Chapter 5

Exponential and Logarithmic Functions

Exercise Set 5.1

1. We interchange the first and second coordinates of each ordered pair to find the inverse of the relation. It is

$\{(8,7), (8,-2), (-4,3), (-8,8)\}$.

2. $\{(1,0), (6,5), (-4,-2)\}$

3. We interchange the first and second coordinates of each ordered pair to find the inverse of the relation. It is

$\{(-1,-1), (4,-3)\}$.

4. $\{(3,-1), (5,2), (5,-3), (0,2)\}$

5. Interchange x and y.

$$\begin{array}{l} y = 4x{-}5 \\ \downarrow \quad\;\; \downarrow \\ x = 4y{-}5 \end{array}$$

6. $2y^2 + 5x^2 = 4$

7. Interchange x and y.

$$\begin{array}{l} x^3 y = -5 \\ \downarrow\downarrow \\ y^3 x = -5 \end{array}$$

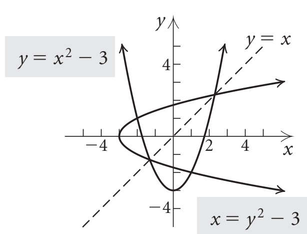
8. $x = 3y^2 - 5y + 9$

9. Interchange x and y.

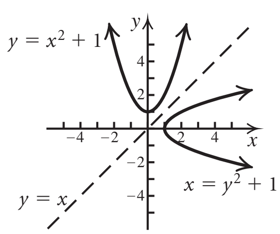
$$\begin{array}{l} x = y^2{-}2y \\ \downarrow \quad\; \downarrow \;\; \downarrow \\ y = x^2{-}2x \end{array}$$

10. $y = \dfrac{1}{2}x + 4$

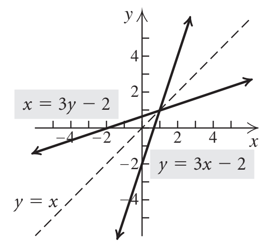
11. Graph $x = y^2 - 3$. Some points on the graph are $(-3,0)$, $(-2,-1)$, $(-2,1)$, $(1,-2)$, and $(1,2)$. Plot these points and draw the curve. Then reflect the graph across the line $y = x$.



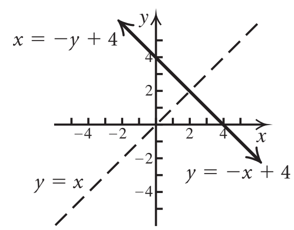
12.



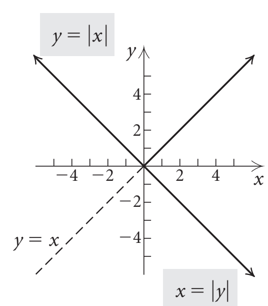
13. Graph $y = 3x - 2$. The intercepts are $(0,-2)$ and $\left(\dfrac{2}{3}, 0\right)$. Plot these points and draw the line. Then reflect the graph across the line $y = x$.



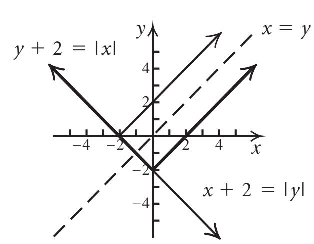
14.



15. Graph $y = |x|$. Some points on the graph are $(0,0)$, $(-2,2)$, $(2,2)$, $(-5,5)$, and $(5,5)$. Plot these points and draw the graph. Then reflect the graph across the line $y = x$.



16.



17. We show that if $f(a) = f(b)$, then $a = b$.

$$\begin{aligned} \frac{1}{3}a - 6 &= \frac{1}{3}b - 6 \\ \frac{1}{3}a &= \frac{1}{3}b \qquad \text{Adding 6} \\ a &= b \qquad \text{Multiplying by 3} \end{aligned}$$

Thus f is one-to-one.

18. Assume $f(a) = f(b)$.

$$4 - 2a = 4 - 2b$$
$$-2a = -2b$$
$$a = b$$

Then f is one-to-one.

19. We show that if $f(a) = f(b)$, then $a = b$.

$$a^3 + \frac{1}{2} = b^3 + \frac{1}{2}$$
$$a^3 = b^3 \qquad \text{Subtracting } \frac{1}{2}$$
$$a = b \qquad \text{Taking cube roots}$$

Thus f is one-to-one.

20. Assume $f(a) = f(b)$.

$$\sqrt[3]{a} = \sqrt[3]{b}$$
$$a = b \qquad \text{Using the principle of powers}$$

Then f is one-to-one.

21. $g(-1) = 1 - (-1)^2 = 1 - 1 = 0$ and $g(1) = 1 - 1^2 = 1 - 1 = 0$, so $g(-1) = g(1)$ but $-1 \neq 1$. Thus the function is not one-to-one.

22. $g(-1) = 4$ and $g(1) = 4$, so $g(-1) = g(1)$ but $-1 \neq 1$. Thus the function is not one-to-one.

23. $f(-2) = (-2)^4 - (-2)^2 = 16 - 4 = 12$ and $f(2) = 2^4 - 2^2 = 16 - 4 = 12$, so $f(-2) = f(2)$ but $-2 \neq 2$. Thus the function is not one-to-one.

24. $g(-1) = 1$ and $g(1) = 1$ so $g(-1) = g(1)$ but $-1 \neq 1$. Thus the function is not one-to-one.

25. The function is one-to-one, because no horizontal line crosses the graph more than once.

26. The function is one-to-one, because no horizontal line crosses the graph more than once.

27. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.

28. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.

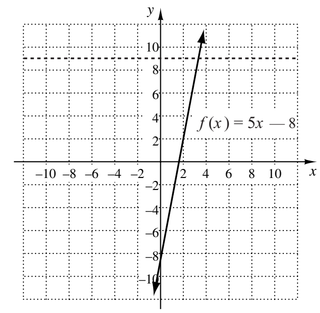
29. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.

30. The function is one-to-one, because no horizontal line crosses the graph more than once.

31. The function is one-to-one, because no horizontal line crosses the graph more than once.

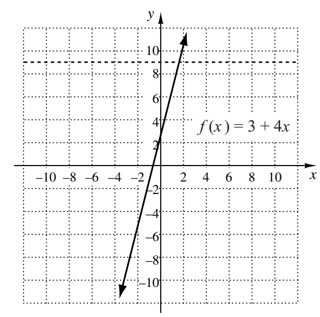
32. The function is one-to-one, because no horizontal line crosses the graph more than once.

33. The graph of $f(x) = 5x - 8$ is shown below.



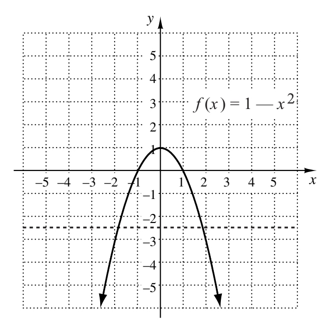
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

34. The graph of $f(x) = 3 + 4x$ is shown below.



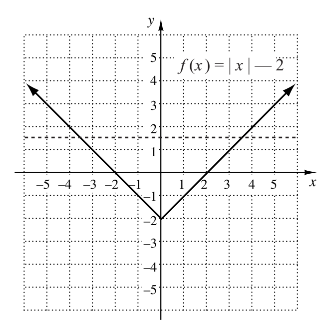
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

35. The graph of $f(x) = 1 - x^2$ is shown below.



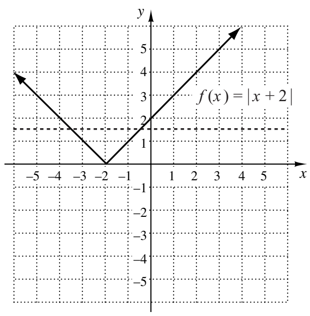
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

36. The graph of $f(x) = |x| - 2$ is shown below.



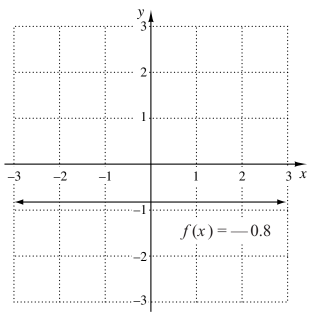
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

37. The graph of $f(x) = |x+2|$ is shown below.



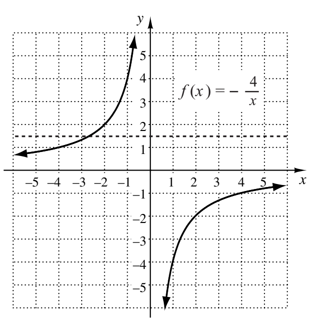
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

38. The graph of $f(x) = -0.8$ is shown below.



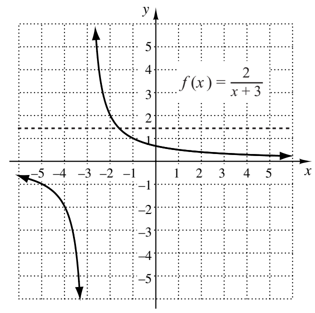
Since the horizontal line $y = -0.8$ crosses the graph more than once, the function is not one-to-one.

39. The graph of $f(x) = -\dfrac{4}{x}$ is shown below.



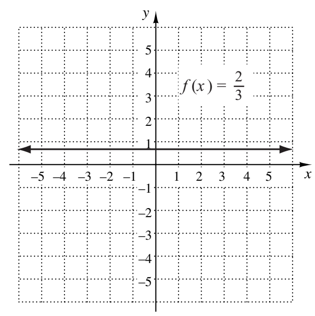
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

40. The graph of $f(x) = \dfrac{2}{x+3}$ is shown below.



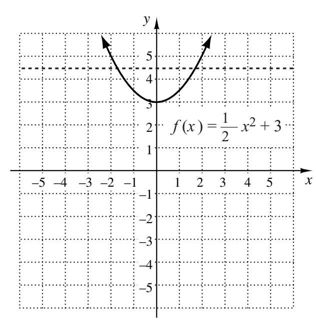
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

41. The graph of $f(x) = \dfrac{2}{3}$ is shown below.



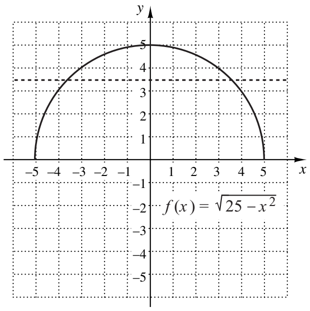
Since the horizontal line $y = \dfrac{2}{3}$ crosses the graph more than once, the function is not one-to-one.

42. The graph of $f(x) = \dfrac{1}{2}x^2 + 3$ is shown below.



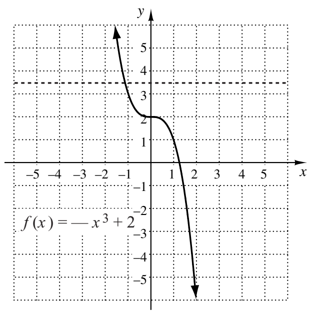
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

43. The graph of $f(x) = \sqrt{25 - x^2}$ is shown below.



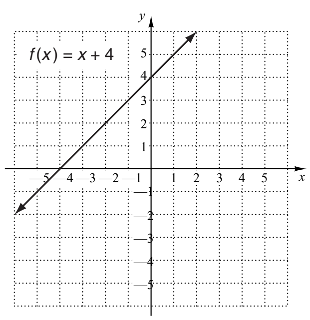
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

44. The graph of $f(x) = -x^3 + 2$ is shown below.



Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

45. a) The graph of $f(x) = x+4$ is shown below. It passes the horizontal-line test, so it is one-to-one.



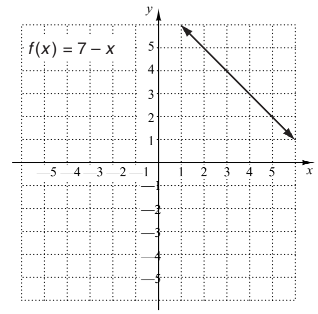
b) Replace $f(x)$ with y: $y = x + 4$

Interchange x and y: $x = y + 4$

Solve for y: $x - 4 = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x - 4$

46. a) The graph of $f(x) = 7-x$ is shown below. It passes the horizontal-line test, so it is one-to-one.



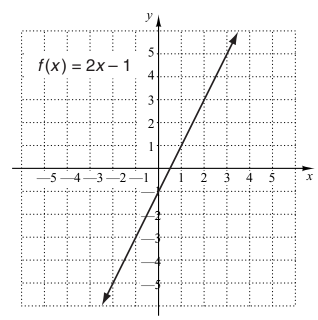
b) Replace $f(x)$ with y: $y = 7 - x$

Interchange x and y: $x = 7 - y$

Solve for y: $y = 7 - x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = 7 - x$

47. a) The graph of $f(x) = 2x-1$ is shown below. It passes the horizontal-line test, so it is one-to-one.



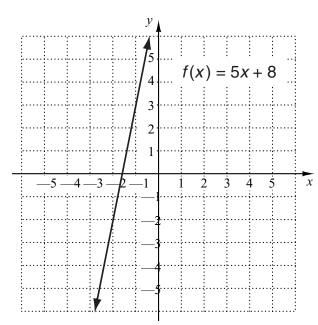
b) Replace $f(x)$ with y: $y = 2x - 1$

Interchange x and y: $x = 2y - 1$

Solve for y: $\dfrac{x+1}{2} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x+1}{2}$

48. a) The graph of $f(x) = 5x+8$ is shown below. It passes the horizontal-line test, so it is one-to-one.



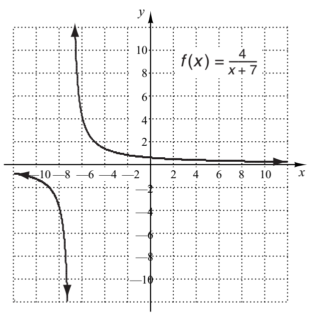
b) Replace $f(x)$ with y: $y = 5x + 8$

Interchange x and y: $x = 5y + 8$

Solve for y: $\dfrac{x-8}{5} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x-8}{5}$

49. a) The graph of $f(x) = \dfrac{4}{x+7}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \dfrac{4}{x+7}$

Interchange x and y: $x = \dfrac{4}{y+7}$

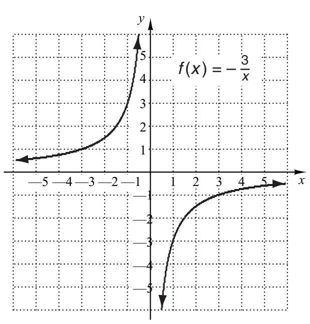
Solve for y: $x(y+7) = 4$

$$y + 7 = \frac{4}{x}$$

$$y = \frac{4}{x} - 7$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{4}{x} - 7$

50. a) The graph of $f(x) = -\dfrac{3}{x}$ is shown below. It passes the horizontal-line test, so it is one-to-one.



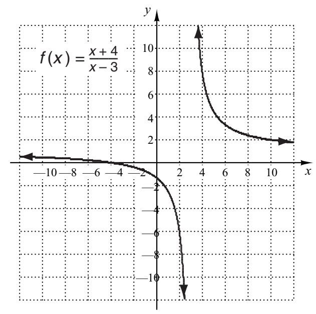
b) Replace $f(x)$ with y: $y = -\dfrac{3}{x}$

Interchange x and y: $x = -\dfrac{3}{y}$

Solve for y: $y = -\dfrac{3}{x}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = -\dfrac{3}{x}$

51. a) The graph of $f(x) = \dfrac{x+4}{x-3}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \dfrac{x+4}{x-3}$

Interchange x and y: $x = \dfrac{y+4}{y-3}$

Solve for y: $(y-3)x = y + 4$

$$xy - 3x = y + 4$$

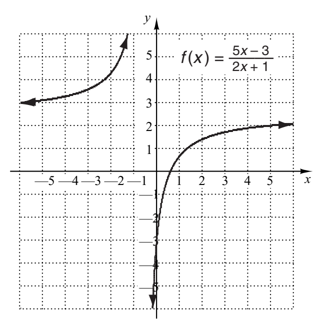
$$xy - y = 3x + 4$$

$$y(x-1) = 3x + 4$$

$$y = \frac{3x+4}{x-1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{3x+4}{x-1}$

52. a) The graph of $f(x) = \dfrac{5x-3}{2x+1}$ is shown below. It passes the horizontal-line test, so it is one-to-one.



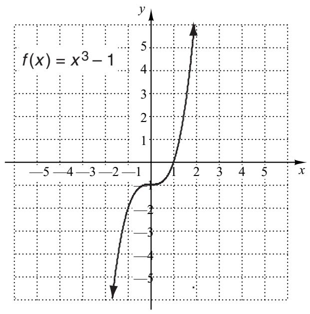
b) Replace $f(x)$ with y: $y = \dfrac{5x-3}{2x+1}$

Interchange x and y: $x = \dfrac{5y-3}{2y+1}$

Solve for y: $\dfrac{x+3}{5-2x} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x+3}{5-2x}$

53. a) The graph of $f(x) = x^3 - 1$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = x^3 - 1$

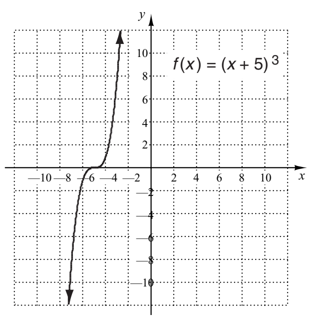
Interchange x and y: $x = y^3 - 1$

Solve for y: $x + 1 = y^3$

$\sqrt[3]{x+1} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x+1}$

54. a) The graph of $f(x) = (x + 5)^3$ is shown below. It passes the horizontal-line test, so it is one-to-one.



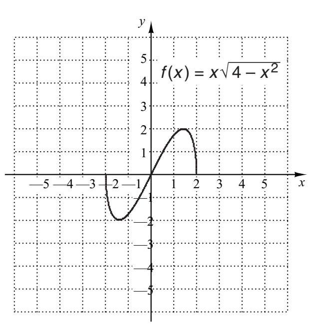
b) Replace $f(x)$ with y: $y = (x + 5)^3$

Interchange x and y: $x = (y + 5)^3$

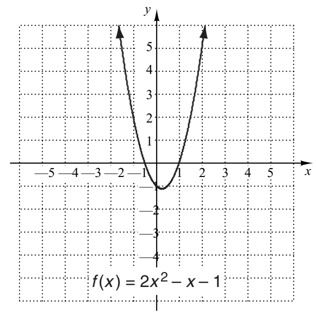
Solve for y: $\sqrt[3]{x} - 5 = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x} - 5$

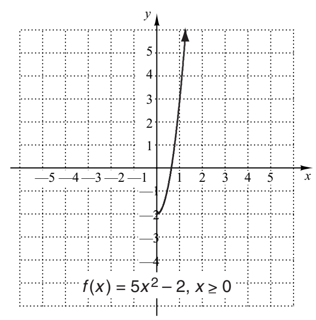
55. a) The graph of $f(x) = x\sqrt{4 - x^2}$ is shown below. Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one and thus does not have an inverse that is a function.



56. a) The graph of $f(x) = 2x^2 - x - 1$ is shown below. Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one and thus does not have an inverse that is a function.



57. a) The graph of $f(x) = 5x^2 - 2$, $x \geq 0$ is shown below. It passes the horizontal-line test, so it is one-to-one.



b) Replace $f(x)$ with y: $y = 5x^2 - 2$

Interchange x and y: $x = 5y^2 - 2$

Solve for y: $x + 2 = 5y^2$

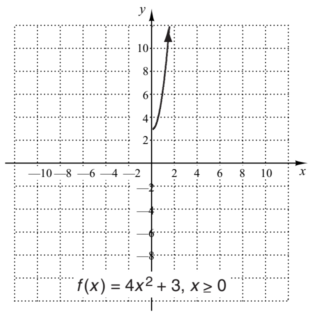
$$\frac{x+2}{5} = y^2$$

$$\sqrt{\frac{x+2}{5}} = y$$

(We take the principal square root, because $x \geq 0$ in the original equation.)

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt{\dfrac{x+2}{5}}$ for all x in the range of $f(x)$, or $f^{-1}(x) = \sqrt{\dfrac{x+2}{5}}$, $x \geq -2$

58. a) The graph of $f(x) = 4x^2 + 3$, $x \geq 0$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = 4x^2 + 3$

Interchange x and y: $x = 4y^2 + 3$

Solve for y: $x - 3 = 4y^2$

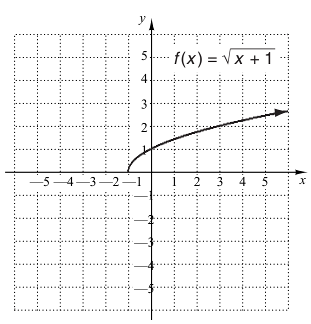
$$\frac{x-3}{4} = y^2$$

$$\frac{\sqrt{x-3}}{2} = y$$

(We take the principal square root since $x \geq 0$ in the original function.)

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{\sqrt{x-3}}{2}$ for all x in the range of $f(x)$, or $f^{-1}(x) = \dfrac{\sqrt{x-3}}{2}$, $x \geq 3$

59. a) The graph of $f(x) = \sqrt{x+1}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \sqrt{x+1}$

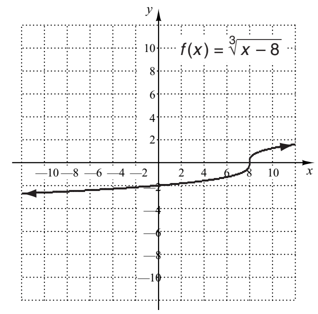
Interchange x and y: $x = \sqrt{y+1}$

Solve for y: $x^2 = y + 1$

$$x^2 - 1 = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x^2 - 1$ for all x in the range of $f(x)$, or $f^{-1}(x) = x^2 - 1$, $x \geq 0$.

60. a) The graph of $f(x) = \sqrt[3]{x-8}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \sqrt[3]{x-8}$

Interchange x and y: $x = \sqrt[3]{y-8}$

Solve for y: $x^3 + 8 = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x^3 + 8$

61. $f(x) = 3x$

The function f multiplies an input by 3. Then to reverse this procedure, f^{-1} would divide each of its inputs by 3. Thus, $f^{-1}(x) = \dfrac{x}{3}$, or $f^{-1}(x) = \dfrac{1}{3}x$.

62. $f(x) = \dfrac{1}{4}x + 7$

The function f multiplies an input by $\dfrac{1}{4}$ and then adds 7. To reverse this procedure, f^{-1} would subtract 7 from each of its inputs and then multiply by 4. Thus, $f^{-1}(x) = 4(x - 7)$.

63. $f(x) = -x$

The outputs of f are the opposites, or additive inverses, of the inputs. Then the outputs of f^{-1} are the opposites of its inputs. Thus, $f^{-1}(x) = -x$.

64. $f(x) = \sqrt[3]{x} - 5$

The function f takes the cube root of an input and then subtracts 5. To reverse this procedure, f^{-1} would add 5 to each of its inputs and then raise the result to the third power. Thus, $f^{-1}(x) = (x + 5)^3$.

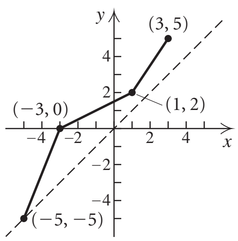
65. $f(x) = \sqrt[3]{x-5}$

The function f subtracts 5 from each input and then takes the cube root of the result. To reverse this procedure, f^{-1} would raise each input to the third power and then add 5 to the result. Thus, $f^{-1}(x) = x^3 + 5$.

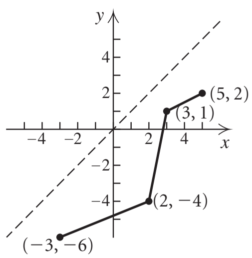
66. $f(x) = x^{-1}$

The outputs of f are the reciprocals of the inputs. Then the outputs of f^{-1} are the reciprocals of its inputs. Thus, $f^{-1}(x) = x^{-1}$.

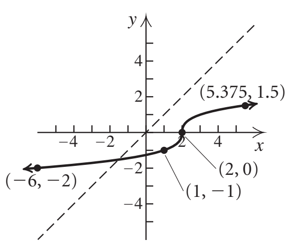
67. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-5, -5)$, $(-3, 0)$, $(1, 2)$, and $(3, 5)$.



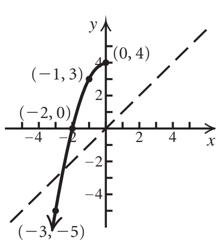
68. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-3, -6)$, $(2, -4)$, $(3, 1)$, and $(5, 2)$.



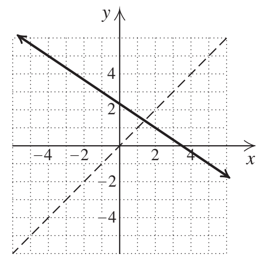
69. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-6, -2)$, $(1, -1)$, $(2, 0)$, and $(5.375, 1.5)$.



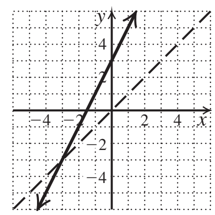
70. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-3, -5)$, $(-2, 0)$, $(-1, 3)$, and $(0, 4)$.



71. We reflect the graph of f across the line $y = x$.



72. We reflect the graph of f across the line $y = x$.



73. We find $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(x)$ and check to see that each is x.

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{7}{8}x\right) =$

$\frac{8}{7}\left(\frac{7}{8}x\right) = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{8}{7}x\right) = \frac{7}{8}\left(\frac{8}{7}x\right) = x$

74. $(f^{-1} \circ f)(x) = 4\left(\frac{x+5}{4}\right) - 5 = x + 5 - 5 = x$

$(f \circ f^{-1})(x) = \frac{4x - 5 + 5}{4} = \frac{4x}{4} = x$

75. We find $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(x)$ and check to see that each is x.

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{1-x}{x}\right) =$

$\frac{1}{\frac{1-x}{x} + 1} = \frac{1}{\frac{1-x+x}{x}} = \frac{1}{\frac{1}{x}} = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{1}{x+1}\right) =$

$\frac{1 - \frac{1}{x+1}}{\frac{1}{x+1}} = \frac{\frac{x+1-1}{x+1}}{\frac{1}{x+1}} = \frac{\frac{x}{x+1}}{\frac{1}{x+1}} = x$

76. $(f^{-1} \circ f)(x) = (\sqrt[3]{x+4})^3 - 4 = x + 4 - 4 = x$

$(f \circ f^{-1})(x) = \sqrt[3]{x^3 - 4 + 4} = \sqrt[3]{x^3} = x$

77. $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = \dfrac{5\left(\frac{2}{5}x + 1\right) - 5}{2} =$

$\frac{2x + 5 - 5}{2} = \frac{2x}{2} = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = \frac{2}{5}\left(\frac{5x-5}{2}\right) + 1 =$

$x - 1 + 1 = x$

78. $(f^{-1}\circ f)(x) = \dfrac{4\left(\dfrac{x+6}{3x-4}\right)+6}{3\left(\dfrac{x+6}{3x-4}\right)-1} = \dfrac{\dfrac{4x+24}{3x-4}+6}{\dfrac{3x+18}{3x-4}-1} =$

$\dfrac{\dfrac{4x+24+18x-24}{3x-4}}{\dfrac{3x+18-3x+4}{3x-4}} = \dfrac{22x}{3x-4}\cdot\dfrac{3x-4}{22} = \dfrac{22x}{22} = x$

$(f\circ f^{-1})(x) = \dfrac{\dfrac{4x+6}{3x-1}+6}{3\left(\dfrac{4x+6}{3x-1}\right)-4} = \dfrac{\dfrac{4x+6+18x-6}{3x-1}}{\dfrac{12x+18}{3x-1}-4} =$

$\dfrac{\dfrac{22x}{3x-1}}{\dfrac{12x+18-12x+4}{3x-1}} = \dfrac{22x}{3x-1}\cdot\dfrac{3x-1}{22} = \dfrac{22x}{22} = x$

79. Replace $f(x)$ with y: $y = 5x-3$

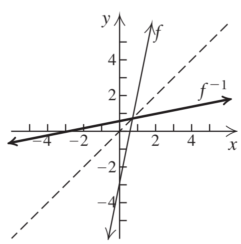
Interchange x and y: $x = 5y-3$

Solve for y: $x+3 = 5y$

$$\frac{x+3}{5} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x+3}{5}$, or $\dfrac{1}{5}x+\dfrac{3}{5}$

The domain and range of f are $(-\infty,\infty)$, so the domain and range of f^{-1} are also $(-\infty,\infty)$.



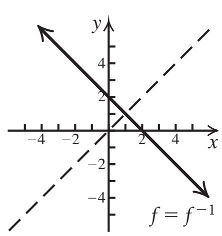
80. Replace $f(x)$ with y: $y = 2-x$

Interchange x and y: $x = 2-y$

Solve for y: $y = 2-x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = 2-x$

The domain and range of f are $(-\infty,\infty)$, so the domain and range of f^{-1} are also $(-\infty,\infty)$.



81. Replace $f(x)$ with y: $y = \dfrac{2}{x}$

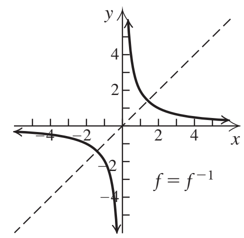
Interchange x and y: $x = \dfrac{2}{y}$

Solve for y: $xy = 2$

$$y = \frac{2}{x}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{2}{x}$

The domain and range of f are $(-\infty,0)\cup(0,\infty)$, so the domain and range of f^{-1} are also $(-\infty,0)\cup(0,\infty)$.



82. Replace $f(x)$ with y: $y = -\dfrac{3}{x+1}$

Interchange x and y: $x = -\dfrac{3}{y+1}$

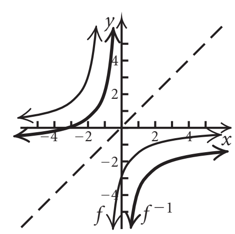
Solve for y: $xy + x = -3$

$$xy = -3-x$$

$$y = \frac{-3-x}{x}, \textit{or } -\frac{3}{x}-1$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = -\dfrac{3}{x}-1$

The domain of f is $(-\infty,-1)\cup(-1,\infty)$ and the range of f is $(-\infty,0)\cup(0,\infty)$. Thus the domain of f^{-1} is $(-\infty,0)\cup(0,\infty)$ and the range of f^{-1} is $(-\infty,-1)\cup(-1,\infty)$.



83. Replace $f(x)$ with y: $y = \dfrac{1}{3}x^3-2$

Interchange x and y: $x = \dfrac{1}{3}y^3-2$

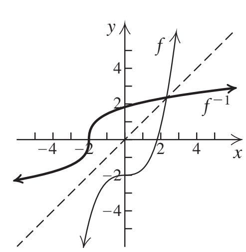
Solve for y: $x+2 = \dfrac{1}{3}y^3$

$$3x+6 = y^3$$

$$\sqrt[3]{3x+6} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{3x+6}$

The domain and range of f are $(-\infty,\infty)$, so the domain and range of f^{-1} are also $(-\infty,\infty)$.



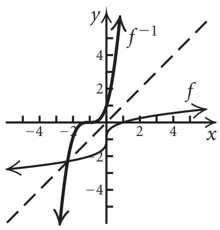
84. Replace $f(x)$ with y: $y = \sqrt[3]{x} - 1$

Interchange x and y: $x = \sqrt[3]{y} - 1$

Solve for y:
$$x + 1 = \sqrt[3]{y}$$
$$(x+1)^3 = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = (x+1)^3$

The domain and range of f are $(-\infty, \infty)$, so the domain and range of f^{-1} are also $(-\infty, \infty)$.



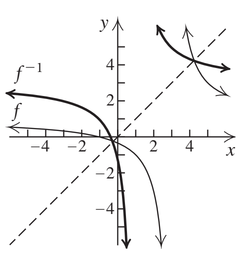
85. Replace $f(x)$ with y: $y = \dfrac{x+1}{x-3}$

Interchange x and y: $x = \dfrac{y+1}{y-3}$

Solve for y:
$$xy - 3x = y + 1$$
$$xy - y = 3x + 1$$
$$y(x-1) = 3x + 1$$
$$y = \frac{3x+1}{x-1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{3x+1}{x-1}$

The domain of f is $(-\infty, 3) \cup (3, \infty)$ and the range of f is $(-\infty, 1) \cup (1, \infty)$. Thus the domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$ and the range of f^{-1} is $(-\infty, 3) \cup (3, \infty)$.



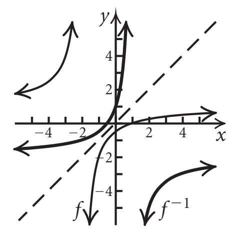
86. Replace $f(x)$ with y: $y = \dfrac{x-1}{x+2}$

Interchange x and y: $x = \dfrac{y-1}{y+2}$

Solve for y:
$$xy + 2x = y - 1$$
$$2x + 1 = y - xy$$
$$2x + 1 = y(1-x)$$
$$\frac{2x+1}{1-x} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{2x+1}{1-x}$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$ and the range of f is $(-\infty, 1) \cup (1, \infty)$. Thus the domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$ and the range of f^{-1} is $(-\infty, -2) \cup (-2, \infty)$.



87. Since $f(f^{-1}(x)) = f^{-1}(f(x)) = x$, then $f(f^{-1}(5)) = 5$ and $f^{-1}(f(a)) = a$.

88. Since $f^{-1}(f(x)) = f(f^{-1}(x)) = x$, then $f^{-1}(f(p)) = p$ and $f(f^{-1}(1253)) = 1253$.

89. a) $s(x) = \dfrac{2x-3}{2}$

$$s(5) = \frac{2 \cdot 5 - 3}{2} = \frac{10-3}{2} = \frac{7}{2} = 3\frac{1}{2}$$
$$s\left(7\frac{1}{2}\right) = s(7.5) = \frac{2 \cdot 7.5 - 3}{2} = \frac{15-3}{2} = \frac{12}{2} = 6$$
$$s(8) = \frac{2 \cdot 8 - 3}{2} = \frac{16-3}{2} = \frac{13}{2} = 6\frac{1}{2}$$

b) The graph of $s(x)$ passes the horizontal-line test and thus has an inverse that is a function.

Replace $f(x)$ with y: $y = \dfrac{2x-3}{2}$

Interchange x and y: $x = \dfrac{2y-3}{2}$

Solve for y:
$$2x = 2y - 3$$
$$2x + 3 = 2y$$
$$\frac{2x+3}{2} = y$$

Replace y with $s^{-1}(x)$: $s^{-1}(x) = \dfrac{2x+3}{2}$

c) $s^{-1}(3) = \dfrac{2 \cdot 3 + 3}{2} = \dfrac{6+3}{2} = \dfrac{9}{2} = 4\dfrac{1}{2}$

$$s^{-1}\left(5\frac{1}{2}\right) = s^{-1}(5.5) = \frac{2 \cdot 5.5 + 3}{2} = \frac{11+3}{2} = \frac{14}{2} = 7$$
$$s^{-1}(7) = \frac{2 \cdot 7 + 3}{2} = \frac{14+3}{2} = \frac{17}{2} = 8\frac{1}{2}$$

90. Replace $C(x)$ with y: $y = \dfrac{60 + 2x}{x}$

Interchange x and y: $x = \dfrac{60 + 2y}{y}$

Solve for y:
$$xy = 60 + 2y$$
$$xy - 2y = 60$$
$$y(x-2) = 60$$
$$y = \frac{60}{x-2}$$

Replace y with $C^{-1}(x)$: $C^{-1}(x) = \dfrac{60}{x-2}$

$C^{-1}(x)$ represents the number of people in the group, where x is the cost per person in dollars.

91. a) In 2005, $x = 2005 - 2000 = 5$.

$P(5) = 2.1782(5) + 25.3 \approx \$36.2$ billion

In 2010, $x = 2010 - 2000 = 10$.

$P(10) = 2.1782(10) + 25.3 \approx \$47.1$ billion

b) Replace $P(x)$ with y: $y = 2.1782x + 25.3$

Interchange x and y: $x = 2.1782y + 25.3$

Solve for y: $x - 25.3 = 2.1782y$

$$\frac{x - 25.3}{2.1782} = y$$

Replace y with $P^{-1}(x)$: $P^{-1}(x) = \dfrac{x - 25.3}{2.1782}$

$P^{-1}(x)$ represents how many years after 2000 x billion dollars are spent on pets.

92. a) $T(-13°) = \frac{5}{9}(-13° - 32°) = \frac{5}{9}(-45°) = -25°$

$T(86°) = \frac{5}{9}(86° - 32°) = \frac{5}{9}(54°) = 30°$

b) Replace $T(x)$ with y: $y = \frac{5}{9}(x - 32)$

Interchange x and y: $x = \frac{5}{9}(y - 32)$

Solve for y: $\frac{9}{5}x = y - 32$

$$\frac{9}{5}x + 32 = y$$

Replace y with $T^{-1}(x)$: $T^{-1}(x) = \frac{9}{5}x + 32$

$T^{-1}(x)$ represents the Fahrenheit temperature when the Celsius temperature is x.

93. The functions for which the coefficient of x^2 is negative have a maximum value. These are (b), (d), (f), and (h).

94. The graphs of the functions for which the coefficient of x^2 is positive open up. These are (a), (c), (e), and (g).

95. Since $|2| > 1$ the graph of $f(x) = 2x^2$ can be obtained by stretching the graph of $f(x) = x^2$ vertically. Since $0 < \left|\frac{1}{4}\right| < 1$, the graph of $f(x) = \frac{1}{4}x^2$ can be obtained by shrinking the graph of $y = x^2$ vertically. Thus the graph of $f(x) = 2x^2$, or (a) is narrower.

96. Since $|-5| > 0$ and $0 < \left|\frac{2}{3}\right| < 1$, the graph of (d) is narrower.

97. We can write (f) as $f(x) = -2[x - (-3)]^2 + 1$. Thus the graph of (f) has vertex $(-3, 1)$.

98. For the functions that can be written in the form $f(x) = a(x - 0)^2 + k$, or $f(x) = ax^2 + k$, the line of symmetry is $x = 0$. These are (a), (b), (c), and (d).

99. The graph of $f(x) = x^2 - 3$ is a parabola with vertex $(0, -3)$. If we consider x-values such that $x \geq 0$, then the graph is the right-hand side of the parabola and it passes the horizontal line test. We find the inverse of $f(x) = x^2 - 3$, $x \geq 0$.

Replace $f(x)$ with y: $y = x^2 - 3$

Interchange x and y: $x = y^2 - 3$

Solve for y: $x + 3 = y^2$

$$\sqrt{x+3} = y$$

(We take the principal square root, because $x \geq 0$ in the original equation.)

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt{x+3}$ for all x in the range of $f(x)$, or $f^{-1}(x) = \sqrt{x+3}$, $x \geq -3$.

Answers may vary. There are other restrictions that also make $f(x)$ one-to-one.

100. No; the graph of f does not pass the horizontal line test.

101. Answers may vary. $f(x) = \frac{3}{x}$, $f(x) = 1 - x$, $f(x) = x$.

102. First find $f^{-1}(x)$.

Replace $f^{-1}(x)$ with y: $y = ax + b$

Interchange x and y: $x = ay + b$

Solve for y: $x - b = ay$

$$\frac{x - b}{a} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x - b}{a} = \dfrac{1}{a}x - \dfrac{b}{a}$

Now we find the values of a and b for which $ax + b = \frac{1}{a}x - \frac{b}{a}$. We see that $a = \frac{1}{a}$ for $a = \pm 1$. If $a = 1$, we have $x + b = x - b$, so $b = 0$. If $a = -1$, we have $-x + b = -x + b$, so b can be any real number.

Exercise Set 5.2

1. $e^4 \approx 54.5982$

2. $e^{10} \approx 22,026.4658$

3. $e^{-2.458} \approx 0.0856$

4. $\left(\dfrac{1}{e^3}\right)^2 \approx 0.0025$

5. $f(x) = -2^x - 1$

$f(0) = -2^0 - 1 = -1 - 1 = -2$

The only graph with y-intercept $(0, -2)$ is (f).

6. $f(x) = -\left(\dfrac{1}{2}\right)^x$

$f(0) = -\left(\dfrac{1}{2}\right)^0 = -1$

Since the y-intercept is $(0, -1)$, the correct graph is (a) or (c). Check another point on the graph.

$f(-1) = -\left(\dfrac{1}{2}\right)^{-1} = -2$, so the point $(-1, -2)$ is on the graph. Thus (c) is the correct choice.

7. $f(x) = e^x + 3$

This is the graph of $f(x) = e^x$ shifted up 3 units. Then (e) is the correct choice.

8. $f(x) = e^{x+1}$

This is the graph of $f(x) = e^x$ shifted left 1 unit. Then (b) is the correct choice.

9. $f(x) = 3^{-x} - 2$

$f(0) = 3^{-0} - 2 = 1 - 2 = -1$

Since the y-intercept is $(0, -1)$, the correct graph is (a) or (c). Check another point on the graph. $f(-1) = 3^{-(-1)} - 2 = 3 - 2 = 1$, so $(-1, 1)$ is on the graph. Thus (a) is the correct choice.

10. $f(x) = 1 - e^x$

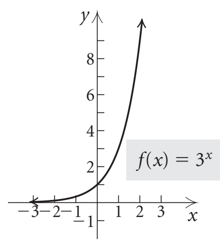
$f(0) = 1 - e^0 = 1 - 1 = 0$

The only graph with y-intercept $(0, 0)$ is (d).

11. Graph $f(x) = 3^x$.

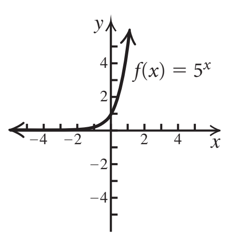
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	$y = f(x)$	(x, y)
-3	$\frac{1}{27}$	$\left(-3, \frac{1}{27}\right)$
-2	$\frac{1}{9}$	$\left(-2, \frac{1}{9}\right)$
-1	$\frac{1}{3}$	$\left(-1, \frac{1}{3}\right)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	9	$(2, 9)$
3	27	$(3, 27)$



12. Graph $f(x) = 5^x$.

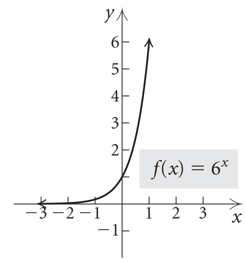
x	$y = f(x)$	(x, y)
-3	$\frac{1}{125}$	$\left(-3, \frac{1}{125}\right)$
-2	$\frac{1}{25}$	$\left(-2, \frac{1}{25}\right)$
-1	$\frac{1}{5}$	$\left(-1, \frac{1}{5}\right)$
0	1	$(0, 1)$
1	5	$(1, 5)$
2	25	$(2, 25)$
3	125	$(3, 125)$



13. Graph $f(x) = 6^x$.

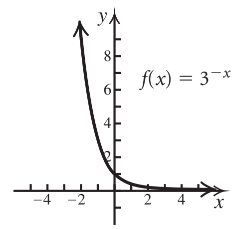
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	$y = f(x)$	(x, y)
-3	$\frac{1}{216}$	$\left(-3, \frac{1}{216}\right)$
-2	$\frac{1}{36}$	$\left(-2, \frac{1}{36}\right)$
-1	$\frac{1}{6}$	$\left(-1, \frac{1}{6}\right)$
0	1	$(0, 1)$
1	6	$(1, 6)$
2	36	$(2, 36)$
3	216	$(3, 216)$



14. Graph $f(x) = 3^{-x}$.

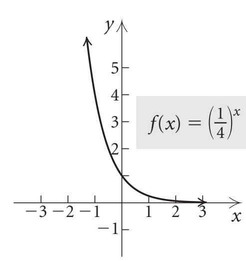
x	$y = f(x)$	(x, y)
-3	27	$(-3, 27)$
-2	9	$(-2, 9)$
-1	3	$(-1, 3)$
0	1	$(0, 1)$
1	$\frac{1}{3}$	$\left(1, \frac{1}{3}\right)$
2	$\frac{1}{9}$	$\left(2, \frac{1}{9}\right)$
3	$\frac{1}{27}$	$\left(3, \frac{1}{27}\right)$



15. Graph $f(x) = \left(\frac{1}{4}\right)^x$.

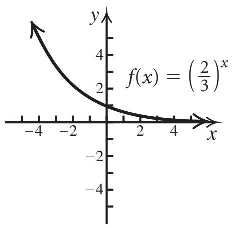
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	$y = f(x)$	(x, y)
-3	64	$(-3, 64)$
-2	16	$(-2, 16)$
-1	4	$(-1, 4)$
0	1	$(0, 1)$
1	$\frac{1}{4}$	$\left(1, \frac{1}{4}\right)$
2	$\frac{1}{16}$	$\left(2, \frac{1}{16}\right)$
3	$\frac{1}{64}$	$\left(3, \frac{1}{64}\right)$



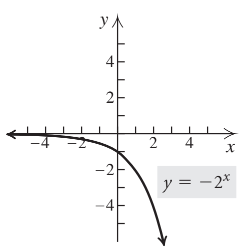
16. Graph $f(x) = \left(\dfrac{2}{3}\right)^x$.

x	$y = f(x)$	(x, y)
-3	$\frac{27}{8}$	$\left(-3, \frac{27}{8}\right)$
-2	$\frac{9}{4}$	$\left(-2, \frac{9}{4}\right)$
-1	$\frac{3}{2}$	$\left(-1, \frac{3}{2}\right)$
0	1	$(0, 1)$
1	$\frac{2}{3}$	$\left(1, \frac{2}{3}\right)$
2	$\frac{4}{9}$	$\left(2, \frac{4}{9}\right)$
3	$\frac{8}{27}$	$\left(3, \frac{8}{27}\right)$



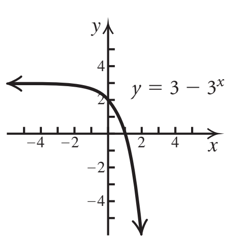
17. Graph $y = -2^x$.

x	y	(x, y)
-3	$-\frac{1}{8}$	$\left(-3, -\frac{1}{8}\right)$
-2	$-\frac{1}{4}$	$\left(-2, -\frac{1}{4}\right)$
-1	$-\frac{1}{2}$	$\left(-1, -\frac{1}{2}\right)$
0	-1	$(0, -1)$
1	-2	$(1, -2)$
2	-4	$(2, -4)$
3	-8	$(3, -8)$



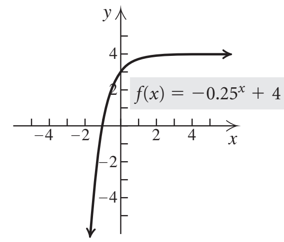
18. Graph $y = 3 - 3^x$.

x	y	(x, y)
-3	$\frac{80}{27}$	$\left(-3, \frac{80}{27}\right)$
-2	$\frac{26}{9}$	$\left(-2, \frac{26}{9}\right)$
-1	$\frac{8}{3}$	$\left(-1, \frac{8}{3}\right)$
0	2	$(0, 2)$
1	0	$(1, 0)$
2	-6	$(2, -6)$
3	-24	$(3, -24)$



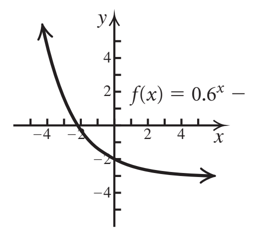
19. Graph $f(x) = -0.25^x + 4$.

x	$y = f(x)$	(x, y)
-3	-60	$(-3, -60)$
-2	-12	$(-2, -12)$
-1	0	$(-1, 0)$
0	3	$(0, 3)$
1	3.75	$(1, 3.75)$
2	3.94	$(2, 3.94)$
3	3.98	$(3, 3.98)$



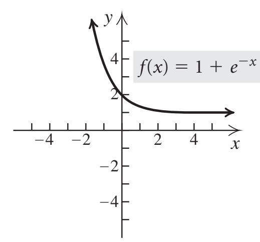
20. Graph $f(x) = 0.6^x - 3$.

x	$y = f(x)$	(x, y)
-3	1.63	$(-3, 1.63)$
-2	-0.22	$(-2, -0.22)$
-1	-1.33	$(-1, -1.33)$
0	-2	$(0, -2)$
1	-2.4	$(1, -2.4)$
2	-2.64	$(2, -2.64)$
3	-2.78	$(3, -2.78)$



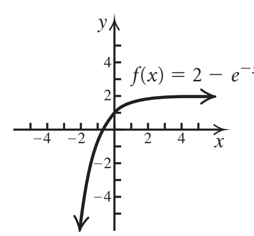
21. Graph $f(x) = 1 + e^{-x}$.

x	$y = f(x)$	(x, y)
-3	21.1	$(-3, 21.1)$
-2	8.4	$(-2, 8.4)$
-1	3.7	$(-1, 3.7)$
0	2	$(0, 2)$
1	1.4	$(1, 1.4)$
2	1.1	$(2, 1.1)$
3	1.0	$(3, 1.0)$



22. Graph $f(x) = 2 - e^{-x}$.

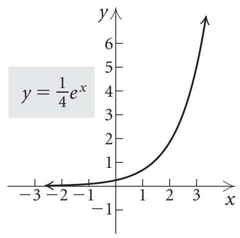
x	$y = f(x)$	(x, y)
-3	-18.1	$(-3, -18.1)$
-2	-5.4	$(-2, -5.4)$
-1	-0.8	$(-1, -0.8)$
0	1	$(0, 1)$
1	1.6	$(1, 1.6)$
2	1.9	$(2, 1.9)$
3	2.0	$(3, 2.0)$



23. Graph $y = \dfrac{1}{4}e^x$.

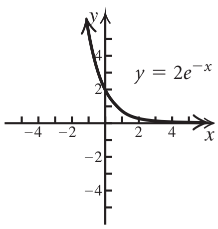
Choose values for x and compute the corresponding y-values. Plot the points (x, y) and connect them with a smooth curve.

x	y	(x, y)
-3	0.0124	$(-3, 0.0124)$
-2	0.0338	$(-2, 0.0338)$
-1	0.0920	$(-1, 0.0920)$
0	0.25	$(0, 0.25)$
1	0.6796	$(1, 0.6796)$
2	1.8473	$(2, 1.8473)$
3	5.0214	$(3, 5.0214)$



24. Graph $y = 2e^{-x}$.

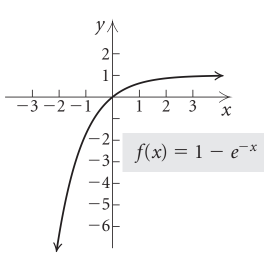
x	y	(x, y)
-3	40.1711	$(-3, 40.1711)$
-2	14.7781	$(-2, 14.7781)$
-1	5.4366	$(-1, 5.4366)$
0	2	$(0, 2)$
1	0.7358	$(1, 0.7358)$
2	0.2707	$(2, 0.2707)$
3	0.0996	$(3, 0.0996)$



25. Graph $f(x) = 1 - e^{-x}$.

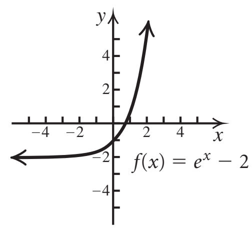
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	y	(x, y)
-3	-19.0855	$(-3, -19.0855)$
-2	-6.3891	$(-2, -6.3891)$
-1	-1.7183	$(-1, -1.7183)$
0	0	$(0, 0)$
1	0.6321	$(1, 0.6321)$
2	0.8647	$(2, 0.8647)$
3	0.9502	$(3, 0.9502)$

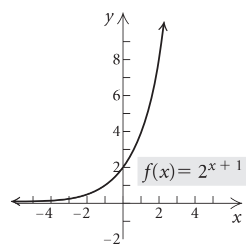


26. Graph $f(x) = e^x - 2$.

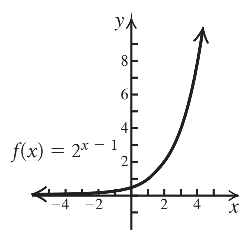
x	y	(x, y)
-3	-1.9502	$(-3, -1.9502)$
-2	-1.8647	$(-2, -1.8647)$
-1	-1.6321	$(-1, -1.6321)$
0	-1	$(0, -1)$
1	0.7183	$(1, 0.7183)$
2	5.3891	$(2, 5.3891)$
3	18.0855	$(3, 18.0855)$



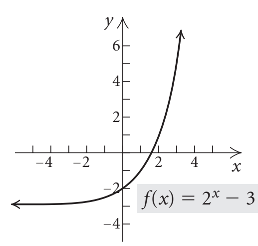
27. Shift the graph of $y = 2^x$ left 1 unit.



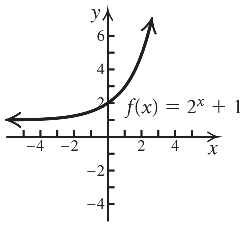
28. Shift the graph of $y = 2^x$ right 1 unit.



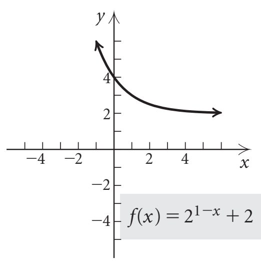
29. Shift the graph of $y = 2^x$ down 3 units.



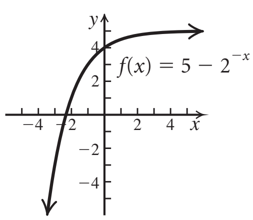
30. Shift the graph of $y = 2^x$ up 1 unit.



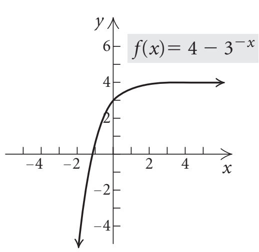
31. Shift the graph of $y = 2^x$ left 1 unit, reflect it across the y-axis, and shift it up 2 units.



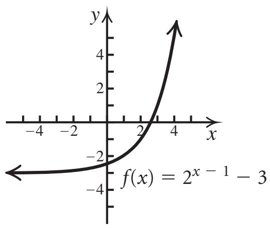
32. Reflect the graph of $y = 2^x$ across the y-axis, reflect it across the x-axis, and shift it up 5 units.



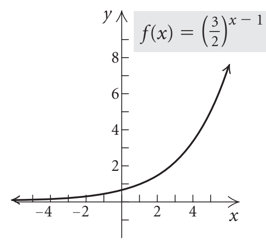
33. Reflect the graph of $y = 3^x$ across the y-axis, then across the x-axis, and then shift it up 4 units.



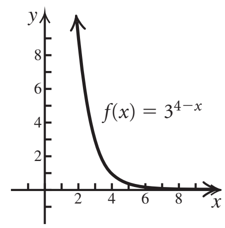
34. Shift the graph of $y = 2^x$ right 1 unit and down 3 units.



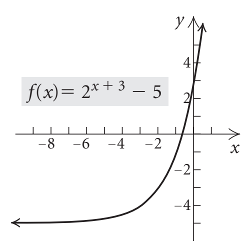
35. Shift the graph of $y = \left(\frac{3}{2}\right)^x$ right 1 unit.



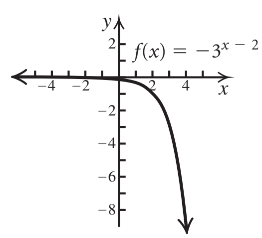
36. Reflect the graph of $y = 3^x$ across the y-axis and then shift it right 4 units.



37. Shift the graph of $y = 2^x$ left 3 units and down 5 units.



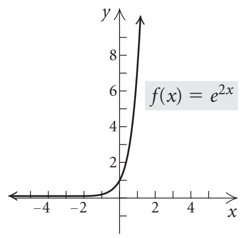
38. Shift the graph of $y = 3^x$ right 2 units and reflect it across the x-axis.



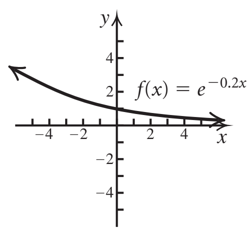
39. Shift the graph of $y = 2^x$ right 1 unit, stretch it vertically, and shift it up 1 unit. The graph is in the answer section in the text.

40. Shift the graph of $y = 3^x$ left 1 unit, stretch it vertically, and shift it down 2 units. The graph is in the Additional Instructor's Answers in the text.

41. Shrink the graph of $y = e^x$ horizontally.



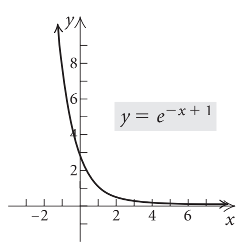
42. Stretch the graph of $y = e^x$ horizontally and reflect it across the y-axis.



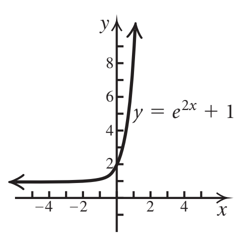
43. Reflect the graph of $y = e^x$ across the x-axis, shift it up 1 unit, and shrink it vertically. The graph is in the answer section in the text.

44. Shift the graph of $y = e^x$ up 1 unit, stretch it vertically, and shift it down 2 units. The graph is in the Additional Instructor's Answers in the text.

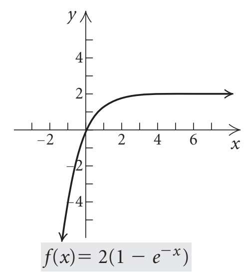
45. Shift the graph of $y = e^x$ left 1 unit and reflect it across the y-axis.



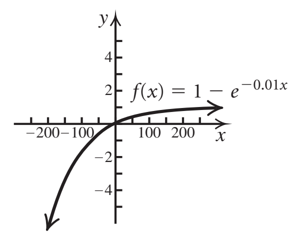
46. Shrink the graph of $y = e^x$ horizontally and shift it up 1 unit.



47. Reflect the graph of $y = e^x$ across the y-axis and then across the x-axis; shift it up 1 unit and then stretch it vertically.



48. Stretch the graph of $y = e^x$ horizontally and reflect it across the y-axis; then reflect it across the x-axis and shift it up 1 unit.



49. We graph $f(x) = e^{-x} - 4$ for $x < -2$, $f(x) = x + 3$ for $-2 \le x < 1$, and $f(x) = x^2$ for $x \ge 1$.

The graph is in the answer section in the text.

50. The graph is in the Additional Instructor's Answers in the text.

51. a) We use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and substitute 82,000 for P, 0.045 for r, and 4 for n.

$A(t) = 82,000\left(1 + \frac{0.045}{4}\right)^{4t} = 82,000(1.01125)^{4t}$

b) $A(0) = 82,000(1.01125)^{4 \cdot 0} = \$82,000$

$A(2) = 82,000(1.01125)^{4 \cdot 2} \approx \$89,677.22$

$A(5) = 82,000(1.01125)^{4 \cdot 5} \approx \$102,561.54$

$A(10) = 82,000(1.01125)^{4 \cdot 10} \approx \$128,278.90$

52. a) $A(t) = 750\left(1 + \frac{0.07}{2}\right)^{2t} = 750(1.035)^{2t}$

b) $A(1) = 750(1.035)^{2 \cdot 1} \approx \$803.42$

$A(6) = 750(1.035)^{2 \cdot 6} \approx \$1133.30$

$A(10) = 750(1.035)^{2 \cdot 10} \approx \$1492.34$

$A(15) = 750(1.035)^{2 \cdot 15} \approx \$2105.10$

$A(25) = 750(1.035)^{2 \cdot 25} \approx \$4188.70$

53. We use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and substitute 3000 for P, 0.05 for r, and 4 for n.

$A(t) = 3000\left(1 + \frac{0.05}{4}\right)^{4t} = 3000(1.0125)^{4t}$

On Jacob's sixteenth birthday, $t = 16 - 6 = 10$.

$A(10) = 3000(1.0125)^{4 \cdot 10} = 4930.86$

When the CD matures \$4930.86 will be available.

54. a) $A(t) = 10,000\left(1 + \frac{0.039}{2}\right)^{2t} = 10,000(1.0195)^{2t}$

b) $A(0) = 10,000(1.0195)^{2\cdot 0} = \$10,000$

$A(4) = 10,000(1.0195)^{2\cdot 4} \approx \$11,670.73$

$A(8) = 10,000(1.0195)^{2\cdot 8} \approx \$13,620.58$

$A(10) = 10,000(1.0195)^{2\cdot 10} \approx \$14,714.47$

$A(18) = 10,000(1.0195)^{2\cdot 18} \approx \$20,041.96$

$A(21) = 10,000(1.0195)^{2\cdot 21} \approx \$22,504.20$

55. We use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and substitute 3000 for P, 0.04 for r, 2 for n, and 2 for t.

$A = 3000\left(1 + \frac{0.04}{2}\right)^{2\cdot 2} \approx \$3247.30$

56. $A = 12,500\left(1 + \frac{0.03}{4}\right)^{4\cdot 3} \approx \$13,672.59$

57. We use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and substitute 120,000 for P, 0.025 for r, 1 for n, and 10 for t.

$A = 120,000\left(1 + \frac{0.025}{1}\right)^{1\cdot 10} \approx \$153,610.15$

58. $A = 120,000\left(1 + \frac{0.025}{4}\right)^{4\cdot 10} \approx \$153,963.22$

59. We use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and substitute 53,500 for P, 0.055 for r, 4 for n, and 6.5 for t.

$A = 53,500\left(1 + \frac{0.055}{4}\right)^{4(6.5)} \approx \$76,305.59$

60. $A = 6250\left(1 + \frac{0.0675}{2}\right)^{2(4.5)} \approx \$8425.97$

61. We use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and substitute 17,400 for P, 0.081 for r, 365 for n, and 5 for t.

$A = 17,400\left(1 + \frac{0.081}{365}\right)^{365\cdot 5} \approx \$26,086.69$

62. $A = 900\left(1 + \frac{0.073}{365}\right)^{365(7.25)} \approx \$1527.81$

63. $W(x) = 23,672.16(1.112)^x$

In 2000, $x = 2000 - 1998 = 2$.

$W(2) = 23,672.16(1.112)^2 \approx 29,272$ service members

In 2008, $x = 2008 - 1998 = 10$.

$W(10) = 23,672.16(1.112)^{10} \approx 68,436$ service members

In 2011, $x = 2011 - 1998 = 13$.

$W(13) = 23,672.16(1.112)^{13} \approx 94,102$ service members

64. $I(t) = 9000(1.0878)^t$

In 1985, $t = 1985 - 1980 = 5$.

$I(5) = 9000(1.0878)^5 \approx 13,708$ students

In 1999, $t = 1999 - 1980 = 19$.

$I(19) = 9000(1.0878)^{19} \approx 44,532$ students

In 2006, $t = 2006 - 1980 = 26$.

$I(26) = 9000(1.0878)^{26} \approx 80,263$ students

In 2012, $t = 2012 - 1980 = 32$.

$I(32) = 9000(1.0878)^{32} \approx 132,988$ students

65. $T(x) = 400(1.055)^x$

In 1950, $x = 1950 - 1913 = 37$.

$T(37) = 400(1.055)^{37} \approx 2900$ pages

In 1990, $x = 1990 - 1913 = 77$.

$T(77) = 400(1.055)^{77} \approx 24,689$ pages

In 2000, $x = 2000 - 1913 = 87$.

$T(87) = 400(1.055)^{87} \approx 42,172$ pages

66. $M(x) = 7(1.082)^x$

In 1985, $x = 1985 - 1975 = 10$.

$M(10) = 7(1.082)^{10} \approx \$15$

In 1992, $x = 1992 - 1975 = 17$.

$M(17) = 7(1.082)^{17} \approx \$27$

In 2002, $x = 2002 - 1975 = 27$.

$M(27) = 7(1.082)^{27} \approx \$59$

67. $M(x) = (200,000)(1.1802)^x$

In 2003, $x = 2003 - 2001 = 2$.

$M(2) = (200,000)(1.1802)^2 \approx \$278,574$

In 2007, $x = 2007 - 2001 = 6$.

$M(6) = (200,000)(1.1802)^6 \approx \$540,460$

In 2013, $x = 2013 - 2001 = 12$.

$M(12) = (200,000)(1.1802)^{12} \approx \$1,460,486$

68. $C(x) = 15.5202(1.0508)^x$

In 1990, $x = 1990 - 1970 = 20$.

$C(20) = 15.5202(1.0508)^{20} \approx \$41.81$

In 2010, $x = 2010 - 1970 = 40$.

$C(40) = 15.5202(1.0508)^{40} \approx \$112.64$

69. $R(x) = 80,000(1.1522)^x$

In 1999, $x = 1999 - 1996 = 3$.

$R(3) = 80,000(1.1522)^3 \approx 122,370$ tons

In 2007, $x = 2007 - 1996 = 11$.

$R(11) = 80,000(1.1522)^{11} \approx 380,099$ tons

In 2012, $x = 2012 - 1996 = 16$.

$R(16) = 80,000(1.1522)^{16} \approx 771,855$ tons

70. $D(t) = 43.1224(1.0475)^t$

In 1984, $t = 1984 - 1960 = 24$.

$D(24) = 43.1224(1.0475)^{24} \approx 131.341$ thousand degrees, or 131,341 degrees

In 2002, $t = 2002 - 1960 = 42$.

$D(42) = 43.1224(1.0475)^{42} \approx 302.811$ thousand degrees, or 302,811 degrees

In 2015, $t = 2015 - 1960 = 55$.

$D(55) = 43.1224(1.0475)^{55} \approx 553.573$ thousand degrees, or 553,573 degrees

71. $T(x) = 23.7624(1.0752)^x$

In 2007, $x = 2007 - 2006 = 1$.

$T(1) = 23.7624(1.0752)^1 \approx 25.5$ million people, or 25,500,000 people

In 2014, $x = 2014 - 2006 = 8$.

$T(8) = 23.7624(1.0752)^8 \approx 42.4$ million people, or 42,400,000 people

72. $N(10) = 3000(2)^{10/20} \approx 4243;$

$N(20) = 3000(2)^{20/20} = 6000;$

$N(30) = 3000(2)^{30/20} \approx 8485;$

$N(40) = 3000(2)^{40/20} = 12,000;$

$N(60) = 3000(2)^{60/20} = 24,000$

73. $V(t) = 56,395(0.9)^t$

$V(0) = 56,395(0.9)^0 = \$56,395$

$V(1) = 56,395(0.9)^1 \approx \$50,756$

$V(3) = 56,395(0.9)^3 \approx \$41,112$

$V(6) = 56,395(0.9)^6 \approx \$29,971$

$V(10) = 56,395(0.9)^{10} \approx \$19,664$

74. $S(10) = 200[1 - (0.86)^{10}] \approx 155.7$ words per minute

$S(20) = 200[1 - (0.86)^{20}] \approx 190.2$ words per minute

$S(40) = 200[1 - (0.86)^{40}] \approx 199.5$ words per minute

$S(100) \approx 200[1 - (0.86)^{100}] \approx 200.0$ words per minute

75. $f(25) = 100(1 - e^{-0.04(25)}) \approx 63\%$.

76. $V(1) = \$58(1 - e^{-1.1(1)}) + \$20 \approx \$58.69$

$V(2) = \$58(1 - e^{-1.1(2)}) + \$20 \approx \$71.57$

$V(4) = \$58(1 - e^{-1.1(4)}) + \$20 \approx \$77.29$

$V(6) = \$58(1 - e^{-1.1(6)}) + \$20 \approx \$77.92$

$V(12) = \$58(1 - e^{-1.1(12)}) + \$20 \approx \$78.00$

77.
$$\begin{aligned}(1-4i)(7+6i) &= 7+6i-28i-24i^2\\ &= 7+6i-28i+24\\ &= 31-22i\end{aligned}$$

78.
$$\begin{aligned}\frac{2-i}{3+1} &= \frac{2-i}{3+i}\cdot\frac{3-i}{3-i}\\ &= \frac{6-5i+i^2}{9-i^2}\\ &= \frac{6-5i-1}{9+1}\\ &= \frac{5-5i}{10}\\ &= \frac{1}{2}-\frac{1}{2}i\end{aligned}$$

79.
$$\begin{aligned}2x^2-13x-7&=0 \quad \text{Setting } f(x)=0\\ (2x+1)(x-7)&=0\\ 2x+1=0 \quad &or \quad x-7=0\\ 2x=-1 \quad &or \quad x=7\\ x=-\frac{1}{2} \quad &or \quad x=7\end{aligned}$$

The zeros of the function are $-\frac{1}{2}$ and 7, and the x-intercepts are $\left(-\frac{1}{2},0\right)$ and $(7,0)$.

80. $h(x) = x^3 - 3x^2 + 3x - 1$

The possible real-number solutions are of the form p/q where $p = \pm 1$ and $q = \pm 1$. Then the possibilities for p/q are 1 and -1. We try 1.

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1\\ & & 1 & -2 & 1\\ \hline & 1 & -2 & 1 & 0\end{array}$$

$h(x) = (x-1)(x^2 - 2x + 1)$

Now find the zeros of $h(x)$.

$$\begin{aligned}(x-1)(x^2-2x+1)&=0\\ (x-1)(x-1)(x-1)&=0\\ x-1=0 \ or \ x-1=0 \ &or \ x-1=0\\ x=1 \ or \ x=1 \ &or \ x=1\end{aligned}$$

The zero of the function is 1 and the x-intercept is $(1,0)$.

81.
$$\begin{aligned}x^4-x^2&=0 \quad \text{Setting } h(x)=0\\ x^2(x^2-1)&=0\\ x^2(x+1)(x-1)&=0\\ x^2=0 \quad or \quad x+1=0 \quad &or \quad x-1=0\\ x=0 \quad or \quad x=-1 \quad &or \quad x=1\end{aligned}$$

The zeros of the function are 0, -1, and 1, and the x-intercepts are $(0,0)$, $(-1,0)$, and $(1,0)$.

82.
$$\begin{aligned}x^3+x^2-12x&=0\\ x(x^2+x-12)&=0\\ x(x+4)(x-3)&=0\\ x=0 \quad or \quad x+4=0 \quad &or \quad x-3=0\\ x=0 \quad or \quad x=-4 \quad &or \quad x=3\end{aligned}$$

The zeros of the function are 0, -4, and 3, and the x-intercepts are $(0,0)$, $(-4,0)$, and $(3,0)$.

83.
$$\begin{aligned}x^3+6x^2-16x&=0\\ x(x^2+6x-16)&=0\\ x(x+8)(x-2)&=0\\ x=0 \quad or \quad x+8=0 \quad &or \quad x-2=0\\ x=0 \quad or \quad x=-8 \quad &or \quad x=2\end{aligned}$$

The solutions are 0, -8, and 2.

84.
$$\begin{aligned} 3x^2-6&=5x\\ 3x^2-5x-6&=0\\ x&=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 3\cdot(-6)}}{2\cdot 3}\\ &=\frac{5\pm\sqrt{97}}{6}\end{aligned}$$

85. $7^\pi \approx 451.8078726$ and $\pi^7 \approx 3020.293228$, so π^7 is larger.

$70^{80} \approx 4.054 \times 10^{147}$ and $80^{70} \approx 1.646 \times 10^{133}$, so 70^{80} is larger.

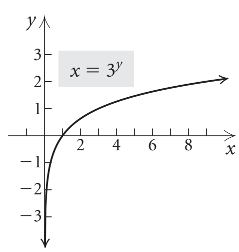
86.
$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{(2e^{x+h}-3)-(2e^x-3)}{h}\\ &=\frac{2e^{x+h}-3-2e^x+3}{h}\\ &=\frac{2e^{x+h}-2e^x}{h}\\ &=\frac{2e^x(e^h-1)}{h}\qquad (e^x\cdot e^h=e^{x+h})\end{aligned}$$

Exercise Set 5.3

1. Graph $x = 3^y$.

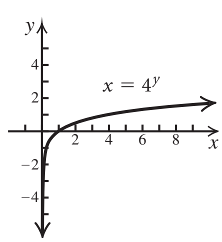
Choose values for y and compute the corresponding x-values. Plot the points (x, y) and connect them with a smooth curve.

x	y	(x,y)
$\frac{1}{27}$	-3	$\left(\frac{1}{27},-3\right)$
$\frac{1}{9}$	-2	$\left(\frac{1}{9},-2\right)$
$\frac{1}{3}$	-1	$\left(\frac{1}{3},-1\right)$
1	0	$(1,0)$
3	1	$(3,1)$
9	2	$(9,2)$
27	3	$(27,3)$



2. Graph $x = 4^y$.

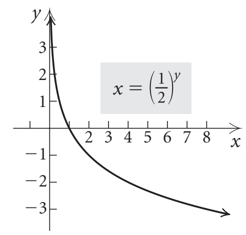
x	y	(x,y)
$\frac{1}{64}$	-3	$\left(\frac{1}{64},-3\right)$
$\frac{1}{16}$	-2	$\left(\frac{1}{16},-2\right)$
$\frac{1}{4}$	-1	$\left(\frac{1}{4},-1\right)$
1	0	$(1,0)$
4	1	$(4,1)$
16	2	$(16,2)$
64	3	$(64,3)$



3. Graph $x = \left(\frac{1}{2}\right)^y$.

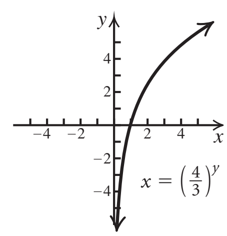
Choose values for y and compute the corresponding x-values. Plot the points (x, y) and connect them with a smooth curve.

x	y	(x,y)
8	-3	$(8,-3)$
4	-2	$(4,-2)$
2	-1	$(2,-1)$
1	0	$(1,0)$
$\frac{1}{2}$	1	$\left(\frac{1}{2},1\right)$
$\frac{1}{4}$	2	$\left(\frac{1}{4},2\right)$
$\frac{1}{8}$	3	$\left(\frac{1}{8},3\right)$



4. Graph $x = \left(\frac{4}{3}\right)^y$.

x	y	(x,y)
$\frac{27}{64}$	-3	$\left(\frac{27}{64},-3\right)$
$\frac{9}{16}$	-2	$\left(\frac{9}{16},-2\right)$
$\frac{3}{4}$	-1	$\left(\frac{3}{4},-1\right)$
1	0	$(1,0)$
$\frac{4}{3}$	1	$\left(\frac{4}{3},1\right)$
$\frac{16}{9}$	2	$\left(\frac{16}{9},2\right)$
$\frac{64}{27}$	3	$\left(\frac{64}{27},3\right)$



5. Graph $y = \log_3 x$.

The equation $y = \log_3 x$ is equivalent to $x = 3^y$. We can find ordered pairs that are solutions by choosing values for y and computing the corresponding x-values.

For $y = -2$, $x = 3^{-2} = \frac{1}{9}$.

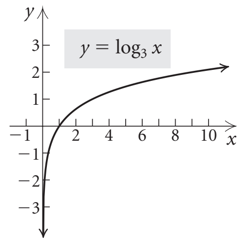
For $y = -1$, $x = 3^{-1} = \frac{1}{3}$.

For $y = 0$, $x = 3^0 = 1$.

For $y = 1$, $x = 3^1 = 3$.

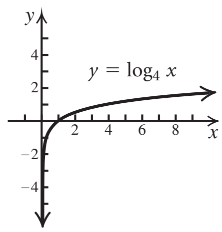
For $y = 2$, $x = 3^2 = 9$.

x, or 3^y	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2



6. $y = \log_4 x$ is equivalent to $x = 4^y$.

x, or 4^y	y
$\frac{1}{16}$	-2
$\frac{1}{4}$	-1
1	0
4	1
16	2



7. Graph $f(x) = \log x$.

Think of $f(x)$ as y. The equation $y = \log x$ is equivalent to $x = 10^y$. We can find ordered pairs that are solutions by choosing values for y and computing the corresponding x-values.

For $y = -2$, $x = 10^{-2} = 0.01$.

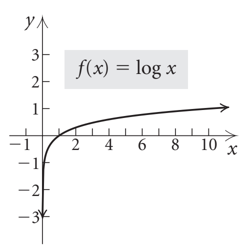
For $y = -1$, $x = 10^{-1} = 0.1$.

For $y = 0$, $x = 10^0 = 1$.

For $y = 1$, $x = 10^1 = 10$.

For $y = 2$, $x = 10^2 = 100$.

x, or 10^y	y
0.01	-2
0.1	-1
1	0
10	1
100	2



8. See Example 10.

9. $\log_2 16 = 4$ because the exponent to which we raise 2 to get 16 is 4.

10. $\log_3 9 = 2$, because the exponent to which we raise 3 to get 9 is 2.

11. $\log_5 125 = 3$, because the exponent to which we raise 5 to get 125 is 3.

12. $\log_2 64 = 6$, because the exponent to which we raise 2 to get 64 is 6.

13. $\log 0.001 = -3$, because the exponent to which we raise 10 to get 0.001 is -3.

14. $\log 100 = 2$, because the exponent to which we raise 10 to get 100 is 2.

15. $\log_2 \frac{1}{4} = -2$, because the exponent to which we raise 2 to get $\frac{1}{4}$ is -2.

16. $\log_8 2 = \frac{1}{3}$, because the exponent to which we raise 8 to get 2 is $\frac{1}{3}$.

17. $\ln 1 = 0$, because the exponent to which we raise e to get 1 is 0.

18. $\ln e = 1$, because the exponent to which we raise e to get e is 1.

19. $\log 10 = 1$, because the exponent to which we raise 10 to get 10 is 1.

20. $\log 1 = 0$, because the exponent to which we raise 10 to get 1 is 0.

21. $\log_5 5^4 = 4$, because the exponent to which we raise 5 to get 5^4 is 4.

22. $\log \sqrt{10} = \log 10^{1/2} = \frac{1}{2}$, because the exponent to which we raise 10 to get $10^{1/2}$ is $\frac{1}{2}$.

23. $\log_3 \sqrt[4]{3} = \log_3 3^{1/4} = \frac{1}{4}$, because the exponent to which we raise 3 to get $3^{1/4}$ is $\frac{1}{4}$.

24. $\log 10^{8/5} = \frac{8}{5}$, because the exponent to which we raise 10 to get $10^{8/5}$ is $\frac{8}{5}$.

25. $\log 10^{-7} = -7$, because the exponent to which we raise 10 to get 10^{-7} is -7.

26. $\log_5 1 = 0$, because the exponent to which we raise 5 to get 1 is 0.

27. $\log_{49} 7 = \frac{1}{2}$, because the exponent to which we raise 49 to get 7 is $\frac{1}{2}$. $(49^{1/2} = \sqrt{49} = 7)$

28. $\log_3 3^{-2} = -2$, because the exponent to which we raise 3 to get 3^{-2} is -2.

29. $\ln e^{3/4} = \frac{3}{4}$, because the exponent to which we raise e to get $e^{3/4}$ is $\frac{3}{4}$.

30. $\log_2 \sqrt{2} = \log_2 2^{1/2} = \frac{1}{2}$, because the exponent to which we raise 2 to get $2^{1/2}$ is $\frac{1}{2}$.

31. $\log_4 1 = 0$, because the exponent to which we raise 4 to get 1 is 0.

32. $\ln e^{-5} = -5$, because the exponent to which we raise e to get e^{-5} is -5.

33. $\ln\sqrt{e} = \ln e^{1/2} = \dfrac{1}{2}$, because the exponent to which we raise e to get $e^{1/2}$ is $\dfrac{1}{2}$.

34. $\log_{64} 4 = \dfrac{1}{3}$, because the exponent to which we raise 64 to get 4 is $\dfrac{1}{3}$. $(64^{1/3} = \sqrt[3]{64} = 4)$

35.

The exponent is the logarithm.

$$10^3 = 1000 \Rightarrow 3 = \log_{10} 1000$$

The base remains the same.

We could also say $3 = \log 1000$.

36. $5^{-3} = \dfrac{1}{125} \Rightarrow \log_5 \dfrac{1}{125} = -3$

37.

The exponent is the logarithm.

$$8^{1/3} = 2 \Rightarrow \log_8 2 = \frac{1}{3}$$

The base remains the same.

38. $10^{0.3010} = 2 \Rightarrow \log_{10} 2 = 0.0310$

We could also say $\log 2 = 0.0310$.

39. $e^3 = t \Rightarrow \log_e t = 3$, or $\ln\ t = 3$

40. $Q^t = x \Rightarrow \log_Q x = t$

41. $e^2 = 7.3891 \Rightarrow \log_e 7.3891 = 2$, or $\ln\ 7.3891 = 2$

42. $e^{-1} = 0.3679 \Rightarrow \log_e 0.3679 = -1$, or $\ln\ 0.3679 = -1$

43. $p^k = 3 \Rightarrow \log_p 3 = k$

44. $e^{-t} = 4000 \Rightarrow \log_e 4000 = -t$, or $\ln\ 4000 = -t$

45.

The logarithm is the exponent.

$$\log_5 5 = 1 \Rightarrow 5^1 = 5$$

The base remains the same.

46. $t = \log_4 7 \Rightarrow 7 = 4^t$

47. $\log 0.01 = -2$ is equivalent to $\log_{10} 0.01 = -2$.

The logarithm is the exponent.

$$\log_{10} 0.01 = -2 \Rightarrow 10^{-2} = 0.01$$

The base remains the same.

48. $\log 7 = 0.845 \Rightarrow 10^{0.845} = 7$

49. $\ln 30 = 3.4012 \Rightarrow e^{3.4012} = 30$

50. $\ln 0.38 = -0.9676 \Rightarrow e^{-0.9676} = 0.38$

51. $\log_a M = -x \Rightarrow a^{-x} = M$

52. $\log_t Q = k \Rightarrow t^k = Q$

53. $\log_a T^3 = x \Rightarrow a^x = T^3$

54. $\ln W^5 = t \Rightarrow e^t = W^5$

55. $\log 3 \approx 0.4771$

56. $\log 8 \approx 0.9031$

57. $\log 532 \approx 2.7259$

58. $\log 93,100 \approx 4.9689$

59. $\log 0.57 \approx -0.2441$

60. $\log 0.082 \approx -1.0862$

61. $\log(-2)$ does not exist. (The calculator gives an error message.)

62. $\ln 50 \approx 3.9120$

63. $\ln 2 \approx 0.6931$

64. $\ln(-4)$ does not exist. (The calculator gives an error message.)

65. $\ln 809.3 \approx 6.6962$

66. $\ln 0.00037 \approx -7.9020$

67. $\ln(-1.32)$ does not exist. (The calculator gives an error message.)

68. $\ln 0$ does not exist. (The calculator gives an error message.)

69. Let $a = 10$, $b = 4$, and $M = 100$ and substitute in the change-of-base formula.

$$\log_4 100 = \frac{\log_{10} 100}{\log_{10} 4} \approx 3.3219$$

70. $\log_3 20 = \dfrac{\log 20}{\log 3} \approx 2.7268$

71. Let $a = 10$, $b = 100$, and $M = 0.3$ and substitute in the change-of-base formula.

$$\log_{100} 0.3 = \frac{\log_{10} 0.3}{\log_{10} 100} \approx -0.2614$$

72. $\log_\pi 100 = \dfrac{\log\ 100}{\log\ \pi} \approx 4.0229$

73. Let $a = 10$, $b = 200$, and $M = 50$ and substitute in the change-of-base formula.

$$\log_{200} 50 = \frac{\log_{10} 50}{\log_{10} 200} \approx 0.7384$$

74. $\log_{5.3} 1700 = \dfrac{\log 1700}{\log 5.3} \approx 4.4602$

75. Let $a = e$, $b = 3$, and $M = 12$ and substitute in the change-of-base formula.

$$\log_3 12 = \frac{\ln 12}{\ln 3} \approx 2.2619$$

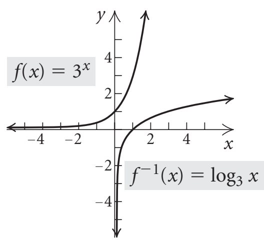
76. $\log_4 25 = \dfrac{\ln 25}{\ln 4} \approx 2.3219$

77. Let $a = e$, $b = 100$, and $M = 15$ and substitute in the change-of-base formula.

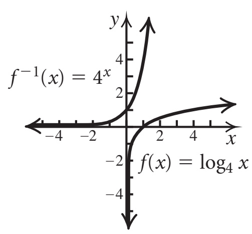
$$\log_{100} 15 = \frac{\ln 15}{\ln 100} \approx 0.5880$$

78. $\log_9 100 = \dfrac{\ln 100}{\ln 9} \approx 2.0959$

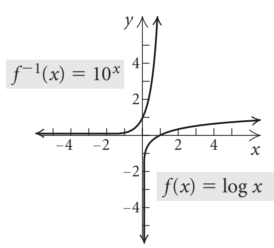
79. Graph $y = 3^x$ and then reflect this graph across the line $y = x$ to get the graph of $y = \log_3 x$.



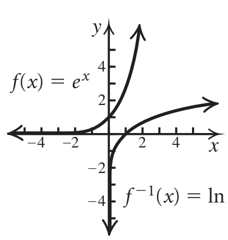
80. Graph $y = \log_4 x$ and then reflect this graph across the line $y = x$ to get the graph of $y = 4^x$.



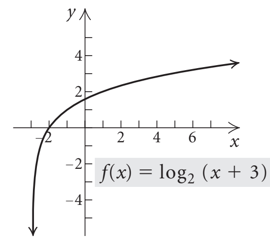
81. Graph $y = \log x$ and then reflect this graph across the line $y = x$ to get the graph of $y = 10^x$.



82. Graph $y = e^x$ and then reflect this graph across the line $y = x$ to get the graph of $y = \ln x$.



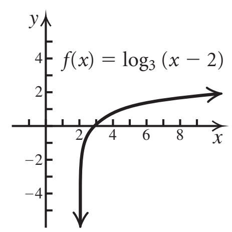
83. Shift the graph of $y = \log_2 x$ left 3 units.



Domain: $(-3, \infty)$

Vertical asymptote: $x = -3$

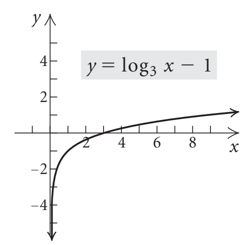
84. Shift the graph of $y = \log_3 x$ right 2 units.



Domain: $(2, \infty)$

Vertical asymptote: $x = 2$

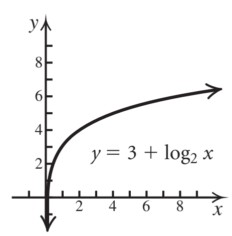
85. Shift the graph of $y = \log_3 x$ down 1 unit.



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

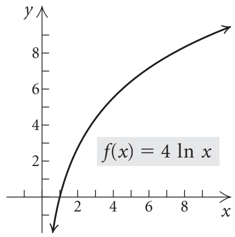
86. Shift the graph of $y = \log_2 x$ up 3 units.



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

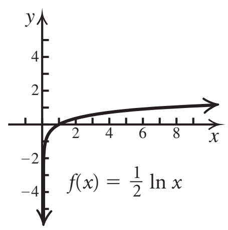
87. Stretch the graph of $y = \ln x$ vertically.



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

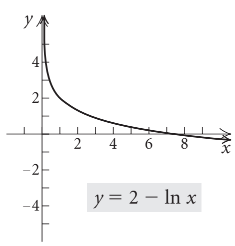
88. Shrink the graph of $y = \ln x$ vertically.



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

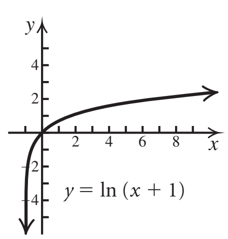
89. Reflect the graph of $y = \ln x$ across the x-axis and then shift it up 2 units.



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

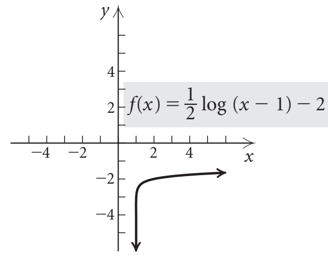
90. Shift the graph of $y = \ln x$ left 1 unit.



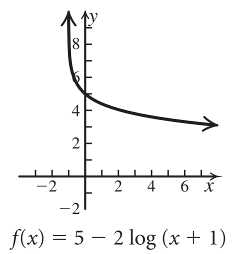
Domain: $(-1, \infty)$

Vertical asymptote: $x = -1$

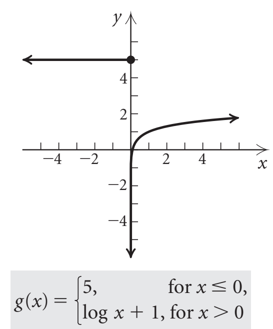
91. Shift the graph of $y = \log x$ right 1 unit, shrink it vertically, and shift it down 2 units.



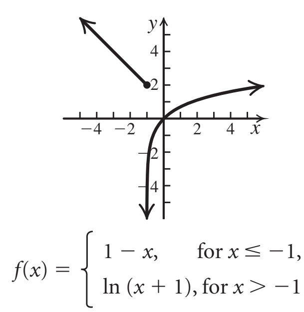
92. Shift the graph of $y = \log x$ left 1 unit, stretch it vertically, reflect it across the x-axis, and shift it up 5 units.



93. Graph $g(x) = 5$ for $x \leq 0$ and $g(x) = \log x + 1$ for $x > 0$.



94.



95. a) We substitute 598.541 for P, since P is in thousands.

$$w(598.541) = 0.37 \ln 598.541 + 0.05$$
$$\approx 2.4 \text{ ft/sec}$$

b) We substitute 3833.995 for P, since P is in thousands.

$$w(3833.995) = 0.37 \ln 3833.995 + 0.05$$
$$\approx 3.1 \text{ ft/sec}$$

c) We substitute 433.746 for P, since P is in thousands.

$$w(433.746) = 0.37\ln 433.746 + 0.05$$
$$\approx 2.3 \text{ ft/sec}$$

d) We substitute 2242.193 for P, since P is in thousands.

$$w(2242.193) = 0.37\ln 2242.193 + 0.05$$
$$\approx 2.9 \text{ ft/sec}$$

e) We substitute 669.651 for P, since P is in thousands.

$$w(669.651) = 0.37\ln 669.651 + 0.05$$
$$\approx 2.5 \text{ ft/sec}$$

f) We substitute 340.882 for P, since P is in thousands.

$$w(340.882) = 0.37\ln 340.882 + 0.05$$
$$\approx 2.2 \text{ ft/sec}$$

g) We substitute 798.382 for P, since P is in thousands.

$$w(798.382) = 0.37\ln 798.382 + 0.05$$
$$\approx 2.5 \text{ ft/sec}$$

h) We substitute 279.243 for P, since P is in thousands.

$$w(279.243) = 0.37\ln 279.243 + 0.05$$
$$\approx 2.1 \text{ ft/sec}$$

96. a)
$$\begin{aligned} S(0) &= 78 - 15\log(0+1) \\ &= 78 - 15\log 1 \\ &= 78 - 15\cdot 0 \\ &= 78\% \end{aligned}$$

b)
$$\begin{aligned} S(4) &= 78 - 15\log(4+1) \\ &= 78 - 15\log 5 \\ &\approx 78 - 15(0.698970) \\ &\approx 67.5\% \end{aligned}$$

$$\begin{aligned} S(24) &= 78 - 15\log(24+1) \\ &= 78 - 15\log 25 \\ &\approx 78 - 15(1.397940) \\ &\approx 57\% \end{aligned}$$

97. a) $R = \log\dfrac{10^{7.7}\cdot I_0}{I_0} = \log 10^{7.7} = 7.7$

b) $R = \log\dfrac{10^{9.5}\cdot I_0}{I_0} = \log 10^{9.5} = 9.5$

c) $R = \log\dfrac{10^{6.6}\cdot I_0}{I_0} = \log 10^{6.6} = 6.6$

d) $R = \log\dfrac{10^{7.4}\cdot I_0}{I_0} = \log 10^{7.4} = 7.4$

e) $R = \log\dfrac{10^{8.0}\cdot I_0}{I_0} = \log 10^{8.0} = 8.0$

f) $R = \log\dfrac{10^{7.9}\cdot I_0}{I_0} = \log 10^{7.9} = 7.9$

g) $R = \log\dfrac{10^{9.1}\cdot I_0}{I_0} = \log 10^{9.1} = 9.1$

h) $R = \log\dfrac{10^{6.9}\cdot I_0}{I_0} = \log 10^{6.9} = 6.9$

98. a) $\text{pH} = -\log[1.6\times 10^{-4}] \approx -(-3.8) \approx 3.8$

b) $\text{pH} = -\log[0.0013] \approx -(-2.9) \approx 2.9$

c) $\text{pH} = -\log[6.3\times 10^{-7}] \approx -(-6.2) \approx 6.2$

d) $\text{pH} = -\log[1.6\times 10^{-8}] \approx -(-7.8) \approx 7.8$

e) $\text{pH} = -\log[6.3\times 10^{-5}] \approx -(-4.2) \approx 4.2$

99. a)
$$\begin{aligned} 7 &= -\log[\text{H}^+] \\ -7 &= \log[\text{H}^+] \\ \text{H}^+ &= 10^{-7} \quad \text{Using the definition of logarithm} \end{aligned}$$

b)
$$\begin{aligned} 5.4 &= -\log[\text{H}^+] \\ -5.4 &= \log[\text{H}^+] \\ \text{H}^+ &= 10^{-5.4} \quad \text{Using the definition of logarithm} \\ \text{H}^+ &\approx 4.0\times 10^{-6} \end{aligned}$$

c)
$$\begin{aligned} 3.2 &= -\log[\text{H}^+] \\ -3.2 &= \log[\text{H}^+] \\ \text{H}^+ &= 10^{-3.2} \quad \text{Using the definition of logarithm} \\ \text{H}^+ &\approx 6.3\times 10^{-4} \end{aligned}$$

d)
$$\begin{aligned} 4.8 &= -\log[\text{H}^+] \\ -4.8 &= \log[\text{H}^+] \\ \text{H}^+ &= 10^{-4.8} \quad \text{Using the definition of logarithm} \\ \text{H}^+ &\approx 1.6\times 10^{-5} \end{aligned}$$

100. a) $N(1) = 1000 + 200\ln 1 = 1000$ units

b) $N(5) = 1000 + 200\ln 5 \approx 1332$ units

101. a)
$$\begin{aligned} L &= 10\log\frac{10^{14}\cdot I_0}{I_0} \\ &= 10\log 10^{14} = 10\cdot 14 \\ &\approx 140 \text{ decibels} \end{aligned}$$

b)
$$\begin{aligned} L &= 10\log\frac{10^{11.5}\cdot I_0}{I_0} \\ &= 10\log 10^{11.5} = 10\cdot 11.5 \\ &\approx 115 \text{ decibels} \end{aligned}$$

c)
$$\begin{aligned} L &= 10\log\frac{10^{9}\cdot I_0}{I_0} \\ &= 10\log 10^{9} = 10\cdot 9 \\ &= 90 \text{ decibels} \end{aligned}$$

d)
$$\begin{aligned} L &= 10\log\frac{10^{6.5}\cdot I_0}{I_0} \\ &= 10\log 10^{6.5} = 10\cdot 6.5 \\ &= 65 \text{ decibels} \end{aligned}$$

e)
$$\begin{aligned} L &= 10\log\frac{10^{10}\cdot I_0}{I_0} \\ &= 10\log 10^{10}\cdot 10\cdot 10 \\ &= 100 \text{ decibels} \end{aligned}$$

f)
$$\begin{aligned} L &= 10\log\frac{10^{19.4}\cdot I_0}{I_0} \\ &= 10\log 10^{19.4}\cdot 10\cdot 19.4 \\ &= 194 \text{ decibels} \end{aligned}$$

102.
$$\begin{aligned} 3x - 10y &= 14 \\ 3x - 14 &= 10y \\ \frac{3}{10}x - \frac{7}{5} &= y \end{aligned}$$
Slope: $\frac{3}{10}$; y-intercept: $\left(0, -\frac{7}{5}\right)$

103. $y = 6 = 0 \cdot x + 6$

Slope: 0; y-intercept $(0, 6)$

104. $x = -4$

Slope: not defined; y-intercept: none

105.
$$\begin{array}{r|rrrr} -5 & 1 & -6 & 3 & 10 \\ & & -5 & 55 & -290 \\ \hline & 1 & -11 & 58 & \vert\, -280 \end{array}$$
The remainder is -280, so $f(-5) = -280$.

106.
$$\begin{array}{r|rrrrr} -1 & 1 & -2 & 0 & 1 & -6 \\ & & -1 & 3 & -3 & 2 \\ \hline & 1 & -3 & 3 & -2 & \vert\, -4 \end{array}$$
$f(-1) = -4$

107.
$$\begin{aligned} f(x) &= (x - \sqrt{7})(x + \sqrt{7})(x - 0) \\ &= (x^2 - 7)(x) \\ &= x^3 - 7x \end{aligned}$$

108.
$$\begin{aligned} f(x) &= (x - 4i)(x + 4i)(x - 1) \\ &= (x^2 + 16)(x - 1) \\ &= x^3 - x^2 + 16x - 16 \end{aligned}$$

109. Using the change-of-base formula, we get

$\frac{\log_5 8}{\log_2 8} = \log_2 8 = 3.$

110. Using the change-of-base formula, we get

$\frac{\log_3 64}{\log_3 16} = \log_{16} 64.$

Let $\log_{16} 64 = x$. Then we have
$$\begin{aligned} 16^x &= 64 \quad \text{Using the definition of logarithm} \\ (2^4)^x &= 2^6 \\ 2^{4x} &= 2^6, \text{ so} \\ 4x &= 6 \\ x &= \frac{6}{4} = \frac{3}{2} \end{aligned}$$
Thus, $\frac{\log_3 64}{\log_3 16} = \frac{3}{2}$.

111. $f(x) = \log_5 x^3$

x^3 must be positive. Since $x^3 > 0$ for $x > 0$, the domain is $(0, \infty)$.

112. $f(x) = \log_4 x^2$

x^2 must be positive, so the domain is $(-\infty, 0) \cup (0, \infty)$.

113. $f(x) = \ln|x|$

$|x|$ must be positive. Since $|x| > 0$ for $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

114. $f(x) = \log(3x - 4)$

$3x - 4$ must be positive. We have
$$\begin{aligned} 3x - 4 &> 0 \\ x &> \frac{4}{3} \end{aligned}$$
The domain is $\left(\frac{4}{3}, \infty\right)$.

115. Graph $y = \log_2(2x + 5) = \frac{\log(2x + 5)}{\log 2}$. Observe that outputs are negative for inputs between $-\frac{5}{2}$ and -2. Thus, the solution set is $\left(-\frac{5}{2}, -2\right)$.

116. Graph $y_1 = \log_2(x - 3) = \frac{\log(x - 3)}{\log 2}$ and $y_2 = 4$.

Observe that the graph of y_1 lies on or above the graph of y_2 for all inputs greater than or equal to 19. Thus, the solution set is $[19, \infty)$.

117. Graph (d) is the graph of $f(x) = \ln|x|$.

118. Graph (c) is the graph of $f(x) = |\ln x|$.

119. Graph (b) is the graph of $f(x) = \ln x^2$.

120. Graph (a) is the graph of $g(x) = |\ln(x - 1)|$.

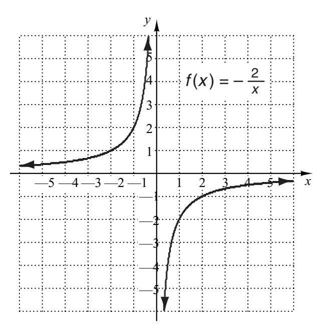
Chapter 5 Mid-Chapter Mixed Review

1. The statement is false. The domain of $y = \log x$, for instance, is $(0, \infty)$.

2. The statement is true. See page 391 in the text.

3. $f(0) = e^{-0} = 1$, so the y-intercept is $(0, 1)$. The given statement is false.

4. The graph of $f(x) = -\frac{2}{x}$ is shown below. It passes the horizontal-line test, so it is one-to-one.



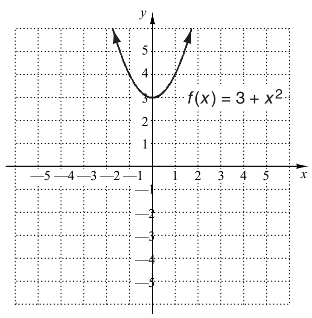
Replace $f(x)$ with y: $y = -\frac{2}{x}$

Interchange x and y: $x = -\frac{2}{y}$

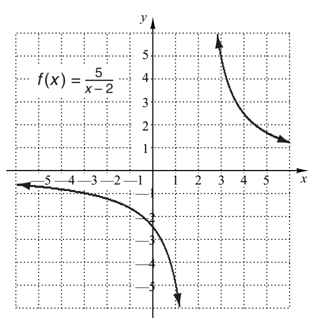
Solve for y:
$$\begin{aligned} xy &= -2 \\ y &= -\frac{2}{x} \end{aligned}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = -\frac{2}{x}$

5. The graph of $f(x) = 3 + x^2$ is shown below. Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one and thus does not have an inverse that is a function.



6. The graph of $f(x) = \dfrac{5}{x-2}$ is shown below. It passes the horizontal-line test, so it is one-to-one.



Replace $f(x)$ with y: $y = \dfrac{5}{x-2}$

Interchange x and y: $x = \dfrac{5}{y-2}$

Solve for y:
$$\begin{aligned} x(y-2) &= 5 \\ xy - 2x &= 5 \\ xy &= 2x + 5 \\ y &= \frac{2x+5}{x}, \text{ or } y = 2 + \frac{5}{x} \end{aligned}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{2x+5}{x}$, or $f^{-1}(x) = 2 + \dfrac{5}{x}$

7. $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = (\sqrt{x-5})^2 + 5 = x - 5 + 5 = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = \sqrt{x^2 + 5 - 5} = \sqrt{x^2} = x$

(This assumes that the domain of $f^{-1}(x)$ is restricted to $\{x|x \geq 0\}$.)

Since $(f^{-1} \circ f)(x) = x = (f \circ f^{-1})(x)$, we know that $f^{-1}(x) = x^2 + 5$.

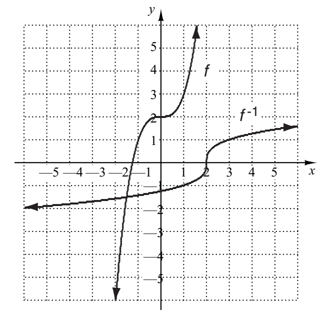
8. Replace $f(x)$ with y: $y = x^3 + 2$

Interchange x and y: $x = y^3 + 2$

Solve for y:
$$\begin{aligned} x - 2 &= y^3 \\ \sqrt[3]{x-2} &= y \end{aligned}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x-2}$

The domain of f is $(-\infty, \infty)$ and the range of f is also $(-\infty, \infty)$. Thus the domain and range of f^{-1} are both $(-\infty, \infty)$.



9. The graph of $y = \log_2 x$ is (d).

10. The graph of $f(x) = 2^x + 2$ is (h).

11. The graph of $f(x) = e^{x-1}$ is (c).

12. The graph of $f(x) = \ln x - 2$ is (g).

13. The graph of $f(x) = \ln(x-2)$ is (b).

14. The graph of $y = 2^{-x}$ is (f).

15. The graph of $f(x) = |\log x|$ is (e).

16. The graph of $f(x) = e^x + 1$ is (a).

17. $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$$A = 3200\left(1 + \frac{0.045}{4}\right)^{4 \cdot 6} \approx \$4185.57$$

18. $\log_4 1 = 0$ because the exponent to which we raise 4 to get 1 is 0.

19. $\ln e^{-4/5}$ is $-\dfrac{4}{5}$ because the exponent to which we raise e to get $e^{-4/5}$ is $-\dfrac{4}{5}$.

20. $\log 0.01 = -2$ because the exponent to which we raise 10 to get 0.01, or 10^{-2}, is -2.

21. $\ln e^2 = 2$ because the exponent to which we raise e to get e^2 is 2.

22. $\ln 1 = 0$ because the exponent to which we raise e to get 1 is 0.

23. $\log_2 \dfrac{1}{16} = -4$ because the exponent to which we raise 2 to get $\dfrac{1}{16}$, or 2^{-4}, is -4.

24. $\log 1 = 0$ because the exponent to which we raise 10 to get 1 is 0.

25. $\log_3 27 = 3$ because the exponent to which we raise 3 to get 27 is 3.

26. $\log \sqrt[4]{10} = \dfrac{1}{4}$ because the exponent to which we raise 10 to get $\sqrt[4]{10}$, or $10^{1/4}$ is $\dfrac{1}{4}$.

27. $\ln e = 1$ because the exponent to which we raise e to get e is 1.

28. $e^{-6} = 0.0025$ is equivalent to $\ln 0.0025 = -6$.

29. $\log T = r$ is equivalent to $10^r = T$.

30. $\log_3 20 = \dfrac{\ln 20}{\ln 3} \approx 2.7268$

31. $\log_\pi 10 = \dfrac{\log 10}{\log \pi} = \dfrac{1}{\log \pi} \approx 2.0115$

32. For an even function f, $f(x) = f(-x)$ so we have $f(x) = f(-x)$ but $x \neq -x$ (for $x \neq 0$). Thus f is not one-to-one and hence it does not have an inverse.

33. The most interest will be earned the eighth year, because the principle is greatest during that year.

34. Some differences are as follows: The range of f is $(-\infty, \infty)$ whereas the range of g is $(0, \infty)$; f has no asymptotes but g has a horizontal asymptote, the x-axis; the y-intercept of f is $(0, 0)$ and the y-intercept of g is $(0, 1)$.

35. If $\log b < 0$, then $b < 1$.

Exercise Set 5.4

1. Use the product rule.

$\log_3(81 \cdot 27) = \log_3 81 + \log_3 27 = 4 + 3 = 7$

2. $\log_2(8 \cdot 64) = \log_2 8 + \log_2 64 = 3 + 6 = 9$

3. Use the product rule.

$\log_5(5 \cdot 125) = \log_5 5 + \log_5 125 = 1 + 3 = 4$

4. $\log_4(64 \cdot 4) = \log_4 64 + \log_4 4 = 3 + 1 = 4$

5. Use the product rule.

$\log_t 8Y = \log_t 8 + \log_t Y$

6. $\log 0.2x = \log 0.2 + \log x$

7. Use the product rule.

$\ln xy = \ln x + \ln y$

8. $\ln ab = \ln a + \ln b$

9. Use the power rule.

$\log_b t^3 = 3 \log_b t$

10. $\log_a x^4 = 4 \log_a x$

11. Use the power rule.

$\log y^8 = 8 \log y$

12. $\ln y^5 = 5 \ln y$

13. Use the power rule.

$\log_c K^{-6} = -6 \log_c K$

14. $\log_b Q^{-8} = -8 \log_b Q$

15. Use the power rule.

$\ln \sqrt[3]{4} = \ln 4^{1/3} = \dfrac{1}{3} \ln 4$

16. $\ln \sqrt{a} = \ln a^{1/2} = \dfrac{1}{2} \ln a$

17. Use the quotient rule.

$\log_t \dfrac{M}{8} = \log_t M - \log_t 8$

18. $\log_a \dfrac{76}{13} = \log_a 76 - \log_a 13$

19. Use the quotient rule.

$\log \dfrac{x}{y} = \log x - \log y$

20. $\ln \dfrac{a}{b} = \ln a - \ln b$

21. Use the quotient rule.

$\ln \dfrac{r}{s} = \ln r - \ln s$

22. $\log_b \dfrac{3}{w} = \log_b 3 - \log_b w$

23.
$$\begin{aligned}
&\log_a 6xy^5z^4 \\
&= \log_a 6 + \log_a x + \log_a y^5 + \log_a z^4 \\
&\qquad \text{Product rule} \\
&= \log_a 6 + \log_a x + 5\log_a y + 4\log_a z \\
&\qquad \text{Power rule}
\end{aligned}$$

24.
$$\begin{aligned}
&\log_a x^3y^2z \\
&= \log_a x^3 + \log_a y^2 + \log_a z \\
&= 3\log_a x + 2\log_a y + \log_a z
\end{aligned}$$

25.
$$\begin{aligned}
&\log_b \frac{p^2q^5}{m^4b^9} \\
&= \log_b p^2q^5 - \log_b m^4b^9 \quad \text{Quotient rule} \\
&= \log_b p^2 + \log_b q^5 - (\log_b m^4 + \log_b b^9) \\
&\qquad \text{Product rule} \\
&= \log_b p^2 + \log_b q^5 - \log_b m^4 - \log_b b^9 \\
&= \log_b p^2 + \log_b q^5 - \log_b m^4 - 9 \quad (\log_b b^9 = 9) \\
&= 2\log_b p + 5\log_b q - 4\log_b m - 9 \quad \text{Power rule}
\end{aligned}$$

26.
$$\begin{aligned}
\log_b \frac{x^2y}{b^3} &= \log_b x^2y - \log_b b^3 \\
&= \log_b x^2 + \log_b y - \log_b b^3 \\
&= \log_b x^2 + \log_b y - 3 \\
&= 2\log_b x + \log_b y - 3
\end{aligned}$$

27.
$$\begin{aligned}
&\ln \frac{2}{3x^3y} \\
&= \ln 2 - \ln 3x^3y \quad \text{Quotient rule} \\
&= \ln 2 - (\ln 3 + \ln x^3 + \ln y) \quad \text{Product rule} \\
&= \ln 2 - \ln 3 - \ln x^3 - \ln y \\
&= \ln 2 - \ln 3 - 3\ln x - \ln y \quad \text{Power rule}
\end{aligned}$$

28. $\log \dfrac{5a}{4b^2} = \log 5a - \log 4b^2$
$= \log 5 + \log a - (\log 4 + \log b^2)$
$= \log 5 + \log a - \log 4 - \log b^2$
$= \log 5 + \log a - \log 4 - 2\log b$

29. $\log \sqrt{r^3 t}$
$= \log (r^3 t)^{1/2}$
$= \dfrac{1}{2}\log r^3 t$ Power rule
$= \dfrac{1}{2}(\log r^3 + \log t)$ Product rule
$= \dfrac{1}{2}(3\log r + \log t)$ Power rule
$= \dfrac{3}{2}\log r + \dfrac{1}{2}\log t$

30. $\ln \sqrt[3]{5x^5} = \ln (5x^5)^{1/3}$
$= \dfrac{1}{3}\ln 5x^5$
$= \dfrac{1}{3}(\ln 5 + \ln x^5)$
$= \dfrac{1}{3}(\ln 5 + 5\ln x)$
$= \dfrac{1}{3}\ln 5 + \dfrac{5}{3}\ln x$

31. $\log_a \sqrt{\dfrac{x^6}{p^5 q^8}}$
$= \dfrac{1}{2}\log_a \dfrac{x^6}{p^5 q^8}$
$= \dfrac{1}{2}[\log_a x^6 - \log_a (p^5 q^8)]$ Quotient rule
$= \dfrac{1}{2}[\log_a x^6 - (\log_a p^5 + \log_a q^8)]$ Product rule
$= \dfrac{1}{2}(\log_a x^6 - \log_a p^5 - \log_a q^8)$
$= \dfrac{1}{2}(6\log_a x - 5\log_a p - 8\log_a q)$ Power rule
$= 3\log_a x - \dfrac{5}{2}\log_a p - 4\log_a q$

32. $\log_c \sqrt[3]{\dfrac{y^3 z^2}{x^4}}$
$= \dfrac{1}{3}\log_c \dfrac{y^3 z^2}{x^4}$
$= \dfrac{1}{3}(\log_c y^3 z^2 - \log_c x^4)$
$= \dfrac{1}{3}(\log_c y^3 + \log_c z^2 - \log_c x^4)$
$= \dfrac{1}{3}(3\log_c y + 2\log_c z - 4\log_c x)$
$= \log_c y + \dfrac{2}{3}\log_c z - \dfrac{4}{3}\log_c x$

33. $\log_a \sqrt[4]{\dfrac{m^8 n^{12}}{a^3 b^5}}$
$= \dfrac{1}{4}\log_a \dfrac{m^8 n^{12}}{a^3 b^5}$ Power rule
$= \dfrac{1}{4}(\log_a m^8 n^{12} - \log_a a^3 b^5)$ Quotient rule
$= \dfrac{1}{4}[\log_a m^8 + \log_a n^{12} - (\log_a a^3 + \log_a b^5)]$
Product rule
$= \dfrac{1}{4}(\log_a m^8 + \log_a n^{12} - \log_a a^3 - \log_a b^5)$
$= \dfrac{1}{4}(\log_a m^8 + \log_a n^{12} - 3 - \log_a b^5)$
$(\log_a a^3 = 3)$
$= \dfrac{1}{4}(8\log_a m + 12\log_a n - 3 - 5\log_a b)$
Power rule

$= 2\log_a m + 3\log_a n - \dfrac{3}{4} - \dfrac{5}{4}\log_a b$

34. $\log_a \sqrt{\dfrac{a^6 b^8}{a^2 b^5}} = \log_a \sqrt{a^4 b^3}$
$= \dfrac{1}{2}(\log_a a^4 + \log_a b^3)$
$= \dfrac{1}{2}(4 + 3\log_a b)$
$= 2 + \dfrac{3}{2}\log_a b$

35. $\log_a 75 + \log_a 2$
$= \log_a (75 \cdot 2)$ Product rule
$= \log_a 150$

36. $\log 0.01 + \log 1000 = \log(0.01 \cdot 1000) = \log 10 = 1$

37. $\log 10{,}000 - \log 100$
$= \log \dfrac{10{,}000}{100}$ Quotient rule
$= \log 100$
$= 2$

38. $\ln 54 - \ln 6 = \ln \dfrac{54}{6} = \ln 9$

39. $\dfrac{1}{2}\log n + 3\log m$
$= \log n^{1/2} + \log m^3$ Power rule
$= \log n^{1/2} m^3$, or Product rule
$\log m^3 \sqrt{n}$ $n^{1/2} = \sqrt{n}$

40. $\dfrac{1}{2}\log a - \log 2 = \log a^{1/2} - \log 2$
$= \log \dfrac{a^{1/2}}{2}$, or $\log \dfrac{\sqrt{a}}{2}$

41. $\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$

$= \log_a x^{1/2} + \log_a y^4 - \log_a x^3$ Power rule

$= \log_a x^{1/2}y^4 - \log_a x^3$ Product rule

$= \log_a \frac{x^{1/2}y^4}{x^3}$ Quotient rule

$= \log_a x^{-5/2}y^4$, or $\log_a \frac{y^4}{x^{5/2}}$ Simplifying

42. $\frac{2}{5}\log_a x - \frac{1}{3}\log_a y = \log_a x^{2/5} - \log_a y^{1/3} = \log_a \frac{x^{2/5}}{y^{1/3}}$

43. $\ln x^2 - 2\ln\sqrt{x}$

$= \ln x^2 - \ln(\sqrt{x})^2$ Power rule

$= \ln x^2 - \ln x$ $[(\sqrt{x})^2 = x]$

$= \ln \frac{x^2}{x}$ Quotient rule

$= \ln x$

44. $\ln 2x + 3(\ln x - \ln y) = \ln 2x + 3\ln\frac{x}{y}$

$= \ln 2x + \ln\left(\frac{x}{y}\right)^3$

$= \ln 2x\left(\frac{x}{y}\right)^3$

$= \ln\frac{2x^4}{y^3}$

45. $\ln(x^2-4) - \ln(x+2)$

$= \ln\frac{x^2-4}{x+2}$ Quotient rule

$= \ln\frac{(x+2)(x-2)}{x+2}$ Factoring

$= \ln(x-2)$ Removing a factor of 1

46. $\log(x^3-8) - \log(x-2)$

$= \log\frac{x^3-8}{x-2}$

$= \log\frac{(x-2)(x^2+2x+4)}{x-2}$

$= \log(x^2+2x+4)$

47. $\log(x^2-5x-14) - \log(x^2-4)$

$= \log\frac{x^2-5x-14}{x^2-4}$ Quotient rule

$= \log\frac{(x+2)(x-7)}{(x+2)(x-2)}$ Factoring

$= \log\frac{x-7}{x-2}$ Removing a factor of 1

48. $\log_a \frac{a}{\sqrt{x}} - \log_a\sqrt{ax} = \log_a\frac{a}{\sqrt{x}\sqrt{ax}}$

$= \log_a\frac{\sqrt{a}}{x}$

$= \log_a\sqrt{a} - \log_a x$

$= \frac{1}{2}\log_a a - \log_a x$

$= \frac{1}{2} - \log_a x$

49. $\ln x - 3[\ln(x-5) + \ln(x+5)]$

$= \ln x - 3\ln[(x-5)(x+5)]$ Product rule

$= \ln x - 3\ln(x^2-25)$

$= \ln x - \ln(x^2-25)^3$ Power rule

$= \ln\frac{x}{(x^2-25)^3}$ Quotient rule

50. $\frac{2}{3}[\ln(x^2-9) - \ln(x+3)] + \ln(x+y)$

$= \frac{2}{3}\ln\frac{x^2-9}{x+3} + \ln(x+y)$

$= \frac{2}{3}\ln\frac{(x+3)(x-3)}{x+3} + \ln(x+y)$

$= \frac{2}{3}\ln(x-3) + \ln(x+y)$

$= \ln(x-3)^{2/3} + \ln(x+y)$

$= \ln[(x-3)^{2/3}(x+y)]$

51. $\frac{3}{2}\ln 4x^6 - \frac{4}{5}\ln 2y^{10}$

$= \frac{3}{2}\ln 2^2x^6 - \frac{4}{5}\ln 2y^{10}$ Writing 4 as 2^2

$= \ln(2^2x^6)^{3/2} - \ln(2y^{10})^{4/5}$ Power rule

$= \ln(2^3x^9) - \ln(2^{4/5}y^8)$

$= \ln\frac{2^3x^9}{2^{4/5}y^8}$ Quotient rule

$= \ln\frac{2^{11/5}x^9}{y^8}$

52. $120(\ln\sqrt[5]{x^3} + \ln\sqrt[3]{y^2} - \ln\sqrt[4]{16z^5})$

$= 120\left(\ln\frac{\sqrt[5]{x^3}\sqrt[3]{y^2}}{\sqrt[4]{16z^5}}\right)$

$= 120\left(\ln\frac{x^{3/5}y^{2/3}}{2z^{5/4}}\right)$

$= \ln\left(\frac{x^{3/5}y^{2/3}}{2z^{5/4}}\right)^{120}$

$= \ln\frac{x^{72}y^{80}}{2^{120}z^{150}}$

53. $\log_a\frac{2}{11} = \log_a 2 - \log_a 11$ Quotient rule

$\approx 0.301 - 1.041$

≈ -0.74

54. $\log_a 14 = \log_a(2 \cdot 7)$
$= \log_a 2 + \log_a 7$
$\approx 0.301 + 0.845$
≈ 1.146

55. $\log_a 98 = \log_a(7^2 \cdot 2)$
$= \log_a 7^2 + \log_a 2$ Product rule
$= 2\log_a 7 + \log_a 2$ Power rule
$\approx 2(0.845) + 0.301$
≈ 1.991

56. $\log_a \frac{1}{7} = \log_a 1 - \log_a 7$
$\approx 0 - 0.845$
≈ -0.845

57. $\frac{\log_a 2}{\log_a 7} \approx \frac{0.301}{0.845} \approx 0.356$

58. $\log_a 9$ cannot be found using the given information.

59. $\log_b 125 = \log_b 5^3$
$= 3\log_b 5$ Power rule
$\approx 3(1.609)$
≈ 4.827

60. $\log_b \frac{5}{3} = \log_b 5 - \log_b 3$
$\approx 1.609 - 1.099$
≈ 0.51

61. $\log_b \frac{1}{6} = \log_b 1 - \log_b 6$ Quotient rule
$= \log_b 1 - \log_b(2 \cdot 3)$
$= \log_b 1 - (\log_b 2 + \log_b 3)$ Product rule
$= \log_b 1 - \log_b 2 - \log_b 3$
$\approx 0 - 0.693 - 1.099$
≈ -1.792

62. $\log_b 30 = \log_b(2 \cdot 3 \cdot 5)$
$= \log_b 2 + \log_b 3 + \log_b 5$
$\approx 0.693 + 1.099 + 1.609$
≈ 3.401

63. $\log_b \frac{3}{b} = \log_b 3 - \log_b b$ Quotient rule
$\approx 1.099 - 1$
≈ 0.099

64. $\log_b 15b = \log_b(3 \cdot 5 \cdot b)$
$= \log_b 3 + \log_b 5 + \log_b b$
$\approx 1.099 + 1.609 + 1$
≈ 3.708

65. $\log_p p^3 = 3$ $(\log_a a^x = x)$

66. $\log_t t^{2713} = 2713$

67. $\log_e e^{|x-4|} = |x-4|$ $(\log_a a^x = x)$

68. $\log_q q^{\sqrt{3}} = \sqrt{3}$

69. $3^{\log_3 4x} = 4x$ $(a^{\log_a x} = x)$

70. $5^{\log_5 (4x-3)} = 4x - 3$

71. $10^{\log w} = w$ $(a^{\log_a x} = x)$

72. $e^{\ln x^3} = x^3$

73. $\ln e^{8t} = 8t$ $(\log_a a^x = x)$

74. $\log 10^{-k} = -k$

75. $\log_b \sqrt{b} = \log_b b^{1/2}$
$= \frac{1}{2}\log_b b$ Power rule
$= \frac{1}{2} \cdot 1$ $(\log_b b = 1)$
$= \frac{1}{2}$

76. $\log_b \sqrt{b^3} = \log_b b^{3/2}$
$= \frac{3}{2}\log_b b$
$= \frac{3}{2} \cdot 1$
$= \frac{3}{2}$

77. The degree of $f(x) = 5 - x^2 + x^4$ is 4, so the function is quartic.

78. The variable in $f(x) = 2^x$ is in the exponent, so $f(x)$ is an exponential function.

79. $f(x) = -\frac{3}{4}$ is of the form $f(x) = mx + b$ $\left(\text{with } m = 0 \text{ and } b = -\frac{3}{4}\right)$, so it is a linear function. In fact, it is a constant function.

80. The variable in $f(x) = 4^x - 8$ is in the exponent, so $f(x)$ is an exponential function.

81. $f(x) = -\frac{3}{x}$ is of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x)$ is not the zero polynomial, so $f(x)$ is a rational function.

82. $f(x) = \log x + 6$ is a logarithmic function.

83. The degree of $f(x) = -\frac{1}{3}x^3 - 4x^2 + 6x + 42$ is 3, so the function is cubic.

84. $f(x) = \frac{x^2 - 1}{x^2 + x - 6}$ is of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x)$ is not the zero polynomial, so $f(x)$ is a rational function.

85. $f(x) = \frac{1}{2}x + 3$ is of the form $f(x) = mx + b$, so it is a linear function.

86. The degree of $f(x) = 2x^2 - 6x + 3$ is 2, so the function is quadratic.

87. $5^{\log_5 8} = 2x$

$8 = 2x \quad (a^{\log_a x} = x)$

$4 = x$

The solution is 4.

88. $\ln e^{3x-5} = -8$

$3x - 5 = -8$

$3x = -3$

$x = -1$

The solution is -1.

89. $\log_a(x^2 + xy + y^2) + \log_a(x - y)$

$= \log_a[(x^2 + xy + y^2)(x - y)]$ Product rule

$= \log_a(x^3 - y^3)$ Multiplying

90. $\log_a(a^{10} - b^{10}) - \log_a(a + b)$

$= \log_a \dfrac{a^{10} - b^{10}}{a + b}$, or

$\log_a(a^9 - a^8b + a^7b^2 - a^6b^3 + a^5b^4 - a^4b^5 + a^3b^6 - a^2b^7 + ab^8 - b^9)$

91. $\log_a \dfrac{x - y}{\sqrt{x^2 - y^2}}$

$= \log_a \dfrac{x - y}{(x^2 - y^2)^{1/2}}$

$= \log_a(x - y) - \log_a(x^2 - y^2)^{1/2}$ Quotient rule

$= \log_a(x - y) - \dfrac{1}{2}\log_a(x^2 - y^2)$ Power rule

$= \log_a(x - y) - \dfrac{1}{2}\log_a[(x + y)(x - y)]$

$= \log_a(x - y) - \dfrac{1}{2}[\log_a(x + y) + \log_a(x - y)]$ Product rule

$= \log_a(x - y) - \dfrac{1}{2}\log_a(x + y) - \dfrac{1}{2}\log_a(x - y)$

$= \dfrac{1}{2}\log_a(x - y) - \dfrac{1}{2}\log_a(x + y)$

92. $\log_a \sqrt{9 - x^2}$

$= \log_a(9 - x^2)^{1/2}$

$= \dfrac{1}{2}\log_a(9 - x^2)$

$= \dfrac{1}{2}\log_a[(3 + x)(3 - x)]$

$= \dfrac{1}{2}[\log_a(3 + x) + \log_a(3 - x)]$

$= \dfrac{1}{2}\log_a(3 + x) + \dfrac{1}{2}\log_a(3 - x)$

93. $\log_a \dfrac{\sqrt[4]{y^2z^5}}{\sqrt[4]{x^3z^{-2}}}$

$= \log_a \sqrt[4]{\dfrac{y^2z^5}{x^3z^{-2}}}$

$= \log_a \sqrt[4]{\dfrac{y^2z^7}{x^3}}$

$= \log_a \left(\dfrac{y^2z^7}{x^3}\right)^{1/4}$

$= \dfrac{1}{4}\log_a \left(\dfrac{y^2z^7}{x^3}\right)$ Power rule

$= \dfrac{1}{4}(\log_a y^2z^7 - \log_a x^3)$ Quotient rule

$= \dfrac{1}{4}(\log_a y^2 + \log_a z^7 - \log_a x^3)$ Product rule

$= \dfrac{1}{4}(2\log_a y + 7\log_a z - 3\log_a x)$ Power rule

$= \dfrac{1}{4}(2 \cdot 3 + 7 \cdot 4 - 3 \cdot 2)$

$= \dfrac{1}{4} \cdot 28$

$= 7$

94. $\log_a M + \log_a N = \log_a(M + N)$

Let $a = 10$, $M = 1$, and $N = 10$. Then $\log_{10} 1 + \log_{10} 10 = 0 + 1 = 1$, but $\log_{10}(1 + 10) = \log_{10} 11 \approx 1.0414$. Thus, the statement is false.

95. $\log_a M - \log_a N = \log_a \dfrac{M}{N}$

This is the quotient rule, so it is true.

96. $\dfrac{\log_a M}{\log_a N} = \log_a M - \log_a N$

Let $M = a^2$ and $N = a$. Then $\dfrac{\log_a a^2}{\log_a a} = \dfrac{2}{1} = 2$, but $\log_a a^2 - \log_a a = 2 - 1 = 1$. Thus, the statement is false.

97. $\dfrac{\log_a M}{x} = \dfrac{1}{x}\log_a M = \log_a M^{1/x}$. The statement is true by the power rule.

98. $\log_a x^3 = 3\log_a x$ is true by the power rule.

99. $\log_a 8x = \log_a 8 + \log_a x = \log_a x + \log_a 8$. The statement is true by the product rule and the commutative property of addition.

100. $\log_N(M \cdot N)^x = x\log_N(M \cdot N)$

$= x(\log_N M + \log_N N)$

$= x(\log_N M + 1)$

$= x\log_N M + x$

The statement is true.

101. $\log_a \left(\dfrac{1}{x}\right) = \log_a x^{-1} = -1 \cdot \log_a x = -1 \cdot 2 = -2$

102. $\log_a x = 2$

$a^2 = x$

Let $\log_{1/a} x = n$ and solve for n.

$\log_{1/a} a^2 = n$ Substituting a^2 for x

$\left(\dfrac{1}{a}\right)^n = a^2$

$(a^{-1})^n = a^2$

$a^{-n} = a^2$

$-n = 2$

$n = -2$

Thus, $\log_{1/a} x = -2$ when $\log_a x = 2$.

103. We use the change-of-base formula.

$$\log_{10} 11 \cdot \log_{11} 12 \cdot \log_{12} 13 \cdots \log_{998} 999 \cdot \log_{999} 1000$$

$$= \log_{10} 11 \cdot \frac{\log_{10} 12}{\log_{10} 11} \cdot \frac{\log_{10} 13}{\log_{10} 12} \cdots \frac{\log_{10} 999}{\log_{10} 998} \cdot \frac{\log_{10} 1000}{\log_{10} 999}$$

$$= \frac{\log_{10} 11}{\log_{10} 11} \cdot \frac{\log_{10} 12}{\log_{10} 12} \cdots \frac{\log_{10} 999}{\log_{10} 999} \cdot \log_{10} 1000$$

$$= \log_{10} 1000$$

$$= 3$$

104. $\log_a x + \log_a y - mz = 0$

$\log_a x + \log_a y = mz$

$\log_a xy = mz$

$a^{mz} = xy$

105. $\ln a - \ln b + xy = 0$

$\ln a - \ln b = -xy$

$\ln \dfrac{a}{b} = -xy$

Then, using the definition of a logarithm, we have $e^{-xy} = \dfrac{a}{b}$.

106. $\log_a \left(\dfrac{1}{x}\right) = \log_a 1 - \log_a x = -\log_a x.$

Let $-\log_a x = y$. Then $\log_a x = -y$ and $x = a^{-y} = a^{-1 \cdot y} = \left(\dfrac{1}{a}\right)^y$, so $\log_{1/a} x = y$. Thus, $\log_a \left(\dfrac{1}{x}\right) = -\log_a x = \log_{1/a} x$.

107.

$$\log_a \left(\frac{x + \sqrt{x^2 - 5}}{5}\right)$$

$$= \log_a \left(\frac{x + \sqrt{x^2 - 5}}{5} \cdot \frac{x - \sqrt{x^2 - 5}}{x - \sqrt{x^2 - 5}}\right)$$

$$= \log_a \left(\frac{5}{5(x - \sqrt{x^2 - 5})}\right) = \log_a \left(\frac{1}{x - \sqrt{x^2 - 5}}\right)$$

$$= \log_a 1 - \log_a (x - \sqrt{x^2 - 5})$$

$$= -\log_a (x - \sqrt{x^2 - 5})$$

Exercise Set 5.5

1. $3^x = 81$

$3^x = 3^4$

$x = 4$ The exponents are the same.

The solution is 4.

2. $2^x = 32$

$2^x = 2^5$

$x = 5$

The solution is 5.

3. $2^{2x} = 8$

$2^{2x} = 2^3$

$2x = 3$ The exponents are the same.

$x = \dfrac{3}{2}$

The solution is $\dfrac{3}{2}$.

4. $3^{7x} = 27$

$3^{7x} = 3^3$

$7x = 3$

$x = \dfrac{3}{7}$

The solution is $\dfrac{3}{7}$.

5. $2^x = 33$

$\log 2^x = \log 33$ Taking the common logarithm on both sides

$x \log 2 = \log 33$ Power rule

$x = \dfrac{\log 33}{\log 2}$

$x \approx \dfrac{1.5185}{0.3010}$

$x \approx 5.044$

The solution is 5.044.

6. $2^x = 40$

$\log 2^x = \log 40$

$x \log 2 = \log 40$

$x = \dfrac{\log 40}{\log 2}$

$x \approx \dfrac{1.6021}{0.3010}$

$x \approx 5.322$

The solution is 5.322.

7.
$$\begin{aligned} 5^{4x-7} &= 125 \\ 5^{4x-7} &= 5^3 \\ 4x-7 &= 3 \\ 4x &= 10 \\ x &= \frac{10}{4} = \frac{5}{2} \end{aligned}$$

The solution is $\frac{5}{2}$.

8.
$$\begin{aligned} 4^{3x-5} &= 16 \\ 4^{3x-5} &= 4^2 \\ 3x-5 &= 2 \\ 3x &= 7 \\ x &= \frac{7}{3} \end{aligned}$$

The solution is $\frac{7}{3}$.

9.
$$\begin{aligned} 27 &= 3^{5x} \cdot 9^{x^2} \\ 3^3 &= 3^{5x} \cdot (3^2)^{x^2} \\ 3^3 &= 3^{5x} \cdot 3^{2x^2} \\ 3^3 &= 3^{5x+2x^2} \\ 3 &= 5x + 2x^2 \\ 0 &= 2x^2 + 5x - 3 \\ 0 &= (2x-1)(x+3) \end{aligned}$$

$x = \frac{1}{2}$ *or* $x = -3$

The solutions are -3 and $\frac{1}{2}$.

10.
$$\begin{aligned} 3^{x^2+4x} &= \frac{1}{27} \\ 3^{x^2+4x} &= 3^{-3} \\ x^2+4x &= -3 \\ x^2+4x+3 &= 0 \\ (x+3)(x+1) &= 0 \end{aligned}$$

$x = -3$ *or* $x = -1$

The solutions are -3 and -1.

11.
$$\begin{aligned} 84^x &= 70 \\ \log 84^x &= \log 70 \\ x \log 84 &= \log 70 \\ x &= \frac{\log 70}{\log 84} \\ x &\approx \frac{1.8451}{1.9243} \\ x &\approx 0.959 \end{aligned}$$

The solution is 0.959.

12.
$$\begin{aligned} 28^x &= 10^{-3x} \\ \log 28^x &= \log 10^{-3x} \\ x \log 28 &= -3x \\ x \log 28 + 3x &= 0 \\ x(\log 28 + 3) &= 0 \\ x &= 0 \end{aligned}$$

The solution is 0.

13.
$$\begin{aligned} 10^{-x} &= 5^{2x} \\ \log\ 10^{-x} &= \log\ 5^{2x} \\ -x &= 2x\ \log\ 5 \\ 0 &= x + 2x\ \log\ 5 \\ 0 &= x(1 + 2\ \log\ 5) \\ 0 &= x \qquad \text{Dividing by } 1 + 2\ \log\ 5 \end{aligned}$$

The solution is 0.

14.
$$\begin{aligned} 15^x &= 30 \\ \log 15^x &= \log 30 \\ x \log 15 &= \log 30 \\ x &= \frac{\log 30}{\log 15} \\ x &\approx 1.256 \end{aligned}$$

The solution is 1.256.

15.
$$\begin{aligned} e^{-c} &= 5^{2c} \\ \ln\ e^{-c} &= \ln 5^{2c} \\ -c &= 2c\ \ln\ 5 \\ 0 &= c + 2c\ \ln\ 5 \\ 0 &= c(1 + 2\ \ln\ 5) \\ 0 &= c \qquad \text{Dividing by } 1 + 2\ \ln\ 5 \end{aligned}$$

The solution is 0.

16.
$$\begin{aligned} e^{4t} &= 200 \\ \ln\ e^{4t} &= \ln\ 200 \\ 4t &= \ln\ 200 \\ t &= \frac{\ln\ 200}{4} \\ t &\approx 1.325 \end{aligned}$$

The solution is 1.325.

17.
$$\begin{aligned} e^t &= 1000 \\ \ln e^t &= \ln 1000 \\ t &= \ln 1000 \quad \text{Using } \log_a a^x = x \\ t &\approx 6.908 \end{aligned}$$

The solution is 6.908.

18.
$$\begin{aligned} e^{-t} &= 0.04 \\ \ln e^{-t} &= \ln 0.04 \\ -t &= \ln 0.04 \\ t &= -\ln 0.04 \approx 3.219 \end{aligned}$$

The solution is 3.219.

19.
$$\begin{aligned} e^{-0.03t} &= 0.08 \\ \ln e^{-0.03t} &= \ln 0.08 \\ -0.03t &= \ln 0.08 \\ t &= \frac{\ln 0.08}{-0.03} \\ t &\approx \frac{-2.5257}{-0.03} \\ t &\approx 84.191 \end{aligned}$$

The solution is 84.191.

20.
$$\begin{aligned} 1000e^{0.09t} &= 5000 \\ e^{0.09t} &= 5 \\ \ln e^{0.09t} &= \ln 5 \\ 0.09t &= \ln 5 \\ t &= \frac{\ln 5}{0.09} \\ t &\approx 17.883 \end{aligned}$$

The solution is 17.883.

21.
$$\begin{aligned} 3^x &= 2^{x-1} \\ \ln 3^x &= \ln 2^{x-1} \\ x\ln 3 &= (x-1)\ln 2 \\ x\ln 3 &= x\ln 2 - \ln 2 \\ \ln 2 &= x\ln 2 - x\ln 3 \\ \ln 2 &= x(\ln 2 - \ln 3) \\ \frac{\ln 2}{\ln 2 - \ln 3} &= x \\ \frac{0.6931}{0.6931 - 1.0986} &\approx x \\ -1.710 &\approx x \end{aligned}$$

The solution is -1.710.

22.
$$\begin{aligned} 5^{x+2} &= 4^{1-x} \\ \log 5^{x+2} &= \log 4^{1-x} \\ (x+2)\log 5 &= (1-x)\log 4 \\ x\log 5 + 2\log 5 &= \log 4 - x\log 4 \\ x\log 5 + x\log 4 &= \log 4 - 2\log 5 \\ x(\log 5 + \log 4) &= \log 4 - 2\log 5 \\ x &= \frac{\log 4 - 2\log 5}{\log 5 + \log 4} \\ x &\approx -0.612 \end{aligned}$$

The solution is -0.612.

23.
$$\begin{aligned} (3.9)^x &= 48 \\ \log(3.9)^x &= \log 48 \\ x\log 3.9 &= \log 48 \\ x &= \frac{\log 48}{\log 3.9} \\ x &\approx \frac{1.6812}{0.5911} \\ x &\approx 2.844 \end{aligned}$$

The solution is 2.844.

24.
$$\begin{aligned} 250 - (1.87)^x &= 0 \\ 250 &= (1.87)^x \\ \log 250 &= \log(1.87)^x \\ \log 250 &= x\log 1.87 \\ \frac{\log 250}{\log 1.87} &= x \\ 8.821 &\approx x \end{aligned}$$

The solution is 8.821.

25.
$$\begin{aligned} e^x + e^{-x} &= 5 \\ e^{2x} + 1 &= 5e^x \quad \text{Multiplying by } e^x \\ e^{2x} - 5e^x + 1 &= 0 \quad \text{This equation is quadratic in } e^x. \\ e^x &= \frac{5 \pm \sqrt{21}}{2} \\ x &= \ln\left(\frac{5 \pm \sqrt{21}}{2}\right) \approx \pm 1.567 \end{aligned}$$

The solutions are -1.567 and 1.567.

26.
$$\begin{aligned} e^x - 6e^{-x} &= 1 \\ e^{2x} - 6 &= e^x \\ e^{2x} - e^x - 6 &= 0 \\ (e^x - 3)(e^x + 2) &= 0 \\ e^x = 3 \quad &or \quad e^x = -2 \\ \ln e^x = \ln 3 \quad &\quad \text{No solution} \\ x &= \ln 3 \\ x &\approx 1.099 \end{aligned}$$

The solution is 1.099.

27.
$$\begin{aligned} 3^{2x-1} &= 5^x \\ \log 3^{2x-1} &= \log 5^x \\ (2x-1)\log 3 &= x \log 5 \\ 2x \log 3 - \log 3 &= x \log 5 \\ -\log 3 &= x \log 5 - 2x \log 3 \\ -\log 3 &= x(\log 5 - 2 \log 3) \\ \frac{-\log 3}{\log 5 - 2 \log 3} &= x \\ 1.869 &\approx x \end{aligned}$$

The solution is 1.869.

28.
$$\begin{aligned} 2^{x+1} &= 5^{2x} \\ \ln 2^{x+1} &= \ln 5^{2x} \\ (x+1)\ln 2 &= 2x \ln 5 \\ x \ln 2 + \ln 2 &= 2x \ln 5 \\ \ln 2 &= 2x \ln 5 - x \ln 2 \\ \ln 2 &= x(2 \ln 5 - \ln 2) \\ \frac{\ln 2}{2 \ln 5 - \ln 2} &= x \\ 0.274 &\approx x \end{aligned}$$

The solution is 0.274.

29.
$$\begin{aligned} 2e^x &= 5 - e^{-x} \\ 2e^x - 5 + e^{-x} &= 0 \\ e^x(2e^x - 5 + e^{-x}) &= e^x \cdot 0 \quad \text{Multiplying by } e^x \\ 2e^{2x} - 5e^x + 1 &= 0 \end{aligned}$$

Let $u = e^x$.

$2u^2 - 5u + 1 = 0$ Substituting

$a = 2$, $b = -5$, $c = 1$

$$u=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$u=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 2\cdot 1}}{2\cdot 2}$$

$$u=\frac{5\pm\sqrt{17}}{4}$$

Replace u with e^x.

$$e^x=\frac{5-\sqrt{17}}{4}\quad or\quad e^x=\frac{5+\sqrt{17}}{4}$$

$$\ln e^x=\ln\left(\frac{5-\sqrt{17}}{4}\right)\quad or\quad \ln e^x=\ln\left(\frac{5+\sqrt{17}}{4}\right)$$

$$x\approx -1.518\quad or\quad x\approx 0.825$$

The solutions are -1.518 and 0.825.

30. $e^x+e^{-x}=4$

$e^x-4+e^{-x}=0$

$e^{2x}-4e^x+1=0$ Multiplying by e^x

Let $u=e^x$.

$$u=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot 1\cdot 1}}{2\cdot 1}=\frac{4\pm\sqrt{12}}{2}$$

$$e^x=\frac{4-\sqrt{12}}{2}\quad or\quad e^x=\frac{4+\sqrt{12}}{2}$$

$$\ln e^x=\ln\left(\frac{4-\sqrt{12}}{2}\right)\quad or\quad \ln e^x=\ln\left(\frac{4+\sqrt{12}}{2}\right)$$

$$x\approx -1.317\quad or\quad x\approx 1.317$$

The solutions are -1.317 and 1.317.

31. $\log_5 x=4$

$x=5^4$ Writing an equivalent exponential equation

$x=625$

The solution is 625.

32. $\log_2 x=-3$

$x=2^{-3}$

$x=\frac{1}{8}$

The solution is $\frac{1}{8}$.

33. $\log x=-4$ The base is 10.

$x=10^{-4}$, or 0.0001

The solution is 0.0001.

34. $\log x=1$

$x=10^1=10$

The solution is 10.

35. $\ln x=1$ The base is e.

$x=e^1=e$

The solution is e.

36. $\ln x=-2$

$x=e^{-2}$, or $\frac{1}{e^2}$

The solution is e^{-2}, or $\frac{1}{e^2}$.

37. $\log_{64}\frac{1}{4}=x$

$\frac{1}{4}=64^x$

$\frac{1}{4}=(4^3)^x$

$4^{-1}=4^{3x}$

$-1=3x$

$-\frac{1}{3}=x$

The solution is $-\frac{1}{3}$.

38. $\log_{125}\frac{1}{25}=x$

$\frac{1}{25}=125^x$

$\frac{1}{5^2}=(5^3)^x$

$5^{-2}=5^{3x}$

$-2=3x$

$-\frac{2}{3}=x$

The solution is $-\frac{2}{3}$.

39. $\log_2(10+3x)=5$

$2^5=10+3x$

$32=10+3x$

$22=3x$

$\frac{22}{3}=x$

The answer checks. The solution is $\frac{22}{3}$.

40. $\log_5(8-7x)=3$

$5^3=8-7x$

$125=8-7x$

$117=-7x$

$-\frac{117}{7}=x$

The answer checks. The solution is $-\frac{117}{7}$.

41. $\log x+\log(x-9)=1$ The base is 10.

$\log_{10}[x(x-9)]=1$

$x(x-9)=10^1$

$x^2-9x=10$

$x^2-9x-10=0$

$(x-10)(x+1)=0$

$x=10$ or $x=-1$

Check: For 10:

$\log x+\log(x-9)=1$	
$\log 10+\log(10-9)$?	1
$\log 10+\log 1$	
$1+0$	
1	1 TRUE

For -1:

$$\begin{array}{c|c} \log x + \log(x-9) = 1 & \\ \hline \log(-1) + \log(-1-9) \ ? \ 1 & \end{array}$$

The number -1 does not check, because negative numbers do not have logarithms. The solution is 10.

42.
$$\begin{aligned} \log_2(x+1) + \log_2(x-1) &= 3 \\ \log_2[(x+1)(x-1)] &= 3 \\ (x+1)(x-1) &= 2^3 \\ x^2 - 1 &= 8 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

The number 3 checks, but -3 does not. The solution is 3.

43.
$$\begin{aligned} \log_2(x+20) - \log_2(x+2) &= \log_2 x \\ \log_2 \frac{x+20}{x+2} &= \log_2 x \\ \frac{x+20}{x+2} &= x \quad \text{Using the property of logarithmic equality} \\ x + 20 &= x^2 + 2x \quad \text{Multiplying by } x+2 \\ 0 &= x^2 + x - 20 \\ 0 &= (x+5)(x-4) \end{aligned}$$

$x + 5 = 0$ *or* $x - 4 = 0$

$x = -5$ *or* $x = 4$

Check: For -5:

$$\begin{array}{c|c} \log_2(x+20) - \log_2(x+2) = \log_2 x & \\ \hline \log_2(-5+20) - \log_2(-5+2) \ ? \ \log_2(-5) & \end{array}$$

The number -5 does not check, because negative numbers do not have logarithms.

For 4:

$$\begin{array}{r|l} \log_2(x+20) - \log_2(x+2) & \log_2 x \\ \hline \log_2(4+20) - \log_2(4+2) & \log_2 4 \\ \log_2 24 - \log_2 6 & \\ \log_2 \frac{24}{6} & \\ \log_2 4 & \log_2 4 \quad \text{TRUE} \end{array}$$

The solution is 4.

44.
$$\begin{aligned} \log(x+5) - \log(x-3) &= \log 2 \\ \log \frac{x+5}{x-3} &= \log 2 \\ \frac{x+5}{x-3} &= 2 \\ x + 5 &= 2x - 6 \\ 11 &= x \end{aligned}$$

The answer checks. The solution is 11.

45.
$$\begin{aligned} \log_8(x+1) - \log_8 x &= 2 \\ \log_8 \left(\frac{x+1}{x}\right) &= 2 \quad \text{Quotient rule} \\ \frac{x+1}{x} &= 8^2 \\ \frac{x+1}{x} &= 64 \\ x + 1 &= 64x \\ 1 &= 63x \\ \frac{1}{63} &= x \end{aligned}$$

The answer checks. The solution is $\frac{1}{63}$.

46.
$$\begin{aligned} \log x - \log(x+3) &= -1 \\ \log_{10} \frac{x}{x+3} &= -1 \\ \frac{x}{x+3} &= 10^{-1} \\ \frac{x}{x+3} &= \frac{1}{10} \\ 10x &= x + 3 \\ 9x &= 3 \\ x &= \frac{1}{3} \end{aligned}$$

The answer checks. The solution is $\frac{1}{3}$.

47.
$$\begin{aligned} \log x + \log(x+4) &= \log 12 \\ \log x(x+4) &= \log 12 \\ x(x+4) &= 12 \quad \text{Using the property of logarithmic equality} \\ x^2 + 4x &= 12 \\ x^2 + 4x - 12 &= 0 \\ (x+6)(x-2) &= 0 \end{aligned}$$

$x + 6 = 0$ *or* $x - 2 = 0$

$x = -6$ *or* $x = 2$

Check: For -6:

$$\begin{array}{c|c} \log x + \log(x+4) = \log 12 & \\ \hline \log(-6) + \log(-6+4) \ ? \ \log 12 & \end{array}$$

The number -6 does not check, because negative numbers do not have logarithms.

For 2:

$$\begin{array}{r|l} \log x + \log(x+4) & \log 12 \\ \hline \log 2 + \log(2+4) & \log 12 \\ \log 2 + \log 6 & \\ \log(2 \cdot 6) & \\ \log 12 & \log 12 \quad \text{TRUE} \end{array}$$

The solution is 2.

48.
$$\begin{aligned}\log_3(x+14)-\log_3(x+6) &= \log_3 x\\ \log_3\frac{x+14}{x+6} &= \log_3 x\\ \frac{x+14}{x+6} &= x\\ x+14 &= x^2+6x\\ 0 &= x^2+5x-14\\ 0 &= (x+7)(x-2)\end{aligned}$$

$x=-7$ *or* $x=2$

Only 2 checks. It is the solution.

49.
$$\begin{aligned}\log(x+8)-\log(x+1) &= \log\ 6\\ \log\frac{x+8}{x+1} &= \log\ 6 \quad \text{Quotient rule}\\ \frac{x+8}{x+1} &= 6 \quad \text{Using the property of logarithmic equality}\\ x+8 &= 6x+6 \quad \text{Multiplying by } x+1\\ 2 &= 5x\\ \frac{2}{5} &= x\end{aligned}$$

The answer checks. The solution is $\frac{2}{5}$.

50.
$$\begin{aligned}\ln x-\ln(x-4) &= \ln 3\\ \ln\frac{x}{x-4} &= \ln 3\\ \frac{x}{x-4} &= 3\\ x &= 3x-12\\ 12 &= 2x\\ 6 &= x\end{aligned}$$

The answer checks. The solution is 6.

51.
$$\begin{aligned}\log_4(x+3)+\log_4(x-3) &= 2\\ \log_4[(x+3)(x-3)] &= 2 \quad \text{Product rule}\\ (x+3)(x-3) &= 4^2\\ x^2-9 &= 16\\ x^2 &= 25\\ x &= \pm 5\end{aligned}$$

The number 5 checks, but -5 does not. The solution is 5.

52.
$$\begin{aligned}\ln(x+1)-\ln x &= \ln 4\\ \ln\frac{x+1}{x} &= \ln 4\\ \frac{x+1}{x} &= 4\\ x+1 &= 4x\\ 1 &= 3x\\ \frac{1}{3} &= x\end{aligned}$$

The answer checks. The solution is $\frac{1}{3}$.

53.
$$\begin{aligned}\log(2x+1)-\log(x-2) &= 1\\ \log\left(\frac{2x+1}{x-2}\right) &= 1 \quad \text{Quotient rule}\\ \frac{2x+1}{x-2} &= 10^1 = 10\\ 2x+1 &= 10x-20\\ &\quad \text{Multiplying by } x-2\\ 21 &= 8x\\ \frac{21}{8} &= x\end{aligned}$$

The answer checks. The solution is $\frac{21}{8}$.

54.
$$\begin{aligned}\log_5(x+4)+\log_5(x-4) &= 2\\ \log_5[(x+4)(x-4)] &= 2\\ x^2-16 &= 25\\ x^2 &= 41\\ x &= \pm\sqrt{41}\end{aligned}$$

Only $\sqrt{41}$ checks. The solution is $\sqrt{41}$.

55.
$$\begin{aligned}\ln(x+8)+\ln(x-1) &= 2\ln x\\ \ln(x+8)(x-1) &= \ln x^2\\ (x+8)(x-1) &= x^2 \quad \text{Using the property of logarithmic equality}\\ x^2+7x-8 &= x^2\\ 7x-8 &= 0\\ 7x &= 8\\ x &= \frac{8}{7}\end{aligned}$$

The answer checks. The solution is $\frac{8}{7}$.

56.
$$\begin{aligned}\log_3 x+\log_3(x+1) &= \log_3 2+\log_3(x+3)\\ \log_3 x(x+1) &= \log_3 2(x+3)\\ x(x+1) &= 2(x+3)\\ x^2+x &= 2x+6\\ x^2-x-6 &= 0\\ (x-3)(x+2) &= 0\end{aligned}$$

$x=3$ *or* $x=-2$

The number 3 checks, but -2 does not. The solution is 3.

57.
$$\begin{aligned}\log_6 x &= 1-\log_6(x-5)\\ \log_6 x+\log_6(x-5) &= 1\\ \log_6 x(x-5) &= 1\\ 6^1 &= x(x-5)\\ 6 &= x^2-5x\\ 0 &= x^2-5x-6\\ 0 &= (x+1)(x-6)\end{aligned}$$

$x+1=0$ *or* $x-6=0$

$x=-1$ *or* $x=6$

The number -1 does not check, but 6 does. The answer is 6.

58.
$$\begin{aligned} 2^{x^2-9x} &= \frac{1}{256} \\ 2^{x^2-9x} &= 2^{-8} \\ x^2-9x &= -8 \\ x^2-9x+8 &= 0 \\ (x-1)(x-8) &= 0 \end{aligned}$$
$x = 1$ *or* $x = 8$

The solutions are 1 and 8.

59.
$$\begin{aligned} 9^{x-1} &= 100(3^x) \\ (3^2)^{x-1} &= 100(3^x) \\ 3^{2x-2} &= 100(3^x) \\ \frac{3^{2x-2}}{3^x} &= 100 \\ 3^{x-2} &= 100 \\ \log\ 3^{x-2} &= \log\ 100 \\ (x-2)\log\ 3 &= 2 \\ x-2 &= \frac{2}{\log\ 3} \\ x &= 2+\frac{2}{\log\ 3} \\ x &\approx 6.192 \end{aligned}$$

The solution is 6.192.

60.
$$\begin{aligned} 2\ \ln\ x - \ln\ 5 &= \ln\ (x+10) \\ \ln\ x^2 - \ln\ 5 &= \ln\ (x+10) \\ \ln \frac{x^2}{5} &= \ln\ (x+10) \\ \frac{x^2}{5} &= x+10 \\ x^2 &= 5x+50 \\ x^2-5x-50 &= 0 \\ (x-10)(x+5) &= 0 \end{aligned}$$
$x = 10$ *or* $x = -5$

Only 10 checks. It is the solution.

61.
$$\begin{aligned} e^x - 2 &= -e^{-x} \\ e^x - 2 &= -\frac{1}{e^x} \\ e^{2x} - 2e^x &= -1 \quad \text{Multiplying by } e^x \\ e^{2x} - 2^{e^x} + 1 &= 0 \end{aligned}$$

Let $u = e^x$.
$$\begin{aligned} u^2-2u+1 &= 0 \\ (u-1)(u-1) &= 0 \end{aligned}$$
$$\begin{aligned} u-1 = 0 \quad &or \quad u-1 = 0 \\ u = 1 \quad &or \quad u = 1 \\ e^x = 1 \quad &or \quad e^x = 1 \quad \text{Replacing } u \text{ with } e^x \\ x = 0 \quad &or \quad x = 0 \end{aligned}$$

The solution is 0.

62.
$$\begin{aligned} 2\ \log\ 50 &= 3\ \log\ 25 + \log\ (x-2) \\ \log\ 50^2 &= \log\ 25^3 + \log\ (x-2) \\ \log\ 2500 &= \log\ 15,625 + \log\ (x-2) \\ \log\ 2500 &= \log\ [15,625(x-2)] \\ 2500 &= 15,625(x-2) \\ 2500 &= 15,625x - 31,250 \\ 33,750 &= 15,625x \\ \frac{54}{25} &= x \end{aligned}$$

The answer checks. The solution is $\frac{54}{25}$.

63. $g(x) = x^2 - 6$

a) $-\frac{b}{2a} = -\frac{0}{2\cdot 1} = 0$

$g(0) = 0^2 - 6 = -6$

The vertex is $(0, -6)$.

b) The axis of symmetry is $x = 0$.

c) Since the coefficient of the x^2-term is positive, the function has a minimum value. It is the second coordinate of the vertex, -6, and it occurs when $x = 0$.

64. $f(x) = -x^2 + 6x - 8$

a) $-\frac{b}{2a} = -\frac{6}{2(-1)} = 3$

$f(3) = -3^2 + 6\cdot 3 - 8 = 1$

The vertex is $(3, 1)$.

b) $x = 3$

c) Maximum: 1 at $x = 3$

65. $G(x) = -2x^2 - 4x - 7$

a) $-\frac{b}{2a} = -\frac{-4}{2(-2)} = -1$

$G(-1) = -2(-1)^2 - 4(-1) - 7 = -5$

The vertex is $(-1, -5)$.

b) The axis of symmetry is $x = -1$.

c) Since the coefficient of the x^2-term is negative, the function has a maximum value. It is the second coordinate of the vertex, -5, and it occurs when $x = -1$.

66. $H(x) = 3x^2 - 12x + 16$

a) $-\frac{b}{2a} = -\frac{-12}{2\cdot 3} = 2$

$H(2) = 3\cdot 2^2 - 12\cdot 2 + 16 = 4$

The vertex is $(2, 4)$.

b) $x = 2$

c) Minimum: 4 at $x = 2$

67. $$\frac{e^x+e^{-x}}{e^x-e^{-x}}=3$$
$$e^x+e^{-x}=3e^x-3e^{-x}\quad\text{Multiplying by } e^x-e^{-x}$$
$$4e^{-x}=2e^x\quad\text{Subtracting } e^x \text{ and adding } 3e^{-x}$$
$$2e^{-x}=e^x$$
$$2=e^{2x}\quad\text{Multiplying by } e^x$$
$$\ln 2=\ln e^{2x}$$
$$\ln 2=2x$$
$$\frac{\ln 2}{2}=x$$
$$0.347\approx x$$
The solution is 0.347.

68. $$\frac{5^x-5^{-x}}{5^x+5^{-x}}=8$$
$$5^x-5^{-x}=8\cdot 5^x+8\cdot 5^{-x}$$
$$-9\cdot 5^{-x}=7\cdot 5^x$$
$$-9=7\cdot 5^{2x}\quad\text{Multiplying by } 5^x$$
$$-\frac{9}{7}=5^{2x}$$
The number 5 raised to any power is non-negative. Thus, the equation has no solution.

69. $$\ln(\log x)=0$$
$$\log x=e^0$$
$$\log x=1$$
$$x=10^1=10$$
The answer checks. The solution is 10.

70. $$\ln(\ln x)=2$$
$$\ln x=e^2$$
$$x=e^{e^2}\approx 1618.178$$
The answer checks. The solution is e^{e^2}, or 1618.178.

71. $$\sqrt{\ln x}=\ln\sqrt{x}$$
$$\sqrt{\ln x}=\frac{1}{2}\ln x\quad\text{Power rule}$$
$$\ln x=\frac{1}{4}(\ln x)^2\quad\text{Squaring both sides}$$
$$0=\frac{1}{4}(\ln x)^2-\ln x$$
Let $u=\ln x$ and substitute.
$$\frac{1}{4}u^2-u=0$$
$$u\left(\frac{1}{4}u-1\right)=0$$
$$u=0\quad\textit{or}\quad \frac{1}{4}u-1=0$$
$$u=0\quad\textit{or}\quad \frac{1}{4}u=1$$
$$u=0\quad\textit{or}\quad u=4$$
$$\ln x=0\quad\textit{or}\quad \ln x=4$$
$$x=e^0=1\quad\textit{or}\quad x=e^4\approx 54.598$$
Both answers check. The solutions are 1 and e^4, or 1 and 54.598.

72. $$\ln\sqrt[4]{x}=\sqrt{\ln x}$$
$$\frac{1}{4}\ln x=\sqrt{\ln x}$$
$$\frac{1}{16}(\ln x)^2=\ln x\quad\text{Squaring both sides}$$
$$\frac{1}{16}(\ln x)^2-\ln x=0$$
Let $u=\ln x$ and substitute.
$$\frac{1}{16}u^2-u=0$$
$$u\left(\frac{1}{16}u-1\right)=0$$
$$u=0\quad\textit{or}\quad \frac{1}{16}u-1=0$$
$$u=0\quad\textit{or}\quad u=16$$
$$\ln x=0\quad\textit{or}\quad \ln x=16$$
$$x=e^0\quad\textit{or}\quad x=e^{16}$$
$$x=1\quad\textit{or}\quad x=e^{16}\approx 8,886,110.521$$
Both answers check. The solutions are 1 and e^{16}, or 1 and 8,886,110.521.

73. $$(\log_3 x)^2-\log_3 x^2=3$$
$$(\log_3 x)^2-2\log_3 x-3=0$$
Let $u=\log_3 x$ and substitute:
$$u^2-2u-3=0$$
$$(u-3)(u+1)=0$$
$$u=3\quad\textit{or}\quad u=-1$$
$$\log_3 x=3\quad\textit{or}\quad \log_3 x=-1$$
$$x=3^3\quad\textit{or}\quad x=3^{-1}$$
$$x=27\quad\textit{or}\quad x=\frac{1}{3}$$
Both answers check. The solutions are $\frac{1}{3}$ and 27.

74. $$\log_3(\log_4 x)=0$$
$$\log_4 x=3^0$$
$$\log_4 x=1$$
$$x=4^1$$
$$x=4$$
The answer checks. The solution is 4.

75. $$\ln x^2=(\ln x)^2$$
$$2\ln x=(\ln x)^2$$
$$0=(\ln x)^2-2\ln x$$
Let $u=\ln x$ and substitute.
$$0=u^2-2u$$
$$0=u(u-2)$$
$$u=0\quad\textit{or}\quad u=2$$
$$\ln x=0\quad\textit{or}\quad \ln x=2$$
$$x=1\quad\textit{or}\quad x=e^2\approx 7.389$$

Both answers check. The solutions are 1 and e^2, or 1 and 7.389.

76. $$\begin{aligned}(\log x)^2 - \log x^2 &= 3\\ (\log x)^2 - 2\log x - 3 &= 0\end{aligned}$$
Let $u = \log x$ and substitute.
$$\begin{aligned}u^2 - 2u - 3 &= 0\\ (u+1)(u-3) &= 0\\ u = -1 \quad &or \quad u = 3\\ \log x = -1 \quad &or \quad \log x = 3\\ x = \frac{1}{10} \quad &or \quad x = 1000\end{aligned}$$
Both answers check. The solutions are $\frac{1}{10}$ and 1000.

77. $$\begin{aligned}5^{2x} - 3\cdot 5^x + 2 &= 0\\ (5^x - 1)(5^x - 2) &= 0 \quad \text{This equation is quadratic in } 5^x.\\ 5^x = 1 \quad &or \quad 5^x = 2\\ \log 5^x = \log 1 \quad &or \quad \log 5^x = \log 2\\ x\log 5 = 0 \quad &or \quad x\log 5 = \log 2\\ x = 0 \quad &or \quad x = \frac{\log 2}{\log 5} \approx 0.431\end{aligned}$$
The solutions are 0 and 0.431.

78. $$\begin{aligned}e^{2x} - 9\cdot e^x + 14 &= 0\\ (e^x - 2)(e^x - 7) &= 0\\ e^x = 2 \quad &or \quad e^x = 7\\ \ln e^x = \ln 2 \quad &or \quad \ln e^x = \ln 7\\ x = \ln 2 \quad &or \quad x = \ln 7\\ x \approx 0.693 \quad &or \quad x \approx 1.946\end{aligned}$$
The solutions are 0.693 and 1.946.

79. $$\begin{aligned}\log_3 |x| &= 2\\ |x| &= 3^2\\ |x| &= 9\\ x = -9 \quad &or \quad x = 9\end{aligned}$$
Both answers check. The solutions are -9 and 9.

80. $$\begin{aligned}x\left(\ln\frac{1}{6}\right) &= \ln 6\\ x(\ln 1 - \ln 6) &= \ln 6\\ -x\ln 6 &= \ln 6 \quad (\ln 1 = 0)\\ x &= -1\end{aligned}$$
The solution is -1.

81. $$\begin{aligned}\ln x^{\ln x} &= 4\\ \ln x\cdot\ln x &= 4\\ (\ln x)^2 &= 4\\ \ln x &= \pm 2\\ \ln x = -2 \quad &or \quad \ln x = 2\\ x = e^{-2} \quad &or \quad x = e^2\\ x \approx 0.135 \quad &or \quad x \approx 7.389\end{aligned}$$
Both answers check. The solutions are e^{-2} and e^2, or 0.135 and 7.389.

82. $$\begin{aligned}x^{\log x} &= \frac{x^3}{100}\\ \log x^{\log x} &= \log\frac{x^3}{100}\\ \log x\cdot\log x &= \log x^3 - \log 100\\ (\log x)^2 &= 3\log x - 2\\ (\log x)^2 - 3\log x + 2 &= 0\end{aligned}$$
Let $u = \log x$ and substitute.
$$\begin{aligned}u^2 - 3u + 2 &= 0\\ (u-1)(u-2) &= 0\\ u = 1 \quad &or \quad u = 2\\ \log x = 1 \quad &or \quad \log x = 2\\ x = 10 \quad &or \quad x = 10^2 = 100\end{aligned}$$
Both answers check. The solutions are 10 and 100.

83. $$\begin{aligned}\frac{\sqrt{(e^{2x}\cdot e^{-5x})^{-4}}}{e^x \div e^{-x}} &= e^7\\ \frac{\sqrt{e^{12x}}}{e^{x-(-x)}} &= e^7\\ \frac{e^{6x}}{e^{2x}} &= e^7\\ e^{4x} &= e^7\\ 4x &= 7\\ x &= \frac{7}{4}\end{aligned}$$
The solution is $\frac{7}{4}$.

84. $$\begin{aligned}\frac{(e^{3x+1})^2}{e^4} &= e^{10x}\\ \frac{e^{6x+2}}{e^4} &= e^{10x}\\ e^{6x-2} &= e^{10x}\\ 6x - 2 &= 10x\\ -2 &= 4x\\ -\frac{1}{2} &= x\end{aligned}$$
The solution is $-\frac{1}{2}$.

85. $|\log_5 x| + 3\log_5 |x| = 4$

Note that we must have $x > 0$. First consider the case when $0 < x < 1$. When $0 < x < 1$, then $\log_5 x < 0$, so $|\log_5 x| = -\log_5 x$ and $|x| = x$. Thus we have:
$$\begin{aligned}-\log_5 x + 3\log_5 x &= 4\\ 2\log_5 x &= 4\\ \log_5 x^2 &= 4\\ x^2 &= 5^4\\ x &= 5^2\\ x &= 25 \quad (\text{Recall that } x > 0.)\end{aligned}$$

25 cannot be a solution since we assumed $0 < x < 1$.

Now consider the case when $x > 1$. In this case $\log_5 x > 0$, so $|\log_5 x| = \log_5 x$ and $|x| = x$. Thus we have:

$$\begin{aligned} \log_5 x + 3\log_5 x &= 4 \\ 4\log_5 x &= 4 \\ \log_5 x &= 1 \\ x &= 5 \end{aligned}$$

This answer checks. The solution is 5.

86. $$\begin{aligned} e^x &< \frac{4}{5} \\ \ln e^x &< \ln 0.8 \\ x &< -0.223 \end{aligned}$$

The solution set is $(-\infty, -0.223)$.

87. $|2^{x^2} - 8| = 3$

$$\begin{aligned} 2^{x^2} - 8 &= -3 &\quad or \quad 2^{x^2} - 8 &= 3 \\ 2^{x^2} &= 5 &\quad or \quad 2^{x^2} &= 11 \\ \log 2^{x^2} &= \log 5 &\quad or \quad \log 2^{x^2} &= \log 11 \\ x^2 \log 2 &= \log 5 &\quad or \quad x^2\log 2 &= \log 11 \\ x^2 &= \frac{\log 5}{\log 2} &\quad or \quad x^2 &= \frac{\log 11}{\log 2} \\ x &= \pm 1.524 &\quad or \quad x &= \pm 1.860 \end{aligned}$$

The solutions are -1.860, -1.524, 1.524, and 1.860.

88. $a = \log_8 225$, so $8^a = 225 = 15^2$.

$b = \log_2 15$, so $2^b = 15$.

Then $$\begin{aligned} 8^a &= (2^b)^2 \\ (2^3)^a &= 2^{2b} \\ 2^{3a} &= 2^{2b} \\ 3a &= 2b \\ a &= \frac{2}{3}b. \end{aligned}$$

89. $\log_2[\log_3(\log_4 x)] = 0$ yields $x = 64$.

$\log_3[\log_2(\log_4 y)] = 0$ yields $y = 16$.

$\log_4[\log_3(\log_2 z)] = 0$ yields $z = 8$.

Then $x + y + z = 64 + 16 + 8 = 88$.

90. $\log_5 125 = 3$ and $\log_{125} 5 = \dfrac{1}{3}$, so $a = (\log_{125} 5)^{\log_5 125}$ is equivalent to $a = \left(\dfrac{1}{3}\right)^3 = \dfrac{1}{27}$. Then $\log_3 a = \log_3 \dfrac{1}{27} = -3$.

91. $f(x) = e^x - e^{-x}$

Replace $f(x)$ with y: $y = e^x - e^{-x}$

Interchange x and y: $x = e^y - e^{-y}$

Solve for y: $xe^y = e^{2y} - 1$ Multiplying by e^y

$$0 = e^{2y} - xe^y - 1$$

Using the quadratic formula with $a = 1$, $b = -x$, and $c = -1$ and taking the positive square root (since $e^y > 0$), we get $e^y = \dfrac{x + \sqrt{x^2+4}}{2}$. Then we have

$$\begin{aligned} \ln e^y &= \ln\left(\frac{x + \sqrt{x^2+4}}{2}\right) \\ y &= \ln\left(\frac{x + \sqrt{x^2+4}}{2}\right) \end{aligned}$$

Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \ln\left(\frac{x + \sqrt{x^2+4}}{2}\right).$$

Exercise Set 5.6

1. a) Substitute 6.8 for P_0 and 0.0113 for k in $P(t) = P_0e^{kt}$. We have:

$P(t) = 6.8e^{0.0113t}$, where $P(t)$ is in billions and t is the number of years after 2009.

b) In 2012, $t = 2012 - 2009 = 3$.

$P(3) = 6.8e^{0.0113(3)} \approx 7.0$ billion

In 2020, $t = 2020 - 2009 = 11$.

$P(11) = 6.8e^{0.0113(11)} \approx 7.7$ billion

c) Substitute 8 for $P(t)$ and solve for t.

$$\begin{aligned} 8 &= 6.8e^{0.0113t} \\ \frac{8}{6.8} &= e^{0.0113t} \\ \ln\frac{8}{6.8} &= \ln e^{0.0113t} \\ \ln\frac{8}{6.8} &= 0.0113t \\ \frac{\ln\frac{8}{6.8}}{0.0113} &= t \\ 14.4 &\approx t \end{aligned}$$

The world population will be 8 billion about 14.4 yr after 2009.

d) $T = \dfrac{\ln 2}{0.0113} \approx 61.3$ yr

2. a) $P(t) = 100e^{0.117t}$

b) $P(7) = 100e^{0.117(7)} \approx 227$

Note that 2 weeks $= 2 \cdot 7$ days $= 14$ days.

$P(14) = 100e^{0.117(14)} \approx 514$

c) $t = \dfrac{\ln 2}{0.117} \approx 5.9$ days

3. a) $k = \dfrac{\ln 2}{70.7} \approx 0.98\%$

b) $k = \dfrac{\ln 2}{45.9} \approx 1.51\%$

c) $T = \dfrac{\ln 2}{0.0321} \approx 21.6$ yr

d) $T = \dfrac{\ln 2}{0.012} \approx 57.8$ yr

e) $k = \dfrac{\ln 2}{248} \approx 0.28\%$

f) $T = \dfrac{\ln 2}{0.0232} \approx 29.9$ per yr

g) $T = \dfrac{\ln 2}{0.014} \approx 49.5$ yr

h) $k = \dfrac{\ln 2}{105.0} \approx 0.66\%$

i) $k = \dfrac{\ln 2}{34.0} \approx 2.04\%$

j) $T = \dfrac{\ln 2}{0.0184} \approx 37.7$ yr

4. a)
$$\begin{aligned} 1550 &= 12e^{k\cdot 59} \\ \frac{1550}{12} &= e^{59k} \\ \ln \frac{1550}{12} &= \ln e^{59k} \\ \ln \frac{1550}{12} &= 59k \\ \frac{\ln \frac{1550}{12}}{59} &= k \\ 0.0824 &\approx k \end{aligned}$$

Then we have $E(t) = 12e^{0.0824t}$, where $E(t)$ is in billions of dollars and t is the number of years after 1950.

b) $E(22) = 12e^{0.0824(22)} \approx \$73.5$ billion

$E(51) = 12e^{0.0824(51)} \approx \$802.2$ billion

$E(65) = 12e^{0.0824(65)} \approx \$2542.5$ billion

5.
$$\begin{aligned} P(t) &= P_0e^{kt} \\ 32,961,561,600 &= 9,035,536e^{0.016t} \\ \frac{32,961,561,600}{9,035,536} &= e^{0.016t} \\ \ln\left(\frac{32,961,561,600}{9,035,536}\right) &= \ln e^{0.016t} \\ \ln\left(\frac{32,961,561,600}{9,035,536}\right) &= 0.016t \\ \frac{\ln\left(\frac{32,961,561,600}{9,035,536}\right)}{0.016} &= t \\ 513 &\approx t \end{aligned}$$

There will be one person for every square yard of land about 513 yr after 2009.

6. a)
$$\begin{aligned} 106,500,000 &= 17,000e^{-k\cdot 58} \\ \frac{106,500,000}{17,000} &= e^{58k} \\ \ln\left(\frac{106,500}{17}\right) &= \ln e^{58k} \\ \ln\left(\frac{106,500}{17}\right) &= 58k \\ \frac{\ln\left(\frac{106,500}{58}\right)}{58} &= k \\ 0.1507 &\approx k \end{aligned}$$

Then we have $A(t) = 17,000e^{0.1507t}$, where $A(t)$ is in dollars and t is the number of years after 1952.

b) $A(68) = 17,000e^{0.1507(68)} \approx \$479,650,669$

c) $T = \dfrac{\ln 2}{0.1507} \approx 4.6$ yr

d)
$$\begin{aligned} 240,000,000 &= 17,000e^{0.1507t} \\ \frac{240,000}{17} &= e^{0.1507t} \\ \ln\left(\frac{240,000}{17}\right) &= \ln e^{0.1507t} \\ \ln\left(\frac{240,000}{17}\right) &= 0.1507t \\ \frac{\ln\left(\frac{240,000}{17}\right)}{0.1507} &= t \\ 63.4 \text{ yr} &\approx t \end{aligned}$$

7. a) Substitute 10,000 for P_0 and 5.4%, or 0.054 for k.

$P(t) = 10,000e^{0.054t}$

b)
$$\begin{aligned} P(1) &= 10,000e^{0.054(1)} \approx \$10,554.85 \\ P(2) &= 10,000e^{0.054(2)} \approx \$11,140.48 \\ P(5) &= 10,000e^{0.054(5)} \approx \$13,099.64 \\ P(10) &= 10,000e^{0.054(10)} \approx \$17,160.07 \end{aligned}$$

c) $T = \dfrac{\ln 2}{0.054} \approx 12.8$ yr

8. a) $T = \dfrac{\ln 2}{0.032} \approx 21.7$ yr

$P(5) = 35,000e^{0.032(5)} \approx \$41,072.88$

b)
$$\begin{aligned} 7130.90 &= 5000e^{5k} \\ 1.4618 &= e^{5k} \\ \ln 1.4618 &= \ln e^{5k} \\ \ln 1.4618 &= 5k \\ \frac{\ln 1.4618}{5} &= k \\ 0.071 &\approx k \\ 7.1\% &\approx k \end{aligned}$$

$T = \dfrac{\ln 2}{0.071} \approx 9.8$ yr

c)
$$\begin{aligned} 9923.47 &= P_0e^{0.056(5)} \\ \frac{9923.47}{e^{0.056(5)}} &= P_0 \\ \$7500 &\approx P_0 \end{aligned}$$

$T = \dfrac{\ln 2}{0.056} \approx 12.4$ yr

d) $k = \dfrac{\ln 2}{11} \approx 0.063$, or 6.3%

$$\begin{aligned} 17,539.32 &= P_0e^{0.063(5)} \\ \frac{17,539.32}{e^{0.063(5)}} &= P_0 \\ \$12,800 &\approx P_0 \end{aligned}$$

e)
$$136,503.18 = 109,000e^{k\cdot 5}$$
$$\frac{136,503.18}{109,000} = e^{5k}$$
$$\ln\left(\frac{136,503.18}{109,000}\right) = \ln e^{5k}$$
$$\ln\left(\frac{136,503.18}{109,000}\right) = 5k$$
$$\frac{\ln\left(\frac{136,503.18}{109,000}\right)}{5} = k$$
$$0.045 \approx k$$
$$4.5\% \approx k$$
$$T = \frac{\ln 2}{0.045} \approx 15.4 \text{ yr}$$

f) $k = \dfrac{\ln 2}{46.2} \approx 0.015$, or 1.5%
$$19,552.82 = P_0e^{0.015(5)}$$
$$\frac{19,552.82}{e^{0.075}} = P_0$$
$$\$18,140 \approx P_0$$

9. We use the function found in Example 5. If the bones have lost 77.2% of their carbon-14 from an initial amount P_0, then $22.8\%P_0$, or $0.228P_0$ remains. We substitute in the function.
$$0.228P_0 = P_0e^{-0.00012t}$$
$$0.228 = e^{-0.00012t}$$
$$\ln 0.228 = \ln e^{-0.00012t}$$
$$\ln 0.228 = -0.00012t$$
$$\frac{\ln 0.228}{-0.00012} = t$$
$$12,320 \approx t$$
The bones are about 12,320 years old.

10. Let x = the percent of carbon-14 remaining in the mummies.

For 3300 yr:
$$xP_0 = P_0e^{-0.00012(3300)}$$
$$x = e^{-0.00012(3300)}$$
$$x \approx 0.673, \text{ or } 67.3\%$$
If 63.7% of the carbon-14 remains, the mummies have lost 100% − 67.3%, or 32.7%, of their carbon-14.

For 3500 yr:
$$xP_0 = P_0e^{-0.00012(3500)}$$
$$x = e^{-0.00012(3500)}$$
$$x \approx 0.657, \text{ or } 65.7\%$$
If 65.7% of the carbon-14 remains, the mummies have lost 100% − 65.7%, or 34.3%, of their carbon-14.

The mummies have lost about 32.7% to 34.3% of their carbon-14.

11. a) $K = \dfrac{\ln 2}{3.1} \approx 0.224$, or 22.4% per min

b) $k = \dfrac{\ln 2}{22.3} \approx 0.031$, or 3.1% per yr

c) $T = \dfrac{\ln 2}{0.0115} \approx 60.3$ days

d) $T = \dfrac{\ln 2}{0.065} \approx 10.7$ yr

e) $k = \dfrac{\ln 2}{29.1} \approx 0.024$, or 2.4% per yr

f) $k = \dfrac{\ln 2}{70.0} \approx 0.010$, or 1.0% per yr

g) $k = \dfrac{\ln 2}{24,100} \approx 0.000029$, or 0.0029% per yr

12. a) $t = 2008 - 1950 = 58$
$$N(t) = N_0e^{-kt}$$
$$2,200,000 = 5,600,000e^{-k(58)}$$
$$\frac{2,200,000}{5,600,000} = e^{-58k}$$
$$\ln\left(\frac{2,200,000}{5,600,000}\right) = \ln e^{-58k}$$
$$\ln\left(\frac{2,200,000}{5,600,000}\right) = -58k$$
$$\frac{\ln\left(\frac{2,200,000}{5,600,000}\right)}{-58} = k$$
$$0.016 \approx k$$
$$N(t) = 5,600,000e^{-0.016t}$$

b) In 2011, $t = 2011 - 1950 = 61$.

$N(61) = 5,600,000e^{-0.016(61)} \approx 2.1$ million farms

In 2015, $t = 2015 - 1950 = 65$.

$N(65) = 5,600,000e^{-0.016(65)} \approx 2.0$ million farms

c)
$$1,000,000 = 5,600,000e^{-0.016t}$$
$$\frac{1,000,000}{5,600,000} = e^{-0.016t}$$
$$\ln\left(\frac{1,000,000}{5,600,000}\right) = \ln e^{-0.016t}$$
$$\ln\left(\frac{1,000,000}{5,600,000}\right) = -0.016t$$
$$\frac{\ln\left(\frac{1,000,000}{5,600,000}\right)}{-0.016} = t$$
$$108 \approx t$$
Only 1,000,000 farms will remain about 108 years after 1950, or in 2058.

13. a)
$$\begin{aligned}C(t) &= C_0e^{-kt}\\ 2.92 &= 4.85e^{-k\cdot 4}\\ \frac{2.92}{4.85} &= e^{-4k}\\ \ln\left(\frac{2.92}{4.85}\right) &= \ln e^{-4k}\\ \ln\left(\frac{2.92}{4.85}\right) &= -4k\\ \frac{\ln\left(\frac{2.92}{4.85}\right)}{-4} &= k\\ 0.1268 &\approx k\end{aligned}$$

Then we have $C(t) = 4.85e^{-0.1268t}$, where C is in dollars and t is the number of yeaers since 2009.

b) In 2015, $t = 2015 - 2009 = 6$.

$C(6) = 4.85e^{-0.1268(6)} \approx \$2.27$

In 2018, $t = 2018 - 2009 = 9$.

$C(9) = 4.85e^{-0.1268(9)} \approx \$1.55$

c)
$$\begin{aligned}1.85 &= 4.85e^{-0.1268t}\\ \frac{1.85}{4.85} &= e^{-0.1268t}\\ \ln\left(\frac{1.85}{4.85}\right) &= \ln e^{-0.1268t}\\ \ln\left(\frac{1.85}{4.85}\right) &= -0.1268t\\ \frac{\ln\left(\frac{1.85}{4.85}\right)}{-0.1268} &= t\\ 8 &\approx t\end{aligned}$$

At the given rate of decay, the average cost per watt will be \$1.85 about 8 yr after 2009, or in 2017.

14. a)
$$\begin{aligned}220,000 &= 66,000e^{k\cdot 10}\\ \frac{220}{66} &= e^{10k}\\ \ln\left(\frac{220}{66}\right) &= \ln\, e^{10k}\\ \ln\left(\frac{220}{66}\right) &= 10k\\ \frac{\ln\left(\frac{220}{66}\right)}{10} &= k\\ 0.1204 &\approx k\end{aligned}$$

Then we have $L(t) = 66,000e^{0.1204t}$, where L is in dollars and t is the number of years after 1999.

b) $L(12) = 66,000e^{0.1204(12)} \approx \$279,906$

c)
$$\begin{aligned}300,000 &= 66,000e^{0.1204t}\\ \frac{300}{66} &= e^{0.1204t}\\ \ln\left(\frac{300}{66}\right) &= \ln\, e^{0.1204t}\\ \ln\left(\frac{300}{66}\right) &= 0.1204t\\ \frac{\ln\left(\frac{300}{66}\right)}{0.1204} &= t\\ 12.6 &\approx t\end{aligned}$$

The value of the car will be \$300,000 about 12.6 yr after 1999.

15. a) In 2006, $t = 2006 - 1960 = 46$
$$\begin{aligned}R(t) &= R_0e^{kt}\\ 15,400,000 &= 900e^{k(46)}\\ \frac{154,000}{9} &= e^{46k}\\ \ln\left(\frac{154,000}{9}\right) &= \ln e^{46k}\\ \ln\left(\frac{154,000}{9}\right) &= 46k\\ \frac{\ln\left(\frac{154,000}{9}\right)}{46} &= k\\ 0.2119 &\approx k\end{aligned}$$

$R(t) = 900e^{0.2119t}$

b) In 2010, $t = 2010 - 1960 = 50$.

$R(50) = 900e^{0.2119(50)} \approx \$35,941,198 \approx \$35.9$ million

c) $T = \dfrac{\ln 2}{0.2119} \approx 3.3$ yr

d)
$$\begin{aligned}25,000,000 &= 900e^{0.2119t}\\ \frac{250,000}{9} &= e^{0.2119t}\\ \ln\left(\frac{250,000}{9}\right) &= \ln e^{0.2119t}\\ \ln\left(\frac{250,000}{9}\right) &= 0.2119t\\ \frac{\ln\left(\frac{250,000}{9}\right)}{0.2119} &= t\\ 48 &\approx t\end{aligned}$$

The value of the painting was \$25 million about 48 yr after 1960, or in 2008.

16. a) $t = 2007 - 1971 = 36$

$$\begin{aligned} 2,800,000 &= 1000e^{k(36)} \\ 2800 &= e^{36k} \\ \ln 2800 &= \ln e^{36k} \\ \ln 2800 &= 36k \\ \frac{\ln 2800}{36} &= k \\ 0.2205 &\approx k \end{aligned}$$

$W(t) = 1000e^{0.2205(t)}$

b) $t = 2011 - 1971 = 40$

$W(40) = 1000e^{0.2205(40)} \approx \$6,768,265 \approx \$6.77$ million

c) $T = \dfrac{\ln 2}{0.2205} \approx 3.1$ yr

d)
$$\begin{aligned} 3,000,000 &= 1000e^{0.2205t} \\ 3000 &= e^{0.2205t} \\ \ln 3000 &= \ln e^{0.2205t} \\ \ln 3000 &= 0.2205t \\ \frac{\ln 3000}{0.2205} &= t \\ 36.3 &\approx t \end{aligned}$$

The value of the card was \$3 million about 36.3 yr after 1971, or in 2007.

17. a) $N(0) = \dfrac{3500}{1 + 19.9e^{-0.6(0)}} \approx 167$

b) $N(2) = \dfrac{3500}{1 + 19.9e^{-0.6(2)}} \approx 500$

$N(5) = \dfrac{3500}{1 + 19.9e^{-0.6(5)}} \approx 1758$

$N(8) = \dfrac{3500}{1 + 19.9e^{-0.6(8)}} \approx 3007$

$N(12) = \dfrac{3500}{1 + 19.9e^{-0.6(12)}} \approx 3449$

$N(16) = \dfrac{3500}{1 + 19.9e^{-0.6(16)}} \approx 3495$

c) As $t \to \infty$, $N(t) \to 3500$; the number of people infected approaches 3500 but never actually reaches it.

18. $P(0) = \dfrac{3040}{1 + 3.75e^{-0.32(0)}} = 640$

$P(1) = \dfrac{3040}{1 + 3.75e^{-0.32(1)}} \approx 817$

$P(5) = \dfrac{3040}{1 + 3.75e^{-0.32(5)}} \approx 1730$

$P(10) = \dfrac{3040}{1 + 3.75e^{-0.32(10)}} \approx 2637$

$P(15) = \dfrac{3040}{1 + 3.75e^{-0.32(15)}} \approx 2949$

$P(20) = \dfrac{3040}{1 + 3.75e^{-0.32(20)}} \approx 3021$

19. To find k we substitute 105 for T_1, 0 for T_0, 5 for t, and 70 for $T(t)$ and solve for k.

$$\begin{aligned} 70 &= 0 + (105 - 0)e^{-5k} \\ 70 &= 105e^{-5k} \\ \frac{70}{105} &= e^{-5k} \\ \ln \frac{70}{105} &= \ln e^{-5k} \\ \ln \frac{70}{105} &= -5k \\ \frac{\ln \frac{70}{105}}{-5} &= k \\ 0.081 &\approx k \end{aligned}$$

The function is $T(t) = 105e^{-0.081t}$.

Now we find $T(10)$.

$T(10) = 105e^{-0.081(10)} \approx 46.7\ ^\circ\text{F}$

20. To find k we substitute 375 for T_1, 72 for T_0, 3 for t, and 365 for $T(t)$.

$$\begin{aligned} 365 &= 72 + (375 - 72)e^{-3k} \\ 293 &= 303e^{-3k} \\ \frac{293}{303} &= e^{-3k} \\ \ln \frac{293}{303} &= \ln e^{-3k} \\ \ln \frac{293}{303} &= -3k \\ \frac{\ln \frac{293}{303}}{-3} &= k \\ 0.011 &\approx k \end{aligned}$$

The function is $T(t) = 72 + 303e^{-0.011t}$.

$T(15) = 72 + 303e^{-0.011(15)} \approx 329^\circ\text{F}$

(Answers may vary slightly due to rounding differences.)

21. To find k we substitute 43 for T_1, 68 for T_0, 12 for t, and 55 for $T(t)$ and solve for k.

$$\begin{aligned} 55 &= 68 + (43 - 68)e^{-12k} \\ -13 &= -25e^{-12k} \\ 0.52 &= e^{-12k} \\ \ln 0.52 &= \ln e^{-12k} \\ \ln 0.52 &= -12k \\ 0.0545 &\approx k \end{aligned}$$

The function is $T(t) = 68 - 25e^{-0.0545t}$.

Now we find $T(20)$.

$T(20) = 68 - 25e^{-0.0545(20)} \approx 59.6^\circ\text{F}$

22. To find k we substitute 94.6 for T_1, 70 for T_0, 60 for t (1 hr = 60 min), and 93.4 for $T(t)$.

$$93.4 = 70 + |94.6 - 70|e^{-k(60)}$$
$$23.4 = 24.6e^{-60k}$$
$$\frac{23.4}{24.6} = e^{-60k}$$
$$\ln\frac{23.4}{24.6} = \ln e^{-60k}$$
$$\ln\frac{23.4}{24.6} = -60k$$
$$k = \frac{\ln\frac{23.4}{24.6}}{-60} \approx 0.0008$$

The function is $T(t) = 70 + 24.6e^{-0.0008t}$.

We substitute 98.6 for $T(t)$ and solve for t.

$$98.6 = 70 + 24.6e^{-0.0008t}$$
$$28.6 = 24.6e^{-0.0008t}$$
$$\frac{28.6}{24.6} = e^{-0.0008t}$$
$$\ln\frac{28.6}{24.6} = \ln e^{-0.0008t}$$
$$\ln\frac{28.6}{24.6} = -0.0008t$$
$$t = \frac{\ln\frac{28.6}{24.6}}{-0.0008} \approx -188$$

The murder was committed at approximately 188 minutes, or about 3 hours, before 12:00 PM, or at about 9:00 AM. (Answers may vary slightly due to rounding differences.)

23. Multiplication principle for inequalities

24. Product rule

25. Principle of zero products

26. Principle of square roots

27. Power rule

28. Multiplication principle for equations

29.
$$480e^{-0.003p} = 150e^{0.004p}$$
$$\frac{480}{150} = \frac{e^{0.004p}}{e^{-0.003p}}$$
$$3.2 = e^{0.007p}$$
$$\ln 3.2 = \ln e^{0.007p}$$
$$\ln 3.2 = 0.007p$$
$$\frac{\ln 3.2}{0.007} = p$$
$$\$166.16 \approx p$$

30.
$$P(4000) = P_0e^{-0.00012(4000)}$$
$$= 0.619P_0, \text{ or } 61.9\%P_0$$

Thus, about 61.9% of the carbon-14 remains, so about 38.1% has been lost.

31.
$$P(t) = P_0e^{kt}$$
$$50{,}000 = P_0e^{0.07(18)}$$
$$\frac{50{,}000}{e^{0.07(18)}} = P_0$$
$$\$14{,}182.70 \approx P_0$$

32. a)
$$P = P_0e^{kt}$$
$$\frac{P}{e^{kt}} = P_0, \text{ or}$$
$$Pe^{-kt} = P_0$$

b) $P_0 = 50{,}000e^{-0.064(18)} \approx \$15{,}800.21$

33.
$$i = \frac{V}{R}\left[1 - e^{-(R/L)t}\right]$$
$$\frac{iR}{V} = 1 - e^{-(R/L)t}$$
$$e^{-(R/L)t} = 1 - \frac{iR}{V}$$
$$\ln e^{-(R/L)t} = \ln\left(1 - \frac{iR}{V}\right)$$
$$-\frac{R}{L}t = \ln\left(1 - \frac{iR}{V}\right)$$
$$t = -\frac{L}{R}\left[\ln\left(1 - \frac{iR}{V}\right)\right]$$

34. a) At 1 m: $I = I_0e^{-1.4(1)} \approx 0.247I_0$

24.7% of I_0 remains.

At 3 m: $I = I_0e^{-1.4(3)} \approx 0.015I_0$

1.5% of I_0 remains.

At 5 m: $I = I_0e^{-1.4(5)} \approx 0.0009I_0$

0.09% of I_0 remains.

At 50 m: $I = I_0e^{-1.4(50)} \approx (3.98 \times 10^{-31})I_0$

Now, $3.98 \times 10^{-31} = (3.98 \times 10^{-29}) \times 10^{-2}$, so $(3.98 \times 10^{-29})\%$ remains.

b) $I = I_0e^{-1.4(10)} \approx 0.0000008I_0$

Thus, 0.00008% remains.

35.
$$y = ae^x$$
$$\ln y = \ln(ae^x)$$
$$\ln y = \ln a + \ln e^x$$
$$\ln y = \ln a + x$$
$$Y = x + \ln a$$

This function is of the form $y = mx + b$, so it is linear.

36.
$$y = ax^b$$
$$\ln y = \ln(ax^b)$$
$$\ln y = \ln a + b\ln x$$
$$Y = \ln a + bX$$

This function is of the form $y = mx + b$, so it is linear.

Chapter 5 Review Exercises

1. The statement is true. See page 391 in the text.

2. Solving $0 = \log x$, we get $x = 10^0 = 1$, so the x-intercept is $(1, 0)$. The given statement is false.

3. The graph of f^{-1} is a reflection of the graph of f across $y = x$, so the statement is false.

4. The statement is true. See page 392 in the text.

5. The statement is false. The range of $y = a^x$, for instance, is $(0, \infty)$.

6. The statement is true. See page 416 in the text.

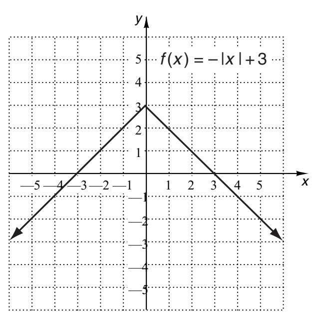
7. We interchange the first and second coordinates of each pair to find the inverse of the relation. It is $\{(-2.7, 1.3), (-3, 8), (3, -5), (-3, 6), (-5, 7)\}$.

8. a) $x = -2y + 3$

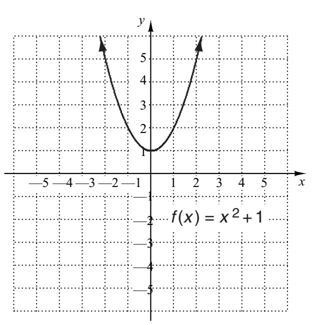
   b) $x = 3y^2 + 2y - 1$

   c) $0.8y^3 - 5.4x^2 = 3y$

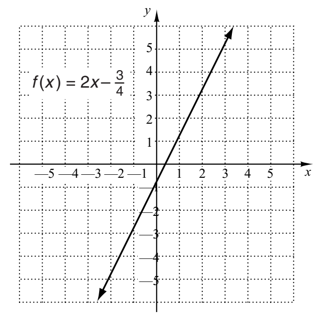
9. The graph of $f(x) = -|x|+3$ is shown below. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.



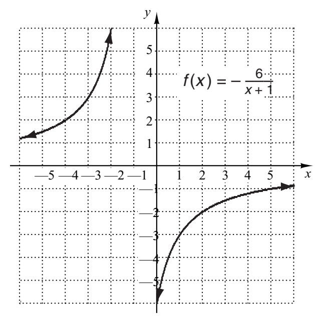
10. The graph of $f(x) = x^2 + 1$ is shown below. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.



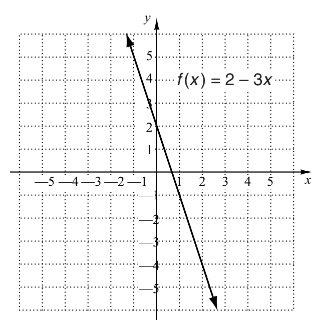
11. The graph of $f(x) = 2x - \frac{3}{4}$ is shown below. The function is one-to-one, because no horizontal line crosses the graph more than once.



12. The graph of $f(x) = -\frac{6}{x+1}$ is shown below. The function is one-to-one, because no horizontal line crosses the graph more than once.



13. a) The graph of $f(x) = 2-3x$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



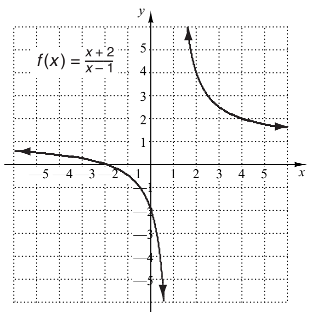
b) Replace $f(x)$ with y: $y = 2 - 3x$

Interchange x and y: $x = 2 - 3y$

Solve for y: $y = \dfrac{-x+2}{3}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{-x+2}{3}$

14. a) The graph of $f(x) = \dfrac{x+2}{x-1}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \dfrac{x+2}{x-1}$

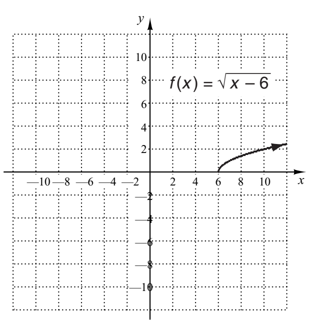
Interchange x and y: $x = \dfrac{y+2}{y-1}$

Solve for y: $(y-1)x = y+2$

$$xy - x = y + 2$$
$$xy - y = x + 2$$
$$y(x-1) = x + 2$$
$$y = \frac{x+2}{x-1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x+2}{x-1}$

15. a) The graph of $f(x) = \sqrt{x-6}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \sqrt{x-6}$

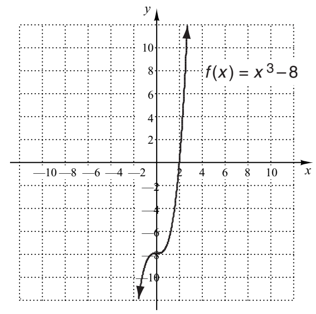
Interchange x and y: $x = \sqrt{y-6}$

Solve for y: $x^2 = y - 6$

$$x^2 + 6 = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x^2 + 6$, for all x in the range of $f(x)$, or $f^{-1}(x) = x^2 + 6$, $x \geq 0$.

16. a) The graph of $f(x) = x^3 - 8$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = x^3 - 8$

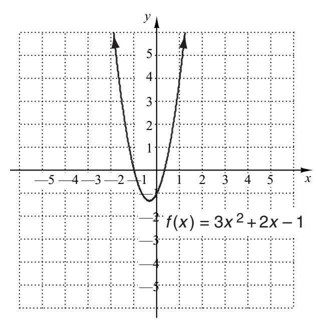
Interchange x and y: $x = y^3 - 8$

Solve for y: $x + 8 = y^3$

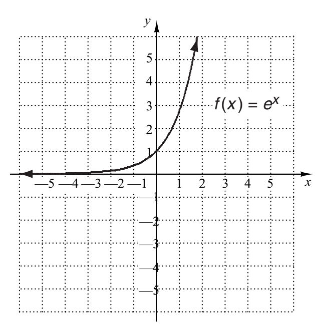
$$\sqrt[3]{x+8} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x+8}$

17. a) The graph of $f(x) = 3x^2 + 2x - 1$ is shown below. It is not one-to-one since there are many horizontal lines that cross the graph more than once. The function does not have an inverse that is a function.



18. a) The graph of $f(x) = e^x$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = e^x$

Interchange x and y: $x = e^y$

Solve for y: $y = \ln x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \ln x$

19. We find $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(x)$ and check to see that each is x.

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(6x - 5) =$

$$\frac{(6x-5)+5}{6} = \frac{6x}{6} = x$$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x+5}{6}\right) =$

$$6\left(\frac{x+5}{6}\right) - 5 = x + 5 - 5 = x$$

20. $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{x+1}{x}\right) =$

$$\frac{1}{\left(\frac{x+1}{x}\right) - 1} = \frac{x}{x+1-x} = \frac{x}{1} = x$$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{1}{x-1}\right) =$

$$\frac{\left(\frac{1}{x-1}\right)+1}{\frac{1}{x-1}} = \frac{1+(x-1)}{1} = \frac{x}{1} = x$$

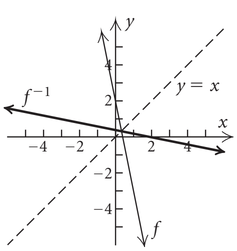
21. Replace $f(x)$ with y: $y = 2 - 5x$

Interchange x and y: $x = 2 - 5y$

Solve for y: $y = \frac{2-x}{5}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{2-x}{5}$

The domain and range of f are $(-\infty, \infty)$, so the domain and range of f^{-1} are also $(-\infty, \infty)$.



22. Replace $f(x)$ with y: $y = \frac{x-3}{x+2}$

Interchange x and y: $x = \frac{y-3}{y+2}$

Solve for y: $xy + 2x = y - 3$

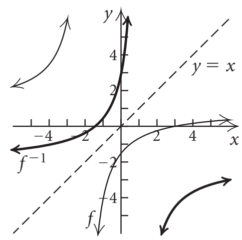
$$2x + 3 = y - xy$$

$$2x + 3 = y(1 - x)$$

$$\frac{2x+3}{1-x} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{2x+3}{1-x}$, or $\frac{-2x-3}{x-1}$

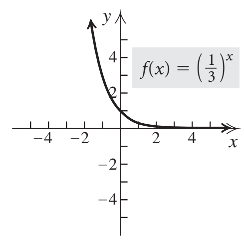
The domain of f is $(-\infty, -2) \cup (-2, \infty)$, and the range of f is $(-\infty, 1) \cup (1, \infty)$. Thus the domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$ and the range of f^{-1} is $(-\infty, -2) \cup (-2, \infty)$.



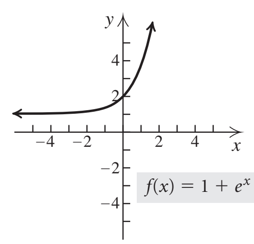
23. Since $f(f^{-1}(x)) = x$, then $f(f^{-1}(657)) = 657$.

24. Since $f(f^{-1}(x)) = x$, then $f(f^{-1}(a)) = a$.

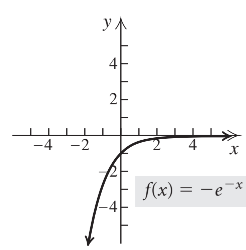
25.



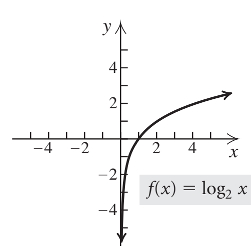
26.



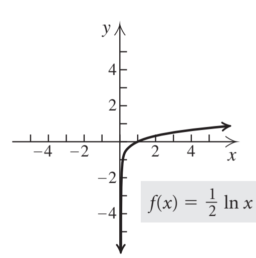
27.



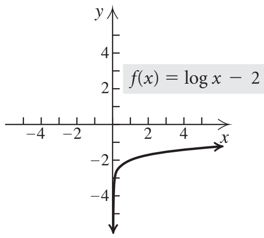
28.



29.



30.



31. $f(x) = e^{x-3}$

This is the graph of $f(x) = e^x$ shifted right 3 units. The correct choice is (c).

32. $f(x) = \log_3 x$

$f(1) = \log_3(1) = 0$

The only graph with x-intercept $(1, 0)$ is (a).

33. $f(x) = -\log_3(x + 1)$

This is the graph of $\log_3 x$ shifted left 1 unit and reflected across the y-axis. The correct choice is (b).

34. $y = \left(\frac{1}{2}\right)^x$

At $x = 0$, $y = \left(\frac{1}{2}\right)^0 = 1$

The only graph with y-intercept $(0, 1)$ is (f).

35. $f(x) = 3(1 - e^{-x})$, $x \geq 0$

This is the graph of $f(x) = e^x$ reflected across the y-axis, reflected across the x-axis, shifted up 1 unit, and stretched by a factor of 3. The correct choice is (e).

36. $f(x) = |\ln(x - 4)|$

This is the graph of $f(x) = \ln x$ shifted right 4 units. The absolute value reflects negative outputs across the x-axis. The line $x = 4$ is a vertical asymptote. The correct choice is (d).

37. $\log_5 125 = 3$ because the exponent to which we raise 5 to get 125 is 3.

38. $\log 100{,}000 = 5$ because the exponent to which we raise 10 to get 100,000 is 5.

39. $\ln e = 1$ because the exponent to which we raise e to get e is 1.

40. $\ln 1 = 0$ because the exponent to which we raise e to get 1 is 0.

41. $\log 10^{1/4} = \frac{1}{4}$ because the exponent to which we raise 10 to get $10^{1/4}$ is $\frac{1}{4}$.

42. $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2}$ because the exponent to which we raise 3 to get $3^{1/2}$ is $\frac{1}{2}$.

43. $\log 1 = 0$ because the exponent to which we raise 10 to get 1 is 0.

44. $\log 10 = 1$ because the exponent to which we raise 10 to get 10 is 1.

45. $\log_2 \sqrt[3]{2} = \log_2 2^{1/3} = \frac{1}{3}$ because the exponent to which we raise 2 to get $2^{1/3}$ is $\frac{1}{3}$.

46. $\log 0.01 = -2$ because the exponent to which we raise 10 to get 0.01 is -2.

47. $\log_4 x = 2 \Rightarrow 4^2 = x$

48. $\log_a Q = k \Rightarrow a^k = Q$

49. $4^{-3} = \frac{1}{64} \Rightarrow \log_4 \frac{1}{64} = -3$

50. $e^x = 80 \Rightarrow \ln 80 = x$, or $\log_e 80 = x$

51. $\log 11 \approx 1.0414$

52. $\log 0.234 \approx -0.6308$

53. $\ln 3 \approx 1.0986$

54. $\ln 0.027 \approx -3.6119$

55. $\log(-3)$ does not exist. (The calculator gives an error message.)

56. $\ln 0$ does not exist. (The calculator gives an error message.)

57. $\log_5 24 = \frac{\log 24}{\log 5} \approx 1.9746$

58. $\log_8 3 = \frac{\log 3}{\log 8} \approx 0.5283$

59.
$$\begin{aligned} & 3\log_b x - 4\log_b y + \frac{1}{2}\log_b z \\ &= \log_b x^3 - \log_b y^4 + \log_b z^{1/2} \\ &= \log_b \frac{x^3 z^{1/2}}{y^4}, \text{ or } \log_b \frac{x^3\sqrt{z}}{y^4} \end{aligned}$$

60.
$$\begin{aligned} & \ln(x^3 - 8) - \ln(x^2 + 2x + 4) + \ln(x + 2) \\ &= \ln \frac{(x^3 - 8)(x + 2)}{x^2 + 2x + 4} \\ &= \ln \frac{(x - 2)(x^2 + 2x + 4)(x + 2)}{x^2 + 2x + 4} \\ &= \ln(x - 2)(x + 2) \\ &= \ln(x^2 - 4) \end{aligned}$$

61.
$$\begin{aligned} \ln \sqrt[4]{wr^2} &= \ln(wr^2)^{1/4} \\ &= \frac{1}{4}\ln wr^2 \\ &= \frac{1}{4}(\ln w + \ln r^2) \\ &= \frac{1}{4}(\ln w + 2\ln r) \\ &= \frac{1}{4}\ln w + \frac{1}{2}\ln r \end{aligned}$$

62. $$\begin{aligned}\log\sqrt[3]{\frac{M^2}{N}} &= \log\left(\frac{M^2}{N}\right)^{1/3}\\ &= \frac{1}{3}\log\frac{M^2}{N}\\ &= \frac{1}{3}(\log M^2 - \log N)\\ &= \frac{1}{3}(2\log M - \log N)\\ &= \frac{2}{3}\log M - \frac{1}{3}\log N\end{aligned}$$

63. $$\begin{aligned}\log_a 3 &= \log_a\left(\frac{6}{2}\right)\\ &= \log_a 6 - \log_a 2\\ &\approx 0.778 - 0.301\\ &\approx 0.477\end{aligned}$$

64. $$\begin{aligned}\log_a 50 &= \log_a(2\cdot 5^2)\\ &= \log_a 2 + \log_a 5^2\\ &= \log_a 2 + 2\log_a 5\\ &\approx 0.301 + 2(0.699)\\ &\approx 1.699\end{aligned}$$

65. $$\begin{aligned}\log_a\frac{1}{5} &= \log_a 5^{-1}\\ &= -\log_a 5\\ &\approx -0.699\end{aligned}$$

66. $$\begin{aligned}\log_a\sqrt[3]{5} &= \log_a 5^{1/3}\\ &= \frac{1}{3}\log_a 5\\ &\approx \frac{1}{3}(0.699)\\ &\approx 0.233\end{aligned}$$

67. $\ln e^{-5k} = -5k \quad (\log_a a^x = x)$

68. $\log_5 5^{-6t} = -6t$

69. $$\begin{aligned}\log_4 x &= 2\\ x &= 4^2 = 16\end{aligned}$$

The solution is 16.

70. $$\begin{aligned}3^{1-x} &= 9^{2x}\\ 3^{1-x} &= (3^2)^{2x}\\ 3^{1-x} &= 3^{4x}\\ 1-x &= 4x\\ 1 &= 5x\\ \frac{1}{5} &= x\end{aligned}$$

The solution is $\frac{1}{5}$.

71. $$\begin{aligned}e^x &= 80\\ \ln e^x &= \ln 80\\ x &= \ln 80\\ x &\approx 4.382\end{aligned}$$

The solution is 4.382.

72. $$\begin{aligned}4^{2x-1} - 3 &= 61\\ 4^{2x-1} &= 64\\ 4^{2x-1} &= 4^3\\ 2x-1 &= 3\\ 2x &= 4\\ x &= 2\end{aligned}$$

The solution is 2.

73. $$\begin{aligned}\log_{16} 4 &= x\\ 16^x &= 4\\ (4^2)^x &= 4^1\\ 4^{2x} &= 4^1\\ 2x &= 1\\ x &= \frac{1}{2}\end{aligned}$$

The solution is $\frac{1}{2}$.

74. $$\begin{aligned}\log_x 125 &= 3\\ x^3 &= 125\\ x &= \sqrt[3]{125}\\ x &= 5\end{aligned}$$

The solution is 5.

75. $$\begin{aligned}\log_2 x + \log_2(x-2) &= 3\\ \log_2 x(x-2) &= 3\\ x(x-2) &= 2^3\\ x^2 - 2x &= 8\\ x^2 - 2x - 8 &= 0\\ (x+2)(x-4) &= 0\end{aligned}$$

$$x+2=0 \quad \text{or} \quad x-4=0$$
$$x=-2 \quad \text{or} \quad x=4$$

The number 4 checks, but -2 does not. The solution is 4.

76. $$\begin{aligned}\log(x^2-1) - \log(x-1) &= 1\\ \log\frac{x^2-1}{x-1} &= 1\\ \frac{(x+1)(x-1)}{x-1} &= 10^1\\ x+1 &= 10\\ x &= 9\end{aligned}$$

The answer checks. The solution is 9.

77. $$\begin{aligned}\log x^2 &= \log x\\ x^2 &= x\\ x^2 - x &= 0\\ x(x-1) &= 0\end{aligned}$$

$$x=0 \quad \text{or} \quad x-1=0$$
$$x=0 \quad \text{or} \quad x=1$$

The number 1 checks, but 0 does not. The solution is 1.

78.
$$\begin{aligned} e^{-x} &= 0.02 \\ \ln e^{-x} &= \ln 0.02 \\ -x &= \ln 0.02 \\ x &= -\ln 0.02 \\ x &\approx 3.912 \end{aligned}$$

The answer checks. The solution is 3.912.

79. a) $A(t) = 16,000(1.0105)^{4t}$

b)
$$\begin{aligned} A(0) &= 16,000(1.0105)^{4\cdot 0} = \$16,000 \\ A(6) &= 16,000(1.0105)^{4\cdot 6} \approx \$20,558.51 \\ A(12) &= 16,000(1.0105)^{4\cdot 12} \approx \$26,415.77 \\ A(18) &= 16,000(1.0105)^{4\cdot 18} \approx \$33,941.80 \end{aligned}$$

80. $B(3) = 27.9(1.3299)^3 \approx \$65.6$ million

$B(9) = 27.9(1.3299)^9 \approx \$363.1$ million

81. $T = \dfrac{\ln 2}{0.086} \approx 8.1$ years

82. $k = \dfrac{\ln 2}{30} \approx 0.023$, or 2.3%

83.
$$\begin{aligned} P(t) &= P_0 e^{kt} \\ 0.73P_0 &= P_0 e^{-0.00012t} \\ 0.73 &= e^{-0.00012t} \\ \ln 0.73 &= \ln e^{-0.00012t} \\ \ln 0.73 &= -0.00012t \\ \frac{\ln 0.73}{-0.00012} &= t \\ 2623 &\approx t \end{aligned}$$

The skeleton is about 2623 years old.

84. $\mathrm{pH} = -\log[2.3 \times 10^{-6}] \approx -(-5.6) = 5.6$

85. $R = \log \dfrac{10^{6.3} \cdot I_0}{I_0} = \log 10^{6.3} = 6.3$

86.
$$\begin{aligned} L &= 10 \log \frac{1000 \cdot I_0}{I_0} \\ &= 10 \log 1000 \\ &= 10 \cdot 3 \\ &= 30 \text{ decibels} \end{aligned}$$

87. a) We substitute 353.823 for P, since P is in thousands.

$$\begin{aligned} W(353.823) &= 0.37 \ln 353.823 + 0.05 \\ &\approx 2.2 \text{ ft/sec} \end{aligned}$$

b) We substitute 3.4 for W and solve for P.

$$\begin{aligned} 3.4 &= 3.7 \ln P + 0.05 \\ 3.35 &= 0.37 \ln P \\ \frac{3.35}{0.37} &= \ln P \\ e^{3.35/0.37} &= P \\ P &\approx 8553.143 \end{aligned}$$

The population is about 8553.143 thousand, or 8,553,143. (Answers may vary due to rounding differences.)

88. a)
$$\begin{aligned} 492 &= 0.035e^{k\cdot 64} \\ \frac{492}{0.035} &= e^{64k} \\ \ln \frac{492}{0.035} &= \ln e^{64k} \\ \ln \frac{492}{0.035} &= 64k \\ \frac{\ln \frac{492}{0.035}}{64} &= k \\ 0.1492 &\approx k \end{aligned}$$

b) We have $S(t) = 0.035e^{0.1492t}$, where $S(t)$ is in billions of dollars and t is the number of years after 1940.

c) $S(25) \approx \$1.459$ billion

$S(55) \approx \$128.2$ billion

$S(75) \approx \$2534$ billion, or \$2.534 trillion

d) 1 trillion is 1000 billion.

$$\begin{aligned} 1000 &= 0.035e^{0.1492t} \\ \frac{1000}{0.035} &= e^{0.1492t} \\ \ln \frac{1000}{0.035} &= \ln e^{0.1492t} \\ \ln \frac{1000}{0.035} &= 0.1492t \\ \frac{\ln \frac{1000}{0.035}}{0.1492} &= t \\ 69 &\approx t \end{aligned}$$

Cash benefits will reach \$1 trillion about 69 yr after 1940, or in 2009.

89. a) $P(t) = 3.039e^{0.013t}$, where $P(t)$ is in millions and t is the number of years after 2005.

b) In 2009, $t = 2009 - 2005 = 4$.

$P(4) = 3.039e^{0.013(4)} \approx 3.201$ million

In 2015, $t = 2015 - 2005 = 10$.

$P(10) = 3.039e^{0.013(10)} \approx 3.461$ million

c)
$$\begin{aligned} 10 &= 3.039e^{0.013t} \\ \frac{10}{3.039} &= e^{0.013t} \\ \ln\left(\frac{10}{3.039}\right) &= \ln e^{0.013t} \\ \ln\left(\frac{10}{3.039}\right) &= 0.013t \\ \frac{\ln\left(\frac{10}{3.039}\right)}{0.013} &= t \\ 92 &\approx t \end{aligned}$$

The population will be 10 million about 92 yr after 2005.

d) $T = \dfrac{\ln 2}{0.013} \approx 53.3$ yr

90. The graph of $f(x) = e^{x-3} + 2$ is a translation of the graph of $y = e^x$ right three units and up 2 units. The horizontal asymptote of $y = e^x$ is $y = 0$, so the horizontal asymptote of $f(x) = e^{x-3} + 2$ is translated up 2 units from $y = 0$. Thus, it is $y = 2$, and answer D is correct.

91. We must have $2x - 3 > 0$, or $x > \frac{3}{2}$, so answer A is correct.

92. The graph of $f(x) = 2^{x-2}$ is the graph of $g(x) = 2^x$ shifted 2 units to the right. Thus D is the correct graph.

93. The graph of $f(x) = \log_2 x$ is the graph of $g(x) = 2^x$ reflected across the line $y = x$. Thus B is the correct graph.

94. $|\log_4 x| = 3$

$$\log_4 x = -3 \quad or \quad \log_4 x = 3$$
$$x = 4^{-3} \quad or \quad x = 4^3$$
$$x = \frac{1}{64} \quad or \quad x = 64$$

Both answers check. The solutions are $\frac{1}{64}$ and 64.

95. $\log x = \ln x$

Graph $y_1 = \log x$ and $y_2 = \ln x$ and find the first coordinates of the points of intersection of the graph. We see that the only solution is 1.

96.
$$5^{\sqrt{x}} = 625$$
$$5^{\sqrt{x}} = 5^4$$
$$\sqrt{x} = 4$$
$$x = 16$$

97. $f(x) = \log_3(\ln x)$

$\ln x$ must be positive, so $x > 1$. The domain is $(1, \infty)$.

98. Reflect the graph of $f(x) = \ln x$ across the line $y = x$ to obtain the graph of $h(x) = e^x$. Then shift this graph 2 units right to obtain the graph of $g(x) = e^{x-2}$.

99. Measure the atmospheric pressure P at the top of the building. Substitute that value in the equation $P = 14.7e^{-0.00005a}$, and solve for the height, or altitude, a, of the top of the building. Also measure the atmospheric pressure at the base of the building and solve for the altitude of the base. Then subtract to find the height of the building.

100. $\log_a ab^3 \neq (\log_a a)(\log_a b^3)$. If the first step had been correct, then so would the second step. The correct procedure follows.

$\log_a ab^3 = \log_a a + \log_a b^3 = 1 + 3\log_a b$

101. The inverse of a function $f(x)$ is written $f^{-1}(x)$, whereas $[f(x)]^{-1}$ means $\frac{1}{f(x)}$.

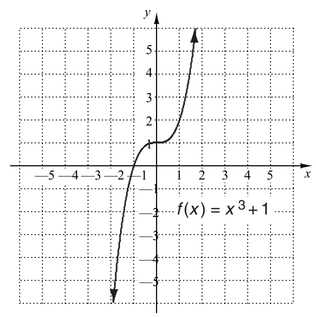
Chapter 5 Test

1. We interchange the first and second coordinates of each pair to find the inverse of the relation. It is $\{(5,-2),(3,4)(-1,0),(-3,-6)\}$.

2. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.

3. The function is one-to-one, because no horizontal line crosses the graph more than once.

4. a) The graph of $f(x) = x^3 + 1$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = x^3 + 1$

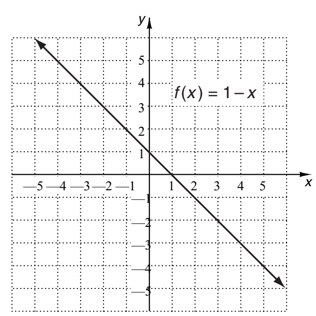
Interchange x and y: $x = y^3 + 1$

Solve for y: $y^3 = x - 1$

$y = \sqrt[3]{x-1}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x-1}$

5. a) The graph of $f(x) = 1 - x$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



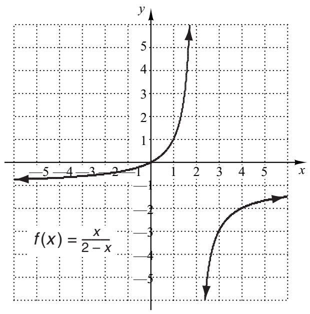
b) Replace $f(x)$ with y: $y = 1 - x$

Interchange x and y: $x = 1 - y$

Solve for y: $y = 1 - x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = 1 - x$

6. a) The graph of $f(x) = \dfrac{x}{2-x}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \dfrac{x}{2-x}$

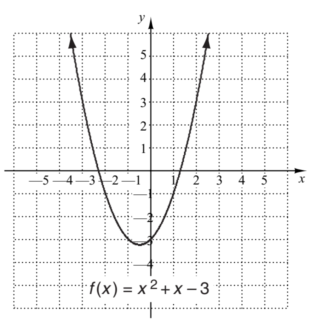
Interchange x and y: $x = \dfrac{y}{2-y}$

Solve for y: $(2-y)x = y$

$$2x - xy = y$$
$$xy + y = 2x$$
$$y(x+1) = 2x$$
$$y = \frac{2x}{x+1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{2x}{x+1}$

7. a) The graph of $f(x) = x^2 + x - 3$ is shown below. It is not one-to-one since there are many horizontal lines that cross the graph more than once. The function does not have an inverse that is a function.



8. We find $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(x)$ and check to see that each is x.

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(-4x+3) =$

$\dfrac{3-(-4x+3)}{4} = \dfrac{3+4x-3}{4} = \dfrac{4x}{4} = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\dfrac{3-x}{4}\right) =$

$-4\left(\dfrac{3-x}{4}\right) + 3 = -3 + x + 3 = x$

9. Replace $f(x)$ with y: $y = \dfrac{1}{x-4}$

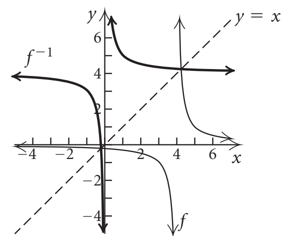
Interchange x and y: $x = \dfrac{1}{y-4}$

Solve for y: $x(y-4) = 1$

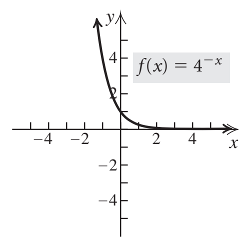
$$xy - 4x = 1$$
$$xy = 4x + 1$$
$$y = \frac{4x+1}{x}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{4x+1}{x}$

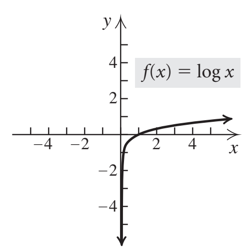
The domain of $f(x)$ is $(-\infty, 4) \cup (4, \infty)$ and the range of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$. Thus, the domain of f^{-1} is $(-\infty, 0) \cup (0, \infty)$ and the range of f^{-1} is $(-\infty, 4) \cup (4, \infty)$.



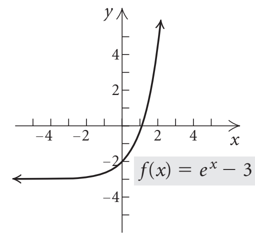
10.



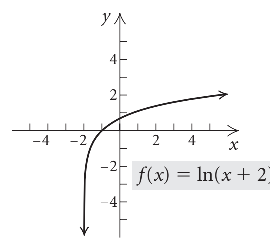
11.



12.



13.



14. $\log 0.00001 = -5$ because the exponent to which we raise 10 to get 0.00001 is -5.

15. $\ln e = 1$ because the exponent to which we raise e to get e is 1.

16. $\ln 1 = 0$ because the exponent to which we raise e to get 1 is 0.

17. $\log_4 \sqrt[5]{4} = \log_4 4^{1/5} = \frac{1}{5}$ because the exponent to which we raise 4 to get $4^{1/5}$ is $\frac{1}{5}$.

18. $\ln x = 4 \Rightarrow x = e^4$

19. $3^x = 5.4 \Rightarrow x = \log_3 5.4$

20. $\ln 16 \approx 2.7726$

21. $\log 0.293 \approx -0.5331$

22. $\log_6 10 = \frac{\log 10}{\log 6} \approx 1.2851$

23.
$$\begin{aligned} &2\log_a x - \log_a y + \frac{1}{2}\log_a z \\ &= \log_a x^2 - \log_a y + \log_a z^{1/2} \\ &= \log_a \frac{x^2 z^{1/2}}{y}, \text{ or } \log_a \frac{x^2\sqrt{z}}{y} \end{aligned}$$

24.
$$\begin{aligned} \ln \sqrt[5]{x^2 y} &= \ln(x^2 y)^{1/5} \\ &= \frac{1}{5}\ln x^2 y \\ &= \frac{1}{5}(\ln x^2 + \ln y) \\ &= \frac{1}{5}(2\ln x + \ln y) \\ &= \frac{2}{5}\ln x + \frac{1}{5}\ln y \end{aligned}$$

25.
$$\begin{aligned} \log_a 4 &= \log_a \left(\frac{8}{2}\right) \\ &= \log_a 8 - \log_a 2 \\ &\approx 0.984 - 0.328 \\ &\approx 0.656 \end{aligned}$$

26. $\ln e^{-4t} = -4t \quad (\log_a a^x = x)$

27.
$$\begin{aligned} \log_{25} 5 &= x \\ 25^x &= 5 \\ (5^2)^x &= 5^1 \\ 5^{2x} &= 5^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

The solution is $\frac{1}{2}$.

28.
$$\begin{aligned} \log_3 x + \log_3(x+8) &= 2 \\ \log_3 x(x+8) &= 2 \\ x(x+8) &= 3^2 \\ x^2 + 8x &= 9 \\ x^2 + 8x - 9 &= 0 \\ (x+9)(x-1) &= 0 \\ x = -9 \quad &or \quad x = 1 \end{aligned}$$

The number 1 checks, but -9 does not. The solution is 1.

29.
$$\begin{aligned} 3^{4-x} &= 27^x \\ 3^{4-x} &= (3^3)^x \\ 3^{4-x} &= 3^{3x} \\ 4 - x &= 3x \\ 4 &= 4x \\ x &= 1 \end{aligned}$$

The solution is 1.

30.
$$\begin{aligned} e^x &= 65 \\ \ln e^x &= \ln 65 \\ x &= \ln 65 \\ x &\approx 4.174 \end{aligned}$$

The solution is 4.174.

31. $R = \log \frac{10^{6.6} \cdot I_0}{I_0} = \log\ 10^{6.6} = 6.6$

32. $k = \frac{\ln 2}{45} \approx 0.0154 \approx 1.54\%$

33. a)
$$\begin{aligned} 1144.54 &= 1000e^{3k} \\ 1.14454 &= e^{3k} \\ \ln 1.14454 &= \ln e^{3k} \\ \ln 1.14454 &= 3k \\ \frac{\ln 1.14454}{3} &= k \\ 0.045 &\approx k \end{aligned}$$

The interest rate is about 4.5%.

b) $P(t) = 1000e^{0.045t}$

c) $P(8) = 1000e^{0.045 \cdot 8} \approx \$1433.33$

d) $T = \frac{\ln 2}{0.045} \approx 15.4$ yr

34. The graph of $f(x) = 2^{x-1} + 1$ is the graph of $g(x) = 2^x$ shifted right 1 unit and up 1 unit. Thus C is the correct graph.

35.
$$\begin{aligned} 4^{\sqrt[3]{x}} &= 8 \\ (2^2)^{\sqrt[3]{x}} &= 2^3 \\ 2^{2\sqrt[3]{x}} &= 2^3 \\ 2\sqrt[3]{x} &= 3 \\ \sqrt[3]{x} &= \frac{3}{2} \\ x &= \left(\frac{3}{2}\right)^3 \\ x &= \frac{27}{8} \end{aligned}$$

The solution is $\frac{27}{8}$.