that the x-intercepts of the related function occur at $x = -4$, $x = -3$, $x = 2$, and $x = 5$. They divide the x-axis into the intervals $(-\infty, -4)$, $(-4, -3)$, $(-3, 2)$, $(2, 5)$, and $(5, \infty)$. The function has positive values only on $(-\infty, -4)$, $(-3, 2)$, and $(5, \infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals are included in the solution set. It is $(-\infty, -4] \cup [-3, 2] \cup [5, \infty)$.

23. By observing the graph or the denominator of the function, we see that the function is not defined for $x = -2$ or $x = 2$. We also see that 0 is a zero of the function. These numbers divide the x-axis into the intervals $(-\infty, -2)$, $(-2, 0)$, $(0, 2)$, and $(2, \infty)$. From the graph we see that the function has positive values only on $(-2, 0)$ and $(2, \infty)$. Since the inequality symbol is $\geq$, 0 must be included in the solution set. It is $(-2, 0] \cup (2, \infty)$.

24. By observing the graph or the denominator of the function, we see that the function is not defined for $x = -2$ or $x = 2$. We also see that the function has no zeros. Thus, the numbers divide the x-axis into the intervals $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$. From the graph we see that the function has negative values only on $(-2, 2)$. Thus, the solution set is $(-2, 2)$.

25. $(x - 1)(x + 4) < 0$

The related equation is $(x - 1)(x + 4) = 0$. Using the principle of zero products, we find that the solutions of the related equation are 1 and -4. These numbers divide the x-axis into the intervals $(-\infty, -4)$, $(-4, 1)$, and $(1, \infty)$. We let $f(x) = (x - 1)(x + 4)$ and test a value in each interval.

$(-\infty, -4)$: $f(-5) = 6 > 0$

$(-4, 1)$: $f(0) = -4 < 0$

$(1, \infty)$: $f(2) = 6 > 0$

Function values are negative only in the interval $(-4, 1)$. The solution set is $(-4, 1)$.

26. $(x + 3)(x - 5) < 0$

The related equation is $(x + 3)(x - 5) = 0$. Its solutions are -3 and 5. These numbers divide the x-axis into the intervals $(-\infty, -3)$, $(-3, 5)$, and $(5, \infty)$. Let $f(x) = (x + 3)(x - 5)$ and test a value in each interval.

$(-\infty, -3)$: $f(-4) = 9 > 0$

$(-3, 5)$: $f(0) = -15 < 0$

$(5, \infty)$: $f(6) = 9 > 0$

Function values are negative only in the interval $(-3, 5)$. The solution set is $(-3, 5)$.

27.

$x^2 + x - 2 > 0$	Polynomial inequality
$x^2 + x - 2 = 0$	Related equation
$(x + 2)(x - 1) = 0$	Factoring

Using the principle of zero products, we find that the solutions of the related equation are -2 and 1. These numbers divide the x-axis into the intervals $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$. We let $f(x) = x^2 + x - 2$ and test a value in each interval.

$(-\infty, -2)$: $f(-3) = 4 > 0$

$(-2, 1)$: $f(0) = -2 < 0$

$(1, \infty)$: $f(2) = 4 > 0$

Function values are positive on $(-\infty, -2)$ and $(1, \infty)$. The solution set is $(-\infty, -2) \cup (1, \infty)$.

28.

$x^2 - x - 6 > 0$	Polynomial inequality
$x^2 - x - 6 = 0$	Related equation
$(x - 3)(x + 2) = 0$	Factoring

The solutions of the related equation are 3 and -2. These numbers divide the x-axis into the intervals $(-\infty, -2)$, $(-2, 3)$, and $(3, \infty)$. We let $f(x) = x^2 - x - 6$ and test a value in each interval.

$(-\infty, -2)$: $f(-3) = 6 > 0$

$(-2, 3)$: $f(0) = -6 < 0$

$(3, \infty)$: $f(4) = 6 > 0$

Function values are positive on $(-\infty, -2)$ and $(3, \infty)$. The solution set is $(-\infty, -2) \cup (3, \infty)$.

29.

$$x^2 - x - 5 \geq x - 2$$

$x^2 - 2x - 3 \geq 0$	Polynomial inequality
$x^2 - 2x - 3 = 0$	Related equation
$(x + 1)(x - 3) = 0$	Factoring

Using the principle of zero products, we find that the solutions of the related equation are -1 and 3. The numbers divide the x-axis into the intervals $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$. We let $f(x) = x^2 - 2x - 3$ and test a value in each interval.

$(-\infty, -1)$: $f(-2) = 5 > 0$

$(-1, 3)$: $f(0) = -3 < 0$

$(3, \infty)$: $f(4) = 5 > 0$

Function values are positive on $(-\infty, -1)$ and on $(3, \infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty, -1] \cup [3, \infty)$.

30.

$$x^2 + 4x + 7 \geq 5x + 9$$
$$x^2 - x - 2 \geq 0$$
$$(x - 2)(x + 1) \geq 0$$

The related equation is $(x - 2)(x + 1) = 0$. Its solutions are 2 and -1. These numbers divide the x-axis into the intervals $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$. We let $f(x) = (x - 2)(x + 1)$ and test a value in each interval.

$(-\infty, -1)$: $f(-2) = 4 > 0$

$(-1, 2)$: $f(0) = -2 < 0$

$(2, \infty)$: $f(3) = 4 > 0$

Function values are positive on $(-\infty, -1)$ and $(2, \infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty, -1] \cup [2, \infty)$.

31.

$x^2 > 25$	Polynomial inequality
$x^2 - 25 > 0$	Equivalent inequality with 0 on one side
$x^2 - 25 = 0$	Related equation
$(x + 5)(x - 5) = 0$	Factoring

Using the principle of zero products, we find that the solutions of the related equation are -5 and 5. These numbers divide the x-axis into the intervals $(-\infty, -5)$, $(-5, 5)$, and $(5, \infty)$. We let $f(x) = x^2 - 25$ and test a value in each interval.

$(-\infty, -5)$: $f(-6) = 11 > 0$

$(-5, 5)$: $f(0) = -25 < 0$

$(5, \infty)$: $f(6) = 11 > 0$

Function values are positive on $(-\infty, -5)$ and $(5, \infty)$. The solution set is $(-\infty, -5) \cup (5, \infty)$.

32.

$x^2 \leq 1$	Polynomial inequality
$x^2 - 1 \leq 0$	Equivalent inequality with 0 on one side
$x^2 - 1 = 0$	Related equation
$(x + 1)(x - 1) = 0$	Factoring

The solutions of the related equation are -1 and 1. These numbers divide the x-axis into the intervals $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$. We let $f(x) = x^2 - 1$ and test a value in each interval.

$(-\infty, -1)$: $f(-2) = 3 > 0$

$(-1, 1)$: $f(0) = -1 < 0$

$(1, \infty)$: $f(2) = 3 > 0$

Function values are negative only on $(-1, 1)$. Since the inequality symbol is $\leq$, the endpoints of the interval must be included in the solution set. It is $[-1, 1]$.

33.

$4 - x^2 \leq 0$	Polynomial inequality
$4 - x^2 = 0$	Related equation
$(2 + x)(2 - x) = 0$	Factoring

Using the principle of zero products, we find that the solutions of the related equation are -2 and 2. These numbers divide the x-axis into the intervals $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$. We let $f(x) = 4 - x^2$ and test a value in each interval.

$(-\infty, -2)$: $f(-3) = -5 < 0$

$(-2, 2)$: $f(0) = 4 > 0$

$(2, \infty)$: $f(3) = -5 < 0$

Function values are negative on $(-\infty, -2)$ and $(2, \infty)$. Since the inequality symbol is $\leq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty, -2] \cup [2, \infty)$.

34.

$11 - x^2 \geq 0$	Polynomial inequality
$11 - x^2 = 0$	Related equation

The solutions of the related equation are $\pm\sqrt{11}$. These numbers divide the x-axis into the intervals $(-\infty, -\sqrt{11})$, $(-\sqrt{11}, \sqrt{11})$, and $(\sqrt{11}, \infty)$. We let $f(x) = 11 - x^2$ and test a value in each interval.

$(-\infty, -\sqrt{11})$: $f(-4) = -5 < 0$

$(-\sqrt{11}, \sqrt{11})$: $f(0) = 11 > 0$

$(\sqrt{11}, \infty)$: $f(4) = -5 < 0$

Function values are positive only on $(-\sqrt{11}, \sqrt{11})$. Since the inequality symbol is $\geq$, the endpoints of the interval must be included in the solution set. It is $[-\sqrt{11}, \sqrt{11}]$.

35.

$6x - 9 - x^2 < 0$	Polynomial inequality
$6x - 9 - x^2 = 0$	Related equation
$-(x^2 - 6x + 9) = 0$	Factoring out -1 and rearranging
$-(x - 3)(x - 3) = 0$	Factoring

Using the principle of zero products, we find that the solution of the related equation is 3. This number divides the x-axis into the intervals $(-\infty, 3)$ and $(3, \infty)$. We let $f(x) = 6x - 9 - x^2$ and test a value in each interval.

$(-\infty, 3)$: $f(-4) = -49 < 0$

$(3, \infty)$: $f(4) = -1 < 0$

Function values are negative on both intervals. The solution set is $(-\infty, 3) \cup (3, \infty)$.

36.

$x^2 + 2x + 1 \leq 0$	Polynomial inequality
$x^2 + 2x + 1 = 0$	Related equation
$(x + 1)(x + 1) = 0$	Factoring

The solution of the related equation is -1. This number divides the x-axis into the intervals $(-\infty, -1)$, and $(-1, \infty)$. We let $f(x) = x^2 + 2x + 1$ and test a value in each interval.

$(-\infty, -1)$: $f(-2) = 1 > 0$

$(-1, \infty)$: $f(0) = 1 > 0$

Function values are negative in neither interval. The function is equal to 0 when $x = -1$. Thus, the solution set is $\{-1\}$.

37.

$x^2 + 12 < 4x$	Polynomial inequality
$x^2 - 4x + 12 < 0$	Equivalent inequality with 0 on one side
$x^2 - 4x + 12 = 0$	Related equation

Using the quadratic formula, we find that the related equation has no real-number solutions. The graph lies entirely above the x-axis, so the inequality has no solution. We could determine this algebraically by letting $f(x) = x^2 - 4x + 12$ and testing any real number (since there are no real-number solutions of $f(x) = 0$ to divide the x-axis into intervals). For example, $f(0) = 12 > 0$, so we see algebraically that the inequality has no solution. The solution set is $\emptyset$.

38.

$x^2 - 8 > 6x$ Polynomial inequality

$x^2 - 6x - 8 > 0$ Equivalent inequality with 0 on one side

$x^2 - 6x - 8 = 0$ Related equation

Using the quadratic formula, we find that the solutions of the related equation are $3 - \sqrt{17}$ and $3 + \sqrt{17}$. These numbers divide the x-axis into the intervals $(-\infty, 3-\sqrt{17})$, $(3 - \sqrt{17}, 3 + \sqrt{17})$, and $(3 + \sqrt{17}, \infty)$. We let $f(x) = x^2 - 6x - 8$ and test a value in each interval.

$(-\infty, 3 - \sqrt{17})$: $f(-2) = 8 > 0$

$(3 - \sqrt{17}, 3 + \sqrt{17})$: $f(0) = -8 < 0$

$(3 + \sqrt{17}, \infty)$: $f(8) = 8 > 0$

Function values are positive on $(-\infty, 3 - \sqrt{17})$ and $(3 + \sqrt{17}, \infty)$. The solution set is $(-\infty, 3 - \sqrt{17}) \cup (3 + \sqrt{17}, \infty)$ or approximately $(-\infty, -1.123) \cup (7.123, \infty)$.

39.

$4x^3 - 7x^2 \le 15x$ Polynomial inequality

$4x^3 - 7x^2 - 15x \le 0$ Equivalent inequality with 0 on one side

$4x^3 - 7x^2 - 15x = 0$ Related equation

$x(4x + 5)(x - 3) = 0$ Factoring

Using the principle of zero products, we find that the solutions of the related equation are 0, $-\frac{5}{4}$, and 3. These numbers divide the x-axis into the intervals $\left(-\infty, -\frac{5}{4}\right)$, $\left(-\frac{5}{4}, 0\right)$, $(0, 3)$, and $(3, \infty)$. We let $f(x) = 4x^3 - 7x^2 - 15x$ and test a value in each interval.

$\left(-\infty, -\frac{5}{4}\right)$: $f(-2) = -30 < 0$

$\left(-\frac{5}{4}, 0\right)$: $f(-1) = 4 > 0$

$(0, 3)$ $f(1) = -18 < 0$

$(3, \infty)$: $f(4) = 84 > 0$

Function values are negative on $\left(-\infty, -\frac{5}{4}\right)$ and $(0, 3)$. Since the inequality symbol is $\le$, the endpoints of the intervals must be included in the solution set. It is $\left(-\infty, -\frac{5}{4}\right] \cup [0, 3]$.

40.

$2x^3 - x^2 < 5x$ Polynomial inequality

$2x^3 - x^2 - 5x < 0$ Equivalent inequality with 0 on one side

$2x^3 - x^2 - 5x = 0$ Related equation

$x(2x^2 - x - 5) = 0$

$x = 0$ *or* $2x^2 - x - 5 = 0$

$x = 0$ *or* $x = \frac{1 \pm \sqrt{41}}{4}$

The solutions of the related equation divide the x-axis into four intervals. We let $f(x) = 2x^3 - x^2 - 5x$ and test a value in each interval. Note that $\frac{1 - \sqrt{41}}{4} \approx -1.3508$ and $\frac{1 + \sqrt{41}}{4} \approx 1.8508$.

$\left(-\infty, \frac{1 - \sqrt{41}}{4}\right)$: $f(-2) = -10 < 0$

$\left(\frac{1 - \sqrt{41}}{4}, 0\right)$: $f(-1) = 2 > 0$

$\left(0, \frac{1 + \sqrt{41}}{4}\right)$: $f(1) = -4 < 0$

$\left(\frac{1 + \sqrt{41}}{4}, \infty\right)$: $f(2) = 2 > 0$

Function values are negative on $\left(-\infty, \frac{1 - \sqrt{41}}{4}\right)$ and $\left(0, \frac{1 + \sqrt{41}}{4}\right)$. The solution set is $\left(-\infty, \frac{1 - \sqrt{41}}{4}\right) \cup \left(0, \frac{1 + \sqrt{41}}{4}\right)$, or approximately $(-\infty, -1.3508) \cup (0, 1.8508)$.

41.

$x^3 + 3x^2 - x - 3 \ge 0$ Polynomial inequality

$x^3 + 3x^2 - x - 3 = 0$ Related equation

$x^2(x + 3) - (x + 3) = 0$ Factoring

$(x^2 - 1)(x + 3) = 0$

$(x + 1)(x - 1)(x + 3) = 0$

Using the principle of zero products, we find that the solutions of the related equation are -1, 1, and -3. These numbers divide the x-axis into the intervals $(-\infty, -3)$, $(-3, -1)$, $(-1, 1)$, and $(1, \infty)$. We let $f(x) = x^3 + 3x^2 - x - 3$ and test a value in each interval.

$(-\infty, -3)$: $f(-4) = -15 < 0$

$(-3, -1)$: $f(-2) = 3 > 0$

$(-1, 1)$: $f(0) = -3 < 0$

$(1, \infty)$: $f(2) = 15 > 0$

Function values are positive on $(-3, -1)$ and $(1, \infty)$. Since the inequality symbol is $\ge$, the endpoints of the intervals must be included in the solution set. It is $[-3, -1] \cup [1, \infty)$.

42.

$x^3 + x^2 - 4x - 4 \ge 0$ Polynomial inequality

$x^3 + x^2 - 4x - 4 = 0$ Related equation

$(x + 2)(x - 2)(x + 1) = 0$ Factoring

The solutions of the related equation are -2, 2, and -1. These numbers divide the x-axis into the intervals $(-\infty, -2)$, $(-2, -1)$, $(-1, 2)$, and $(2, \infty)$. We let $f(x) = x^3 + x^2 - 4x - 4$ and test a value in each interval.

$(-\infty, -2)$: $f(-3) = -10 < 0$

$(-2, -1)$: $f(-1.5) = 0.875 > 0$

$(-1, 2)$: $f(0) = -4 < 0$

$(2, \infty)$: $f(3) = 20 > 0$

Function values are positive only on $(-2,-1)$ and $(2,\infty)$. Since the inequality symbol is $\geq$, the endpoints of the interval must be included in the solution set. It is $[-2,-1]\cup[2,\infty)$.

43.

$x^3-2x^2<5x-6$	Polynomial inequality
$x^3-2x^2-5x+6<0$	Equivalent inequality with 0 on one side
$x^3-2x^2-5x+6=0$	Related equation

Using the techniques of Section 3.3, we find that the solutions of the related equation are -2, 1, and 3. They divide the x-axis into the intervals $(-\infty,-2)$, $(-2,1)$, $(1,3)$, and $(3,\infty)$. Let $f(x)=x^3-2x^2-5x+6$ and test a value in each interval.

$(-\infty,-2)$: $f(-3)=-24<0$

$(-2,1)$: $f(0)=6>0$

$(1,3)$: $f(2)=-4<0$

$(3,\infty)$: $f(4)=18>0$

Function values are negative on $(-\infty,-2)$ and $(1,3)$. The solution set is $(-\infty,-2)\cup(1,3)$.

44.

$x^3+x\leq 6-4x^2$	Polynomial inequality
$x^3+4x^2+x-6\leq 0$	Equivalent inequality with 0 on one side
$x^3+4x^2+x-6=0$	Related equation

Using the techniques of Section 3.3, we find that the solutions of the related equation are -3, -2, and 1. They divide the x-axis into the intervals $(-\infty,-3)$, $(-3,-2)$, $(-2,1)$, and $(1,\infty)$. Let $f(x)=x^3+4x^2+x-6$ and test a value in each interval.

$(-\infty,-3)$: $f(-4)=-10<0$

$(-3,-2)$: $f(-2.5)=0.875>0$

$(-2,1)$: $f(0)=-6<0$

$(1,\infty)$: $f(2)=20>0$

Function values are negative on $(-\infty,-3)$ and $(-2,1)$. Since the inequality symbol is $\leq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty,-3]\cup[-2,1]$.

45.

$x^5+x^2\geq 2x^3+2$	Polynomial inequality
$x^5-2x^3+x^2-2\geq 0$	Related inequality with 0 on one side
$x^5-2x^3+x^2-2=0$	Related equation
$x^3(x^2-2)+x^2-2=0$	Factoring
$(x^3+1)(x^2-2)=0$	

Using the principle of zero products, we find that the real-number solutions of the related equation are -1, $-\sqrt{2}$, and $\sqrt{2}$. These numbers divide the x-axis into the intervals $(-\infty,-\sqrt{2})$, $(-\sqrt{2},-1)$, $(-1,\sqrt{2})$, and $(\sqrt{2},\infty)$. We let $f(x)=x^5-2x^3+x^2-2$ and test a value in each interval.

$(-\infty,-\sqrt{2})$: $f(-2)=-14<0$

$(-\sqrt{2},-1)$: $f(-1.3)\approx 0.37107>0$

$(-1,\sqrt{2})$: $f(0)=-2<0$

$(\sqrt{2},\infty)$: $f(2)=18>0$

Function values are positive on $(-\sqrt{2},-1)$ and $(\sqrt{2},\infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $[-\sqrt{2},-1]\cup[\sqrt{2},\infty)$.

46.

$x^5+24>3x^3+8x^2$	Polynomial inequality
$x^5-3x^3-8x^2+24>0$	Equivalent inequality with 0 on one side
$x^5-3x^3-8x^2+24=0$	Related equation
$x^3(x^2-3)-8(x^2-3)=0$	Factoring
$(x^3-8)(x^2-3)=0$	

Using the principle of zero products, we find that the real-number solutions of the related equation are 2, $-\sqrt{3}$, and $\sqrt{3}$. These numbers divide the x-axis into the intervals $(-\infty,-\sqrt{3})$, $(-\sqrt{3},\sqrt{3})$, $(\sqrt{3},2)$, and $(2,\infty)$. We let $f(x)=x^5-3x^3-8x^2+24$ and test a value in each interval.

$(-\infty,-\sqrt{3})$: $f(-2)=-16<0$

$(-\sqrt{3},\sqrt{3})$: $f(0)=24>0$

$(\sqrt{3},2)$: $f(1.8)\approx -0.5203<0$

$(2,\infty)$: $f(3)=114>0$

Function values are positive on $(-\sqrt{3},\sqrt{3})$ and $(2,\infty)$. The solution set is $(-\sqrt{3},\sqrt{3})\cup(2,\infty)$.

47.

$2x^3+6\leq 5x^2+x$	Polynomial inequality
$2x^3-5x^2-x+6\leq 0$	Equivalent inequality with 0 on one side
$2x^3-5x^2-x+6=0$	Related equation

Using the techniques of Section 3.3, we find that the solutions of the related equation are -1, $\frac{3}{2}$, and 2. We can also use the graph of $y=2x^3-5x^2-x+6$ to find these solutions. They divide the x-axis into the intervals $(-\infty,-1)$, $\left(-1,\frac{3}{2}\right)$, $\left(\frac{3}{2},2\right)$, and $(2,\infty)$. Let $f(x)=2x^3-5x^2-x+6$ and test a value in each interval.

$(-\infty,-1)$: $f(-2)=-28<0$

$\left(-1,\frac{3}{2}\right)$: $f(0)=6>0$

$\left(\frac{3}{2},2\right)$: $f(1.6)=-0.208<0$

$(2,\infty)$: $f(3)=12>0$

Function values are negative in $(-\infty,-1)$ and $\left(\frac{3}{2},2\right)$. Since the inequality symbol is $\leq$, the endpoints of the intervals must be included in the solution set. The solution set is $(-\infty,-1]\cup\left[\frac{3}{2},2\right]$.

48.

$$\begin{array}{ll} 2x^3+x^2 < 10+11x & \text{Polynomial inequality} \\ 2x^3+x^2-11x-10 < 0 & \text{Equivalent inequality with 0 on one side} \\ 2x^3+x^2-11x-10 = 0 & \text{Related equation} \end{array}$$

Using the techniques of Section 3.3, we find that the real-number solutions of the related equation are -2, -1, and $\frac{5}{2}$. These numbers divide the x-axis into the intervals $(-\infty,-2)$, $(-2,-1)$, $\left(-1,\frac{5}{2}\right)$, and $\left(\frac{5}{2},\infty\right)$. We let $f(x)=2x^3+x^2-11x-10$ and test a value in each interval.

$(-\infty,-2)$: $f(-3)=-22<0$

$(-2,-1)$: $f(-1.5)=2>0$

$\left(-1,\frac{5}{2}\right)$: $f(0)=-10<0$

$\left(\frac{5}{2},\infty\right)$: $f(3)=20>0$

Function values are negative on $(-\infty,-2)$ and $\left(-1,\frac{5}{2}\right)$. The solution set is $(-\infty,-2)\cup\left(-1,\frac{5}{2}\right)$.

49.

$$\begin{array}{ll} x^3+5x^2-25x \le 125 & \text{Polynomial inequality} \\ x^3+5x^2-25x-125 \le 0 & \text{Equivalent inequality with 0 on one side} \\ x^3+5x^2-25x-125 = 0 & \text{Related equation} \\ x^2(x+5)-25(x+5)=0 & \text{Factoring} \\ (x^2-25)(x+5)=0 & \\ (x+5)(x-5)(x+5)=0 & \end{array}$$

Using the principle of zero products, we find that the solutions of the related equation are -5 and 5. These numbers divide the x-axis into the intervals $(-\infty,-5)$, $(-5,5)$, and $(5,\infty)$. We let $f(x)=x^3+5x^2-25x-125$ and test a value in each interval.

$(-\infty,-5)$: $f(-6)=-11<0$

$(-5,5)$: $f(0)=-125<0$

$(5,\infty)$: $f(6)=121>0$

Function values are negative on $(-\infty,-5)$ and $(-5,5)$. Since the inequality symbol is $\le$, the endpoints of the intervals must be included in the solution set. It is $(-\infty,-5]\cup[-5,5]$ or $(-\infty,5]$.

50.

$$\begin{array}{ll} x^3-9x+27 \ge 3x^2 & \text{Polynomial inequality} \\ x^3-3x^2-9x+27 \ge 0 & \text{Equivalent inequality with 0 on one side} \\ x^3-3x^2-9x+27 = 0 & \text{Related equation} \\ x^2(x-3)-9(x-3)=0 & \text{Factoring} \\ (x^2-9)(x-3)=0 & \\ (x+3)(x-3)(x-3)=0 & \end{array}$$

The solutions of the related equation are -3 and 3. These numbers divide the x-axis into the intervals $(-\infty,-3)$, $(-3,3)$, and $(3,\infty)$. We let $f(x)=x^3-3x^2-9x+27$ and test a value in each interval.

$(-\infty,-3)$: $f(-4)=-49<0$

$(-3,3)$: $f(0)=27>0$

$(3,\infty)$: $f(4)=7>0$

Function values are positive only on $(-3,3)$ and $(3,\infty)$. Since the inequality symbol is $\ge$, the endpoints of the intervals must be included in the solution set. It is $[-3,3]\cup[3,\infty)$, or $[-3,\infty)$.

51.

$$\begin{array}{ll} 0.1x^3-0.6x^2-0.1x+2<0 & \text{Polynomial inequality} \\ 0.1x^3-0.6x^2-0.1x+2=0 & \text{Related equation} \end{array}$$

After trying all the possibilities, we find that the related equation has no rational zeros. Using the graph of $y=0.1x^3-0.6x^2-0.1x+2$, we find that the only real-number solutions of the related equation are approximately -1.680, 2.154, and 5.526. These numbers divide the x-axis into the intervals $(-\infty,-1.680)$, $(-1.680,2.154)$, $(2.154,5.526)$, and $(5.526,\infty)$. We let $f(x)=0.1x^3-0.6x^2-0.1x+2$ and test a value in each interval.

$(-\infty,-1.680)$: $f(-2)=-1<0$

$(-1.680,2.154)$: $f(0)=2>0$

$(2.154,5.526)$: $f(3)=-1<0$

$(5.526,\infty)$: $f(6)=1.4>0$

Function values are negative on $(-\infty,-1.680)$ and $(2.154,5.526)$. The graph can also be used to determine this. The solution set is $(-\infty,-1.680)\cup(2.154,5.526)$.

52.

$$19.2x^3+12.8x^2+144 \ge 172.8x+3.2x^4$$

$$-3.2x^4+19.2x^3+12.8x^2-172.8x+144 \ge 0$$

$$-3.2x^4+19.2x^3+12.8x^2-172.8x+144 = 0$$

Related equation

The solutions of the related equation are -3, 1, 3, and 5. These numbers divide the x-axis into the intervals $(-\infty,-3)$, $(-3,1)$, $(1,3)$, $(3,5)$, and $(5,\infty)$. We let $f(x)=-3.2x^4+19.2x^3+12.8x^2-172.8x+144$ and test a value in each interval.

$(-\infty,-3)$: $f(-4)=-1008<0$

$(-3,1)$: $f(0)=144>0$

$(1,3)$: $f(2)=-48<0$

$(3,5)$: $f(4)=67.2>0$

$(5, \infty)$: $f(6) = -432 < 0$

Function values are positive only on $(-3, 1)$ and $(3, 5)$. The graph of $y = -3.2x^4 + 19.2x^3 + 12.8x^2 - 172.8x + 144$ can also be used to determine this. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $[-3, 1] \cup [3, 5]$.

53. $\frac{1}{x+4} > 0$ Rational inequality

$\frac{1}{x+4} = 0$ Related equation

The denominator of $f(x) = \frac{1}{x+4}$ is 0 when $x = -4$, so the function is not defined for $x = -4$. The related equation has no solution. Thus, the only critical value is -4. It divides the x-axis into the intervals $(-\infty, -4)$ and $(-4, \infty)$. We test a value in each interval.

$(-\infty, -4)$: $f(-5) = -1 < 0$

$(-4, \infty)$: $f(0) = \frac{1}{4} > 0$

Function values are positive on $(-4, \infty)$. This can also be determined from the graph of $y = \frac{1}{x+4}$. The solution set is $(-4, \infty)$.

54. $\frac{1}{x-3} \leq 0$ Rational inequality

$\frac{1}{x-3} = 0$ Related equation

The denominator of $f(x) = \frac{1}{x-3}$ is 0 when $x = 3$, so the function is not defined for $x = 3$. The related equation has no solution. Thus, the only critical value is 3. It divides the x-axis into the intervals $(-\infty, 3)$ and $(3, \infty)$. We test a value in each interval.

$(-\infty, 3)$: $f(0) = -\frac{1}{3} < 0$

$(3, \infty)$: $f(4) = 1 > 0$

Function values are negative on $(-\infty, 3)$. Note that since 3 is not in the domain of $f(x)$, it cannot be included in the solution set. It is $(-\infty, 3)$.

55. $\frac{-4}{2x+5} < 0$ Rational inequality

$\frac{-4}{2x+5} = 0$ Related equation

The denominator of $f(x) = \frac{-4}{2x+5}$ is 0 when $x = -\frac{5}{2}$, so the function is not defined for $x = -\frac{5}{2}$. The related equation has no solution. Thus, the only critical value is $-\frac{5}{2}$. It divides the x-axis into the intervals $\left(-\infty, -\frac{5}{2}\right)$ and $\left(-\frac{5}{2}, \infty\right)$. We test a value in each interval.

$\left(-\infty, -\frac{5}{2}\right)$: $f(-3) = 4 > 0$

$\left(-\frac{5}{2}, \infty\right)$: $f(0) = -\frac{4}{5} < 0$

Function values are negative on $\left(-\frac{5}{2}, \infty\right)$. The solution set is $\left(-\frac{5}{2}, \infty\right)$.

56. $\frac{-2}{5-x} \geq 0$ Rational inequality

$\frac{-2}{5-x} = 0$ Related equation

The denominator of $f(x) = \frac{-2}{5-x}$ is 0 when $x = 5$, so the function is not defined for $x = 5$. The related equation has no solution. Thus, the only critical value is 5. It divides the x-axis into the intervals $(-\infty, 5)$ and $(5, \infty)$. We test a value in each interval.

$(-\infty, 5)$: $f(0) = -\frac{2}{5} < 0$

$(5, \infty)$: $f(6) = 2 > 0$

Function values are positive on $(5, \infty)$. Note that since 5 is not in the domain of $f(x)$, it cannot be included in the solution set. It is $(5, \infty)$.

57. $\frac{2x}{x-4} \geq 0$ Rational inequality

$\frac{2x}{x-4} = 0$ Related equation

The denominator of $f(x) = \frac{2x}{x-4}$ is 0 when $x = 4$, so the function is not defined for $x = 4$.

We solve the related equation $f(x) = 0$.

$$\frac{2x}{x-4} = 0$$

$$2x = 0 \quad \text{Multiplying by } x-4$$

$$x = 0$$

The critical values are 0 and 4. They divide the x-axis into the intervals $(-\infty, 0)$, $(0, 4)$, and $(4, \infty)$. We test a value in each interval.

$(-\infty, 0)$: $f(-1) = \frac{2}{5} > 0$

$(0, 4)$: $f(1) = -\frac{2}{3} < 0$

$(4, \infty)$: $f(5) = 10 > 0$

Function values are positive on $(-\infty, 0)$ and $(4, \infty)$. Since the inequality symbol is $\geq$ and $f(0) = 0$, then 0 must be included in the solution set. And since 4 is not in the domain of $f(x)$, 4 is not included in the solution set. It is $(-\infty, 0] \cup (4, \infty)$.

58. $\frac{5x}{x+1} < 0$

The denominator of $f(x) = \frac{5x}{x+1}$ is 0 when $x = -1$, so the function is not defined for $x = -1$. We solve the related equation $f(x) = 0$.

$$\frac{5x}{x+1} = 0$$

$$5x = 0 \quad \text{Multiplying by } x+1$$

$$x = 0$$

The critical values are -1 and 0. They divide the x-axis into the intervals $(-\infty, -1)$, $(-1, 0)$, and $(0, \infty)$. We test a value in each interval.

$(-\infty, -1)$: $f(-2) = 10 > 0$

$(-1, 0)$: $f\left(-\frac{1}{2}\right) = -5 < 0$

$(0, \infty)$: $f(1) = \frac{5}{2} > 0$

Function values are negative on $(-1, 0)$. The solution set is $(-1, 0)$.

59. $\frac{x-4}{x+3} - \frac{x+2}{x-1} \le 0$

The denominator of $f(x) = \frac{x-4}{x+3} - \frac{x+2}{x-1}$ is 0 when $x = -3$ or $x = 1$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\begin{aligned}
\frac{x-4}{x+3} - \frac{x+2}{x-1} &= 0\\
(x+3)(x-1)\left(\frac{x-4}{x+3} - \frac{x+2}{x-1}\right) &= (x+3)(x-1)\cdot 0\\
(x-1)(x-4) - (x+3)(x+2) &= 0\\
x^2 - 5x + 4 - (x^2 + 5x + 6) &= 0\\
-10x - 2 &= 0\\
-10x &= 2\\
x &= -\frac{1}{5}
\end{aligned}$$

The critical values are -3, $-\frac{1}{5}$, and 1. They divide the x-axis into the intervals $(-\infty, -3)$, $\left(-3, -\frac{1}{5}\right)$, $\left(-\frac{1}{5}, 1\right)$, and $(1, \infty)$. We test a value in each interval.

$(-\infty, -3)$: $f(-4) = 7.6 > 0$

$\left(-3, -\frac{1}{5}\right)$: $f(-1) = -2 < 0$

$\left(-\frac{1}{5}, 1\right)$: $f(0) = \frac{2}{3} > 0$

$(1, \infty)$: $f(2) = -4.4 < 0$

Function values are negative on $\left(-3, -\frac{1}{5}\right)$ and $(1, \infty)$. Note that since the inequality symbol is $\le$ and $f\left(-\frac{1}{5}\right) = 0$, then $-\frac{1}{5}$ must be included in the solution set. Note also that since neither -3 nor 1 is in the domain of $f(x)$, they are not included in the solution set. It is $\left(-3, -\frac{1}{5}\right] \cup (1, \infty)$.

60. $\frac{x+1}{x-2} - \frac{x-3}{x-1} < 0$

The denominator of $f(x) = \frac{x+1}{x-2} - \frac{x-3}{x-1}$ is 0 when $x = 2$ or $x = 1$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\begin{aligned}
\frac{x+1}{x-2} - \frac{x-3}{x-1} &= 0\\
(x-1)(x+1) - (x-2)(x-3) &= 0\\
&\text{Multiplying by } (x-2)(x-1)\\
x^2 - 1 - x^2 + 5x - 6 &= 0\\
5x - 7 &= 0\\
5x &= 7\\
x &= \frac{7}{5}
\end{aligned}$$

The critical values are 1, $\frac{7}{5}$, and 2. They divide the x-axis into the intervals $(-\infty, 1)$, $\left(1, \frac{7}{5}\right)$, $\left(\frac{7}{5}, 2\right)$, and $(2, \infty)$. We test a value in each interval.

$(-\infty, 1)$: $f(0) = -3.5 < 0$

$\left(1, \frac{7}{5}\right)$: $f\left(\frac{6}{5}\right) = 6.25 > 0$

$\left(\frac{7}{5}, 2\right)$: $f\left(\frac{9}{5}\right) = -12.5 < 0$

$(2, \infty)$: $f(3) = 4 > 0$

Function values are negative on $(-\infty, 1,)$ and $\left(\frac{7}{5}, 2\right)$. The solution set is $(-\infty, 1) \cup \left(\frac{7}{5}, 2\right)$.

61. $\frac{x+6}{x-2} > \frac{x-8}{x-5}$ Rational inequality

$\frac{x+6}{x-2} - \frac{x-8}{x-5} > 0$ Equivalent inequality with 0 on one side

The denominator of $f(x) = \frac{x+6}{x-2} - \frac{x-8}{x-5}$ is 0 when $x = 2$ or $x = 5$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\begin{aligned}
\frac{x+6}{x-2} - \frac{x-8}{x-5} &= 0\\
(x-2)(x-5)\left(\frac{x+6}{x-2} - \frac{x-8}{x-5}\right) &= (x-2)(x-5)\cdot 0\\
(x-5)(x+6) - (x-2)(x-8) &= 0\\
x^2 + x - 30 - (x^2 - 10x + 16) &= 0\\
x^2 + x - 30 - x^2 + 10x - 16 &= 0\\
11x - 46 &= 0\\
11x &= 46\\
x &= \frac{46}{11}
\end{aligned}$$

The critical values are 2, $\frac{46}{11}$, and 5. They divide the x-axis into the intervals $(-\infty, 2)$, $\left(2, \frac{46}{11}\right)$, $\left(\frac{46}{11}, 5\right)$, and $(5, \infty)$. We test a value in each interval.

$(-\infty, 2)$: $f(0) = -4.6 < 0$

$\left(2, \frac{46}{11}\right)$: $f(4) = 1 > 0$

$\left(\frac{46}{11}, 5\right)$: $f(4.5) = -2.8 < 0$

$(5, \infty)$: $f(6) = 5 > 0$

Function values are positive on $\left(2, \frac{46}{11}\right)$ and $(5, \infty)$. The solution set is $\left(2, \frac{46}{11}\right) \cup (5, \infty)$.

62. $\frac{x-7}{x+2} \geq \frac{x-9}{x+3}$

$\frac{x-7}{x+2} - \frac{x-9}{x+3} \geq 0$

The denominator of $f(x) = \frac{x-7}{x+2} - \frac{x-9}{x+3}$ is 0 when $x = -2$ or $x = -3$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{x-7}{x+2} - \frac{x-9}{x+3} = 0$$

$$(x+3)(x-7) - (x+2)(x-9) = 0$$

Multiplying by $(x+2)(x+3)$

$$x^2 - 4x - 21 - (x^2 - 7x - 18) = 0$$
$$x^2 - 4x - 21 - x^2 + 7x + 18 = 0$$
$$3x - 3 = 0$$
$$3x = 3$$
$$x = 1$$

The critical values are -3, -2, and 1. They divide the x-axis into the intervals $(-\infty, -3)$, $(-3, -2)$, $(-2, 1)$, and $(1, \infty)$. We test a value in each interval.

$(-\infty, -3)$: $f(-4) = -7.5 < 0$

$(-3, -2)$: $f(-2.5) = 42 > 0$

$(-2, 1)$: $f(0) = -0.5 < 0$

$(1, \infty)$: $f(2) = 0.15 > 0$

Function values are positive on $(-3, -2)$ and $(1, \infty)$. Note that since the inequality symbol is ≥ 0 and $f(1) = 0$, then 1 must be included in the solution set. The solution set is $(-3, -2) \cup [1, \infty)$.

63. $\frac{x+1}{x-2} \geq 3$ Rational inequality

$\frac{x+1}{x-2} - 3 \geq 0$ Equivalent inequality with 0 on one side

The denominator of $f(x) = \frac{x+1}{x-2} - 3$ is 0 when $x = 2$, so the function is not defined for this value of x. We solve the related equation $f(x) = 0$.

$$\frac{x+1}{x-2} - 3 = 0$$
$$(x-2)\left(\frac{x+1}{x-2} - 3\right) = (x-2) \cdot 0$$
$$x + 1 - 3(x-2) = 0$$
$$x + 1 - 3x + 6 = 0$$
$$-2x + 7 = 0$$
$$-2x = -7$$
$$x = \frac{7}{2}$$

The critical values are 2 and $\frac{7}{2}$. They divide the x-axis into the intervals $(-\infty, 2)$, $\left(2, \frac{7}{2}\right)$, and $\left(\frac{7}{2}, \infty\right)$. We test a value in each interval.

$(-\infty, 2)$: $f(0) = -3.5 < 0$

$\left(2, \frac{7}{2}\right)$: $f(3) = 1 > 0$

$\left(\frac{7}{2}, \infty\right)$: $f(4) = -0.5 < 0$

Function values are positive on $\left(2, \frac{7}{2}\right)$. Note that since the inequality symbol is $\geq$ and $f\left(\frac{7}{2}\right) = 0$, then $\frac{7}{2}$ must be included in the solution set. Note also that since 2 is not in the domain of $f(x)$, it is not included in the solution set. It is $\left(2, \frac{7}{2}\right]$.

64. $\frac{x}{x-5} < 2$ Rational inequality

$\frac{x}{x-5} - 2 < 0$ Equivalent inequality with 0 on one side

The denominator of $f(x) = \frac{x}{x-5} - 2$ is 0 when $x = 5$, so the function is not defined for this value of x. We solve the related equation $f(x) = 0$.

$$\frac{x}{x-5} - 2 = 0$$

$x - 2(x-5) = 0$ Multiplying by $x - 5$

$$x - 2x + 10 = 0$$
$$-x + 10 = 0$$
$$x = 10$$

The critical values are 5 and 10. They divide the x-axis into the intervals $(-\infty, 5)$, $(5, 10)$, and $(10, \infty)$. We test a value in each interval.

$(-\infty, 5)$: $f(0) = -2 < 0$

$(5, 10)$: $f(6) = 4 > 0$

$(10, \infty)$: $f(11) = -\frac{1}{6} < 0$

Function values are negative on $(-\infty, 5)$ and $(10, \infty)$. The solution set is $(-\infty, 5) \cup (10, \infty)$.

65. $x - 2 > \frac{1}{x}$ Rational inequality

$x - 2 - \frac{1}{x} > 0$ Equivalent inequality with 0 on one side

The denominator of $f(x) = x - 2 - \frac{1}{x}$ is 0 when $x = 0$, so the function is not defined for this value of x. We solve the related equation $f(x) = 0$.

$$x - 2 - \frac{1}{x} = 0$$
$$x\left(x - 2 - \frac{1}{x}\right) = x \cdot 0$$
$$x^2 - 2x - x \cdot \frac{1}{x} = 0$$
$$x^2 - 2x - 1 = 0$$

Using the quadratic formula we find that $x = 1 \pm \sqrt{2}$. The critical values are $1 - \sqrt{2}$, 0, and $1 + \sqrt{2}$. They divide the x-axis into the intervals $(-\infty, 1 - \sqrt{2})$, $(1 - \sqrt{2}, 0)$, $(0, 1 + \sqrt{2})$, and $(1 + \sqrt{2}, \infty)$. We test a value in each interval.

$(-\infty, 1 - \sqrt{2})$: $f(-1) = -2 < 0$

$(1 - \sqrt{2}, 0)$: $f(-0.1) = 7.9 > 0$

$(0, 1 + \sqrt{2})$: $f(1) = -2 < 0$

$(1 + \sqrt{2}, \infty)$: $f(3) = \frac{2}{3} > 0$

Function values are positive on $(1 - \sqrt{2}, 0)$ and $(1 + \sqrt{2}, \infty)$. The solution set is $(1 - \sqrt{2}, 0) \cup (1 + \sqrt{2}, \infty)$.

66. $4 \geq \frac{4}{x} + x$ Rational inequality

$4 - \frac{4}{x} - x \geq 0$ Equivalent inequality with 0 on one side

The denominator of $f(x) = 4 - \frac{4}{x} - x$ is 0 when $x = 0$, so the function is not defined for this value of x. We solve the related equation $f(x) = 0$.

$$4 - \frac{4}{x} - x = 0$$
$$4x - 4 - x^2 = 0 \quad \text{Multiplying by } x$$
$$-(x - 2)^2 = 0$$
$$x = 2$$

The critical values are 0 and 2. They divide the x-axis into the intervals $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$. We test a value in each interval.

$(-\infty, 0)$: $f(-1) = 9 > 0$

$(0, 2)$: $f(1) = -1 < 0$

$(2, \infty)$: $f(3) = -\frac{1}{3} < 0$

Function values are positive on $(-\infty, 0)$. Note that since the inequality symbol is $\geq$ and $f(2) = 0$, then 2 must be included in the solution set. Note also that since 0 is not in the domain of $f(x)$, it is not included in the solution set. It is $(-\infty, 0) \cup \{2\}$.

67.
$$\frac{2}{x^2 - 4x + 3} \leq \frac{5}{x^2 - 9}$$
$$\frac{2}{x^2 - 4x + 3} - \frac{5}{x^2 - 9} \leq 0$$
$$\frac{2}{(x - 1)(x - 3)} - \frac{5}{(x + 3)(x - 3)} \leq 0$$

The denominator of $f(x) = \frac{2}{(x - 1)(x - 3)} - \frac{5}{(x + 3)(x - 3)}$ is 0 when $x = 1$, 3, or -3, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{2}{(x - 1)(x - 3)} - \frac{5}{(x + 3)(x - 3)} = 0$$
$$(x-1)(x-3)(x+3)\left(\frac{2}{(x-1)(x-3)} - \frac{5}{(x+3)(x-3)}\right) = (x - 1)(x - 3)(x + 3) \cdot 0$$
$$2(x + 3) - 5(x - 1) = 0$$
$$2x + 6 - 5x + 5 = 0$$
$$-3x + 11 = 0$$
$$-3x = -11$$
$$x = \frac{11}{3}$$

The critical values are -3, 1, 3, and $\frac{11}{3}$. They divide the x-axis into the intervals $(-\infty, -3)$, $(-3, 1)$, $(1, 3)$, $\left(3, \frac{11}{3}\right)$, and $\left(\frac{11}{3}, \infty\right)$. We test a value in each interval.

$(-\infty, -3)$: $f(-4) \approx -0.6571 < 0$

$(-3, 1)$: $f(0) \approx 1.2222 > 0$

$(1, 3)$: $f(2) = -1 < 0$

$\left(3, \frac{11}{3}\right)$: $f(3.5) \approx 0.6154 > 0$

$\left(\frac{11}{3}, \infty\right)$: $f(4) \approx -0.0476 < 0$

Function values are negative on $(-\infty, -3)$, $(1, 3)$, and $\left(\frac{11}{3}, \infty\right)$. Note that since the inequality symbol is $\leq$ and $f\left(\frac{11}{3}\right) = 0$, then $\frac{11}{3}$ must be included in the solution set. Note also that since -3, 1, and 3 are not in the domain of $f(x)$, they are not included in the solution set. It is $(-\infty, -3) \cup (1, 3) \cup \left[\frac{11}{3}, \infty\right)$.

68.
$$\frac{3}{x^2 - 4} \leq \frac{5}{x^2 + 7x + 10}$$
$$\frac{3}{(x + 2)(x - 2)} - \frac{5}{(x + 2)(x + 5)} \leq 0$$

The denominator of $f(x) = \frac{3}{(x + 2)(x - 2)} - \frac{5}{(x + 2)(x + 5)}$ is 0 when $x = -2$, 2, or -5, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{3}{(x+2)(x-2)} - \frac{5}{(x+2)(x+5)} = 0$$
$$3(x + 5) - 5(x - 2) = 0 \quad \text{Multiplying by } (x+2)(x-2)(x+5)$$
$$3x + 15 - 5x + 10 = 0$$
$$-2x + 25 = 0$$
$$x = \frac{25}{2}$$

The critical values are -5, -2, 2, and $\frac{25}{2}$. They divide the x-axis into the intervals $(-\infty, -5)$, $(-5, -2)$, $(-2, 2)$, $\left(2, \frac{25}{2}\right)$, and $\left(\frac{25}{2}, \infty\right)$. We test a value in each interval.

$(-\infty, -5)$: $f(-6) \approx -1.156 < 0$

$(-5, -2)$: $f(-3) = 3.1 > 0$

$(-2, 2)$: $f(0) = -1.25 < 0$

$\left(2, \frac{25}{2}\right)$: $f(3) = 0.475 > 0$

$\left(\frac{25}{2}, \infty\right)$: $f(13) \approx -0.0003 < 0$

Function values are negative on $(-\infty, -5)$, $(-2, 2)$, and $\left(\frac{25}{2}, \infty\right)$. Note that since the inequality symbol is $\leq$ and $f\left(\frac{25}{2}\right) = 0$, then $\frac{25}{2}$ must be included in the solution set. Note also that since -5, -2, and 2 are not in the domain of $f(x)$, they are not included in the solution set. It is $(-\infty, -5) \cup (-2, 2) \cup \left[\frac{25}{2}, \infty\right)$.

69.
$$\frac{3}{x^2+1} \geq \frac{6}{5x^2+2}$$
$$\frac{3}{x^2+1} - \frac{6}{5x^2+2} \geq 0$$

The denominator of $f(x) = \frac{3}{x^2+1} - \frac{6}{5x^2+2}$ has no real-number zeros. We solve the related equation $f(x) = 0$.

$$\frac{3}{x^2+1} - \frac{6}{5x^2+2} = 0$$
$$(x^2+1)(5x^2+2)\left(\frac{3}{x^2+1} - \frac{6}{5x^2+2}\right) = (x^2+1)(5x^2+2) \cdot 0$$
$$3(5x^2+2) - 6(x^2+1) = 0$$
$$15x^2 + 6 - 6x^2 - 6 = 0$$
$$9x^2 = 0$$
$$x^2 = 0$$
$$x = 0$$

The only critical value is 0. It divides the x-axis into the intervals $(-\infty, 0)$ and $(0, \infty)$. We test a value in each interval.

$(-\infty, 0)$: $f(-1) \approx 0.64286 > 0$

$(0, \infty)$: $f(1) \approx 0.64286 > 0$

Function values are positive on both intervals. Note that since the inequality symbol is $\geq$ and $f(0) = 0$, then 0 must be included in the solution set. It is $(-\infty, 0] \cup [0, \infty)$, or $(-\infty, \infty)$.

70.
$$\frac{4}{x^2-9} < \frac{3}{x^2-25}$$
$$\frac{4}{x^2-9} - \frac{3}{x^2-25} < 0$$
$$\frac{4}{(x+3)(x-3)} - \frac{3}{(x+5)(x-5)} < 0$$

The denominator of $f(x) = \frac{4}{(x+3)(x-3)} - \frac{3}{(x+5)(x-5)}$ is 0 when $x = -3$, 3, -5, or 5, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{4}{(x+3)(x-3)} - \frac{3}{(x+5)(x-5)} = 0$$
$$4(x+5)(x-5) - 3(x+3)(x-3) = 0$$

Multiplying by $(x+3)(x-3)(x+5)(x-5)$

$$4x^2 - 100 - 3x^2 + 27 = 0$$
$$x^2 - 73 = 0$$
$$x = \pm\sqrt{73}$$

The critical values are $-\sqrt{73}$, -5, -3, 3, 5, and $\sqrt{73}$. They divide the x-axis into the intervals $(-\infty, -\sqrt{73})$, $(-\sqrt{73}, -5)$, $(-5, -3)$, $(-3, 3)$, $(3, 5)$, $(5, \sqrt{73})$, and $(\sqrt{73}, \infty)$. We test a value in each interval.

$(-\infty, -\sqrt{73})$: $f(-9) \approx 0.00198 > 0$

$(-\sqrt{73}, -5)$: $f(-6) \approx -0.1246 < 0$

$(-5, -3)$: $f(-4) \approx 0.90476 > 0$

$(-3, 3)$: $f(0) \approx -0.3244 < 0$

$(3, 5)$: $f(4) \approx 0.90476 > 0$

$(5, \sqrt{73})$: $f(6) \approx -0.1246 < 0$

$(\sqrt{73}, \infty)$: $f(9) \approx 0.00198 > 0$

Function values are negative on $(-\sqrt{73}, -5)$, $(-3, 3)$, and $(5, \sqrt{73})$. The solution set is $(-\sqrt{73}, -5) \cup (-3, 3) \cup (5, \sqrt{73})$.

71.
$$\frac{5}{x^2+3x} < \frac{3}{2x+1}$$
$$\frac{5}{x^2+3x} - \frac{3}{2x+1} < 0$$
$$\frac{5}{x(x+3)} - \frac{3}{2x+1} < 0$$

The denominator of $f(x) = \frac{5}{x(x+3)} - \frac{3}{2x+1}$ is 0 when $x = 0$, -3, or $-\frac{1}{2}$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{5}{x(x+3)} - \frac{3}{2x+1} = 0$$
$$x(x+3)(2x+1)\left(\frac{5}{x(x+3)} - \frac{3}{2x+1}\right) = x(x+3)(2x+1) \cdot 0$$
$$5(2x+1) - 3x(x+3) = 0$$
$$10x + 5 - 3x^2 - 9x = 0$$
$$-3x^2 + x + 5 = 0$$

Using the quadratic formula we find that $x = \frac{1 \pm \sqrt{61}}{6}$. The critical values are -3, $\frac{1-\sqrt{61}}{6}$, $-\frac{1}{2}$, 0, and $\frac{1+\sqrt{61}}{6}$. They divide the x-axis into the intervals $(-\infty, -3)$, $\left(-3, \frac{1-\sqrt{61}}{6}\right)$,

$\left(\frac{1-\sqrt{61}}{6}, -\frac{1}{2}\right)$, $\left(-\frac{1}{2}, 0\right)$, $\left(0, \frac{1+\sqrt{61}}{6}\right)$, and $\left(\frac{1+\sqrt{61}}{6}, \infty\right)$.

We test a value in each interval.

$(-\infty, -3)$: $f(-4) \approx 1.6786 > 0$

$\left(-3, \frac{1-\sqrt{61}}{6}\right)$: $f(-2) = -1.5 < 0$

$\left(\frac{1-\sqrt{61}}{6}, -\frac{1}{2}\right)$: $f(-1) = 0.5 > 0$

$\left(-\frac{1}{2}, 0\right)$: $f(-0.1) \approx -20.99 < 0$

$\left(0, \frac{1+\sqrt{61}}{6}\right)$: $f(1) = 0.25 > 0$

$\left(\frac{1+\sqrt{61}}{6}, \infty\right)$: $f(2) = -0.1 < 0$

Function values are negative on $\left(-3, \frac{1-\sqrt{61}}{6}\right)$, $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1+\sqrt{61}}{6}, \infty\right)$. The solution set is $\left(-3, \frac{1-\sqrt{61}}{6}\right) \cup \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1+\sqrt{61}}{6}, \infty\right)$.

72. $$\frac{2}{x^2+3} > \frac{3}{5+4x^2}$$

$$\frac{2}{x^2+3} - \frac{3}{5+4x^2} > 0$$

The denominator of $f(x) = \frac{2}{x^2+3} - \frac{3}{5+4x^2}$ has no real-number zeros. We solve the related equation $f(x) = 0$.

$$\frac{2}{x^2+3} - \frac{3}{5+4x^2} = 0$$

$$2(5+4x^2) - 3(x^2+3) = 0 \quad \text{Multiplying by } (x^2+3)(5+4x^2)$$

$$10 + 8x^2 - 3x^2 - 9 = 0$$

$$5x^2 + 1 = 0$$

This equation has no real-number solutions. Thus, there are no critical values. We test a value in $(-\infty, \infty)$: $f(0) = \frac{1}{15} > 0$. The function is positive on $(-\infty, \infty)$. This is the solution set.

73. $$\frac{5x}{7x-2} > \frac{x}{x+1}$$

$$\frac{5x}{7x-2} - \frac{x}{x+1} > 0$$

The denominator of $f(x) = \frac{5x}{7x-2} - \frac{x}{x+1}$ is 0 when $x = \frac{2}{7}$ or $x = -1$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{5x}{7x-2} - \frac{x}{x+1} = 0$$

$$(7x-2)(x+1)\left(\frac{5x}{7x-2} - \frac{x}{x+1}\right) = (7x-2)(x+1) \cdot 0$$

$$5x(x+1) - x(7x-2) = 0$$

$$5x^2 + 5x - 7x^2 + 2x = 0$$

$$-2x^2 + 7x = 0$$

$$-x(2x-7) = 0$$

$x = 0$ *or* $x = \frac{7}{2}$

The critical values are -1, 0, $\frac{2}{7}$, and $\frac{7}{2}$. They divide the x-axis into the intervals $(-\infty, -1)$, $(-1, 0)$, $\left(0, \frac{2}{7}\right)$, $\left(\frac{2}{7}, \frac{7}{2}\right)$, and $\left(\frac{7}{2}, \infty\right)$. We test a value in each interval.

$(-\infty, -1)$: $f(-2) = -1.375 < 0$

$(-1, 0)$: $f(-0.5) \approx 1.4545 > 0$

$\left(0, \frac{2}{7}\right)$: $f(0.1) \approx -0.4755 < 0$

$\left(\frac{2}{7}, \frac{7}{2}\right)$: $f(1) = 0.5 > 0$

$\left(\frac{7}{2}, \infty\right)$: $f(4) \approx -0.0308 < 0$

Function values are positive on $(-1, 0)$ and $\left(\frac{2}{7}, \frac{7}{2}\right)$. The solution set is $(-1, 0) \cup \left(\frac{2}{7}, \frac{7}{2}\right)$.

74. $$\frac{x^2-x-2}{x^2+5x+6} < 0$$

$$\frac{x^2-x-2}{(x+3)(x+2)} < 0$$

The denominator of $f(x) = \frac{x^2-x-2}{(x+3)(x+2)}$ is 0 when $x = -3$ or $x = -2$, so the function is not defined for these values of x. We solve the related equation $f(x) = 0$.

$$\frac{x^2-x-2}{(x+3)(x+2)} = 0$$

$$(x-2)(x+1) = 0$$

$x = 2$ *or* $x = -1$

The critical values are -3, -2, -1, and 2. They divide the x-axis into the intervals $(-\infty, -3)$, $(-3, -2)$, $(-2, -1)$, $(-1, 2)$, and $(2, \infty)$. We test a value in each interval.

$(-\infty, -3)$: $f(-4) = 9 > 0$

$(-3, -2)$: $f(-2.5) = -27 < 0$

$(-2, -1)$: $f(-1.5) \approx 2.3333 > 0$

$(-1, 2)$: $f(0) \approx -0.3333 < 0$

$(2, \infty)$: $f(3) \approx 0.13333 > 0$

Function values are negative on $(-3, -2)$ and $(-1, 2)$. The solution set is $(-3, -2) \cup (-1, 2)$.

75.

$$\frac{x}{x^2+4x-5}+\frac{3}{x^2-25} \leq \frac{2x}{x^2-6x+5}$$

$$\frac{x}{x^2+4x-5}+\frac{3}{x^2-25}-\frac{2x}{x^2-6x+5} \leq 0$$

$$\frac{x}{(x+5)(x-1)}+\frac{3}{(x+5)(x-5)}-\frac{2x}{(x-5)(x-1)} \leq 0$$

The denominator of

$$f(x)=\frac{x}{(x+5)(x-1)}+\frac{3}{(x+5)(x-5)}-\frac{2x}{(x-5)(x-1)}$$

is 0 when $x=-5$, 1, or 5, so the function is not defined for these values of x. We solve the related equation $f(x)=0$.

$$\frac{x}{(x+5)(x-1)}+\frac{3}{(x+5)(x-5)}-\frac{2x}{(x-5)(x-1)}=0$$

$$x(x-5)+3(x-1)-2x(x+5)=0$$

Multiplying by $(x+5)(x-1)(x-5)$

$$x^2-5x+3x-3-2x^2-10x=0$$

$$-x^2-12x-3=0$$

$$x^2+12x+3=0$$

Using the quadratic formula, we find that $x=-6\pm\sqrt{33}$. The critical values are $-6-\sqrt{33}$, -5, $-6+\sqrt{33}$, 1, and 5. They divide the x-axis into the intervals $(-\infty,-6-\sqrt{33})$, $(-6-\sqrt{33},-5)$, $(-5,-6+\sqrt{33})$, $(-6+\sqrt{33},1)$, $(1,5)$, and $(5,\infty)$. We test a value in each interval.

$(-\infty,-6-\sqrt{33})$: $f(-12)\approx 0.00194>0$

$(-6-\sqrt{33},-5)$: $f(-6)\approx -0.4286<0$

$(-5,-6+\sqrt{33})$: $f(-1)\approx 0.16667>0$

$(-6+\sqrt{33},1)$: $f(0)=-0.12<0$

$(1,5)$: $f(2)\approx 1.4762>0$

$(5,\infty)$: $f(6)\approx -2.018<0$

Function values are negative on $(-6-\sqrt{33},-5)$, $(-6+\sqrt{33},1)$, and $(5,\infty)$. Note that since the inequality symbol is $\leq$ and $f(-6\pm\sqrt{33})=0$, then $-6-\sqrt{33}$ and $-6+\sqrt{33}$ must be included in the solution set. Note also that since -5, 1, and 5 are not in the domain of $f(x)$, they are not included in the solution set. It is $[-6-\sqrt{33},-5)\cup[-6+\sqrt{33},1)\cup(5,\infty)$.

76.

$$\frac{2x}{x^2-9}+\frac{x}{x^2+x-12} \geq \frac{3x}{x^2+7x+12}$$

$$\frac{2x}{(x+3)(x-3)}+\frac{x}{(x+4)(x-3)}-\frac{3x}{(x+4)(x+3)} \geq 0$$

The denominator of $f(x)=$

$$\frac{2x}{(x+3)(x-3)}+\frac{x}{(x+4)(x-3)}-\frac{3x}{(x+4)(x+3)}$$

is 0 when $x=-3$, 3, or -4, so the function is not defined for these values of x. We solve the related equation $f(x)=0$.

$$\frac{2x}{(x+3)(x-3)}+\frac{x}{(x+4)(x-3)}-\frac{3x}{(x+4)(x+3)}=0$$

$$2x(x+4)+x(x+3)-3x(x-3)=0$$

$$2x^2+8x+x^2+3x-3x^2+9x=0$$

$$20x=0$$

$$x=0$$

The critical values are -4, -3, 0, and 3. They divide the x-axis into the intervals $(-\infty,-4)$, $(-4,-3)$, $(-3,0)$, $(0,3)$, and $(3,\infty)$. We test a value in each interval.

$(-\infty,-4)$: $f(-5)=6.25>0$

$(-4,-3)$: $f(-3.5)\approx -43.08<0$

$(-3,0)$: $f(-1)\approx 0.83333>0$

$(0,3)$: $f(1)=-0.5<0$

$(3,\infty)$: $f(4)\approx 1.4286>0$

Function values are positive on $(-\infty,-4)$, $(-3,0)$, and $(3,\infty)$. Note that since the inequality symbol is $\geq$ and $f(0)=0$, then 0 must be included in the solution set. Note also that since -4, -3, and 3 are not in the domain of $f(x)$, they are not included in the solution set. It is $(-\infty,-4)\cup(-3,0]\cup(3,\infty)$.

77. We write and solve a rational inequality.

$$\frac{4t}{t^2+1}+98.6>100$$

$$\frac{4t}{t^2+1}-1.4>0$$

The denominator of $f(t)=\dfrac{4t}{t^2+1}-1.4$ has no real-number zeros. We solve the related equation $f(t)=0$.

$$\frac{4t}{t^2+1}-1.4=0$$

$$4t-1.4(t^2+1)=0 \quad \text{Multiplying by } t^2+1$$

$$4t-1.4t^2-1.4=0$$

Using the quadratic formula, we find that $t=\dfrac{4\pm\sqrt{8.16}}{2.8}$; that is, $t\approx 0.408$ or $t\approx 2.449$. These numbers divide the t-axis into the intervals $(-\infty,0.408)$, $(0.408,2.449)$, and $(2.449,\infty)$. We test a value in each interval.

$(-\infty,0.408)$: $f(0)=-1.4<0$

$(0.408,2.449)$: $f(1)=0.6>0$

$(2.449,\infty)$: $f(3)=-0.2<0$

Function values are positive on $(0.408,2.449)$. The solution set is $(0.408,2.449)$.

78. We write and solve a rational inequality.

$$\frac{500t}{2t^2+9}\geq 40$$

$$\frac{500t}{2t^2+9}-40\geq 0$$

The denominator of $f(t)=\dfrac{500t}{2t^2+9}-40$ has no real-number zeros. We solve the related equation $f(t)=0$.

$$\frac{500t}{2t^2+9} - 40 = 0$$
$$500t - 80t^2 - 360 = 0 \quad \text{Multiplying by } 2t^2+9$$

Using the quadratic formula, we find that $t = \frac{25 \pm \sqrt{337}}{8}$; that is, $t \approx 0.830$ or $t \approx 5.420$. These numbers divide the t-axis into the intervals $(-\infty, 0.830)$, $(0.830, 5.420)$, and $(5.420, \infty)$. We test a value in each interval.

$(-\infty, 0.830)$: $f(0) = -40 < 0$

$(0.830, 5.420)$: $f(1) \approx 5.4545 > 0$

$(5.420, \infty)$: $f(6) \approx -2.963 < 0$

Function values are positive on $(0.830, 5.420)$. The solution set is $[0.830, 5.420]$.

79. a) We write and solve a polynomial inequality.

$$-3x^2 + 630x - 6000 > 0 \quad (x \geq 0)$$

We first solve the related equation.

$$-3x^2 + 630x - 6000 = 0$$
$$x^2 - 210x + 2000 = 0 \quad \text{Dividing by } -3$$
$$(x-10)(x-200) = 0 \quad \text{Factoring}$$

Using the principle of zero products or by observing the graph of $y = -3x^2 + 630 - 6000$, we see that the solutions of the related equation are 10 and 200. These numbers divide the x-axis into the intervals $(-\infty, 10)$, $(10, 200)$, and $(200, \infty)$. Since we are restricting our discussion to nonnegative values of x, we consider the intervals $[0, 10)$, $(10, 200)$, and $(200, \infty)$.

We let $f(x) = -3x^2 + 630x - 6000$ and test a value in each interval.

$[0, 10)$: $f(0) = -6000 < 0$

$(10, 200)$: $f(11) = 567 > 0$

$(200, \infty)$: $f(201) = -573 < 0$

Function values are positive only on $(10, 200)$. The solution set is $\{x | 10 < x < 200\}$, or $(10, 200)$.

b) From part (a), we see that function values are negative on $[0, 10)$ and $(200, \infty)$. Thus, the solution set is $\{x | 0 < x < 10 \text{ or } x > 200\}$, or $(0, 10) \cup (200, \infty)$.

80. a) We write and solve a polynomial inequality.

$$-16t^2 + 32t + 1920 > 1920$$
$$-16t^2 + 32t > 0$$
$$-16t^2 + 32t = 0 \quad \text{Related equation}$$
$$-16t(t-2) = 0 \quad \text{Factoring}$$

The solutions of the related equation are 0 and 2. This could also be determined from the graph of $y = -16x^2 + 32x$. These numbers divide the t-axis into the intervals $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$.

Since only nonnegative values of t have meaning in this application, we restrict our discussion to the intervals $(0, 2)$ and $(2, \infty)$. We let $f(x) = -16t^2 + 32t$ and test a value in each interval.

$(0, 2)$: $f(1) = 16 > 0$

$(2, \infty)$: $f(3) = -48 < 0$

Function values are positive on $(0, 2)$. The solution set is $\{t | 0 < t < 2\}$, or $(0, 2)$.

b) We write and solve a polynomial inequality.

$$-16t^2 + 32t + 1920 < 640$$
$$-16t^2 + 32t + 1280 < 0$$
$$-16t^2 + 32t^2 + 1280 = 0 \quad \text{Related equation}$$
$$t^2 - 2t - 80 = 0$$
$$(t-10)(t+8) = 0$$

The solutions of the related equation are 10 and -8. These numbers divide the t-axis into the intervals $(-\infty, -8)$, $(-8, 10)$, and $(10, \infty)$. As in part (a), we will not consider negative values of t. In addition, note that the nonnegative solution of $S(t) = 0$ is 12. This means that the object reaches the ground in 12 sec. Thus, we also restrict our discussion to values of t such that $t \leq 12$. We consider the intervals $[0, 10)$ and $(10, 12]$. We let $f(x) = -16t^2 + 32t + 1280$ and test a value in each interval.

$[0, 10)$: $f(0) = 1280 > 0$

$(10, 12]$: $f(11) = -304 < 0$

Function values are negative on $(10, 12]$. The solution set is $\{x | 10 < x \leq 12\}$, or $(10, 12]$.

81. We write an inequality.

$$27 \leq \frac{n(n-3)}{2} \leq 230$$
$$54 \leq n(n-3) \leq 460 \quad \text{Multiplying by 2}$$
$$54 \leq n^2 - 3n \leq 460$$

We write this as two inequalities.

$$54 \leq n^2 - 3n \quad \textit{and} \quad n^2 - 3n \leq 460$$

Solve each inequality.

$$n^2 - 3n \geq 54$$
$$n^2 - 3n - 54 \geq 0$$
$$n^2 - 3n - 54 = 0 \quad \text{Related equation}$$
$$(n+6)(n-9) = 0$$
$$n = -6 \quad \textit{or} \quad n = 9$$

Since only positive values of n have meaning in this application, we consider the intervals $(0, 9)$ and $(9, \infty)$. Let $f(n) = n^2 - 3n - 54$ and test a value in each interval.

$(0, 9)$: $f(1) = -56 < 0$

$(9, \infty)$: $f(10) = 16 > 0$

Function values are positive on $(9, \infty)$. Since the inequality symbol is $\geq$, 9 must also be included in the solution set for this portion of the inequality. It is $\{n | n \geq 9\}$.

Now solve the second inequality.

$$n^2 - 3n \leq 460$$
$$n^2 - 3n - 460 \leq 0$$
$$n^2 - 3n - 460 = 0 \quad \text{Related equation}$$
$$(n+20)(n-23) = 0$$

$n = -20 \quad or \quad n = 23$

We consider only positive values of n as above. Thus, we consider the intervals $(0, 23)$ and $(23, \infty)$. Let $f(n) = n^2 - 3n - 460$ and test a value in each interval.

$(0, 23)$: $f(1) = -462 < 0$

$(23, \infty)$: $f(24) = 44 > 0$

Function values are negative on $(0, 23)$. Since the inequality symbol is $\leq$, 23 must also be included in the solution set for this portion of the inequality. It is $\{n|0 < n \leq 23\}$.

The solution set of the original inequality is $\{n|n \geq 9 \; and \; 0 < n \leq 23\}$, or $\{n|9 \leq n \leq 23\}$.

82. We write an inequality.

$$66 \leq \frac{n(n-1)}{2} \leq 300$$

$$132 \leq n^2 - n \leq 600$$

We write this as two inequalities.

$$132 \leq n^2 - n \quad and \quad n^2 - n \leq 600$$

Solve each inequality.

$$n^2 - n \geq 132$$
$$n^2 - n - 132 \geq 0$$
$$n^2 - n - 132 = 0 \quad \text{Related equation}$$
$$(n + 11)(n - 12) = 0$$
$$n = -11 \quad or \quad n = 12$$

Since only positive values of n have meaning in this application, we consider the intervals $(0, 12)$ and $(12, \infty)$. Let $f(n) = n^2 - n - 132$ and test a value in each interval.

$(0, 12)$: $f(1) = -132 < 0$

$(12, \infty)$: $f(13) = 24 > 0$

Function values are positive on $(12, \infty)$. Since the inequality symbol is $\geq$, 12 must also be included in the solution set for this portion of the inequality. It is $\{n|n \geq 12\}$.

Now solve the second inequality.

$$n^2 - n \leq 600$$
$$n^2 - n - 600 \leq 0$$
$$n^2 - n - 600 = 0 \quad \text{Related equation}$$
$$(n + 24)(n - 25) = 0$$
$$n = -24 \quad or \quad n = 25$$

We consider only positive values of n as above. Thus, we consider the intervals $(0, 25)$ and $(25, \infty)$. Let $f(n) = n^2 - n - 600$ and test a value in each interval.

$(0, 25)$: $f(1) = -600 < 0$

$(25, \infty)$: $f(26) = 50 > 0$

Function values are negative on $(0, 25)$. Since the inequality symbol is $\leq$, 25 must also be included in the solution set for this portion of the inequality. It is $\{n|0 < n \leq 25\}$.

The solution set of the original inequality is $\{n|n \geq 12 \; and \; 0 < n \leq 25\}$, or $\{n|12 \leq n \leq 25\}$.

83.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-2)]^2 + (y - 4)^2 = 3^2$$
$$(x + 2)^2 + (y - 4)^2 = 9$$

84. $r = \dfrac{7/2}{2} = \dfrac{7}{4}$

$$(x - 0)^2 + [y - (-3)]^2 = \left(\frac{7}{4}\right)^2$$
$$x^2 + (y + 3)^2 = \frac{49}{16}$$

85. $h(x) = -2x^2 + 3x - 8$

a) $-\dfrac{b}{2a} = -\dfrac{3}{2(-2)} = \dfrac{3}{4}$

$$h\left(\frac{3}{4}\right) = -2\left(\frac{3}{4}\right)^2 + 3 \cdot \frac{3}{4} - 8 = -\frac{55}{8}$$

The vertex is $\left(\dfrac{3}{4}, -\dfrac{55}{8}\right)$.

b) The coefficient of x^2 is negative, so there is a maximum value. It is the second coordinate of the vertex, $-\dfrac{55}{8}$. It occurs at $x = \dfrac{3}{4}$.

c) The range is $\left(-\infty, -\dfrac{55}{8}\right]$.

86. $g(x) = x^2 - 10x + 2$

a) $-\dfrac{b}{2a} = -\dfrac{-10}{2 \cdot 1} = 5$

$g(5) = 5^2 - 10 \cdot 5 + 2 = -23$

The vertex is $(5, -23)$.

b) Minimum: -23 at $x = 5$

c) $[-23, \infty)$

87. $|x^2 - 5| = |5 - x^2| = 5 - x^2$ when $5 - x^2 \geq 0$. Thus we solve $5 - x^2 \geq 0$.

$$5 - x^2 \geq 0$$
$$5 - x^2 = 0 \quad \text{Related equation}$$
$$5 = x^2$$
$$\pm\sqrt{5} = x$$

Let $f(x) = 5 - x^2$ and test a value in each of the intervals determined by the solutions of the related equation.

$(-\infty, -\sqrt{5})$: $f(-3) = -4 < 0$

$(-\sqrt{5}, \sqrt{5})$: $f(0) = 5 > 0$

$(\sqrt{5}, \infty)$: $f(3) = -4 < 0$

Function values are positive on $(-\sqrt{5}, \sqrt{5})$. Since the inequality symbol is $\geq$, the endpoints of the interval must be included in the solution set. It is $\left[-\sqrt{5}, \sqrt{5}\right]$.

88.

$$x^4 - 6x^2 + 5 > 0$$
$$x^4 - 6x^2 + 5 = 0 \quad \text{Related equation}$$
$$(x^2 - 1)(x^2 - 5) = 0$$
$$x = \pm 1 \quad or \quad x = \pm\sqrt{5}$$

Let $f(x) = x^4 - 6x^2 + 5$ and test a value in each of the intervals determined by the solutions of the related equation.

$(-\infty, -\sqrt{5})$: $f(-3) = 32 > 0$

$(-\sqrt{5}, -1)$: $f(-2) = -3 < 0$

$(-1, 1)$: $f(0) = 5 > 0$

$(1, \sqrt{5})$: $f(2) = -3 < 0$

$(\sqrt{5}, \infty)$: $f(3) = 32 > 0$

The solution set is $(-\infty, -\sqrt{5}) \cup (-1, 1) \cup (\sqrt{5}, \infty)$.

89.
$$\begin{aligned} 2|x|^2 - |x| + 2 &\le 5 \\ 2|x|^2 - |x| - 3 &\le 0 \\ 2|x|^2 - |x| - 3 &= 0 \quad \text{Related equation} \\ (2|x| - 3)(|x| + 1) &= 0 \quad \text{Factoring} \\ 2|x| - 3 = 0 \quad &or \quad |x| + 1 = 0 \\ |x| = \frac{3}{2} \quad &or \quad |x| = -1 \end{aligned}$$

The solution of the first equation is $x = -\frac{3}{2}$ or $x = \frac{3}{2}$. The second equation has no solution. Let $f(x) = 2|x|^2 - |x| - 3$ and test a value in each interval determined by the solutions of the related equation.

$\left(-\infty, -\frac{3}{2}\right)$: $f(-2) = 3 > 0$

$\left(-\frac{3}{2}, \frac{3}{2}\right)$: $f(0) = -3 < 0$

$\left(\frac{3}{2}, \infty\right)$: $f(2) = 3 > 0$

Function values are negative on $\left(-\frac{3}{2}, \frac{3}{2}\right)$. Since the inequality symbol is $\le$, the endpoints of the interval must also be included in the solution set. It is $\left[-\frac{3}{2}, \frac{3}{2}\right]$.

90.
$$\begin{aligned} (7 - x)^{-2} &< 0 \\ \frac{1}{(7-x)^2} &< 0 \end{aligned}$$

Since $(7-x)^2 \ge 0$ for all real numbers x, then $\frac{1}{(7-x)^2} > 0$ for all values of x in the domain of $f(x) = \frac{1}{(7-x)^2}$. Thus, the solution set is $\emptyset$.

91.
$$\begin{aligned} \left|1 + \frac{1}{x}\right| &< 3 \\ -3 < 1 + \frac{1}{x} &< 3 \\ -3 < 1 + \frac{1}{x} \quad &and \quad 1 + \frac{1}{x} < 3 \end{aligned}$$

First solve $-3 < 1 + \frac{1}{x}$.

$$0 < 4 + \frac{1}{x}, \text{ or } \frac{1}{x} + 4 > 0$$

The denominator of $f(x) = \frac{1}{x} + 4$ is 0 when $x = 0$, so the function is not defined for this value of x. Now solve the related equation.

$$\begin{aligned} \frac{1}{x} + 4 &= 0 \\ 1 + 4x &= 0 \quad \text{Multiplying by } x \\ x &= -\frac{1}{4} \end{aligned}$$

The critical values are $-\frac{1}{4}$ and 0. Test a value in each of the intervals determined by them.

$\left(-\infty, -\frac{1}{4}\right)$: $f(-1) = 3 > 0$

$\left(-\frac{1}{4}, 0\right)$: $f(-0.1) = -6 < 0$

$(0, \infty)$: $f(1) = 5 > 0$

The solution set for this portion of the inequality is $\left(-\infty, -\frac{1}{4}\right) \cup (0, \infty)$.

Next solve $1 + \frac{1}{x} < 3$, or $\frac{1}{x} - 2 < 0$. The denominator of $f(x) = \frac{1}{x} - 2$ is 0 when $x = 0$, so the function is not defined for this value of x. Now solve the related equation.

$$\begin{aligned} \frac{1}{x} - 2 &= 0 \\ 1 - 2x &= 0 \quad \text{Multiplying by } x \\ x &= \frac{1}{2} \end{aligned}$$

The critical values are 0 and $\frac{1}{2}$. Test a value in each of the intervals determined by them.

$(-\infty, 0)$: $f(-1) = -3 < 0$

$\left(0, \frac{1}{2}\right)$: $f(0.1) = 8 > 0$

$\left(\frac{1}{2}, \infty\right)$: $f(1) = -1 < 0$

The solution set for this portion of the inequality is $(-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$.

The solution set of the original inequality is

$\left(\left(-\infty, -\frac{1}{4}\right) \cup (0, \infty)\right)$ *and* $\left((-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)\right)$,

or $\left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{1}{2}, \infty\right)$.

92. $\left|2 - \frac{1}{x}\right| \le 2 + \left|\frac{1}{x}\right|$

Note that $\frac{1}{x}$ is not defined when $x = 0$. Thus, $x \ne 0$. Also note that $2 - \frac{1}{x} = 0$ when $x = \frac{1}{2}$. The numbers 0 and $\frac{1}{2}$ divide the x-axis into the intervals $(-\infty, 0)$, $\left(0, \frac{1}{2}\right)$, and $\left(\frac{1}{2}, \infty\right)$. Find the solution set of the inequality for each interval. Then find the union of the three solution sets.

If $x < 0$, then $2 - \frac{1}{x} > 0$ and $\frac{1}{x} < 0$, so

$\left|2 - \frac{1}{x}\right| = 2 - \frac{1}{x}$ and $\left|\frac{1}{x}\right| = -\frac{1}{x}$.

Solve: $x < 0$ *and* $2 - \frac{1}{x} \le 2 - \frac{1}{x}$

$x < 0$ *and* $2 \le 2$

The solution set for this interval is $(-\infty, 0)$.

If $0 < x < \frac{1}{2}$, then $2 - \frac{1}{x} < 0$ and $\frac{1}{x} > 0$, so

$\left|2 - \frac{1}{x}\right| = -\left(2 - \frac{1}{x}\right) = -2 + \frac{1}{x}$ and $\left|\frac{1}{x}\right| = \frac{1}{x}$.

Solve: $0 < x < \frac{1}{2}$ *and* $-2 + \frac{1}{x} \le 2 + \frac{1}{x}$

$0 < x < \frac{1}{2}$ *and* $-2 \le 2$

The solution set for this interval is $\left(0, \frac{1}{2}\right)$.

If $x \ge \frac{1}{2}$, then $2 - \frac{1}{x} > 0$ and $\frac{1}{x} > 0$, so

$\left|2 - \frac{1}{x}\right| = 2 - \frac{1}{x}$ and $\left|\frac{1}{x}\right| = \frac{1}{x}$.

Solve: $x \ge \frac{1}{2}$ *and* $2 - \frac{1}{x} \le 2 + \frac{1}{x}$

$x \ge \frac{1}{2}$ *and* $-\frac{1}{x} \le \frac{1}{x}$

$x \ge \frac{1}{2}$ *and* $-1 \le -1$

Then the solution set for this interval is $\left[\frac{1}{2}, \infty\right)$.

Then the solution set of the original inequality is

$(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left[\frac{1}{2}, \infty\right)$, or $(-\infty, 0) \cup (0, \infty)$.

93. First find a quadratic equation with solutions -4 and 3.

$$(x + 4)(x - 3) = 0$$
$$x^2 + x - 12 = 0$$

Test a point in each of the three intervals determined by -4 and 3.

$(-\infty, -4)$: $(-5 + 4)(-5 - 3) = 8 > 0$

$(-4, 3)$: $(0 + 4)(0 - 3) = -12 < 0$

$(3, \infty)$: $(4 + 4)(4 - 3) = 8 > 0$

Then a quadratic inequality for which the solution set is $(-4, 3)$ is $x^2 + x - 12 < 0$. Answers may vary.

94. First find the polynomial with solutions -4, 3, and 7.

$$(x + 4)(x - 3)(x - 7) = 0$$
$$x^3 - 6x^2 - 19x + 84 = 0$$

Test a point in each of the four intervals determined by -4, 3, and 7.

$(-\infty, -4)$: $(-5 + 4)(-5 - 3)(-5 - 7) = -96 < 0$

$(-4, 3)$: $(0 + 4)(0 - 3)(0 - 7) = 84 > 0$

$(3, 7)$: $(4 + 4)(4 - 3)(4 - 7) = -24 < 0$

$(7, \infty)$: $(8 + 4)(8 - 3)(8 - 7) = 60 > 0$

Then a polynomial inequality for which the solution set is $[-4, 3] \cup [7, \infty)$ is $x^3 - 6x^2 - 19x + 84 \ge 0$. Answers may vary.

95. $f(x) = \sqrt{\dfrac{72}{x^2 - 4x - 21}}$

The radicand must be nonnegative and the denominator must be nonzero. Thus, the values of x for which $x^2 - 4x - 21 > 0$ comprise the domain. By inspecting the graph of $y = x^2 – 4x – 21$ we see that the domain is $\{x | x < -3 \text{ or } x > 7\}$, or $(-\infty, -3) \cup (7, \infty)$.

96. $f(x) = \sqrt{x^2 - 4x - 21}$

The radicand must be nonnegative. By inspecting the graph of $y = x^2 - 4x - 21$ we see that the domain is $\{x | x \le -3 \text{ or } x \ge 7\}$, or $(-\infty, -3] \cup [7, \infty)$.

Chapter 4 Review Exercises

1. $f(-b) = (-b+a)(-b+b)(-b-c) = (-b+a) \cdot 0 \cdot (-b-c) = 0$, so the statement is true.

2. The statement is true. See page 349 in the text.

3. In addition to the given possibilities, 9 and -9 are also possible rational zeros. The statement is false.

4. The degree of P is 8, so the graph of $P(x)$ has at most 8 x-intercepts. The statement is false.

5. The domain of the function is the set of all real numbers except -2 and 3, or $\{x | x \ne -2 \text{ and } x \ne 3\}$. The statement is false.

6. $f(x) = 7x^2 - 5 + 0.45x^4 - 3x^3$
$= 0.45x^4 - 3x^3 + 7x^2 - 5$

The leading term is $0.45x^4$ and the leading coefficient is 0.45. The degree of the polynomial is 4, so the polynomial is quartic.

7. $h(x) = -25$

The leading term is -25 and the leading coefficient is -25. The degree of the polynomial is 0, so the polynomial is constant.

8. $g(x) = 6 - 0.5x$
$= -0.5x + 6$

The leading term is $-0.5x$ and the leading coefficient is -0.5. The degree of the polynomial is 1, so the polynomial is linear.

9. $f(x) = \frac{1}{3}x^3 - 2x + 3$

The leading term is $\frac{1}{3}x^3$ and the leading coefficient is $\frac{1}{3}$. The degree of the polynomial is 3, so the polynomial is cubic.

10. $f(x) = -\frac{1}{2}x^4 + 3x^2 + x - 6$

The leading term is $-\frac{1}{2}x^4$. The degree, 4, is even and the leading coefficient, $-\frac{1}{2}$, is negative. As $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

11. $f(x) = x^5 + 2x^3 - x^2 + 5x + 4$

The leading term is x^5. The degree, 5, is odd and the leading coefficient, 1, is positive. As $x \to \infty$, $f(x) \to \infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

12. $g(x) = \left(x - \frac{2}{3}\right)(x+2)^3(x-5)^2$

$\frac{2}{3}$, multiplicity 1;

-2, multiplicity 3;

5, multiplicity 2

13.
$$\begin{aligned} f(x) &= x^4 - 26x^2 + 25 \\ &= (x^2-1)(x^2-25) \\ &= (x+1)(x-1)(x+5)(x-5) \end{aligned}$$

± 1, ± 5; each has multiplicity 1

14.
$$\begin{aligned} h(x) &= x^3 + 4x^2 - 9x - 36 \\ &= x^2(x+4) - 9(x+4) \\ &= (x+4)(x^2-9) \\ &= (x+4)(x+3)(x-3) \end{aligned}$$

-4, ± 3; each has multiplicity 1

15. $A = P(1+r)^t$

a)
$$\begin{aligned} 6760 &= 6250(1+r)^2 \\ \frac{6760}{6250} &= (1+r)^2 \\ \pm 1.04 &= 1+r \\ -1 \pm 1.04 &= r \\ -2.04 = r \quad &or \quad 0.04 = r \end{aligned}$$

Only 0.04 has meaning in this application. The interest rate is 0.04 or 4%.

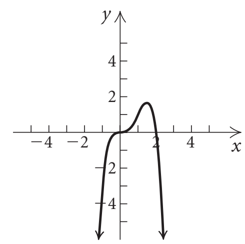
b)
$$\begin{aligned} 1,215,506.25 &= 1,000,000(1+r)^4 \\ \frac{1,215,506.25}{1,000,000} &= (1+r)^4 \\ \pm 1.05 &= 1+r \\ -1 \pm 1.05 &= r \\ -2.05 = r \quad &or \quad 0.05 = r \end{aligned}$$

Only 0.05 has meaning in this application. The interest rate is 0.05 or 5%.

16. $f(x) = -x^4 + 2x^3$

1. The leading term is $-x^4$. The degree, 4, is even and the leading coefficient, -1, is negative so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
2. We solve $f(x) = 0$.
$$\begin{aligned} -x^4 + 2x^3 &= 0 \\ -x^3(x-2) &= 0 \\ -x^3 = 0 \quad or \quad x-2 &= 0 \\ x = 0 \quad or \quad x &= 2 \end{aligned}$$
The zeros of the function are 0 and 2, so the x-intercepts of the graph are $(0,0)$ and $(2,0)$.
3. The zeros divide the x-axis into 3 intervals, $(-\infty, 0)$, $(0,2)$, and $(2,\infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.
In $(-\infty, 0)$, test -1:
$f(-1) = -(-1)^4 + 2(-1)^3 = -3 < 0$
In $(0,2)$, test 1:
$f(1) = -1^4 + 2 \cdot 1^3 = 1 > 0$
In $(2,\infty)$, test 3:
$f(3) = -3^4 + 2 \cdot 3^3 = -27 < 0$
Thus the graph lies below the x-axis on $(-\infty, 0)$ and on $(2,\infty)$ and above the x-axis on $(0,2)$. We also know the points $(-1,-3)$, $(1,1)$, and $(3,-27)$ are on the graph.
4. From Step 2 we know that the y-intercept is $(0,0)$.
5. We find additional points on the graph and then draw the graph.

x	$f(x)$
-2	-32
-0.5	-0.3125
0.5	0.1875
1.5	1.6875



$f(x) = -x^4 + 2x^3$

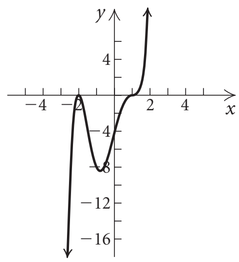
6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

17. $g(x) = (x-1)^3(x+2)^2$

1. The leading term is $x \cdot x \cdot x \cdot x \cdot x$, or x^5. The degree, 5, is odd and the leading coefficient, 1, is positive so as $x \to \infty$, $g(x) \to \infty$ and as $x \to -\infty$, $g(x) \to -\infty$.
2. We see that the zeros of the function are 1 and -2, so the x-intercepts of the graph are $(1,0)$ and $(-2,0)$.
3. The zeros divide the x-axis into 3 intervals, $(-\infty,-2)$, $(-2,1)$, and $(1,\infty)$. We choose a value for x from each interval and find $g(x)$. This tells us the sign of $g(x)$ for all values of x in that interval.
In $(-\infty,-2)$, test -3:
$g(-3) = (-3-1)^3(-3+2)^2 = -64 < 0$
In $(-2,1)$, test 0:
$g(0) = (0-1)^3(0+2)^2 = -4 < 0$
In $(1,\infty)$, test 2:
$g(2) = (2-1)^3(2+2)^2 = 16 > 0$
Thus the graph lies below the x-axis on $(-\infty,-2)$ and on $(-2,1)$ and above the x-axis on $(1,\infty)$. We also know that the points $(-3,-64)$, $(0,-4)$, and $(2,16)$ are on the graph.
4. From Step 3 we know that $g(0) = -4$, so the y-intercept is $(0,-4)$.

5. We find additional points on the graph and then draw the graph.

x	$g(x)$
-2.5	-10.7
-1	-8
-0.5	-7.6
0.5	-0.8



$g(x) = (x-1)^3(x+2)^2$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

18. $h(x) = x^3 + 3x^2 - x - 3$

1. The leading term is x^3. The degree, 3, is odd and the leading coefficient, 1, is positive so as $x \to \infty$, $h(x) \to \infty$ and as $x \to -\infty$, $h(x) \to -\infty$.

2. We solve $h(x) = 0$.

$$x^3 + 3x^2 - x - 3 = 0$$
$$x^2(x+3) - (x+3) = 0$$
$$(x+3)(x^2-1) = 0$$
$$(x+3)(x+1)(x-1) = 0$$
$$x+3 = 0 \quad or \quad x+1 = 0 \quad or \quad x-1 = 0$$
$$x = -3 \quad or \quad x = -1 \quad or \quad x = 1$$

The zeros of the function are -3, -1, and 1, so the x-intercepts of the graph are $(-3,0)$, $(-1,0)$, and $(1,0)$.

3. The zeros divide the x-axis into 4 intervals, $(-\infty,-3)$, $(-3,-1)$, $(-1,1)$, and $(1,\infty)$. We choose a value for x from each interval and find $h(x)$. This tells us the sign of $h(x)$ for all values of x in that interval.

In $(-\infty,-3)$, test -4:

$h(-4) = (-4)^3 + 3(-4)^2 - (-4) - 3 = -15 < 0$

In $(-3,-1)$, test -2:

$h(-2) = (-2)^3 + 3(-2)^2 - (-2) - 3 = 3 > 0$

In $(-1,1)$, test 0:

$h(0) = 0^3 + 3\cdot 0^2 - 0 - 3 = -3 < 0$

In $(1,\infty)$, test 2:

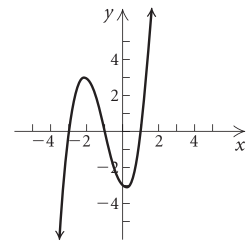
$h(2) = 2^3 + 3\cdot 2^2 - 2 - 3 = 15 > 0$

Thus the graph lies below the x-axis on $(-\infty,-3)$ and on $(-1,1)$ and above the x-axis on $(-3,-1)$ and on $(1,\infty)$. We also know the points $(-4,-15)$, $(-2,3)$, $(0,-3)$, and $(2,15)$ are on the graph.

4. From Step 3 we know that $h(0) = -3$, so the y-intercept is $(0,-3)$.

5. We find additional points on the graph and then draw the graph.

x	$h(x)$
-2.5	2.625
-0.5	-1.875
0.5	-1.875
1.5	5.625



$h(x) = x^3 + 3x^2 - x - 3$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

19. $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$

1. The leading term is x^4. The degree, 4, is even and the leading coefficient, 1, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.

2. We solve $f(x) = 0$, or $x^4 - 5x^3 + 6x^2 + 4x - 8 = 0$. The possible rational zeros are ±1, ±2, ±4, and ±8. We try -1.

-1	1	-5	6	4	-8
		-1	6	-12	8
	1	-6	12	-8	0

Now we have $(x+1)(x^3 - 6x^2 + 12x - 8) = 0$. We use synthetic division to determine if 2 is a zero of $x^3 - 6x^2 + 12x - 8 = 0$.

2	1	-6	12	-8
		2	-8	8
	1	-4	4	0

We have $(x+1)(x-2)(x^2-4x+4) = 0$, or $(x+1)(x-2)(x-2)^2 = 0$. Thus the zeros of $f(x)$ are -1 and 2 and the x-intercepts of the graph are $(-1,0)$ and $(2,0)$.

3. The zeros divide the x-axis into 3 intervals, $(-\infty,-1)$, $(-1,2)$, and $(2,\infty)$. We choose a value for x from each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

In $(-\infty,-1)$, test -2:

$f(-2) = (-2)^4 - 5(-2)^3 + 6(-2)^2 + 4(-2) - 8 = 64 > 0$

In $(-1,2)$, test 0:

$f(0) = 0^4 - 5\cdot 0^3 + 6\cdot 0^2 + 4\cdot 0 - 8 = -8 < 0$

In $(2,\infty)$, test 3:

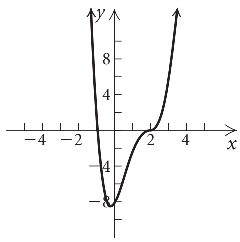
$f(3) = 3^4 - 5\cdot 3^3 + 6\cdot 3^2 + 4\cdot 3 - 8 = 4 > 0$

Thus the graph lies above the x-axis on $(-\infty,-1)$ and on $(2,\infty)$ and below the x-axis on $(-1,2)$. We also know that the points $(-2,64)$, $(0,-8)$, and $(3,4)$ are on the graph.

4. From Step 3 we know that $f(0) = -8$, so the y-intercept is $(0,-8)$.

5. We find additional points on the graph and then draw the graph.

x	$f(x)$
-1.5	21.4
-0.5	-7.8
1	-2
4	40



$f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

20. $g(x) = 2x^3 + 7x^2 - 14x + 5$

1. The leading term is x^3. The degree, 3, is odd and the leading coefficient, 2, is positive so as $x \to \infty$, $g(x) \to \infty$ and as $x \to -\infty$, $h(x) \to -\infty$.

2. We solve $g(x) = 0$, or $2x^3 + 7x^2 - 14x + 5 = 0$. The possible rational zeros are ± 1, ± 5, $\pm\frac{1}{2}$, and $\pm\frac{5}{2}$.

   We try 1.

$$\begin{array}{r|rrrr} 1 & 2 & 7 & -14 & 5 \\ & & 2 & 9 & -5 \\ \hline & 2 & 9 & -5 & 0 \end{array}$$

   Now we have:

   $(x-1)(2x^2+9x-5) = 0$

   $(x-1)(2x-1)(x+5) = 0$

   $x - 1 = 0$ *or* $2x - 1 = 0$ *or* $x + 5 = 0$

   $x = 1$ *or* $2x = 1$ *or* $x = -5$

   $x = 1$ *or* $x = \frac{1}{2}$ *or* $x = -5$

   The zeros of the function are 1, $\frac{1}{2}$, and -5, so the x-intercepts are $(1,0)$, $\left(\frac{1}{2}, 0\right)$, and $(-5, 0)$.

3. The zeros divide the x-axis into 4 intervals, $(-\infty, -5)$, $\left(-5, \frac{1}{2}\right)$, $\left(\frac{1}{2}, 1\right)$, and $(1, \infty)$. We choose a value for x from each interval and find $g(x)$. This tells us the sign of $g(x)$ for all values of x in that interval.

   In $(-\infty, -5)$, test -6:

   $g(-6) = 2(-6)^3 + 7(-6)^2 - 14(-6) + 5 = -91 < 0$

   In $\left(-5, \frac{1}{2}\right)$, test 0:

   $g(0) = 2 \cdot 0^3 + 7 \cdot 0^2 - 14 \cdot 0 + 5 = 5 > 0$

   In $\left(\frac{1}{2}, 1\right)$, test $\frac{3}{4}$:

   $g\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right)^3 + 7\left(\frac{3}{4}\right)^2 - 14 \cdot \frac{3}{4} + 5 = -0.71875 < 0$

   In $(1, \infty)$, test 2:

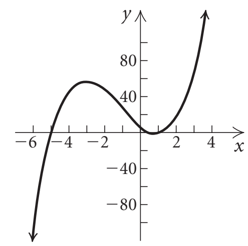
   $g(2) = 2 \cdot 2^3 + 7 \cdot 2^2 - 14 \cdot 2 + 5 = 21 > 0$

   Thus the graph lies below the x-axis on $(-\infty, -5)$ and on $\left(\frac{1}{2}, 1\right)$ and above the x-axis on $\left(-5, \frac{1}{2}\right)$ and on $(1, \infty)$. We also know the points $(-6, -91)$, $(0, 5)$, $(0.75, -0.71875)$, and $(2, 21)$ are on the graph.

4. From Step 3 we know that $g(0) = 5$, so the y-intercept is $(0, 5)$.

5. We find additional points on the graph and then draw the graph.

x	$g(x)$
-3	56
-1	24
1.5	6.5
3	80



$g(x) = 2x^3 + 7x^2 - 14x + 5$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

21. $f(1) = 4 \cdot 1^2 - 5 \cdot 1 - 3 = -4$

$f(2) = 4 \cdot 2^2 - 5 \cdot 2 - 3 = 3$

By the intermediate value theorem, since $f(1)$ and $f(2)$ have opposite signs, $f(x)$ has a zero between 1 and 2.

22. $f(-1) = (-1)^3 - 4(-1)^2 + \frac{1}{2}(-1) + 2 = -3.5$

$f(1) = 1^3 - 4 \cdot 1^2 + \frac{1}{2} \cdot 1 + 2 = -0.5$

Since $f(-1)$ and $f(1)$ have the same sign, the intermediate value theorem does not allow us to determine if there is a real zero between -1 and 1.

23.

$$\begin{array}{r|l} & 6x^2 + 16x + 52 \\ \hline x-3 & 6x^3 - 2x^2 + 4x - 1 \\ & \underline{6x^3 - 18x^2} \\ & \qquad 16x^2 + 4x \\ & \qquad \underline{16x^2 - 48x} \\ & \qquad\qquad 52x - 1 \\ & \qquad\qquad \underline{52x - 156} \\ & \qquad\qquad\qquad 155 \end{array}$$

$Q(x) = 6x^2 + 16x + 52$; $R(x) = 155$;

$P(x) = (x-3)(6x^2 + 16x + 52) + 155$

24.

$$\begin{array}{r|l} & x^3 - 3x^2 + 3x - 2 \\ \hline x+1 & x^4 - 2x^3 + 0x^2 + x + 5 \\ & \underline{x^4 + x^3} \\ & \quad -3x^3 + 0x^2 \\ & \quad \underline{-3x^3 - 3x^2} \\ & \qquad 3x^2 + x \\ & \qquad \underline{3x^2 + 3x} \\ & \qquad\quad -2x + 5 \\ & \qquad\quad \underline{-2x - 2} \\ & \qquad\qquad 7 \end{array}$$

$Q(x) = x^3 - 3x^2 + 3x - 2$; $R(x) = 7$;

$P(x) = (x+1)(x^3 - 3x^2 + 3x - 2) + 7$

25.
$$\begin{array}{r|rrrr} 5 & 1 & 2 & -13 & 10 \\ & & 5 & 35 & 110 \\ \hline & 1 & 7 & 22 & \vert\ 120 \end{array}$$

The quotient is $x^2 + 7x + 22$; the remainder is 120.

26. $(x^4 + 3x^3 + 3x^2 + 3x + 2) \div (x + 2) =$
$(x^4 + 3x^3 + 3x^2 + 3x + 2) \div [x - (-2)]$

$$\begin{array}{r|rrrrr} -2 & 1 & 3 & 3 & 3 & 2 \\ & & -2 & -2 & -2 & -2 \\ \hline & 1 & 1 & 1 & 1 & \vert\ 0 \end{array}$$

The quotient is $x^3 + x^2 + x + 1$; the remainder is 0.

27.
$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 0 & 0 & -2 & 0 \\ & & -1 & 1 & -1 & 1 & 1 \\ \hline & 1 & -1 & 1 & -1 & -1 & \vert\ 1 \end{array}$$

The quotient is $x^4 - x^3 + x^2 - x - 1$; the remainder is 1.

28.
$$\begin{array}{r|rrrr} -2 & 1 & 2 & -13 & 10 \\ & & -2 & 0 & 26 \\ \hline & 1 & 0 & -13 & \vert\ 36 \end{array}$$

$f(-2) = 36$

29.
$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 0 & 0 & -16 \\ & & -2 & 4 & -8 & 16 \\ \hline & 1 & -2 & 4 & -8 & \vert\ 0 \end{array}$$

$f(-2) = 0$

30.
$$\begin{array}{r|rrrrrr} -10 & 1 & -4 & 1 & -1 & 2 & -100 \\ & & -10 & 140 & -1410 & 14,110 & -141,120 \\ \hline & 1 & -14 & 141 & -1411 & 14,112 & \vert\ -141,220 \end{array}$$

$f(-10) = -141,220$

31.
$$\begin{array}{r|rrrr} -i & 1 & -5 & 1 & -5 \\ & & -i & -1+5i & 5 \\ \hline & 1 & -5-i & 5i & \vert\ 0 \end{array}$$

$f(-i) = 0$, so $-i$ is a zero of $f(x)$.

$$\begin{array}{r|rrrr} -5 & 1 & -5 & 1 & -5 \\ & & -5 & 50 & -255 \\ \hline & 1 & -10 & 51 & \vert\ -260 \end{array}$$

$f(-5) \neq 0$, so -5 is not a zero of $f(x)$.

32.
$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -3 & 14 & -8 \\ & & -1 & 5 & -2 & -12 \\ \hline & 1 & -5 & 2 & 12 & \vert\ -20 \end{array}$$

$f(-1) \neq 0$, so -1 is not a zero of $f(x)$.

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -3 & 14 & -8 \\ & & -2 & 12 & -18 & 8 \\ \hline & 1 & -6 & 9 & -4 & \vert\ 0 \end{array}$$

$f(-2) = 0$, so -2 is a zero of $f(x)$.

33.
$$\begin{array}{r|rrrr} \frac{1}{3} & 1 & -\frac{4}{3} & -\frac{5}{3} & \frac{2}{3} \\ & & \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \hline & 1 & -1 & -2 & \vert\ 0 \end{array}$$

$f\left(\dfrac{1}{3}\right) = 0$, so $\dfrac{1}{3}$ is a zero of $f(x)$.

$$\begin{array}{r|rrrr} 1 & 1 & -\frac{4}{3} & -\frac{5}{3} & \frac{2}{3} \\ & & 1 & -\frac{1}{3} & -2 \\ \hline & 1 & -\frac{1}{3} & -2 & \vert\ -\frac{4}{3} \end{array}$$

$f(1) \neq 0$, so 1 is not a zero of $f(x)$.

34.
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -5 & 0 & 6 \\ & & 2 & 4 & -2 & -4 \\ \hline & 1 & 2 & -1 & -2 & \vert\ 2 \end{array}$$

$Q(2) \neq 0$, so 2 is not a zero of $f(x)$.

$$\begin{array}{r|rrrrr} -\sqrt{3} & 1 & 0 & -5 & 0 & 6 \\ & & -\sqrt{3} & 3 & 2\sqrt{3} & -6 \\ \hline & 1 & -\sqrt{3} & -2 & 2\sqrt{3} & \vert\ 0 \end{array}$$

$f(-\sqrt{3}) = 0$, so $-\sqrt{3}$ is a zero of $f(x)$.

35. $f(x) = x^3 + 2x^2 - 7x + 4$

Try $x + 1$ and $x + 2$. Using synthetic division we find that $f(-1) \neq 0$ and $f(-2) \neq 0$. Thus $x + 1$ and $x + 2$ are not factors of $f(x)$. Try $x + 4$.

$$\begin{array}{r|rrrr} -4 & 1 & 2 & -7 & 4 \\ & & -4 & 8 & -4 \\ \hline & 1 & -2 & 1 & \vert\ 0 \end{array}$$

Since $f(-4) = 0$, $x + 4$ is a factor of $f(x)$. Thus $f(x) = (x + 4)(x^2 - 2x + 1) = (x + 4)(x - 1)^2$.

Now we solve $f(x) = 0$.

$x + 4 = 0$ *or* $(x - 1)^2 = 0$

$x = -4$ *or* $x - 1 = 0$

$x = -4$ *or* $x = 1$

The solutions of $f(x) = 0$ are -4 and 1.

36. $f(x) = x^3 + 4x^2 - 3x - 18$

Try $x + 1$, $x - 1$, and $x + 2$. Using synthetic division we find that $f(-1) \neq 0$, $f(1) \neq 0$, and $f(-2) \neq 0$. Thus $x+1$, $x - 1$, and $x + 2$ are not factors of $f(x)$. Try $x - 2$.

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -3 & -18 \\ & & 2 & 12 & 18 \\ \hline & 1 & 6 & 9 & \vert\ 0 \end{array}$$

Since $f(2) = 0$, $x - 2$ is a factor of $f(x)$. Thus
$f(x) = (x - 2)(x^2 + 6x + 9) = (x - 2)(x + 3)^2$.

Now solve $f(x) = 0$.

$(x - 2)(x + 3)^2 = 0$

$x - 2 = 0$ *or* $(x + 3)^2 = 0$

$x = 2$ *or* $x + 3 = 0$

$x = 2$ *or* $x = -3$

The solutions of $f(x) = 0$ are 2 and -3.

37. $f(x) = x^4 - 4x^3 - 21x^2 + 100x - 100$

Using synthetic division we find that $f(2) = 0$:

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & -21 & 100 & -100 \\ & & 2 & -4 & -50 & 100 \\ \hline & 1 & -2 & -25 & 50 & \vert\ 0 \end{array}$$

Then we have:

$$\begin{aligned} f(x) &= (x-2)(x^3-2x^2-25x+50) \\ &= (x-2)[x^2(x-2)-25(x-2)] \\ &= (x-2)(x-2)(x^2-25) \\ &= (x-2)^2(x+5)(x-5) \end{aligned}$$

Now solve $f(x)=0$.

$$\begin{array}{lll} (x-2)^2=0 \ \ or & x+5=0 \ \ or & x-5=0 \\ x-2=0 \ \ or & x=-5 \ \ or & x=5 \\ x=2 \ \ or & x=-5 \ \ or & x=5 \end{array}$$

The solutions of $f(x)=0$ are 2, -5, and 5.

38. $f(x)=x^4-3x^2+2=x^4+0\cdot x^3-3x^2+0\cdot x+2$

Try $x+1$.

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & -3 & 0 & 2 \\ & & -1 & 1 & 2 & -2 \\ \hline & 1 & -1 & -2 & 2 & 0 \end{array}$$

Since $f(-1)=0$, $(x+1)$ is a factor of $f(x)$. We have:

$$\begin{aligned} f(x) &= (x+1)(x^3-x^2-2x+2) \\ &= (x+1)[x^2(x-1)-2(x-1)] \\ &= (x+1)(x-1)(x^2-2) \\ &= (x+1)(x-1)(x+\sqrt{2})(x-\sqrt{2}) \end{aligned}$$

Now solve $f(x)=0$.

$(x+1)(x-1)(x+\sqrt{2})(x-\sqrt{2})=0$

$$\begin{array}{llll} x+1=0 \ \ or & x-1=0 \ \ or & x+\sqrt{2}=0 \ \ or & x-\sqrt{2}=0 \\ x=-1 \ \ or & x=1 \ \ or & x=-\sqrt{2} \ \ or & x=\sqrt{2} \end{array}$$

The solutions of $f(x)=0$ are -1, 1, $-\sqrt{2}$, and $\sqrt{2}$.

39. A polynomial function of degree 3 with -4, -1, and 2 as zeros has factors $x+4$, $x+1$, and $x-2$ so we have $f(x)=a_n(x+4)(x+1)(x-2)$.

The simplest polynomial is obtained if we let $a_n=1$.

$$\begin{aligned} f(x) &= (x+4)(x+1)(x-2) \\ &= (x^2+5x+4)(x-2) \\ &= x^3-2x^2+5x^2-10x+4x-8 \\ &= x^3+3x^2-6x-8 \end{aligned}$$

40. A polynomial function of degree 3 with -3, $1+i$, and $1-i$ as zeros has factors $x+3$, $x-(1+i)$, and $x-(1-i)$ so we have $f(x)=a_n(x+3)[x-(1+i)][x-(1-i)]$.

The simplest polynomial is obtained if we let $a_n=1$.

$$\begin{aligned} f(x) &= (x+3)[x-(1+i)][x-(1-i)] \\ &= (x+3)[(x-1)-i][(x-1)+i) \\ &= (x+3)[(x-1)^2-i^2] \\ &= (x+3)(x^2-2x+1+1) \\ &= (x+3)(x^2-2x+2) \\ &= x^3-2x^2+2x+3x^2-6x+6 \\ &= x^3+x^2-4x+6 \end{aligned}$$

41. A polynomial function of degree 3 with $\frac{1}{2}$, $1-\sqrt{2}$, and $1+\sqrt{2}$ as zeros has factors $x-\frac{1}{2}$, $x-(1-\sqrt{2})$, and $x-(1+\sqrt{2})$ so we have

$$f(x)=a_n\left(x-\frac{1}{2}\right)[x-(1-\sqrt{2})][x-(1+\sqrt{2})].$$

Let $a_n=1$.

$$\begin{aligned} f(x) &= \left(x-\frac{1}{2}\right)[x-(1-\sqrt{2})][x-(1+\sqrt{2})] \\ &= \left(x-\frac{1}{2}\right)[(x-1)+\sqrt{2}][(x-1)-\sqrt{2}] \\ &= \left(x-\frac{1}{2}\right)(x^2-2x+1-2) \\ &= \left(x-\frac{1}{2}\right)(x^2-2x-1) \\ &= x^3-2x^2-x-\frac{1}{2}x^2+x+\frac{1}{2} \\ &= x^3-\frac{5}{2}x^2+\frac{1}{2} \end{aligned}$$

If we let $a_n=2$, we obtain $f(x)=2x^3-5x^2+1$.

42. A polynomial function of degree 4 has at most 4 real zeros. Since 4 zeros are given, these are all of the zeros of the desired function. We proceed as in Exercise 39 above, letting $a_n=1$.

$$\begin{aligned} f(x) &= (x+5)^3\left(x-\frac{1}{2}\right) \\ &= (x^3+15x^2+75x+125)\left(x-\frac{1}{2}\right) \\ &= x^4+15x^3+75x^2+125x-\frac{1}{2}x^3-\frac{15}{2}x^2-\frac{75}{2}x-\frac{125}{2} \\ &= x^4+\frac{29}{2}x^3+\frac{135}{2}x^2+\frac{175}{2}x-\frac{125}{2} \end{aligned}$$

If we had let $a_n=2$, the result would have been $f(x)=2x^4+29x^3+135x^2+175x-125$.

43. A polynomial function of degree 5 has at most 5 real zeros. Since 5 zeros are given, these are all of the zeros of the desired function. We proceed as in Exercise 39 above, letting $a_n=1$.

$$\begin{aligned} f(x) &= (x+3)^2(x-2)(x-0)^2 \\ &= (x^2+6x+9)(x^3-2x^2) \\ &= x^5+6x^4+9x^3-2x^4-12x^3-18x^2 \\ &= x^5+4x^4-3x^3-18x^2 \end{aligned}$$

44. A polynomial function of degree 5 can have at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $\sqrt{5}$ and i, or $-\sqrt{5}$ and $-i$.

45. A polynomial function of degree 5 can have at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $1+\sqrt{3}$ and $-\sqrt{3}$, or $1-\sqrt{3}$ and $\sqrt{3}$.

46. A polynomial function of degree 5 can have at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 4 given zeros, the other zero is the conjugate of $-\sqrt{2}$, or $\sqrt{2}$.

47. $-\sqrt{11}$ is also a zero.

$$\begin{aligned} f(x) &= (x-\sqrt{11})(x+\sqrt{11}) \\ &= x^2-11 \end{aligned}$$

48. i is also a zero.

$$\begin{aligned} f(x) &= (x+i)(x-i)(x-6) \\ &= (x^2+1)(x-6) \\ &= x^3-6x^2+x-6 \end{aligned}$$

49. $1-i$ is also a zero.

$$\begin{aligned} f(x) &= (x+1)(x-4)[x-(1+i)][x-(1-i)] \\ &= (x^2-3x-4)(x^2-2x+2) \\ &= x^4-2x^3+2x^2-3x^3+6x^2-6x-4x^2+8x-8 \\ &= x^4-5x^3+4x^2+2x-8 \end{aligned}$$

50. $-\sqrt{5}$ and $2i$ are also zeros.

$$\begin{aligned} f(x) &= (x-\sqrt{5})(x+\sqrt{5})(x+2i)(x-2i) \\ &= (x^2-5)(x^2+4) \\ &= x^4-x^2-20 \end{aligned}$$

51.

$$\begin{aligned} f(x) &= \left(x-\frac{1}{3}\right)(x-0)(x+3) \\ &= \left(x^2-\frac{1}{3}x\right)(x+3) \\ &= x^3+\frac{8}{3}x^2-x \end{aligned}$$

52. $h(x)=4x^5-2x^3+6x-12$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm1,\pm2,\pm3,\pm4,\pm6,\pm12}{\pm1,\pm2,\pm4}$

Possibilities for p/q: $\pm1,\pm2,\pm3,\pm4,\pm6,\pm12,$ $\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{1}{4},\pm\frac{3}{4}$

53. $g(x)=3x^4-x^3+5x^2-x+1$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm1}{\pm1,\pm3}$

Possibilities for p/q: $\pm1,\pm\frac{1}{3}$

54. $f(x)=x^3-2x^2+x-24$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}: \quad \dfrac{\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24}{\pm1}$

Possibilities for p/q: $\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

55. $f(x)=3x^5+2x^4-25x^3-28x^2+12x$

a) We know that 0 is a zero since
$f(x)=x(3x^4+2x^3-25x^2-28x+12)$.
Now consider $g(x)=3x^4+2x^3-25x^2-28x+12$.
Possibilities for p/q: $\pm1,\pm2,\pm3,\pm4,\pm6,\pm12,$ $\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$

From the graph of $y=3x^4+2x^3-25x^2-28x+12$, we see that, of all the possibilities above, only -2, $\frac{1}{3}$, $\frac{2}{3}$, and 3 might be zeros. We use synthetic division to determine if -2 is a zero.

$$\begin{array}{r|rrrrr} -2 & 3 & 2 & -25 & -28 & 12 \\ & & -6 & 8 & 34 & -12 \\ \hline & 3 & -4 & -17 & 6 & 0 \end{array}$$

Now try 3 in the quotient above.

$$\begin{array}{r|rrrr} 3 & 3 & -4 & -17 & 6 \\ & & 9 & 15 & -6 \\ \hline & 3 & 5 & -2 & 0 \end{array}$$

We have $f(x)=(x+2)(x-3)(3x^2+5x-2)$.

We find the other zeros.

$$\begin{aligned} 3x^3+5x-2&=0 \\ (3x-1)(x+2)&=0 \\ 3x-1=0 \quad &or \quad x+2=0 \\ 3x=1 \quad &or \quad x=-2 \\ x=\frac{1}{3} \quad &or \quad x=-2 \end{aligned}$$

The rational zeros of $g(x)=3x^4+2x^3-25x^2-28x+12$ are -2, 3, and $\frac{1}{3}$. Since 0 is also a zero of $f(x)$, the zeros of $f(x)$ are -2, 3, $\frac{1}{3}$, and 0. (The zero -2 has multiplicity 2.) These are the only zeros.

b) From our work above we see
$f(x)=x(x+2)(x-3)(3x-1)(x+2)$, or
$x(x+2)^2(x-3)(3x-1)$.

56. $f(x)=x^3-2x^2-3x+6$

a) Possibilities for p/q: $\pm1,\pm2,\pm3,\pm6$

From the graph of $f(x)$, we see that -2 and 2 might be zeros. Synthetic division shows that -2 is not a zero. Try 2.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$f(x)=(x-2)(x^2-3)$

We find the other zeros.

$$\begin{aligned} x^2-3&=0 \\ x^2&=3 \\ x&=\pm\sqrt{3} \end{aligned}$$

There is only one rational zero, 2. The other zeros are $\pm\sqrt{3}$. (Note that we could have used factoring by grouping to find this result.)

b) $f(x)=(x-2)(x+\sqrt{3})(x-\sqrt{3})$

57. $f(x)=x^4-6x^3+9x^2+6x-10$

a) Possibilities for p/q: $\pm1,\pm2,\pm5,\pm10$

From the graph of $f(x)$, we see that -1 and 1 might be zeros.

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 9 & 6 & -10 \\ & & -1 & 7 & -16 & 10 \\ \hline & 1 & -7 & 16 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 16 & -10 \\ & & 1 & -6 & 10 \\ \hline & 1 & -6 & 10 & 0 \end{array}$$

$f(x)=(x+1)(x-1)(x^2-6x+10)$

Using the quadratic formula, we find that the other zeros are $3\pm i$.

The rational zeros are -1 and 1. The other zeros are $3\pm i$.

b) $f(x) = (x+1)(x-1)[x-(3+i)][x-(3-i)]$
$= (x+1)(x-1)(x-3-i)(x-3+i)$

58. $f(x) = x^3 + 3x^2 - 11x - 5$

a) Possibilities for p/q: ± 1, ± 5

From the graph of $f(x)$, we see that -5 might be a zero.

$$\begin{array}{r|rrrr} -5 & 1 & 3 & -11 & -5 \\ & & -5 & 10 & 5 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$f(x) = (x+5)(x^2 - 2x - 1)$

Use the quadratic formula to find the other zeros.

$x^2 - 2x - 1 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

The rational zero is -5. The other zeros are $1 \pm \sqrt{2}$.

b) $f(x) = (x+5)[x-(1+\sqrt{2})][x-(1-\sqrt{2})]$
$= (x+5)(x-1-\sqrt{2})(x-1+\sqrt{2})$

59. $f(x) = 3x^3 - 8x^2 + 7x - 2$

a) Possibilities for p/q: $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

From the graph of $f(x)$, we see that $\frac{2}{3}$ and 1 might be zeros.

$$\begin{array}{r|rrrr} 1 & 3 & -8 & 7 & -2 \\ & & 3 & -5 & 2 \\ \hline & 3 & -5 & 2 & 0 \end{array}$$

We have $f(x) = (x-1)(3x^2 - 5x + 2)$.

We find the other zeros.

$$3x^2 - 5x + 2 = 0$$
$$(3x-2)(x-1) = 0$$
$$3x - 2 = 0 \quad or \quad x - 1 = 0$$
$$x = \frac{2}{3} \quad or \quad x = 1$$

The rational zeros are 1 and $\frac{2}{3}$. (The zero 1 has multiplicity 2.) These are the only zeros.

b) $f(x) = (x-1)^2(3x-2)$

60. $f(x) = x^5 - 8x^4 + 20x^3 - 8x^2 - 32x + 32$

a) Possibilities for p/q: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$

From the graph of $f(x)$, we see that -1, 2, and 4 might be zeros. Synthetic division shows that -1 is not a zero. Try 2.

$$\begin{array}{r|rrrrrr} 2 & 1 & -8 & 20 & -8 & -32 & 32 \\ & & 2 & -12 & 16 & 16 & -32 \\ \hline & 1 & -6 & 8 & 8 & -16 & 0 \end{array}$$

We try 2 again.

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 8 & 8 & -16 \\ & & 2 & -8 & 0 & 16 \\ \hline & 1 & -4 & 0 & 8 & 0 \end{array}$$

We try 2 a third time.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 0 & 8 \\ & & 2 & -4 & -8 \\ \hline & 1 & -2 & -4 & 0 \end{array}$$

We have $f(x) = (x-2)^3(x^2 - 2x - 4)$.

Use the quadratic formula to find the other zeros.

$x^2 - 2x - 4 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$$

The rational zero is 2. (It is a zero of multiplicity 3.) The other zeros are $1 \pm \sqrt{5}$.

b) $f(x) = (x-2)^3[x-(1+\sqrt{5})][x-(1-\sqrt{5})]$
$= (x-2)^3(x-1-\sqrt{5})(x-1+\sqrt{5})$

61. $f(x) = x^6 + x^5 - 28x^4 - 16x^3 + 192x^2$

a) We know that 0 is a zero since

$f(x) = x^2(x^4 + x^3 - 28x^2 - 16x + 192)$.

Consider $g(x) = x^4 + x^3 - 28x^2 - 16x + 192$.

Possibilities for p/q: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 32, \pm 48, \pm 64, \pm 96, \pm 192$

From the graph of $y = g(x)$, we see that -4, 3 and 4 might be zeros.

$$\begin{array}{r|rrrrr} -4 & 1 & 1 & -28 & -16 & 192 \\ & & -4 & 12 & 64 & -192 \\ \hline & 1 & -3 & -16 & 48 & 0 \end{array}$$

We have $f(x) = x^2 \cdot g(x) = x^2(x+4)(x^3 - 3x^2 - 16x + 48)$.

We find the other zeros.

$$x^3 - 3x^2 - 16x + 48 = 0$$
$$x^2(x-3) - 16(x-3) = 0$$
$$(x-3)(x^2 - 16) = 0$$
$$(x-3)(x+4)(x-4) = 0$$
$$x - 3 = 0 \quad or \quad x + 4 = 0 \quad or \quad x - 4 = 0$$
$$x = 3 \quad or \quad x = -4 \quad or \quad x = 4$$

The rational zeros are 0, -4, 3, and 4. (The zeros 0 and -4 each have multiplicity 2.) These are the only zeros.

b) $f(x) = x^2(x+4)^2(x-3)(x-4)$

62. $f(x) = 2x^5 - 13x^4 + 32x^3 - 38x^2 + 22x - 5$

a) Possibilities for p/q: $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}$

From the graph of $y=2x^5-13x^4+32x^3-38x^2+22x-5$, we see that 1 and $\frac{5}{2}$ might be zeros. We try 1.

$$\begin{array}{r|rrrrrr} 1 & 2 & -13 & 32 & -38 & 22 & -5 \\ & & 2 & -11 & 21 & -17 & 5 \\ \hline & 2 & -11 & 21 & -17 & 5 & 0 \end{array}$$

We try 1 again.

$$\begin{array}{r|rrrrr} 1 & 2 & -11 & 21 & -17 & 5 \\ & & 2 & -9 & 12 & -5 \\ \hline & 2 & -9 & 12 & -5 & 0 \end{array}$$

We try 1 a third time.

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 12 & -5 \\ & & 2 & -7 & 5 \\ \hline & 2 & -7 & 5 & 0 \end{array}$$

We have $f(x) = (x-1)(x-1)(x-1)(2x^2 - 7x + 5)$. We find the other zeros.

$$\begin{aligned} 2x^2 - 7x + 5 &= 0 \\ (2x-5)(x-1) &= 0 \\ 2x - 5 = 0 \quad &or \quad x - 1 = 0 \\ 2x = 5 \quad &or \quad x = 1 \\ x = \frac{5}{2} \quad &or \quad x = 1 \end{aligned}$$

The rational zeros are 1 and $\frac{5}{2}$. (The number 1 is a zero of multiplicity 4.) These are the only zeros.

b) $f(x) = (x-1)^4(2x-5)$

63. $f(x) = 2x^6 - 7x^3 + x^2 - x$

There are 3 variations in sign in $f(x)$, so there are 3 or 1 positive real zeros.

$$\begin{aligned} f(-x) &= 2(-x)^6 - 7(-x)^3 + (-x)^2 - (-x) \\ &= 2x^6 + 7x^3 + x^2 + x \end{aligned}$$

There are no variations in sign in $f(-x)$, so there are no negative real zeros.

64. $h(x) = -x^8 + 6x^5 - x^3 + 2x - 2$

There are 4 variations in sign in $h(x)$, so there are 4 or 2 or 0 positive real zeros.

$$\begin{aligned} h(-x) &= -(-x)^8 + 6(-x)^5 - (-x)^3 + 2(-x) - 2 \\ &= -x^8 - 6x^5 + x^3 - 2x - 2 \end{aligned}$$

There are 2 variations in sign in $h(-x)$, so there are 2 or 0 negative real zeros.

65. $g(x) = 5x^5 - 4x^2 + x - 1$

There are 3 variations in sign in $g(x)$, so there are 3 or 1 positive real zeros.

$$\begin{aligned} g(-x) &= 5(-x)^5 - 4(-x)^2 + (-x) - 1 \\ &= -5x^5 - 4x^2 - x - 1 \end{aligned}$$

There is no variation in sign in $g(-x)$, so there are 0 negative real zeros.

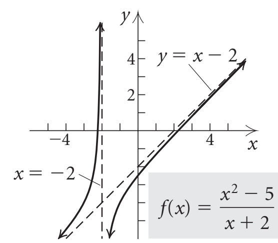
66. $f(x) = \dfrac{x^2 - 5}{x + 2}$

1. The numerator and the denominator have no common factors. The denominator is zero when $x = -2$, so the domain excludes -2. It is $(-\infty, -2) \cup (-2, \infty)$. The line $x = -2$ is the vertical asymptote.
2. The degree of the numerator is 1 greater than the degree of the denominator, so we divide to find the oblique asymptote.

$$\begin{array}{r} x - 2 \\ x+2 \,\overline{)\, x^2 + 0x - 5} \\ \underline{x^2 + 2x} \\ -2x - 5 \\ \underline{-2x - 4} \\ -1 \end{array}$$

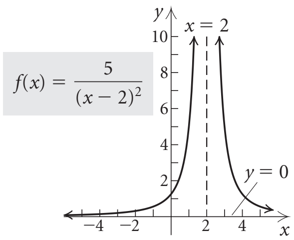
The oblique asymptote is $y = x - 2$. There is no horizontal asymptote.

3. The numerator is zero when $x \pm \sqrt{5}$, so the x-intercepts are $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.
4. $f(0) = \dfrac{0^2 - 5}{0 + 2} = -\dfrac{5}{2}$, so the y-intercept is $\left(0, -\dfrac{5}{2}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



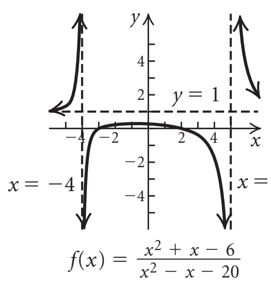
67. $f(x) = \dfrac{5}{(x-2)^2}$

1. The numerator and the denominator have no common factors. The denominator is zero when $x = 2$, so the domain excludes 2. It is $(-\infty, 2) \cup (2, \infty)$. The line $x = 2$ is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. $f(0) = \dfrac{5}{(0-2)^2} = \dfrac{5}{4}$, so the y-intercept is $\left(0, \dfrac{5}{4}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



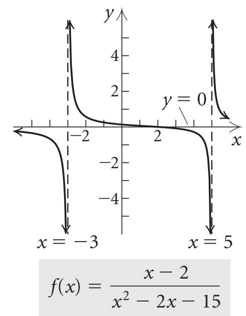
68. $f(x) = \dfrac{x^2+x-6}{x^2-x-20} = \dfrac{(x+3)(x-2)}{(x+4)(x-5)}$

1. The numerator and the denominator have no common factors. The denominator is zero when $x = -4$, or $x = 5$, so the domain excludes -4 and 5. It is $(-\infty, -4) \cup (-4, 5) \cup (5, \infty)$. The lines $x = -4$ and $x = 5$ are vertical asymptotes.
2. The numerator and the denominator have the same degree, so the horizontal asymptote is determined by the ratio of the leading coefficients, 1/1, or 1. Thus, $y = 1$ is the horizontal asymptote. There is no oblique asymptote.
3. The numerator is zero when $x = -3$ or $x = 2$, so the x-intercepts are $(-3, 0)$ and $(2, 0)$.
4. $f(0) = \dfrac{0^2+0-6}{0^2-0-20} = \dfrac{3}{10}$, so the y-intercept is $\left(0, \dfrac{3}{10}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



69. $f(x) = \dfrac{x-2}{x^2-2x-15} = \dfrac{x-2}{(x+3)(x-5)}$

1. The numerator and the denominator have no common factors. The denominator is zero when $x = -3$, or $x = 5$, so the domain excludes -3 and 5. It is $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$. The lines $x = -3$ and $x = 5$ are vertical asymptotes.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote. There is no oblique asymptote.
3. The numerator is zero when $x = 2$, so the x-intercept is $(2, 0)$.
4. $f(0) = \dfrac{0-2}{0^2-2\cdot 0-15} = \dfrac{2}{15}$, so the y-intercept is $\left(0, \dfrac{2}{15}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



70. Answers may vary. The numbers -2 and 3 must be zeros of the denominator.
$f(x) = \dfrac{1}{(x+2)(x-3)}$, or $f(x) = \dfrac{1}{x^2-x-6}$

71. Answers may vary. The numbers -2 and 3 must be zeros of the denominator, and -3 must be zero of the numerator. In addition, the numerator and denominator must have the same degree and the ratio of the leading coefficients must be 4.
$f(x) = \dfrac{4x(x+3)}{(x+2)(x-3)}$, or $f(x) = \dfrac{4x^2+12x}{x^2-x-6}$

72. a) The horizontal asymptote of $N(t)$ is the ratio of the leading coefficients of the numerator and denominator, 0.7/8, or 0.0875. Thus, $N(t) \to 0.0875$ as $t \to \infty$.

b) The medication never completely disappears from the body; a trace amount remains.

73.

$x^2-9<0$	Polynomial inequality
$x^2-9=0$	Related equation
$(x+3)(x-3)=0$	Factoring

The solutions of the related equation are -3 and 3. These numbers divide the x-axis into the intervals $(-\infty, -3)$, $(-3, 3)$, and $(3, \infty)$.

We let $f(x) = (x+3)(x-3)$ and test a value in each interval.

$(-\infty, -3)$: $f(-4) = 7 > 0$
$(-3, 3)$: $f(0) = -9 < 0$
$(3, \infty)$: $f(4) = 7 > 0$

Function values are negative only on $(-3, 3)$. The solution set is $(-3, 3)$.

74.

$2x^2>3x+2$	Polynomial inequality
$2x^2-3x-2>0$	Equivalent inequality
$2x^2-3x-2=0$	Related equation
$(2x+1)(x-2)=0$	Factoring

The solutions of the related equation are $-\dfrac{1}{2}$ and 2.

These numbers divide the x-axis into the intervals $\left(-\infty, -\dfrac{1}{2}\right)$, $\left(-\dfrac{1}{2}, 2\right)$, and $(2, \infty)$. We let $f(x) = (2x+1)(x-2)$ and test a value in each interval.

$\left(-\infty,-\frac{1}{2}\right)$: $f(-1)=3>0$

$\left(-\frac{1}{2},2\right)$: $f(0)=-2<0$

$(2,\infty)$: $f(3)=7>0$

Function values are positive on $\left(-\infty,-\frac{1}{2}\right)$ and $(2,\infty)$.

The solution set is $\left(-\infty,-\frac{1}{2}\right)\cup(2,\infty)$.

75. $(1-x)(x+4)(x-2)\le 0$ Polynomial inequality

$(1-x)(x+4)(x-2)=0$ Related equation

The solutions of the related equation are 1, -4 and 2. These numbers divide the x-axis into the intervals $(-\infty,-4)$, $(-4,1)$, $(1,2)$ and $(2,\infty)$.

We let $f(x)=(1-x)(x+4)(x-2)$ and test a value in each interval.

$(-\infty,-4):\ f(-5)=42>0$

$(-4,1):\ f(0)=-8<0$

$(1,2):\ f\left(\frac{3}{2}\right)=\frac{11}{8}>0$

$(2,\infty):\ f(3)=-14<0$

Function values are negative on $(-4,1)$ and $(2,\infty)$. Since the inequality symbol is $\le$, the endpoints of the intervals must be included in the solution set. It is $[-4,1]\cup[2,\infty)$.

76. $\frac{x-2}{x+3}<4$ Rational inequality

$\frac{x-2}{x+3}-4<0$ Equivalent inequality

$\frac{x-2}{x+3}-4=0$ Related equation

The denominator of $f(x)=\frac{x-2}{x+3}-4$ is zero when $x=-3$, so the function is not defined for this value of x. We solve the related equation $f(x)=0$.

$$\frac{x-2}{x+3}-4=0$$
$$(x+3)\left(\frac{x-2}{x+3}-4\right)=(x+3)\cdot 0$$
$$x-2-4(x+3)=0$$
$$x-2-4x-12=0$$
$$-3x-14=0$$
$$-3x=14$$
$$x=-\frac{14}{3}$$

The critical values are $-\frac{14}{3}$ and -3. They divide the x-axis into the intervals $\left(-\infty,-\frac{14}{3}\right)$, $\left(-\frac{14}{3},-3\right)$ and $(-3,\infty)$.

We test a value in each interval.

$\left(-\infty,-\frac{14}{3}\right):\ f(-8)=-2<0$

$\left(-\frac{14}{3},-3\right):\ f(-4)=2>0$

$(-3,\infty):\ f(2)=-4<0$

Function values are negative for $\left(-\infty,-\frac{14}{3}\right)$ and $(-3,\infty)$. The solution set is $\left(-\infty,-\frac{14}{3}\right)\cup(-3,\infty)$.

77. a) We write and solve a polynomial equation.

$$-16t^2+80t+224=0$$
$$-16(t^2-5t-14)=0$$
$$-16(t+2)(t-7)=0$$

The solutions are $t=-2$ and $t=7$. Only $t=7$ has meaning in this application. The rocket reaches the ground at $t=7$ seconds.

b) We write and solve a polynomial inequality.

$-16t^2+80t+224>320$ Polynomial inequality

$-16t^2+80t-96>0$ Equivalent inequality

$-16t^2+80t-96=0$ Related equation

$-16(t^2-5t+6)=0$

$-16(t-2)(t-3)=0$

The solutions of the related equation are 2 and 3. These numbers divide the t-axis into the intervals $(-\infty,2)$, $(2,3)$ and $(3,\infty)$. We restrict our discussion to values of t such that $0\le t\le 7$ since we know from part (a) the rocket is in the air for 7 sec. We consider the intervals $[0,2)$, $(2,3)$ and $(3,7]$. We let $f(t)=-16t^2+80t-96$ and test a value in each interval.

$[0,2)$: $f(1)=-32<0$

$(2,3)$: $f\left(\frac{5}{2}\right)=4>0$

$(3,7]$: $f(4)=-32<0$

Function values are positive on $(2,3)$. The solution set is $(2,3)$

78. We write and solve a rational inequality.

$$\frac{8000t}{4t^2+10}\ge 400$$
$$\frac{8000t}{4t^2+10}-400\ge 0$$

The denominator of $f(t)=\frac{8000t}{4t^2+10}-400$ has no real number zeros. We solve the related equation $f(t)=0$. Keep in mind that the formula gives the population in thousands.

$$\frac{8000t}{4t^2+10} - 400 = 0$$
$$(4t^2+10)\left(\frac{8000t}{4t^2+10} - 400\right) = (4t^2+10)\cdot 0$$
$$8000t - 1600t^2 - 4000 = 0$$
$$-1600t^2 + 8000t - 4000 = 0$$
$$-800(2t^2 - 10t + 5) = 0$$

Using the quadratic formula, we find that $t = \frac{5 \pm \sqrt{15}}{2}$.

These numbers divide the t-axis into the intervals $\left(-\infty, \frac{5-\sqrt{15}}{2}\right)$, $\left(\frac{5-\sqrt{15}}{2}, \frac{5+\sqrt{15}}{2}\right)$ and $\left(\frac{5+\sqrt{15}}{2}, \infty\right)$.

We test a value in each interval, restricting our discussion to values of $t \geq 0$.

$$\left[0, \frac{5-\sqrt{15}}{2}\right):\ f(0.5) = -\frac{400}{11} < 0$$
$$\left(\frac{5-\sqrt{15}}{2}, \frac{5+\sqrt{15}}{2}\right):\ f(1) = \frac{1200}{7} > 0$$
$$\left(\frac{5+\sqrt{15}}{2}, \infty\right):\ f(5) = -\frac{400}{11} < 0$$

Function values are positive on $\left(\frac{5-\sqrt{15}}{2}, \frac{5+\sqrt{15}}{2}\right)$.

Since the inequality symbol is $\geq$, the endpoints of the interval must be included in the solution set. It is $\left[\frac{5-\sqrt{15}}{2}, \frac{5+\sqrt{15}}{2}\right]$.

79. $g(x) = \frac{x^2+2x-3}{x^2-5x+6} = \frac{x^2+2x-3}{(x-2)(x-3)}$

The values of x that make the denominator 0 are 2 and 3, so the domain is $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$. Answer A is correct.

80. $f(x) = \frac{x-4}{(x+1)(x-2)(x+4)}$

The zeros of the denominator are -1, 2, and -4, so the vertical asymptotes are $x = -1$, $x = 2$, and $x = 4$. Answer C is correct.

81. $f(x) = -\frac{1}{2}x^4 + x^3 + 1$

The degree of the function is even and the leading coefficient is negative, so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$. In addition, $f(0) = 1$, so the y-intercept is $(0, 1)$. Thus B is the correct graph.

82.

$x^2 \geq 5 - 2x$ Polynomial inequality

$x^2 + 2x - 5 \geq 0$ Equivalent inequality

$x^2 + 2x - 5 = 0$ Related equation

Using the quadratic formula, we find that $x = -1 \pm \sqrt{6}$; that is, $x \approx -3.449$ or $x \approx 1.449$. These numbers divide the x-axis into the intervals $(-\infty, -1-\sqrt{6})$, $(-1-\sqrt{6}, -1+\sqrt{6})$ and $(-1+\sqrt{6}, \infty)$. We let $f(x) = x^2 + 2x - 5$ and test a value in each interval.

$$(-\infty, -1-\sqrt{6}):\ f(-4) = 3 > 0$$
$$(-1-\sqrt{6}, -1+\sqrt{6}):\ f(0) = -5 < 0$$
$$(-1+\sqrt{6}, \infty):\ f(2) = 3 > 0$$

Function values are positive on $(-\infty, -1-\sqrt{6})$ and $(-1+\sqrt{6}, \infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty, -1-\sqrt{6}] \cup [-1+\sqrt{6}, \infty)$.

83.

$$\left|1 - \frac{1}{x^2}\right| < 3$$
$$-3 < 1 - \frac{1}{x^2} < 3$$
$$-3 < \frac{x^2-1}{x^2} < 3$$
$$-3 < \frac{(x+1)(x-1)}{x^2} < 3$$
$$-3 < \frac{(x+1)(x-1)}{x^2} \ \textit{and}\ \frac{(x+1)(x-1)}{x^2} < 3$$

First, solve

$$-3 < \frac{(x+1)(x-1)}{x^2}$$
$$0 < \frac{(x+1)(x-1)}{x^2} + 3$$

The denominator of $f(x) = \frac{(x+1)(x-1)}{x^2} + 3$ is zero when $x = 0$, so the function is not defined for this value of x. Solve the related equation.

$$\frac{(x+1)(x-1)}{x^2} + 3 = 0$$
$$(x+1)(x-1) + 3x^2 = 0 \quad \text{Multiplying by } x^2$$
$$x^2 - 1 + 3x^2 = 0$$
$$4x^2 = 1$$
$$x^2 = \frac{1}{4}$$
$$x = \pm\frac{1}{2}$$

The critical values are $-\frac{1}{2}$, 0 and $\frac{1}{2}$. Test a value in each of the intervals determined by them.

$$\left(-\infty, -\frac{1}{2}\right):\ f(-1) = 3 > 0$$
$$\left(-\frac{1}{2}, 0\right):\ f\left(-\frac{1}{4}\right) = -12 < 0$$
$$\left(0, \frac{1}{2}\right):\ f\left(\frac{1}{4}\right) = -12 < 0$$
$$\left(\frac{1}{2}, \infty\right):\ f(1) = 3 > 0$$

The solution set for this portion of the inequality is $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

Next, solve

$$\frac{(x+1)(x-1)}{x^2} < 3$$
$$\frac{(x+1)(x-1)}{x^2} - 3 < 0$$

The denominator of $f(x)=\dfrac{(x+1)(x-1)}{x^2}-3$ is zero when $x=0$, so the function is not defined for this value of x. Now solve the related equation.

$$\begin{aligned}\frac{(x+1)(x-1)}{x^2}-3&=0\\(x+1)(x-1)-3x^2&=0\quad\text{Multiplying by } x^2\\x^2-1-3x^2&=0\\2x^2&=-1\\x^2&=-\frac{1}{2}\end{aligned}$$

There are no real solutions for this portion of the inequality. The solution set of the original inequality is $\left(-\infty,-\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$.

84. $x^4-2x^3+3x^2-2x+2=0$

The only possible rational solutions are ±1, ±2. Synthetic division shows that none are solutions. Try complex numbers.

$$\begin{array}{r|rrrrr} i & 1 & -2 & 3 & -2 & 2\\ & & i & -1-2i & 2+2i & -2\\\hline & 1 & -2+i & 2-2i & 2i & 0\end{array}$$

$$\begin{array}{r|rrrr} -i & 1 & -2+i & 2-2i & 2i\\ & & -i & 2i & -2i\\\hline & 1 & -2 & 2 & 0\end{array}$$

We can use the quadratic formula to find that the solutions of $x^2-2x+2=0$ are $1+i$ and $1-i$. The solutions are $1+i$, $1-i$, i, $-i$.

85. $(x-2)^{-3}<0$

$$\frac{1}{(x-2)^3}<0$$

The denominator of $f(x)=\dfrac{1}{(x-2)^3}$ is zero when $x=2$, so the function is not defined for this value of x. The related equation $\dfrac{1}{(x-2)^3}=0$ has no solution, so 2 is the only critical point. Test a value in each of the intervals determined by this critical point.

$(-\infty,2):\ f(1)=-1<0$

$(2,\infty):\ f(3)=1>0$

Function values are negative on $(-\infty,2)$. The solution set is $(-\infty,2)$.

86. $f(x)=x^3-1$

The possibilities for p/q are ±1. From the graph of $y=x^3-1$ we see that 1 might be a zero.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1\\ & & 1 & 1 & 1\\\hline & 1 & 1 & 1 & 0\end{array}$$

$f(x)=(x-1)(x^2+x+1)$

Using the quadratic formula, we find that the other zeros are $-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$.

$$\begin{aligned}f(x)&=(x-1)\left[x-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right]\left[x-\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right]\\&=(x-1)\left(x+\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\left(x+\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\end{aligned}$$

87. Divide $x^3+kx^2+kx-15$ by $x+3$.

$$\begin{array}{r|rrrr} -3 & 1 & k & k & -15\\ & & -3 & 9-3k & -27+6k\\\hline & 1 & -3+k & 9-2k & -42+6k\end{array}$$

Thus $f(-3)=-42+6k$.

We know that if $x+3$ is a factor of $f(x)$, then $f(-3)=0$. We solve $-42+6k=0$ for k.

$$\begin{aligned}-42+6k&=0\\6k&=42\\k&=7\end{aligned}$$

88. Divide $x^2-4x+3k$ by $x+5$.

$$\begin{array}{r|rrr} -5 & 1 & -4 & 3k\\ & & -5 & 45\\\hline & 1 & -9 & 45+3k\end{array}$$

Since the remainder is 33, we solve $45+3k=33$.

$$\begin{aligned}45+3k&=33\\3k&=-12\\k&=-4\end{aligned}$$

89. $f(x)=\sqrt{x^2+3x-10}$

Since we cannot take the square root of a negative number, then $x^2+3x-10\ge0$.

$$\begin{aligned}x^2+3x-10&\ge0\quad\text{Polynomial inequality}\\x^2+3x-10&=0\quad\text{Related equation}\\(x+5)(x-2)&=0\quad\text{Factoring}\end{aligned}$$

The solutions of the related equation are -5 and 2. These numbers divide the x-axis into the intervals $(-\infty,-5)$, $(-5,2)$ and $(2,\infty)$.

We let $g(x)=(x+5)(x-2)$ and test a value in each interval.

$(-\infty,-5):\ g(-6)=8>0$

$(-5,2):\ g(0)=-10<0$

$(2,\infty):\ g(3)=8>0$

Functions values are positive on $(-\infty,-5)$ and $(2,\infty)$. Since the equality symbol is $\ge$, the endpoints of the intervals must be included in the solution set. It is $(-\infty,-5]\cup[2,\infty)$.

90. $f(x)=\sqrt{x^2-3.1x+2.2}+1.75$

Since we cannot take the square root of a negative number, then $x^2-3.1x+2.2\ge0$.

$$\begin{aligned}x^2-3.1x+2.2&\ge0\quad\text{Polynomial inequality}\\x^2-3.1x+2.2&=0\quad\text{Related equation}\\(x-1.1)(x-2)&=0\quad\text{Factoring}\end{aligned}$$

The solutions of the related equation are 1.1 and 2. These numbers divide the x-axis into the intervals $(-\infty,1.1)$, $(1.1,2)$, and $(2,\infty)$.

We let $g(x) = (x - 1.1)(x - 2)$ and test a value in each interval.

$(-\infty, 1.1):\ g(0) = 2.2 > 0$

$(1.1, 2):\ g(1.5) = -0.2 < 0$

$(2, \infty):\ g(3) = 1.9 > 0$

Function values are positive on $(-\infty, 1.1)$ and $(2, \infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty, 1.1] \cup [2, \infty)$.

91. $f(x) = \dfrac{1}{\sqrt{5 - |7x + 2|}}$

We cannot take the square root of a negative number; neither can the denominator be zero. Thus we have $5 - |7x + 2| > 0$.

$$\begin{aligned} 5 - |7x + 2| &> 0 \quad \text{Polynomial inequality} \\ |7x + 2| &< 5 \\ -5 < 7x + 2 &< 5 \\ -7 < 7x &< 3 \\ -1 < x &< \frac{3}{7} \end{aligned}$$

The solution set is $\left(-1, \dfrac{3}{7}\right)$.

92. A polynomial function is a function that can be defined by a polynomial expression. A rational function is a function that can be defined as a quotient of two polynomials.

93. No; since imaginary zeros of polynomials with rational coefficients occur in conjugate pairs, a third-degree polynomial with rational coefficients can have at most two imaginary zeros. Thus, there must be at least one real zero.

94. Vertical asymptotes occur at any x-values that make the denominator zero. The graph of a rational function does not cross any vertical asymptotes. Horizontal asymptotes occur when the degree of the numerator is less than or equal to the degree of the denominator. Oblique asymptotes occur when the degree of the numerator is greater than the degree of the denominator. Graphs of rational functions may cross horizontal or oblique asymptotes.

95. If $P(x)$ is an even function, then $P(-x) = P(x)$ and thus $P(-x)$ has the same number of sign changes as $P(x)$. Hence, $P(x)$ has one negative real zero also.

96. A horizontal asymptote occurs when the degree of the numerator of a rational function is less than or equal to the degree of the denominator. An oblique asymptote occurs when the degree of the numerator is 1 greater than the degree of the denominator. Thus, a rational function cannot have both a horizontal asymptote and an oblique asymptote.

97. A quadratic inequality $ax^2+bx+c \leq 0$, $a > 0$, or $ax^2+bx+c \geq 0$, $a < 0$, has a solution set that is a closed interval.

Chapter 4 Test

1. $$\begin{aligned} f(x) &= 2x^3 + 6x^2 - x^4 + 11 \\ &= -x^4 + 2x^3 + 6x^2 + 11 \end{aligned}$$

The leading term is $-x^4$ and the leading coefficient is -1. The degree of the polynomial is 4, so the polynomial is quartic.

2. $h(x) = -4.7x + 29$

The leading term is $-4.7x$ and the leading coefficient is -4.7. The degree of the polynomial is 1, so the polynomial is linear.

3. $f(x) = x(3x - 5)(x - 3)^2(x + 1)^3$

The zeros of the function are 0, $\dfrac{5}{3}$, 3, and -1.

The factors x and $3x - 5$ each occur once, so the zeros 0 and $\dfrac{5}{3}$ have multiplicity 1.

The factor $x - 3$ occurs twice, so the zero 3 has multiplicity 2.

The factor $x + 1$ occurs three times, so the zero -1 has multiplicity 3.

4. In 1930, $x = 1930 - 1900 = 30$.

$f(30) = -0.0000007623221(30)^4 + 0.00021189064(30)^3 - 0.016314058(30)^2 + 0.2440779643(30) + 13.59260684 \approx 11.3\%$

In 1990, $x = 1990 - 1900 = 90$.

$f(90) = -0.0000007623221(90)^4 + 0.00021189064(90)^3 - 0.016314058(90)^2 + 0.2440779643(90) + 13.59260684 \approx 7.9\%$

In 2000, $x = 2000 - 1900 = 100$.

$f(100) = -0.0000007623221(100)^4$
$+ 0.00021189064(100)^3 - 0.016314058(100)^2$
$+ 0.2440779643(100) + 13.59260684 \approx 10.5\%$

5. $f(x) = x^3 - 5x^2 + 2x + 8$

1. The leading term is x^3. The degree, 3, is odd and the leading coefficient, 1, is positive so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.

2. We solve $f(x) = 0$. By the rational zeros theorem, we know that the possible rational zeros are 1, -1, 2, -2, 4, -4, 8, and -8. Synthetic division shows that 1 is not a zero. We try -1.

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8 \\ & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

We have $f(x) = (x + 1)(x^2 - 6x + 8) = (x + 1)(x - 2)(x - 4)$.

Now we find the zeros of $f(x)$.

$$\begin{aligned} x + 1 = 0 \quad &or \quad x - 2 = 0 \quad or \quad x - 4 = 0 \\ x = -1 \quad &or \quad x = 2 \quad or \quad x = 4 \end{aligned}$$

The zeros of the function are -1, 2, and 4, so the x-intercepts are $(-1, 0)$, $(2, 0)$, and $(4, 0)$.

3. The zeros divide the x-axis into 4 intervals, $(-\infty,-1)$, $(-1,2)$, $(2,4)$, and $(4,\infty)$. We choose a value for x in each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

   In $(-\infty,-1)$, test -3:

   $f(-3)=(-3)^3-5(-3)^2+2(-3)+8=-70<0$

   In $(-1,2)$, test 0:

   $f(0)=0^3-5(0)^2+2(0)+8=8>0$

   In $(2,4)$, test 3:

   $f(3)=3^3-5(3)^2+2(3)+8=-4<0$

   In $(4,\infty)$, test 5:

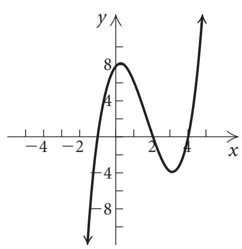
   $f(5)=5^3-5(5)^2+2(5)+8=18>0$

   Thus the graph lies below the x-axis on $(-\infty,-1)$ and on $(2,4)$ and above the x-axis on $(-1,2)$ and $(4,\infty)$. We also know the points $(-3,-70)$, $(0,8)$, $(3,-4)$, and $(5,18)$ are on the graph.

4. From Step 3 we know that $f(0)=8$, so the y-intercept is $(0,8)$.

5. We find additional points on the graph and draw the graph.

x	$f(x)$
-0.5	5.625
0.5	7.875
2.5	-2.625
6	56



$f(x)=x^3-5x^2+2x+8$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

6. $f(x)=-2x^4+x^3+11x^2-4x-12$

1. The leading term is $-2x^4$. The degree, 4, is even and the leading coefficient, -2, is negative so $x\to\infty$, $f(x)\to-\infty$ and as $x\to-\infty$, $f(x)\to-\infty$.

2. We solve $f(x)=0$.

   The possible rational zeros are ±1, ±2, ±3, ±4, ±6, ±12, $\pm\frac{1}{2}$, and $\pm\frac{3}{2}$. We try -1.

$$\begin{array}{r|rrrrr} -1 & -2 & 1 & 11 & -4 & -12 \\ & & 2 & -3 & -8 & 12 \\ \hline & -2 & 3 & 8 & -12 & 0 \end{array}$$

   We have $f(x)=(x+1)(-2x^3+3x^2+8x-12)$. Find the other zeros.

$$-2x^3+3x^2+8x-12=0$$
$$2x^3-3x^2-8x+12=0 \quad \text{Multiplying by } -1$$
$$x^2(2x-3)-4(2x-3)=0$$
$$(2x-3)(x^2-4)=0$$
$$(2x-3)(x+2)(x-2)=0$$
$$2x-3=0 \quad or \quad x+2=0 \quad or \quad x-2=0$$
$$2x=3 \quad or \quad x=-2 \quad or \quad x=2$$
$$x=\frac{3}{2} \quad or \quad x=-2 \quad or \quad x=2$$

   The zeros of the function are -1, $\frac{3}{2}$, -2, and 2, so the x-intercepts are $(-2,0)$, $(-1,0)$, $\left(\frac{3}{2},0\right)$, and $(2,0)$.

3. The zeros divide the x-axis into 5 intervals, $(-\infty,-2)$, $(-2,-1)$, $\left(-1,\frac{3}{2}\right)$, $\left(\frac{3}{2},2\right)$, and $(2,\infty)$. We choose a value for x in each interval and find $f(x)$. This tells us the sign of $f(x)$ for all values of x in that interval.

   In $(-\infty,-2)$, test -3: $f(-3)=-90<0$

   In $(-2,-1)$, test -1.5: $f(-1.5)=5.25>0$

   In $\left(-1,\frac{3}{2}\right)$, test 0: $f(0)=-12<0$

   In $\left(\frac{3}{2},2\right)$, test 1.75: $f(1.75)\approx1.29>0$

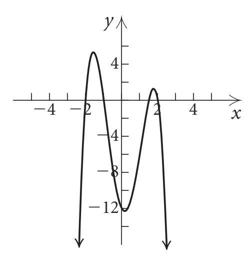
   In $(2,\infty)$, test 3: $f(3)=-60<0$

   Thus the graph lies below the x-axis on $(-\infty,-2)$, on $\left(-1,\frac{3}{2}\right)$, and on $(2,\infty)$ and above the x-axis on $(-2,-1)$ and on $\left(\frac{3}{2},2\right)$. We also know the points $(-3,-90)$, $(-1.5,5.25)$, $(0,-12)$, $(1.75,1.29)$, and $(3,-60)$ are on the graph.

4. From Step 3 we know that $f(0)=-12$, so the y-intercept is $(0,-12)$.

5. We find additional points on the graph and draw the graph.

x	$f(x)$
-0.5	-7.5
0.5	-11.25
2.5	-15.75



$f(x)=-2x^4+x^3+11x^2-4x-12$

6. Checking the graph as described on page 311 in the text, we see that it appears to be correct.

7. $f(0)=-5\cdot0^2+3=3$

$f(2)=-5\cdot2^2+3=-17$

By the intermediate value theorem, since $f(0)$ and $f(2)$ have opposite signs, $f(x)$ has a zero between 0 and 2.

8. $g(-2)=2(-2)^3+6(-2)^2-3=5$

$g(-1)=2(-1)^3+6(-1)^2-3=1$

Since both $g(-2)$ and $g(-1)$ are positive, we cannot use the intermediate value theorem to determine if there is a zero between -2 and -1.

9.
$$\begin{array}{r l}
 & x^3+4x^2+4x+6 \\
x-1 & \overline{)\,x^4+3x^3+0x^2+2x-5} \\
 & \underline{x^4-x^3} \\
 & 4x^3+0x^2 \\
 & \underline{4x^3-4x^2} \\
 & 4x^2+2x \\
 & \underline{4x^2-4x} \\
 & 6x-5 \\
 & \underline{6x-6} \\
 & 1
\end{array}$$

The quotient is x^3+4x^2+4x+6; the remainder is 1.

$P(x)=(x-1)(x^3+4x^2+4x+6)+1$

10.
$$\begin{array}{r|rrrr}
5 & 3 & 0 & -12 & 7 \\
 & & 15 & 75 & 315 \\
\hline
 & 3 & 15 & 63 & 322
\end{array}$$

$Q(x)=3x^2+15x+63$; $R(x)=322$

11.
$$\begin{array}{r|rrrr}
-3 & 2 & -6 & 1 & -4 \\
 & & -6 & 36 & -111 \\
\hline
 & 2 & -12 & 37 & -115
\end{array}$$

$P(-3)=-115$

12.
$$\begin{array}{r|rrrr}
-2 & 1 & 4 & 1 & -6 \\
 & & -2 & -4 & 6 \\
\hline
 & 1 & 2 & -3 & 0
\end{array}$$

$f(-2)=0$, so -2 is a zero of $f(x)$.

13. The function can be written in the form

$f(x)=a_n(x+3)^2(x)(x-6)$.

The simplest polynomial is obtained if we let $a_n=1$.

$$\begin{aligned}
f(x)&=(x+3)^2(x)(x-6)\\
&=(x^2+6x+9)(x^2-6x)\\
&=x^4+6x^3+9x^2-6x^3-36x^2-54x\\
&=x^4-27x^2-54x
\end{aligned}$$

14. A polynomial function of degree 5 can have at most 5 zeros. Since $f(x)$ has rational coefficients, in addition to the 3 given zeros, the other zeros are the conjugates of $\sqrt{3}$ and $2-i$, or $-\sqrt{3}$ and $2+i$.

15. $-3i$ is also a zero.

$$\begin{aligned}
f(x)&=(x+10)(x-3i)(x+3i)\\
&=(x+10)(x^2+9)\\
&=x^3+10x^2+9x+90
\end{aligned}$$

16. $\sqrt{3}$ and $1+i$ are also zeros.

$$\begin{aligned}
f(x)&=(x-0)(x+\sqrt{3})(x-\sqrt{3})[x-(1-i)][x-(1+i)]\\
&=x(x^2-3)[(x-1)+i][(x-1)-i]\\
&=(x^3-3x)(x^2-2x+1+1)\\
&=(x^3-3x)(x^2-2x+2)\\
&=x^5-2x^4+2x^3-3x^3+6x^2-6x\\
&=x^5-2x^4-x^3+6x^2-6x
\end{aligned}$$

17. $f(x)=2x^3+x^2-2x+12$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}:\ \dfrac{\pm1,\pm2,\pm3,\pm4,\pm6,\pm12}{\pm1,\pm2}$

Possibilities for p/q: $\pm1,\pm2,\pm3,\pm4,\pm6,\pm12,\pm\frac{1}{2},\pm\frac{3}{2}$

18. $h(x)=10x^4-x^3+2x-5$

$\dfrac{\text{Possibilities for } p}{\text{Possibilities for } q}:\ \dfrac{\pm1,\pm5}{\pm1,\pm2,\pm5,\pm10}$

Possibilities for p/q: $\pm1,\pm5,\pm\frac{1}{2},\pm\frac{5}{2},\pm\frac{1}{5},\pm\frac{1}{10}$

19. $f(x)=x^3+x^2-5x-5$

a) Possibilities for p/q: $\pm1,\pm5$

From the graph of $y=f(x)$, we see that -1 might be a zero.

$$\begin{array}{r|rrrr}
-1 & 1 & 1 & -5 & -5 \\
 & & -1 & 0 & 5 \\
\hline
 & 1 & 0 & -5 & 0
\end{array}$$

We have $f(x)=(x+1)(x^2-5)$. We find the other zeros.

$$\begin{aligned}
x^2-5&=0\\
x^2&=5\\
x&=\pm\sqrt{5}
\end{aligned}$$

The rational zero is -1. The other zeros are $\pm\sqrt{5}$.

b) $f(x)=(x+1)(x-\sqrt{5})(x+\sqrt{5})$

20. $g(x)=2x^4-11x^3+16x^2-x-6$

a) Possibilities for p/q: $\pm1,\pm2,\pm3,\pm6,\pm\frac{1}{2},\pm\frac{3}{2}$

From the graph of $y=g(x)$, we see that $-\frac{1}{2}$, 1, 2, and 3 might be zeros. We try $-\frac{1}{2}$.

$$\begin{array}{r|rrrrr}
-\frac{1}{2} & 2 & -11 & 16 & -1 & -6 \\
 & & -1 & 6 & -11 & 6 \\
\hline
 & 2 & -12 & 22 & -12 & 0
\end{array}$$

Now we try 1.

$$\begin{array}{r|rrrr}
1 & 2 & -12 & 22 & -12 \\
 & & 2 & -10 & 12 \\
\hline
 & 2 & -10 & 12 & 0
\end{array}$$

We have $g(x)=\left(x+\frac{1}{2}\right)(x-1)(2x^2-10x+12)=$ $2\left(x+\frac{1}{2}\right)(x-1)(x^2-5x+6)$. We find the other zeros.

$$\begin{aligned}
x^2-5x+6&=0\\
(x-2)(x-3)&=0\\
x-2=0 \quad &or \quad x-3=0\\
x=2 \quad &or \quad x=3
\end{aligned}$$

The rational zeros are $-\frac{1}{2}$, 1, 2, and 3. These are the only zeros.

b) $$\begin{aligned}
g(x)&=2\left(x+\frac{1}{2}\right)(x-1)(x-2)(x-3)\\
&=(2x+1)(x-1)(x-2)(x-3)
\end{aligned}$$

21. $h(x) = x^3 + 4x^2 + 4x + 16$

a) Possibilities for p/q: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

From the graph of $h(x)$, we see that -4 might be a zero.

-4	1	4	4	16
		-4	0	-16
	1	0	4	0

We have $h(x) = (x + 4)(x^2 + 4)$. We find the other zeros.

$$x^2 + 4 = 0$$
$$x^2 = -4$$
$$x = \pm 2i$$

The rational zero is -4. The other zeros are $\pm 2i$.

b) $h(x) = (x + 4)(x + 2i)(x - 2i)$

22. $f(x) = 3x^4 - 11x^3 + 15x^2 - 9x + 2$

a) Possibilities for p/q: $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

From the graph of $f(x)$, we see that $\frac{2}{3}$ and 1 might be zeros. We try $\frac{2}{3}$.

$\frac{2}{3}$	3	-11	15	-9	2
		2	-6	6	-2
	3	-9	9	-3	0

Now we try 1.

1	3	-9	9	-3
		3	-6	3
	3	-6	3	0

We have $f(x) = \left(x - \frac{2}{3}\right)(x - 1)(3x^2 - 6x + 3) = 3\left(x - \frac{2}{3}\right)(x - 1)(x^2 - 2x + 1)$. We find the other zeros.

$$x^2 - 2x + 1 = 0$$
$$(x - 1)(x - 1) = 0$$
$$x - 1 = 0 \quad or \quad x - 1 = 0$$
$$x = 1 \quad or \quad x = 1$$

The rational zeros are $\frac{2}{3}$ and 1. (The zero 1 has multiplicity 3.) These are the only zeros.

b) $$f(x) = 3\left(x - \frac{2}{3}\right)(x - 1)(x - 1)(x - 1)$$
$$= (3x - 2)(x - 1)^3$$

23. $g(x) = -x^8 + 2x^6 - 4x^3 - 1$

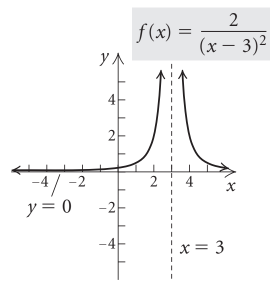
There are 2 variations in sign in $g(x)$, so there are 2 or 0 positive real zeros.

$$g(-x) = -(-x)^8 + 2(-x)^6 - 4(-x)^3 - 1$$
$$= -x^8 + 2x^6 + 4x^3 - 1$$

There are 2 variations in sign in $g(-x)$, so there are 2 or 0 negative real zeros.

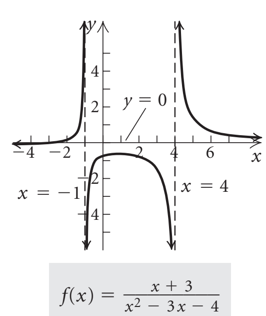
24. $f(x) = \dfrac{2}{(x - 3)^2}$

1. The numerator and the denominator have no common factors. The denominator is zero when $x = 3$, so the domain excludes 3. it is $(-\infty, 3) \cup (3, \infty)$. The line $x = 3$ is the vertical asymptote.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote.
3. The numerator has no zeros, so there is no x-intercept.
4. $f(0) = \dfrac{2}{(0 - 3)^2} = \dfrac{2}{9}$, so the y-intercept is $\left(0, \dfrac{2}{9}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



25. $f(x) = \dfrac{x + 3}{x^2 - 3x - 4} = \dfrac{x + 3}{(x + 1)(x - 4)}$

1. The numerator and the denominator have no common factors. The denominator is zero when $x = -1$ or $x = 4$, so the domain excludes -1 and 4. It is $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$. The lines $x = -1$ and $x = 4$ are vertical asymptotes.
2. Because the degree of the numerator is less than the degree of the denominator, the x-axis, or $y = 0$, is the horizontal asymptote.
3. The numerator is zero at $x = -3$, so the x-intercept is $(-3, 0)$.
4. $f(0) = \dfrac{0 + 3}{(0 + 1)(0 - 4)} = -\dfrac{3}{4}$, so the y-intercept is $\left(0, -\dfrac{3}{4}\right)$.
5. Find other function values to determine the shape of the graph and then draw it.



26. Answers may vary. The numbers -1 and 2 must be zeros of the denominator, and -4 must be a zero of the numerator.

$f(x) = \dfrac{x+4}{(x+1)(x-2)}$, or $f(x) = \dfrac{x+4}{x^2-x-2}$

27.

$$\begin{aligned} 2x^2 &> 5x+3 && \text{Polynomial inequality} \\ 2x^2-5x-3 &> 0 && \text{Equivalent inequality} \\ 2x^2-5x-3 &= 0 && \text{Related equation} \\ (2x+1)(x-3) &= 0 && \text{Factoring} \end{aligned}$$

The solutions of the related equation are $-\frac{1}{2}$ and 3. These numbers divide the x-axis into the intervals $\left(-\infty, -\frac{1}{2}\right)$, $\left(-\frac{1}{2}, 3\right)$, and $(3, \infty)$.

We let $f(x) = (2x+1)(x-3)$ and test a value in each interval.

$\left(-\infty, -\frac{1}{2}\right)$: $f(-1) = 4 > 0$

$\left(-\frac{1}{2}, 3\right)$: $f(0) = -3 < 0$

$(3, \infty)$: $f(4) = 9 > 0$

Function values are positive on $\left(-\infty, -\frac{1}{2}\right)$ and $(3, \infty)$. The solution set is $\left(-\infty, -\frac{1}{2}\right) \cup (3, \infty)$.

28.

$$\begin{aligned} \frac{x+1}{x-4} &\le 3 && \text{Rational inequality} \\ \frac{x+1}{x-4} - 3 &\le 0 && \text{Equivalent inequality} \end{aligned}$$

The denominator of $f(x) = \dfrac{x+1}{x-4} - 3$ is zero when $x = 4$, so the function is not defined for this value of x. We solve the related equation $f(x) = 0$.

$$\begin{aligned} \frac{x+1}{x-4} - 3 &= 0 \\ (x-4)\left(\frac{x+1}{x-4} - 3\right) &= (x-4)\cdot 0 \\ x+1-3(x-4) &= 0 \\ x+1-3x+12 &= 0 \\ -2x+13 &= 0 \\ 2x &= 13 \\ x &= \frac{13}{2} \end{aligned}$$

The critical values are 4 and $\frac{13}{2}$. They divide the x-axis into the intervals $(-\infty, 4)$, $\left(4, \frac{13}{2}\right)$ and $\left(\frac{13}{2}, \infty\right)$. We test a value in each interval.

$(-\infty, 4)$: $f(3) = -7 < 0$

$\left(4, \frac{13}{2}\right)$: $f(5) = 3 > 0$

$\left(\frac{13}{2}, \infty\right)$: $f(9) = -1 < 0$

Function values are negative for $(-\infty, 4)$ and $\left(\frac{13}{2}, \infty\right)$. Since the inequality symbol is $\le$, the endpoint of the interval $\left(\frac{13}{2}, \infty\right)$ must be included in the solution set. It is $(-\infty, 4) \cup \left[\frac{13}{2}, \infty\right)$.

29. a) We write and solve a polynomial equation.

$$\begin{aligned} -16t^2+64t+192 &= 0 \\ -16(t^2-4t-12) &= 0 \\ -16(t+2)(t-6) &= 0 \end{aligned}$$

The solutions are $t = -2$ and $t = 6$. Only $t = 6$ has meaning in this application. The rocket reaches the ground at $t = 6$ seconds.

b) We write and solve a polynomial inequality.

$$\begin{aligned} -16t^2+64t+192 &> 240 \\ -16t^2+64t-48 &> 0 \\ -16t^2+64t-48 &= 0 && \text{Related equation} \\ -16(t^2-4t+3) &= 0 \\ -16(t-1)(t-3) &= 0 \end{aligned}$$

The solutions of the related equation are 1 and 3. These numbers divide the t-axis into the intervals $(-\infty, 1)$, $(1, 3)$ and $(3, \infty)$. Because the rocket returns to the ground at $t = 6$, we restrict our discussion to values of t such that $0 \le t \le 6$. We consider the intervals $[0, 1)$, $(1, 3)$ and $(3, 6]$. We let $f(t) = -16t^2 + 64t - 48$ and test a value in each interval.

$[0, 1)$: $f\left(\frac{1}{2}\right) = -20 < 0$

$(1, 3)$: $f(2) = 16 > 0$

$(3, 6]$: $f(4) = -48 < 0$

Function values are positive on $(1, 3)$. The solution set is $(1, 3)$.

30. $f(x) = x^3 - x^2 - 2$

The degree of the function is odd and the leading coefficient is positive, so as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$. In addition, $f(0) = -2$, so the y-intercept is $(0, -2)$. Thus D is the correct graph.

31. $f(x) = \sqrt{x^2+x-12}$

Since we cannot take the square root of a negative number, then $x^2 + x - 12 \ge 0$.

$$\begin{aligned} x^2+x-12 &\ge 0 && \text{Polynomial inequality} \\ x^2+x-12 &= 0 && \text{Related equation} \\ (x+4)(x-3) &= 0 && \text{Factoring} \end{aligned}$$

The solutions of the related equation are -4 and 3. These numbers divide the x-axis into the intervals $(-\infty, -4)$, $(-4, 3)$ and $(3, \infty)$.

We let $f(x) = (x+4)(x-3)$ and test a value in each interval.

$(-\infty, -4)$: $g(-5) = 8 > 0$

$(-4, 3)$: $g(0) = -12 < 0$

$(3, \infty)$: $g(4) = 8 > 0$

Function values are positive on $(-\infty, -4)$ and $(3, \infty)$. Since the inequality symbol is $\geq$, the endpoints of the intervals must be included in the solution set. It is $(-\infty, -4] \cup [3, \infty)$.