# Chapter 9

# Systems of Equations and Matrices

## Exercise Set 9.1

1. Graph (c) is the graph of this system.

2. Graph (e) is the graph of this system.

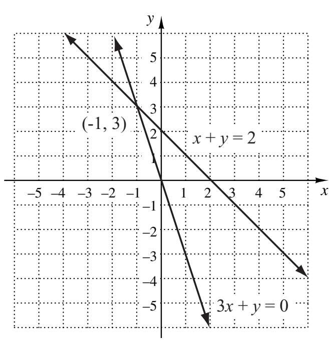
3. Graph (f) is the graph of this system.

4. Graph (a) is the graph of this system.

5. Graph (b) is the graph of this system.

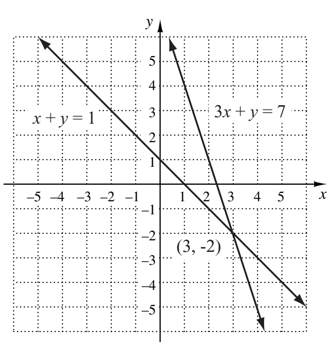
6. Graph (d) is the graph of this system.

7. Graph $x + y = 2$ and $3x + y = 0$ and find the coordinates of the point of intersection.



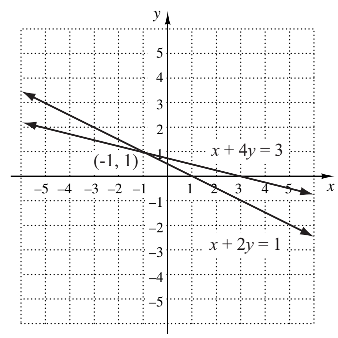

The solution is $(-1, 3)$.

8. Graph $x + y = 1$ and $3x + y = 7$ and find the coordinates of the point of intersection.



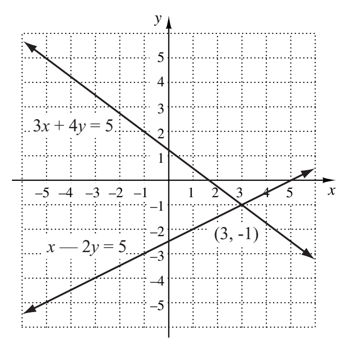

The solution is $(3, -2)$.

9. Graph $x + 2y = 1$ and $x + 4y = 3$ and find the coordinates of the point of intersection.



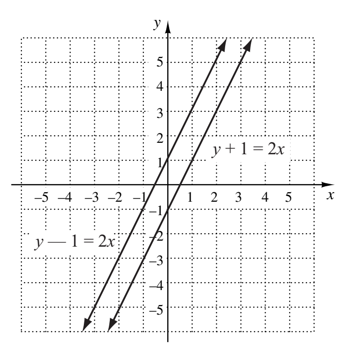

The solution is $(-1, 1)$.

10. Graph $3x+4y = 5$ and $x-2y = 5$ and find the coordinates of the point of intersection.



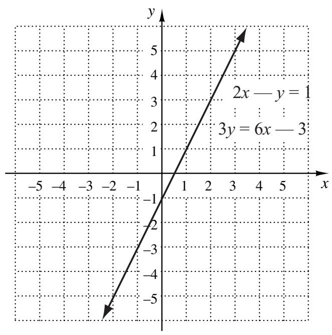

The solution is $(3, -1)$.

11. Graph $y + 1 = 2x$ and $y - 1 = 2x$ and find the coordinates of the point of intersection.



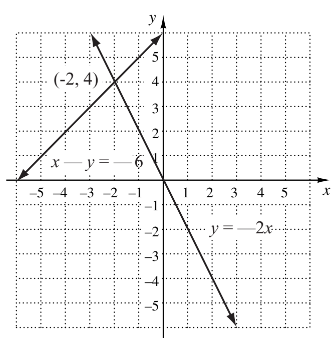

The graphs do not intersect, so there is no solution.

**12.** Graph $2x-y=1$ and $3y=6x-3$ and find the coordinates of the point of intersection.



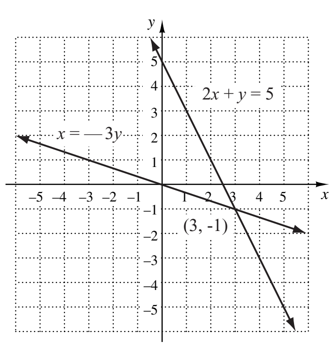

The graphs coincide so there are infinitely many solutions. Solving either equation for $y$, we have $y=2x-1$, so the solutions can be expressed as $(x, 2x-1)$. Similarly, solving either equation for $x$, we get $x=\dfrac{y+1}{2}$, so the solutions can also be expressed as $\left(\dfrac{y+1}{2}, y\right)$.

**13.** Graph $x-y=-6$ and $y=-2x$ and find the coordinates of the point of intersection.



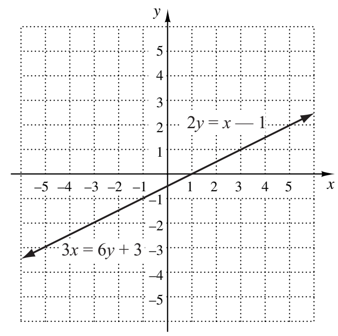

The solution is $(-2, 4)$.

**14.** Graph $2x+y=5$ and $x=-3y$ and find the coordinates of the point of intersection.



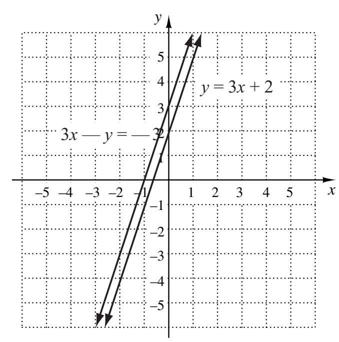

The solution is $(3, -1)$.

**15.** Graph $2y=x-1$ and $3x=6y+3$ and find the coordinates of the point of intersection.




The graphs coincide so there are infinitely many solutions. Solving either equation for $y$, we get $y=\dfrac{x-1}{2}$, so the solutions can be expressed as $\left(x, \dfrac{x-1}{2}\right)$. Similarly, solving either equation for $x$, we get $x=2y+1$, so the solutions can also be expressed as $(2y+1, y)$.

**16.** Graph $y=3x+2$ and $3x-y=-3$ and find the coordinates of the point of intersection.




The graphs do not intersect, so there is no solution.

**17.**
$$\begin{aligned} x + \phantom{3}y &= 9, \quad (1) \\ 2x - 3y &= -2 \quad (2) \end{aligned}$$

Solve equation (1) for either $x$ or $y$. We choose to solve for $y$.

$$y = 9 - x$$

Then substitute $9-x$ for $y$ in equation (2) and solve the resulting equation.

$$\begin{aligned} 2x - 3(9-x) &= -2 \\ 2x - 27 + 3x &= -2 \\ 5x - 27 &= -2 \\ 5x &= 25 \\ x &= 5 \end{aligned}$$

Now substitute 5 for $x$ in either equation (1) or (2) and solve for $y$.

$$\begin{aligned} 5 + y &= 9 \quad \text{Using equation (1).} \\ y &= 4 \end{aligned}$$

The solution is $(5, 4)$.

**18.** $3x - y = 5,\quad (1)$

$x + y = \frac{1}{2}\quad (2)$

Solve equation (2) for $y$.

$$y = \frac{1}{2} - x$$

Substitute in equation (1) and solve for $x$.

$$3x - \left(\frac{1}{2} - x\right) = 5$$
$$3x - \frac{1}{2} + x = 5$$
$$4x = \frac{11}{2}$$
$$x = \frac{11}{8}$$

Back-substitute to find $y$.

$$\frac{11}{8} + y = \frac{1}{2}\qquad \text{Using equation (2)}$$
$$y = -\frac{7}{8}$$

The solution is $\left(\frac{11}{8}, -\frac{7}{8}\right)$.

**19.** $x - 2y = 7,\quad (1)$

$x = y + 4\quad (2)$

Use equation (2) and substitute $y+4$ for $x$ in equation (1). Then solve for $y$.

$$y + 4 - 2y = 7$$
$$-y + 4 = 7$$
$$-y = 3$$
$$y = -3$$

Substitute $-3$ for $y$ in equation (2) to find $x$.

$$x = -3 + 4 = 1$$

The solution is $(1, -3)$.

**20.** $x + 4y = 6,\quad (1)$

$x = -3y + 3\quad (2)$

Substitute $-3y + 3$ for $x$ in equation (1) and solve for $y$.

$$-3y + 3 + 4y = 6$$
$$y = 3$$

Back-substitute to find $x$.

$$x = -3 \cdot 3 + 3 = -6$$

The solution is $(-6, 3)$.

**21.** $y = 2x - 6,\quad (1)$

$5x - 3y = 16\quad (2)$

Use equation (1) and substitute $2x - 6$ for $y$ in equation (2). Then solve for $x$.

$$5x - 3(2x - 6) = 16$$
$$5x - 6x + 18 = 16$$
$$-x + 18 = 16$$
$$-x = -2$$
$$x = 2$$

Substitute 2 for $x$ in equation (1) to find $y$.

$$y = 2 \cdot 2 - 6 = 4 - 6 = -2$$

The solution is $(2, -2)$.

**22.** $3x + 5y = 2,\quad (1)$

$2x - y = -3\quad (2)$

Solve equation (2) for $y$.

$$y = 2x + 3$$

Substitute $2x + 3$ for $y$ in equation (1) and solve for $x$.

$$3x + 5(2x + 3) = 2$$
$$3x + 10x + 15 = 2$$
$$13x = -13$$
$$x = -1$$

Back-substitute to find $y$.

$$2(-1) - y = -3\quad \text{Using equation (2)}$$
$$-2 - y = -3$$
$$1 = y$$

The solution is $(-1, 1)$.

**23.** $x + y = 3,\quad (1)$

$y = 4 - x\quad (2)$

Use equation (2) and substitute $4-x$ for $y$ in equation (1).

$$x + 4 - x = 3$$
$$4 = 3$$

There are no values of $x$ and $y$ for which $4 = 3$, so the system of equations has no solution.

**24.** $x - 2y = 3,\quad (1)$

$2x = 4y + 6\quad (2)$

Solve equation (1) for $x$.

$$x = 2y + 3$$

Substitute $2y + 3$ for $x$ in equation (2) and solve for $y$.

$$2(2y + 3) = 4y + 6$$
$$4y + 6 = 4y + 6$$
$$6 = 6$$

The equation $6 = 6$ is true for all values of $x$ and $y$, so the system of equations has infinitely many solutions. We know that $x = 2y + 3$, so we can write the solutions in the form $(2y + 3, y)$.

If we solve either equation for $y$, we get $y = \frac{1}{2}x - \frac{3}{2}$, so we can also write the solutions in the form $\left(x, \frac{1}{2}x - \frac{3}{2}\right)$.

**25.** $x - 5y = 4,\quad (1)$

$y = 7 - 2x\quad (2)$

Use equation (2) and substitute $7 - 2x$ for $y$ in equation (1). Then solve for $x$.

$$\begin{aligned} x - 5(7 - 2x) &= 4 \\ x - 35 + 10x &= 4 \\ 11x - 35 &= 4 \\ 11x &= 39 \\ x &= \frac{39}{11} \end{aligned}$$

Substitute $\frac{39}{11}$ for $x$ in equation (2) to find $y$.

$$y = 7 - 2 \cdot \frac{39}{11} = 7 - \frac{78}{11} = -\frac{1}{11}$$

The solution is $\left(\frac{39}{11}, -\frac{1}{11}\right)$.

**26.** $5x + 3y = -1,\quad (1)$

$x + \ \ y = 1\quad (2)$

Solve equation (2) for either $x$ or $y$. We choose to solve for $x$.

$$x = 1 - y$$

Substitute $1 - y$ for $x$ in equation (1) and solve for $y$.

$$\begin{aligned} 5(1 - y) + 3y &= -1 \\ 5 - 5y + 3y &= -1 \\ -2y &= -6 \\ y &= 3 \end{aligned}$$

Back-substitute to find $x$.

$x + 3 = 1$ Using equation (2)

$x = -2$

The solution is $(-2, 3)$.

**27.** $x + 2y = 2,\quad (1)$

$4x + 4y = 5\quad (2)$

Solve one equation for either $x$ or $y$. We choose to solve equation (1) for $x$ since $x$ has a coefficient of 1 in that equation.

$$\begin{aligned} x + 2y &= 2 \\ x &= -2y + 2 \end{aligned}$$

Substitute $-2y + 2$ for $x$ in equation (2) and solve for $y$.

$$\begin{aligned} 4(-2y + 2) + 4y &= 5 \\ -8y + 8 + 4y &= 5 \\ -4y + 8 &= 5 \\ -4y &= -3 \\ y &= \frac{3}{4} \end{aligned}$$

Substitute $\frac{3}{4}$ for $y$ in either equation (1) or equation (2) and solve for $x$.

$x + 2y = 2$ Using equation (1)

$$\begin{aligned} x + 2 \cdot \frac{3}{4} &= 2 \\ x + \frac{3}{2} &= 2 \\ x &= \frac{1}{2} \end{aligned}$$

The solution is $\left(\frac{1}{2}, \frac{3}{4}\right)$.

**28.** $2x - y = 2,\quad (1)$

$4x + y = 3\quad (2)$

Solve one equation for either $x$ or $y$. We choose to solve equation (2) for $y$ since $y$ has a coefficient of 1 in that equation.

$$y = -4x + 3$$

Substitute $-4x + 3$ for $y$ in equation (1) and solve for $x$.

$$\begin{aligned} 2x - (-4x + 3) &= 2 \\ 2x + 4x - 3 &= 2 \\ 6x &= 5 \\ x &= \frac{5}{6} \end{aligned}$$

Back-substitute to find $y$.

$$\begin{aligned} 4 \cdot \frac{5}{6} + y &= 3 \\ \frac{10}{3} + y &= 3 \\ y &= -\frac{1}{3} \end{aligned}$$

The solution is $\left(\frac{5}{6}, -\frac{1}{3}\right)$.

**29.** $3x - y = 5,\quad (1)$

$3y = 9x - 15\quad (2)$

Solve one equation for $x$ or $y$. We will solve equation (1) for $y$.

$$\begin{aligned} 3x - y &= 5 \\ 3x &= y + 5 \\ 3x - 5 &= y \end{aligned}$$

Substitute $3x - 5$ for $y$ in equation (2) and solve for $x$.

$$\begin{aligned} 3(3x - 5) &= 9x - 15 \\ 9x - 15 &= 9x - 15 \\ -15 &= -15 \end{aligned}$$

The equation $-15 = -15$ is true for all values of $x$ and $y$, so the system of equations has infinitely many solutions. We know that $y = 3x - 5$, so we can write the solutions in the form $(x, 3x - 5)$.

If we solve either equation for $x$, we get $x = \frac{1}{3}y + \frac{5}{3}$, so we can also write the solutions in the form $\left(\frac{1}{3}y + \frac{5}{3}, y\right)$.

**30.** $2x - y = 7,\quad (1)$

$y = 2x - 5\quad (2)$

Substitute $2x - 5$ for $y$ in equation (1).

$$\begin{aligned} 2x - (2x - 5) &= 7 \\ 2x - 2x + 5 &= 7 \\ 5 &= 7 \end{aligned}$$

There are no values of $x$ and $y$ for which $5 = 7$, so the system of equations has no solution.

**31.** $x + 2y = 7,\quad (1)$

$x - 2y = -5\quad (2)$

We add the equations to eliminate $y$.

$$\begin{array}{r} x + 2y = 7 \\ x - 2y = -5 \\ \hline 2x \qquad = 2 \end{array}\quad \text{Adding}$$

$$x = 1$$

Back-substitute in either equation and solve for $y$.

$$\begin{aligned} 1 + 2y &= 7 \quad \text{Using equation (1)} \\ 2y &= 6 \\ y &= 3 \end{aligned}$$

The solution is $(1, 3)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**32.** $3x + 4y = -2\quad (1)$

$$\begin{array}{r} -3x - 5y = 1 \quad (2) \\ \hline -y = -1 \end{array}\quad \text{Adding}$$

$$y = 1$$

Back-substitute to find $x$.

$$\begin{aligned} 3x + 4 \cdot 1 &= -2 \quad \text{Using equation (1)} \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

The solution is $(-2, 1)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**33.** $x - 3y = 2,\quad (1)$

$6x + 5y = -34\quad (2)$

Multiply equation (1) by $-6$ and add it to equation (2) to eliminate $x$.

$$\begin{array}{r} -6x + 18y = -12 \\ 6x + 5y = -34 \\ \hline 23y = -46 \end{array}$$

$$y = -2$$

Back-substitute to find $x$.

$$\begin{aligned} x - 3(-2) &= 2 \quad \text{Using equation (1)} \\ x + 6 &= 2 \\ x &= -4 \end{aligned}$$

The solution is $(-4, -2)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**34.** $x + 3y = 0,\quad (1)$

$20x - 15y = 75\quad (2)$

Multiply equation (1) by 5 and add.

$$\begin{array}{r} 5x + 15y = 0 \\ 20x - 15y = 75 \\ \hline 25x \qquad = 75 \end{array}$$

$$x = 3$$

Back-substitute to find $y$.

$$\begin{aligned} 3 + 3y &= 0 \quad \text{Using equation (1)} \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

The solution is $(3, -1)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**35.** $3x - 12y = 6,\quad (1)$

$2x - 8y = 4\quad (2)$

Multiply equation (1) by 2 and equation (2) by $-3$ and add.

$$\begin{array}{r} 6x - 24y = 12 \\ -6x + 24y = -12 \\ \hline 0 = 0 \end{array}$$

The equation $0 = 0$ is true for all values of $x$ and $y$. Thus, the system of equations has infinitely many solutions. Solving either equation for $y$, we can write $y = \frac{1}{4}x - \frac{1}{2}$ so the solutions are ordered pairs of the form $\left(x, \frac{1}{4}x - \frac{1}{2}\right)$. Equivalently, if we solve either equation for $x$ we get $x = 4y + 2$ so the solutions can also be expressed as $(4y + 2, y)$. Since there are infinitely many solutions, the system of equations is consistent and the equations are dependent.

**36.** $2x + 6y = 7,\quad (1)$

$3x + 9y = 10\quad (2)$

Multiply equation (1) by 3 and equation (2) by $-2$ and add.

$$\begin{array}{r} 6x + 18y = 21 \\ -6x - 18y = -20 \\ \hline 0 = 1 \end{array}$$

We get a false equation so there is no solution. Since there is no solution the system of equations is inconsistent and the equations are independent.

**37.** $4x - 2y = 3,\quad (1)$

$2x - y = 4\quad (2)$

Multiply equation (2) by $-2$ and add.

$$\begin{array}{r} 4x - 2y = 3 \\ -4x + 2y = -8 \\ \hline 0 = -5 \end{array}$$

We get a false equation so there is no solution. Since there is no solution the system of equations is inconsistent and the equations are independent.

**38.** $6x + 9y = 12$, (1)

$4x + 6y = 8$ (2)

Multiply equation (1) by 2 and equation (2) by $-3$ and add.

$$\begin{array}{r} 12x + 18y = 24 \\ -12x - 18y = -24 \\ \hline 0 = 0 \end{array}$$

The equation $0 = 0$ is true for all values of $x$ and $y$. Thus, the system of equations has infinitely many solutions. Solving either equation for $y$, we get $y = \frac{4}{3} - \frac{2}{3}x$ so the solutions are ordered pairs of the form $\left(x, \frac{4}{3} - \frac{2}{3}x\right)$. Equivalently, if we solve either equation for $x$ we get $x = 2 - \frac{3}{2}y$, so the solutions can also be expressed as $\left(2 - \frac{3}{2}y, y\right)$. Since there are infinitely many solutions, the system of equations is consistent and the equations are independent.

**39.** $2x = 5 - 3y$, (1)

$4x = 11 - 7y$ (2)

We rewrite the equations.

$2x + 3y = 5$, (1a)

$4x + 7y = 11$ (2a)

Multiply equation (2a) by $-2$ and add to eliminate $x$.

$$\begin{array}{r} -4x - 6y = -10 \\ 4x + 7y = 11 \\ \hline y = 1 \end{array}$$

Back-substitute to find $x$.

$2x = 5 - 3 \cdot 1$ Using equation (1)

$2x = 2$

$x = 1$

The solution is $(1, 1)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**40.** $7(x - y) = 14$, (1)

$2x = y + 5$ (2)

$x - y = 2$, (1a) Dividing equation (1) by 7

$2x - y = 5$ (2a) Rewriting equation (2)

Multiply equation (1a) by $-1$ and add.

$$\begin{array}{r} -x + y = -2 \\ 2x - y = 5 \\ \hline x \quad = 3 \end{array}$$

Back-substitute to find $y$.

$3 - y = 2$ Using equation (1a)

$1 = y$

The solution is $(3, 1)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**41.** $0.3x - 0.2y = -0.9$,

$0.2x - 0.3y = -0.6$

First, multiply each equation by 10 to clear the decimals.

$3x - 2y = -9$ (1)

$2x - 3y = -6$ (2)

Now multiply equation (1) by 3 and equation (2) by $-2$ and add to eliminate $y$.

$$\begin{array}{r} 9x - 6y = -27 \\ -4x + 6y = 12 \\ \hline 5x \quad = -15 \end{array}$$

$x = -3$

Back-substitute to find $y$.

$3(-3) - 2y = -9$ Using equation (1)

$-9 - 2y = -9$

$-2y = 0$

$y = 0$

The solution is $(-3, 0)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**42.** $0.2x - 0.3y = 0.3$,

$0.4x + 0.6y = -0.2$

First, multiply each equation by 10 to clear the decimals.

$2x - 3y = 3$ (1)

$4x + 6y = -2$ (2)

Now multiply equation (1) by 2 and add.

$$\begin{array}{r} 4x - 6y = 6 \\ 4x + 6y = -2 \\ \hline 8x \quad = 4 \end{array}$$

$x = \frac{1}{2}$

Back-substitute to find $y$.

$4 \cdot \frac{1}{2} + 6y = -2$ Using equation (2)

$2 + 6y = -2$

$6y = -4$

$y = -\frac{2}{3}$

The solution is $\left(\frac{1}{2}, -\frac{2}{3}\right)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**43.** $\frac{1}{5}x + \frac{1}{2}y = 6$, (1)

$\frac{3}{5}x - \frac{1}{2}y = 2$ (2)

We could multiply both equations by 10 to clear fractions, but since the $y$-coefficients differ only by sign we will just add to eliminate $y$.

$$\begin{array}{rl} \frac{1}{5}x + \frac{1}{2}y = 6 \\ \frac{3}{5}x - \frac{1}{2}y = 2 \\ \hline \frac{4}{5}x \qquad = 8 \\ x = 10 \end{array}$$

Back-substitute to find $y$.

$$\frac{1}{5}\cdot 10 + \frac{1}{2}y = 6 \quad \text{Using equation (1)}$$
$$2 + \frac{1}{2}y = 6$$
$$\frac{1}{2}y = 4$$
$$y = 8$$

The solution is $(10, 8)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**44.** $\frac{2}{3}x + \frac{3}{5}y = -17,$

$\frac{1}{2}x - \frac{1}{3}y = -1$

Multiply the first equation by 15 and the second by 6 to clear fractions.

$$10x + 9y = -255 \quad (1)$$
$$3x - 2y = -6 \quad (2)$$

Now multiply equation (1) by 2 and equation (2) by 9 and add.

$$\begin{array}{rl} 20x + 18y = -510 \\ 27x - 18y = -54 \\ \hline 47x \qquad = -564 \\ x = -12 \end{array}$$

Back-substitute to find $y$.

$$3(-12) - 2y = -6 \quad \text{Using equation (2)}$$
$$-36 - 2y = -6$$
$$-2y = 30$$
$$y = -15$$

The solution is $(-12, -15)$. Since the system of equations has exactly one solution it is consistent and the equations are independent.

**45.** The statement is true. See page 739 of the text.

**46.** False; if we obtain $0 = 0$, which is true for all values of $x$ and $y$, there are infinitely many solutions.

**47.** False; a consistent system of equations can have exactly one solution or infinitely many solutions. See page 739 of the text.

**48.** True; see page 739 of the text.

**49.** True; a system of equations that has infinitely many solutions is consistent and dependent. See page 739 of the text.

**50.** False; an inconsistent system of equations has no solution whereas a system with dependent equations has infinitely many solutions. See page 739 of the text.

**51.** ***Familiarize***. Let $x$ = the number of complaints about cell-phone providers and $y$ = the number of complaints about cable/satellite TV providers.

***Translate***. The total number of complaints was 70,093 so we have one equation.

$$x + y = 70,093$$

Cable/satellite TV providers received 4861 fewer complaints than cell-phne providers so we have a second equation.

$$x - 4861 = y$$

***Carry out***. We solve the system of equations

$$x + y = 70,093, \quad (1)$$
$$x - 4861 = y. \quad (2)$$

Substitute $x - 4861$ for $y$ in equation (1) and solve for $x$.

$$x + (x - 4861) = 70,093$$
$$2x - 4861 = 70,093$$
$$2x = 74,954$$
$$x = 37,477$$

Back-substitute in equation (2) to find $y$.

$$37,477 - 4861 = y$$
$$32,616 = y$$

***Check***. $37,477 + 32,616 = 70,093$ complaints and 32,616 is 4861 less than 37,477. The answer checks.

***State***. There were 37,477 complaints about cell-phone providers and 32,616 complaints about cable/satellite TV providers.

**52.** Let $x$ = the total average medical bills for an obese adult and $y$ = the total average medical bills for a person with a healthy weight.

Solve: $x = y + 2460,$

$x + y = 14,170$

$x = \$8315$, $y = \$5855$

**53.** ***Familiarize***. Let $k$ = the number of knee replacements, in millions, and $h$ = the number of hip replacements, in millions, in 2030.

***Translate***. The total number of knee and hip replacements will be 4.072 million so we have one equation.

$$k + h = 4.072$$

There will be 2.982 million more knee replacements than hip replacements, so we have a second equation.

$$k = h + 2.928$$

***Carry out***. We solve the system of equations

$$k + h = 4.072, \quad (1)$$
$$k = h + 2.928. \quad (2)$$

Substitute $h + 2.928$ for $k$ in equation (1) and solve for $h$.

$$\begin{aligned} h + 2.928 + h &= 4.072 \\ 2h + 2.928 &= 4.072 \\ 2h &= 1.144 \\ h &= 0.572 \end{aligned}$$

Back-substitute in equation (2) to find $k$.

$$k = 0.572 + 2.928 = 3.5$$

***Check***. $3.5 + 0.572 = 4.072$ replacements and 3.5 is 2.928 more than 0.572. The answer checks.

***State***. In 2030 there will be 3.5 million knee replacements and 0.572 million, or 572,000, hip replacements.

**54.** Let $m =$ the number of streets named Main Street and $s =$ the number of streets named Second Street.

$$\text{Solve: } \begin{aligned} m + s &= 15,684, \\ s &= m + 260 \end{aligned}$$

$m = 7712$ and $s = 7972$, so there are 7712 streets named Main Street and 7972 streets named Second Street in the U.S.

**55.** ***Familiarize***. Let $x =$ the number of injuries that occur in snowboarding each winter and $y =$ the number of injuries that occur in skiing.

***Translate***. The total number of snowboarding and skiing injuries is 288,400 so we have one equation:

$$x + y = 288,400$$

Skiing accounts for 400 more injuries than snowboarding, so we have a second equation:

$$y = x + 400$$

***Carry out***. We solve the system of equations

$$\begin{aligned} x + y &= 288,400, \quad (1) \\ y &= x + 400. \quad (2) \end{aligned}$$

Substitute $x + 400$ for $y$ in equation (1) and solve for $x$.

$$\begin{aligned} x + (x + 400) &= 288,400 \\ 2x + 400 &= 288,400 \\ 2x &= 288,000 \\ x &= 144,000 \end{aligned}$$

Back-substitute in equation (2) to find $y$.

$$y = 144,000 + 400 = 144,400$$

***Check***. $144,400 + 144,000 = 288,400$ injuries and 144,400 is 400 more than 144,000. The answer checks.

***State***. Snowboarding accounts for 144,000 injuries each winter and skiing accounts for about 144,400 injuries.

**56.** Let $c$ and $p$ represent the number of teaspoons of added sugar in the Coca-Cola and the Pepsi, respectively.

$$\text{Solve: } \begin{aligned} c + p &= 34, \\ c &= p - 1 \end{aligned}$$

$c = 16.5$ tsp, $p = 17.5$ tsp

**57.** ***Familiarize***. Let $x =$ the number of standard-delivery packages and $y =$ the number of express-delivery packages.

***Translate***. A total of 120 packages were shipped, so we have one equation.

$$x + y = 120$$

Total shipping charges were \$596, so we have a second equation.

$$3.50x + 7.50y = 596$$

***Carry out***. We solve the system of equations:

$$\begin{aligned} x + \quad y &= 120, \\ 3.50x + 7.50y &= 596. \end{aligned}$$

First we multiply both sides of the second equation by 10 to clear the decimals. This gives us the system of equations

$$\begin{aligned} x + \quad y &= 120, \quad (1) \\ 35x + 75y &= 5960. \quad (2) \end{aligned}$$

Now multiply equation (1) by $-35$ and add.

$$\begin{aligned} -35x - 35y &= -4200 \\ 35x + 75y &= 5960 \\ \hline 40y &= 1760 \\ y &= 44 \end{aligned}$$

Back-substitute to find $x$.

$$\begin{aligned} x + 44 &= 120 \quad \text{Using equation (1)} \\ x &= 76 \end{aligned}$$

***Check***. $76 + 44 = 120$ packages. Total shipping charges are $\$3.50(76) + \$7.50(44) = \$266 + \$330 = \$596$. The answer checks.

***State***. The business shipped 76 standard-delivery packages and 44 express-delivery packages.

**58.** Let $x =$ the number of tickets sold for pavilion seats and $y =$ the number sold for lawn seats.

$$\text{Solve: } \begin{aligned} x + y &= 1500, \\ 25x + 15y &= 28,500. \end{aligned}$$

$x = 600, \ y = 900$

**59.** ***Familiarize***. Let $x =$ the amount invested at 4% and $y =$ the amount invested at 5%. Then the interest from the investments is $4\%x$ and $5\%y$, or $0.04x$ and $0.05y$.

***Translate***.

The total investment is \$15,000.

$$x + y = 15,000$$

The total interest is \$690.

$$0.04x + 0.05y = 690$$

We have a system of equations:

$$\begin{aligned} x + \quad y &= 15,000, \\ 0.04x + 0.05y &= 690 \end{aligned}$$

Multiplying the second equation by 100 to clear the decimals, we have:

$$\begin{aligned} x + \ y &= 15,000, \quad (1) \\ 4x + 5y &= 69,000. \quad (2) \end{aligned}$$

***Carry out***. We begin by multiplying equation (1) by $-4$ and adding.

$$\begin{array}{r} -4x - 4y = -60,000 \\ 4x + 5y = 69,000 \\ \hline y = 9000 \end{array}$$

Back-substitute to find $x$.

$x + 9000 = 15,000$ Using equation (1)

$x = 6000$

***Check***. The total investment is \$6000+\$9000, or \$15,000. The total interest is $0.04(\$6000) + 0.05(\$9000)$, or $\$240 + \$450$, or \$690. The solution checks.

***State***. \$6000 was invested at 4% and \$9000 was invested at 5%.

**60.** Let $x$ = the number of short-sleeved shirts sold and $y$ = the number of long-sleeved shirts sold.

Solve: $x + y = 36$,

$12x + 18y = 522$.

$x = 21$, $y = 15$

**61.** ***Familiarize***. Let $x$ = the number of pounds of French roast coffee used and $y$ = the number of pounds of Kenyan coffee. We organize the information in a table.

| | French roast | Kenyan | Mixture |
|---|---|---|---|
| Amount | $x$ | $y$ | 10 lb |
| Price per pound | \$9.00 | \$7.50 | \$8.40 |
| Total cost | \$9$x$ | \$7.50$y$ | \$8.40(10), or \$84 |

***Translate***. The first and third rows of the table give us a system of equations.

$x + y = 10$,

$9x + 7.5y = 84$

Multiply the second equation by 10 to clear the decimals.

$x + y = 10$, (1)

$90x + 75y = 840$ (2)

***Carry out***. Begin by multiplying equation (1) by $-75$ and adding.

$$\begin{array}{r} -75x - 75y = -750 \\ 90x + 75y = 840 \\ \hline 15x \qquad = 90 \\ x = 6 \end{array}$$

Back-substitute to find $y$.

$6 + y = 10$ Using equation (1)

$y = 4$

***Check***. The total amount of coffee in the mixture is $6 + 4$, or 10 lb. The total value of the mixture is $6(\$9) + 4(\$7.50)$, or $\$54 + \$30$, or \$84. The solution checks.

***State***. 6 lb of French roast coffee and 4 lb of Kenyan coffee should be used.

**62.** Let $x$ = the monthly sales, $C$ = the earnings with the straight-commission plan, and $S$ = the earnings with the salary-plus-commission plan. Then 8% of sales is represented by $8\%x$, or $0.08x$, and 1% of sales is represented by $1\%x$, or $0.01x$.

Solve: $C = 0.08x$,

$S = 1500 + 0.01x$.

$x \approx 21,428.57$, so the two plans pay the same amount for monthly sales of about \$21,428.57.

**63.** ***Familiarize***. Let $x$ = the number of servings of spaghetti and meatballs required and $y$ = the number of servings of iceberg lettuce required. Then $x$ servings of spaghetti contain $260x$ Cal and $32x$ g of carbohydrates; $y$ servings of lettuce contain $5y$ Cal and $1 \cdot y$ or $y$, g of carbohydrates.

***Translate***. One equation comes from the fact that 400 Cal are desired:

$260x + 5y = 400$.

A second equation comes from the fact that 50g of carbohydrates are required:

$32x + y = 50$.

***Carry out***. We solve the system

$260x + 5y = 400$, (1)

$32x + y = 50$. (2)

Multiply equation (2) by $-5$ and add.

$$\begin{array}{r} 260x + 5y = 400 \\ -160x - 5y = -250 \\ \hline 100x \qquad = 150 \\ x = 1.5 \end{array}$$

Back-substitute to find $y$.

$32(1.5) + y = 50$ Using equation (2)

$48 + y = 50$

$y = 2$

***Check***. 1.5 servings of spaghetti contain 260(1.5), or 390 Cal and 32(1.5), or 48 g of carbohydrates; 2 servings of lettuce contain $5 \cdot 2$, or 10 Cal and $1 \cdot 2$, or 2 g of carbohydrates. Together they contain $390 + 10$, or 400 Cal and $48 + 2$, or 50 g of carbohydrates. The solution checks.

***State***. 1.5 servings of spaghetti and meatballs and 2 servings of iceberg lettuce are required.

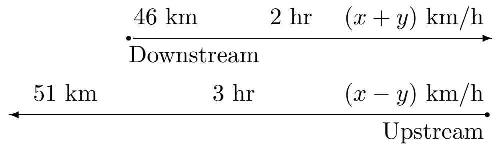
**64.** Let $x$ = the number of servings of tomato soup and $y$ = the number of slices of whole wheat bread required.

Solve: $100x + 70y = 230$,

$18x + 13y = 42$.

$x = 1.25$, $y = 1.5$

**65.** ***Familiarize***. It helps to make a drawing. Then organize the information in a table. Let $x$ = the speed of the boat and $y$ = the speed of the stream. The speed upstream is $x - y$. The speed downstream is $x + y$.




| | Distance | Speed | Time |
|---|---|---|---|
| Downstream | 46 | $x+y$ | 2 |
| Upstream | 51 | $x-y$ | 3 |

***Translate***. Using $d = rt$ in each row of the table, we get a system of equations.

$$46 = (x+y)2 \qquad x+y = 23, \quad (1)$$
$$\text{or}$$
$$51 = (x-y)3 \qquad x-y = 17 \quad (2)$$

***Carry out***. We begin by adding equations (1) and (2).

$$\begin{aligned} x + y &= 23 \\ x - y &= 17 \\ \hline 2x \quad &= 40 \\ x &= 20 \end{aligned}$$

Back-substitute to find $y$.

$$20 + y = 23 \quad \text{Using equation (1)}$$
$$y = 3$$

***Check***. The speed downstream is $20+3$, or 23 km/h. The distance traveled downstream in 2 hr is $23 \cdot 2$, or 46 km. The speed upstream is $20-3$, or 17 km/h. The distance traveled upstream in 3 hr is $17 \cdot 3$, or 51 km. The solution checks.

***State***. The speed of the boat is 20 km/h. The speed of the stream is 3 km/h.

**66.** Let $x$ = the speed of the plane and $y$ = the speed of the wind.

| | Speed | Time | Distance |
|---|---|---|---|
| Downwind | $x+y$ | 3 | 3000 |
| Upwind | $x-y$ | 4 | 3000 |

Solve: $(x+y)3 = 3000,$
$(x-y)4 = 3000.$

$x = 875$ km/h, $y = 125$ km/h

**67.** ***Familiarize***. Let $d$ = the distance the slower plane travels, in km. Then $780 - d$ = the distance the faster plane travels. Let $t$ = the number of hours each plane travels. We organize the information in a table.

| | Distance | Speed | Time |
|---|---|---|---|
| Slower plane | $d$ | 190 | $t$ |
| Faster plane | $780-d$ | 200 | $t$ |

***Translate***. Using $d = rt$ in each row of the table, we get a system of equations.

$$d = 190t, \quad (1)$$
$$780 - d = 200t \quad (2)$$

***Carry out***. We begin by adding the equations.

$$\begin{aligned} d &= 190t \\ 780 - \quad d &= 200t \\ \hline 780 &= 390t \\ 2 &= t \end{aligned}$$

***Check***. In 2 hr, the slower plane travels $190 \cdot 2$, or 380 km, and the faster plane travels $200 \cdot 2$, or 400 km. The total distance traveled is 380 km + 400 km, or 780 km, so the answer checks.

***State***. The planes will meet in 2 hr.

**68.** Let $b$ = the speed of the boat in still water, in mph, and $c$ = the speed of the current.

| | Distance | Speed | Time |
|---|---|---|---|
| Downstream | 45 | $b+c$ | 3 |
| Upstream | 45 | $b-c$ | 5 |

Solve: $45 = (b+c)3,$
$45 = (b-c)5.$

$b = 12$ mph, $c = 3$ mph

**69.** ***Familiarize and Translate***. We use the system of equations given in the problem.

$$y = 70 + 2x \quad (1)$$
$$y = 175 - 5x, \quad (2)$$

***Carry out***. Substitute $175 - 5x$ for $y$ in equation (1) and solve for $x$.

$$175 - 5x = 70 + 2x$$
$$105 = 7x \quad \text{Adding } 5x \text{ and subtracting } 70$$
$$15 = x$$

Back-substitute in either equation to find $y$. We choose equation (1).

$$y = 70 + 2 \cdot 15 = 70 + 30 = 100$$

***Check***. Substituting 15 for $x$ and 100 for $y$ in both of the original equations yields true equations, so the solution checks.

***State***. The equilibrium point is $(15, \$100)$.

**70.** Solve: $y = 240 + 40x,$
$y = 500 - 25x.$

$x = 4$, $y = 400$, so the equilibrium point is $(4, \$400)$.

**71.** ***Familiarize and Translate***. We find the value of $x$ for which $C = R$, where

$$C = 14x + 350,$$
$$R = 16.5x.$$

***Carry out***. When $C = R$ we have:

$$14x + 350 = 16.5x$$
$$350 = 2.5x$$
$$140 = x$$

***Check***. When $x = 140$, $C = 14 \cdot 140 + 350$, or 2310 and $R = 16.5(140)$, or 2310. Since $C = R$, the solution checks.

***State***. 140 units must be produced and sold in order to break even.

**72.** Solve $C = R$, where

$$C = 8.5x + 75,$$
$$R = 10x.$$

$x = 50$

**73.** ***Familiarize and Translate***. We find the value of $x$ for which $C = R$, where

$$C = 15x + 12,000,$$
$$R = 18x - 6000.$$

***Carry out***. When $C = R$ we have:

$$15x + 12,000 = 18x - 6000$$
$$18,000 = 3x \quad \text{Subtracting } 15x \text{ and adding } 6000$$
$$6000 = x$$

***Check***. When $x = 6000$, $C = 15\cdot 6000 + 12,000$, or 102,000 and $R = 18 \cdot 6000 - 6000$, or 102,000. Since $C = R$, the solution checks.

***State***. 6000 units must be produced and sold in order to break even.

**74.** Solve $C = R$, where

$$C = 3x + 400,$$
$$R = 7x - 600.$$

$x = 250$

**75.** ***Familiarize***. Let $t$ = the number of air travels in 2004, in millions. Then a decrease of 8.2% from this number is $t - 8.2\%t$, or $t - 0.082t$, or $0.918t$. This represents the number of air travelers in 2009.

***Translate***.

$$\underbrace{\text{Number of air travelers in 2009}}_{\downarrow\; 769.6} \quad \underset{\downarrow\;=}{\text{is}} \quad \underbrace{\text{8.2\% less than number in 2004}}_{\downarrow\; 0.918t}$$

***Carry out***. We solve the equation.

$$769.6 = 0.918t$$
$$838.3 \approx t$$

***Check***. A decrease of 8.2% from 838.3 million is $0.918(838.3) = 769.5594 \approx 769.6$ million. The answer checks.

***State***. There were about 838.3 million air travelers in 2004.

**76.** Let $s$ = the number of online retail sales in 2002, in billions of dollars.

Solve: $133.6 = 3s$

$$44.5 \approx s$$

Online retail sales totaled about \$44.5 billion in 2002.

**77.** Substituting 15 for $f(x)$, we solve the following equation.

$$15 = x^2 - 4x + 3$$
$$0 = x^2 - 4x - 12$$
$$0 = (x - 6)(x + 2)$$
$$x - 6 = 0 \quad or \quad x + 2 = 0$$
$$x = 6 \quad or \quad x = -2$$

If the output is 15, the input is 6 or $-2$.

**78.** Substituting 8 for $f(x)$, we solve the following equation.

$$8 = x^2 - 4x + 3$$
$$0 = x^2 - 4x - 5$$
$$0 = (x - 5)(x + 1)$$
$$x - 5 = 0 \quad or \quad x + 1 = 0$$
$$x = 5 \quad or \quad x = -1$$

Given an output of 8, the corresponding inputs are 5 and $-1$.

**79.** $f(-2) = (-2)^2 - 4(-2) + 3 = 15$

If the input is $-2$, the output is 15.

**80.**
$$x^2 - 4x + 3 = 0$$
$$(x - 1)(x - 3) = 0$$
$$x = 1 \quad or \quad x = 3$$

**81.** ***Familiarize***. Let $x$ = the time spent jogging and $y$ = the time spent walking. Then Nancy jogs $8x$ km and walks $4y$ km.

***Translate***.

The total time is 1 hr.

$$x + y = 1$$

The total distance is 6 km.

$$8x + 4y = 6$$

***Carry out***. Solve the system

$$x + y = 1, \quad (1)$$
$$8x + 4y = 6. \quad (2)$$

Multiply equation (1) by $-4$ and add.

$$\begin{array}{r} -4x - 4y = -4 \\ 8x + 4y = 6 \\ \hline 4x \qquad = 2 \end{array}$$
$$x = \frac{1}{2}$$

This is the time we need to find the distance spent jogging, so we could stop here. However, we will not be able to check the solution unless we find $y$ also so we continue. We back-substitute.

$$\frac{1}{2} + y = 1 \quad \text{Using equation (1)}$$
$$y = \frac{1}{2}$$

Then the distance jogged is $8 \cdot \frac{1}{2}$, or 4 km, and the distance walked is $4 \cdot \frac{1}{2}$, or 2 km.

***Check***. The total time is $\frac{1}{2}$ hr $+ \frac{1}{2}$ hr, or 1 hr. The total distance is 4 km + 2 km, or 6 km. The solution checks.

***State***. Nancy jogged 4 km on each trip.

**82.** Let $x =$ the number of one-turtleneck orders and $y =$ the number of two-turtleneck orders.

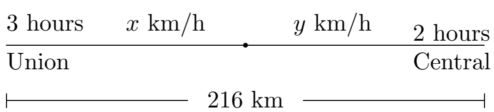
Solve: $x + 2y = 1250$,

$15x + 25y = 16,750$.

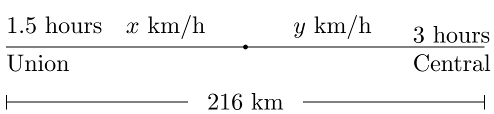
$y = 400$

**83.** ***Familiarize and Translate***. We let $x$ and $y$ represent the speeds of the trains. Organize the information in a table. Using $d = rt$, we let $3x$, $2y$, $1.5x$, and $3y$ represent the distances the trains travel.

First situation:




Second situation:




| | Distance traveled in first situation | Distance traveled in second situation |
|---|---|---|
| $\text{Train}_1$ (from Union to Central) | $3x$ | $1.5x$ |
| $\text{Train}_2$ (from Central to Union) | $2y$ | $3y$ |
| Total | 216 | 216 |

The total distance in each situation is 216 km. Thus, we have a system of equations.

$$3x + 2y = 216, \quad (1)$$
$$1.5x + 3y = 216 \quad (2)$$

***Carry out***. Multiply equation (2) by $-2$ and add.

$$\begin{array}{r} 3x + 2y = 216 \\ -3x - 6y = -432 \\ \hline -4y = -216 \\ y = 54 \end{array}$$

Back-substitute to find $x$.

$3x + 2 \cdot 54 = 216$ Using equation (1)

$3x + 108 = 216$

$3x = 108$

$x = 36$

***Check***. If $x = 36$ and $y = 54$, the total distance the trains travel in the first situation is $3 \cdot 36 + 2 \cdot 54$, or 216 km. The total distance they travel in the second situation is $1.5 \cdot 36 + 3 \cdot 54$, or 216 km. The solution checks.

***State***. The speed of the first train is 36 km/h. The speed of the second train is 54 km/h.

**84.** Let $x =$ the amount of mixture replaced by 100% antifreeze and $y =$ the amount of 30% mixture retained.

| | Replaced | Retained | Total |
|---|---|---|---|
| Amount | $x$ | $y$ | 16 L |
| Percent of antifreeze | 100% | 30% | 50% |
| Amount of antifreeze | $100\%x$ | $30\%y$ | $50\% \times 16$, or 8 L |

Solve: $x + y = 16$,

$x + 0.3y = 8$.

$x = 4\frac{4}{7}$ L

**85.** Substitute the given solutions in the equation $Ax + By = 1$ to get a system of equations.

$3A - B = 1, \quad (1)$

$-4A - 2B = 1 \quad (2)$

Multiply equation (1) by $-2$ and add.

$$\begin{array}{r} -6A + 2B = -2 \\ -4A - 2B = 1 \\ \hline -10A \qquad = -1 \\ A = \dfrac{1}{10} \end{array}$$

Back-substitute to find $B$.

$3\left(\frac{1}{10}\right) - B = 1$ Using equation (1)

$\frac{3}{10} - B = 1$

$-B = \frac{7}{10}$

$B = -\frac{7}{10}$

We have $A = \frac{1}{10}$ and $B = -\frac{7}{10}$.

**86.** Let $x =$ the number of people ahead of you and $y =$ the number of people behind you.

Solve: $x = 2 + y$,

$x + 1 + y = 3y$.

$x = 5$

**87.** ***Familiarize***. Let $x$ and $y$ represent the number of gallons of gasoline used in city driving and in highway driving, respectively. Then $49x$ and $51y$ represent the number of miles driven in the city and on the highway, respectively.

***Translate***. The fact that 9 gal of gasoline were used gives us one equation:

$x + y = 9$.

A second equation comes from the fact that the car is driven 447 mile:

$49x + 51y = 447$.

***Carry out***. We solve the system of equations

$x + y = 9, \quad (1)$

$49x + 51y = 447. \quad (2)$

Multiply equation (1) by $-49$ and add.

$$\begin{aligned} -49x - 49y &= -441 \\ 49x + 51y &= 447 \\ \hline 2y &= 6 \\ y &= 3 \end{aligned}$$

Back-substitute to find $x$.

$$\begin{aligned} x+3 &= 9 \\ x &= 6 \end{aligned}$$

Then in the city the car is driven 49(6), or 294 mi; on the highway it is driven 51(3), or 153 mi.

***Check***. The number of gallons of gasoline used is $6+3$, or 9. The number of miles driven is $294+153=447$. The answer checks.

***State***. The car was driven 294 mi in the city and 153 mi on the highway.

**88.** First we convert the given distances to miles:

$$300 \text{ ft} = \frac{300}{5280} \text{ mi} = \frac{5}{88} \text{ mi},$$

$$500 \text{ ft} = \frac{500}{5280} \text{ mi} = \frac{25}{264} \text{ mi}$$

Then at 10 mph, Heather can run to point $P$ in $\dfrac{\frac{5}{88}}{10}$, or $\dfrac{1}{176}$ hr, and she can run to point $Q$ in $\dfrac{\frac{25}{264}}{10}$, or $\dfrac{5}{528}$ hr (using $d=rt$, or $\dfrac{d}{r}=t$).

Let $d$ = the distance, in miles, from the train to point $P$ in the drawing in the text, and let $r$ the speed of the train, in miles per hour.

| | Distance | Speed | Time |
|---|---|---|---|
| Going to $P$ | $d$ | $r$ | $\frac{1}{176}$ |
| Going to $Q$ | $d+\frac{5}{88}+\frac{25}{264}$ | $r$ | $\frac{5}{528}$ |

Solve: $$d = r\left(\frac{1}{176}\right),$$

$$d + \frac{5}{88} + \frac{25}{264} = r\left(\frac{5}{528}\right).$$

$r = 40$ mph

## Exercise Set 9.2

**1.**

$$\begin{aligned} x + y + z &= 2, && (1) \\ 6x - 4y + 5z &= 31, && (2) \\ 5x + 2y + 2z &= 13 && (3) \end{aligned}$$

Multiply equation (1) by $-6$ and add it to equation (2). We also multiply equation (1) by $-5$ and add it to equation (3).

$$\begin{aligned} x + y + z &= 2 && (1) \\ -10y - z &= 19 && (4) \\ -3y - 3z &= 3 && (5) \end{aligned}$$

Multiply the last equation by 10 to make the $y$-coefficient a multiple of the $y$-coefficient in equation (4).

$$\begin{aligned} x + y + z &= 2 && (1) \\ -10y - z &= 19 && (4) \\ -30y - 30z &= 30 && (6) \end{aligned}$$

Multiply equation (4) by $-3$ and add it to equation (6).

$$\begin{aligned} x + y + z &= 2 && (1) \\ -10y - z &= 19 && (4) \\ -27z &= -27 && (7) \end{aligned}$$

Solve equation (7) for $z$.

$$\begin{aligned} -27z &= -27 \\ z &= 1 \end{aligned}$$

Back-substitute 1 for $z$ in equation (4) and solve for $y$.

$$\begin{aligned} -10y - 1 &= 19 \\ -10y &= 20 \\ y &= -2 \end{aligned}$$

Back-substitute 1 for $z$ for $-2$ and $y$ in equation (1) and solve for $x$.

$$\begin{aligned} x + (-2) + 1 &= 2 \\ x - 1 &= 2 \\ x &= 3 \end{aligned}$$

The solution is $(3, -2, 1)$.

**2.**

$$\begin{aligned} x + 6y + 3z &= 4, && (1) \\ 2x + y + 2z &= 3, && (2) \\ 3x - 2y + z &= 0 && (3) \end{aligned}$$

Multiply equation (1) by $-2$ and add it to equation (2). We also multiply equation (1) by $-3$ and add it to equation (3).

$$\begin{aligned} x + 6y + 3z &= 4, && (1) \\ -11y - 4z &= -5, && (4) \\ -20y - 8z &= -12 && (5) \end{aligned}$$

Multiply equation (5) by 11.

$$\begin{aligned} x + 6y + 3z &= 4, && (1) \\ -11y - 4z &= -5, && (4) \\ -220y - 88z &= -132 && (6) \end{aligned}$$

Multiply equation (4) by $-20$ and add it to equation (6).

$$\begin{aligned} x + 6y + 3z &= 4, && (1) \\ -11y - 4z &= -5, && (4) \\ -8z &= -32 && (7) \end{aligned}$$

Complete the solution.

$$\begin{aligned} -8z &= -32 \\ z &= 4 \\ -11y - 4\cdot 4 &= -5 \\ -11y &= 11 \\ y &= -1 \\ x + 6(-1) + 3\cdot 4 &= 4 \\ x &= -2 \end{aligned}$$

The solution is $(-2, -1, 4)$.

**3.** 
$$\begin{aligned} x - y + 2z &= -3 \quad (1)\\ x + 2y + 3z &= 4 \quad (2)\\ 2x + y + z &= -3 \quad (3) \end{aligned}$$

Multiply equation (1) by $-1$ and add it to equation (2). We also multiply equation (1) by $-2$ and add it to equation (3).

$$\begin{aligned} x - y + 2z &= -3 \quad (1)\\ 3y + z &= 7 \quad (4)\\ 3y - 3z &= 3 \quad (5) \end{aligned}$$

Multiply equation (4) by $-1$ and add it to equation (5).

$$\begin{aligned} x - y + 2z &= -3 \quad (1)\\ 3y + z &= 7 \quad (4)\\ -4z &= -4 \quad (6) \end{aligned}$$

Solve equation (6) for $z$.

$$\begin{aligned} -4z &= -4\\ z &= 1 \end{aligned}$$

Back-substitute 1 for $z$ in equation (4) and solve for $y$.

$$\begin{aligned} 3y + 1 &= 7\\ 3y &= 6\\ y &= 2 \end{aligned}$$

Back-substitute 1 for $z$ and 2 for $y$ in equation (1) and solve for $x$.

$$\begin{aligned} x - 2 + 2 \cdot 1 &= -3\\ x &= -3 \end{aligned}$$

The solution is $(-3, 2, 1)$.

**4.** 
$$\begin{aligned} x + y + z &= 6, \quad (1)\\ 2x - y - z &= -3, \quad (2)\\ x - 2y + 3z &= 6 \quad (3) \end{aligned}$$

Multiply equation (1) by $-2$ and add it to equation (2). Also, multiply equation (1) by $-1$ and add it to equation (3).

$$\begin{aligned} x + y + z &= 6, \quad (1)\\ -3y - 3z &= -15, \quad (4)\\ -3y + 2z &= 0 \quad (5) \end{aligned}$$

Multiply equation (4) by $-1$ and add it to equation (5).

$$\begin{aligned} x + y + z &= 6, \quad (1)\\ -3y - 3z &= -15, \quad (4)\\ 5z &= 15 \quad (6) \end{aligned}$$

Complete the solution.

$$\begin{aligned} 5z &= 15\\ z &= 3\\ -3y - 3 \cdot 3 &= -15\\ -3y &= -6\\ y &= 2\\ x + 2 + 3 &= 6\\ x &= 1 \end{aligned}$$

The solution is $(1, 2, 3)$.

**5.** 
$$\begin{aligned} x + 2y - z &= 5, \quad (1)\\ 2x - 4y + z &= 0, \quad (2)\\ 3x + 2y + 2z &= 3 \quad (3) \end{aligned}$$

Multiply equation (1) by $-2$ and add it to equation (2). Also, multiply equation (1) by $-3$ and add it to equation (3).

$$\begin{aligned} x + 2y - z &= 5, \quad (1)\\ -8y + 3z &= -10, \quad (4)\\ -4y + 5z &= -12 \quad (5) \end{aligned}$$

Multiply equation (5) by 2 to make the $y$-coefficient a multiple of the $y$-coefficient of equation (4).

$$\begin{aligned} x + 2y - z &= 5, \quad (1)\\ -8y + 3z &= -10, \quad (4)\\ -8y + 10z &= -24 \quad (6) \end{aligned}$$

Multiply equation (4) by $-1$ and add it to equation (6).

$$\begin{aligned} x + 2y - z &= 5, \quad (1)\\ -8y + 3z &= -10, \quad (4)\\ 7z &= -14 \quad (7) \end{aligned}$$

Solve equation (7) for $z$.

$$\begin{aligned} 7z &= -14\\ z &= -2 \end{aligned}$$

Back-substitute $-2$ for $z$ in equation (4) and solve for $y$.

$$\begin{aligned} -8y + 3(-2) &= -10\\ -8y - 6 &= -10\\ -8y &= -4\\ y &= \frac{1}{2} \end{aligned}$$

Back-substitute $\frac{1}{2}$ for $y$ and $-2$ for $z$ in equation (1) and solve for $x$.

$$\begin{aligned} x + 2 \cdot \frac{1}{2} - (-2) &= 5\\ x + 1 + 2 &= 5\\ x &= 2 \end{aligned}$$

The solution is $\left(2, \frac{1}{2}, -2\right)$.

**6.** 
$$\begin{aligned} 2x + 3y - z &= 1, \quad (1)\\ x + 2y + 5z &= 4, \quad (2)\\ 3x - y - 8z &= -7 \quad (3) \end{aligned}$$

Interchange equations (1) and (2).

$$\begin{aligned} x + 2y + 5z &= 4, \quad (2)\\ 2x + 3y - z &= 1, \quad (1)\\ 3x - y - 8z &= -7 \quad (3) \end{aligned}$$

Multiply equation (2) by $-2$ and add it to equation (1). Multiply equation (2) by $-3$ and add it to equation (3).

$$\begin{aligned} x + 2y + 5z &= 4, \quad (2)\\ -y - 11z &= -7, \quad (4)\\ -7y - 23z &= -19 \quad (5) \end{aligned}$$

Multiply equation (4) by $-7$ and add it to equation (5).

$$\begin{aligned} x + 2y + \phantom{1}5z &= 4, \quad (2)\\ -\phantom{2}y - 11z &= -7 \quad (4)\\ 54z &= 30 \end{aligned}$$

Complete the solution.

$$54z = 30$$
$$z = \frac{5}{9}$$
$$-y - 11\cdot\frac{5}{9} = -7$$
$$-y = -\frac{8}{9}$$
$$y = \frac{8}{9}$$
$$x + 2\cdot\frac{8}{9} + 5\cdot\frac{5}{9} = 4$$
$$x = -\frac{5}{9}$$

The solution is $\left(-\frac{5}{9}, \frac{8}{9}, \frac{5}{9}\right)$.

**7.**

$$\begin{aligned} x + 2y - z &= -8, \quad (1)\\ 2x - y + z &= 4, \quad (2)\\ 8x + y + z &= 2 \quad (3) \end{aligned}$$

Multiply equation (1) by $-2$ and add it to equation (2). Also, multiply equation (1) by $-8$ and add it to equation (3).

$$\begin{aligned} x + 2y - z &= -8, \quad (1)\\ -5y + 3z &= 20, \quad (4)\\ -15y + 9z &= 66 \quad (5) \end{aligned}$$

Multiply equation (4) by $-3$ and add it to equation (5).

$$\begin{aligned} x + 2y - z &= -8, \quad (1)\\ -5y + 3z &= 20, \quad (4)\\ 0 &= 6 \quad (6) \end{aligned}$$

Equation (6) is false, so the system of equations has no solution.

**8.**

$$\begin{aligned} x + 2y - z &= 4, \quad (1)\\ 4x - 3y + z &= 8, \quad (2)\\ 5x - y \phantom{+z} &= 12 \quad (3) \end{aligned}$$

Multiply equation (1) by $-4$ and add it to equation (2). Also, multiply equation (1) by $-5$ and add it to equation (3).

$$\begin{aligned} x + 2y - z &= 4, \quad (1)\\ -11y + 5z &= -8, \quad (4)\\ -11y + 5z &= -8 \quad (5) \end{aligned}$$

Multiply equation (4) by $-1$ and add it to equation (5).

$$\begin{aligned} x + 2y - z &= 4, \quad (1)\\ -11y + 5z &= -8, \quad (4)\\ 0 &= 0 \quad (6) \end{aligned}$$

The equation $0 = 0$ tells us that equation (3) of the original system is dependent on the first two equations. The system of equations has infinitely many solutions and is equivalent to

$$\begin{aligned} x + 2y - z &= 4, \quad (1)\\ 4x - 3y + z &= 8. \quad (2) \end{aligned}$$

To find an expression for the solutions, we first solve equation (4) for either $y$ or $z$. We choose to solve for $y$.

$$-11y + 5z = -8$$
$$-11y = -5z - 8$$
$$y = \frac{5z+8}{11}$$

Back-substitute in equation (1) to find an expression for $x$ in terms of $z$.

$$x + 2\left(\frac{5z+8}{11}\right) - z = 4$$
$$x + \frac{10z}{11} + \frac{16}{11} - z = 4$$
$$x = \frac{z}{11} + \frac{28}{11} = \frac{z+28}{11}$$

The solutions are given by $\left(\frac{z+28}{11}, \frac{5z+8}{11}, z\right)$, where $z$ is any real number.

**9.**

$$\begin{aligned} 2x + y - 3z &= 1, \quad (1)\\ x - 4y + z &= 6, \quad (2)\\ 4x - 7y - z &= 13 \quad (3) \end{aligned}$$

Interchange equations (1) and (2).

$$\begin{aligned} x - 4y + z &= 6, \quad (2)\\ 2x + y - 3z &= 1, \quad (1)\\ 4x - 7y - z &= 13 \quad (3) \end{aligned}$$

Multiply equation (2) by $-2$ and add it to equation (1). Also, multiply equation (2) by $-4$ and add it to equation (3).

$$\begin{aligned} x - 4y + z &= 6, \quad (2)\\ 9y - 5z &= -11, \quad (4)\\ 9y - 5z &= -11 \quad (5) \end{aligned}$$

Multiply equation (4) by $-1$ and add it to equation (5).

$$\begin{aligned} x - 4y + z &= 6, \quad (1)\\ 9y - 5z &= -11, \quad (4)\\ 0 &= 0 \quad (6) \end{aligned}$$

The equation $0 = 0$ tells us that equation (3) of the original system is dependent on the first two equations. The system of equations has infinitely many solutions and is equivalent to

$$\begin{aligned} 2x + y - 3z &= 1, \quad (1)\\ x - 4y + z &= 6. \quad (2) \end{aligned}$$

To find an expression for the solutions, we first solve equation (4) for either $y$ or $z$. We choose to solve for $z$.

$$9y - 5z = -11$$
$$-5z = -9y - 11$$
$$z = \frac{9y+11}{5}$$

Back-substitute in equation (2) to find an expression for $x$ in terms of $y$.

$$x - 4y + \frac{9y+11}{5} = 6$$
$$x - 4y + \frac{9}{5}y + \frac{11}{5} = 6$$
$$x = \frac{11}{5}y + \frac{19}{5} = \frac{11y+19}{5}$$

The solutions are given by $\left(\frac{11y+19}{5}, y, \frac{9y+11}{5}\right)$, where $y$ is any real number.

**10.**
$$\begin{aligned} x + 3y + 4z &= 1, & (1)\\ 3x + 4y + 5z &= 3, & (2)\\ x + 8y + 11z &= 2 & (3)\end{aligned}$$

Multiply equation (1) by $-3$ and add it to equation (2). Also, multiply equation (1) by $-1$ and add it to equation (3).

$$\begin{aligned} x + 3y + 4z &= 1, & (1)\\ -5y - 7z &= 0, & (4)\\ 5y + 7z &= 1 & (5)\end{aligned}$$

Add equation (4) to equation (5).

$$\begin{aligned} x + 3y + 4z &= 1, & (1)\\ -5y - 7z &= 0, & (4)\\ 0 &= 1 & (6)\end{aligned}$$

Equation (6) is false, so the system of equations has no solution.

**11.**
$$\begin{aligned} 4a + 9b &= 8, & (1)\\ 8a + 6c &= -1, & (2)\\ 6b + 6c &= -1 & (3)\end{aligned}$$

Multiply equation (1) by $-2$ and add it to equation (2).

$$\begin{aligned} 4a + 9b &= 8, & (1)\\ -18b + 6c &= -17, & (4)\\ 6b + 6c &= -1 & (3)\end{aligned}$$

Multiply equation (3) by 3 to make the $b$-coefficient a multiple of the $b$-coefficient in equation (4).

$$\begin{aligned} 4a + 9b &= 8, & (1)\\ -18b + 6c &= -17, & (4)\\ 18b + 18c &= -3 & (5)\end{aligned}$$

Add equation (4) to equation (5).

$$\begin{aligned} 4a + 9b &= 8, & (1)\\ -18b + 6c &= -17, & (4)\\ 24c &= -20 & (6)\end{aligned}$$

Solve equation (6) for $c$.

$$24c = -20$$
$$c = -\frac{20}{24} = -\frac{5}{6}$$

Back-substitute $-\frac{5}{6}$ for $c$ in equation (4) and solve for $b$.

$$-18b + 6c = -17$$
$$-18b + 6\left(-\frac{5}{6}\right) = -17$$
$$-18b - 5 = -17$$
$$-18b = -12$$
$$b = \frac{12}{18} = \frac{2}{3}$$

Back-substitute $\frac{2}{3}$ for $b$ in equation (1) and solve for $a$.

$$4a + 9b = 8$$
$$4a + 9 \cdot \frac{2}{3} = 8$$
$$4a + 6 = 8$$
$$4a = 2$$
$$a = \frac{1}{2}$$

The solution is $\left(\frac{1}{2}, \frac{2}{3}, -\frac{5}{6}\right)$.

**12.**
$$\begin{aligned} 3p + 2r &= 11, & (1)\\ q - 7r &= 4, & (2)\\ p - 6q &= 1 & (3)\end{aligned}$$

Interchange equations (1) and (3).

$$\begin{aligned} p - 6q &= 1, & (3)\\ q - 7r &= 4, & (2)\\ 3p + 2r &= 11 & (1)\end{aligned}$$

Multiply equation (3) by $-3$ and add it to equation (1).

$$\begin{aligned} p - 6q &= 1, & (3)\\ q - 7r &= 4, & (2)\\ 18q + 2r &= 8 & (4)\end{aligned}$$

Multiply equation (2) by $-18$ and add it to equation (4).

$$\begin{aligned} p - 6q &= 1, & (3)\\ q - 7r &= 4, & (2)\\ 128r &= -64 & (5)\end{aligned}$$

Complete the solution.

$$128r = -64$$
$$r = -\frac{1}{2}$$
$$q - 7\left(-\frac{1}{2}\right) = 4$$
$$q = \frac{1}{2}$$
$$p - 6 \cdot \frac{1}{2} = 1$$
$$p = 4$$

The solution is $\left(4, \frac{1}{2}, -\frac{1}{2}\right)$.

**13.** $\begin{aligned} 2x \quad\quad + z &= 1, &(1)\\ 3y - 2z &= 6, &(2)\\ x - 2y \quad\quad &= -9 &(3)\end{aligned}$

Interchange equations (1) and (3).

$\begin{aligned} x - 2y \quad\quad &= -9, &(3)\\ 3y - 2z &= 6, &(2)\\ 2x \quad\quad + z &= 1 &(1)\end{aligned}$

Multiply equation (3) by $-2$ and add it to equation (1).

$\begin{aligned} x - 2y \quad\quad &= -9, &(3)\\ 3y - 2z &= 6, &(2)\\ 4y + z &= 19 &(4)\end{aligned}$

Multiply equation (4) by 3 to make the $y$-coefficient a multiple of the $y$-coefficient in equation (2).

$\begin{aligned} x - 2y \quad\quad &= -9, &(3)\\ 3y - 2z &= 6, &(2)\\ 12y + 3z &= 57 &(5)\end{aligned}$

Multiply equation (2) by $-4$ and add it to equation (5).

$\begin{aligned} x - 2y \quad\quad &= -9, &(3)\\ 3y - 2z &= 6, &(2)\\ 11z &= 33 &(6)\end{aligned}$

Solve equation (6) for $z$.

$$\begin{aligned} 11z &= 33\\ z &= 3\end{aligned}$$

Back-substitute 3 for $z$ in equation (2) and solve for $y$.

$$\begin{aligned} 3y - 2z &= 6\\ 3y - 2\cdot 3 &= 6\\ 3y - 6 &= 6\\ 3y &= 12\\ y &= 4\end{aligned}$$

Back-substitute 4 for $y$ in equation (3) and solve for $x$.

$$\begin{aligned} x - 2y &= -9\\ x - 2\cdot 4 &= -9\\ x - 8 &= -9\\ x &= -1\end{aligned}$$

The solution is $(-1, 4, 3)$.

**14.** $\begin{aligned} 3x \quad\quad + 4z &= -11, &(1)\\ x - 2y \quad\quad &= 5, &(2)\\ 4y - z &= -10 &(3)\end{aligned}$

Interchange equations (1) and (2).

$\begin{aligned} x - 2y \quad\quad &= 5, &(2)\\ 3x \quad\quad + 4z &= -11, &(1)\\ 4y - z &= -10 &(3)\end{aligned}$

Multiply equation (2) by $-3$ and add it to equation (1).

$\begin{aligned} x - 2y \quad\quad &= 5, &(2)\\ 6y + 4z &= -26, &(4)\\ 4y - z &= -10 &(3)\end{aligned}$

Multiply equation (3) by 3 to make the $y$-coefficient a multiple of the $y$-coefficient in equation (4).

$\begin{aligned} x - 2y \quad\quad &= 5, &(2)\\ 6y + 4z &= -26, &(4)\\ 12y - 3z &= -30 &(5)\end{aligned}$

Multiply equation (4) by $-2$ and add it to equation (5).

$\begin{aligned} x - 2y \quad\quad &= 5, &(2)\\ 6y + 4z &= -26, &(4)\\ -11z &= 22 &(6)\end{aligned}$

Solve equation (6) for $z$.

$$\begin{aligned} -11z &= 22\\ z &= -2\end{aligned}$$

Back-substitute $-2$ for $z$ in equation (4) and solve for $y$.

$$\begin{aligned} 6y + 4z &= -26\\ 6y + 4(-2) &= -26\\ 6y - 8 &= -26\\ 6y &= -18\\ y &= -3\end{aligned}$$

Back-substitute $-3$ for $y$ in equation (2) and solve for $x$.

$$\begin{aligned} x - 2y &= 5\\ x - 2(-3) &= 5\\ x + 6 &= 5\\ x &= -1\end{aligned}$$

The solution is $(-1, -3, -2)$.

**15.** $\begin{aligned} w + x + y + z &= 2 &(1)\\ w + 2x + 2y + 4z &= 1 &(2)\\ -w + x - y - z &= -6 &(3)\\ -w + 3x + y - z &= -2 &(4)\end{aligned}$

Multiply equation (1) by $-1$ and add to equation (2). Add equation (1) to equation (3) and to equation (4).

$\begin{aligned} w + x + y + z &= 2 &(1)\\ x + y + 3z &= -1 &(5)\\ 2x \quad\quad\quad &= -4 &(6)\\ 4x + 2y \quad\quad &= 0 &(7)\end{aligned}$

Solve equation (6) for $x$.

$$\begin{aligned} 2x &= -4\\ x &= -2\end{aligned}$$

Back-substitute $-2$ for $x$ in equation (7) and solve for $y$.

$$\begin{aligned} 4(-2) + 2y &= 0\\ -8 + 2y &= 0\\ 2y &= 8\\ y &= 4\end{aligned}$$

Back-substitute $-2$ for $x$ and 4 for $y$ in equation (5) and solve for $z$.

$$\begin{aligned} -2 + 4 + 3z &= -1\\ 3z &= -3\\ z &= -1\end{aligned}$$

Back-substitute $-2$ for $x$, 4 for $y$, and $-1$ for $z$ in equation (1) and solve for $w$.

$$w - 2 + 4 - 1 = 2$$
$$w = 1$$

The solution is $(1, -2, 4, -1)$.

**16.**
$$\begin{aligned} w + x - y + z &= 0, && (1)\\ -w + 2x + 2y + z &= 5, && (2)\\ -w + 3x + y - z &= -4, && (3)\\ -2w + x + y - 3z &= -7 && (4) \end{aligned}$$

Add equation (1) to equation (2) and equation (3). Also, multiply equation (1) by 2 and add it to equation (4).

$$\begin{aligned} w + x - y + z &= 0, && (1)\\ 3x + y + 2z &= 5, && (5)\\ 4x &= -4, && (6)\\ 3x - y - z &= -7 && (7) \end{aligned}$$

Multiply equation (6) by 3.

$$\begin{aligned} w + x - y + z &= 0, && (1)\\ 3x + y + 2z &= 5, && (5)\\ 12x &= -12, && (8)\\ 3x - y - z &= -7 && (7) \end{aligned}$$

Multiply equation (5) by $-4$ and add it to equation (8). Also, multiply equation (5) by $-1$ and add it to equation (7).

$$\begin{aligned} w + x - y + z &= 0, && (1)\\ 3x + y + 2z &= 5, && (5)\\ -4y - 8z &= -32, && (9)\\ -2y - 3z &= -12 && (10) \end{aligned}$$

Multiply equation (10) by 2.

$$\begin{aligned} w + x - y + z &= 0, && (1)\\ 3x + y + 2z &= 5, && (5)\\ -4y - 8z &= -32, && (9)\\ -4y - 6z &= -24 && (11) \end{aligned}$$

Multiply equation (9) by $-1$ and add it to equation (11).

$$\begin{aligned} w + x - y + z &= 0, && (1)\\ 3x + y + 2z &= 5, && (5)\\ -4y - 8z &= -32, && (9)\\ 2z &= 8 && (12) \end{aligned}$$

Complete the solution.

$$2z = 8$$
$$z = 4$$
$$-4y - 8 \cdot 4 = -32$$
$$-4y = 0$$
$$y = 0$$
$$3x + 0 + 2 \cdot 4 = 5$$
$$3x = -3$$
$$x = -1$$
$$w - 1 - 0 + 4 = 0$$
$$w = -3$$

The solution is $(-3, -1, 0, 4)$.

**17.** ***Familiarize***. Let $x$, $y$, and $z$ represent the number of Winter Olympics sites in North America, Europe, and Asia, respectively.

***Translate***. The total number of sites is 21.

$$x + y + z = 21$$

The number of European sites is 5 more than the total number of sites in North America and Asia.

$$y = x + z + 5$$

There are 4 more sites in North America than in Asia.

$$x = z + 4$$

We have

$$\begin{aligned} x + y + z &= 21,\\ y &= x + z + 5,\\ x &= z + 4 \end{aligned}$$

or

$$\begin{aligned} x + y + z &= 21,\\ -x + y - z &= 5,\\ x - z &= 4. \end{aligned}$$

***Carry out***. Solving the system of equations, we get (6, 13, 2).

***Check***. The total number of sites is $6+13+2$, or 21. The total number of sites in North America and Asia is $6+2$, or 8, and 5 more than this is $8+5$, or 13, the number of sites in Europe. Also, the number of sites in North America, 6, is 4 more than 2, the number of sites in Asia. The answer checks.

***State***. The Winter Olympics have been held in 6 North American sites, 13 European sites, and 2 Asian sites.

**18.** Let $x$, $y$, and $z$ represent the number of billions of pounds of apples prduced in China, the United States, and Turkey, respectively.

Solve: $x + y + z = 74,$
$x = y + z + 44,$
$y = 2z.$

$x = 59$, $y = 10$, $z = 5$

**19.** ***Familiarize***. Let $x$, $y$, and $z$ represent the number of restaurant-purchased meals that will be eaten in a restaurant, in a car, and at home, respectively.

***Translate***. The total number of meals is 170.

$$x + y + z = 170$$

The total number of restaurant-purchased meals eaten in a car or at home is 14 more than the number eaten in a restaurant.

$$y + z = x + 14$$

Twenty more restaurant-purchased meals will be eaten in a restaurant than at home.

$$x = z + 20$$

We have

$$x+y+z=170,$$
$$y+z=x+14,$$
$$x=z+20$$

or

$$\begin{aligned} x+y+z&=170,\\ -x+y+z&=14,\\ x\quad\ -z&=20.\end{aligned}$$

***Carry out***. Solving the system of equations, we get (78, 34, 58).

***Check***. The total number of meals is $78+34+58$, or 170. The total number of meals eaten in a car or at home is $34+58$, or 92. This is 14 more than 78, the number of meals eaten in a restaurant. Finally, 20 more than the number of restaurant-purchased meals eaten at home, 58, is 78, the number of meals eaten in a restaurant. The answer checks.

***State***. The number of restaurant-purchased meals eaten in a restaurant, in a car, and at home is 78, 34, and 58, respectively.

**20.** Let $x$, $y$, and $z$ represent the number of children adopted from China, Ethiopia, and Russia, respectively.

Solve: $x+y+z=6864$,
$x=y+z-862$,
$2z=x+171$.

$x=3001$, $y=2277$, $z=1586$

**21.** ***Familiarize***. Let $x$ $y$, and $z$ represent the number of milligrams of caffeine in an 8-oz serving of brewed coffee, Red Bull energy drink, and Mountain Dew, respectively.

***Translate***. The total amount of caffeine in one serving of each beverage is 197 mg.

$$x+y+z=197$$

One serving of brewed coffee has 6 mg more caffeine than two servings of Mountain Dew.

$$x=2z+6$$

One serving of Red Bull contains 37 mg less caffeine than one serving each of brewed coffee and Mountain Dew.

$$y=x+z-37$$

We have a system of equations

$$\begin{aligned} x+y+z&=197, \\ x&=2z+6, \\ y&=x+z-37 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x+y+\ z&=197,\\ x\quad -2z&=6,\\ -x+y-\ z&=-37\end{aligned}$$

***Carry out***. Solving the system of equations, we get $(80, 80, 37)$.

***Check***. The total amount of caffeine is $80+80+37$, or 197 mg. Also, 80 mg is 6 mg more than twice 37 mg, and 80 mg is 37 mg less than the total of 80 mg and 37 mg, or 117 mg. The answer checks.

***State***. One serving each of brewed coffee, Red Bull energy drink, and Mountain Dew contains 80 mg, 80 mg, and 37 mg of caffeine, respectively.

**22.** Let $x$, $y$, and $z$ represent the average amount spent on flowers, jewelry, and gift certificates for Mother's Day, respectively.

Solve: $x+y+z=53.42$,
$y=z+4.40$,
$x+z=y+15.58$

$x=\$19.98$, $y=\$18.92$, $z=\$14.52$

**23.** ***Familiarize***. Let $x$, $y$, and $z$ represent the number of fish, cats, and dogs owned by Americans, in millions, respectively.

***Translate***. The total number of fish, cats, and dogs owned is 355 million.

$$x+y+z=355$$

The number of fish owned is 11 million more than the total number of cats and dogs owned.

$$x=y+z+11$$

There are 16 million more cats than dogs.

$$y=z+16$$

We have

$$x+y+z=355,$$
$$x=y+z+11,$$
$$y=z+16$$

or

$$\begin{aligned} x+y+z&=355,\\ x-y-z&=11,\\ y-z&=16.\end{aligned}$$

***Carry out***. Solving the system of equations, we get (183, 94, 78).

***Check***. The total number of fish, cats, and dogs is $183+94+78$, or 355 million. The total number of cats and dogs owned is $94+78$, or 172 million, and 172 million + 11 million is 183 million, the number of fish owned. The number of cats owned, 94 million, is 16 million more than 78 million, the number of dogs owned. The solution checks.

***State***. Americans own 183 million fish, 94 million cats, and 78 million dogs.

**24.** Let $x$, $y$, and $z$ represent the number of orders of \$25 or less, from \$25.01 to \$75, and over \$75, respectively.

Solve: $x+y+z=600$,
$4x+6y+7z=3340$,
$x=z+80$.

$x=180$, $y=320$, $z=100$

**25.** ***Familiarize***. Let $x$, $y$, and $z$ represent the amounts earned by *The Dark Knight*, *Spider-Man 3*, and *The Twilight Saga: New Moon*, respectively, in millions of dollars.

***Translate***. Total earnings are \$452 million.

$$x+y+z=452$$

Together, *Spider-Man 3* and *New Moon* earned \$136 million more than *The Dark Knight*.

$$y+z=x+136$$

*New Moon* earned \$15 million less than *The Dark Knight*.

$z = x - 15$

We have

$$x + y + z = 452,$$
$$y + z = x + 136,$$
$$z = x - 15$$

or

$$\begin{aligned} x + y + z &= 452, \\ -x + y + z &= 136, \\ -x \quad + z &= -15. \end{aligned}$$

***Carry out***. Solving the system of equations, we get (158, 151, 143).

***Check***. The earnings total \$158+\$151+\$143, or \$452 million. Together, *Spider-Man 3* and *New Moon* earned \$151 + \$143, or \$294 million. This is \$136 million more than \$158 million, the earnings of *The Dark Knight*. Also, \$15 million less than \$158 million, the earnings of *The Dark Knight* is \$158 − \$15 or \$143 million, the earnings of *New Moon*. The answer checks.

***State***. *The Dark Knight*, *Spider-Man 3*, and *New Moon* grossed \$158 million, \$151 million, and \$143 million, respectively, in a weekend.

**26.** Let $x$, $y$, and $z$ represent the number of adults who do spring housecleaning in March, April, and May, respectively.

Solve: $x + y + z = 70,$

$y = x + z + 14,$

$y + z = 3x + 2$

$x = 17$, $y = 42$, $z = 11$

**27.** ***Familiarize***. Let $x$, $y$, and $z$ represent the number of servings of ground beef, baked potato, and strawberries required, respectively. One serving of ground beef contains $245x$ Cal, $0x$ or 0 g of carbohydrates, and $9x$ mg of calcium. One baked potato contains $145y$ Cal, $34y$ g of carbohydrates, and $8y$ mg of calcium. One serving of strawberries contains $45z$ Cal, $10z$ g of carbohydrates, and $21z$ mg of calcium.

***Translate***.

The total number of calories is 485.

$245x + 145y + 45z = 485$

A total of 41.5 g of carbohydrates is required.

$34y + 10z = 41.5$

A total of 35 mg of calcium is required.

$9x + 8y + 21z = 35$

We have a system of equations.

$$\begin{aligned} 245x + 145y + 45z &= 485, \\ 34y + 10z &= 41.5, \\ 9x + \quad 8y + 21z &= 35 \end{aligned}$$

***Carry out***. Solving the system of equations, we get $(1.25, 1, 0.75)$.

***Check***. 1.25 servings of ground beef contains 306.25 Cal, no carbohydrates, and 11.25 mg of calcium; 1 baked potato contains 145 Cal, 34 g of carbohydrates, and 8 mg of calcium; 0.75 servings of strawberries contains 33.75 Cal, 7.5 g of carbohydrates, and 15.75 mg of calcium. Thus, there are a total of $306.25 + 145 + 33.75$, or 485 Cal, $34 + 7.5$, or 41.5 g of carbohydrates, and $11.25 + 8 + 15.75$, or 35 mg of calcium. The solution checks.

***State***. 1.25 servings of ground beef, 1 baked potato, and 0.75 serving of strawberries are required.

**28.** Let $x$, $y$, and $z$ represent the number of servings of chicken, mashed potatoes, and peas to be used, respectively.

Solve: $140x + 160y + 125z = 415,$

$27x + 4y + 8z = 50.5,$

$64x + 636y + 139z = 553.$

$x = 1.5$, $y = 0.5$, $z = 1$

**29.** ***Familiarize***. Let $x$, $y$, and $z$ represent the amounts invested at 3%, 4%, and 6%, respectively. Then the annual interest from the investments is $3\%x$, $4\%y$, and $6\%z$, or $0.03x$, $0.04y$, and $0.06z$.

***Translate***.

A total of \$5000 was invested.

$x + y + z = 5000$

The total interest is \$243.

$0.03x + 0.04y + 0.06z = 243$

The amount invested at 6% is \$1500 more than the amount invested at 3%.

$z = x + 1500$

We have a system of equations.

$$x + y + z = 5000,$$
$$0.03x + 0.04y + 0.06z = 243,$$
$$z = x + 1500$$

or

$$\begin{aligned} x + \ y + \ z &= 5000, \\ 3x + 4y + 6z &= 24,300, \\ -x \qquad + \ z &= 1500 \end{aligned}$$

***Carry out***. Solving the system of equations, we get $(1300, 900, 2800)$.

***Check***. The total investment was \$1300+\$900+\$2800, or \$5000. The total interest was $0.03(\$1300) + 0.04(\$900) + 0.06(\$2800) = \$39 + \$36 + \$168$, or \$243. The amount invested at 6%, \$2800, is \$1500 more than the amount invested at 3%, \$1300. The solution checks.

***State***. \$1300 was invested at 3%, \$900 at 4%, and \$2800 at 6%.

**30.** Let $x$, $y$, and $z$ represent the amounts invested at 2%, 3%, and 4%, respectively.

Solve: $0.02x + 0.03y + 0.04z = 126,$

$y = x + 500,$

$z = 3y.$

$x = \$300$, $y = \$800$, $z = \$2400$

**31.** ***Familiarize***. Let $x$, $y$, and $z$ represent the prices of orange juice, a raisin bagel, and a cup of coffee, respectively. The new price for orange juice is $x + 25\%x$, or $x + 0.25x$, or $1.25x$; the new price of a bagel is $y + 20\%y$, or $y + 0.2y$, or $1.2y$.

***Translate***.

Orange juice, a raisin bagel, and a cup of coffee cost \$5.35.

$$x + y + z = 5.35$$

After the price increase, orange juice, a raisin bagel, and a cup of coffee will cost \$6.20.

$$1.25x + 1.2y + z = 6.20$$

After the price increases, orange juice will cost 50¢ (or \$0.50) more than coffee.

$$1.25x = z + 0.50$$

We have a system of equations.

$$\begin{aligned} x + y + z &= 5.35, & 100x + 100y + 100z &= 535,\\ 1.25x + 1.2y + z &= 6.20, \text{ or } & 125x + 120y + 100z &= 620,\\ 1.25x &= z + 0.50 & 125x \qquad - 100z &= 50 \end{aligned}$$

***Carry out***. Solving the system of equations, we get $(1.6, 2.25, 1.5)$.

***Check***. If orange juice costs \$1.60, a bagel costs \$2.25, and a cup of coffee costs \$1.50, then together they cost \$1.60 + \$2.25 + \$1.50, or \$5.35. After the price increases orange juice will cost 1.25(\$1.60), or \$2 and a bagel will cost 1.2(\$2.25) or \$2.70. Then orange juice, a bagel, and coffee will cost \$2 + \$2.70 + \$1.50, or \$6.20. After the price increase the price of orange juice, \$2, will be 50¢ more than the price of coffee, \$1.50. The solution checks.

***State***. Before the increase orange juice cost \$1.60, a raisin bagel cost \$2.25, and a cup of coffee cost \$1.50.

**32.** Let $x$, $y$, and $z$ represent the prices of a carton of milk, a donut, and a cup of coffee, respectively.

Solve: $x + 2y + z = 6.75,$

$$3y + 2z = 8.50,$$

$$x + y + 2z = 7.25.$$

$x = \$1.75$, $y = \$1.50$, $z = \$2$

Then 2 cartons of milk and 2 donuts will cost 2(\$1.75) + 2(\$1.50), or \$6.50. They will not have enough money. They need 5¢ more.

**33.** a) Substitute the data points $(0, 16)$, $(7, 9)$, and $(20, 21)$ in the function $f(x) = ax^2 + bx + c$.

$$16 = a \cdot 0^2 + b \cdot 0 + c$$
$$9 = a \cdot 7^2 + b \cdot 7 + c$$
$$21 = a \cdot 20^2 + b \cdot 20 + c$$

We have a system of equations.

$$\begin{aligned} c &= 16,\\ 49a + 7b + c &= 9,\\ 400a + 20b + c &= 21. \end{aligned}$$

Solving the system of equations, we get $\left(\frac{5}{52}, -\frac{87}{52}, 16\right)$ so $f(x) = \frac{5}{52}x^2 - \frac{87}{52}x + 16$, where $x$ is the number of years after 1990 and $f(x)$ is a percent.

b) In 2003, $x = 2003 - 1990$, or 13.

$$f(13) = \frac{5}{52} \cdot 13^2 - \frac{87}{52} \cdot 13 + 16 = 10.5\%$$

**34.** a) Substituting the data points $(0, 15.1)$, $(2, 20.5)$, and $(4, 11.0)$ in the function $f(x) = ax^2 + bx + c$, we get a system of equations.

$$\begin{aligned} c &= 15.1,\\ 4a + 2b + c &= 20.5,\\ 16a + 4b + c &= 11.0 \end{aligned}$$

Solving the system of equations we get $(-1.8625, 6.425, 15.1)$, or $\left(-\frac{149}{80}, \frac{257}{40}, \frac{151}{10}\right)$, so $f(x) = -1.8625x^2 + 6.425x + 15.1$, or $f(x) = -\frac{149}{80}x^2 + \frac{257}{40}x + \frac{151}{10}$, where $x$ is the number of years after 2004 and $f(x)$ is in billions of dollars.

b) In 2007, $x = 2007 - 2004$, or 3.

$$f(3) = -1.8625(3)^2 + 6.425(3) + 15.1 \approx \$17.6 \text{ billion}$$

**35.** a) Substitute the data points $(0, 431)$, $(5, 441)$, and $(10, 418)$ in the function $f(x) = ax^2 + bx + c$.

$$431 = a \cdot 0^2 + b \cdot 0 + c$$
$$441 = a \cdot 5^2 + b \cdot 5 + c$$
$$418 = a \cdot 10^2 + b \cdot 10 + c$$

We have a system of equations.

$$\begin{aligned} c &= 431,\\ 25a + 5b + c &= 441,\\ 100a + 10b + c &= 418 \end{aligned}$$

Solving the system of equations, we get $(-0.66, 5.3, 431)$, or $\left(-\frac{33}{50}, \frac{53}{10}, 431\right)$, so

$f(x) = -0.66x^2 + 5.3x + 431$, or $-\frac{33}{50}x^2 + \frac{53}{10}x + 431$, where $x$ is the number of years after 1997.

b) In 2009, $x = 2009 - 1997$, or 12.

$$f(12) = -0.66(12)^2 + 5.3(12) + 431 \approx 400 \text{ acres}$$

**36.** a) Substituting the data points $(0, 3.127)$, $(1, 3.079)$, and $(2, 3.101)$ in the function $f(x) = ax^2 + bx + c$, we get a system of equations.

$$\begin{aligned} c &= 3.127,\\ a + b + c &= 3.079,\\ 4a + 2b + c &= 3.101 \end{aligned}$$

Solving the system of equations we get $(0.035, -0.083, 3.127)$, or $\left(\frac{7}{200}, -\frac{83}{1000}, 3.127\right)$, so $f(x) = 0.035x^2 - 0.083x + 3.127$, or $f(x) = \frac{7}{200}x^2 - \frac{83}{1000}x + 3.127$, where $x$ is the number of years after 2007 and $f(x)$ is in billions.

b) In 2012, $x = 2012 - 2007$, or 5.
$f(5) = 0.035(5)^2 - 0.083(5) + 3.127 = 3.587$ billion books

**37.** Perpendicular

**38.** The leading-term test

**39.** A vertical line

**40.** A one-to-one function

**41.** A rational function

**42.** Inverse variation

**43.** A vertical asymptote

**44.** A horizontal asymptote

**45.** $\frac{2}{x} - \frac{1}{y} - \frac{3}{z} = -1,$
$\frac{2}{x} - \frac{1}{y} + \frac{1}{z} = -9,$
$\frac{1}{x} + \frac{2}{y} - \frac{4}{z} = 17$

First substitute $u$ for $\frac{1}{x}$, $v$ for $\frac{1}{y}$, and $w$ for $\frac{1}{z}$ and solve for $u$, $v$, and $w$.

$2u - v - 3w = -1,$
$2u - v + w = -9,$
$u + 2v - 4w = 17$

Solving this system we get $(-1, 5, -2)$.

If $u = -1$, and $u = \frac{1}{x}$, then $-1 = \frac{1}{x}$, or $x = -1$.

If $v = 5$ and $v = \frac{1}{y}$, then $5 = \frac{1}{y}$, or $y = \frac{1}{5}$.

If $w = -2$ and $w = \frac{1}{z}$, then $-2 = \frac{1}{z}$, or $z = -\frac{1}{2}$.

The solution of the original system is $\left(-1, \frac{1}{5}, -\frac{1}{2}\right)$.

**46.** $\frac{2}{x} + \frac{2}{y} - \frac{3}{z} = 3,$
$\frac{1}{x} - \frac{2}{y} - \frac{3}{z} = 9,$
$\frac{7}{x} - \frac{2}{y} + \frac{9}{z} = -39$

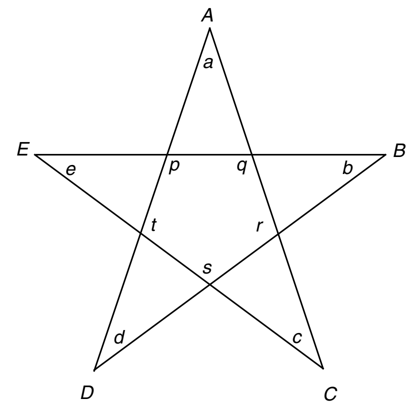
Substitute $u$ for $\frac{1}{x}$, $v$ for $\frac{1}{y}$, and $w$ for $\frac{1}{z}$ and solve for $u$, $v$, and $w$.

$2u + 2v - 3w = 3,$
$u - 2v - 3w = 9,$
$7u - 2v + 9w = -39$

Solving this system we get $(-2, -1, -3)$.

Then $\frac{1}{x} = -2$, or $x = -\frac{1}{2}$; $\frac{1}{y} = -1$, or $y = -1$; and $\frac{1}{z} = -3$, or $z = -\frac{1}{3}$. The solution of the original system is $\left(-\frac{1}{2}, -1, -\frac{1}{3}\right)$.

**47.** Label the angle measures at the tips of the stars $a$, $b$, $c$, $d$, and $e$. Also label the angles of the pentagon $p$, $q$, $r$, $s$, and $t$.




Using the geometric fact that the sum of the angle measures of a triangle is 180°, we get 5 equations.

$p + b + d = 180$
$q + c + e = 180$
$r + a + d = 180$
$s + b + e = 180$
$t + a + c = 180$

Adding these equations, we get

$(p + q + r + s + t) + 2a + 2b + 2c + 2d + 2e = 5(180).$

The sum of the angle measures of any convex polygon with $n$ sides is given by the formula $S = (n - 2)180$. Thus $p + q + r + s + t = (5 - 2)180$, or 540. We substitute and solve for $a + b + c + d + e$.

$540 + 2(a + b + c + d + e) = 900$
$2(a + b + c + d + e) = 360$
$a + b + c + d + e = 180$

The sum of the angle measures at the tips of the star is 180°.

**48.** Let $h$, $t$, and $u$ represent the hundred's, ten's, and unit's digit of the year, respectively. (We know the thousand's digit is 1.) Using the given information we know the following:

$1 + h + t + u = 24,$
$u = 1 + h,$
$t = k \cdot 3$ and $u = m \cdot 3$
where $k$, $m$ are positive integers.

We know $h > 5$ (there was no transcontinental railroad before 1600); and, since

$u = 1 + h = m \cdot 3$, $h \neq 6$, $h \neq 7$, $h \neq 9$.

Thus $h = 8$ and $u = 9$. Then $1 + 8 + t + 9 = 24$, or $t = 6$.

The year is 1869.

**49.** Substituting, we get

$$\begin{aligned} A + \frac{3}{4}B + 3C &= 12, \\ \frac{4}{3}A + \quad B + 2C &= 12, \\ 2A + \quad B + \quad C &= 12, \text{ or} \end{aligned}$$

$$\begin{aligned} 4A + 3B + 12C &= 48, \\ 4A + 3B + \ \ 6C &= 36, \quad \text{Clearing fractions} \\ 2A + \ \ B + \quad C &= 12. \end{aligned}$$

Solving the system of equations, we get $(3, 4, 2)$. The equation is $3x + 4y + 2z = 12$.

**50.** Solve: $1 = B - M - 2N$,

$$\begin{aligned} 2 &= B - 3M + 6N, \\ 1 &= B - \frac{3}{2}M - N \end{aligned}$$

$B = 2$, $M = \frac{1}{2}$, $N = \frac{1}{4}$, so $y = 2 - \frac{1}{2}x - \frac{1}{4}z$.

**51.** Substituting, we get

$$\begin{aligned} 59 &= a(-2)^3 + b(-2)^2 + c(-2) + d, \\ 13 &= a(-1)^3 + b(-1)^2 + c(-1) + d, \\ -1 &= a \cdot 1^3 + b \cdot 1^2 + c \cdot 1 + d, \\ -17 &= a \cdot 2^3 + b \cdot 2^2 + c \cdot 2 + d, \text{ or} \end{aligned}$$

$$\begin{aligned} -8a + 4b - 2c + d &= 59, \\ -a + \ b - \ c + d &= 13, \\ a + \ b + \ c + d &= -1, \\ 8a + 4b + 2c + d &= -17. \end{aligned}$$

Solving the system of equations, we get

$(-4, 5, -3, 1)$, so $y = -4x^3 + 5x^2 - 3x + 1$.

**52.** Solve: $-39 = a(-2)^3 + b(-2)^2 + c(-2) + d$,

$$\begin{aligned} -12 &= a(-1)^3 + b(-1)^2 + c(-1) + d, \\ -6 &= a \cdot 1^3 + b \cdot 1^2 + c \cdot 1 + d, \\ 16 &= a \cdot 3^3 + b \cdot 3^2 + c \cdot 3 + d, \text{ or} \end{aligned}$$

$$\begin{aligned} -8a + 4b - 2c + d &= -39, \\ -a + \ b - \ c + d &= -12, \\ a + \ b + \ c + d &= -6, \\ 27a + 9b + 3c + d &= 16. \end{aligned}$$

$a = 2$, $b = -4$, $c = 1$, $d = -5$, so $y = 2x^3 - 4x^2 + x - 5$.

**53.** ***Familiarize and Translate***. Let $a$, $s$, and $c$ represent the number of adults, students, and children in attendance, respectively.

The total attendance was 100.

$$a + s + c = 100$$

The total amount of money taken in was \$100.

(Express 50 cents as $\frac{1}{2}$ dollar.)

$$10a + 3s + \frac{1}{2}c = 100$$

The resulting system is

$$\begin{aligned} a + \ s + \ c &= 100, \\ 10a + 3s + \frac{1}{2}c &= 100. \end{aligned}$$

***Carry out***. Multiply the first equation by $-3$ and add it to the second equation to obtain

$7a - \frac{5}{2}c = -200$ or $a = \frac{5}{14}(c - 80)$ where $(c-80)$ is a positive multiple of 14 (because $a$ must be a positive integer). That is $(c - 80) = k \cdot 14$ or $c = 80 + k \cdot 14$, where $k$ is a positive integer. If $k > 1$, then $c > 100$. This is impossible since the total attendance is 100. Thus $k = 1$, so $c = 80 + 1 \cdot 14 = 94$. Then $a = \frac{5}{14}(94 - 80) = \frac{5}{14} \cdot 14 = 5$, and $5 + s + 94 = 100$, or $s = 1$.

***Check***. The total attendance is $5 + 1 + 94$, or 100.

The total amount of money taken in was

$\$10 \cdot 5 + \$3 \cdot 1 + \$\frac{1}{2} \cdot 94 = \$100$. The result checks.

***State***. There were 5 adults, 1 student, and 94 children in attendance.

## Exercise Set 9.3

**1.** The matrix has 3 rows and 2 columns, so its order is $3 \times 2$.

**2.** The matrix has 4 rows and 1 column, so its order is $4 \times 1$.

**3.** The matrix has 1 row and 4 columns, so its order is $1 \times 4$.

**4.** The matrix has 1 row and 1 column, so its order is $1 \times 1$.

**5.** The matrix has 3 rows and 3 columns, so its order is $3 \times 3$.

**6.** The matrix has 2 rows and 4 columns, so its order is $2 \times 4$.

**7.** We omit the variables and replace the equals signs with a vertical line.

$$\left[\begin{array}{rr|r} 2 & -1 & 7 \\ 1 & 4 & -5 \end{array}\right]$$

**8.** $\left[\begin{array}{rr|r} 3 & 2 & 8 \\ 2 & -3 & 15 \end{array}\right]$

**9.** We omit the variables, writing zeros for the missing terms, and replace the equals signs with a vertical line.

$$\left[\begin{array}{rrr|r} 1 & -2 & 3 & 12 \\ 2 & 0 & -4 & 8 \\ 0 & 3 & 1 & 7 \end{array}\right]$$

**10.** $\left[\begin{array}{rrr|r} 1 & 1 & -1 & 7 \\ 0 & 3 & 2 & 1 \\ -2 & -5 & 0 & 6 \end{array}\right]$

**11.** Insert variables and replace the vertical line with equals signs.

$$\begin{aligned} 3x - 5y &= 1, \\ x + 4y &= -2 \end{aligned}$$

**12.** $x + 2y = -6,$
$4x + y = -3$

**13.** Insert variables and replace the vertical line with equals signs.

$2x + y - 4z = 12,$
$3x + 5z = -1,$
$x - y + z = 2$

**14.** $-x - 2y + 3z = 6,$
$4y + z = 2,$
$2x - y = 9$

**15.** $4x + 2y = 11,$
$3x - y = 2$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rr|r} 4 & 2 & 11 \\ 3 & -1 & 2 \end{array}\right]$$

Multiply row 2 by 4 to make the first number in row 2 a multiple of 4.

$$\left[\begin{array}{rr|r} 4 & 2 & 11 \\ 12 & -4 & 8 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{rr|r} 4 & 2 & 11 \\ 0 & -10 & -25 \end{array}\right]$$

Multiply row 1 by $\frac{1}{4}$ and row 2 by $-\frac{1}{10}$.

$$\left[\begin{array}{rr|r} 1 & \frac{1}{2} & \frac{11}{4} \\ 0 & 1 & \frac{5}{2} \end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$x + \frac{1}{2}y = \frac{11}{4}, \quad (1)$$
$$y = \frac{5}{2} \quad (2)$$

Back-substitute in equation (1) and solve for $x$.

$$x + \frac{1}{2} \cdot \frac{5}{2} = \frac{11}{4}$$
$$x + \frac{5}{4} = \frac{11}{4}$$
$$x = \frac{6}{4} = \frac{3}{2}$$

The solution is $\left(\frac{3}{2}, \frac{5}{2}\right)$.

**16.** $2x + y = 1,$
$3x + 2y = -2$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rr|r} 2 & 1 & 1 \\ 3 & 2 & -2 \end{array}\right]$$

Multiply row 2 by 2.

$$\left[\begin{array}{rr|r} 2 & 1 & 1 \\ 6 & 4 & -4 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{rr|r} 2 & 1 & 1 \\ 0 & 1 & -7 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1.

$$\left[\begin{array}{rr|r} 2 & 0 & 8 \\ 0 & 1 & -7 \end{array}\right]$$

Multiply row 1 by $\frac{1}{2}$.

$$\left[\begin{array}{rr|r} 1 & 0 & 4 \\ 0 & 1 & -7 \end{array}\right]$$

The solution is $(4, -7)$.

**17.** $5x - 2y = -3,$
$2x + 5y = -24$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rr|r} 5 & -2 & -3 \\ 2 & 5 & -24 \end{array}\right]$$

Multiply row 2 by 5 to make the first number in row 2 a multiple of 5.

$$\left[\begin{array}{rr|r} 5 & -2 & -3 \\ 10 & 25 & -120 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

$$\left[\begin{array}{rr|r} 5 & -2 & -3 \\ 0 & 29 & -114 \end{array}\right]$$

Multiply row 1 by $\frac{1}{5}$ and row 2 by $\frac{1}{29}$.

$$\left[\begin{array}{rr|r} 1 & -\frac{2}{5} & -\frac{3}{5} \\ 0 & 1 & -\frac{114}{29} \end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$x - \frac{2}{5}y = -\frac{3}{5}, \quad (1)$$
$$y = -\frac{114}{29} \quad (2)$$

Back-substitute in equation (1) and solve for $x$.

$$x - \frac{2}{5}\left(-\frac{114}{29}\right) = -\frac{3}{5}$$
$$x + \frac{228}{145} = -\frac{3}{5}$$
$$x = -\frac{315}{145} = -\frac{63}{29}$$

The solution is $\left(-\frac{63}{29}, -\frac{114}{29}\right)$.

**18.** $2x + \ \ y = 1,$
$3x - 6y = 4$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rr|r} 2 & 1 & 1 \\ 3 & -6 & 4 \end{array}\right]$$

Multiply row 2 by 2.

$$\left[\begin{array}{rr|r} 2 & 1 & 1 \\ 6 & -12 & 8 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{rr|r} 2 & 1 & 1 \\ 0 & -15 & 5 \end{array}\right]$$

Multiply row 1 by $\frac{1}{2}$ and row 2 by $-\frac{1}{15}$.

$$\left[\begin{array}{rr|r} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} \end{array}\right]$$

We have:

$$x + \frac{1}{2}y = 1, \quad (1)$$
$$y = -\frac{1}{3} \quad (2)$$

Back-substitute in equation (1) and solve for $x$.

$$x + \frac{1}{2}\left(-\frac{1}{3}\right) = \frac{1}{2}$$
$$x = \frac{2}{3}$$

The solution is $\left(\frac{2}{3}, -\frac{1}{3}\right)$.

**19.** $3x + 4y = 7,$
$-5x + 2y = 10$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rr|r} 3 & 4 & 7 \\ -5 & 2 & 10 \end{array}\right]$$

Multiply row 2 by 3 to make the first number in row 2 a multiple of 3.

$$\left[\begin{array}{rr|r} 3 & 4 & 7 \\ -15 & 6 & 30 \end{array}\right]$$

Multiply row 1 by 5 and add it to row 2.

$$\left[\begin{array}{rr|r} 3 & 4 & 7 \\ 0 & 26 & 65 \end{array}\right]$$

Multiply row 1 by $\frac{1}{3}$ and row 2 by $\frac{1}{26}$.

$$\left[\begin{array}{rr|r} 1 & \frac{4}{3} & \frac{7}{3} \\ 0 & 1 & \frac{5}{2} \end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$x + \frac{4}{3}y = \frac{7}{3}, \quad (1)$$
$$y = \frac{5}{2} \quad (2)$$

Back-substitute in equation (1) and solve for $x$.

$$x + \frac{4}{3} \cdot \frac{5}{2} = \frac{7}{3}$$
$$x + \frac{10}{3} = \frac{7}{3}$$
$$x = -\frac{3}{3} = -1$$

The solution is $\left(-1, \frac{5}{2}\right)$.

**20.** $5x - 3y = -2,$
$4x + 2y = 5$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rr|r} 5 & -3 & -2 \\ 4 & 2 & 5 \end{array}\right]$$

Multiply row 2 by 5.

$$\left[\begin{array}{rr|r} 5 & -3 & -2 \\ 20 & 10 & 25 \end{array}\right]$$

Multiply row 1 by $-4$ and add it to row 2.

$$\left[\begin{array}{rr|r} 5 & -3 & -2 \\ 0 & 22 & 33 \end{array}\right]$$

Multiply row 1 by $\dfrac{1}{5}$ and row 2 by $\dfrac{1}{22}$.

$$\left[\begin{array}{rr|r} 1 & -\frac{3}{5} & -\frac{2}{5} \\ 0 & 1 & \frac{3}{2} \end{array}\right]$$

We have:

$$x - \frac{3}{5}y = -\frac{2}{5}, \quad (1)$$
$$y = \frac{3}{2} \qquad (2)$$

Back-substitute in (1) and solve for $x$.

$$x - \frac{3}{5}\cdot\frac{3}{2} = -\frac{2}{5}$$
$$x - \frac{9}{10} = -\frac{2}{5}$$
$$x = \frac{1}{2}$$

The solution is $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$.

**21.** $3x + 2y = 6,$

$2x - 3y = -9$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rr|r} 3 & 2 & 6 \\ 2 & -3 & -9 \end{array}\right]$$

Multiply row 2 by 3 to make the first number in row 2 a multiple of 3.

$$\left[\begin{array}{rr|r} 3 & 2 & 6 \\ 6 & -9 & -27 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

$$\left[\begin{array}{rr|r} 3 & 2 & 6 \\ 0 & -13 & -39 \end{array}\right]$$

Multiply row 2 by $-\dfrac{1}{13}$.

$$\left[\begin{array}{rr|r} 3 & 2 & 6 \\ 0 & 1 & 3 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 1.

$$\left[\begin{array}{rr|r} 3 & 0 & 0 \\ 0 & 1 & 3 \end{array}\right]$$

Multiply row 1 by $\dfrac{1}{3}$.

$$\left[\begin{array}{rr|r} 1 & 0 & 0 \\ 0 & 1 & 3 \end{array}\right]$$

We have $x = 0$, $y = 3$. The solution is $(0, 3)$.

**22.** $x - 4y = 9,$

$2x + 5y = 5$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rr|r} 1 & -4 & 9 \\ 2 & 5 & 5 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

$$\left[\begin{array}{rr|r} 1 & -4 & 9 \\ 0 & 13 & -13 \end{array}\right]$$

Multiply row 2 by $\dfrac{1}{13}$.

$$\left[\begin{array}{rr|r} 1 & -4 & 9 \\ 0 & 1 & -1 \end{array}\right]$$

Multiply row 2 by 4 and add it to row 1.

$$\left[\begin{array}{rr|r} 1 & 0 & 5 \\ 0 & 1 & -1 \end{array}\right]$$

The solution is $(5, -1)$.

**23.** $x - 3y = 8,$

$2x - 6y = 3$

Write the augmented matrix.

$$\left[\begin{array}{rr|r} 1 & -3 & 8 \\ 2 & -6 & 3 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

$$\left[\begin{array}{rr|r} 1 & -3 & 8 \\ 0 & 0 & -13 \end{array}\right]$$

The last row corresponds to the false equation $0 = -13$, so there is no solution.

**24.** $4x - 8y = 12,$

$-x + 2y = -3$

Write the augmented matrix.

$$\left[\begin{array}{rr|r} 4 & -8 & 12 \\ -1 & 2 & -3 \end{array}\right]$$

Interchange the rows.

$$\left[\begin{array}{rr|r} -1 & 2 & -3 \\ 4 & -8 & 12 \end{array}\right]$$

Multiply row 1 by 4 and add it to row 2.

$$\left[\begin{array}{rr|r} -1 & 2 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

The last row corresponds to $0 = 0$, so the system is dependent and equivalent to $-x+2y = -3$. Solving this system for $x$ gives us $x = 2y + 3$. Then the solutions are of the form $(2y + 3, y)$, where $y$ is any real number.

**25.** $-2x + 6y = 4,$

$3x - 9y = -6$

Write the augmented matrix.

$$\left[\begin{array}{rr|r} -2 & 6 & 4 \\ 3 & -9 & -6 \end{array}\right]$$

Multiply row 1 by $-\dfrac{1}{2}$.

$$\left[\begin{array}{rr|r} 1 & -3 & -2 \\ 3 & -9 & -6 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{rr|r} 1 & -3 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

The last row corresponds to the equation $0 = 0$ which is true for all values of $x$ and $y$. Thus, the system of equations is dependent and is equivalent to the first equation $-2x + 6y = 4$, or $x - 3y = -2$. Solving for $x$, we get $x = 3y - 2$. Then the solutions are of the form $(3y - 2, y)$, where $y$ is any real number.

**26.** $6x + 2y = -10,$

$-3x - y = 6$

Write the augmented matrix.

$$\left[\begin{array}{rr|r} 6 & 2 & -10 \\ -3 & -1 & 6 \end{array}\right]$$

Interchange the rows.

$$\left[\begin{array}{rr|r} -3 & -1 & 6 \\ 6 & 2 & -10 \end{array}\right]$$

Multiply row 1 by 2 and add it to row 2.

$$\left[\begin{array}{rr|r} -3 & -1 & 6 \\ 0 & 0 & 2 \end{array}\right]$$

The last row corresponds to the false equation $0 = 2$, so there is no solution.

**27.** $x + 2y - 3z = 9,$

$2x - y + 2z = -8,$

$3x - y - 4z = 3$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrr|r} 1 & 2 & -3 & 9 \\ 2 & -1 & 2 & -8 \\ 3 & -1 & -4 & 3 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 2 & -3 & 9 \\ 0 & -5 & 8 & -26 \\ 0 & -7 & 5 & -24 \end{array}\right]$$

Multiply row 2 by $-\dfrac{1}{5}$ to get a 1 in the second row, second column.

$$\left[\begin{array}{rrr|r} 1 & 2 & -3 & 9 \\ 0 & 1 & -\frac{8}{5} & \frac{26}{5} \\ 0 & -7 & 5 & -24 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 1. Also, multiply row 2 by 7 and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 0 & \frac{1}{5} & -\frac{7}{5} \\ 0 & 1 & -\frac{8}{5} & \frac{26}{5} \\ 0 & 0 & -\frac{31}{5} & \frac{62}{5} \end{array}\right]$$

Multiply row 3 by $-\dfrac{5}{31}$ to get a 1 in the third row, third column.

$$\left[\begin{array}{rrr|r} 1 & 0 & \frac{1}{5} & -\frac{7}{5} \\ 0 & 1 & -\frac{8}{5} & \frac{26}{5} \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Multiply row 3 by $-\dfrac{1}{5}$ and add it to row 1. Also, multiply row 3 by $\dfrac{8}{5}$ and add it to row 2.

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

We have $x = -1$, $y = 2$, $z = -2$. The solution is $(-1, 2, -2)$.

**28.** $x - y + 2z = 0,$
$x - 2y + 3z = -1,$
$2x - 2y + z = -3$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 0 \\ 1 & -2 & 3 & -1 \\ 2 & -2 & 1 & -3 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 2. Also, multiply row 1 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

Multiply row 2 by $-1$.

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

Add row 2 to row 1.

$$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

Multiply row 3 by $-\dfrac{1}{3}$.

$$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Multiply row 3 by $-1$ and add it to row 1. Also, add row 3 to row 2.

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

The solution is $(0, 2, 1)$.

**29.** $4x - y - 3z = 1,$
$8x + y - z = 5,$
$2x + y + 2z = 5$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrr|r} 4 & -1 & -3 & 1 \\ 8 & 1 & -1 & 5 \\ 2 & 1 & 2 & 5 \end{array}\right]$$

First interchange rows 1 and 3 so that each number below the first number in the first row is a multiple of that number.

$$\left[\begin{array}{rrr|r} 2 & 1 & 2 & 5 \\ 8 & 1 & -1 & 5 \\ 4 & -1 & -3 & 1 \end{array}\right]$$

Multiply row 1 by $-4$ and add it to row 2. Also, multiply row 1 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 2 & 1 & 2 & 5 \\ 0 & -3 & -9 & -15 \\ 0 & -3 & -7 & -9 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 2 & 1 & 2 & 5 \\ 0 & -3 & -9 & -15 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

Multiply row 2 by $-\dfrac{1}{3}$ to get a 1 in the second row, second column.

$$\left[\begin{array}{rrr|r} 2 & 1 & 2 & 5 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1.

$$\left[\begin{array}{rrr|r} 2 & 0 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

Multiply row 3 by $\dfrac{1}{2}$ to get a 1 in the third row, third column.

$$\left[\begin{array}{rrr|r} 2 & 0 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Add row 3 to row 1. Also multiply row 3 by $-3$ and add it to row 2.

$$\left[\begin{array}{rrr|r} 2 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Finally, multiply row 1 by $\dfrac{1}{2}$.

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & \frac{3}{2} \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

We have $x = \dfrac{3}{2}$, $y = -4$, $z = 3$. The solution is $\left(\dfrac{3}{2}, -4, 3\right)$.

**30.** $3x + 2y + 2z = 3,$
$x + 2y - z = 5,$
$2x - 4y + z = 0$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrr|r} 3 & 2 & 2 & 3 \\ 1 & 2 & -1 & 5 \\ 2 & -4 & 1 & 0 \end{array}\right]$$

Interchange the first two rows.

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 5 \\ 3 & 2 & 2 & 3 \\ 2 & -4 & 1 & 0 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2. Also, multiply row 1 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 5 \\ 0 & -4 & 5 & -12 \\ 0 & -8 & 3 & -10 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{4}$.

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 5 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & -8 & 3 & -10 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 1. Also, multiply row 2 by 8 and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 0 & \frac{3}{2} & -1 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & 0 & -7 & 14 \end{array}\right]$$

Multiply row 3 by $-\frac{1}{7}$.

$$\left[\begin{array}{rrr|r} 1 & 0 & \frac{3}{2} & -1 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Multiply row 3 by $-\frac{3}{2}$ and add it to row 1. Also, multiply row 3 by $\frac{5}{4}$ and add it to row 2.

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -2 \end{array}\right]$$

The solution is $\left(2, \frac{1}{2}, -2\right)$.

**31.** $x - 2y + 3z = 4,$
$3x + y - z = 0,$
$2x + 3y - 5z = 1$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rrr|r} 1 & -2 & 3 & -4 \\ 3 & 1 & -1 & 0 \\ 2 & 3 & -5 & 1 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2. Also, multiply row 1 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -2 & 3 & -4 \\ 0 & 7 & -10 & 12 \\ 0 & 7 & -11 & 9 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -2 & 3 & -4 \\ 0 & 7 & -10 & 12 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

Multiply row 2 by $\frac{1}{7}$ and multiply row 3 by $-1$.

$$\left[\begin{array}{rrr|r} 1 & -2 & 3 & -4 \\ 0 & 1 & -\frac{10}{7} & \frac{12}{7} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Now write the system of equations that corresponds to the last matrix.

$$\begin{aligned} x - 2y + 3z &= -4, \quad (1) \\ y - \frac{10}{7}z &= \frac{12}{7}, \quad (2) \\ z &= 3 \quad (3) \end{aligned}$$

Back-substitute 3 for $z$ in equation (2) and solve for $y$.

$$\begin{aligned} y - \frac{10}{7}\cdot 3 &= \frac{12}{7} \\ y - \frac{30}{7} &= \frac{12}{7} \\ y &= \frac{42}{7} = 6 \end{aligned}$$

Back-substitute 6 for $y$ and 3 for $z$ in equation (1) and solve for $x$.

$$\begin{aligned} x - 2\cdot 6 + 3\cdot 3 &= -4 \\ x - 3 &= -4 \\ x &= -1 \end{aligned}$$

The solution is $(-1, 6, 3)$.

**32.** $$\begin{aligned} 2x - 3y + 2z &= 2, \\ x + 4y - z &= 9, \\ -3x + y - 5z &= 5 \end{aligned}$$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rrr|r} 2 & -3 & 2 & 2 \\ 1 & 4 & -1 & 9 \\ -3 & 1 & -5 & 5 \end{array}\right]$$

Interchange the first two rows.

$$\left[\begin{array}{rrr|r} 1 & 4 & -1 & 9 \\ 2 & -3 & 2 & 2 \\ -3 & 1 & -5 & 5 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by 3 and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 4 & -1 & 9 \\ 0 & -11 & 4 & -16 \\ 0 & 13 & -8 & 32 \end{array}\right]$$

Multiply row 3 by 11.

$$\left[\begin{array}{rrr|r} 1 & 4 & -1 & 9 \\ 0 & -11 & 4 & -16 \\ 0 & 143 & -88 & 352 \end{array}\right]$$

Multiply row 2 by 13 and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 4 & -1 & 9 \\ 0 & -11 & 4 & -16 \\ 0 & 0 & -36 & 144 \end{array}\right]$$

Multiply row 2 by $-\dfrac{1}{11}$ and multiply row 3 by $-\dfrac{1}{36}$.

$$\left[\begin{array}{rrr|r} 1 & 4 & -1 & 9 \\ 0 & 1 & -\frac{4}{11} & \frac{16}{11} \\ 0 & 0 & 1 & -4 \end{array}\right]$$

We have

$$\begin{aligned} x + 4y - z &= 9, && (1) \\ y - \frac{4}{11}z &= \frac{16}{11}, && (2) \\ z &= -4. && (3) \end{aligned}$$

Back-substitute in (2) to find $y$.

$$\begin{aligned} y - \frac{4}{11}(-4) &= \frac{16}{11} \\ y + \frac{16}{11} &= \frac{16}{11} \\ y &= 0 \end{aligned}$$

Back-substitute in (1) to find $x$.

$$\begin{aligned} x - 4 \cdot 0 - (-4) &= 9 \\ x &= 5 \end{aligned}$$

The solution is $(5, 0, -4)$.

**33.** $$\begin{aligned} 2x - 4y - 3z &= 3, \\ x + 3y + z &= -1, \\ 5x + y - 2z &= 2 \end{aligned}$$

Write the augmented matrix.

$$\left[\begin{array}{rrr|r} 2 & -4 & -3 & 3 \\ 1 & 3 & 1 & -1 \\ 5 & 1 & -2 & 2 \end{array}\right]$$

Interchange the first two rows to get a 1 in the first row, first column.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & -1 \\ 2 & -4 & -3 & 3 \\ 5 & 1 & -2 & 2 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by $-5$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & -1 \\ 0 & -10 & -5 & 5 \\ 0 & -14 & -7 & 7 \end{array}\right]$$

Multiply row 2 by $-\dfrac{1}{10}$ to get a 1 in the second row, second column.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & -1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -14 & -7 & 7 \end{array}\right]$$

Multiply row 2 by 14 and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & -1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The last row corresponds to the equation $0 = 0$. This indicates that the system of equations is dependent. It is equivalent to

$$\begin{aligned} x + 3y + z &= -1, \\ y + \frac{1}{2}z &= -\frac{1}{2} \end{aligned}$$

We solve the second equation for $y$.

$$y = -\frac{1}{2}z - \frac{1}{2}$$

Substitute for $y$ in the first equation and solve for $x$.

$$\begin{aligned} x + 3\left(-\frac{1}{2}z - \frac{1}{2}\right) + z &= -1 \\ x - \frac{3}{2}z - \frac{3}{2} + z &= -1 \\ x &= \frac{1}{2}z + \frac{1}{2} \end{aligned}$$

The solution is $\left(\dfrac{1}{2}z + \dfrac{1}{2}, -\dfrac{1}{2}z - \dfrac{1}{2}, z\right)$, where $z$ is any real number.

**34.** $x + \ \ y - 3z = 4,$

$4x + 5y + \ \ z = 1,$

$2x + 3y + 7z = -7$

Write the augmented matrix.

$$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 4 \\ 4 & 5 & 1 & 1 \\ 2 & 3 & 7 & -7 \end{array}\right]$$

Multiply row 1 by $-4$ and add it to row 2. Also, multiply row 1 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 1 & 13 & -15 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 4 \\ 0 & 1 & 13 & -15 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

We have a dependent system of equations that is equivalent to

$x + y - \ \ 3z = 4,$

$\quad y + 13z = -15.$

Solve the second equation for $y$.

$y = -13z - 15$

Substitute for $y$ in the first equation and solve for $x$.

$x - 13z - 15 - 3z = 4$

$x = 16z + 19$

The solution is $(16z+19, -13z-15, z)$, where $z$ is any real number.

**35.** $p + \ \ q + \ \ r = 1,$

$p + 2q + 3r = 4,$

$4p + 5q + 6r = 7$

Write the augmented matrix.

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 2. Also, multiply row 1 by $-4$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The last row corresponds to the equation $0 = 0$. This indicates that the system of equations is dependent. It is equivalent to

$p + q + \ \ r = 1,$

$\quad q + 2r = 3.$

We solve the second equation for $q$.

$q = -2r + 3$

Substitute for $y$ in the first equation and solve for $p$.

$p - 2r + 3 + r = 1$

$p - r + 3 = 1$

$p = r - 2$

The solution is $(r - 2, -2r + 3, r)$, where $r$ is any real number.

**36.** $m + n + t = 9,$

$m - n - t = -15,$

$3m + n + t = 2$

Write the augmented matrix.

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 9 \\ 1 & -1 & -1 & -15 \\ 3 & 1 & 1 & 2 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 2. Also, multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & -2 & -2 & -25 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 9 \\ 0 & -2 & -2 & -24 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

The last row corresponds to the false equation $0 = -1$. Thus, the system of equations has no solution.

**37.** $a + b - c = 7,$

$a - b + c = 5,$

$3a + b - c = -1$

Write the augmented matrix.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 7 \\ 1 & -1 & 1 & 5 \\ 3 & 1 & -1 & -1 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 2. Also, multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & -2 & 2 & -22 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 7 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & 0 & -20 \end{array}\right]$$

The last row corresponds to the false equation $0 = -20$. Thus, the system of equations has no solution.

**38.** $$\begin{aligned} a - b + \phantom{3}c &= 3, \\ 2a + b - 3c &= 5, \\ 4a + b - \phantom{3}c &= 11 \end{aligned}$$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 3 \\ 2 & 1 & -3 & 5 \\ 4 & 1 & -1 & 11 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by $-4$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 3 \\ 0 & 3 & -5 & -1 \\ 0 & 5 & -5 & -1 \end{array}\right]$$

Multiply row 3 by 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 3 \\ 0 & 3 & -5 & -1 \\ 0 & 15 & -15 & -3 \end{array}\right]$$

Multiply row 2 by $-5$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 3 \\ 0 & 3 & -5 & -1 \\ 0 & 0 & 10 & 2 \end{array}\right]$$

Multiply row 2 by $\frac{1}{3}$ and multiply row 3 by $\frac{1}{10}$.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 3 \\ 0 & 1 & -\frac{5}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right]$$

We have

$$\begin{aligned} x - y + \phantom{\tfrac{5}{3}}z &= 3, && (1) \\ y - \frac{5}{3}z &= -\frac{1}{3}, && (2) \\ z &= \frac{1}{5}. && (3) \end{aligned}$$

Back-substitute in equation (2) to find $y$.

$$\begin{aligned} y - \frac{5}{3} \cdot \frac{1}{5} &= -\frac{1}{3} \\ y &= 0 \end{aligned}$$

Back-substitute in equation (1) to find $x$.

$$\begin{aligned} x - 0 + \frac{1}{5} &= 3 \\ x &= \frac{14}{5} \end{aligned}$$

The solution is $\left(\frac{14}{5}, 0, \frac{1}{5}\right)$.

**39.** $$\begin{aligned} -2w + 2x + 2y - 2z &= -10, \\ w + \phantom{2}x + \phantom{2}y + \phantom{2}z &= -5, \\ 3w + \phantom{2}x - \phantom{2}y + 4z &= -2, \\ w + 3x - 2y + 2z &= -6 \end{aligned}$$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rrrr|r} -2 & 2 & 2 & -2 & -10 \\ 1 & 1 & 1 & 1 & -5 \\ 3 & 1 & -1 & 4 & -2 \\ 1 & 3 & -2 & 2 & -6 \end{array}\right]$$

Interchange rows 1 and 2.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -5 \\ -2 & 2 & 2 & -2 & -10 \\ 3 & 1 & -1 & 4 & -2 \\ 1 & 3 & -2 & 2 & -6 \end{array}\right]$$

Multiply row 1 by 2 and add it to row 2. Multiply row 1 by $-3$ and add it to row 3. Multiply row 1 by $-1$ and add it to row 4.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -5 \\ 0 & 4 & 4 & 0 & -20 \\ 0 & -2 & -4 & 1 & 13 \\ 0 & 2 & -3 & 1 & -1 \end{array}\right]$$

Interchange rows 2 and 3.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -5 \\ 0 & -2 & -4 & 1 & 13 \\ 0 & 4 & 4 & 0 & -20 \\ 0 & 2 & -3 & 1 & -1 \end{array}\right]$$

Multiply row 2 by 2 and add it to row 3. Add row 2 to row 4.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -5 \\ 0 & -2 & -4 & 1 & 13 \\ 0 & 0 & -4 & 2 & 6 \\ 0 & 0 & -7 & 2 & 12 \end{array}\right]$$

Multiply row 4 by 4.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -5 \\ 0 & -2 & -4 & 1 & 13 \\ 0 & 0 & -4 & 2 & 6 \\ 0 & 0 & -28 & 8 & 48 \end{array}\right]$$

Multiply row 3 by $-7$ and add it to row 4.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -5 \\ 0 & -2 & -4 & 1 & 13 \\ 0 & 0 & -4 & 2 & 6 \\ 0 & 0 & 0 & -6 & 6 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{2}$, row 3 by $-\frac{1}{4}$, and row 6 by $-\frac{1}{6}$.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & -5 \\ 0 & 1 & 2 & -\frac{1}{2} & -\frac{13}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$\begin{aligned} w + x + y + z &= -5, && (1) \\ x + 2y - \frac{1}{2}z &= -\frac{13}{2}, && (2) \\ y - \frac{1}{2}z &= -\frac{3}{2}, && (3) \\ z &= -1 && (4) \end{aligned}$$

Back-substitute in equation (3) and solve for $y$.

$$\begin{aligned} y - \frac{1}{2}(-1) &= -\frac{3}{2} \\ y + \frac{1}{2} &= -\frac{3}{2} \\ y &= -2 \end{aligned}$$

Back-substitute in equation (2) and solve for $x$.

$$\begin{aligned} x + 2(-2) - \frac{1}{2}(-1) &= -\frac{13}{2} \\ x - 4 + \frac{1}{2} &= -\frac{13}{2} \\ x &= -3 \end{aligned}$$

Back-substitute in equation (1) and solve for $w$.

$$\begin{aligned} w - 3 - 2 - 1 &= -5 \\ w &= 1 \end{aligned}$$

The solution is $(1, -3, -2, -1)$.

**40.** 
$$\begin{aligned} -w + 2x - 3y + z &= -8, \\ -w + x + y - z &= -4, \\ w + x + y + z &= 22, \\ -w + x - y - z &= -14 \end{aligned}$$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{cccc|c} -1 & 2 & -3 & 1 & -8 \\ -1 & 1 & 1 & -1 & -4 \\ 1 & 1 & 1 & 1 & 22 \\ -1 & 1 & -1 & -1 & -14 \end{array}\right]$$

Multiply row 1 by $-1$.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ -1 & 1 & 1 & -1 & -4 \\ 1 & 1 & 1 & 1 & 22 \\ -1 & 1 & -1 & -1 & -14 \end{array}\right]$$

Add row 1 to row 2 and to row 4. Multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & -1 & 4 & -2 & 4 \\ 0 & 3 & -2 & 2 & 14 \\ 0 & -1 & 2 & -2 & -6 \end{array}\right]$$

Multiply row 2 by $-1$.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & 1 & -4 & 2 & -4 \\ 0 & 3 & -2 & 2 & 14 \\ 0 & -1 & 2 & -2 & -6 \end{array}\right]$$

Multiply row 2 by $-3$ and add it to row 3. Also, add row 2 to row 4.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & 1 & -4 & 2 & -4 \\ 0 & 0 & 10 & -4 & 26 \\ 0 & 0 & -2 & 0 & -10 \end{array}\right]$$

Interchange row 3 and row 4.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & 1 & -4 & 2 & -4 \\ 0 & 0 & -2 & 0 & -10 \\ 0 & 0 & 10 & -4 & 26 \end{array}\right]$$

Multiply row 3 by $-\frac{1}{2}$.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & 1 & -4 & 2 & -4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 10 & -4 & 26 \end{array}\right]$$

Multiply row 3 by $-10$ and add it to row 4.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & 1 & -4 & 2 & -4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & -4 & -24 \end{array}\right]$$

Multiply row 4 by $-\frac{1}{4}$.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 8 \\ 0 & 1 & -4 & 2 & -4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

Add row 4 to row 1. Also, multiply row 4 by $-2$ and add it to row 2.

$$\left[\begin{array}{rrrr|r} 1 & -2 & 3 & 0 & 14 \\ 0 & 1 & -4 & 0 & -16 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

Multiply row 3 by $-3$ and add it to row 1. Also, multiply row 3 by 4 and add it to row 2.

$$\left[\begin{array}{rrrr|r} 1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

Multiply row 2 by 2 and add it to row 1.

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

The solution is $(7, 4, 5, 6)$.

**41.** ***Familiarize***. Let $x$, $y$, and $z$ represent the amounts borrowed at 8%, 10%, and 12%, respectively. Then the annual interest is $8\%x$, $10\%y$, and $12\%z$, or $0.08x$, $0.1y$, and $0.12z$.

***Translate***.

The total amount borrowed was \$30,000.

$$x + y + z = 30,000$$

The total annual interest was \$3040.

$$0.08x + 0.1y + 0.12z = 3040$$

The total amount borrowed at 8% and 10% was twice the amount borrowed at 12%.

$$x + y = 2z$$

We have a system of equations.

$$\begin{aligned} &x + y + z = 30,000, \\ &0.08x + 0.1y + 0.12z = 3040, \\ &x + y = 2z, \text{ or} \end{aligned}$$

$$\begin{aligned} x + \quad y + \quad z &= 30,000, \\ 0.08x + 0.1y + 0.12z &= 3040, \\ x + \quad y - \quad 2z &= 0 \end{aligned}$$

***Carry out***. Using Gaussian elimination or Gauss-Jordan elimination, we find that the solution is $(8000, 12,000, 10,000)$.

***Check***. The total amount borrowed was \$8000+\$12,000+ \$10,000, or \$30,000. The total annual interest was $0.08(\$8000) + 0.1(\$12,000)+$ $0.12(\$10,000)$, or $\$640 + \$1200 + \$1200$, or \$3040. The total amount borrowed at 8% and 10%, $\$8000 + \$12,000$ or \$20,000, was twice the amount borrowed at 12%, \$10,000. The solution checks.

***State***. The amounts borrowed at 8%, 10%, and 12% were \$8000, \$12,000 and \$10,000, respectively.

**42.** Let $x$ and $y$ represent the number of 37¢ and 23¢ stamps purchased, respectively.

Solve:
$$\begin{aligned} x + \quad y &= 60, \\ 0.41x + 0.17y &= 21 \end{aligned}$$

$x = 45$, $y = 15$

**43.** ***Familiarize***. Let $x =$ the number of hours the Houlihans were out before 11 P.M. and $y =$ the number of hours after 11 P.M. Then they pay the babysitter $\$5x$ before 11 P.M. and $\$7.50y$ after 11 P.M.

***Translate***.

The Houlihans were out for a total of 5 hr.

$$x + y = 5$$

They paid the sitter a total of \$30.

$$5x + 7.5y = 30$$

***Carry out***. Use Gaussian elimination or Gauss-Jordan elimination to solve the system of equations.

$$\begin{aligned} x + \quad y &= 5, \\ 5x + 7.5y &= 30. \end{aligned}$$

The solution is $(3, 2)$. The coordinate $y = 2$ indicates that the Houlihans were out 2 hr after 11 P.M., so they came home at 1 A.M.

***Check***. The total time is $3 + 2$, or 5 hr. The total pay is $\$5 \cdot 3 + \$7.50(2)$, or $\$15 + \$15$, or \$30. The solution checks.

***State***. The Houlihans came home at 1 A.M.

**44.** Let $x$, $y$, and $z$ represent the amount spent on advertising in fiscal years 2010, 2011, and 2012, respectively, in millions of dollars.

Solve:
$$\begin{aligned} &x + y + z = 11, \\ &z = 3x, \\ &y = z - 3 \end{aligned}$$

$x = \$2$ million, $y = \$3$ million, $z = \$6$ million

**45.** The function has a variable in the exponent, so it is an exponential function.

**46.** The function is of the form $f(x) = mx + b$, so it is linear.

**47.** The function is the quotient of two polynomials, so it is a rational function.

**48.** This is a polynomial function of degree 4, so it is a quartic function.

**49.** The function is of the form $f(x) = \log_a x$, so it is logarithmic.

**50.** This is a polynomial function of degree 3, so it is a cubic function.

**51.** The function is of the form $f(x) = mx + b$, so it is linear.

**52.** This is a polynomial function of degree 2, so it is quadratic.

**53.** Substitute to find three equations.

$$\begin{aligned} 12 &= a(-3)^2 + b(-3) + c \\ -7 &= a(-1)^2 + b(-1) + c \\ -2 &= a \cdot 1^2 + b \cdot 1 + c \end{aligned}$$

We have a system of equations.

$$\begin{aligned} 9a - 3b + c &= 12, \\ a - \phantom{3}b + c &= -7, \\ a + \phantom{3}b + c &= -2 \end{aligned}$$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rrr|r} 9 & -3 & 1 & 12 \\ 1 & -1 & 1 & -7 \\ 1 & 1 & 1 & -2 \end{array}\right]$$

Interchange the first two rows.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -7 \\ 9 & -3 & 1 & 12 \\ 1 & 1 & 1 & -2 \end{array}\right]$$

Multiply row 1 by $-9$ and add it to row 2. Also, multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -7 \\ 0 & 6 & -8 & 75 \\ 0 & 2 & 0 & 5 \end{array}\right]$$

Interchange row 2 and row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -7 \\ 0 & 2 & 0 & 5 \\ 0 & 6 & -8 & 75 \end{array}\right]$$

Multiply row 2 by $-3$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -7 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & -8 & 60 \end{array}\right]$$

Multiply row 2 by $\frac{1}{2}$ and row 3 by $-\frac{1}{8}$.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & -7 \\ 0 & 1 & 0 & \frac{5}{2} \\ 0 & 0 & 1 & -\frac{15}{2} \end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$\begin{aligned} x - y + z &= -7, \\ y \qquad &= \frac{5}{2}, \\ z &= -\frac{15}{2} \end{aligned}$$

Back-substitute $\frac{5}{2}$ for $y$ and $-\frac{15}{2}$ for $z$ in the first equation and solve for $x$.

$$\begin{aligned} x - \frac{5}{2} - \frac{15}{2} &= -7 \\ x - 10 &= -7 \\ x &= 3 \end{aligned}$$

The solution is $\left(3, \frac{5}{2}, -\frac{15}{2}\right)$, so the equation is $y = 3x^2 + \frac{5}{2}x - \frac{15}{2}$.

**54.** Solve:

$$\begin{aligned} 0 &= a(-1)^2 + b(-1) + c, \\ -3 &= a \cdot 1^2 + b \cdot 1 + c, \\ -22 &= a \cdot 3^2 + b \cdot 3 + c, \text{ or} \end{aligned}$$

$$\begin{aligned} a - b + c &= 0, \\ a + b + c &= -3, \\ 9a + 3b + c &= -22 \end{aligned}$$

$a = -2$, $b = -\frac{3}{2}$, $c = \frac{1}{2}$, so $y = -2x^2 - \frac{3}{2}x + \frac{1}{2}$.

**55.** $\begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$

Multiply row 1 by $-3$ and add it to row 2.

$$\begin{bmatrix} 1 & 5 \\ 0 & -13 \end{bmatrix}$$

Multiply row 2 by $-\frac{1}{13}$.

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \quad \text{Row-echelon form}$$

Multiply row 2 by $-5$ and add it to row 1.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Reduced row-echelon form}$$

**56.**

$$\begin{aligned} x - \phantom{3}y + \phantom{2k}3z &= -8, \\ 2x + 3y - \phantom{2k3}z &= 5, \\ 3x + 2y + 2kz &= -3k \end{aligned}$$

Write the augmented matrix.

$$\left[\begin{array}{rrr|r} 1 & -1 & 3 & -8 \\ 2 & 3 & -1 & 5 \\ 3 & 2 & 2k & -3k \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 3 & -8 \\ 0 & 5 & -7 & 21 \\ 0 & 5 & 2k-9 & -3k+24 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 3 & -8 \\ 0 & 5 & -7 & 21 \\ 0 & 0 & 2k-2 & -3k+3 \end{array}\right]$$

a) If the system has no solution we have:

$$\begin{aligned} 2k-2=0 &\quad\text{and}\quad -3k+3\neq 0 \\ k=1 &\quad\text{and}\quad k\neq 1 \end{aligned}$$

This is impossible, so there is no value of $k$ for which the system has no solution.

b) If the system has exactly one solution, we have:

$$\begin{aligned} 2k-2 &\neq -3k+3 \\ 5k &\neq 5 \\ k &\neq 1 \end{aligned}$$

c) If the system has infinitely many solutions, we have:

$$\begin{aligned} 2k-2=0 &\quad\text{and}\quad -3k+3=0 \\ k=1 &\quad\text{and}\quad k=1,\ \text{or} \end{aligned}$$

$k=1$

**57.** $y=x+z$,

$3y+5z=4$,

$x+4=y+3z$, or

$$\begin{aligned} x-\ y+\ z&=0, \\ 3y+5z&=4, \\ x-\ y-3z&=-4 \end{aligned}$$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 0 & 3 & 5 & 4 \\ 1 & -1 & -3 & -4 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 0 & 3 & 5 & 4 \\ 0 & 0 & -4 & -4 \end{array}\right]$$

Multiply row 3 by $-\frac{1}{4}$.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 0 & 3 & 5 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Multiply row 3 by $-1$ and add it to row 1. Also, multiply row 3 by $-5$ and add it to row 2.

$$\left[\begin{array}{rrr|r} 1 & -1 & 0 & -1 \\ 0 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Multiply row 2 by $\frac{1}{3}$.

$$\left[\begin{array}{rrr|r} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Add row 2 to row 1.

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Read the solution from the last matrix. It is $\left(-\frac{4}{3}, -\frac{1}{3}, 1\right)$.

**58.** $x+y=2z$,

$2x-5z=4$,

$x-z=y+8$, or

$$\begin{aligned} x+\ y-2z&=0, \\ 2x\qquad -5z&=4, \\ x-\ y-\ z&=8 \end{aligned}$$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrr|r} 1 & 1 & -2 & 0 \\ 2 & 0 & -5 & 4 \\ 1 & -1 & -1 & 8 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 1 & -2 & 0 \\ 0 & -2 & -1 & 4 \\ 0 & -2 & 1 & 8 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{2}$.

$$\left[\begin{array}{rrr|r} 1 & 1 & -2 & 0 \\ 0 & 1 & \frac{1}{2} & -2 \\ 0 & -2 & 1 & 8 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1. Also, multiply row 2 by 2 and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 0 & -\frac{5}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -2 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

Multiply row 3 by $\frac{1}{2}$.

$$\left[\begin{array}{rrr|r} 1 & 0 & -\frac{5}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Multiply row 3 by $\frac{5}{2}$ and add it to row 1. Also, multiply row 3 by $-\frac{1}{2}$ and add it to row 2.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

The solution is $(7, -3, 2)$.

**59.** $x - 4y + 2z = 7,$
$3x + y + 3z = -5$

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 3 & 1 & 3 & -5 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 0 & 13 & -3 & -26 \end{array}\right]$$

Multiply row 2 by $\frac{1}{13}$.

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 0 & 1 & -\frac{3}{13} & -2 \end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$x - 4y + 2z = 7,$$
$$y - \frac{3}{13}z = -2$$

Solve the second equation for $y$.

$$y = \frac{3}{13}z - 2$$

Substitute in the first equation and solve for $x$.

$$x - 4\left(\frac{3}{13}z - 2\right) + 2z = 7$$
$$x - \frac{12}{13}z + 8 + 2z = 7$$
$$x = -\frac{14}{13}z - 1$$

The solution is $\left(-\frac{14}{13}z - 1, \frac{3}{13}z - 2, z\right)$, where $z$ is any real number.

**60.** $x - y - 3z = 3,$
$-x + 3y + z = -7$

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ -1 & 3 & 1 & -7 \end{array}\right]$$

Add row 1 to row 2.

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 0 & 2 & -2 & -4 \end{array}\right]$$

Multiply row 2 by $\frac{1}{2}$.

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

We have

$$x - y - 3z = 3,$$
$$y - z = -2.$$

Then $y = z - 2$. Substitute in the first equation and solve for $x$.

$$x - (z - 2) - 3z = 3$$
$$x - z + 2 - 3z = 3$$
$$x = 4z + 1$$

The solution is $(4z+1, z-2, z)$, where $z$ is any real number.

**61.** $4x + 5y = 3,$
$-2x + y = 9,$
$3x - 2y = -15$

Write the augmented matrix.

$$\left[\begin{array}{cc|c} 4 & 5 & 3 \\ -2 & 1 & 9 \\ 3 & -2 & -15 \end{array}\right]$$

Multiply row 2 by 2 and row 3 by 4.

$$\left[\begin{array}{cc|c} 4 & 5 & 3 \\ -4 & 2 & 18 \\ 12 & -8 & -60 \end{array}\right]$$

Add row 1 to row 2. Also, multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{cc|c} 4 & 5 & 3 \\ 0 & 7 & 21 \\ 0 & -23 & -69 \end{array}\right]$$

Multiply row 2 by $\frac{1}{7}$ and row 3 by $-\frac{1}{23}$.

$$\left[\begin{array}{cc|c} 4 & 5 & 3 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{cc|c} 4 & 5 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

The last row corresponds to the equation $0 = 0$. Thus we have a dependent system that is equivalent to

$$4x + 5y = 3, \quad (1)$$
$$y = 3. \quad (2)$$

Back-substitute in equation (1) to find $x$.

$$\begin{aligned} 4x + 5 \cdot 3 &= 3 \\ 4x + 15 &= 3 \\ 4x &= -12 \\ x &= -3 \end{aligned}$$

The solution is $(-3, 3)$.

**62.** $2x - 3y = -1,$
$-x + 2y = -2,$
$3x - 5y = 1$

Write the augmented matrix.

$$\left[\begin{array}{rr|r} 2 & -3 & -1 \\ -1 & 2 & -2 \\ 3 & -5 & 1 \end{array}\right]$$

Interchange the first two rows.

$$\left[\begin{array}{rr|r} -1 & 2 & -2 \\ 2 & -3 & -1 \\ 3 & -5 & 1 \end{array}\right]$$

Multiply row 1 by 2 and add it to row 2. Also, multiply row 1 by 3 and add it to row 3.

$$\left[\begin{array}{rr|r} -1 & 2 & -2 \\ 0 & 1 & -5 \\ 0 & 1 & -5 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 3.

$$\left[\begin{array}{rr|r} -1 & 2 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{array}\right]$$

We have a dependent system that is equivalent to

$$\begin{aligned} -x + 2y &= -2, \quad (1) \\ y &= -5. \quad (2) \end{aligned}$$

Back-substitute in equation (1) to find $x$.

$$\begin{aligned} -x + 2(-5) &= -2 \\ -x - 10 &= -2 \\ -x &= 8 \\ x &= -8 \end{aligned}$$

The solution is $(-8, -5)$.

## Exercise Set 9.4

**1.** $\begin{bmatrix} 5 & x \end{bmatrix} = \begin{bmatrix} y & -3 \end{bmatrix}$

Corresponding entries of the two matrices must be equal. Thus we have $5 = y$ and $x = -3$.

**2.** $\begin{bmatrix} 6x \\ 25 \end{bmatrix} = \begin{bmatrix} -9 \\ 5y \end{bmatrix}$

$$\begin{aligned} 6x &= -9 \quad \text{and} \quad 25 = 5y \\ x &= -\frac{3}{2} \quad \text{and} \quad 5 = y \end{aligned}$$

**3.** $\begin{bmatrix} 3 & 2x \\ y & -8 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & -8 \end{bmatrix}$

Corresponding entries of the two matrices must be equal. Thus, we have:

$$\begin{aligned} 2x &= -2 \quad \text{and} \quad y = 1 \\ x &= -1 \quad \text{and} \quad y = 1 \end{aligned}$$

**4.** $\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$

$$\begin{aligned} x - 1 &= 0 \quad \text{and} \quad y + 3 = -2 \\ x &= 1 \quad \text{and} \quad y = -5 \end{aligned}$$

**5.** $\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 5 \\ 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1 + (-3) & 2 + 5 \\ 4 + 2 & 3 + (-1) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 7 \\ 6 & 2 \end{bmatrix}$$

**6.** $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} -2 & 7 \\ 6 & 2 \end{bmatrix}$ (See Exercise 5.)

**7.** $\mathbf{E} + \mathbf{0} = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 + 0 & 3 + 0 \\ 2 + 0 & 6 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

**8.** $2\mathbf{A} = \begin{bmatrix} 2 \cdot 1 & 2 \cdot 2 \\ 2 \cdot 4 & 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix}$

**9.** $3\mathbf{F} = 3\begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 3 \cdot 3 & 3 \cdot 3 \\ 3 \cdot (-1) & 3 \cdot (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 9 \\ -3 & -3 \end{bmatrix}$$

**10.** $(-1)\mathbf{D} = \begin{bmatrix} -1 \cdot 1 & -1 \cdot 1 \\ -1 \cdot 1 & -1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

**11.** $3\mathbf{F} = 3\begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ -3 & -3 \end{bmatrix},$

$2\mathbf{A} = 2\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix}$

$3\mathbf{F} + 2\mathbf{A} = \begin{bmatrix} 9 & 9 \\ -3 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 9 + 2 & 9 + 4 \\ -3 + 8 & -3 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 13 \\ 5 & 3 \end{bmatrix}$$

**12.** $\mathbf{A} - \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} -3 & 5 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 2 & 4 \end{bmatrix}$

**13.** $\mathbf{B}-\mathbf{A}=\begin{bmatrix}-3 & 5\\ 2 & -1\end{bmatrix}-\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$

$=\begin{bmatrix}-3 & 5\\ 2 & -1\end{bmatrix}+\begin{bmatrix}-1 & -2\\ -4 & -3\end{bmatrix}$

$[\mathbf{B}-\mathbf{A}=\mathbf{B}+(-\mathbf{A})]$

$=\begin{bmatrix}-3+(-1) & 5+(-2)\\ 2+(-4) & -1+(-3)\end{bmatrix}$

$=\begin{bmatrix}-4 & 3\\ -2 & -4\end{bmatrix}$

**14.** $\mathbf{AB}=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}\begin{bmatrix}-3 & 5\\ 2 & -1\end{bmatrix}$

$=\begin{bmatrix}1(-3)+2\cdot 2 & 1\cdot 5+2(-1)\\ 4(-3)+3\cdot 2 & 4\cdot 5+3(-1)\end{bmatrix}$

$=\begin{bmatrix}1 & 3\\ -6 & 17\end{bmatrix}$

**15.** $\mathbf{BA}=\begin{bmatrix}-3 & 5\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$

$=\begin{bmatrix}-3\cdot 1+5\cdot 4 & -3\cdot 2+5\cdot 3\\ 2\cdot 1+(-1)4 & 2\cdot 2+(-1)3\end{bmatrix}$

$=\begin{bmatrix}17 & 9\\ -2 & 1\end{bmatrix}$

**16.** $\mathbf{0F}=\mathbf{0}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

**17.** $\mathbf{CD}=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

$=\begin{bmatrix}1\cdot 1+(-1)\cdot 1 & 1\cdot 1+(-1)\cdot 1\\ -1\cdot 1+1\cdot 1 & -1\cdot 1+1\cdot 1\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

**18.** $\mathbf{EF}=\begin{bmatrix}1 & 3\\ 2 & 6\end{bmatrix}\begin{bmatrix}3 & 3\\ -1 & -1\end{bmatrix}$

$=\begin{bmatrix}1\cdot 3+3(-1) & 1\cdot 3+3(-1)\\ 2\cdot 3+6(-1) & 2\cdot 3+6(-1)\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

**19.** $\mathbf{AI}=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}1\cdot 1+2\cdot 0 & 1\cdot 0+2\cdot 1\\ 4\cdot 1+3\cdot 0 & 4\cdot 0+3\cdot 1\end{bmatrix}$

$=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$

**20.** $\mathbf{IA}=\mathbf{A}=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$

**21.** $\begin{bmatrix}-1 & 0 & 7\\ 3 & -5 & 2\end{bmatrix}\begin{bmatrix}6\\ -4\\ 1\end{bmatrix}$

$=\begin{bmatrix}-1\cdot 6+0(-4)+7\cdot 1\\ 3\cdot 6+(-5)(-4)+2\cdot 1\end{bmatrix}$

$=\begin{bmatrix}1\\ 40\end{bmatrix}$

**22.** $\begin{bmatrix}6 & -1 & 2\end{bmatrix}\begin{bmatrix}1 & 4\\ -2 & 0\\ 5 & -3\end{bmatrix}$

$=\begin{bmatrix}6\cdot 1+(-1)(-2)+2\cdot 5 & 6\cdot 4+(-1)\cdot 0+2(-3)\end{bmatrix}$

$=\begin{bmatrix}18 & 18\end{bmatrix}$

**23.** $\begin{bmatrix}-2 & 4\\ 5 & 1\\ -1 & -3\end{bmatrix}\begin{bmatrix}3 & -6\\ -1 & 4\end{bmatrix}$

$=\begin{bmatrix}-2\cdot 3+4(-1) & -2(-6)+4\cdot 4\\ 5\cdot 3+1(-1) & 5(-6)+1\cdot 4\\ -1\cdot 3+(-3)(-1) & -1(-6)+(-3)\cdot 4\end{bmatrix}$

$=\begin{bmatrix}-10 & 28\\ 14 & -26\\ 0 & -6\end{bmatrix}$

**24.** $\begin{bmatrix}2 & -1 & 0\\ 0 & 5 & 4\end{bmatrix}\begin{bmatrix}-3 & 1 & 0\\ 0 & 2 & -1\\ 5 & 0 & 4\end{bmatrix}$

$=\begin{bmatrix}-6+0+0 & 2-2+0 & 0+1+0\\ 0+0+20 & 0+10+0 & 0-5+16\end{bmatrix}$

$=\begin{bmatrix}-6 & 0 & 1\\ 20 & 10 & 11\end{bmatrix}$

**25.** $\begin{bmatrix}1\\ -5\\ 3\end{bmatrix}\begin{bmatrix}-6 & 5 & 8\\ 0 & 4 & -1\end{bmatrix}$

This product is not defined because the number of columns of the first matrix, 1, is not equal to the number of rows of the second matrix, 2.

**26.** $\begin{bmatrix}2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}0 & -4 & 3\\ 2 & 1 & 0\\ -1 & 0 & 6\end{bmatrix}$

$=\begin{bmatrix}0+0+0 & -8+0+0 & 6+0+0\\ 0-2+0 & 0-1+0 & 0+0+0\\ 0+0-3 & 0+0+0 & 0+0+18\end{bmatrix}$

$=\begin{bmatrix}0 & -8 & 6\\ -2 & -1 & 0\\ -3 & 0 & 18\end{bmatrix}$

**27.** $\begin{bmatrix}1 & -4 & 3\\ 0 & 8 & 0\\ -2 & -1 & 5\end{bmatrix}\begin{bmatrix}3 & 0 & 0\\ 0 & -4 & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}3+0+0 & 0+16+0 & 0+0+3\\ 0+0+0 & 0-32+0 & 0+0+0\\ -6+0+0 & 0+4+0 & 0+0+5\end{bmatrix}$

$=\begin{bmatrix}3 & 16 & 3\\ 0 & -32 & 0\\ -6 & 4 & 5\end{bmatrix}$

**28.** $\begin{bmatrix} 4 \\ -5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 6 & -7 \\ 0 & -3 \end{bmatrix}$

This product is not defined because the number of columns of the first matrix, 1, is not equal to the number of rows of the second matrix, 3.

**29.** a) $\mathbf{B} = \begin{bmatrix} 300 & 80 & 40 \end{bmatrix}$

b) $\$300 + 5\% \cdot \$300 = 1.05(\$300) = \$315$

$\$80 + 5\% \cdot \$80 = 1.05(\$80) = \$84$

$\$40 + 5\% \cdot \$40 = 1.05(\$40) = \$42$

We write the matrix that corresponds to these amounts.

$\mathbf{R} = \begin{bmatrix} 315 & 84 & 42 \end{bmatrix}$

c) $\mathbf{B}+\mathbf{R} = \begin{bmatrix} 300 & 80 & 40 \end{bmatrix} + \begin{bmatrix} 315 & 84 & 42 \end{bmatrix}$
$= \begin{bmatrix} 615 & 164 & 82 \end{bmatrix}$

The entries represent the total budget for each type of expenditure for June and July.

**30.** a) $\mathbf{A} = \begin{bmatrix} 40 & 20 & 30 \end{bmatrix}$

b) $1.1(40) = 44$, $1.1(20) = 22$, $1.1(30) = 33$

$\mathbf{B} = \begin{bmatrix} 44 & 22 & 33 \end{bmatrix}$

c) $\mathbf{A}+\mathbf{B} = \begin{bmatrix} 40 & 20 & 30 \end{bmatrix} + \begin{bmatrix} 44 & 22 & 33 \end{bmatrix}$
$= \begin{bmatrix} 84 & 42 & 63 \end{bmatrix}$

The entries represent the total amount of each type of produce ordered for both weeks.

**31.** a) $\mathbf{C} = \begin{bmatrix} 140 & 27 & 3 & 13 & 64 \end{bmatrix}$

$\mathbf{P} = \begin{bmatrix} 180 & 4 & 11 & 24 & 662 \end{bmatrix}$

$\mathbf{B} = \begin{bmatrix} 50 & 5 & 1 & 82 & 20 \end{bmatrix}$

b) $\mathbf{C} + 2\mathbf{P} + 3\mathbf{B}$

$= \begin{bmatrix} 140 & 27 & 3 & 13 & 64 \end{bmatrix} +$
$\begin{bmatrix} 360 & 8 & 22 & 48 & 1324 \end{bmatrix} +$
$\begin{bmatrix} 150 & 15 & 3 & 246 & 60 \end{bmatrix}$

$= \begin{bmatrix} 650 & 50 & 28 & 307 & 1448 \end{bmatrix}$

The entries represent the total nutritional value of one serving of chicken, 1 cup of potato salad, and 3 broccoli spears.

**32.** a) $\mathbf{P} = \begin{bmatrix} 290 & 15 & 9 & 39 \end{bmatrix}$

$\mathbf{G} = \begin{bmatrix} 70 & 2 & 0 & 17 \end{bmatrix}$

$\mathbf{M} = \begin{bmatrix} 150 & 8 & 8 & 11 \end{bmatrix}$

b) $3\mathbf{P} + 2\mathbf{G} + 2\mathbf{M}$

$= \begin{bmatrix} 870 & 45 & 27 & 117 \end{bmatrix} +$
$\begin{bmatrix} 140 & 4 & 0 & 34 \end{bmatrix} +$
$\begin{bmatrix} 300 & 16 & 16 & 22 \end{bmatrix}$

$= \begin{bmatrix} 1310 & 65 & 43 & 173 \end{bmatrix}$

The entries represent the total nutritional value of 3 slices of pizza, 1 cup of gelatin, and 2 cups of whole milk.

**33.** a) $\mathbf{M} = \begin{bmatrix} 1.50 & 0.15 & 0.26 & 0.23 & 0.64 \\ 1.55 & 0.14 & 0.24 & 0.21 & 0.75 \\ 1.62 & 0.22 & 0.31 & 0.28 & 0.53 \\ 1.70 & 0.20 & 0.29 & 0.33 & 0.68 \end{bmatrix}$

b) $\mathbf{N} = \begin{bmatrix} 65 & 48 & 93 & 57 \end{bmatrix}$

c) $\mathbf{NM} = \begin{bmatrix} 419.46 & 48.33 & 73.78 & 69.88 & 165.65 \end{bmatrix}$

d) The entries of **NM** represent the total cost, in dollars, of each item for the day's meals.

**34.** a) $\mathbf{M} = \begin{bmatrix} 1 & 2.5 & 0.75 & 0.5 \\ 0 & 0.5 & 0.25 & 0 \\ 0.75 & 0.25 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 & 1 \end{bmatrix}$

b) $\mathbf{C} = \begin{bmatrix} 25 & 34 & 54 & 83 \end{bmatrix}$

c) $\mathbf{CM} = \begin{bmatrix} 107 & 93 & 95.75 & 122.5 \end{bmatrix}$

d) The entries of **CM** represent the total cost, in cents, of each menu item.

**35.** a) $\mathbf{S} = \begin{bmatrix} 8 & 15 \\ 6 & 10 \\ 4 & 3 \end{bmatrix}$

b) $\mathbf{C} = \begin{bmatrix} 4 & 2.50 & 3 \end{bmatrix}$

c) $\mathbf{CS} = \begin{bmatrix} 59 & 94 \end{bmatrix}$

d) The entries of **CS** represent the total cost, in dollars, of ingredients for each coffee shop.

**36.** a) $\mathbf{M} = \begin{bmatrix} 900 & 500 \\ 450 & 1000 \\ 600 & 700 \end{bmatrix}$

b) $\mathbf{P} = \begin{bmatrix} 5 & 8 & 4 \end{bmatrix}$

c) $\mathbf{PM} = \begin{bmatrix} 10,500 & 13,300 \end{bmatrix}$

d) The entries of **PM** represent the total profit from each distributor.

**37.** a) $\mathbf{P} = \begin{bmatrix} 6 & 4.5 & 5.2 \end{bmatrix}$

b) $\mathbf{PS} = \begin{bmatrix} 6 & 4.5 & 5.2 \end{bmatrix}\begin{bmatrix} 8 & 15 \\ 6 & 10 \\ 4 & 3 \end{bmatrix}$
$= \begin{bmatrix} 95.8 & 150.6 \end{bmatrix}$

The profit from Mugsey's Coffee Shop is \$95.80, and the profit from The Coffee Club is \$150.60.

**38.** a) $\mathbf{C} = \begin{bmatrix} 20 & 25 & 15 \end{bmatrix}$

b) $\mathbf{CM} = \begin{bmatrix} 20 & 25 & 15 \end{bmatrix}\begin{bmatrix} 900 & 500 \\ 450 & 1000 \\ 600 & 700 \end{bmatrix}$
$= \begin{bmatrix} 38,250 & 45,500 \end{bmatrix}$

The total production costs for the products shipped to Distributors 1 and 2 are \$38,250 and \$45,500, respectively.

**39.** $2x - 3y = 7,$
$x + 5y = -6$

Write the coefficients on the left in a matrix. Then write the product of that matrix and the column matrix containing the variables, and set the result equal to the column matrix containing the constants on the right.

$$\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$$

**40.** $\begin{bmatrix} -1 & 1 \\ 5 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \end{bmatrix}$

**41.** $x + y - 2z = 6,$
$3x - y + z = 7,$
$2x + 5y - 3z = 8$

Write the coefficients on the left in a matrix. Then write the product of that matrix and the column matrix containing the variables, and set the result equal to the column matrix containing the constants on the right.

$$\begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ 2 & 5 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$$

**42.** $\begin{bmatrix} 3 & -1 & 1 \\ 1 & 2 & -1 \\ 4 & 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 11 \end{bmatrix}$

**43.** $3x - 2y + 4z = 17,$
$2x + y - 5z = 13$

Write the coefficients on the left in a matrix. Then write the product of that matrix and the column matrix containing the variables, and set the result equal to the column matrix containing the constants on the right.

$$\begin{bmatrix} 3 & -2 & 4 \\ 2 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ 13 \end{bmatrix}$$

**44.** $\begin{bmatrix} 3 & 2 & 5 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \end{bmatrix}$

**45.** $-4w + x - y + 2z = 12,$
$w + 2x - y - z = 0,$
$-w + x + 4y - 3z = 1,$
$2w + 3x + 5y - 7z = 9$

Write the coefficients on the left in a matrix. Then write the product of that matrix and the column matrix containing the variables, and set the result equal to the column matrix containing the constants on the right.

$$\begin{bmatrix} -4 & 1 & -1 & 2 \\ 1 & 2 & -1 & -1 \\ -1 & 1 & 4 & -3 \\ 2 & 3 & 5 & -7 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 1 \\ 9 \end{bmatrix}$$

**46.** $\begin{bmatrix} 12 & 2 & 4 & -5 \\ -1 & 4 & -1 & 12 \\ 2 & -1 & 4 & 0 \\ 0 & 2 & 10 & 1 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 13 \\ 5 \end{bmatrix}$

**47.** $f(x) = x^2 - x - 6$

a) $-\dfrac{b}{2a} = -\dfrac{-1}{2 \cdot 1} = \dfrac{1}{2}$

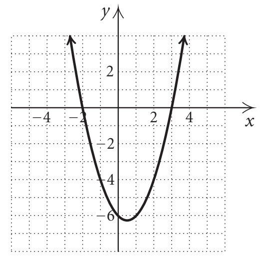
$f\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^2 - \dfrac{1}{2} - 6 = -\dfrac{25}{4}$

The vertex is $\left(\dfrac{1}{2}, -\dfrac{25}{4}\right)$.

b) The axis of symmetry is $x = \dfrac{1}{2}$.

c) Since the coefficient of $x^2$ is positive, the function has a minimum value. It is the second coordinate of the vertex, $-\dfrac{25}{4}$.

d) Plot some points and draw the graph of the function.



$f(x) = x^2 - x - 6$

**48.** $f(x) = 2x^2 - 5x - 3$

a) $-\dfrac{b}{2a} = -\dfrac{-5}{2 \cdot 2} = \dfrac{5}{4}$

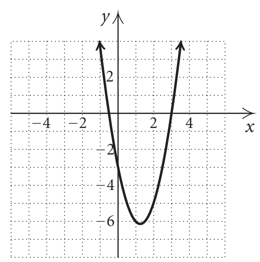
$f\left(\dfrac{5}{4}\right) = 2\left(\dfrac{5}{4}\right)^2 - 5\left(\dfrac{5}{4}\right) - 3 = -\dfrac{49}{8}$

The vertex is $\left(\dfrac{5}{4}, -\dfrac{49}{8}\right)$.

b) $x = \dfrac{5}{4}$

c) Minimum: $-\dfrac{49}{8}$

d)



$f(x) = 2x^2 - 5x - 3$

**49.** $f(x) = -x^2 - 3x + 2$

a) $-\dfrac{b}{2a} = -\dfrac{-3}{2(-1)} = -\dfrac{3}{2}$

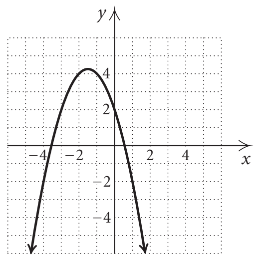
$f\left(-\dfrac{3}{2}\right) = -\left(-\dfrac{3}{2}\right)^2 - 3\left(-\dfrac{3}{2}\right) + 2 = \dfrac{17}{4}$

The vertex is $\left(-\dfrac{3}{2}, \dfrac{17}{4}\right)$.

b) The axis of symmetry is $x = -\dfrac{3}{2}$.

c) Since the coefficient of $x^2$ is negative, the function has a maximum value. It is the second coordinate of the vertex, $\dfrac{17}{4}$.

d) Plot some points and draw the graph of the function.



$f(x) = -x^2 - 3x + 2$

**50.** $f(x) = -3x^2 + 4x + 4$

a) $-\dfrac{b}{2a} = -\dfrac{4}{2(-3)} = \dfrac{2}{3}$

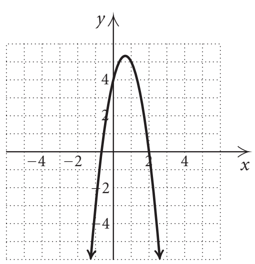
$f\left(\dfrac{2}{3}\right) = -3\left(\dfrac{2}{3}\right)^2 + 4\left(\dfrac{2}{3}\right) + 4 = \dfrac{16}{3}$

The vertex is $\left(\dfrac{2}{3}, \dfrac{16}{3}\right)$.

b) $x = \dfrac{2}{3}$

c) Maximum: $\dfrac{16}{3}$

d)



$f(x) = -3x^2 + 4x + 4$

**51.** $\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$$(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\mathbf{A}^2 - \mathbf{B}^2 = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 \\ 0 & -3 \end{bmatrix}$$

Thus $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) \neq \mathbf{A}^2 - \mathbf{B}^2$.

**52.** $\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$$(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{B}) = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$$

We found $\mathbf{A}^2$ and $\mathbf{B}^2$ in Exercise 51.

$$2\mathbf{AB} = 2\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 4 & 0 \end{bmatrix}$$

$$\mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix}$$

Thus $(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{B}) \neq \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2$.

**53.** In Exercise 51 we found that $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$ and we also found $\mathbf{A}^2$ and $\mathbf{B}^2$.

$$\mathbf{BA} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\mathbf{A}^2 + \mathbf{BA} - \mathbf{AB} - \mathbf{B}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$$

Thus $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = \mathbf{A}^2 + \mathbf{BA} - \mathbf{AB} - \mathbf{B}^2$.

**54.** In Exercise 52 we found that

$(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{B}) = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$.

We found $\mathbf{A}^2$ and $\mathbf{B}^2$ in Exercise 51, and we found $\mathbf{BA}$ and $\mathbf{AB}$ in Exercise 53.

$$\mathbf{A}^2 + \mathbf{BA} + \mathbf{AB} + \mathbf{B}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$$

Thus $(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{B}) = \mathbf{A}^2 + \mathbf{BA} + \mathbf{AB} + \mathbf{B}^2$.

**55.** See the answer section in the text.

**56.** $\mathbf{A}+(\mathbf{B}+\mathbf{C})$

$$= \begin{bmatrix} a_{11}+(b_{11}+c_{11}) & \cdots & a_{1n}+(b_{1n}+c_{1n}) \\ a_{21}+(b_{21}+c_{21}) & \cdots & a_{2n}+(b_{2n}+c_{2n}) \\ \vdots & \vdots & \vdots \\ a_{m1}+(b_{m1}+c_{m1}) & \cdots & a_{mn}+(b_{mn}+c_{mn}) \end{bmatrix}$$

$$= \begin{bmatrix} (a_{11}+b_{11})+c_{11} & \cdots & (a_{1n}+b_{1n})+c_{1n} \\ (a_{21}+b_{21})+c_{21} & \cdots & (a_{2n}+b_{2n})+c_{2n} \\ \vdots & \vdots & \vdots \\ (a_{m1}+b_{m1})+c_{m1} & \cdots & (a_{mn}+b_{mn})+c_{mn} \end{bmatrix}$$

$= (\mathbf{A}+\mathbf{B})+\mathbf{C}$

**57.** See the answer section in the text.

**58.** $k(\mathbf{A}+\mathbf{B})$

$$= \begin{bmatrix} k(a_{11}+b_{11}) & \cdots & k(a_{1n}+b_{1n}) \\ k(a_{21}+b_{21}) & \cdots & k(a_{2n}+b_{2n}) \\ \vdots & & \vdots \\ k(a_{m1}+b_{m1}) & \cdots & k(a_{mn}+b_{mn}) \end{bmatrix}$$

$$= \begin{bmatrix} ka_{11}+kb_{11} & \cdots & ka_{1n}+kb_{1n} \\ ka_{21}+kb_{21} & \cdots & ka_{2n}+kb_{2n} \\ \vdots & & \vdots \\ ka_{m1}+kb_{m1} & \cdots & ka_{mn}+kb_{mn} \end{bmatrix}$$

$= k\mathbf{A}+k\mathbf{B}$

**59.** See the answer section in the text.

## Chapter 9 Mid-Chapter Mixed Review

1. False; see page 739 in the text.
2. True; see page 767 in the text.
3. True; see page 776 in the text.
4. False; see page 775 in the text.
5. $2x+y=-4,\quad (1)$
   $x=y-5\quad (2)$

   Substitute $y-5$ for $x$ in equation (1) and solve for $y$.

$$\begin{aligned} 2(y-5)+y &= -4 \\ 2y-10+y &= -4 \\ 3y-10 &= -4 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

Back-substitute in equation (2) to find $x$.

$x = 2-5 = -3$

The solution is $(-3, 2)$.

**6.** $x+y=4,\quad (1)$
$y=2-x\quad (2)$

Substitute $2-x$ for $y$ in equation (1) and solve for $x$.

$$\begin{aligned} x+2-x &= 4 \\ 2 &= 4 \end{aligned}$$

We get a false equation. The system of equations has no solution.

**7.** $2x-3y=8,\quad (1)$
$3x+2y=-1\quad (2)$

Multiply equation (1) by 2 and equation (2) by 3 and add.

$$\begin{aligned} 4x-6y &= 16 \\ 9x+6y &= -3 \\ \hline 13x \quad &= 13 \\ x &= 1 \end{aligned}$$

Back-substitute and solve for $y$.

$$\begin{aligned} 3\cdot 1+2y &= -1 \quad \text{Using equation (2)} \\ 3+2y &= -1 \\ 2y &= -4 \\ y &= -2 \end{aligned}$$

The solution is $(1, -2)$.

**8.** $x-3y=1,\quad (1)$
$6y=2x-2\quad (2)$

Solve equation (1) for $x$ and then substitute in equation (2).

$$\begin{aligned} x-3y &= 1 \\ x &= 3y+1 \quad \text{Solving for } x \end{aligned}$$

$$\begin{aligned} 6y &= 2(3y+1)-2 \quad \text{Substituting} \\ 6y &= 6y+2-2 \\ 6y &= 6y \\ 0 &= 0 \end{aligned}$$

We get an equation that is true for all values of $x$ and $y$. We can express the solutions in terms of $y$ as $(3y+1, y)$. We can also express them in terms of $x$ as shown below.

$$\begin{aligned} x-3y &= 1 \quad (1) \\ x-1 &= 3y \\ \frac{x-1}{3} &= y \end{aligned}$$

Then we have solutions of the form $\left(x, \dfrac{x-1}{3}\right)$.

**9.** $x+2y+3z=4,\quad (1)$
$x-2y+z=2,\quad (2)$
$2x-6y+4z=7\quad (3)$

Multiply equation (1) by $-1$ and add it to equation (2).

Multiply equation (1) by $-2$ and add it to equation (3).

$$\begin{aligned} x+2y+3z &= 4 \quad (1) \\ -4y-2z &= -2 \quad (4) \\ -10y-2z &= -1 \quad (5) \end{aligned}$$

Multiply equation (5) by 2.

$$\begin{aligned} x + 2y + 3z &= 4 \quad (1)\\ -4y - 2z &= -2 \quad (4)\\ -20y - 4z &= -2 \quad (6)\end{aligned}$$

Multiply equation (4) by $-5$ and add it to equation (6).

$$\begin{aligned} x + 2y + 3z &= 4 \quad (1)\\ -4y - 2z &= -2 \quad (4)\\ 6z &= 8 \quad (7)\end{aligned}$$

Solve equation (7) for $z$.

$$6z = 8$$
$$z = \frac{4}{3}$$

Back-substitute $\frac{4}{3}$ for $z$ in equation (4) and solve for $y$.

$$\begin{aligned} -4y - 2\left(\frac{4}{3}\right) &= -2\\ -4y - \frac{8}{3} &= -2\\ -4y &= \frac{2}{3}\\ y &= -\frac{1}{6}\end{aligned}$$

Back-substitute $-\frac{1}{6}$ for $y$ and $\frac{4}{3}$ for $z$ in equation (1) and solve for $x$.

$$\begin{aligned} x + 2\left(-\frac{1}{6}\right) + 3\cdot\frac{4}{3} &= 4\\ x - \frac{1}{3} + 4 &= 4\\ x + \frac{11}{3} &= 4\\ x &= \frac{1}{3}\end{aligned}$$

The solution is $\left(\frac{1}{3}, -\frac{1}{6}, \frac{4}{3}\right)$.

**10.** Let $x$, $y$, and $z$ represent the number of orders up to 10 lb, from 10 lb up to 15 lb, and of 15 lb or more, respectively.

Solve: $x + y + z = 150,$
$3x + 5y + 7.50z = 680,$
$x = 3z$

$x = 60$, $y = 70$, $z = 20$

**11.** $2x + y = 5,$
$3x + 2y = 6$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{cc|c} 2 & 1 & 5\\ 3 & 2 & 6\end{array}\right]$$

Multiply row 2 by 2.

$$\left[\begin{array}{cc|c} 2 & 1 & 5\\ 6 & 4 & 12\end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{cc|c} 2 & 1 & 5\\ 0 & 1 & -3\end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1.

$$\left[\begin{array}{cc|c} 2 & 0 & 8\\ 0 & 1 & -3\end{array}\right]$$

Multiply row 1 by $\frac{1}{2}$.

$$\left[\begin{array}{cc|c} 1 & 0 & 4\\ 0 & 1 & -3\end{array}\right]$$

The solution is $(4, -3)$.

**12.** $3x + 2y - 3z = -2,$
$2x + 3y + 2z = -2,$
$x + 4y + 4z = 1$

$$\left[\begin{array}{ccc|c} 3 & 2 & -3 & -2\\ 2 & 3 & 2 & -2\\ 1 & 4 & 4 & 1\end{array}\right]$$

Interchange row 1 and row 3.

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 1\\ 2 & 3 & 2 & -2\\ 3 & 2 & -3 & -2\end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.
Multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 1\\ 0 & -5 & -6 & -4\\ 0 & -10 & -15 & -5\end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 3.

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 1\\ 0 & -5 & -6 & -4\\ 0 & 0 & -3 & 3\end{array}\right]$$

Multiply row 2 by $-\frac{1}{5}$ and row 3 by $-\frac{1}{3}$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 4 & 1\\ 0 & 1 & \frac{6}{5} & \frac{4}{5}\\ 0 & 0 & 1 & -1\end{array}\right]$$

Write the system of equations that corresponds to the last matrix.

$$x + 4y + 4z = 1, \quad (1)$$
$$y + \frac{6}{5}z = \frac{4}{5}, \quad (2)$$
$$z = -1 \quad (3)$$

Back-substitute $-1$ for $z$ in equation (2) and solve for $y$.

$$y + \frac{6}{5}(-1) = \frac{4}{5}$$
$$y - \frac{6}{5} = \frac{4}{5}$$
$$y = 2$$

Back-substitute 2 for $y$ and $-1$ for $z$ in equation (1) and solve for $x$.

$$x + 4 \cdot 2 + 4(-1) = 1$$
$$x + 8 - 4 = 1$$
$$x + 4 = 1$$
$$x = -3$$

The solution is $(-3, 2, -1)$.

**13.** $\mathbf{A} + \mathbf{B} = \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3 + (-2) & -1 + 6 \\ 5 + 1 & 4 + (-3) \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 \\ 6 & 1 \end{bmatrix}$

**14.** $\mathbf{B} - \mathbf{A} = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 7 \\ -4 & -7 \end{bmatrix}$

**15.** $4\mathbf{D} = 4\begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -8 & 12 & 0 \\ 4 & -4 & 8 \\ -12 & 16 & 4 \end{bmatrix}$

**16.** $2\mathbf{A} + 3\mathbf{B} = \begin{bmatrix} 6 & -2 \\ 10 & 8 \end{bmatrix} + \begin{bmatrix} -6 & 18 \\ 3 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 16 \\ 13 & -1 \end{bmatrix}$

**17.** $\mathbf{AB} = \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3(-2) + 9 + (-1) \cdot 1 & 3 \cdot 6 + (-1)(-3) \\ 5(-2) + 4 \cdot 1 & 5 \cdot 6 + 4(-3) \end{bmatrix}$

$= \begin{bmatrix} -7 & 21 \\ -6 & 18 \end{bmatrix}$

**18.** $\mathbf{BA} = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2 \cdot 3 + 6 \cdot 5 & -2(-1) + 6 \cdot 4 \\ 1 \cdot 3 + (-3) \cdot 5 & 1(-1) + (-3) \cdot 4 \end{bmatrix}$

$= \begin{bmatrix} 24 & 26 \\ -12 & -13 \end{bmatrix}$

**19.** $\mathbf{BC}$

$= \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -4 & 1 & -1 \\ 2 & 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} -2(-4)+6 \cdot 2 & -2 \cdot 1 + 6 \cdot 3 & -2(-1) + 6(-2) \\ 1(-4)+(-3) \cdot 2 & 1 \cdot 1+(-3) \cdot 3 & 1(-1)+(-3)(-2) \end{bmatrix}$

$= \begin{bmatrix} 20 & 16 & -10 \\ -10 & -8 & 5 \end{bmatrix}$

**20.** The number of columns in **D** is not the same as the number of rows in **C** so **DC** is not defined.

**21.** A matrix equation equivalent to the given system is
$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 3 & -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$.

**22.** When a variable is not alone on one side of an equation or when solving for a variable is difficult or produces an expression containing fractions, the elimination method is preferable to the substitution method.

**23.** Add a non-zero multiple of one equation to a non-zero multiple of the other equation, where the multiples are not opposites.

**24.** See Example 4 on page 768 in the text.

**25.** No; see Exercise 17 on page 780 in the text, for example.

## Exercise Set 9.5

**1.** $\mathbf{BA} = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mathbf{AB} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $\mathbf{BA} = \mathbf{I} = \mathbf{AB}$, **B** is the inverse of **A**.

**2.** $\mathbf{BA} = \mathbf{I} = \mathbf{AB}$, so **B** is the inverse of **A**.

**3.** $\mathbf{BA} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 6 \\ 1 & 0 & -2 \\ 1 & 0 & -3 \end{bmatrix} =$
$\begin{bmatrix} 3 & -2 & 0 \\ 4 & -3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

Since $\mathbf{BA} \neq \mathbf{I}$, **B** is not the inverse of **A**.

**4.** $\mathbf{BA} = \begin{bmatrix} 1 & 0 & -24 \\ 0 & 1 & 8 \\ 0 & 0 & 17 \end{bmatrix} \neq \mathbf{I}$, so **B** is not the inverse of **A**.

**5.** $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

Write the augmented matrix.

$\left[\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$

Multiply row 2 by 3.

$\left[\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ 15 & 9 & 0 & 3 \end{array}\right]$

Multiply row 1 by $-5$ and add it to row 2.

$$\left[\begin{array}{rr|rr} 3 & 2 & 1 & 0 \\ 0 & -1 & -5 & 3 \end{array}\right]$$

Multiply row 2 by 2 and add it to row 1.

$$\left[\begin{array}{rr|rr} 3 & 0 & -9 & 6 \\ 0 & -1 & -5 & 3 \end{array}\right]$$

Multiply row 1 by $\frac{1}{3}$ and row 2 by $-1$.

$$\left[\begin{array}{rr|rr} 1 & 0 & -3 & 2 \\ 0 & 1 & 5 & -3 \end{array}\right]$$

Then $\mathbf{A}^{-1} = \left[\begin{array}{rr} -3 & 2 \\ 5 & -3 \end{array}\right]$.

**6.** $\mathbf{A} = \left[\begin{array}{rr} 3 & 5 \\ 1 & 2 \end{array}\right]$

Write the augmented matrix.

$$\left[\begin{array}{rr|rr} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$$

Interchange the rows.

$$\left[\begin{array}{rr|rr} 1 & 2 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2.

$$\left[\begin{array}{rr|rr} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -3 \end{array}\right]$$

Multiply row 2 by 2 and add it to row 1.

$$\left[\begin{array}{rr|rr} 1 & 0 & 2 & -5 \\ 0 & -1 & 1 & -3 \end{array}\right]$$

Multiply row 2 by $-1$.

$$\left[\begin{array}{rr|rr} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

Then $\mathbf{A}^{-1} = \left[\begin{array}{rr} 2 & -5 \\ -1 & 3 \end{array}\right]$.

**7.** $\mathbf{A} = \left[\begin{array}{rr} 6 & 9 \\ 4 & 6 \end{array}\right]$

Write the augmented matrix.

$$\left[\begin{array}{rr|rr} 6 & 9 & 1 & 0 \\ 4 & 6 & 0 & 1 \end{array}\right]$$

Multiply row 2 by 3.

$$\left[\begin{array}{rr|rr} 6 & 9 & 1 & 0 \\ 12 & 18 & 0 & 3 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

$$\left[\begin{array}{rr|rr} 6 & 9 & 1 & 0 \\ 0 & 0 & -2 & 3 \end{array}\right]$$

We cannot obtain the identity matrix on the left since the second row contains only zeros to the left of the vertical line. Thus, $\mathbf{A}^{-1}$ does not exist.

**8.** $\mathbf{A} = \left[\begin{array}{rr} -4 & -6 \\ 2 & 3 \end{array}\right]$

Write the augmented matrix.

$$\left[\begin{array}{rr|rr} -4 & -6 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$$

Multiply row 2 by 2.

$$\left[\begin{array}{rr|rr} -4 & -6 & 1 & 0 \\ 4 & 6 & 0 & 2 \end{array}\right]$$

Add row 1 to row 2.

$$\left[\begin{array}{rr|rr} -4 & -6 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

We cannot obtain the identity matrix on the left since the second row contains only zeros to the left of the vertical line. Thus, $\mathbf{A}^{-1}$ does not exist.

**9.** $\mathbf{A} = \left[\begin{array}{rr} 4 & -3 \\ 1 & -2 \end{array}\right]$

Write the augmented matrix.

$$\left[\begin{array}{rr|rr} 4 & -3 & 1 & 0 \\ 1 & -2 & 0 & 1 \end{array}\right]$$

Interchange the rows.

$$\left[\begin{array}{rr|rr} 1 & -2 & 0 & 1 \\ 4 & -3 & 1 & 0 \end{array}\right]$$

Multiply row 1 by $-4$ and add it to row 2.

$$\left[\begin{array}{rr|rr} 1 & -2 & 0 & 1 \\ 0 & 5 & 1 & -4 \end{array}\right]$$

Multiply row 2 by $\frac{1}{5}$.

$$\left[\begin{array}{rr|rr} 1 & -2 & 0 & 1 \\ 0 & 1 & \frac{1}{5} & -\frac{4}{5} \end{array}\right]$$

Multiply row 2 by 2 and add it to row 1.

$$\left[\begin{array}{rr|rr} 1 & 0 & \frac{2}{5} & -\frac{3}{5} \\ 0 & 1 & \frac{1}{5} & -\frac{4}{5} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \left[\begin{array}{rr} \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & -\frac{4}{5} \end{array}\right]$, or $\left[\begin{array}{rr} 0.4 & -0.6 \\ 0.2 & -0.8 \end{array}\right]$.

**10.** $\mathbf{A} = \left[\begin{array}{rr} 0 & -1 \\ 1 & 0 \end{array}\right]$

Write the augmented matrix.

$$\left[\begin{array}{rr|rr} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$

Interchange the rows.

$$\left[\begin{array}{rr|rr} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

Multiply row 2 by $-1$.

$$\left[\begin{array}{rr|rr} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

Then $\mathbf{A}^{-1} = \left[\begin{array}{rr} 0 & 1 \\ -1 & 0 \end{array}\right]$.

**11.** $\mathbf{A} = \left[\begin{array}{rrr} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{array}\right]$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

Interchange the first two rows.

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2. Also, multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & -3 & 1 & -3 & 0 \\ 0 & -2 & 1 & 0 & -1 & 1 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{2}$.

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & -2 & 1 & 0 & -1 & 1 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1. Also, multiply row 2 by 2 and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 4 & -1 & 2 & 1 \end{array}\right]$$

Multiply row 3 by $\frac{1}{4}$.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{array}\right]$$

Multiply row 3 by $\frac{1}{2}$ and add it to row 1. Also, multiply row 3 by $-\frac{3}{2}$ and add it to row 2.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & \frac{3}{8} & -\frac{1}{4} & \frac{1}{8} \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{3}{4} & -\frac{3}{8} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} \frac{3}{8} & -\frac{1}{4} & \frac{1}{8} \\ -\frac{1}{8} & \frac{3}{4} & -\frac{3}{8} \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$.

**12.** $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{array}\right]$$

Add row 2 to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -2 & -3 & 1 & 1 \end{array}\right]$$

Multiply row 3 by $-\frac{1}{2}$.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

Multiply row 3 by $-1$ and add it to row 1. Also, multiply row 3 by 2 and add it to row 2.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & -1 \\ \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$.

**13.** $\mathbf{A} = \begin{bmatrix} 1 & -4 & 8 \\ 1 & -3 & 2 \\ 2 & -7 & 10 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 1 & -4 & 8 & 1 & 0 & 0 \\ 1 & -3 & 2 & 0 & 1 & 0 \\ 2 & -7 & 10 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 2. Also, multiply row 1 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & -4 & 8 & 1 & 0 & 0 \\ 0 & 1 & -6 & -1 & 1 & 0 \\ 0 & 1 & -6 & -2 & 0 & 1 \end{array}\right]$$

Since the second and third rows are identical left of the vertical line, it will not be possible to obtain the identity matrix on the left side. Thus, $\mathbf{A}^{-1}$ does not exist.

**14.** $\mathbf{A} = \begin{bmatrix} -2 & 5 & 3 \\ 4 & -1 & 3 \\ 7 & -2 & 5 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} -2 & 5 & 3 & 1 & 0 & 0 \\ 4 & -1 & 3 & 0 & 1 & 0 \\ 7 & -2 & 5 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 3 by 2.

$$\left[\begin{array}{rrr|rrr} -2 & 5 & 3 & 1 & 0 & 0 \\ 4 & -1 & 3 & 0 & 1 & 0 \\ 14 & -4 & 10 & 0 & 0 & 2 \end{array}\right]$$

Multiply row 1 by 2 and add it to row 2. Also, multiply row 1 by 7 and add it to row 3.

$$\left[\begin{array}{rrr|rrr} -2 & 5 & 3 & 1 & 0 & 0 \\ 0 & 9 & 9 & 2 & 1 & 0 \\ 0 & 31 & 31 & 7 & 0 & 2 \end{array}\right]$$

To the left of the vertical line, row 3 is a multiple of row 2 so it will not be possible to obtain the identity matrix on the left. Thus, $\mathbf{A}^{-1}$ does not exist.

**15.** $\mathbf{A} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 3 & 4 \\ -1 & -1 & -1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 2 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 4 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 \end{array}\right]$$

Interchange rows 1 and 3.

$$\left[\begin{array}{rrr|rrr} -1 & -1 & -1 & 0 & 0 & 1 \\ 3 & 3 & 4 & 0 & 1 & 0 \\ 2 & 3 & 2 & 1 & 0 & 0 \end{array}\right]$$

Multiply row 1 by 3 and add it to row 2. Also, multiply row 1 by 2 and add it to row 3.

$$\left[\begin{array}{rrr|rrr} -1 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & 0 & 2 \end{array}\right]$$

Multiply row 1 by $-1$.

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & 0 & 2 \end{array}\right]$$

Interchange rows 2 and 3.

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 3 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & -1 & 0 & -3 \\ 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 3 \end{array}\right]$$

Multiply row 3 by $-1$ and add it to row 1.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -1 & -1 & -6 \\ 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 3 \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} -1 & -1 & -6 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$.

**16.** $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & -2 \\ -1 & 3 & 3 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & -1 & -2 & 0 & 1 & 0 \\ -1 & 3 & 3 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2. Also, add row 1 to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -5 & -8 & -2 & 1 & 0 \\ 0 & 5 & 6 & 1 & 0 & 1 \end{array}\right]$$

Multiply row 2 by $-\frac{1}{5}$.

$$\left[\begin{array}{rrr|rrr} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{8}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 5 & 6 & 1 & 0 & 1 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 1. Also, multiply row 2 by $-5$ and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 1 & \frac{8}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & -2 & -1 & 1 & 1 \end{array}\right]$$

Multiply row 3 by $-\frac{1}{2}$.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 1 & \frac{8}{5} & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

Multiply row 3 by $\frac{1}{5}$ and add it to row 1. Also, multiply row 3 by $-\frac{8}{5}$ and add it to row 2.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & \frac{3}{10} & \frac{3}{10} & -\frac{1}{10} \\ 0 & 1 & 0 & -\frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} \frac{3}{10} & \frac{3}{10} & -\frac{1}{10} \\ -\frac{2}{5} & \frac{3}{5} & \frac{4}{5} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$, or

$$\begin{bmatrix} 0.3 & 0.3 & -0.1 \\ -0.4 & 0.6 & 0.8 \\ 0.5 & -0.5 & -0.5 \end{bmatrix}.$$

**17.** $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by 2 and add it to row 2. Also, multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & 1 & 0 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right]$$

Add row 3 to row 1 and also to row 2.

$$\left[\begin{array}{rrr|rrr} 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right]$$

Add row 2 to row 1. Also, multiply row 2 by 3 and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$.

**18.** $\mathbf{A} = \begin{bmatrix} 7 & -1 & -9 \\ 2 & 0 & -4 \\ -4 & 0 & 6 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 7 & -1 & -9 & 1 & 0 & 0 \\ 2 & 0 & -4 & 0 & 1 & 0 \\ -4 & 0 & 6 & 0 & 0 & 1 \end{array}\right]$$

Interchange row 1 and row 2.

$$\left[\begin{array}{rrr|rrr} 2 & 0 & -4 & 0 & 1 & 0 \\ 7 & -1 & -9 & 1 & 0 & 0 \\ -4 & 0 & 6 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 2 by 2.

$$\left[\begin{array}{rrr|rrr} 2 & 0 & -4 & 0 & 1 & 0 \\ 14 & -2 & -18 & 2 & 0 & 0 \\ -4 & 0 & 6 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-7$ and add it to row 2. Also, multiply row 1 by 2 and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 2 & 0 & -4 & 0 & 1 & 0 \\ 0 & -2 & 10 & 2 & -7 & 0 \\ 0 & 0 & -2 & 0 & 2 & 1 \end{array}\right]$$

Multiply row 3 by $-2$ and add it to row 1. Also, multiply row 3 by 5 and add it to row 2.

$$\left[\begin{array}{rrr|rrr} 2 & 0 & 0 & 0 & -3 & -2 \\ 0 & -2 & 0 & 2 & 3 & 5 \\ 0 & 0 & -2 & 0 & 2 & 1 \end{array}\right]$$

Multiply row 1 by $\frac{1}{2}$. Also, multiply rows 2 and 3 by $-\frac{1}{2}$.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 0 & -\frac{3}{2} & -1 \\ 0 & 1 & 0 & -1 & -\frac{3}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 0 & -1 & -\frac{1}{2} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} 0 & -\frac{3}{2} & -1 \\ -1 & -\frac{3}{2} & -\frac{5}{2} \\ 0 & -1 & -\frac{1}{2} \end{bmatrix}$, or

$$\begin{bmatrix} 0 & -1.5 & -1 \\ -1 & -1.5 & -2.5 \\ 0 & -1 & -0.5 \end{bmatrix}.$$

**19.** $\mathbf{A} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} 1 & 3 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 3.

$$\left[\begin{array}{rrr|rrr} 1 & 3 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right]$$

Add row 3 to row 1 and also to row 2.

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -2 & 1 & -1 & 0 & 1 \end{array}\right]$$

Since the second row consists only of zeros to the left of the vertical line, it will not be possible to obtain the identity matrix on the left side. Thus, $\mathbf{A}^{-1}$ does not exist. A graphing calculator will return an error message when we try to find $\mathbf{A}^{-1}$.

**20.** $\mathbf{A} = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrr|rrr} -1 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-1$.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Add row 1 to row 2.

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Since the second and third rows are identical to the left of the vertical line, it will not be possible to obtain the identity matrix on the left side. Thus, $\mathbf{A}^{-1}$ does not exist. A graphing calculator will return an error message when we try to find $\mathbf{A}^{-1}$.

**21.** $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rrrr|rrrr} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & -5 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 4 by $-1$.

$$\left[\begin{array}{rrrr|rrrr} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & -5 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right]$$

Multiply row 4 by $-4$ and add it to row 1. Multiply row 4 by 5 and add it to row 2. Also, multiply row 4 by 2 and add it to row 3.

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 0 & 1 & 0 & 0 & 4 \\ 0 & 1 & 3 & 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right]$$

Multiply row 3 by $-3$ and add it to row 1 and to row 2.

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & -3 & 10 \\ 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 1.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -2 & 3 & 8 \\ 0 & 1 & 0 & 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} 1 & -2 & 3 & 8 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$.

**22.** $\mathbf{A} = \begin{bmatrix} -2 & -3 & 4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & -2 & 5 & 1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{cccc|cccc} -2 & -3 & 4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & -6 & 1 & 0 & 0 & 1 & 0 \\ -2 & -2 & 5 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 1 by $-1$ and add it to row 4.

$$\left[\begin{array}{cccc|cccc} -2 & -3 & 4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & -6 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

Since the second and fourth rows are identical to the left of the vertical line, it will not be possible to get the identity matrix on the left side. Thus, $\mathbf{A}^{-1}$ does not exist. A graphing calculator will return an error message when we try to find $\mathbf{A}^{-1}$.

**23.** $\mathbf{A} = \begin{bmatrix} 1 & -14 & 7 & 38 \\ -1 & 2 & 1 & -2 \\ 1 & 2 & -1 & -6 \\ 1 & -2 & 3 & 6 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{cccc|cccc} 1 & -14 & 7 & 38 & 1 & 0 & 0 & 0 \\ -1 & 2 & 1 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & -1 & -6 & 0 & 0 & 1 & 0 \\ 1 & -2 & 3 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

Add row 1 to row 2. Also, multiply row 1 by $-1$ and add it to row 3 and to row 4.

$$\left[\begin{array}{cccc|cccc} 1 & -14 & 7 & 38 & 1 & 0 & 0 & 0 \\ 0 & -12 & 8 & 36 & 1 & 1 & 0 & 0 \\ 0 & 16 & -8 & -44 & -1 & 0 & 1 & 0 \\ 0 & 12 & -4 & -32 & -1 & 0 & 0 & 1 \end{array}\right]$$

Add row 2 to row 4.

$$\left[\begin{array}{cccc|cccc} 1 & -14 & 7 & 38 & 1 & 0 & 0 & 0 \\ 0 & -12 & 8 & 36 & 1 & 1 & 0 & 0 \\ 0 & 16 & -8 & -44 & -1 & 0 & 1 & 0 \\ 0 & 0 & 4 & 4 & 0 & 1 & 0 & 1 \end{array}\right]$$

Multiply row 4 by $\frac{1}{4}$.

$$\left[\begin{array}{cccc|cccc} 1 & -14 & 7 & 38 & 1 & 0 & 0 & 0 \\ 0 & -12 & 8 & 36 & 1 & 1 & 0 & 0 \\ 0 & 16 & -8 & -44 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array}\right]$$

Multiply row 4 by $-38$ and add it to row 1. Multiply row 4 by $-36$ and add it to row 2. Also, multiply row 4 by 44 and add it to row 3.

$$\left[\begin{array}{cccc|cccc} 1 & -14 & -31 & 0 & 1 & -\frac{19}{2} & 0 & -\frac{19}{2} \\ 0 & -12 & -28 & 0 & 1 & -8 & 0 & -9 \\ 0 & 16 & 36 & 0 & -1 & 11 & 1 & 11 \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array}\right]$$

Multiply row 3 by $\frac{1}{36}$.

$$\left[\begin{array}{cccc|cccc} 1 & -14 & -31 & 0 & 1 & -\frac{19}{2} & 0 & -\frac{19}{2} \\ 0 & -12 & -28 & 0 & 1 & -8 & 0 & -9 \\ 0 & \frac{4}{9} & 1 & 0 & -\frac{1}{36} & \frac{11}{36} & \frac{1}{36} & \frac{11}{36} \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \end{array}\right]$$

Multiply row 3 by 31 and add it to row 1. Multiply row 3 by 28 and add it to row 2. Also, multiply row 3 by $-1$ and add it to row 4.

$$\left[\begin{array}{cccc|cccc} 1 & -\frac{2}{9} & 0 & 0 & \frac{5}{36} & -\frac{1}{36} & \frac{31}{36} & -\frac{1}{36} \\ 0 & \frac{4}{9} & 0 & 0 & \frac{2}{9} & \frac{5}{9} & \frac{7}{9} & -\frac{4}{9} \\ 0 & \frac{4}{9} & 1 & 0 & -\frac{1}{36} & \frac{11}{36} & \frac{1}{36} & \frac{11}{36} \\ 0 & -\frac{4}{9} & 0 & 1 & \frac{1}{36} & -\frac{1}{18} & -\frac{1}{36} & -\frac{1}{18} \end{array}\right]$$

Multiply row 2 by $\frac{1}{2}$ and add it to row 1. Also, multiply row 2 by $-1$ and add it to row 3. Add row 2 to row 4.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{5}{4} & -\frac{1}{4} \\ 0 & \frac{4}{9} & 0 & 0 & \frac{2}{9} & \frac{5}{9} & \frac{7}{9} & -\frac{4}{9} \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{4} & -\frac{3}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & -\frac{1}{2} \end{array}\right]$$

Multiply row 2 by $\frac{9}{4}$.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{5}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & \frac{1}{2} & \frac{5}{4} & \frac{7}{4} & -1 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{4} & -\frac{3}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & -\frac{1}{2} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{5}{4} & -\frac{1}{4} \\ \frac{1}{2} & \frac{5}{4} & \frac{7}{4} & -1 \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{3}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & -\frac{1}{2} \end{bmatrix}$, or

$$\begin{bmatrix} 0.25 & 0.25 & 1.25 & -0.25 \\ 0.5 & 1.25 & 1.75 & -1 \\ -0.25 & -0.25 & -0.75 & 0.75 \\ 0.25 & 0.5 & 0.75 & -0.5 \end{bmatrix}.$$

**24.** $\mathbf{A} = \begin{bmatrix} 10 & 20 & -30 & 15 \\ 3 & -7 & 14 & -8 \\ -7 & -2 & -1 & 2 \\ 4 & 4 & -3 & 1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{cccc|cccc} 10 & 20 & -30 & 15 & 1 & 0 & 0 & 0 \\ 3 & -7 & 14 & -8 & 0 & 1 & 0 & 0 \\ -7 & -2 & -1 & 2 & 0 & 0 & 1 & 0 \\ 4 & 4 & -3 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply rows 2 and 3 by 10. Also, multiply row 4 by 5.

$$\left[\begin{array}{cccc|cccc} 10 & 20 & -30 & 15 & 1 & 0 & 0 & 0 \\ 30 & -70 & 140 & -80 & 0 & 10 & 0 & 0 \\ -70 & -20 & -10 & 20 & 0 & 0 & 10 & 0 \\ 20 & 20 & -15 & 5 & 0 & 0 & 0 & 5 \end{array}\right]$$

Multiply row 1 by $-3$ and add it to row 2. Multiply row 1 by 7 and add it to row 3. Also, multiply row 1 by $-2$ and add it to row 4.

$$\left[\begin{array}{cccc|cccc} 10 & 20 & -30 & 15 & 1 & 0 & 0 & 0 \\ 0 & -130 & 230 & -125 & -3 & 10 & 0 & 0 \\ 0 & 120 & -220 & 125 & 7 & 0 & 10 & 0 \\ 0 & -20 & 45 & -25 & -2 & 0 & 0 & 5 \end{array}\right]$$

Interchange rows 2 and 4.

$$\left[\begin{array}{cccc|cccc} 10 & 20 & -30 & 15 & 1 & 0 & 0 & 0 \\ 0 & -20 & 45 & -25 & -2 & 0 & 0 & 5 \\ 0 & 120 & -220 & 125 & 7 & 0 & 10 & 0 \\ 0 & -130 & 230 & -125 & -3 & 10 & 0 & 0 \end{array}\right]$$

Multiply row 4 by 2.

$$\left[\begin{array}{cccc|cccc} 10 & 20 & -30 & 15 & 1 & 0 & 0 & 0 \\ 0 & -20 & 45 & -25 & -2 & 0 & 0 & 5 \\ 0 & 120 & -220 & 125 & 7 & 0 & 10 & 0 \\ 0 & -260 & 460 & -250 & -6 & 20 & 0 & 0 \end{array}\right]$$

Add row 2 to row 1. Multiply row 2 by 6 and add it to row 3. Also, multiply row 2 by $-13$ and add it to row 4.

$$\left[\begin{array}{cccc|cccc} 10 & 0 & 15 & -10 & -1 & 0 & 0 & 5 \\ 0 & -20 & 45 & -25 & -2 & 0 & 0 & 5 \\ 0 & 0 & 50 & -25 & -5 & 0 & 10 & 30 \\ 0 & 0 & -125 & 75 & 20 & 20 & 0 & -65 \end{array}\right]$$

Multiply rows 1 and 2 by 10 and multiply row 4 by 2.

$$\left[\begin{array}{cccc|cccc} 100 & 0 & 150 & -100 & -10 & 0 & 0 & 50 \\ 0 & -200 & 450 & -250 & -20 & 0 & 0 & 50 \\ 0 & 0 & 50 & -25 & -5 & 0 & 10 & 30 \\ 0 & 0 & -250 & 150 & 40 & 40 & 0 & -130 \end{array}\right]$$

Multiply row 3 by $-3$ and add it to row 1. Multiply row 3 by $-9$ and add it to row 2. Also, multiply row 3 by 5 and add it to row 4.

$$\left[\begin{array}{cccc|cccc} 100 & 0 & 0 & -25 & 5 & 0 & -30 & -40 \\ 0 & -200 & 0 & -25 & 25 & 0 & -90 & -220 \\ 0 & 0 & 50 & -25 & -5 & 0 & 10 & 30 \\ 0 & 0 & 0 & 25 & 15 & 40 & 50 & 20 \end{array}\right]$$

Add row 4 to rows 1, 2, and 3.

$$\left[\begin{array}{cccc|cccc} 100 & 0 & 0 & 0 & 20 & 40 & 20 & -20 \\ 0 & -200 & 0 & 0 & 40 & 40 & -40 & -200 \\ 0 & 0 & 50 & 0 & 10 & 40 & 60 & 50 \\ 0 & 0 & 0 & 25 & 15 & 40 & 50 & 20 \end{array}\right]$$

Multiply row 1 by $\frac{1}{100}$, multiply row 2 by $-\frac{1}{200}$, multiply row 3 by $\frac{1}{50}$, and multiply row 4 by $\frac{1}{25}$.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & -\frac{1}{5} \\ 0 & 1 & 0 & 0 & -\frac{1}{5} & -\frac{1}{5} & \frac{1}{5} & 1 \\ 0 & 0 & 1 & 0 & \frac{1}{5} & \frac{4}{5} & \frac{6}{5} & 1 \\ 0 & 0 & 0 & 1 & \frac{3}{5} & \frac{8}{5} & 2 & \frac{4}{5} \end{array}\right]$$

Then $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & \frac{1}{5} & 1 \\ \frac{1}{5} & \frac{4}{5} & \frac{6}{5} & 1 \\ \frac{3}{5} & \frac{8}{5} & 2 & \frac{4}{5} \end{bmatrix}$, or

$$\begin{bmatrix} 0.2 & 0.4 & 0.2 & -0.2 \\ -0.2 & -0.2 & 0.2 & 1 \\ 0.2 & 0.8 & 1.2 & 1 \\ 0.6 & 1.6 & 2 & 0.8 \end{bmatrix}.$$

**25.** Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

Then we have $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} -23 \\ 83 \end{bmatrix}$$

The solution is $(-23, 83)$.

**26.** $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 28 \\ -46 \end{bmatrix}$

The solution is $(28, -46)$.

**27.** Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}$$

Then we have $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 3 & 1 & -2 \\ 0 & -3 & 6 \\ -3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -9 \\ 45 \\ 9 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}$$

The solution is $(-1, 5, 1)$.

**28.** $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -3 & 2 & -1 \\ 12 & -3 & 4 \\ 7 & -3 & 4 \end{bmatrix}\begin{bmatrix} -4 \\ -3 \\ 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 5 \\ -35 \\ -15 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -3 \end{bmatrix}$$

The solution is $(1, -7, -3)$.

**29.** $4x + 3y = 2,$
$x - 2y = 6$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ \frac{1}{11} & -\frac{4}{11} \end{bmatrix}\begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$.

The solution is $(2, -2)$.

**30.** $2x - 3y = 7,$
$4x + y = -7$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -7 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}\begin{bmatrix} 7 \\ -7 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$.

The solution is $(-1, -3)$.

**31.** $5x + y = 2,$
$3x - 2y = -4$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{13} & \frac{1}{13} \\ \frac{3}{13} & -\frac{5}{13} \end{bmatrix}\begin{bmatrix} 2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

The solution is $(0, 2)$.

**32.** $x - 6y = 5,$
$-x + 4y = -5$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 1 & -6 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} -2 & -3 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 5 \\ -5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$.

The solution is $(5, 0)$.

**33.** $x + z = 1,$
$2x + y = 3,$
$x - y + z = 4$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & -1 \\ \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -2 \end{bmatrix}$.

The solution is $(3, -3, -2)$.

**34.** $x + 2y + 3z = -1,$
$2x - 3y + 4z = 2,$
$-3x + 5y - 6z = 4$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 4 \\ -3 & 5 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} -2 & 27 & 17 \\ 0 & 3 & 2 \\ 1 & -11 & -7 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 124 \\ 14 \\ -51 \end{bmatrix}$.

The solution is $(124, 14, -51)$.

**35.** $2x + 3y + 4z = 2,$
$x - 4y + 3z = 2,$
$5x + y + z = -4$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -4 & 3 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} -\frac{1}{16} & \frac{1}{112} & \frac{25}{112} \\ \frac{1}{8} & -\frac{9}{56} & -\frac{1}{56} \\ \frac{3}{16} & \frac{13}{112} & -\frac{11}{112} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

The solution is $(-1, 0, 1)$.

**36.** $$\begin{aligned} x + y \quad\; &= 2,\\ 3x \quad + 2z &= 5,\\ 2x + 3y - 3z &= 9 \end{aligned}$$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & 0 & 2 \\ 2 & 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$$

Then $\mathbf{X}=\mathbf{A}^{-1}\mathbf{B}= \begin{bmatrix} -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{13}{7} & -\frac{3}{7} & -\frac{2}{7} \\ \frac{9}{7} & -\frac{1}{7} & -\frac{3}{7} \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$.

The solution is $(3, -1, -2)$.

**37.** $$\begin{aligned} 2w - 3x + 4y - 5z &= 0,\\ 3w - 2x + 7y - 3z &= 2,\\ w + x - y + z &= 1,\\ -w - 3x - 6y + 4z &= 6 \end{aligned}$$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 2 & -3 & 4 & -5 \\ 3 & -2 & 7 & -3 \\ 1 & 1 & -1 & 1 \\ -1 & -3 & -6 & 4 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 6 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \dfrac{1}{203}\begin{bmatrix} 26 & 11 & 127 & 9 \\ -8 & -19 & 39 & -34 \\ -37 & 39 & -48 & -5 \\ -55 & 47 & -11 & 20 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$.

The solution is $(1, -1, 0, 1)$.

**38.** $$\begin{aligned} 5w - 4x + 3y - 2z &= -6,\\ w + 4x - 2y + 3z &= -5,\\ 2w - 3x + 6y - 9z &= 14,\\ 3w - 5x + 2y - 4z &= -3 \end{aligned}$$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 5 & -4 & 3 & -2 \\ 1 & 4 & -2 & 3 \\ 2 & -3 & 6 & -9 \\ 3 & -5 & 2 & -4 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ -5 \\ 14 \\ -3 \end{bmatrix}$$

Then

$$\mathbf{X}=\mathbf{A}^{-1}\mathbf{B}=\frac{1}{302}\begin{bmatrix} 18 & 75 & 1 & 45 \\ -10 & 59 & 33 & -25 \\ 112 & -87 & 23 & -173 \\ 82 & -61 & -29 & -97 \end{bmatrix} \begin{bmatrix} -6 \\ -5 \\ 14 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \\ -1 \end{bmatrix}.$$

The solution is $(-2, 1, 2, -1)$.

**39.** ***Familiarize***. Let $x$ = the number of hot dogs sold and $y$ = the number of sausages.

***Translate***.

The total number of items sold was 145.

$$x + y = 145$$

The number of hot dogs sold is 45 more than the number of sausages.

$$x = y + 45$$

We have a system of equations:

$$\begin{aligned} x + y &= 145, \\ x &= y + 45, \end{aligned} \quad \text{or} \quad \begin{aligned} x + y &= 145, \\ x - y &= 45. \end{aligned}$$

***Carry out***. Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 145 \\ 45 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 145 \\ 45 \end{bmatrix} = \begin{bmatrix} 95 \\ 50 \end{bmatrix}$, so the solution is $(95, 50)$.

***Check***. The total number of items is $95+50$, or 145. The number of hot dogs, 95, is 45 more than the number of sausages. The solution checks.

***State***. Stefan sold 95 hot dogs and 50 Italian sausages.

**40.** Let $x$ = the price of a lab record book and $y$ = the price of a highlighter.

Solve: $$\begin{aligned} 4x + 3y &= 17.83,\\ 3x + 2y &= 13.05 \end{aligned}$$

Writing $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17.83 \\ 13.05 \end{bmatrix}$, we find that $x = \$3.49$, $y = \$1.29$.

**41.** ***Familiarize***. Let $x$, $y$, and $z$ represent the prices of one ton of topsoil, mulch, and pea gravel, respectively.

***Translate***.

Four tons of topsoil, 3 tons of mulch, and 6 tons of pea gravel costs \$2825.

$$4x + 3y + 6z = 2825$$

Five tons of topsoil, 2 tons of mulch, and 5 tons of pea gravel costs \$2663.

$$5x + 2y + 5z = 2663$$

Pea gravel costs \$17 less per ton than topsoil.

$$z = x - 17$$

We have a system of equations.

$$\begin{aligned} 4x + 3y + 6z &= 2825,\\ 5x + 2y + 5z &= 2663,\\ z &= x - 17, \text{ or} \end{aligned}$$

$$\begin{aligned} 4x + 3y + 6z &= 2825,\\ 5x + 2y + 5z &= 2663,\\ x \qquad\quad - z &= 17 \end{aligned}$$

***Carry out***. Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 4 & 3 & 6 \\ 5 & 2 & 5 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2825 \\ 2663 \\ 17 \end{bmatrix}$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} -\frac{1}{5} & \frac{3}{10} & \frac{3}{10} \\ 1 & -1 & 1 \\ -\frac{1}{5} & \frac{3}{10} & -\frac{7}{10} \end{bmatrix} \begin{bmatrix} 2825 \\ 2663 \\ 17 \end{bmatrix} =$

$\begin{bmatrix} 239 \\ 179 \\ 222 \end{bmatrix}$, so the solution is $(239, 179, 222)$.

***Check***. Four tons of topsoil, 3 tons of mulch, and 6 tons of pea gravel costs $4 \cdot \$239 + 3 \cdot \$179 + 6 \cdot \$222$, or $\$956 + \$537 + \$1332$, or \$2825. Five tons of topsoil, 2 tons of mulch, and 5 tons of pea gravel costs $5 \cdot \$239 + 2 \cdot \$179 + 5 \cdot \$222$, or $\$1195 + \$358 + \$1110$, or \$2663. The price of pea gravel, \$222, is \$17 less than the price of topsoil, \$239. The solution checks.

***State***. The price of topsoil is \$239 per ton, of mulch is \$179 per ton, and of pea gravel is \$222 per ton.

**42.** Let $x$, $y$, and $z$ represent the amounts invested at 2.2%, 2.65%, and 3.05%, respectively.

Solve: $x + y + z = 8500$,
$0.022x + 0.0265y + 0.0305z = 230$,
$z = x + 1500$

Writing

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.022 & 0.0265 & 0.0305 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8500 \\ 230 \\ 1500 \end{bmatrix},$$

we find that $x = \$2500$, $y = \$2000$, $z = \$4000$.

**43.**
$$\begin{array}{r|rrrr} -2 & 1 & -6 & 4 & -8 \\ & & -2 & 16 & -40 \\ \hline & 1 & -8 & 20 & \boxed{-48} \end{array}$$

$f(-2) = -48$

**44.**
$$\begin{array}{r|rrrrr} 3 & 2 & -1 & 5 & 6 & -4 \\ & & 6 & 15 & 60 & 198 \\ \hline & 2 & 5 & 20 & 66 & \boxed{194} \end{array}$$

$f(3) = 194$

**45.**
$$\begin{aligned} 2x^2 + x &= 7 \\ 2x^2 + x - 7 &= 0 \end{aligned}$$
$a = 2$, $b = 1$, $c = -7$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-7)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1 + 56}}{4} \\ &= \frac{-1 \pm \sqrt{57}}{4} \end{aligned}$$

The solutions are $\frac{-1 + \sqrt{57}}{4}$ and $\frac{-1 - \sqrt{57}}{4}$, or $\frac{-1 \pm \sqrt{57}}{4}$.

**46.** $\frac{1}{x+1} - \frac{6}{x-1} = 1$, LCD is $(x+1)(x-1)$
$$\begin{aligned} (x+1)(x-1)\left(\frac{1}{x+1} - \frac{6}{x-1}\right) &= (x+1)(x-1) \cdot 1 \\ x - 1 - 6(x+1) &= x^2 - 1 \\ x - 1 - 6x - 6 &= x^2 - 1 \\ 0 &= x^2 + 5x + 6 \\ 0 &= (x+3)(x+2) \end{aligned}$$
$x = -3$ *or* $x = -2$

Both numbers check.

**47.**
$$\begin{aligned} \sqrt{2x+1} - 1 &= \sqrt{2x-4} \\ (\sqrt{2x+1} - 1)^2 &= (\sqrt{2x-4})^2 \quad \text{Squaring both sides} \\ 2x + 1 - 2\sqrt{2x+1} + 1 &= 2x - 4 \\ 2x + 2 - 2\sqrt{2x+1} &= 2x - 4 \\ 2 - 2\sqrt{2x+1} &= -4 \quad \text{Subtracting } 2x \\ -2\sqrt{2x+1} &= -6 \quad \text{Subtracting 2} \\ \sqrt{2x+1} &= 3 \quad \text{Dividing by } -2 \\ (\sqrt{2x+1})^2 &= 3^2 \quad \text{Squaring both sides} \\ 2x + 1 &= 9 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

The number 4 checks. It is the solution.

**48.** $x - \sqrt{x} - 6 = 0$

Let $u = \sqrt{x}$.
$$\begin{aligned} u^2 - u - 6 &= 0 \\ (u-3)(u+2) &= 0 \end{aligned}$$
$u = 3$ *or* $u = -2$
$\sqrt{x} = 3$ *or* $\sqrt{x} = -2$
$x = 9$ No solution

The number 9 checks. It is the solution.

**49.** $f(x) = x^3 - 3x^2 - 6x + 8$

We use synthetic division to find one factor. We first try $x - 1$.
$$\begin{array}{r|rrrr} 1 & 1 & -3 & -6 & 8 \\ & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$$

Since $f(1) = 0$, $x - 1$ is a factor of $f(x)$. We have $f(x) = (x-1)(x^2 - 2x - 8)$. Factoring the trinomial we get $f(x) = (x-1)(x-4)(x+2)$.

**50.** $f(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$

We try $x - 1$.
$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -16 & -2 & 15 \\ & & 1 & 3 & -13 & -15 \\ \hline & 1 & 3 & -13 & -15 & \boxed{0} \end{array}$$

We have $f(x) = (x-1)(x^3 + 3x^2 - 13x - 15)$. Now we factor the cubic polynomial. We try $x + 1$.

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -13 & -15 \\ & & -1 & -2 & 15 \\ \hline & 1 & 2 & -15 & 0 \end{array}$$

Now we have $f(x) = (x-1)(x+1)(x^2+2x-15) = (x-1)(x+1)(x+5)(x-3)$.

**51.** $\mathbf{A} = [x]$

Write the augmented matrix.

$\left[\begin{array}{c|c} x & 1 \end{array}\right]$

Multiply by $\dfrac{1}{x}$.

$\left[\begin{array}{c|c} 1 & \dfrac{1}{x} \end{array}\right]$

Then $\mathbf{A}^{-1}$ exists if and only if $x \neq 0$. $\mathbf{A}^{-1} = \left[\dfrac{1}{x}\right]$.

**52.** $\mathbf{A} = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$

Write the augmented matrix.

$\left[\begin{array}{cc|cc} x & 0 & 1 & 0 \\ 0 & y & 0 & 1 \end{array}\right]$

Multiply row 1 by $\dfrac{1}{x}$ and row 2 by $\dfrac{1}{y}$.

$\left[\begin{array}{cc|cc} 1 & 0 & \dfrac{1}{x} & 0 \\ 0 & 1 & 0 & \dfrac{1}{y} \end{array}\right]$

Then $\mathbf{A}^{-1}$ exists if and only if $x \neq 0$ and $y \neq 0$, or if and only if $xy \neq 0$.

$\mathbf{A}^{-1} = \begin{bmatrix} \dfrac{1}{x} & 0 \\ 0 & \dfrac{1}{y} \end{bmatrix}$

**53.** $\mathbf{A} = \begin{bmatrix} 0 & 0 & x \\ 0 & y & 0 \\ z & 0 & 0 \end{bmatrix}$

Write the augmented matrix.

$\left[\begin{array}{ccc|ccc} 0 & 0 & x & 1 & 0 & 0 \\ 0 & y & 0 & 0 & 1 & 0 \\ z & 0 & 0 & 0 & 0 & 1 \end{array}\right]$

Interchange row 1 and row 3.

$\left[\begin{array}{ccc|ccc} z & 0 & 0 & 0 & 0 & 1 \\ 0 & y & 0 & 0 & 1 & 0 \\ 0 & 0 & x & 1 & 0 & 0 \end{array}\right]$

Multiply row 1 by $\dfrac{1}{z}$, row 2 by $\dfrac{1}{y}$, and row 3 by $\dfrac{1}{x}$.

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & \dfrac{1}{z} \\ 0 & 1 & 0 & 0 & \dfrac{1}{y} & 0 \\ 0 & 0 & 1 & \dfrac{1}{x} & 0 & 0 \end{array}\right]$

Then $\mathbf{A}^{-1}$ exists if and only if $x \neq 0$ and $y \neq 0$ and $z \neq 0$, or if and only if $xyz \neq 0$.

$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & \dfrac{1}{z} \\ 0 & \dfrac{1}{y} & 0 \\ \dfrac{1}{x} & 0 & 0 \end{bmatrix}$

**54.** $\mathbf{A} = \begin{bmatrix} x & 1 & 1 & 1 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & w \end{bmatrix}$

Write the augmented matrix.

$\left[\begin{array}{cccc|cccc} x & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & y & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w & 0 & 0 & 0 & 1 \end{array}\right]$

Multiply row 4 by $-\dfrac{1}{w}$ and add it to row 1.

$\left[\begin{array}{cccc|cccc} x & 1 & 1 & 0 & 1 & 0 & 0 & -\dfrac{1}{w} \\ 0 & y & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w & 0 & 0 & 0 & 1 \end{array}\right]$

Multiply row 3 by $-\dfrac{1}{z}$ and add it to row 1.

$\left[\begin{array}{cccc|cccc} x & 1 & 0 & 0 & 1 & 0 & -\dfrac{1}{z} & -\dfrac{1}{w} \\ 0 & y & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w & 0 & 0 & 0 & 1 \end{array}\right]$

Multiply row 2 by $-\dfrac{1}{y}$ and add it to row 1.

$\left[\begin{array}{cccc|cccc} x & 0 & 0 & 0 & 1 & -\dfrac{1}{y} & -\dfrac{1}{z} & -\dfrac{1}{w} \\ 0 & y & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w & 0 & 0 & 0 & 1 \end{array}\right]$

Multiply row 1 by $\dfrac{1}{x}$, row 2 by $\dfrac{1}{y}$, row 3 by $\dfrac{1}{z}$, and row 4 by $\dfrac{1}{w}$.

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \dfrac{1}{x} & -\dfrac{1}{xy} & -\dfrac{1}{xz} & -\dfrac{1}{xw} \\ 0 & 1 & 0 & 0 & 0 & \dfrac{1}{y} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \dfrac{1}{z} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dfrac{1}{w} \end{array}\right]$

Then $\mathbf{A}^{-1}$ exists if and only if $w \neq 0$ and $x \neq 0$ and $y \neq 0$ and $z \neq 0$, or if and only if $wxyz \neq 0$.

$\mathbf{A}^{-1} = \begin{bmatrix} \dfrac{1}{x} & -\dfrac{1}{xy} & -\dfrac{1}{xz} & -\dfrac{1}{xw} \\ 0 & \dfrac{1}{y} & 0 & 0 \\ 0 & 0 & \dfrac{1}{z} & 0 \\ 0 & 0 & 0 & \dfrac{1}{w} \end{bmatrix}$

## Exercise Set 9.6

**1.** $\begin{vmatrix} 5 & 3 \\ -2 & -4 \end{vmatrix} = 5(-4) - (-2) \cdot 3 = -20 + 6 = -14$

**2.** $\begin{vmatrix} -8 & 6 \\ -1 & 2 \end{vmatrix} = -8 \cdot 2 - (-1) \cdot 6 = -16 + 6 = -10$

**3.** $\begin{vmatrix} 4 & -7 \\ -2 & 3 \end{vmatrix} = 4 \cdot 3 - (-2)(-7) = 12 - 14 = -2$

**4.** $\begin{vmatrix} -9 & -6 \\ 5 & 4 \end{vmatrix} = -9 \cdot 4 - 5(-6) = -36 + 30 = -6$

**5.** $\begin{vmatrix} -2 & -\sqrt{5} \\ -\sqrt{5} & 3 \end{vmatrix} = -2 \cdot 3 - (-\sqrt{5})(-\sqrt{5}) = -6 - 5 = -11$

**6.** $\begin{vmatrix} \sqrt{5} & -3 \\ 4 & 2 \end{vmatrix} = \sqrt{5}(2) - 4(-3) = 2\sqrt{5} + 12$

**7.** $\begin{vmatrix} x & 4 \\ x & x^2 \end{vmatrix} = x \cdot x^2 - x \cdot 4 = x^3 - 4x$

**8.** $\begin{vmatrix} y^2 & -2 \\ y & 3 \end{vmatrix} = y^2 \cdot 3 - y(-2) = 3y^2 + 2y$

**9.** $\mathbf{A} = \begin{bmatrix} 7 & -4 & -6 \\ 2 & 0 & -3 \\ 1 & 2 & -5 \end{bmatrix}$

$M_{11}$ is the determinant of the matrix formed by deleting the first row and first column of $\mathbf{A}$:

$M_{11} = \begin{vmatrix} 0 & -3 \\ 2 & -5 \end{vmatrix} = 0(-5) - 2(-3) = 0 + 6 = 6$

$M_{32}$ is the determinant of the matrix formed by deleting the third row and second column of $\mathbf{A}$:

$M_{32} = \begin{vmatrix} 7 & -6 \\ 2 & -3 \end{vmatrix} = 7(-3) - 2(-6) = -21 + 12 = -9$

$M_{22}$ is the determinant of the matrix formed by deleting the second row and second column of $\mathbf{A}$:

$M_{22} = \begin{vmatrix} 7 & -6 \\ 1 & -5 \end{vmatrix} = 7(-5) - 1(-6) = -35 + 6 = -29$

**10.** See matrix $\mathbf{A}$ in Exercise 9 above:

$M_{13} = \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = 2 \cdot 2 - 1 \cdot 0 = 4 - 0 = 4$

$M_{31} = \begin{vmatrix} -4 & -6 \\ 0 & -3 \end{vmatrix} = -4(-3) - 0(-6) = 12 - 0 = 12$

$M_{23} = \begin{vmatrix} 7 & -4 \\ 1 & 2 \end{vmatrix} = 7 \cdot 2 - 1(-4) = 14 + 4 = 18$

**11.** In Exercise 9 we found that $M_{11} = 6$.

$A_{11} = (-1)^{1+1} M_{11} = 1 \cdot 6 = 6$

In Exercise 9 we found that $M_{32} = -9$.

$A_{32} = (-1)^{3+2} M_{32} = -1(-9) = 9$

In Exercise 9 we found that $M_{22} = -29$.

$A_{22} = (-1)^{2+2}(-29) = 1(-29) = -29$

**12.** In Exercise 10 we found that $M_{13} = 4$.

$A_{13} = (-1)^{1+3} M_{13} = 1 \cdot 4 = 4$

In Exercise 10 we found that $M_{31} = 12$.

$A_{31} = (-1)^{3+1} M_{31} = 1 \cdot 12 = 12$

In Exercise 10 we found that $M_{23} = 18$.

$A_{23} = (-1)^{2+3} M_{23} = -1 \cdot 18 = -18$

**13.** $\mathbf{A} = \begin{bmatrix} 7 & -4 & -6 \\ 2 & 0 & -3 \\ 1 & 2 & -5 \end{bmatrix}$

$$\begin{aligned}
&|\mathbf{A}| \\
&= 2A_{21} + 0A_{22} + (-3)A_{23} \\
&= 2(-1)^{2+1}\begin{vmatrix} -4 & -6 \\ 2 & -5 \end{vmatrix} + 0 + (-3)(-1)^{2+3}\begin{vmatrix} 7 & -4 \\ 1 & 2 \end{vmatrix} \\
&= 2(-1)[-4(-5) - 2(-6)] + 0 + \\
&\qquad (-3)(-1)[7 \cdot 2 - 1(-4)] \\
&= -2(32) + 0 + 3(18) = -64 + 0 + 54 \\
&= -10
\end{aligned}$$

**14.** See matrix $\mathbf{A}$ in Exercise 13 above.

$$\begin{aligned}
&|\mathbf{A}| \\
&= -4A_{12} + 0A_{22} + 2A_{32} \\
&= -4(-1)^{1+2}\begin{vmatrix} 2 & -3 \\ 1 & -5 \end{vmatrix} + 0 + 2(-1)^{3+2}\begin{vmatrix} 7 & -6 \\ 2 & -3 \end{vmatrix} \\
&= -4(-1)[2(-5) - 1(-3)] + 0 + \\
&\qquad 2(-1)[7(-3) - 2(-6)] \\
&= 4(-7) + 0 - 2(-9) = -28 + 0 + 18 \\
&= -10
\end{aligned}$$

**15.** $\mathbf{A} = \begin{bmatrix} 7 & -4 & -6 \\ 2 & 0 & -3 \\ 1 & 2 & -5 \end{bmatrix}$

$$\begin{aligned}
&|\mathbf{A}| \\
&= -6A_{13} + (-3)A_{23} + (-5)A_{33} \\
&= -6(-1)^{1+3}\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} + (-3)(-1)^{2+3}\begin{vmatrix} 7 & -4 \\ 1 & 2 \end{vmatrix} + \\
&\qquad (-5)(-1)^{3+3}\begin{vmatrix} 7 & -4 \\ 2 & 0 \end{vmatrix} \\
&= -6 \cdot 1(2 \cdot 2 - 1 \cdot 0) + (-3)(-1)[7 \cdot 2 - 1(-4)] + \\
&\qquad -5 \cdot 1(7 \cdot 0 - 2(-4)) \\
&= -6(4) + 3(18) - 5(8) = -24 + 54 - 40 \\
&= -10
\end{aligned}$$

**16.** See matrix **A** in Exercise 15 above.

$$\begin{aligned}&|\mathbf{A}|\\&= 7A_{11} - 4A_{12} - 6A_{13}\\&= 7(-1)^{1+1}\begin{vmatrix}0 & -3\\2 & -5\end{vmatrix} + (-4)(-1)^{1+2}\begin{vmatrix}2 & -3\\1 & -5\end{vmatrix} +\\&\qquad (-6)(-1)^{1+3}\begin{vmatrix}2 & 0\\1 & 2\end{vmatrix}\\&= 7\cdot 1[0(-5)-2(-3)]+(-4)(-1)[2(-5)-1(-3)]+\\&\qquad (-6)(1)(2\cdot 2 - 1\cdot 0)\\&= 7(6) + 4(-7) - 6(4) = 42 - 28 - 24\\&= -10\end{aligned}$$

**17.** $\mathbf{A} = \begin{bmatrix}1 & 0 & 0 & -2\\4 & 1 & 0 & 0\\5 & 6 & 7 & 8\\-2 & -3 & -1 & 0\end{bmatrix}$

$M_{12} = \begin{bmatrix}4 & 0 & 0\\5 & 7 & 8\\-2 & -1 & 0\end{bmatrix}$

We will expand $M_{12}$ across the first row.

$$\begin{aligned}M_{12} &= 4(-1)^{1+1}\begin{vmatrix}7 & 8\\-1 & 0\end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix}5 & 8\\-2 & 0\end{vmatrix} +\\&\qquad 0(-1)^{1+3}\begin{vmatrix}5 & 7\\-2 & -1\end{vmatrix}\\&= 4\cdot 1[7\cdot 0 - (-1)8] + 0 + 0\\&= 4(8) = 32\end{aligned}$$

$M_{44} = \begin{bmatrix}1 & 0 & 0\\4 & 1 & 0\\5 & 6 & 7\end{bmatrix}$

We will expand $M_{44}$ across the first row.

$$\begin{aligned}M_{44} &= 1(-1)^{1+1}\begin{vmatrix}1 & 0\\6 & 7\end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix}4 & 0\\5 & 7\end{vmatrix} +\\&\qquad 0(-1)^{1+3}\begin{vmatrix}4 & 1\\5 & 6\end{vmatrix}\\&= 1\cdot 1(1\cdot 7 - 6\cdot 0) + 0 + 0\\&= 1(7) = 7\end{aligned}$$

**18.** $\mathbf{A} = \begin{bmatrix}1 & 0 & 0 & -2\\4 & 1 & 0 & 0\\5 & 6 & 7 & 8\\-2 & -3 & -1 & 0\end{bmatrix}$

$M_{41}$ is the determinant of the matrix formed by deleting the fourth row and the first column of **A**.

$M_{41} = \begin{bmatrix}0 & 0 & -2\\1 & 0 & 0\\6 & 7 & 8\end{bmatrix}$

We will expand $M_{41}$ across the first row.

$$\begin{aligned}M_{41} &= 0(-1)^{1+1}\begin{vmatrix}0 & 0\\7 & 8\end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix}1 & 0\\6 & 8\end{vmatrix} +\\&\qquad (-2)(-1)^{3+1}\begin{vmatrix}1 & 0\\6 & 7\end{vmatrix}\\&= 0 + 0 + (-2)(1)(1\cdot 7 - 6\cdot 0)\\&= 0 + 0 - 2(7) = -14\end{aligned}$$

$M_{33}$ is the determinant of the matrix formed by deleting the third row and the third column of **A**.

$M_{33} = \begin{bmatrix}1 & 0 & -2\\4 & 1 & 0\\-2 & -3 & 0\end{bmatrix}$

We will expand $M_{33}$ down the third column.

$$\begin{aligned}M_{33} &= -2(-1)^{1+3}\begin{vmatrix}4 & 1\\-2 & -3\end{vmatrix} + 0(-1)^{2+3}\begin{vmatrix}1 & 0\\-2 & -3\end{vmatrix} +\\&\qquad 0(-1)^{3+3}\begin{vmatrix}1 & 0\\4 & 1\end{vmatrix}\\&= -2(1)[4(-3) - (-2)(1)] + 0 + 0\\&= -2(-10) + 0 + 0 = 20\end{aligned}$$

**19.** $\mathbf{A} = \begin{bmatrix}1 & 0 & 0 & -2\\4 & 1 & 0 & 0\\5 & 6 & 7 & 8\\-2 & -3 & -1 & 0\end{bmatrix}$

$$\begin{aligned}A_{22} &= (-1)^{2+2}M_{22} = M_{22}\\&= \begin{vmatrix}1 & 0 & -2\\5 & 7 & 8\\-2 & -1 & 0\end{vmatrix}\end{aligned}$$

We will expand across the first row.

$$\begin{aligned}&\begin{vmatrix}1 & 0 & -2\\5 & 7 & 8\\-2 & -1 & 0\end{vmatrix}\\&= 1(-1)^{1+1}\begin{vmatrix}7 & 8\\-1 & 0\end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix}5 & 8\\-2 & 0\end{vmatrix} +\\&\qquad (-2)(-1)^{1+3}\begin{vmatrix}5 & 7\\-2 & -1\end{vmatrix}\\&= 1\cdot 1[7\cdot 0 - (-1)8] + 0 + (-2)\cdot 1[5(-1) - (-2)7]\\&= 1(8) + 0 - 2(9) = 8 + 0 - 18\\&= -10\end{aligned}$$

$$\begin{aligned}A_{34} &= (-1)^{3+4}M_{34} = -1\cdot M_{34}\\&= -1\cdot\begin{vmatrix}1 & 0 & 0\\4 & 1 & 0\\-2 & -3 & -1\end{vmatrix}\end{aligned}$$

We will expand across the first row.

$$\begin{aligned}&-1\cdot\begin{vmatrix}1 & 0 & 0\\4 & 1 & 0\\-2 & -3 & -1\end{vmatrix}\\&= -1\left[1(-1)^{1+1}\begin{vmatrix}1 & 0\\-3 & -1\end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix}4 & 0\\-2 & -1\end{vmatrix} +\right.\\&\qquad \left. 0(-1)^{1+3}\begin{vmatrix}4 & 1\\-2 & -3\end{vmatrix}\right]\\&= -1[1\cdot 1(1(-1) - (-3)\cdot 0) + 0 + 0]\\&= -1[1(-1)] = 1\end{aligned}$$

**20.** $\mathbf{A} = \begin{bmatrix}1 & 0 & 0 & -2\\4 & 1 & 0 & 0\\5 & 6 & 7 & 8\\-2 & -3 & -1 & 0\end{bmatrix}$

$$A_{24} = (-1)^{2+4}M_{24} = 1 \cdot M_{24} = M_{24}$$
$$= \begin{vmatrix} 1 & 0 & 0 \\ 5 & 6 & 7 \\ -2 & -3 & -1 \end{vmatrix}$$

We will expand across the first row.

$$\begin{vmatrix} 1 & 0 & 0 \\ 5 & 6 & 7 \\ -2 & -3 & -1 \end{vmatrix}$$
$$= 1(-1)^{1+1}\begin{vmatrix} 6 & 7 \\ -3 & -1 \end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix} 5 & 7 \\ -2 & -1 \end{vmatrix} +$$
$$0(-1)^{1+3}\begin{vmatrix} 5 & 6 \\ -2 & -3 \end{vmatrix}$$
$$= 1 \cdot 1[6(-1) - (-3)(7)] + 0 + 0$$
$$= 1(15) = 15$$

$$A_{43} = (-1)^{4+3}M_{43} = -1 \cdot M_{43}$$
$$= -1 \cdot \begin{vmatrix} 1 & 0 & -2 \\ 4 & 1 & 0 \\ 5 & 6 & 8 \end{vmatrix}$$

We will expand across the first row.

$$-1 \cdot \begin{vmatrix} 1 & 0 & -2 \\ 4 & 1 & 0 \\ 5 & 6 & 8 \end{vmatrix}$$
$$= -1\left[1(-1)^{1+1}\begin{vmatrix} 1 & 0 \\ 6 & 8 \end{vmatrix} + 0(-1)^{1+2}\begin{vmatrix} 4 & 0 \\ 5 & 8 \end{vmatrix} + \right.$$
$$\left.(-2)(-1)^{1+3}\begin{vmatrix} 4 & 1 \\ 5 & 6 \end{vmatrix}\right]$$
$$= -1[1 \cdot 1(1 \cdot 8 - 6 \cdot 0) + 0 + (-2) \cdot 1(4 \cdot 6 - 5 \cdot 1)]$$
$$= -1[1(8) + 0 - 2(19)]$$
$$= -1(8 - 38) = -1(-30)$$
$$= 30$$

**21.** $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 4 & 1 & 0 & 0 \\ 5 & 6 & 7 & 8 \\ -2 & -3 & -1 & 0 \end{bmatrix}$

$$|\mathbf{A}|$$
$$= 1 \cdot A_{11} + 0 \cdot A_{12} + 0 \cdot A_{13} + (-2)A_{14}$$
$$= A_{11} + (-2)A_{14}$$
$$= (-1)^{1+1}\begin{vmatrix} 1 & 0 & 0 \\ 6 & 7 & 8 \\ -3 & -1 & 0 \end{vmatrix} +$$
$$(-2)(-1)^{1+4}\begin{vmatrix} 4 & 1 & 0 \\ 5 & 6 & 7 \\ -2 & -3 & -1 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 0 & 0 \\ 6 & 7 & 8 \\ -3 & -1 & 0 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 & 0 \\ 5 & 6 & 7 \\ -2 & -3 & -1 \end{vmatrix}$$

We will expand each determinant across the first row. We have:

$$1(-1)^{1+1}\begin{vmatrix} 7 & 8 \\ -1 & 0 \end{vmatrix} + 0 + 0 +$$
$$2\left[4(-1)^{1+1}\begin{vmatrix} 6 & 7 \\ -3 & -1 \end{vmatrix} + 1(-1)^{1+2}\begin{vmatrix} 5 & 7 \\ -2 & -1 \end{vmatrix} + 0\right]$$
$$= 1 \cdot 1[7 \cdot 0 - (-1)8] + 2[4 \cdot 1[6(-1) - (-3) \cdot 7] +$$
$$1(-1)[5(-1) - (-2) \cdot 7]$$
$$= 1(8) + 2[4(15) - 1(9)] = 8 + 2(51)$$
$$= 8 + 102 = 110$$

**22.** See matrix **A** in Exercise 21 above.

$$|\mathbf{A}| = 0 \cdot A_{13} + 0 \cdot A_{23} + 7A_{33} + (-1)A_{43}$$
$$= 7A_{33} - A_{43}$$
$$= 7(-1)^{3+3}\begin{vmatrix} 1 & 0 & -2 \\ 4 & 1 & 0 \\ -2 & -3 & 0 \end{vmatrix} -$$
$$(-1)^{4+3}\begin{vmatrix} 1 & 0 & -2 \\ 4 & 1 & 0 \\ 5 & 6 & 8 \end{vmatrix}$$

We will expand each determinant down the third column. We have:

$$7\left[-2(-1)^{1+3}\begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix} + 0 + 0\right] +$$
$$\left[-2(-1)^{1+3}\begin{vmatrix} 4 & 1 \\ 5 & 6 \end{vmatrix} + 0 + 8(-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix}\right]$$
$$= 7[-2(4(-3) - (-2) \cdot 1)] + [-2(4 \cdot 6 - 5 \cdot 1) +$$
$$8(1 \cdot 1 - 4 \cdot 0)]$$
$$= 7[-2(-10)] + [-2(19) + 8(1)]$$
$$= 7(20) + (-30) = 140 - 30$$
$$= 110$$

**23.** We will expand across the first row. We could have chosen any other row or column just as well.

$$\begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & 1 \\ 3 & 4 & -6 \end{vmatrix}$$
$$= 3(-1)^{1+1}\begin{vmatrix} 3 & 1 \\ 4 & -6 \end{vmatrix} + 1 \cdot (-1)^{1+2}\begin{vmatrix} -2 & 1 \\ 3 & -6 \end{vmatrix} +$$
$$2(-1)^{1+3}\begin{vmatrix} -2 & 3 \\ 3 & 4 \end{vmatrix}$$
$$= 3 \cdot 1[3(-6) - 4 \cdot 1] + 1(-1)[-2(-6) - 3 \cdot 1] +$$
$$2 \cdot 1(-2 \cdot 4 - 3 \cdot 3)$$
$$= 3(-22) - (9) + 2(-17)$$
$$= -109$$

**24.** We will expand down the third column.

$$\begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & 3 \\ -1 & 5 & 1 \end{vmatrix}$$

$$= 1(-1)^{1+3}\begin{vmatrix} 2 & 4 \\ -1 & 5 \end{vmatrix} + 3(-1)^{2+3}\begin{vmatrix} 3 & -2 \\ -1 & 5 \end{vmatrix} + 1(-1)^{3+3}\begin{vmatrix} 3 & -2 \\ 2 & 4 \end{vmatrix}$$

$$= 1\cdot 1[2\cdot 5-(-1)\cdot 4]+3(-1)[3\cdot 5-(-1)(-2)]+ 1\cdot 1[3\cdot 4-2(-2)]$$

$$= 1\cdot 14 - 3\cdot 13 + 1\cdot 16$$

$$= -9$$

**25.** We will expand down the second column. We could have chosen any other row or column just as well.

$$\begin{vmatrix} x & 0 & -1 \\ 2 & x & x^2 \\ -3 & x & 1 \end{vmatrix}$$

$$= 0(-1)^{1+2}\begin{vmatrix} 2 & x^2 \\ -3 & 1 \end{vmatrix} + x(-1)^{2+2}\begin{vmatrix} x & -1 \\ -3 & 1 \end{vmatrix} + x(-1)^{3+2}\begin{vmatrix} x & -1 \\ 2 & x^2 \end{vmatrix}$$

$$= 0(-1)[2\cdot 1-(-3)x^2]+x\cdot 1[x\cdot 1-(-3)(-1)]+ x(-1)[x\cdot x^2-2(-1)]$$

$$= 0 + x(x-3) - x(x^3+2)$$

$$= x^2-3x-x^4-2x = -x^4+x^2-5x$$

**26.** We will expand across the third row.

$$\begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix}$$

$$= 0(-1)^{3+1}\begin{vmatrix} 1 & -1 \\ x & x \end{vmatrix} + x(-1)^{3+2}\begin{vmatrix} x & -1 \\ x^2 & x \end{vmatrix} + 1(-1)^{3+3}\begin{vmatrix} x & 1 \\ x^2 & x \end{vmatrix}$$

$$= 0 + x(-1)[x\cdot x - x^2(-1)] + 1\cdot 1(x\cdot x - x^2\cdot 1)$$

$$= -x(x^2+x^2)+1(x^2-x^2)$$

$$= -x(2x^2)+1(0) = -2x^3$$

**27.** $-2x + 4y = 3,$
$3x - 7y = 1$

$$D = \begin{vmatrix} -2 & 4 \\ 3 & -7 \end{vmatrix} = -2(-7)-3\cdot 4 = 14-12 = 2$$

$$D_x = \begin{vmatrix} 3 & 4 \\ 1 & -7 \end{vmatrix} = 3(-7)-1\cdot 4 = -21-4 = -25$$

$$D_y = \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = -2\cdot 1 - 3\cdot 3 = -2-9 = -11$$

$$x = \frac{D_x}{D} = \frac{-25}{2} = -\frac{25}{2}$$

$$y = \frac{D_y}{D} = \frac{-11}{2} = -\frac{11}{2}$$

The solution is $\left(-\frac{25}{2}, -\frac{11}{2}\right)$.

**28.** $5x - 4y = -3,$
$7x + 2y = 6$

$$D = \begin{vmatrix} 5 & -4 \\ 7 & 2 \end{vmatrix} = 5\cdot 2 - 7(-4) = 10+28 = 38$$

$$D_x = \begin{vmatrix} -3 & -4 \\ 6 & 2 \end{vmatrix} = -3\cdot 2 - 6(-4) = -6+24 = 18$$

$$D_y = \begin{vmatrix} 5 & -3 \\ 7 & 6 \end{vmatrix} = 5\cdot 6 - 7(-3) = 30+21 = 51$$

$$x = \frac{D_x}{D} = \frac{18}{38} = \frac{9}{19}$$

$$y = \frac{D_y}{D} = \frac{51}{38}$$

The solution is $\left(\frac{9}{19}, \frac{51}{38}\right)$.

**29.** $2x - y = 5,$
$x - 2y = 1$

$$D = \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} = 2(-2)-1(-1) = -4+1 = -3$$

$$D_x = \begin{vmatrix} 5 & -1 \\ 1 & -2 \end{vmatrix} = 5(-2)-1(-1) = -10+1 = -9$$

$$D_y = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 2\cdot 1 - 1\cdot 5 = 2-5 = -3$$

$$x = \frac{D_x}{D} = \frac{-9}{-3} = 3$$

$$y = \frac{D_y}{D} = \frac{-3}{-3} = 1$$

The solution is $(3, 1)$.

**30.** $3x + 4y = -2,$
$5x - 7y = 1$

$$D = \begin{vmatrix} 3 & 4 \\ 5 & -7 \end{vmatrix} = 3(-7)-5\cdot 4 = -21-20 = -41$$

$$D_x = \begin{vmatrix} -2 & 4 \\ 1 & -7 \end{vmatrix} = -2(-7)-1\cdot 4 = 14-4 = 10$$

$$D_y = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} = 3\cdot 1 - 5(-2) = 3+10 = 13$$

$$x = \frac{D_x}{D} = \frac{10}{-41} = -\frac{10}{41}$$

$$y = \frac{D_y}{D} = \frac{13}{-41} = -\frac{13}{41}$$

The solution is $\left(-\frac{10}{41}, -\frac{13}{41}\right)$.

**31.** $2x + 9y = -2,$
$4x - 3y = 3$

$$D = \begin{vmatrix} 2 & 9 \\ 4 & -3 \end{vmatrix} = 2(-3) - 4 \cdot 9 = -6 - 36 = -42$$

$$D_x = \begin{vmatrix} -2 & 9 \\ 3 & -3 \end{vmatrix} = -2(-3) - 3 \cdot 9 = 6 - 27 = -21$$

$$D_y = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 2 \cdot 3 - 4(-2) = 6 + 8 = 14$$

$$x = \frac{D_x}{D} = \frac{-21}{-42} = \frac{1}{2}$$
$$y = \frac{D_y}{D} = \frac{14}{-42} = -\frac{1}{3}$$

The solution is $\left(\frac{1}{2}, -\frac{1}{3}\right)$.

**32.** $2x + 3y = -1,$
$3x + 6y = -0.5$

$$D = \begin{vmatrix} 2 & 3 \\ 3 & 6 \end{vmatrix} = 2 \cdot 6 - 3 \cdot 3 = 12 - 9 = 3$$

$$D_x = \begin{vmatrix} -1 & 3 \\ -0.5 & 6 \end{vmatrix} = -1 \cdot 6 - (-0.5)3 =$$
$$-6 + 1.5 = -4.5$$

$$D_y = \begin{vmatrix} 2 & -1 \\ 3 & -0.5 \end{vmatrix} = 2(-0.5) - 3(-1) = -1 + 3 = 2$$

$$x = \frac{D_x}{D} = \frac{-4.5}{3} = -1.5, \text{ or } -\frac{3}{2}$$
$$y = \frac{D_y}{D} = \frac{2}{3}$$

The solution is $\left(-\frac{3}{2}, \frac{2}{3}\right)$.

**33.** $2x + 5y = 7,$
$3x - 2y = 1$

$$D = \begin{vmatrix} 2 & 5 \\ 3 & -2 \end{vmatrix} = 2(-2) - 3 \cdot 5 = -4 - 15 = -19$$

$$D_x = \begin{vmatrix} 7 & 5 \\ 1 & -2 \end{vmatrix} = 7(-2) - 1 \cdot 5 = -14 - 5 = -19$$

$$D_y = \begin{vmatrix} 2 & 7 \\ 3 & 1 \end{vmatrix} = 2 \cdot 1 - 3 \cdot 7 = 2 - 21 = -19$$

$$x = \frac{D_x}{D} = \frac{-19}{-19} = 1$$
$$y = \frac{D_y}{D} = \frac{-19}{-19} = 1$$

The solution is $(1, 1)$.

**34.** $3x + 2y = 7,$
$2x + 3y = -2$

$$D = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 3 \cdot 3 - 2 \cdot 2 = 9 - 4 = 5$$

$$D_x = \begin{vmatrix} 7 & 2 \\ -2 & 3 \end{vmatrix} = 7 \cdot 3 - (-2) \cdot 2 = 21 + 4 = 25$$

$$D_y = \begin{vmatrix} 3 & 7 \\ 2 & -2 \end{vmatrix} = 3(-2) - 2 \cdot 7 = -6 - 14 = -20$$

$$x = \frac{D_x}{D} = \frac{25}{5} = 5$$
$$y = \frac{D_y}{D} = \frac{-20}{5} = -4$$

The solution is $(5, -4)$.

**35.** $3x + 2y - z = 4,$
$3x - 2y + z = 5,$
$4x - 5y - z = -1$

$$D = \begin{vmatrix} 3 & 2 & -1 \\ 3 & -2 & 1 \\ 4 & -5 & -1 \end{vmatrix} = 42$$

$$D_x = \begin{vmatrix} 4 & 2 & -1 \\ 5 & -2 & 1 \\ -1 & -5 & -1 \end{vmatrix} = 63$$

$$D_y = \begin{vmatrix} 3 & 4 & -1 \\ 3 & 5 & 1 \\ 4 & -1 & -1 \end{vmatrix} = 39$$

$$D_z = \begin{vmatrix} 3 & 2 & 4 \\ 3 & -2 & 5 \\ 4 & -5 & -1 \end{vmatrix} = 99$$

$$x = \frac{D_x}{D} = \frac{63}{42} = \frac{3}{2}$$

$$y = \frac{D_y}{D} = \frac{39}{42} = \frac{13}{14}$$

$$z = \frac{D_z}{D} = \frac{99}{42} = \frac{33}{14}$$

The solution is $\left(\frac{3}{2}, \frac{13}{14}, \frac{33}{14}\right)$.

(Note that we could have used Cramer's rule to find only two of the values and then used substitution to find the remaining value.)

**36.** $3x - y + 2z = 1,$
$x - y + 2z = 3,$
$-2x + 3y + z = 1$

$$D = \begin{vmatrix} 3 & -1 & 2 \\ 1 & -1 & 2 \\ -2 & 3 & 1 \end{vmatrix} = -14$$

$$D_x = \begin{vmatrix} 1 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = 14$$

$$D_y = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 3 & 2 \\ -2 & 1 & 1 \end{vmatrix} = 12$$

$$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 1 & -1 & 3 \\ -2 & 3 & 1 \end{vmatrix} = -22$$

$$x = \frac{D_x}{D} = \frac{14}{-14} = -1$$

$$y = \frac{D_y}{D} = \frac{12}{-14} = -\frac{6}{7}$$

$$z = \frac{D_z}{D} = \frac{-22}{-14} = \frac{11}{7}$$

The solution is $\left(-1, -\frac{6}{7}, \frac{11}{7}\right)$.

**37.** $3x + 5y - z = -2,$
$x - 4y + 2z = 13,$
$2x + 4y + 3z = 1$

$$D = \begin{vmatrix} 3 & 5 & -1 \\ 1 & -4 & 2 \\ 2 & 4 & 3 \end{vmatrix} = -67$$

$$D_x = \begin{vmatrix} -2 & 5 & -1 \\ 13 & -4 & 2 \\ 1 & 4 & 3 \end{vmatrix} = -201$$

$$D_y = \begin{vmatrix} 3 & -2 & -1 \\ 1 & 13 & 2 \\ 2 & 1 & 3 \end{vmatrix} = 134$$

$$D_z = \begin{vmatrix} 3 & 5 & -2 \\ 1 & -4 & 13 \\ 2 & 4 & 1 \end{vmatrix} = -67$$

$$x = \frac{D_x}{D} = \frac{-201}{-67} = 3$$

$$y = \frac{D_y}{D} = \frac{134}{-67} = -2$$

$$z = \frac{D_z}{D} = \frac{-67}{-67} = 1$$

The solution is $(3, -2, 1)$.

(Note that we could have used Cramer's rule to find only two of the values and then used substitution to find the remaining value.)

**38.** $3x + 2y + 2z = 1,$
$5x - y - 6z = 3,$
$2x + 3y + 3z = 4$

$$D = \begin{vmatrix} 3 & 2 & 2 \\ 5 & -1 & -6 \\ 2 & 3 & 3 \end{vmatrix} = 25$$

$$D_x = \begin{vmatrix} 1 & 2 & 2 \\ 3 & -1 & -6 \\ 4 & 3 & 3 \end{vmatrix} = -25$$

$$D_y = \begin{vmatrix} 3 & 1 & 2 \\ 5 & 3 & -6 \\ 2 & 4 & 3 \end{vmatrix} = 100$$

$$D_z = \begin{vmatrix} 3 & 2 & 1 \\ 5 & -1 & 3 \\ 2 & 3 & 4 \end{vmatrix} = -50$$

$$x = \frac{D_x}{D} = \frac{-25}{25} = -1$$

$$y = \frac{D_y}{D} = \frac{100}{25} = 4$$

$$z = \frac{D_z}{D} = \frac{-50}{25} = -2$$

The solution is $(-1, 4, -2)$.

**39.** $x - 3y - 7z = 6,$
$2x + 3y + z = 9,$
$4x + y = 7$

$$D = \begin{vmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{vmatrix} = 57$$

$$D_x = \begin{vmatrix} 6 & -3 & -7 \\ 9 & 3 & 1 \\ 7 & 1 & 0 \end{vmatrix} = 57$$

$$D_y = \begin{vmatrix} 1 & 6 & -7 \\ 2 & 9 & 1 \\ 4 & 7 & 0 \end{vmatrix} = 171$$

$$D_z = \begin{vmatrix} 1 & -3 & 6 \\ 2 & 3 & 9 \\ 4 & 1 & 7 \end{vmatrix} = -114$$

$$x = \frac{D_x}{D} = \frac{57}{57} = 1$$

$$y = \frac{D_y}{D} = \frac{171}{57} = 3$$

$$z = \frac{D_z}{D} = \frac{-114}{57} = -2$$

The solution is $(1, 3, -2)$.

(Note that we could have used Cramer's rule to find only two of the values and then used substitution to find the remaining value.)

**40.** $x - 2y - 3z = 4,$
$3x \quad - 2z = 8,$
$2x + y + 4z = 13$

$$D = \begin{vmatrix} 1 & -2 & -3 \\ 3 & 0 & -2 \\ 2 & 1 & 4 \end{vmatrix} = 25$$

$$D_x = \begin{vmatrix} 4 & -2 & -3 \\ 8 & 0 & -2 \\ 13 & 1 & 4 \end{vmatrix} = 100$$

$$D_y = \begin{vmatrix} 1 & 4 & -3 \\ 3 & 8 & -2 \\ 2 & 13 & 4 \end{vmatrix} = -75$$

$$D_z = \begin{vmatrix} 1 & -2 & 4 \\ 3 & 0 & 8 \\ 2 & 1 & 13 \end{vmatrix} = 50$$

$$x = \frac{D_x}{D} = \frac{100}{25} = 4$$

$$y = \frac{D_y}{D} = \frac{-75}{25} = -3$$

$$z = \frac{D_z}{D} = \frac{50}{25} = 2$$

The solution is $(4, -3, 2)$.

**41.** $6y + 6z = -1,$
$8x \quad + 6z = -1,$
$4x + 9y \quad = 8$

$$D = \begin{vmatrix} 0 & 6 & 6 \\ 8 & 0 & 6 \\ 4 & 9 & 0 \end{vmatrix} = 576$$

$$D_x = \begin{vmatrix} -1 & 6 & 6 \\ -1 & 0 & 6 \\ 8 & 9 & 0 \end{vmatrix} = 288$$

$$D_y = \begin{vmatrix} 0 & -1 & 6 \\ 8 & -1 & 6 \\ 4 & 8 & 0 \end{vmatrix} = 384$$

$$D_z = \begin{vmatrix} 0 & 6 & -1 \\ 8 & 0 & -1 \\ 4 & 9 & 8 \end{vmatrix} = -480$$

$$x = \frac{D_x}{D} = \frac{288}{576} = \frac{1}{2}$$

$$y = \frac{D_y}{D} = \frac{384}{576} = \frac{2}{3}$$

$$z = \frac{D_z}{D} = \frac{-480}{576} = -\frac{5}{6}$$

The solution is $\left(\frac{1}{2}, \frac{2}{3}, -\frac{5}{6}\right)$.

(Note that we could have used Cramer's rule to find only two of the values and then used substitution to find the remaining value.)

**42.** $3x + 5y \quad = 2,$
$2x \quad - 3z = 7,$
$4y + 2z = -1$

$$D = \begin{vmatrix} 3 & 5 & 0 \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{vmatrix} = 16$$

$$D_x = \begin{vmatrix} 2 & 5 & 0 \\ 7 & 0 & -3 \\ -1 & 4 & 2 \end{vmatrix} = -31$$

$$D_y = \begin{vmatrix} 3 & 2 & 0 \\ 2 & 7 & -3 \\ 0 & -1 & 2 \end{vmatrix} = 25$$

$$D_z = \begin{vmatrix} 3 & 5 & 2 \\ 2 & 0 & 7 \\ 0 & 4 & -1 \end{vmatrix} = -58$$

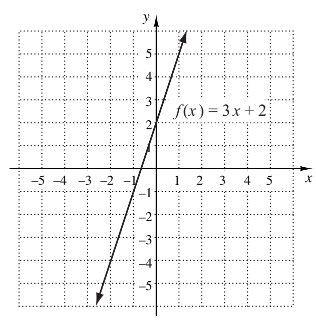
$$x = \frac{D_x}{D} = \frac{-31}{16} = -\frac{31}{16}$$

$$y = \frac{D_y}{D} = \frac{25}{16}$$

$$z = \frac{D_z}{D} = \frac{-58}{16} = -\frac{29}{8}$$

The solution is $\left(-\frac{31}{16}, \frac{25}{16}, -\frac{29}{8}\right)$.

**43.** The graph of $f(x) = 3x+2$ is shown below. Since it passes the horizontal-line test, the function is one-to-one.




We find a formula for $f^{-1}(x)$.

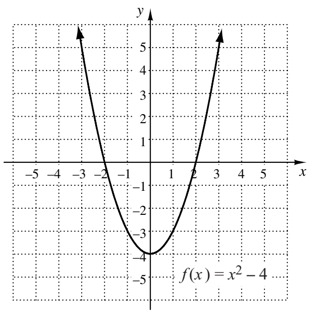
Replace $f(x)$ with $y$: $y = 3x + 2$

Interchange $x$ and $y$: $x = 3y + 2$

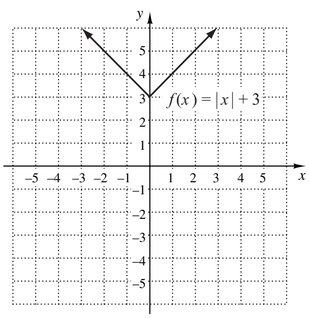
Solve for $y$: $y = \dfrac{x-2}{3}$

Replace $y$ with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x-2}{3}$

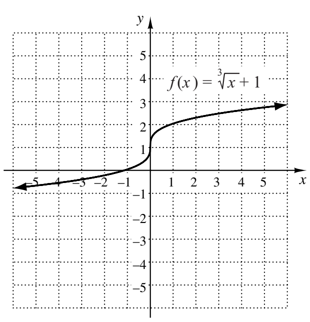
**44.** The graph of $f(x) = x^2 - 4$ is shown below. It fails the horizontal-line test, so it is not one-to-one.




**45.** The graph of $f(x) = |x| + 3$ is shown below. It fails the horizontal-line test, so it is not one-to-one.




**46.** The graph of $f(x) = \sqrt[3]{x} + 1$ is show below. Since it passes the horizontal-line test, the function is one-to-one.




We find a formula for $f^{-1}(x)$.

Replace $f(x)$ with $y$: $y = \sqrt[3]{x} + 1$

Interchange $x$ and $y$: $x = \sqrt[3]{y} + 1$

Solve for $y$: $y = (x - 1)^3$

Replace $y$ with $f^{-1}(x)$: $f^{-1}(x) = (x - 1)^3$

**47.** $(3 - 4i) - (-2 - i) = 3 - 4i + 2 + i =$
$(3 + 2) + (-4 + 1)i = 5 - 3i$

**48.** $(5 + 2i) + (1 - 4i) = 6 - 2i$

**49.** $(1 - 2i)(6 + 2i) = 6 + 2i - 12i - 4i^2 =$
$6 + 2i - 12i + 4 = 10 - 10i$

**50.** $\dfrac{3+i}{4-3i} = \dfrac{3+i}{4-3i} \cdot \dfrac{4+3i}{4+3i} = \dfrac{12 + 9i + 4i + 3i^2}{16 - 9i^2} =$
$\dfrac{12 + 9i + 4i - 3}{16 + 9} = \dfrac{9 + 13i}{25} = \dfrac{9}{25} + \dfrac{13}{25}i$

**51.** $\begin{vmatrix} x & 5 \\ -4 & x \end{vmatrix} = 24$

$x \cdot x - (-4)(5) = 24$ Evaluating the determinant

$x^2 + 20 = 24$

$x^2 = 4$

$x = \pm 2$

The solutions are $-2$ and 2.

**52.** $\begin{vmatrix} y & 2 \\ 3 & y \end{vmatrix} = y$

$y^2 - 6 = y$

$y^2 - y - 6 = 0$

$(y - 3)(y + 2) = 0$

$y - 3 = 0$ *or* $y + 2 = 0$

$y = 3$ *or* $y = -2$

The solutions are 3 and $-2$.

**53.** $\begin{vmatrix} x & -3 \\ -1 & x \end{vmatrix} \geq 0$

$x \cdot x - (-1)(-3) \geq 0$

$x^2 - 3 \geq 0$

We solve the related equation.

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

The numbers $-\sqrt{3}$ and $\sqrt{3}$ divide the $x$-axis into three intervals. We let $f(x) = x^2 - 3$ and test a value in each interval.

$(-\infty, -\sqrt{3})$: $f(-2) = (-2)^2 - 3 = 1 > 0$

$(-\sqrt{3}, \sqrt{3})$: $f(0) = 0^2 - 3 = -3 < 0$

$(\sqrt{3}, \infty)$: $f(2) = 2^2 - 3 = 1 > 0$

The function is positive in $(-\infty, -\sqrt{3})$, and $(\sqrt{3}, \infty)$. We also include the endpoints of the intervals since the inequality symbol is $\geq$. The solution set is $\{x | x \leq -\sqrt{3}$ *or* $x \geq \sqrt{3}\}$, or $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$.

**54.** $\begin{vmatrix} y & -5 \\ -2 & y \end{vmatrix} < 0$

$y^2 - 10 < 0$

Solve the related equation.

$y^2 - 10 = 0$

$y^2 = 10$

$y = \pm\sqrt{10}$

The numbers $-\sqrt{10}$ and $\sqrt{10}$ divide the axis into three intervals. We let $f(y) = y^2 - 10$ and test a value in each interval.

$(-\infty, -\sqrt{10})$: $f(-4) = (-4)^2 - 10 = 6 > 0$

$(-\sqrt{10}, \sqrt{10})$: $f(0) = 0^2 - 10 = -10 < 0$

$(\sqrt{10}, \infty)$: $f(4) = 4^2 - 10 = 6 > 0$

The solution set is $\{y | -\sqrt{10} < y < \sqrt{10}\}$, or $(-\sqrt{10}, \sqrt{10})$.

**55.** $$\begin{vmatrix} x+3 & 4 \\ x-3 & 5 \end{vmatrix} = -7$$
$$\begin{aligned} (x+3)(5)-(x-3)(4) &= -7 \\ 5x+15-4x+12 &= -7 \\ x+27 &= -7 \\ x &= -34 \end{aligned}$$

The solution is $-34$.

**56.** $$\begin{vmatrix} m+2 & -3 \\ m+5 & -4 \end{vmatrix} = 3m-5$$
$$\begin{aligned} -4m-8+3m+15 &= 3m-5 \\ -m+7 &= 3m-5 \\ -4m &= -12 \\ m &= 3 \end{aligned}$$

**57.** $$\begin{vmatrix} 2 & x & 1 \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -6$$
$$\begin{aligned} -x-2 &= -6 \quad \text{Evaluating the determinant} \\ -x &= -4 \\ x &= 4 \end{aligned}$$

The solution is 4.

**58.** $$\begin{vmatrix} x & 2 & x \\ 3 & -1 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -10$$
$$\begin{aligned} -5x-10 &= -10 \\ -5x &= 0 \\ x &= 0 \end{aligned}$$

**59.** Answers may vary.
$$\begin{vmatrix} L & -W \\ 2 & 2 \end{vmatrix}$$

**60.** Answers may vary.
$$\begin{vmatrix} \pi & -\pi \\ h & r \end{vmatrix}$$

**61.** Answers may vary.
$$\begin{vmatrix} a & b \\ -b & a \end{vmatrix}$$

**62.** Answers may vary.
$$\begin{vmatrix} \dfrac{h}{2} & -\dfrac{h}{2} \\ b & a \end{vmatrix}$$

**63.** Answers may vary.
$$\begin{vmatrix} 2\pi r & 2\pi r \\ -h & r \end{vmatrix}$$

**64.** Answers may vary.
$$\begin{vmatrix} xy & Q \\ Q & xy \end{vmatrix}$$

## Exercise Set 9.7

**1.** Graph $(f)$ is the graph of $y > x$.

**2.** Graph $(c)$ is the graph of $y < -2x$.

**3.** Graph $(h)$ is the graph of $y \le x-3$.

**4.** Graph $(a)$ is the graph of $y \ge x+5$.

**5.** Graph $(g)$ is the graph of $2x+y<4$.

**6.** Graph $(d)$ is the graph of $3x+y<-6$.

**7.** Graph $(b)$ is the graph of $2x-5y>10$.

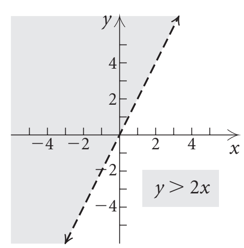
**8.** Graph $(e)$ is the graph of $3x-9y<9$.

**9.** Graph: $y > 2x$

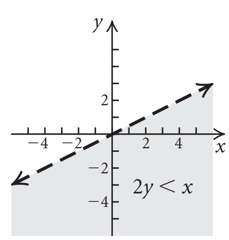
1. We first graph the related equation $y = 2x$. We draw the line dashed since the inequality symbol is $>$.
2. To determine which half-plane to shade, test a point not on the line. We try $(1,1)$ and substitute:

$$\begin{array}{c|l} \multicolumn{2}{c}{y > 2x} \\ \hline 1 \ ? & 2\cdot 1 \\ 1 & 2 \qquad \text{FALSE} \end{array}$$

Since $1 > 2$ is false, $(1,1)$ is not a solution, nor are any points in the half-plane containing $(1,1)$. The points in the opposite half-plane are solutions, so we shade that half-plane and obtain the graph.




**10.**

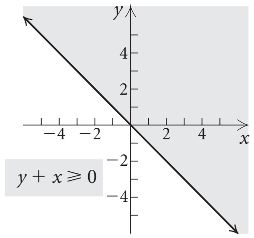



**11.** Graph: $y + x \ge 0$

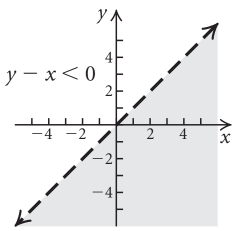
1. First graph the related equation $y+x=0$. Draw the line solid since the inequality is $\ge$.
2. Next determine which half-plane to shade by testing a point not on the line. Here we use $(2,2)$ as a check.

$$\begin{array}{c|l} \multicolumn{2}{c}{y+x \ge 0} \\ \hline 2+2 \ ? & 0 \\ 4 & 0 \quad \text{TRUE} \end{array}$$

Since $4 \geq 0$ is true, $(2,2)$ is a solution. Thus shade the half-plane containing $(2,2)$.




**12.**

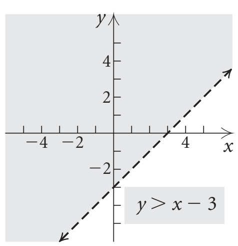



**13.** Graph: $y > x - 3$

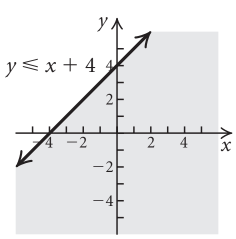
1. We first graph the related equation $y = x - 3$. Draw the line dashed since the inequality symbol is $>$.
2. To determine which half-plane to shade, test a point not on the line. We try $(0,0)$.

$$\begin{array}{c|c}
\multicolumn{2}{c}{y > x - 3} \\
\hline
0 \; ? & 0 - 3 \\
0 & -3 \quad \text{TRUE}
\end{array}$$

Since $0 > -3$ is true, $(0,0)$ is a solution. Thus we shade the half-plane containing $(0,0)$.




**14.**

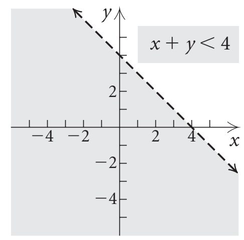



**15.** Graph: $x + y < 4$

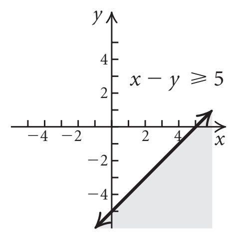
1. First graph the related equation $x + y = 4$. Draw the line dashed since the inequality is $<$.
2. To determine which half-plane to shade, test a point not on the line. We try $(0,0)$.

$$\begin{array}{c|c}
\multicolumn{2}{c}{x + y < 4} \\
\hline
0 + 0 \; ? & 4 \\
0 & 4 \quad \text{TRUE}
\end{array}$$

Since $0 < 4$ is true, $(0,0)$ is a solution. Thus shade the half-plane containing $(0,0)$.




**16.**

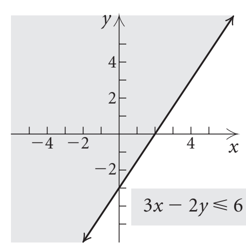



**17.** Graph: $3x - 2y \leq 6$

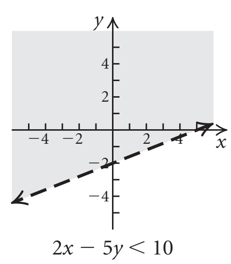
1. First graph the related equation $3x - 2y = 6$. Draw the line solid since the inequality is $\leq$.
2. To determine which half-plane to shade, test a point not on the line. We try $(0,0)$.

$$\begin{array}{c|c}
\multicolumn{2}{c}{3x - 2y \leq 6} \\
\hline
3(0) - 2(0) \; ? & 6 \\
0 & 6 \quad \text{TRUE}
\end{array}$$

Since $0 \leq 6$ is true, $(0,0)$ is a solution. Thus shade the half-plane containing $(0,0)$.




**18.**

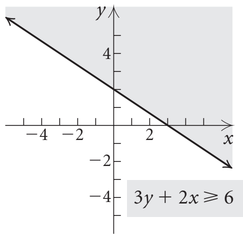


**19.** Graph: $3y + 2x \geq 6$

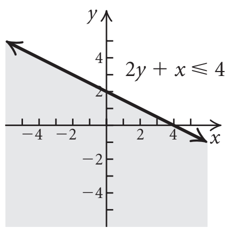
1. First graph the related equation $3y + 2x = 6$. Draw the line solid since the inequality is $\geq$.
2. To determine which half-plane to shade, test a point not on the line. We try $(0, 0)$.

$$\begin{array}{r|l} 3y + 2x & \geq 6 \\ \hline 3 \cdot 0 + 2 \cdot 0 & ?\ 6 \\ 0 & 6 \quad \text{FALSE} \end{array}$$

Since $0 \geq 6$ is false, $(0, 0)$ is not a solution. We shade the half-plane which does not contain $(0, 0)$.




**20.**



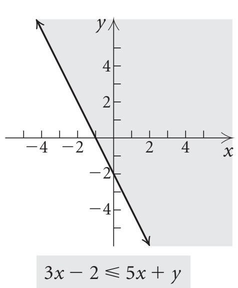

**21.** Graph: $3x - 2 \leq 5x + y$

$$-2 \leq 2x + y \quad \text{Adding } -3x$$

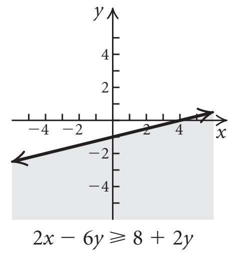
1. First graph the related equation $2x + y = -2$. Draw the line solid since the inequality is $\leq$.
2. To determine which half-plane to shade, test a point not on the line. We try $(0, 0)$.

$$\begin{array}{r|l} 2x + y & \geq -2 \\ \hline 2(0) + 0 & ?\ -2 \\ 0 & -2 \quad \text{TRUE} \end{array}$$

Since $0 \geq -2$ is true, $(0, 0)$ is a solution. Thus shade the half-plane containing the origin.




**22.**

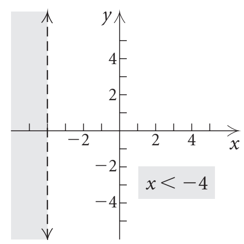



**23.** Graph: $x < -4$

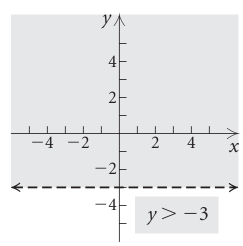
1. We first graph the related equation $x = -4$. Draw the line dashed since the inequality is $<$.
2. To determine which half-plane to shade, test a point not on the line. We try $(0, 0)$.

$$\begin{array}{r|l} x & < -4 \\ \hline 0 & ?\ -4 \quad \text{FALSE} \end{array}$$

Since $0 < -4$ is false, $(0, 0)$ is not a solution. Thus, we shade the half-plane which does not contain the origin.




**24.**

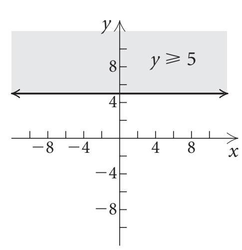



**25.** Graph: $y \geq 5$

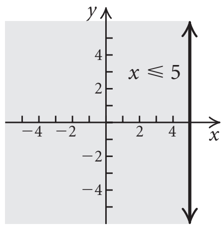
1. First we graph the related equation $y = 5$. Draw the line solid since the inequality is $\geq$.
2. To determine which half-plane to shade we test a point not on the line. We try $(0, 0)$.

$$\begin{array}{r|l} y & \geq 5 \\ \hline 0 & ?\ 5 \quad \text{FALSE} \end{array}$$

Since $0 \geq 5$ is false, $(0, 0)$ is not a solution. We shade the half-plane that does not contain $(0, 0)$.



**26.**

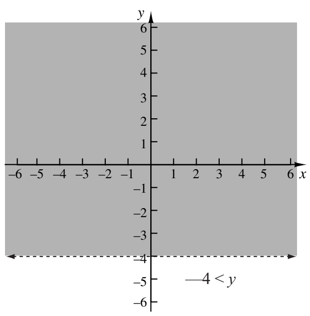


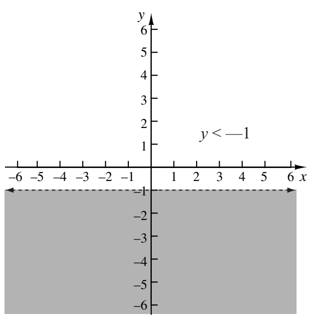

**27.** Graph: $-4 < y < -1$

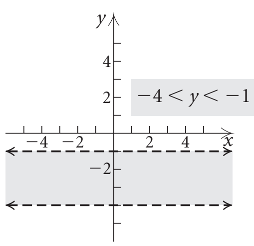
This is a conjunction of two inequalities

$$-4 < y \quad \textit{and} \quad y < -1.$$

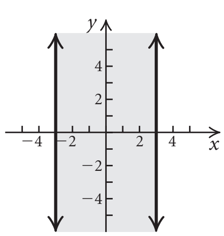
We can graph $-4 < y$ and $y < -1$ separately and then graph the intersection, or region in both solution sets.



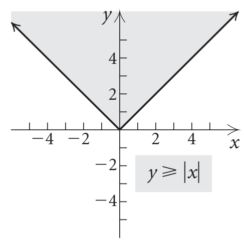

**28.**



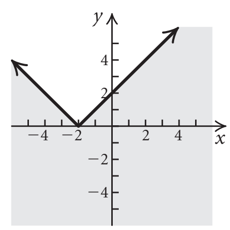
$-3 \leqslant x \leqslant 3$

**29.** Graph: $y \geq |x|$

1. Graph the related equation $y = |x|$. Draw the line solid since the inequality symbol is $\geq$.
2. To determine the region to shade, observe that the solution set consists of all ordered pairs $(x, y)$ where the second coordinate is greater than or equal to the absolute value of the first coordinate. We see that the solutions are the points on or above the graph of $y = |x|$.




**30.**



$y \leqslant |x + 2|$

**31.** Graph $(f)$ is the correct graph.

**32.** Graph $(c)$ is the correct graph.

**33.** Graph $(a)$ is the correct graph.

**34.** Graph $(d)$ is the correct graph.

**35.** Graph $(b)$ is the correct graph.

**36.** Graph $(e)$ is the correct graph.

**37.** First we find the related equations. One line goes through $(0, 4)$ and $(4, 0)$. We find its slope:

$$m = \frac{0-4}{4-0} = \frac{-4}{4} = -1$$

This line has slope $-1$ and $y$-intercept $(0, 4)$, so its equation is $y = -x + 4$.

The other line goes through $(0, 0)$ and $(1, 3)$. We find the slope.

$$m = \frac{3-0}{1-0} = 3$$

This line has slope 3 and $y$-intercept $(0, 0)$, so its equation is $y = 3x + 0$, or $y = 3x$.

Observing the shading on the graph and the fact that the lines are solid, we can write the system of inqualities as

$$y \leq -x + 4,$$
$$y \leq 3x.$$

Answers may vary.

**38.** First we find the related equations. One line goes through $(-1, 2)$ and $(0, 0)$. We find its slope:

$$m = \frac{0-2}{0-(-1)} = -2$$

Then the equation of the line is $y = -2x + 0$, or $y = -2x$.

The other line goes through $(0, -3)$ and $(3, 0)$. We find its slope:

$$m = \frac{0-(-3)}{3-0} = \frac{3}{3} = 1$$

Then the equation of this line is $y = x - 3$.

Observing the shading on the graph and the fact that the lines are solid, we can write the system of inequalities as

$$y \le -2x,$$
$$y \ge x - 3.$$

Answers may vary.

**39.** The equation of the vertical line is $x = 2$ and the equation of the horizontal line is $y = -1$. The lines are dashed and the shaded area is to the left of the vertical line and above the horizontal line, so the system of inequalities can be written

$$x < 2,$$
$$y > -1.$$

**40.** The equation of the vertical line is $x = -1$ and the equation of the horizontal line is $y = 2$. The lines are dashed and the shaded area is to the right of the vertical line and below the horizontal line, so the system of inequalities can be written

$$x > -1,$$
$$y < 2.$$

**41.** First we find the related equations. One line goes through $(0, 3)$ and $(3, 0)$. We find its slope:

$$m = \frac{0-3}{3-0} = \frac{-3}{3} = -1$$

This line has slope $-1$ and $y$-intercept $(0, 3)$, so its equation is $y = -x + 3$.

The other line goes through $(0, 1)$ and $(1, 2)$. We find its slope:

$$m = \frac{2-1}{1-0} = \frac{1}{1} = 1$$

This line has slope 1 and $y$-intercept $(0, 1)$, so its equation is $y = x + 1$.

Observe that both lines are solid and that the shading lies below both lines, to the right of the $y$-axis, and above the $x$-axis. We can write this system of inequalities as

$$y \le -x + 3,$$
$$y \le x + 1,$$
$$x \ge 0,$$
$$y \ge 0.$$

**42.** One line goes through $(-2, 0)$ and $(0, 2)$. We find its slope:

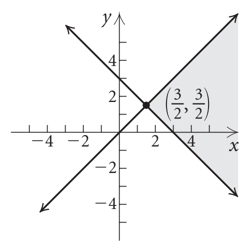
$$m = \frac{2-0}{0-(-2)} = \frac{2}{2} = 1$$

Then the equation of this line is $y = x + 2$. The other line is the vertical line $x = 2$. Observe that both lines are solid and that the shading lies below $y = x + 2$, to the left of $x = 2$, to the right of the $y$-axis, and above the $x$-axis. We can write this system of inequalities as

$$y \le x + 2,$$
$$x \le 2,$$
$$x \ge 0,$$
$$y \ge 0.$$

**43.** Graph: $y \le x$,
$y \ge 3 - x$

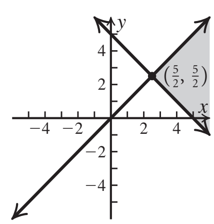
We graph the related equations $y = x$ and $y = 3 - x$ using solid lines and determine the solution set for each inequality. Then we shade the region common to both solution sets.




We find the vertex $\left(\frac{3}{2}, \frac{3}{2}\right)$ by solving the system

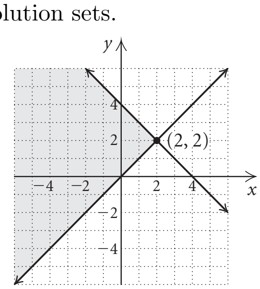
$$y = x,$$
$$y = 3 - x.$$

**44.** Graph: $y \ge x$,
$y \le 5 - x$

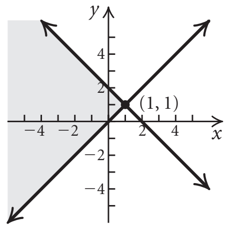



**45.** Graph: $y \ge x$,
$y \le 4 - x$

We graph the related equations $y = x$ and $y = 4 - x$ using solid lines and determine the solution set for each inequality. Then we shade the region common to both solution sets.

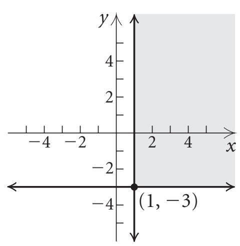


**46.** Graph: $y \geq x$,
$y \leq 2 - x$




**47.** Graph: $y \geq -3$,
$x \geq 1$

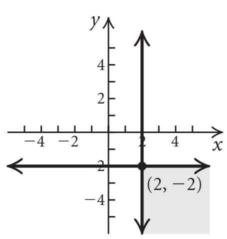
We graph the related equations $y = -3$ and $x = 1$ using solid lines and determine the solution set for each inequality. Then we shade the region common to both solution sets.




We find the vertex $(1, -3)$ by solving the system

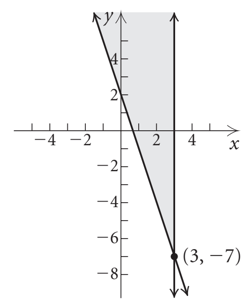
$y = -3$,
$x = 1$.

**48.** Graph: $y \leq -2$,
$x \geq 2$




**49.** Graph: $x \leq 3$,
$y \geq 2 - 3x$

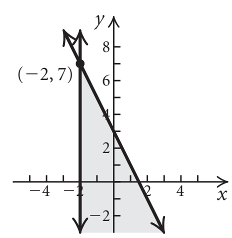
We graph the related equations $x = 3$ and $y = 2 - 3x$ using solid lines and determine the half-plane containing the solution set for each inequality. Then we shade the region common to both solution sets.




We find the vertex $(3, -7)$ by solving the system

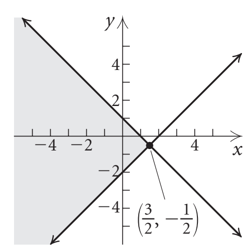
$x = 3$,
$y = 2 - 3x$.

**50.** Graph: $x \geq -2$,
$y \leq 3 - 2x$




**51.** Graph: $x + y \leq 1$,
$x - y \leq 2$

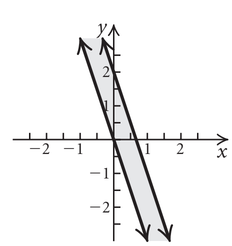
We graph the related equations $x + y = 1$ and $x - y = 2$ using solid lines and determine the half-plane containing the solution set for each inequality. Then we shade the region common to both solution sets.



We find the vertex $\left(\frac{3}{2}, -\frac{1}{2}\right)$ by solving the system

$x + y = 1$,
$x - y = 2$.

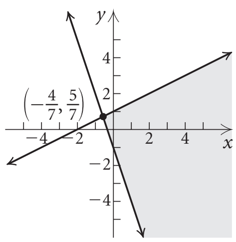
**52.** Graph: $y + 3x \geq 0$,
$y + 3x \leq 2$



**53.** Graph: $2y - x \leq 2$,

$y + 3x \geq -1$

We graph the related equations $2y - x = 2$ and $y + 3x = -1$ using solid lines and determine the half-plane containing the solution set for each inequality. Then we shade the region common to both solution sets.



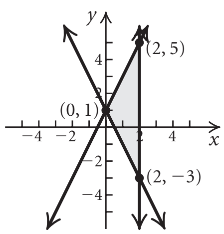

We find the vertex $\left(-\dfrac{4}{7}, \dfrac{5}{7}\right)$ by solving the system

$$2y - x = 2,$$
$$y + 3x = -1.$$

**54.** Graph: $y \leq 2x + 1$,

$y \geq -2x + 1$,

$x \leq 2$

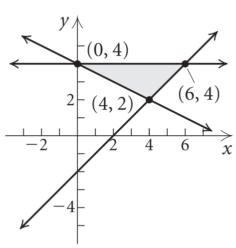



**55.** Graph: $x - y \leq 2$,

$x + 2y \geq 8$,

$y \leq 4$

We graph the related equations $x - y = 2$, $x + 2y = 8$, and $y = 4$ using solid lines and determine the half-plane containing the solution set for each inequality. Then we shade the region common to all three solution sets.




We find the vertex $(0, 4)$ by solving the system

$$x + 2y = 8,$$
$$y = 4.$$

We find the vertex $(6, 4)$ by solving the system

$$x - y = 2,$$
$$y = 4.$$

We find the vertex $(4, 2)$ by solving the system

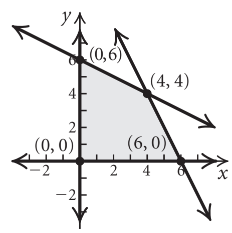
$$x - y = 2,$$
$$x + 2y = 8.$$

**56.** Graph: $x + 2y \leq 12$,

$2x + y \leq 12$,

$x \geq 0$,

$y \geq 0$



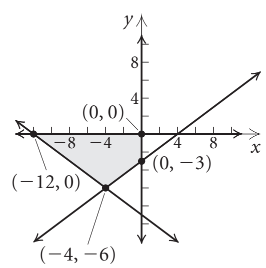

**57.** Graph: $4x - 3y \geq -12$,

$4x + 3y \geq -36$,

$y \leq 0$,

$x \leq 0$

Shade the intersection of the graphs of the four inequalities.




We find the vertex $(-12, 0)$ by solving the system

$$4y + 3x = -36,$$
$$y = 0.$$

We find the vertex $(0, 0)$ by solving the system

$$y = 0,$$
$$x = 0.$$

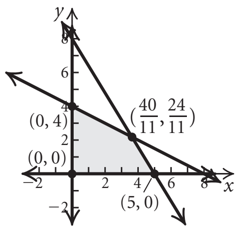
We find the vertex $(0, -3)$ by solving the system

$$4y - 3x = -12,$$
$$x = 0.$$

We find the vertex $(-4, -6)$ by solving the system

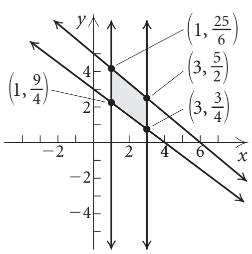
$$4y - 3x = -12,$$
$$4y + 3x = -36.$$

**58.** Graph: $8x+5y\le 40$,
$x+2y\le 8$,
$x\ge 0$,
$y\ge 0$




**59.** Graph: $3x+4y\ge 12$,
$5x+6y\le 30$,
$1\le x\le 3$

Shade the intersection of the graphs of the given inequalities.




We find the vertex $\left(1, \dfrac{25}{6}\right)$ by solving the system

$$5x+6y=30,$$
$$x=1.$$

We find the vertex $\left(3, \dfrac{5}{2}\right)$ by solving the system

$$5x+6y=30,$$
$$x=3.$$

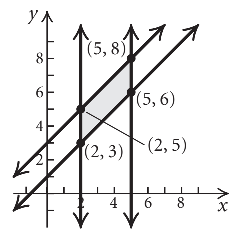
We find the vertex $\left(3, \dfrac{3}{4}\right)$ by solving the system

$$3x+4y=12,$$
$$x=3.$$

We find the vertex $\left(1, \dfrac{9}{4}\right)$ by solving the system

$$3x+4y=12,$$
$$x=1.$$

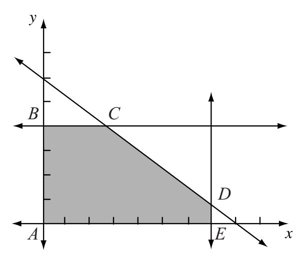
**60.** Graph: $y-x\ge 1$,
$y-x\le 3$,
$2\le x\le 5$




**61.** Find the maximum and minimum values of $P=17x-3y+60$, subject to

$$6x+8y\le 48,$$
$$0\le y\le 4,$$
$$0\le x\le 7.$$

Graph the system of inequalities and determine the vertices.




Vertex $A$: $(0,0)$

Vertex $B$:
We solve the system $x=0$ and $y=4$. The coordinates of point $B$ are $(0,4)$.

Vertex $C$:
We solve the system $6x+8y=48$ and $y=4$. The coordinates of point $C$ are $\left(\dfrac{8}{3}, 4\right)$.

Vertex $D$:
We solve the system $6x+8y=48$ and $x=7$. The coordinates of point $D$ are $\left(7, \dfrac{3}{4}\right)$.

Vertex $E$:
We solve the system $x=7$ and $y=0$. The coordinates of point $E$ are $(7,0)$.

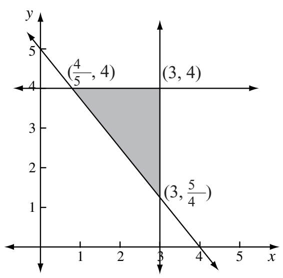
Evaluate the objective function $P$ at each vertex.

| Vertex | $P=17x-3y+60$ |
|---|---|
| $A(0,0)$ | $17\cdot 0-3\cdot 0+60=60$ |
| $B(0,4)$ | $17\cdot 0-3\cdot 4+60=48$ |
| $C\left(\dfrac{8}{3},4\right)$ | $17\cdot\dfrac{8}{3}-3\cdot 4+60=66\dfrac{2}{3}$ |
| $D\left(7,\dfrac{3}{4}\right)$ | $17\cdot 7-3\cdot\dfrac{3}{4}+60=176\dfrac{3}{4}$ |
| $E(7,0)$ | $17\cdot 7-3\cdot 0+60=179$ |

The maximum value of $P$ is 179 when $x=7$ and $y=0$.

The minimum value of $P$ is 48 when $x=0$ and $y=4$.

**62.** We graph the system of inequalities and find the vertices:



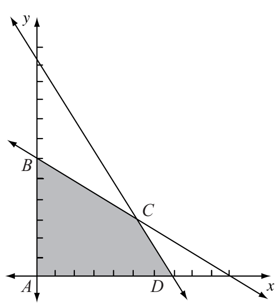

| Vertex | $Q = 28x - 4y + 72$ |
|---|---|
| $\left(\frac{4}{5}, 4\right)$ | $78\frac{2}{5} \longleftarrow$ Minimum |
| $(3, 4)$ | 140 |
| $\left(3, \frac{5}{4}\right)$ | $151 \longleftarrow$ Maximum |

**63.** Find the maximum and minimum values of

$F = 5x + 36y$, subject to

$$5x + 3y \le 34,$$
$$3x + 5y \le 30,$$
$$x \ge 0,$$
$$y \ge 0.$$

Graph the system of inequalities and find the vertices.




Vertex $A$: $(0, 0)$

Vertex $B$:
We solve the system $3x + 5y = 30$ and $x = 0$. The coordinates of point $B$ are $(0, 6)$.

Vertex $C$:
We solve the system $5x + 3y = 34$ and $3x + 5y = 30$. The coordinates of point $C$ are $(5, 3)$.

Vertex $D$:
We solve the system $5x + 3y = 34$ and $y = 0$. The coordinates of point $D$ are $\left(\frac{34}{5}, 0\right)$.

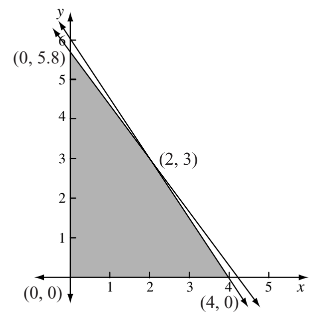
Evaluate the objective function $F$ at each vertex.

| Vertex | $F = 5x + 36y$ |
|---|---|
| $A(0, 0)$ | $5 \cdot 0 + 36 \cdot 0 + = 0$ |
| $B(0, 6)$ | $5 \cdot 0 + 36 \cdot 6 = 216$ |
| $C(5, 3)$ | $5 \cdot 5 + 39 \cdot 3 = 133$ |
| $D\left(\frac{34}{5}, 0\right)$ | $5 \cdot \frac{34}{5} + 36 \cdot 0 = 34$ |

The maximum value of $F$ is 216 when $x = 0$ and $y = 6$.

The minimum value of $F$ is 0 when $x = 0$ and $y = 0$.

**64.** We graph the system of inequalities and find the vertices:




| Vertex | $G = 16x + 14y$ |
|---|---|
| $(0, 0)$ | $0 \longleftarrow$ Minimum |
| $(0, 5.8)$ | $81.2 \longleftarrow$ Maximum |
| $(2, 3)$ | 74 |
| $(4, 0)$ | 64 |

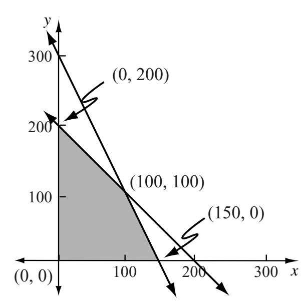
**65.** Let $x =$ the number of jumbo biscuits and $y =$ the number of regular biscuits to be made per day. The income $I$ is given by

$$I = 0.10x + 0.08y$$

subject to the constraints

$$x + y \le 200,$$
$$2x + y \le 300,$$
$$x \ge 0,$$
$$y \ge 0.$$

We graph the system of inequalities, determine the vertices, and find the value if $I$ at each vertex.




| Vertex | $I = 0.10x + 0.08y$ |
|---|---|
| $(0, 0)$ | $0.10(0) + 0.08(0) = 0$ |
| $(0, 200)$ | $0.10(0) + 0.08(200) = 16$ |
| $(100, 100)$ | $0.10(100) + 0.08(100) = 18$ |
| $(150, 0)$ | $0.10(150) + 0.08(0) = 15$ |

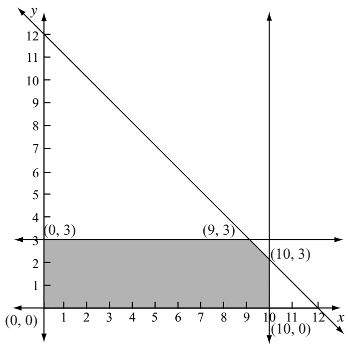
The company will have a maximum income of \$18 when 100 of each type of biscuit are made.

**66.** Let $x =$ the number of gallons the pickup truck uses and $y =$ the number of gallons the moped uses. Find the maximum value of

$$M = 20x + 100y$$

subject to

$$\begin{aligned} x + y &\leq 12, \\ 0 \leq x &\leq 10, \\ 0 \leq y &\leq 3. \end{aligned}$$




| Vertex | $M = 20x + 100y$ |
|---|---|
| $(0, 0)$ | 0 |
| $(0, 3)$ | 300 |
| $(9, 3)$ | 480 |
| $(10, 2)$ | 400 |
| $(10, 0)$ | 200 |

The maximum number of miles is 480 when the pickup truck uses 9 gal and the moped uses 3 gal.

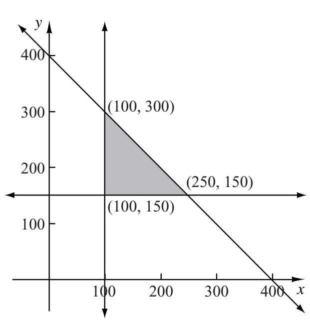
**67.** Let $x$ = the number of units of lumber and $y$ = the number of units of plywood produced per week. The profit $P$ is given by

$$P = 20x + 30y$$

subject to the constraints

$$\begin{aligned} x + y &\leq 400, \\ x &\geq 100, \\ y &\geq 150. \end{aligned}$$

We graph the system of inequalities, determine the vertices and find the value of $P$ at each vertex.




| Vertex | $P = 20x + 30y$ |
|---|---|
| $(100, 150)$ | $20 \cdot 100 + 30 \cdot 150 = 6500$ |
| $(100, 300)$ | $20 \cdot 100 + 30 \cdot 300 = 11,000$ |
| $(250, 150)$ | $20 \cdot 250 + 30 \cdot 150 = 9500$ |

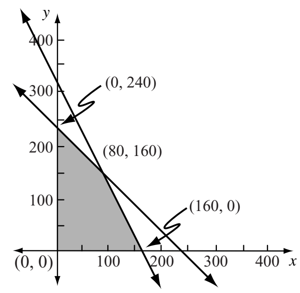
The maximum profit of \$11,000 is achieved by producing 100 units of lumber and 300 units of plywood.

**68.** Let $x$ = the corn acreage and $y$ = the oats acreage. Find the maximum value of

$$P = 40x + 30y$$

subject to

$$\begin{aligned} x + y &\leq 240, \\ 2x + y &\leq 320, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$




| Vertex | $P = 40x + 30y$ |
|---|---|
| $(0, 0)$ | 0 |
| $(0, 240)$ | 7200 |
| $(80, 160)$ | 8000 |
| $(160, 0)$ | 6400 |

The maximum profit of \$8000 occurs when 80 acres of corn and 160 acres of oats are planted.

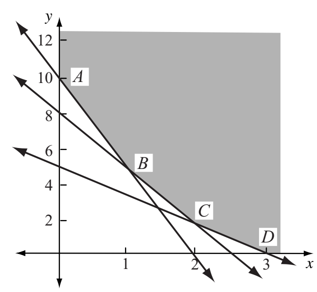
**69.** Let $x$ = the number of sacks of soybean meal to be used and $y$ = the number of sacks of oats. The minimum cost is given by

$$C = 15x + 5y$$

subject to the constraints

$$\begin{aligned} 50x + 15y &\geq 120, \\ 8x + 5y &\geq 24, \\ 5x + y &\geq 10, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $C$ at each vertex.



| Vertex | $C = 15x + 5y$ |
|---|---|
| $A(0,10)$ | $15 \cdot 0 + 5 \cdot 10 = 50$ |
| $B\left(\frac{6}{5}, 4\right)$ | $15 \cdot \frac{6}{5} + 5 \cdot 4 = 38$ |
| $C\left(\frac{24}{13}, \frac{24}{13}\right)$ | $15 \cdot \frac{24}{13} + 5 \cdot \frac{24}{13} = 36\frac{12}{13}$ |
| $D(3,0)$ | $15 \cdot 3 + 5 \cdot 0 = 45$ |

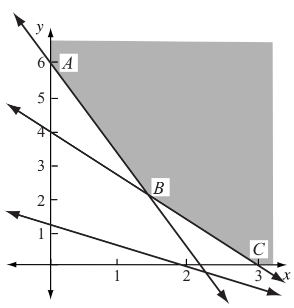
The minimum cost of $\$36\frac{12}{13}$ is achieved by using $\frac{24}{13}$, or $1\frac{11}{13}$ sacks of soybean meal and $\frac{24}{13}$, or $1\frac{11}{13}$ sacks of oats.

**70.** Let $x$ = the number of sacks of soybean meal to be used and $y$ = the number of sacks of alfalfa. Find the minimum value of

$$C = 15x + 8y$$

subject to the constraints

$$50x + 20y \geq 120,$$
$$8x + 6y \geq 24,$$
$$5x + 8y \geq 10,$$
$$x \geq 0,$$
$$y \geq 0.$$




| Vertex | $C = 15x + 8y$ |
|---|---|
| $A(0,6)$ | $15 \cdot 0 + 8 \cdot 6 = 48$ |
| $B\left(\frac{12}{7}, \frac{12}{7}\right)$ | $15 \cdot \frac{12}{7} + 8 \cdot \frac{12}{7} = 39\frac{3}{7}$ |
| $C(3,0)$ | $15 \cdot 3 + 8 \cdot 0 = 45$ |

The minimum cost of $\$39\frac{3}{7}$ is achieved by using $\frac{12}{7}$, or $1\frac{5}{7}$ sacks of soybean meal and $\frac{12}{7}$, or $1\frac{5}{7}$ sacks of alfalfa.

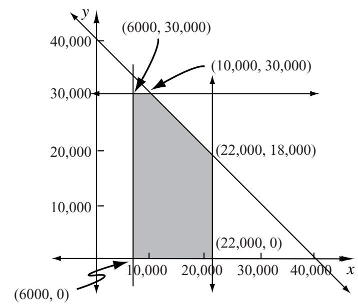
**71.** Let $x$ = the amount invested in corporate bonds and $y$ = the amount invested in municipal bonds. The income $I$ is given by

$$I = 0.08x + 0.075y$$

subject to the constraints

$$x + y \leq 40,000,$$
$$6000 \leq x \leq 22,000,$$
$$0 \leq y \leq 30,000.$$

We graph the system of inequalities, determine the vertices, and find the value of $I$ at each vertex.




| Vertex | $I = 0.08x + 0.075y$ |
|---|---|
| $(6000, 0)$ | 480 |
| $(6000, 30,000)$ | 2730 |
| $(10,000, 30,000)$ | 3050 |
| $(22,000, 18,000)$ | 3110 |
| $(22,000, 0)$ | 1760 |

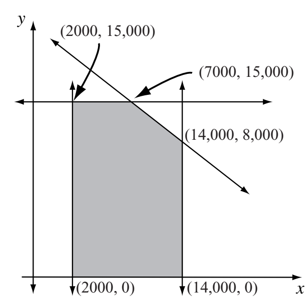
The maximum income of \$3110 occurs when \$22,000 is invested in corporate bonds and \$18,000 is invested in municipal bonds.

**72.** Let $x$ = the amount invested in City Bank and $y$ = the amount invested in People's Bank. Find the maximum value of

$$I = 0.06x + 0.065y$$

subject to

$$x + y \leq 22,000,$$
$$2000 \leq x \leq 14,000$$
$$0 \leq y \leq 15,000.$$




| Vertex | $I = 0.06x + 0.065y$ |
|---|---|
| $(2000, 0)$ | 120 |
| $(2000, 15,000)$ | 1095 |
| $(7000, 15,000)$ | 1395 |
| $(14,000, 8,000)$ | 1360 |
| $(14,000, 0)$ | 840 |

The maximum interest income is \$1395 when \$7000 is invested in City Bank and \$15,000 is invested in People's Bank.

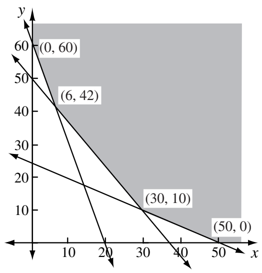
**73.** Let $x$ = the number of $P_1$ airplanes and $y$ = the number of $P_2$ airplanes to be used. The operating cost $C$, in thousands of dollars, is given by

$$C = 12x + 10y$$

subject to the constraints

$$\begin{aligned} 40x + 80y &\geq 2000, \\ 40x + 30y &\geq 1500, \\ 120x + 40y &\geq 2400, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $C$ at each vertex.




| Vertex | $C = 12x + 10y$ |
|---|---|
| $(0, 60)$ | $12 \cdot 0 + 10 \cdot 60 = 600$ |
| $(6, 42)$ | $12 \cdot 6 + 10 \cdot 42 = 492$ |
| $(30, 10)$ | $12 \cdot 30 + 10 \cdot 10 = 460$ |
| $(50, 0)$ | $12 \cdot 50 + 10 \cdot 0 = 600$ |

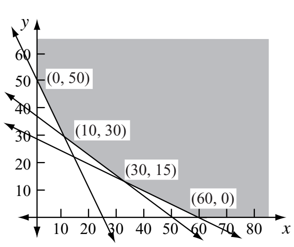
The minimum cost of \$460 thousand is achieved using 30 $P_1$'s and 10 $P_2$'s.

**74.** Let $x$ = the number of $P_2$ airplanes and $y$ = the number of $P_3$ airplanes to be used. Find the minimum value of

$$C = 10x + 15y$$

subject to

$$\begin{aligned} 80x + 40y &\geq 2000, \\ 30x + 40y &\geq 1500, \\ 40x + 80y &\geq 2400, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$




| Vertex | $C = 10x + 15y$ |
|---|---|
| $(0, 50)$ | $10 \cdot 0 + 15 \cdot 50 = 750$ |
| $(10, 30)$ | $10 \cdot 10 + 15 \cdot 30 = 550$ |
| $(30, 15)$ | $10 \cdot 30 + 15 \cdot 15 = 525$ |
| $(60, 0)$ | $10 \cdot 60 + 15 \cdot 0 = 600$ |

The minimum cost of \$525 thousand is achieved using 30 $P_2$'s and 15 $P_3$'s.

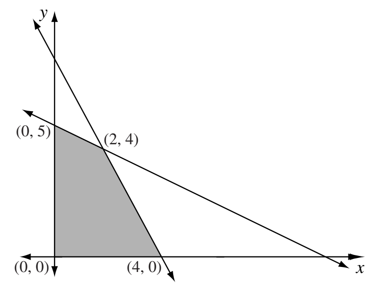
**75.** Let $x$ = the number of knit suits and $y$ = the number of worsted suits made. The profit is given by

$$P = 34x + 31y$$

subject to

$$\begin{aligned} 2x + 4y &\leq 20, \\ 4x + 2y &\leq 16, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $P$ at each vertex.




| Vertex | $P = 34x + 31y$ |
|---|---|
| $(0, 0)$ | $34 \cdot 0 + 31 \cdot 0 = 0$ |
| $(0, 5)$ | $34 \cdot 0 + 31 \cdot 5 = 155$ |
| $(2, 4)$ | $34 \cdot 2 + 31 \cdot 4 = 192$ |
| $(4, 0)$ | $34 \cdot 4 + 31 \cdot 0 = 136$ |

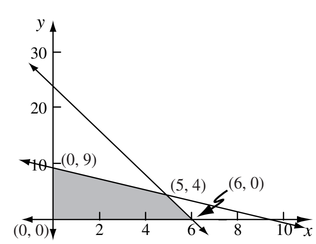
The maximum profit per day is \$192 when 2 knit suits and 4 worsted suits are made.

**76.** Let $x$ = the number of smaller gears and $y$ = the number of larger gears produced each day. Find the maximum value of

$$P = 25x + 10y$$

subject to

$$\begin{aligned} 4x + y &\leq 24, \\ x + y &\leq 9, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$




| Vertex | $P = 25x + 10y$ |
|---|---|
| $(0, 0)$ | $25 \cdot 0 + 10 \cdot 0 = 0$ |
| $(0, 9)$ | $25 \cdot 0 + 10 \cdot 9 = 90$ |
| $(5, 4)$ | $25 \cdot 5 + 10 \cdot 4 = 165$ |
| $(6, 0)$ | $25 \cdot 6 + 10 \cdot 0 = 150$ |

The maximum profit of \$165 per day is achieved when 5 smaller gears and 4 larger gears are produced.

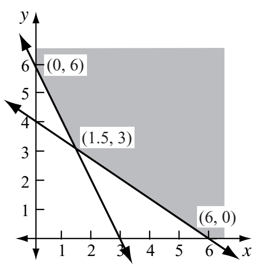
**77.** Let $x$ = the number of pounds of meat and $y$ = the number of pounds of cheese in the diet in a week. The cost is given by

$$C = 3.50x + 4.60y$$

subject to

$$\begin{aligned} 2x + 3y &\geq 12, \\ 2x + y &\geq 6, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $C$ at each vertex.




| Vertex | $C = 3.50x + 4.60y$ |
|---|---|
| $(0, 6)$ | $3.50(0) + 4.60(6) = 27.60$ |
| $(1.5, 3)$ | $3.50(1.5) + 4.60(3) = 19.05$ |
| $(6, 0)$ | $3.50(6) + 4.60(0) = 21.00$ |

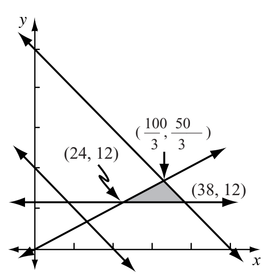
The minimum weekly cost of \$19.05 is achieved when 1.5 lb of meat and 3 lb of cheese are used.

**78.** Let $x$ = the number of teachers and $y$ = the number of teacher's aides. Find the minimum value of

$$C = 35,000x + 18,000y$$

subject to

$$\begin{aligned} x + y &\leq 50, \\ x + y &\geq 20, \\ y &\geq 12, \\ x &\geq 2y. \end{aligned}$$




| Vertex | $C = 35,000x + 18,000y$ |
|---|---|
| $(24, 12)$ | $35,000 \cdot 24 + 18,000 \cdot 12 = 1,056,000$ |
| $\left(\frac{100}{3}, \frac{50}{3}\right)$ | $35,000 \cdot \frac{100}{3} + 18,000 \cdot \frac{50}{3} \approx 1,466,667$ |
| $(38, 12)$ | $35,000 \cdot 38 + 18,000 \cdot 12 = 1,546,000$ |

The minimum cost of \$1,056,000 is achieved when 24 teachers and 12 teacher's aides are hired.

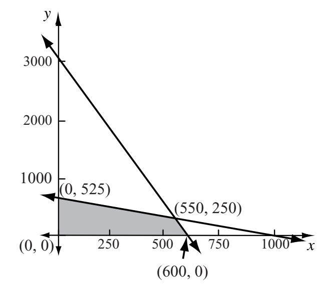
**79.** Let $x$ = the number of animal $A$ and $y$ = the number of animal $B$. The total number of animals is given by

$$T = x + y$$

subject to

$$\begin{aligned} x + 0.2y &\leq 600, \\ 0.5x + y &\leq 525, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $T$ at each vertex.




| Vertex | $T = x + y$ |
|---|---|
| $(0, 0)$ | $0 + 0 = 0$ |
| $(0, 525)$ | $0 + 525 = 525$ |
| $(550, 250)$ | $550 + 250 = 800$ |
| $(600, 0)$ | $600 + 0 = 600$ |

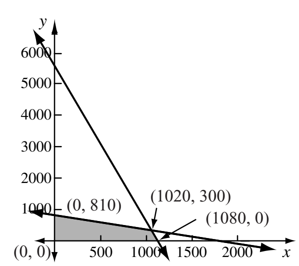
The maximum total number of 800 is achieved when there are 550 of $A$ and 250 of $B$.

**80.** Let $x$ = the number of animal $A$ and $y$ = the number of animal $B$. Find the maximum value of

$$T = x + y$$

subject to

$$\begin{aligned} x + 0.2y &\leq 1080, \\ 0.5x + y &\leq 810, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}$$




| Vertex | $T = x + y$ |
|---|---|
| $(0, 0)$ | $0 + 0 = 0$ |
| $(0, 810)$ | $0 + 810 = 810$ |
| $(1020, 300)$ | $1020 + 300 = 1320$ |
| $(1080, 0)$ | $1080 + 0 = 1080$ |

The maximum total number of 1320 is achieved when there are 1020 of $A$ and 300 of $B$.

**81.** $-5 \le x + 2 < 4$

$-7 \le x < 2$ Subtracting 2

The solution set is $\{x| -7 \le x < 2\}$, or $[-7, 2)$.

**82.** $|x - 3| \ge 2$

$x - 3 \le -2$ *or* $x - 3 \ge 2$

$x \le 1$ *or* $x \ge 5$

The solution set is $\{x|x \le 1$ *or* $x \ge 5\}$, or $(-\infty, 1] \cup [5, \infty)$.

**83.** $x^2 - 2x \le 3$ Polynomial inequality

$x^2 - 2x - 3 \le 0$

$x^2 - 2x - 3 = 0$ Related equation

$(x + 1)(x - 3) = 0$ Factoring

Using the principle of zero products or by observing the graph of $y = x^2 - 2x - 3$, we see that the solutions of the related equation are $-1$ and 3. These numbers divide the $x$-axis into the intervals $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$. We let $f(x) = x^2 - 2x - 3$ and test a value in each interval.

$(-\infty, -1)$: $f(-2) = 5 > 0$

$(-1, 3)$: $f(0) = -3 < 0$

$(3, \infty)$: $f(4) = 5 > 0$

Function values are negative on $(-1, 3)$. This can also be determined from the graph of $y = x^2 - 2x - 3$. Since the inequality symbol is $\le$, the endpoints of the interval must be included in the solution set. It is $\{x| -1 \le x \le 3\}$ or $[-1, 3]$.

**84.** $\dfrac{x-1}{x+2} > 4$ Rational inequality

$\dfrac{x-1}{x+2} - 4 > 0$

$\dfrac{x-1}{x+2} - 4 = 0$ Related equation

The denominator of $f(x) = \dfrac{x-1}{x+2} - 4$ is 0 when $x = -2$, so the function is not defined for $x = -2$. We solve the related equation $f(x) = 0$.

$\dfrac{x-1}{x+2} - 4 = 0$

$x - 1 - 4(x + 2) = 0$ Multiplying by $x + 2$

$x - 1 - 4x - 8 = 0$

$-3x - 9 = 0$

$-3x = 9$

$x = -3$

Thus, the critical values are $-3$ and $-2$. They divide the $x$-axis into the intervals $(-\infty, -3)$, $(-3, -2)$, and $(-2, \infty)$. We test a value in each interval.

$(-\infty, -3)$: $f(-4) = -\dfrac{3}{2} < 0$

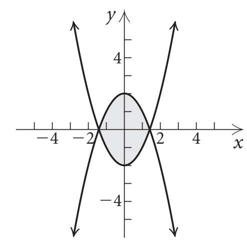
$(-3, -2)$: $f(-2.5) = 3 > 0$

$(-2, \infty)$: $f(0) = -\dfrac{9}{2} < 0$

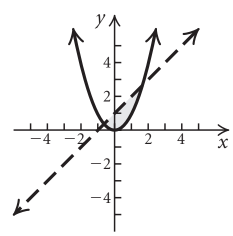
Function values are positive on $(-3, -2)$. This can also be determined from the graph of $y = \dfrac{x-1}{x+2} - 4$. The solution set is $\{x| -3 < x < -2\}$, or $(-3, -2)$.

**85.** Graph: $y \ge x^2 - 2$,
$y \le 2 - x^2$

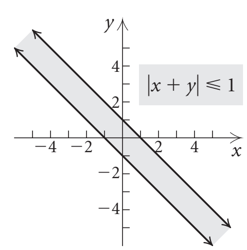
First graph the related equations $y = x^2 - 2$ and $y = 2 - x^2$ using solid lines. The solution set consists of the region above the graph of $y = x^2 - 2$ and below the graph of $y = 2 - x^2$.



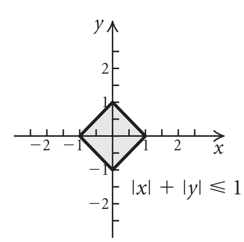
**86.** $y < x + 1$
$y \ge x^2$



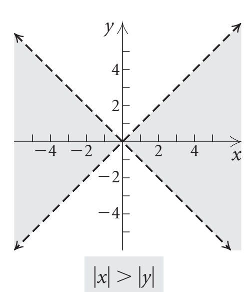
**87.**



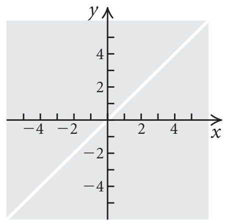

**88.**




**89.**



**90.**




$|x - y| > 0$

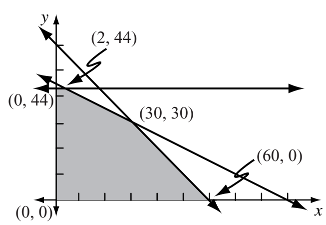
**91.** Let $x$ = the number of less expensive speaker assemblies and $y$ = the number of more expensive assemblies. The income is given by

$$I = 350x + 600y$$

subject to

$$\begin{aligned} y &\le 44 \\ x + y &\le 60, \\ x + 2y &\le 90, \\ x &\ge 0, \\ y &\ge 0. \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $I$ at each vertex.




| Vertex | $I = 350x + 600y$ |
|---|---|
| $(0, 0)$ | $350 \cdot 0 + 600 \cdot 0 = 0$ |
| $(0, 44)$ | $350 \cdot 0 + 600 \cdot 44 = 26,400$ |
| $(2, 44)$ | $350 \cdot 2 + 600 \cdot 44 = 27,100$ |
| $(30, 30)$ | $350 \cdot 30 + 600 \cdot 30 = 28,500$ |
| $(60, 0)$ | $350 \cdot 60 + 600 \cdot 0 = 21,000$ |

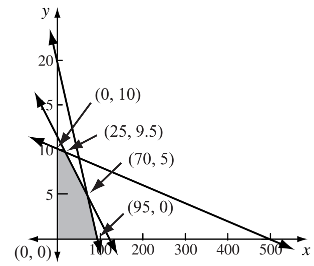
The maximum income of \$28,500 is achieved when 30 less expensive and 30 more expensive assemblies are made.

**92.** Let $x$ = the number of chairs and $y$ = the number of sofas produced. Find the maximum value of

$$I = 80x + 300y$$

subject to

$$\begin{aligned} 20x + 100y &\le 1900, \\ x + 50y &\le 500, \\ 2x + 20y &\le 240, \\ x &\ge 0, \\ y &\ge 0. \end{aligned}$$




| Vertex | $I = 80x + 300y$ |
|---|---|
| $(0, 0)$ | $80 \cdot 0 + 300 \cdot 0 = 0$ |
| $(0, 10)$ | $80 \cdot 0 + 300 \cdot 10 = 3000$ |
| $(25, 9.5)$ | $80 \cdot 25 + 300(9.5) = 4850$ |
| $(70, 5)$ | $80 \cdot 70 + 300 \cdot 5 = 7100$ |
| $(95, 0)$ | $80 \cdot 95 + 300 \cdot 0 = 7600$ |

The maximum income of \$7600 is achieved by making 95 chairs and 0 sofas.

## Exercise Set 9.8

**1.** $$\frac{x+7}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$\frac{x+7}{(x-3)(x+2)} = \frac{A(x+2) + B(x-3)}{(x-3)(x+2)} \quad \text{Adding}$$

Equate the numerators:

$x + 7 = A(x+2) + B(x-3)$

Let $x + 2 = 0$, or $x = -2$. Then we get

$$\begin{aligned} -2 + 7 &= 0 + B(-2-3) \\ 5 &= -5B \\ -1 &= B \end{aligned}$$

Next let $x - 3 = 0$, or $x = 3$. Then we get

$$\begin{aligned} 3 + 7 &= A(3+2) + 0 \\ 10 &= 5A \\ 2 &= A \end{aligned}$$

The decomposition is as follows:

$$\frac{2}{x-3} - \frac{1}{x+2}$$

**2.** $$\frac{2x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\frac{2x}{(x+1)(x-1)} = \frac{A(x-1) + B(x+1)}{(x+1)(x-1)}$$

$$2x = A(x-1) + B(x+1)$$

Let $x = 1$: $2 \cdot 1 = 0 + B(1+1)$

$$\begin{aligned} 2 &= 2B \\ 1 &= B \end{aligned}$$

Let $x = -1$: $2(-1) = A(-1-1) + 0$

$$\begin{aligned} -2 &= -2A \\ 1 &= A \end{aligned}$$

The decomposition is $\dfrac{1}{x+1} + \dfrac{1}{x-1}$.

**3.** $\dfrac{7x-1}{6x^2-5x+1}$

$= \dfrac{7x-1}{(3x-1)(2x-1)}$ Factoring the denominator

$= \dfrac{A}{3x-1}+\dfrac{B}{2x-1}$

$= \dfrac{A(2x-1)+B(3x-1)}{(3x-1)(2x-1)}$ Adding

Equate the numerators:

$7x-1 = A(2x-1)+B(3x-1)$

Let $2x-1=0$, or $x=\dfrac{1}{2}$. Then we get

$$7\left(\frac{1}{2}\right)-1 = 0+B\left(3\cdot\frac{1}{2}-1\right)$$
$$\frac{5}{2} = \frac{1}{2}B$$
$$5 = B$$

Next let $3x-1=0$, or $x=\dfrac{1}{3}$. We get

$$7\left(\frac{1}{3}\right)-1 = A\left(2\cdot\frac{1}{3}-1\right)+0$$
$$\frac{7}{3}-1 = A\left(\frac{2}{3}-1\right)$$
$$\frac{4}{3} = -\frac{1}{3}A$$
$$-4 = A$$

The decomposition is as follows:

$$-\frac{4}{3x-1}+\frac{5}{2x-1}$$

**4.** $\dfrac{13x+46}{12x^2-11x-15} = \dfrac{13x+46}{(4x+3)(3x-5)}$

$= \dfrac{A}{4x+3}+\dfrac{B}{3x-5}$

$= \dfrac{A(3x-5)+B(4x+3)}{(4x+3)(3x-5)}$

Let $x=\dfrac{5}{3}$: $13\cdot\dfrac{5}{3}+46 = 0+B\left(4\cdot\dfrac{5}{3}+3\right)$

$$\frac{203}{3} = \frac{29}{3}B$$
$$7 = B$$

Let $x=-\dfrac{3}{4}$:

$$13\left(-\frac{3}{4}\right)+46 = A\left[3\left(-\frac{3}{4}\right)-5\right]+0$$
$$\frac{145}{4} = -\frac{29}{4}A$$
$$-5 = A$$

The decomposition is $-\dfrac{5}{4x+3}+\dfrac{7}{3x-5}$.

**5.** $\dfrac{3x^2-11x-26}{(x^2-4)(x+1)}$

$= \dfrac{3x^2-11x-26}{(x+2)(x-2)(x+1)}$ Factoring the denominator

$= \dfrac{A}{x+2}+\dfrac{B}{x-2}+\dfrac{C}{x+1}$

$= \dfrac{A(x-2)(x+1)+B(x+2)(x+1)+C(x+2)(x-2)}{(x+2)(x-2)(x+1)}$ Adding

Equate the numerators:

$3x^2-11x-26 = A(x-2)(x+1)+$
$B(x+2)(x+1)+C(x+2)(x-2)$

Let $x+2=0$ or $x=-2$. Then we get

$$3(-2)^2-11(-2)-26 = A(-2-2)(-2+1)+0+0$$
$$12+22-26 = A(-4)(-1)$$
$$8 = 4A$$
$$2 = A$$

Next let $x-2=0$, or $x=2$. Then, we get

$$3\cdot 2^2-11\cdot 2-26 = 0+B(2+2)(2+1)+0$$
$$12-22-26 = B\cdot 4\cdot 3$$
$$-36 = 12B$$
$$-3 = B$$

Finally let $x+1=0$, or $x=-1$. We get

$$3(-1)^2-11(-1)-26 = 0+0+C(-1+2)(-1-2)$$
$$3+11-26 = C(1)(-3)$$
$$-12 = -3C$$
$$4 = C$$

The decomposition is as follows:

$$\frac{2}{x+2}-\frac{3}{x-2}+\frac{4}{x+1}$$

**6.** $\dfrac{5x^2+9x-56}{(x-4)(x-2)(x+1)}$

$= \dfrac{A}{x-4}+\dfrac{B}{x-2}+\dfrac{C}{x+1}$

$= \dfrac{A(x-2)(x+1)+B(x-4)(x+1)+C(x-4)(x-2)}{(x-4)(x-2)(x+1)}$

$5x^2+9x-56 = A(x-2)(x+1)+$
$B(x-4)(x+1)+C(x-4)(x-2)$

Let $x=4$:

$$5\cdot 4^2+9\cdot 4-56 = A(4-2)(4+1)+0+0$$
$$60 = 10A$$
$$6 = A$$

Let $x=2$:

$$5\cdot 2^2+9\cdot 2-56 = 0+B(2-4)(2+1)+0$$
$$-18 = -6B$$
$$3 = B$$

Let $x=-1$:

$$5(-1)^2+9(-1)-56 = 0+0+C(-1-4)(-1-2)$$
$$-60 = 15C$$
$$-4 = C$$

The decomposition is $\dfrac{6}{x-4}+\dfrac{3}{x-2}-\dfrac{4}{x+1}$.

**7.** $$\frac{9}{(x+2)^2(x-1)}$$
$$=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x-1}$$
$$=\frac{A(x+2)(x-1)+B(x-1)+C(x+2)^2}{(x+2)^2(x-1)}\quad\text{Adding}$$

Equate the numerators:

$$9=A(x+2)(x-1)+B(x-1)+C(x+2)^2\qquad(1)$$

Let $x-1=0$, or $x=1$. Then, we get

$$9=0+0+C(1+2)^2$$
$$9=9C$$
$$1=C$$

Next let $x+2=0$, or $x=-2$. Then, we get

$$9=0+B(-2-1)+0$$
$$9=-3B$$
$$-3=B$$

To find $A$ we first simplify equation (1).

$$\begin{aligned}9&=A(x^2+x-2)+B(x-1)+C(x^2+4x+4)\\&=Ax^2+Ax-2A+Bx-B+Cx^2+4Cx+4C\\&=(A+C)x^2+(A+B+4C)x+(-2A-B+4C)\end{aligned}$$

Then we equate the coefficients of $x^2$.

$$0=A+C$$
$$0=A+1\quad\text{Substituting 1 for } C$$
$$-1=A$$

The decomposition is as follows:

$$-\frac{1}{x+2}-\frac{3}{(x+2)^2}+\frac{1}{x-1}$$

**8.** $$\frac{x^2-x-4}{(x-2)^3}$$
$$=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}$$
$$=\frac{A(x-2)^2+B(x-2)+C}{(x-2)^3}$$

$$x^2-x-4=A(x-2)^2+B(x-2)+C\qquad(1)$$

Let $x=2$: $2^2-2-4=0+0+C$

$$-2=C$$

Simplify equation (1).

$$\begin{aligned}x^2-x-4&=Ax^2-4Ax+4A+Bx-2B+C\\&=Ax^2+(-4A+B)x+(4A-2B+C)\end{aligned}$$

Then $1=A$ and

$-1=-4A+B$, so $B=3$.

The decomposition is $\dfrac{1}{x-2}+\dfrac{3}{(x-2)^2}-\dfrac{2}{(x-2)^3}$.

**9.** $$\frac{2x^2+3x+1}{(x^2-1)(2x-1)}$$
$$=\frac{2x^2+3x+1}{(x+1)(x-1)(2x-1)}\quad\text{Factoring the denominator}$$
$$=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{2x-1}$$
$$=\frac{A(x-1)(2x-1)+B(x+1)(2x-1)+C(x+1)(x-1)}{(x+1)(x-1)(2x-1)}\quad\text{Adding}$$

Equate the numerators:

$$2x^2+3x+1=A(x-1)(2x-1)+B(x+1)(2x-1)+C(x+1)(x-1)$$

Let $x+1=0$, or $x=-1$. Then, we get

$$2(-1)^2+3(-1)+1=A(-1-1)[2(-1)-1]+0+0$$
$$2-3+1=A(-2)(-3)$$
$$0=6A$$
$$0=A$$

Next let $x-1=0$, or $x=1$. Then, we get

$$2\cdot1^2+3\cdot1+1=0+B(1+1)(2\cdot1-1)+0$$
$$2+3+1=B\cdot2\cdot1$$
$$6=2B$$
$$3=B$$

Finally we let $2x-1=0$, or $x=\frac{1}{2}$. We get

$$2\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1=0+0+C\left(\frac{1}{2}+1\right)\left(\frac{1}{2}-1\right)$$
$$\frac{1}{2}+\frac{3}{2}+1=C\cdot\frac{3}{2}\cdot\left(-\frac{1}{2}\right)$$
$$3=-\frac{3}{4}C$$
$$-4=C$$

The decomposition is as follows:

$$\frac{3}{x-1}-\frac{4}{2x-1}$$

**10.** $$\frac{x^2-10x+13}{(x^2-5x+6)(x-1)}$$
$$=\frac{x^2-10x+13}{(x-3)(x-2)(x-1)}$$
$$=\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{x-1}$$
$$=\frac{A(x-2)(x-1)+B(x-3)(x-1)+C(x-3)(x-2)}{(x-3)(x-2)(x-1)}$$

$$x^2-10x+13=A(x-2)(x-1)+B(x-3)(x-1)+C(x-3)(x-2)$$

Let $x=3$:

$$3^2-10\cdot3+13=A(3-2)(3-1)+0+0$$
$$-8=2A$$
$$-4=A$$

Let $x = 2$:

$$2^2 - 10 \cdot 2 + 13 = 0 + B(2-3)(2-1) + 0$$
$$-3 = -B$$
$$3 = B$$

Let $x = 1$:

$$1^2 - 10 \cdot 1 + 13 = 0 + 0 + C(1-3)(1-2)$$
$$4 = 2C$$
$$2 = C$$

The decomposition is $-\dfrac{4}{x-3} + \dfrac{3}{x-2} + \dfrac{2}{x-1}$.

**11.** $\dfrac{x^4 - 3x^3 - 3x^2 + 10}{(x+1)^2(x-3)}$

$= \dfrac{x^4 - 3x^3 - 3x^2 + 10}{x^3 - x^2 - 5x - 3}$ Multiplying the denominator

Since the degree of the numerator is greater than the degree of the denominator, we divide.

$$\begin{array}{r@{}l} & \phantom{x^4-}x - \phantom{3}2 \\ x^3 - x^2 - 5x - 3 & \overline{)\,x^4 - 3x^3 - 3x^2 + 0x + 10} \\ & \underline{x^4 - \phantom{3}x^3 - 5x^2 - \phantom{0}3x} \\ & \phantom{x^4}-2x^3 + 2x^2 + \phantom{0}3x + 10 \\ & \phantom{x^4}\underline{-2x^3 + 2x^2 + 10x + \phantom{1}6} \\ & \phantom{x^4 - 2x^3 + 2x^2}-\phantom{1}7x + \phantom{1}4 \end{array}$$

The original expression is thus equivalent to the following:

$x - 2 + \dfrac{-7x+4}{x^3 - x^2 - 5x - 3}$

We proceed to decompose the fraction.

$\dfrac{-7x+4}{(x+1)^2(x-3)}$

$= \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-3}$

$= \dfrac{A(x+1)(x-3) + B(x-3) + C(x+1)^2}{(x+1)^2(x-3)}$

Adding

Equate the numerators:

$$-7x + 4 = A(x+1)(x-3) + B(x-3) + C(x+1)^2 \qquad (1)$$

Let $x - 3 = 0$, or $x = 3$. Then, we get

$$-7 \cdot 3 + 4 = 0 + 0 + C(3+1)^2$$
$$-17 = 16C$$
$$-\frac{17}{16} = C$$

Let $x + 1 = 0$, or $x = -1$. Then, we get

$$-7(-1) + 4 = 0 + B(-1-3) + 0$$
$$11 = -4B$$
$$-\frac{11}{4} = B$$

To find $A$ we first simplify equation (1).

$-7x + 4$

$= A(x^2 - 2x - 3) + B(x-3) + C(x^2 + 2x + 1)$

$= Ax^2 - 2Ax - 3A + Bx - 3B + Cx^2 - 2Cx + C$

$= (A+C)x^2 + (-2A+B-2C)x + (-3A-3B+C)$

Then equate the coefficients of $x^2$.

$0 = A + C$

Substituting $-\dfrac{17}{16}$ for $C$, we get $A = \dfrac{17}{16}$.

The decomposition is as follows:

$\dfrac{17/16}{x+1} - \dfrac{11/4}{(x+1)^2} - \dfrac{17/16}{x-3}$

The original expression is equivalent to the following:

$x - 2 + \dfrac{17/16}{x+1} - \dfrac{11/4}{(x+1)^2} - \dfrac{17/16}{x-3}$

**12.** $\dfrac{10x^3 - 15x^2 - 35x}{x^2 - x - 6} = 10x - 5 + \dfrac{20x - 30}{x^2 - x - 6}$

Dividing

$$\frac{20x-30}{x^2-x-6} = \frac{20x-30}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{A(x+2) + B(x-3)}{(x-3)(x+2)}$$

$20x - 30 = A(x+2) + B(x-3)$

Let $x = 3$: $20 \cdot 3 - 30 = A(3+2) + 0$

$$30 = 5A$$
$$6 = A$$

Let $x = -2$: $20(-2) - 30 = 0 + B(-2-3)$

$$-70 = -5B$$
$$14 = B$$

The decomposition is $10x - 5 + \dfrac{6}{x-3} + \dfrac{14}{x+2}$.

**13.** $\dfrac{-x^2 + 2x - 13}{(x^2+2)(x-1)}$

$= \dfrac{Ax + B}{x^2 + 2} + \dfrac{C}{x-1}$

$= \dfrac{(Ax+B)(x-1) + C(x^2+2)}{(x^2+2)(x-1)}$ Adding

Equate the numerators:

$$-x^2 + 2x - 13 = (Ax+B)(x-1) + C(x^2+2) \qquad (1)$$

Let $x - 1 = 0$, or $x = 1$. Then we get

$$-1^2 + 2 \cdot 1 - 13 = 0 + C(1^2 + 2)$$
$$-1 + 2 - 13 = C(1+2)$$
$$-12 = 3C$$
$$-4 = C$$

To find $A$ and $B$ we first simplify equation (1).

$-x^2 + 2x - 13$

$= Ax^2 - Ax + Bx - B + Cx^2 + 2C$

$= (A+C)x^2 + (-A+B)x + (-B+2C)$

Equate the coefficients of $x^2$:

$-1 = A + C$

Substituting $-4$ for $C$, we get $A = 3$.

Equate the constant terms:

$-13 = -B + 2C$

Substituting $-4$ for $C$, we get $B = 5$.

The decomposition is as follows:

$$\frac{3x+5}{x^2+2} - \frac{4}{x-1}$$

**14.** $$\frac{26x^2+208x}{(x^2+1)(x+5)}$$
$$= \frac{Ax+B}{x^2+1} + \frac{C}{x+5}$$
$$= \frac{(Ax+B)(x+5)+C(x^2+1)}{(x^2+1)(x+5)}$$

$$26x^2+208x = (Ax+B)(x+5)+C(x^2+1) \qquad (1)$$

Let $x = -5$:

$$26(-5)^2 + 208(-5) = 0 + C[(-5)^2+1]$$
$$-390 = 26C$$
$$-15 = C$$

Simplify equation (1).

$$26x^2+208x$$
$$= Ax^2+5Ax+Bx+5B+Cx^2+C$$
$$= (A+C)x^2+(5A+B)x+(5B+C)$$

$26 = A+C$

$26 = A-15$

$41 = A$

$0 = 5B+C$

$0 = 5B-15$

$3 = B$

The decomposition is $\dfrac{41x+3}{x^2+1} - \dfrac{15}{x+5}$.

**15.** $$\frac{6+26x-x^2}{(2x-1)(x+2)^2}$$
$$= \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$
$$= \frac{A(x+2)^2+B(2x-1)(x+2)+C(2x-1)}{(2x-1)(x+2)^2}$$

Adding

Equate the numerators:

$$6+26x-x^2 = A(x+2)^2+B(2x-1)(x+2)+ C(2x-1) \qquad (1)$$

Let $2x-1=0$, or $x = \dfrac{1}{2}$. Then, we get

$$6+26\cdot\frac{1}{2} - \left(\frac{1}{2}\right)^2 = A\left(\frac{1}{2}+2\right)^2 + 0 + 0$$
$$6+13-\frac{1}{4} = A\left(\frac{5}{2}\right)^2$$
$$\frac{75}{4} = \frac{25}{4}A$$
$$3 = A$$

Let $x+2=0$, or $x=-2$. We get

$$6+26(-2)-(-2)^2 = 0+0+C[2(-2)-1]$$
$$6-52-4 = -5C$$
$$-50 = -5C$$
$$10 = C$$

To find $B$ we first simplify equation (1).

$$6+26x-x^2$$
$$= A(x^2+4x+4)+B(2x^2+3x-2)+C(2x-1)$$
$$= Ax^2+4Ax+4A+2Bx^2+3Bx-2B+2Cx-C$$
$$= (A+2B)x^2+(4A+3B+2C)x+(4A-2B-C)$$

Equate the coefficients of $x^2$:

$-1 = A+2B$

Substituting 3 for $A$, we obtain $B=-2$.

The decomposition is as follows:

$$\frac{3}{2x-1} - \frac{2}{x+2} + \frac{10}{(x+2)^2}$$

**16.** $$\frac{5x^3+6x^2+5x}{(x^2-1)(x+1)^3}$$
$$= \frac{5x^3+6x^2+5x}{(x-1)(x+1)^4}$$
$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} + \frac{E}{(x+1)^4}$$
$$= [A(x+1)^4+B(x-1)(x+1)^3+C(x-1)(x+1)^2+ D(x-1)(x+1)+E(x-1)]/[(x-1)(x+1)^4]$$

$$5x^3+6x^2+5x = A(x+1)^4+B(x-1)(x+1)^3+C(x-1)(x+1)^2+ D(x-1)(x+1)+E(x-1) \qquad (1)$$

Let $x=1$: $5\cdot 1^3+6\cdot 1^2+5\cdot 1 = A(1+1)^4$
$$16 = 16A$$
$$A = 1$$

Let $x=-1$:

$$5(-1)^3+6(-1)^2+5(-1) = E(-1-1)$$
$$-4 = -2E$$
$$2 = E$$

Simplify equation (1).

$$5x^3+6x^2+5x = (A+B)x^4+(4A+2B+C)x^3+ (6A+C+D)x^2+(4A-2B-C+E)x+ (A-B-C-D-E)$$

$0 = A+B$

$0 = 1+B$

$-1 = B$

$5 = 4A+2B+C$

$5 = 4\cdot 1+2(-1)+C$

$3 = C$

$0 = A-B-C-D-E$

$0 = 1-(-1)-3-D-2$

$D = -3$

The decomposition is

$$\frac{1}{x-1}-\frac{1}{x+1}+\frac{3}{(x+1)^2}-\frac{3}{(x+1)^3}+\frac{2}{(x+1)^4}.$$

**17.** $\dfrac{6x^3+5x^2+6x-2}{2x^2+x-1}$

Since the degree of the numerator is greater than the degree of the denominator, we divide.

$$\begin{array}{r}
3x+\ \ 1\phantom{x-2}\\
2x^2+x-1\,\overline{)\,6x^3+5x^2+6x-2}\\
\underline{6x^3+3x^2-3x\phantom{-2}}\\
2x^2+9x-2\\
\underline{2x^2+\ x-1}\\
8x-1
\end{array}$$

The original expression is equivalent to

$$3x+1+\frac{8x-1}{2x^2+x-1}$$

We proceed to decompose the fraction.

$$\frac{8x-1}{2x^2+x-1}=\frac{8x-1}{(2x-1)(x+1)}\quad\text{Factoring the denominator}$$
$$=\frac{A}{2x-1}+\frac{B}{x+1}$$
$$=\frac{A(x+1)+B(2x-1)}{(2x-1)(x+1)}\quad\text{Adding}$$

Equate the numerators:

$8x-1=A(x+1)+B(2x-1)$

Let $x+1=0$, or $x=-1$. Then we get

$$\begin{aligned}8(-1)-1&=0+B[2(-1)-1]\\-8-1&=B(-2-1)\\-9&=-3B\\3&=B\end{aligned}$$

Next let $2x-1=0$, or $x=\dfrac{1}{2}$. We get

$$\begin{aligned}8\left(\frac{1}{2}\right)-1&=A\left(\frac{1}{2}+1\right)+0\\4-1&=A\left(\frac{3}{2}\right)\\3&=\frac{3}{2}A\\2&=A\end{aligned}$$

The decomposition is

$$\frac{2}{2x-1}+\frac{3}{x+1}.$$

The original expression is equivalent to

$$3x+1+\frac{2}{2x-1}+\frac{3}{x+1}.$$

**18.** $$\frac{2x^3+3x^2-11x-10}{x^2+2x-3}=2x-1+\frac{-3x-13}{x^2+2x-3}\quad\text{Dividing}$$

$$\begin{aligned}\frac{-3x-13}{x^2+2x-3}&=\frac{A}{x+3}+\frac{B}{x-1}\\&=\frac{A(x-1)+B(x+3)}{(x+3)(x-1)}\\-3x-13&=A(x-1)+B(x+3)\end{aligned}$$

Let $x=-3$: $-3(-3)-13=A(-3-1)+0$
$$\begin{aligned}-4&=-4A\\1&=A\end{aligned}$$

Let $x=1$: $-3\cdot 1-13=0+B(1+3)$
$$\begin{aligned}-16&=4B\\-4&=B\end{aligned}$$

The decomposition is $2x-1+\dfrac{1}{x+3}-\dfrac{4}{x-1}$.

**19.** $$\frac{2x^2-11x+5}{(x-3)(x^2+2x-5)}$$
$$=\frac{A}{x-3}+\frac{Bx+C}{x^2+2x-5}$$
$$=\frac{A(x^2+2x-5)+(Bx+C)(x-3)}{(x-3)(x^2+2x-5)}\quad\text{Adding}$$

Equate the numerators:

$$2x^2-11x+5=A(x^2+2x-5)+(Bx+C)(x-3)\qquad(1)$$

Let $x-3=0$, or $x=3$. Then, we get

$$\begin{aligned}2\cdot 3^2-11\cdot 3+5&=A(3^2+2\cdot 3-5)+0\\18-33+5&=A(9+6-5)\\-10&=10A\\-1&=A\end{aligned}$$

To find $B$ and $C$, we first simplify equation (1).

$$\begin{aligned}2x^2-11x+5&=Ax^2+2Ax-5A+Bx^2-3Bx+Cx-3C\\&=(A+B)x^2+(2A-3B+C)x+(-5A-3C)\end{aligned}$$

Equate the coefficients of $x^2$:

$2=A+B$

Substituting $-1$ for $A$, we get $B=3$.

Equate the constant terms:

$5=-5A-3C$

Substituting $-1$ for $A$, we get $C=0$.

The decomposition is as follows:

$$-\frac{1}{x-3}+\frac{3x}{x^2+2x-5}$$

**20.** $$\frac{3x^2-3x-8}{(x-5)(x^2+x-4)}$$
$$=\frac{A}{x-5}+\frac{Bx+C}{x^2+x-4}$$
$$=\frac{A(x^2+x-4)+(Bx+C)(x-5)}{(x-5)(x^2+x-4)}$$

$$3x^2-3x-8=A(x^2+x-4)+(Bx+C)(x-5)\qquad(1)$$

Let $x=5$: $3\cdot 5^2-3\cdot 5-8=A(5^2+5-4)+0$
$$\begin{aligned}52&=26A\\2&=A\end{aligned}$$

Simplify equation (1).

$$\begin{aligned}
&3x^2 - 3x - 8 \\
&= Ax^2 + Ax - 4A + Bx^2 - 5Bx + Cx - 5C \\
&= (A+B)x^2 + (A - 5B + C)x + (-4A - 5C)
\end{aligned}$$

$$\begin{aligned}
3 &= A + B \\
3 &= 2 + B \\
1 &= B
\end{aligned}$$

$$\begin{aligned}
-8 &= -4A - 5C \\
-8 &= -4 \cdot 2 - 5C \\
0 &= C
\end{aligned}$$

The decomposition is $\dfrac{2}{x-5} + \dfrac{x}{x^2+x-4}$.

**21.** $\dfrac{-4x^2 - 2x + 10}{(3x+5)(x+1)^2}$

The decomposition looks like

$\dfrac{A}{3x+5} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$.

Add and equate the numerators.

$$\begin{aligned}
&-4x^2 - 2x + 10 \\
&= A(x+1)^2 + B(3x+5)(x+1) + C(3x+5) \\
&= A(x^2 + 2x + 1) + B(3x^2 + 8x + 5) + C(3x+5)
\end{aligned}$$

or

$$\begin{aligned}
&-4x^2 - 2x + 10 \\
&= (A + 3B)x^2 + (2A + 8B + 3C)x + (A + 5B + 5C)
\end{aligned}$$

Then equate corresponding coefficients.

$-4 = A + 3B$ Coefficients of $x^2$-terms

$-2 = 2A + 8B + 3C$ Coefficients of $x$-terms

$10 = A + 5B + 5C$ Constant terms

We solve this system of three equations and find $A = 5$, $B = -3$, $C = 4$.

The decomposition is

$\dfrac{5}{3x+5} - \dfrac{3}{x+1} + \dfrac{4}{(x+1)^2}$.

**22.** $\dfrac{26x^2 - 36x + 22}{(x-4)(2x-1)^2} = \dfrac{A}{x-4} + \dfrac{B}{2x-1} + \dfrac{C}{(2x-1)^2}$

Add and equate numerators.

$$\begin{aligned}
&26x^2 - 36x + 22 \\
&= A(2x-1)^2 + B(x-4)(2x-1) + C(x-4) \\
&= A(4x^2 - 4x + 1) + B(2x^2 - 9x + 4) + C(x-4)
\end{aligned}$$

or

$$26x^2 - 36x + 22 = (4A + 2B)x^2 + (-4A - 9B + C)x + (A + 4B - 4C)$$

Solving the system of equations

$$\begin{aligned}
26 &= 4A + 2B, \\
-36 &= -4A - 9B + C, \\
22 &= A + 4B - 4C
\end{aligned}$$

we get $A = 6$, $B = 1$, and $C = -3$.

Then the decomposition is

$\dfrac{6}{x-4} + \dfrac{1}{2x-1} - \dfrac{3}{(2x-1)^2}$.

**23.** $\dfrac{36x + 1}{12x^2 - 7x - 10} = \dfrac{36x + 1}{(4x-5)(3x+2)}$

The decomposition looks like

$\dfrac{A}{4x-5} + \dfrac{B}{3x+2}$.

Add and equate the numerators.

$$36x + 1 = A(3x+2) + B(4x-5)$$

or $36x + 1 = (3A + 4B)x + (2A - 5B)$

Then equate corresponding coefficients.

$36 = 3A + 4B$ Coefficients of $x$-terms

$1 = 2A - 5B$ Constant terms

We solve this system of equations and find $A = 8$ and $B = 3$.

The decomposition is

$\dfrac{8}{4x-5} + \dfrac{3}{3x+2}$.

**24.** $\dfrac{-17x + 61}{6x^2 + 39x - 21} = \dfrac{A}{6x-3} + \dfrac{B}{x+7}$

$$\begin{aligned}
-17x + 61 &= (A + 6B)x + (7A - 3B) \\
-17 &= A + 6B, \\
61 &= 7A - 3B
\end{aligned}$$

Then $A = 7$ and $B = -4$.

The decomposition is

$\dfrac{7}{6x-3} - \dfrac{4}{x+7}$.

**25.** $\dfrac{-4x^2 - 9x + 8}{(3x^2+1)(x-2)}$

The decomposition looks like

$\dfrac{Ax + B}{3x^2+1} + \dfrac{C}{x-2}$.

Add and equate the numerators.

$$\begin{aligned}
&-4x^2 - 9x + 8 \\
&= (Ax + B)(x-2) + C(3x^2 + 1) \\
&= Ax^2 - 2Ax + Bx - 2B + 3Cx^2 + C
\end{aligned}$$

or

$$\begin{aligned}
&-4x^2 - 9x + 8 \\
&= (A + 3C)x^2 + (-2A + B)x + (-2B + C)
\end{aligned}$$

Then equate corresponding coefficients.

$-4 = A + 3C$ Coefficients of $x^2$-terms

$-9 = -2A + B$ Coefficients of $x$-terms

$8 = -2B + C$ Constant terms

We solve this system of equations and find $A = 2$, $B = -5$, $C = -2$.

The decomposition is

$\dfrac{2x-5}{3x^2+1} - \dfrac{2}{x-2}$.

**26.** $\dfrac{11x^2 - 39x + 16}{(x^2+4)(x-8)} = \dfrac{Ax + B}{x^2+4} + \dfrac{C}{x-8}$

$$11x^2 - 39x + 16 = (A + C)x^2 + (-8A + B)x + (-8B + 4C)$$

$$\begin{aligned} 11 &= A + C, \\ -39 &= -8A + B, \\ 16 &= -8B + 4C \end{aligned}$$

Then $A = 5$, $B = 1$, and $C = 6$.

The decomposition is

$$\frac{5x+1}{x^2+4} + \frac{6}{x-8}.$$

**27.**

$$\begin{aligned} x^3 + x^2 + 9x + 9 &= 0 \\ x^2(x+1) + 9(x+1) &= 0 \\ (x+1)(x^2+9) &= 0 \end{aligned}$$

$$\begin{aligned} x + 1 = 0 \quad &or \quad x^2 + 9 = 0 \\ x = -1 \quad &or \quad x^2 = -9 \\ x = -1 \quad &or \quad x = \pm 3i \end{aligned}$$

The solutions are $-1$, $3i$, and $-3i$.

**28.**

$$\begin{aligned} x^3 - 3x^2 + x - 3 &= 0 \\ x^2(x-3) + (x-3) &= 0 \\ (x-3)(x^2+1) &= 0 \end{aligned}$$

$$\begin{aligned} x - 3 = 0 \quad &or \quad x^2 + 1 = 0 \\ x = 3 \quad &or \quad x^2 = -1 \\ x = 3 \quad &or \quad x = \pm i \end{aligned}$$

The solutions are 3, $i$, and $-i$.

**29.** $f(x) = x^3 + x^2 - 3x - 2$

We use synthetic division to factor the polynomial. Using the possibilities found by the rational zeros theorem we find that $x + 2$ is a factor:

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -3 & -2 \\ & & -2 & 2 & 2 \\ \hline & 1 & -1 & -1 & 0 \end{array}$$

We have $x^3 + x^2 - 3x - 2 = (x+2)(x^2 - x - 1)$.

$$\begin{aligned} x^3 + x^2 - 3x - 2 &= 0 \\ (x+2)(x^2 - x - 1) &= 0 \end{aligned}$$

$$x + 2 = 0 \quad or \quad x^2 - x - 1 = 0$$

The solution of the first equation is $-2$. We use the quadratic formula to solve the second equation.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} \\ &= \frac{1 \pm \sqrt{5}}{2} \end{aligned}$$

The solutions are $-2$, $\dfrac{1+\sqrt{5}}{2}$ and $\dfrac{1-\sqrt{5}}{2}$.

**30.** $f(x) = x^4 - x^3 - 5x^2 - x - 6$

$$\begin{array}{r|rrrrr} -2 & 1 & -1 & -5 & -1 & -6 \\ & & -2 & 6 & -2 & 6 \\ \hline & 1 & -3 & 1 & -3 & 0 \end{array}$$

$$\begin{aligned} x^4 - x^3 - 5x^2 - x - 6 &= 0 \\ (x+2)(x^3 - 3x^2 + x - 3) &= 0 \\ (x+2)[x^2(x-3) + (x-3)] &= 0 \\ (x+2)(x-3)(x^2+1) &= 0 \end{aligned}$$

$$\begin{aligned} x + 2 = 0 \quad or \quad x - 3 = 0 \quad &or \quad x^2 + 1 = 0 \\ x = -2 \quad or \quad x = 3 \quad &or \quad x = \pm i \end{aligned}$$

The solutions are $-2$, 3, $i$, and $-i$.

**31.** $f(x) = x^3 + 5x^2 + 5x - 3$

$$\begin{array}{r|rrrr} -3 & 1 & 5 & 5 & -3 \\ & & -3 & -6 & 3 \\ \hline & 1 & 2 & -1 & 0 \end{array}$$

$$\begin{aligned} x^3 + 5x^2 + 5x - 3 &= 0 \\ (x+3)(x^2 + 2x - 1) &= 0 \end{aligned}$$

$$x + 3 = 0 \quad or \quad x^2 + 2x - 1 = 0$$

The solution of the first equation is $-3$. We use the quadratic formula to solve the second equation.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{8}}{2} \\ &= \frac{-2 \pm 2\sqrt{2}}{2} = \frac{2(-1 \pm \sqrt{2})}{2} \\ &= -1 \pm \sqrt{2} \end{aligned}$$

The solutions are $-3$, $-1 + \sqrt{2}$, and $-1 - \sqrt{2}$.

**32.** $\dfrac{9x^3 - 24x^2 + 48x}{(x-2)^4(x+1)} = \dfrac{A}{x+1} + \dfrac{P(x)}{(x-2)^4}$

Add and equate numerators.

$$9x^3 - 24x^2 + 48x = A(x-2)^4 + P(x)(x+1)$$

Let $x = -1$:

$$\begin{aligned} 9(-1)^3 - 24(-1)^2 + 48(-1) &= A(-1-2)^4 + 0 \\ -81 &= 81A \\ -1 &= A \end{aligned}$$

Then

$$\begin{aligned} 9x^3 - 24x^2 + 48x &= -(x-2)^4 + P(x)(x+1) \\ 9x^3 - 24x^2 + 48x &= -x^4 + 8x^3 - 24x^2 + 32x - \\ &\quad\ 16 + P(x)(x+1) \\ x^4 + x^3 + 16x + 16 &= P(x)(x+1) \\ x^3 + 16 &= P(x) \quad \text{Dividing by } x+1 \end{aligned}$$

Now decompose $\dfrac{x^3+16}{(x-2)^4}$.

$$\frac{x^3+16}{(x-2)^4} = \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3} + \frac{E}{(x-2)^4}$$

Add and equate numerators.

$$\begin{aligned} x^3 + 16 = B(x-2)^3 &+ C(x+2)^2 + \\ D(x-2) &+ E \qquad (1) \end{aligned}$$

Let $x = 2$: $2^3 + 16 = 0 + 0 + 0 + E$

$$24 = E$$

Simplify equation (1).

$$\begin{aligned} x^3 + 16 = Bx^3 &+ (-6B+C)x^2 + (12B - 4C + D)x + \\ &(-8B + 4C - 2D + E) \end{aligned}$$

$1 = B$

$0 = -6B + C$

$0 = -6 \cdot 1 + C$

$6 = C$

$0 = 12B - 4C + D$

$0 = 12 \cdot 1 - 4 \cdot 6 + D$

$12 = D$

The decomposition is

$-\dfrac{1}{x+1} + \dfrac{1}{x-2} + \dfrac{6}{(x-2)^2} + \dfrac{12}{(x-2)^3} + \dfrac{24}{(x-2)^4}$.

**33.** $\dfrac{x}{x^4 - a^4}$

$= \dfrac{x}{(x^2+a^2)(x+a)(x-a)}$ Factoring the denominator

$= \dfrac{Ax+B}{x^2+a^2} + \dfrac{C}{x+a} + \dfrac{D}{x-a}$

$= [(Ax+B)(x+a)(x-a) + C(x^2+a^2)(x-a) + D(x^2+a^2)(x+a)]/[(x^2+a^2)(x+a)(x-a)]$

Equate the numerators:

$x = (Ax+B)(x+a)(x-a) + C(x^2+a^2)(x-a) + D(x^2+a^2)(x+a)$

Let $x - a = 0$, or $x = a$. Then, we get

$a = 0 + 0 + D(a^2+a^2)(a+a)$

$a = D(2a^2)(2a)$

$a = 4a^3 D$

$\dfrac{1}{4a^2} = D$

Let $x + a = 0$, or $x = -a$. We get

$-a = 0 + C[(-a)^2 + a^2](-a-a) + 0$

$-a = C(2a^2)(-2a)$

$-a = -4a^3 C$

$\dfrac{1}{4a^2} = C$

Equate the coefficients of $x^3$:

$0 = A + C + D$

Substituting $\dfrac{1}{4a^2}$ for $C$ and for $D$, we get

$A = -\dfrac{1}{2a^2}$.

Equate the constant terms:

$0 = -Ba^2 - Ca^3 + Da^3$

Substitute $\dfrac{1}{4a^2}$ for $C$ and for $D$. Then solve for $B$.

$0 = -Ba^2 - \dfrac{1}{4a^2} \cdot a^3 + \dfrac{1}{4a^2} \cdot a^3$

$0 = -Ba^2$

$0 = B$

The decomposition is as follows:

$-\dfrac{\frac{1}{2a^2}x}{x^2+a^2} + \dfrac{\frac{1}{4a^2}}{x+a} + \dfrac{\frac{1}{4a^2}}{x-a}$

**34.** $\dfrac{1}{e^{-x} + 3 + 2e^x} = \dfrac{e^x}{1 + 3e^x + 2e^{2x}}$

Multiplying by $e^x/e^x$

Let $y = e^x$, decompose $\dfrac{y}{2y^2 + 3y + 1}$, and then substitute $e^x$ for $y$. The result is $\dfrac{1}{e^x+1} - \dfrac{1}{2e^x+1}$.

**35.** $\dfrac{1 + \ln x^2}{(\ln x + 2)(\ln x - 3)^2} = \dfrac{1 + 2\ln x}{(\ln x + 2)(\ln x - 3)^2}$

Let $u = \ln x$. Then we have:

$\dfrac{1+2u}{(u+2)(u-3)^2}$

$= \dfrac{A}{u+2} + \dfrac{B}{u-3} + \dfrac{C}{(u-3)^2}$

$= \dfrac{A(u-3)^2 + B(u+2)(u-3) + C(u+2)}{(u+2)(u-3)^2}$

Equate the numerators:

$1 + 2u = A(u-3)^2 + B(u+2)(u-3) + C(u+2)$

Let $u - 3 = 0$, or $u = 3$.

$1 + 2 \cdot 3 = 0 + 0 + C(5)$

$7 = 5C$

$\dfrac{7}{5} = C$

Let $u + 2 = 0$, or $u = -2$.

$1 + 2(-2) = A(-2-3)^2 + 0 + 0$

$-3 = 25A$

$-\dfrac{3}{25} = A$

To find $B$, we equate the coefficients of $u^2$:

$0 = A + B$

Substituting $-\dfrac{3}{25}$ for $A$ and solving for $B$, we get $B = \dfrac{3}{25}$.

The decomposition of $\dfrac{1+2u}{(u+2)(u-3)^2}$ is as follows:

$-\dfrac{3}{25(u+2)} + \dfrac{3}{25(u-3)} + \dfrac{7}{5(u-3)^2}$

Substituting $\ln x$ for $u$ we get

$-\dfrac{3}{25(\ln x + 2)} + \dfrac{3}{25(\ln x - 3)} + \dfrac{7}{5(\ln x - 3)^2}$.

## Chapter 9 Review Exercises

**1.** The statement is true. See page 739 in the text.

**2.** The statement is false. See page 739 in the text.

**3.** The statement is true. See page 775 in the text.

**4.** The statement is false. See page 777 in the text.

**5.** (a)

**6.** (e)

**7.** (h)

**8.** (d)

**9.** (b)

**10.** (g)

**11.** (c)

**12.** (f)

**13.** $5x - 3y = -4, \quad (1)$
$3x - y = -4 \quad (2)$

Multiply equation (2) by $-3$ and add.

$$\begin{aligned} 5x - 3y &= -4 \\ -9x + 3y &= 12 \\ \hline -4x \qquad &= 8 \\ x &= -2 \end{aligned}$$

Back-substitute to find $y$.

$$\begin{aligned} 3(-2) - y &= -4 \quad \text{Using equation (2)} \\ -6 - y &= -4 \\ -y &= 2 \\ y &= -2 \end{aligned}$$

The solution is $(-2, -2)$.

**14.** $2x + 3y = 2, \quad (1)$
$5x - y = -29 \quad (2)$

Multiply equation (2) by 3 and add.

$$\begin{aligned} 2x + 3y &= 2 \\ 15x - 3y &= -87 \\ \hline 17x \qquad &= -85 \\ x &= -5 \end{aligned}$$

Back-substitute to find $y$.

$$\begin{aligned} 5(-5) - y &= -29 \quad \text{Using equation (2)} \\ -25 - y &= -29 \\ -y &= -4 \\ y &= 4 \end{aligned}$$

The solution is $(-5, 4)$.

**15.** $x + 5y = 12, \quad (1)$
$5x + 25y = 12 \quad (2)$

Solve equation (1) for $x$.

$$x = -5y + 12$$

Substitute in equation (2) and solve for $y$.

$$\begin{aligned} 5(-5y + 12) + 25y &= 12 \\ -25y + 60 + 25y &= 12 \\ 60 &= 12 \end{aligned}$$

We get a false equation, so there is no solution.

**16.** $x + y = -2, \quad (1)$
$-3x - 3y = 6 \quad (2)$

Multiply equation (1) by 3 and add.

$$\begin{aligned} 3x + 3y &= -6 \\ -3x - 3y &= 6 \\ \hline 0 &= 0 \end{aligned}$$

The equation $0 = 0$ is true for all values of $x$ and $y$. Thus the system of equations has infinitely many solutions. Solving either equation for $y$, we get $y = -x - 2$, so the solutions are ordered pairs of the form $(x, -x - 2)$. Equivalently, if we solve either equation for $x$ we get $x = -y - 2$ so the solutions can also be expressed as $(-y - 2, y)$.

**17.** $x + 5y - 3z = 4, \quad (1)$
$3x - 2y + 4z = 3, \quad (2)$
$2x + 3y - z = 5 \quad (3)$

Multiply equation (1) by $-3$ and add it to equation (2).

Multiply equation (1) by $-2$ and add it to equation (3).

$$\begin{aligned} x + 5y + 3z &= 4 \quad (1) \\ -17y + 13z &= -9 \quad (4) \\ -7y + 5z &= -3 \quad (5) \end{aligned}$$

Multiply equation (5) by 17.

$$\begin{aligned} x + 5y + 3z &= 4 \quad (1) \\ -17y + 13z &= -9 \quad (4) \\ -119y + 85z &= -51 \quad (6) \end{aligned}$$

Multiply equation (4) by $-7$ and add it to equation (6).

$$\begin{aligned} x + 5y + 3z &= 4 \quad (1) \\ -17y + 13z &= -9 \quad (4) \\ -6z &= 12 \quad (7) \end{aligned}$$

Now we solve equation (7) for $z$.

$$\begin{aligned} -6z &= 12 \\ z &= -2 \end{aligned}$$

Back-substitute $-2$ for $z$ in equation (4) and solve for $y$.

$$\begin{aligned} -17y + 13(-2) &= -9 \\ -17y - 26 &= -9 \\ -17y &= 17 \\ y &= -1 \end{aligned}$$

Finally, we back-substitute $-1$ for $y$ and $-2$ for $z$ in equation (1) and solve for $x$.

$$\begin{aligned} x + 5(-1) - 3(-2) &= 4 \\ x - 5 + 6 &= 4 \\ x + 1 &= 4 \\ x &= 3 \end{aligned}$$

The solution is $(3, -1, -2)$.

**18.** $2x - 4y + 3z = -3, \quad (1)$
$-5x + 2y - z = 7, \quad (2)$
$3x + 2y - 2z = 4 \quad (3)$

Multiply equations (2) and (3) by 2.

$$\begin{aligned} 2x - 4y + 3z &= -3 \quad (1) \\ -10x + 4y - 2z &= 14 \quad (4) \\ 6x + 4y - 4z &= 8 \quad (5) \end{aligned}$$

Multiply equation (1) by 5 and add it to equation (4).

Multiply equation (1) by $-3$ and add it to equation (5).

$$\begin{aligned} 2x - 4y + 3z &= -3 \quad (1)\\ -16y + 13z &= -1 \quad (6)\\ 16y - 13z &= 17 \quad (7) \end{aligned}$$

Add equation (6) to equation (7).

$$\begin{aligned} 2x - 4y + 3z &= -3 \quad (1)\\ -16y + 13z &= -1 \quad (6)\\ 0 &= 16 \quad (8) \end{aligned}$$

Equation (8) is false, so the system of equations has no solution.

**19.** 
$$\begin{aligned} x - y &= 5, \quad (1)\\ y - z &= 6, \quad (2)\\ z - w &= 7, \quad (3)\\ x + w &= 8 \quad (4) \end{aligned}$$

Multiply equation (1) by $-1$ and add it to equation (4).

$$\begin{aligned} x - y &= 5 \quad (1)\\ y - z &= 6 \quad (2)\\ z - w &= 7 \quad (3)\\ y + w &= 3 \quad (5) \end{aligned}$$

Multiply equation (2) by $-1$ and add it to equation (5).

$$\begin{aligned} x - y &= 5 \quad (1)\\ y - z &= 6 \quad (2)\\ z - w &= 7 \quad (3)\\ z + w &= -3 \quad (6) \end{aligned}$$

Multiply equation (3) by $-1$ and add it to equation (6).

$$\begin{aligned} x - y &= 5 \quad (1)\\ y - z &= 6 \quad (2)\\ z - w &= 7 \quad (3)\\ 2w &= -10 \quad (7) \end{aligned}$$

Solve equation (7) for $w$.

$$\begin{aligned} 2w &= -10\\ w &= -5 \end{aligned}$$

Back-substitute $-5$ for $w$ in equation (3) and solve for $z$.

$$\begin{aligned} z - w &= 7\\ z - (-5) &= 7\\ z + 5 &= 7\\ z &= 2 \end{aligned}$$

Back-substitute 2 for $z$ in equation (2) and solve for $y$.

$$\begin{aligned} y - z &= 6\\ y - 2 &= 6\\ y &= 8 \end{aligned}$$

Back-substitute 8 for $y$ in equation (1) and solve for $x$.

$$\begin{aligned} x - y &= 5\\ x - 8 &= 5\\ x &= 13 \end{aligned}$$

Writing the solution as $(w, x, y, z)$, we have $(-5, 13, 8, 2)$.

**20.** Systems 13, 14, 16, 17, and 19 each have at least one solution, so they are consistent. Systems 15 and 17 have no solution, so they are inconsistent.

**21.** Systems 13, 14, 15, 17, 18, and 19 each have either no solution or exactly one solution, so the equations in those systems are independent. System 16 has infinitely many solutions, so the equations in that system are dependent.

**22.** 
$$\begin{aligned} x + 2y &= 5,\\ 2x - 5y &= -8 \end{aligned}$$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{cc|c} 1 & 2 & 5\\ 2 & -5 & -8 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

$$\left[\begin{array}{cc|c} 1 & 2 & 5\\ 0 & -9 & -18 \end{array}\right]$$

Multiply row 2 by $-\dfrac{1}{9}$.

$$\left[\begin{array}{cc|c} 1 & 2 & 5\\ 0 & 1 & 2 \end{array}\right]$$

We have:

$$\begin{aligned} x + 2y &= 5 \quad (1)\\ y &= 2 \quad (2) \end{aligned}$$

Back-substitute in equation (1) and solve for $x$.

$$\begin{aligned} x + 2(2) &= 5\\ x &= 1 \end{aligned}$$

The solution is $(1, 2)$.

**23.** 
$$\begin{aligned} 3x + 4y + 2z &= 3\\ 5x - 2y - 13z &= 3\\ 4x + 3y - 3z &= 6 \end{aligned}$$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{ccc|c} 3 & 4 & 2 & 3\\ 5 & -2 & -13 & 3\\ 4 & 3 & -3 & 6 \end{array}\right]$$

Multiply row 2 and row 3 by 3.

$$\left[\begin{array}{ccc|c} 3 & 4 & 2 & 3\\ 15 & -6 & -39 & 9\\ 12 & 9 & -9 & 18 \end{array}\right]$$

Multiply row 1 by $-5$ and add it to row 2.

Multiply row 1 by $-4$ and add it to row 3.

$$\left[\begin{array}{ccc|c} 3 & 4 & 2 & 3\\ 0 & -26 & -49 & -6\\ 0 & -7 & -17 & 6 \end{array}\right]$$

Multiply row 3 by 26.

$$\left[\begin{array}{rrr|r} 3 & 4 & 2 & 3 \\ 0 & -26 & -49 & -6 \\ 0 & -182 & -442 & 156 \end{array}\right]$$

Multiply row 2 by $-7$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 3 & 4 & 2 & 3 \\ 0 & -26 & -49 & -6 \\ 0 & 0 & -99 & 198 \end{array}\right]$$

Multiply row 1 by $\frac{1}{3}$, row 2 by $-\frac{1}{26}$, and row 3 by $-\frac{1}{99}$.

$$\left[\begin{array}{rrr|r} 1 & \frac{4}{3} & \frac{2}{3} & 1 \\ 0 & 1 & \frac{49}{26} & \frac{3}{13} \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$x + \frac{4}{3}y + \frac{2}{3}z = 1 \quad (1)$$
$$y + \frac{49}{26}z = \frac{3}{13} \quad (2)$$
$$z = -2$$

Back-substitute in equation (2) and solve for $y$.

$$y + \frac{49}{26}(-2) = \frac{3}{13}$$
$$y - \frac{49}{13} = \frac{3}{13}$$
$$y = \frac{52}{13} = 4$$

Back-substitute in equation (1) and solve for $x$.

$$x + \frac{4}{3}(4) + \frac{2}{3}(-2) = 1$$
$$x + \frac{16}{3} - \frac{4}{3} = 1$$
$$x = -3$$

The solution is $(-3, 4, -2)$.

**24.** $3x + 5y + z = 0,$
$2x - 4y - 3z = 0,$
$x + 3y + z = 0$

Write the augmented matrix. We will use Gaussian elimination.

$$\left[\begin{array}{rrr|r} 3 & 5 & 1 & 0 \\ 2 & -4 & -3 & 0 \\ 1 & 3 & 1 & 0 \end{array}\right]$$

Interchange rows 1 and 3.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & 0 \\ 2 & -4 & -3 & 0 \\ 3 & 5 & 1 & 0 \end{array}\right]$$

Multiply row 1 by $-2$ and add it to row 2.

Multiply row 1 by $-3$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & -4 & -2 & 0 \end{array}\right]$$

Multiply row 3 by 5.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & -20 & -10 & 0 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 3.

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & 0 \\ 0 & -10 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The last row corresponds to the equation $0 = 0$. This indicates that the equations are dependent.

The system of equations is equivalent to

$$x + 3y + z = 0 \quad (1)$$
$$-10y - 5z = 0. \quad (2)$$

Solve equation (2) for $y$.

$$-10y - 5z = 0$$
$$y = -\frac{z}{2}$$

Substitute $-\frac{z}{2}$ for $y$ in equation (1) and solve for $x$.

$$x + 3\left(-\frac{z}{2}\right) + z = 0$$
$$x = \frac{z}{2}$$

The solution is $\left(\frac{z}{2}, -\frac{z}{2}, z\right)$, where $z$ is any real number.

**25.** $w + x + y + z = -2,$
$-3w - 2x + 3y + 2z = 10,$
$2w + 3x + 2y - z = -12,$
$2w + 4x - y + z = 1$

Write the augmented matrix. We will use Gauss-Jordan elimination.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ -3 & -2 & 3 & 2 & 10 \\ 2 & 3 & 2 & -1 & -12 \\ 2 & 4 & -1 & 1 & 1 \end{array}\right]$$

Multiply row 1 by 3 and add it to row 2.

Multiply row 1 by $-2$ and add it to row 3.

Multiply row 1 by $-2$ and add it to row 4.

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 6 & 5 & 4 \\ 0 & 1 & 0 & -3 & -8 \\ 0 & 2 & -3 & -1 & 5 \end{array}\right]$$

Multiply row 2 by $-1$ and add it to row 1.

Multiply row 2 by $-1$ and add it to row 3.

Multiply row 2 by $-2$ and add it to row 4.

$$\left[\begin{array}{cccc|c} 1 & 0 & -5 & -4 & -6 \\ 0 & 1 & 6 & 5 & 4 \\ 0 & 0 & -6 & -8 & -12 \\ 0 & 0 & -15 & -11 & -3 \end{array}\right]$$

Multiply row 1 by 3.

Multiply row 3 by $-\dfrac{1}{2}$.

$$\left[\begin{array}{cccc|c} 3 & 0 & -15 & -12 & -18 \\ 0 & 1 & 6 & 5 & 4 \\ 0 & 0 & 3 & 4 & 6 \\ 0 & 0 & -15 & -11 & -3 \end{array}\right]$$

Multiply row 3 by 5 and add it to row 1.

Multiply row 3 by $-2$ and add it to row 2.

Multiply row 3 by 5 and add it to row 4.

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 8 & 12 \\ 0 & 1 & 0 & -3 & -8 \\ 0 & 0 & 3 & 4 & 6 \\ 0 & 0 & 0 & 9 & 27 \end{array}\right]$$

Multiply row 4 by $\dfrac{1}{9}$.

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 8 & 12 \\ 0 & 1 & 0 & -3 & -8 \\ 0 & 0 & 3 & 4 & 6 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

Multiply row 4 by $-8$ and add it to row 1.

Multiply row 4 by 3 and add it to row 2.

Multiply row 4 by $-4$ and add it to row 3.

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & -12 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

Multiply rows 1 and 3 by $\dfrac{1}{3}$.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

The solution is $(-4, 1, -2, 3)$.

**26.** Let $x$ = the number of nickels and $y$ = the number of dimes.

Solve: $$\begin{aligned} x + \quad y &= 75, \\ 0.05x + 0.10y &= 5.95 \end{aligned}$$

$x = 31$, $y = 44$

**27.** ***Familiarize***. Let $x$ = the amount invested at 3% and $y$ = the amount invested at 3.5%. Then the interest from the investments is $3\%x$ and $3.5\%y$, or $0.03x$ and $0.035y$.

***Translate***.

The total investment is \$5000.

$$x + y = 5000$$

The total interest is \$167.

$$0.03x + 0.035y = 167$$

We have a system of equations.

$$\begin{aligned} x + \quad y &= 5000, \\ 0.03x + 0.035y &= 167 \end{aligned}$$

Multiplying the second equation by 1000 to clear the decimals, we have:

$$\begin{aligned} x + \quad y &= 5000, & (1) \\ 30x + 35y &= 167,000. & (2) \end{aligned}$$

***Carry out***. We begin by multiplying equation (1) by $-30$ and adding.

$$\begin{aligned} -30x - 30y &= -150,000 \\ 30x + 35y &= 167,000 \\ \hline 5y &= 17,000 \\ y &= 3400 \end{aligned}$$

Back-substitute to find $x$.

$$\begin{aligned} x + 3400 &= 5000 \quad \text{Using equation (1)} \\ x &= 1600 \end{aligned}$$

***Check***. The total investment is \$1600 + \$3400, or \$5000. The total interest is 0.03(\$1600) + 0.035(\$3400), or \$48 + \$119, or \$167. The solution checks.

***State***. \$1600 was invested at 3% and \$3400 was invested at 3.5%.

**28.** Let $x$, $y$, and $z$ represent the number of servings of bagels, cream cheese, and bananas, respectively.

Solve: $$\begin{aligned} 200x + 100y + 105z &= 460, \\ 2x + \ 10y + \quad z &= 9, \\ 29x + \ 24y + \ 7z &= 55 \end{aligned}$$

$x = 1$, $y = \dfrac{1}{2}$, $z = 2$

**29.** ***Familiarize***. Let $x$, $y$, and $z$ represent the scores on the first, second, and third tests, respectively.

***Translate***.

The total score on the three tests is 226.

$$x + y + z = 226$$

The sum of the scores on the first and second tests exceeds the score on the third test by 62.

$$x + y = z + 62$$

The first score exceeds the second by 6.

$x = y + 6$

We have a system of equations.

$$x + y + z = 226,$$
$$x + y = z + 62,$$
$$x = y + 6$$

or

$$\begin{aligned} x + y + z &= 226,\\ x + y - z &= 62,\\ x - y \phantom{{}+z} &= 6 \end{aligned}$$

***Carry out***. Solving the system of equations, we get $(75, 69, 82)$.

***Check***. The sum of the scores is $75+69+82$, or 226. The sum of the scores on the first two tests is $75 + 69$, or 144. This exceeds the score on the third test, 82, by 62. The score on the first test, 75, exceeds the score on the second test, 69, by 6. The solution checks.

***State***. The scores on the first, second, and third tests were 75, 69, and 82, respectively.

**30.** a) Solve:
$$\begin{aligned} 40 &= a \cdot 0^2 + b \cdot 0 + c,\\ 48 &= a \cdot 1^2 + b \cdot 1 + c,\\ 40 &= a \cdot 2^2 + b \cdot 2 + c \end{aligned}$$
or
$$\begin{aligned} c &= 40,\\ a + b + c &= 48,\\ 4a + 2b + c &= 40. \end{aligned}$$

$a = -8$, $b = 16$, $c = 40$, so $f(x) = -8x^2 + 16x + 40$, where $x$ is the number of years after 2006 and $f(x)$ is in thousands.

b) In 2009, $x = 2009 - 2006 = 3$.

$f(3) = -8\cdot 3^2+16\cdot 3+40 = 16$ thousand trademarks

**31.** $$\mathbf{A}+\mathbf{B} = \begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 6\\ 1 & -2 & 0\\ 0 & 1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+(-1) & -1+0 & 0+6\\ 2+1 & 3+(-2) & -2+0\\ -2+0 & 0+1 & 1+(-3) \end{bmatrix}$$
$$= \begin{bmatrix} 0 & -1 & 6\\ 3 & 1 & -2\\ -2 & 1 & -2 \end{bmatrix}$$

**32.** $$-3\mathbf{A} = -3\begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 3 & 0\\ -6 & -9 & 6\\ 6 & 0 & -3 \end{bmatrix}$$

**33.** $$-\mathbf{A} = -1\begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0\\ -2 & -3 & 2\\ 2 & 0 & -1 \end{bmatrix}$$

**34.** $$\mathbf{AB} = \begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 6\\ 1 & -2 & 0\\ 0 & 1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} -1-1+0 & 0+2+0 & 6+0+0\\ -2+3+0 & 0-6-2 & 12+0+6\\ 2+0+0 & 0+0+1 & -12+0-3 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 2 & 6\\ 1 & -8 & 18\\ 2 & 1 & -15 \end{bmatrix}$$

**35.** $\mathbf{A}$ and $\mathbf{B}$ do not have the same order, so it is not possible to find $\mathbf{A}+\mathbf{B}$.

**36.** $$\mathbf{A}-\mathbf{B} = \begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 6\\ 1 & -2 & 0\\ 0 & 1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -1 & -6\\ 1 & 5 & -2\\ -2 & -1 & 4 \end{bmatrix}$$

**37.** $$\mathbf{BA} = \begin{bmatrix} -1 & 0 & 6\\ 1 & -2 & 0\\ 0 & 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -1+0-12 & 1+0+0 & 0+0+6\\ 1-4+0 & -1-6+0 & 0+4+0\\ 0+2+6 & 0+3+0 & 0-2-3 \end{bmatrix}$$
$$= \begin{bmatrix} -13 & 1 & 6\\ -3 & -7 & 4\\ 8 & 3 & -5 \end{bmatrix}$$

**38.** $$\mathbf{A}+3\mathbf{B} = \begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 & 6\\ 1 & -2 & 0\\ 0 & 1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -1 & 0\\ 2 & 3 & -2\\ -2 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 18\\ 3 & -6 & 0\\ 0 & 3 & -9 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & -1 & 18\\ 5 & -3 & -2\\ -2 & 3 & -8 \end{bmatrix}$$

**39.** a) $$\mathbf{M} = \begin{bmatrix} 0.98 & 0.23 & 0.30 & 0.28 & 0.45\\ 1.03 & 0.19 & 0.27 & 0.34 & 0.41\\ 1.01 & 0.21 & 0.35 & 0.31 & 0.39\\ 0.99 & 0.25 & 0.29 & 0.33 & 0.42 \end{bmatrix}$$

b) $\mathbf{N} = \begin{bmatrix} 32 & 19 & 43 & 38 \end{bmatrix}$

c) $\mathbf{NM} = \begin{bmatrix} 131.98 & 29.50 & 40.80 & 41.29 & 54.92 \end{bmatrix}$

d) The entries of $\mathbf{NM}$ represent the total cost, in dollars, for each item for the day's meal.

**40.** $\mathbf{A} = \begin{bmatrix} -2 & 0\\ 1 & 3 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{cc|cc} -2 & 0 & 1 & 0\\ 1 & 3 & 0 & 1 \end{array}\right]$$

Interchange rows.

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1\\ -2 & 0 & 1 & 0 \end{array}\right]$$

Multiply row 1 by 2 and add it to row 2.

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 6 & 1 & 2 \end{array}\right]$$

Multiply row 2 by $\frac{1}{6}$.

$$\left[\begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 1 & \frac{1}{6} & \frac{1}{3} \end{array}\right]$$

Multiply row 2 by $-3$ and add it to row 1.

$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{6} & \frac{1}{3} \end{array}\right]$$

$$\mathbf{A}^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

**41.** $\mathbf{A} = \begin{bmatrix} 0 & 0 & 3 \\ 0 & -2 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Interchange rows 1 and 3.

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 \end{array}\right]$$

Multiply row 1 by $\frac{1}{4}$, row 2 by $-\frac{1}{2}$, and row 3 by $\frac{1}{3}$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 & 0 \end{array}\right]$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & \frac{1}{4} \\ 0 & -\frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 \end{bmatrix}$$

**42.** $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -5 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & -5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Interchange rows 2 and 3.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 4 & -5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 2 by $-2$ and add it to row 3.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -9 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 2 by 9.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 18 & 18 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & -9 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 3 by 2 and add it to row 2.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 18 & 0 & 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & -9 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 2 by $\frac{1}{18}$ and row 3 by $-\frac{1}{9}$.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{9} & \frac{5}{18} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{9} & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & \frac{5}{18} & 0 \\ 0 & -\frac{1}{9} & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**43.** $3x - 2y + 4z = 13,$
$x + 5y - 3z = 7,$
$2x - 3y + 7z = -8$

Write the coefficients on the left in a matrix. Then write the product of that matrix and the column matrix containing the variables, and set the result equal to the column matrix containing the constants on the right.

$$\begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & -3 \\ 2 & -3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \\ -8 \end{bmatrix}$$

**44.** $2x + 3y = 5,$
$3x + 5y = 11$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

Then,

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 11 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \end{bmatrix}$$

The solution is $(-8, 7)$.

**45.** $5x - y + 2z = 17,$
$3x + 2y - 3z = -16,$
$4x - 3y - z = 5$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 5 & -1 & 2 \\ 3 & 2 & -3 \\ 4 & -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ -16 \\ 5 \end{bmatrix}$$

Then,

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} \frac{11}{80} & \frac{7}{80} & \frac{1}{80} \\ \frac{9}{80} & \frac{13}{80} & -\frac{21}{80} \\ \frac{17}{80} & -\frac{11}{80} & -\frac{13}{80} \end{bmatrix}\begin{bmatrix} 17 \\ -16 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

The solution is $(1, -2, 5)$.

**46.** $w - x - y + z = -1,$
$2w + 3x - 2y - z = 2,$
$-w + 5x + 4y - 2z = 3,$
$3w - 2x + 5y + 3z = 4$

Write an equivalent matrix equation, $\mathbf{AX} = \mathbf{B}$.

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 2 & 3 & -2 & -1 \\ -1 & 5 & 4 & -2 \\ 3 & -2 & 5 & 3 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Then,

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \frac{1}{47}\begin{bmatrix} -55 & 22 & -19 & 13 \\ 67 & -8 & 24 & -9 \\ -26 & 1 & -3 & 7 \\ 143 & -29 & 40 & -15 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ -3 \end{bmatrix}$$

The solution is $(2, -1, 1, -3)$.

**47.** $\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 3(-2) = 4 + 6 = 10$

**48.** $\begin{vmatrix} \sqrt{3} & -5 \\ -3 & -\sqrt{3} \end{vmatrix} = \sqrt{3}(-\sqrt{3}) - (-3)(-5) = -3 - 15$
$= -18$

**49.** We will expand across the first row.

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 4 & -3 \end{vmatrix}$$
$$= 2(-1)^{1+1}\begin{vmatrix} 2 & -1 \\ 4 & -3 \end{vmatrix} + (-1)(-1)^{1+2}\begin{vmatrix} 1 & -1 \\ 3 & -3 \end{vmatrix} +$$
$$1(-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$
$$= 2 \cdot 1[2(-3) - 4(-1)] + (-1)(-1)[1(-3) - 3(-1)]+$$
$$1 \cdot 1[1(4) - 3(2)]$$
$$= 2(-2) + 1(0) + 1(-2)$$
$$= -6$$

**50.** We will expand down the third column.

$$\begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 0 \\ -1 & 3 & 1 \end{vmatrix}$$
$$= 2(-1)^{1+3}\begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} + 0(-1)^{2+3}\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} +$$
$$1(-1)^{3+3}\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}$$
$$= 2 \cdot 1[-1(3) - (-1)(2)] + 0 + 1 \cdot 1[1(2) - (-1)(-1)]$$
$$= 2(-1) + 0 + 1(1)$$
$$= -1$$

**51.** $5x - 2y = 19,$
$7x + 3y = 15$

$$D = \begin{vmatrix} 5 & -2 \\ 7 & 3 \end{vmatrix} = 5(3) - 7(-2) = 29$$

$$D_x = \begin{vmatrix} 19 & -2 \\ 15 & 3 \end{vmatrix} = 19(3) - 15(-2) = 87$$

$$D_y = \begin{vmatrix} 5 & 19 \\ 7 & 15 \end{vmatrix} = 5(15) - 7(19) = -58$$

$$x = \frac{D_x}{D} = \frac{87}{29} = 3$$
$$y = \frac{D_y}{D} = \frac{-58}{29} = -2$$

The solution is $(3, -2)$.

**52.** $x + y = 4,$
$4x + 3y = 11$

$$D = \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 1(3) - 4(1) = -1$$

$$D_x = \begin{vmatrix} 4 & 1 \\ 11 & 3 \end{vmatrix} = 4(3) - 11(1) = 1$$

$$D_y = \begin{vmatrix} 1 & 4 \\ 4 & 11 \end{vmatrix} = 1(11) - 4(4) = -5$$

$$x = \frac{D_x}{D} = \frac{1}{-1} = -1$$
$$y = \frac{D_y}{D} = \frac{-5}{-1} = 5$$

The solution is $(-1, 5)$.

**53.** $3x - 2y + z = 5,$
$4x - 5y - z = -1,$
$3x + 2y - z = 4$

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 4 & -5 & -1 \\ 3 & 2 & -1 \end{vmatrix} = 42$$

$$D_x = \begin{vmatrix} 5 & -2 & 1 \\ -1 & -5 & -1 \\ 4 & 2 & -1 \end{vmatrix} = 63$$

$$D_y = \begin{vmatrix} 3 & 5 & 1 \\ 4 & -1 & -1 \\ 3 & 4 & -1 \end{vmatrix} = 39$$

$$D_z = \begin{vmatrix} 3 & -2 & 5 \\ 4 & -5 & -1 \\ 3 & 2 & 4 \end{vmatrix} = 99$$

$$x = \frac{D_x}{D} = \frac{63}{42} = \frac{3}{2}$$
$$y = \frac{D_y}{D} = \frac{39}{42} = \frac{13}{14}$$
$$z = \frac{D_z}{D} = \frac{99}{42} = \frac{33}{14}$$

The solution is $\left(\frac{3}{2}, \frac{13}{14}, \frac{33}{14}\right)$.

**54.** $2x - y - z = 2,$
$3x + 2y + 2z = 10,$
$x - 5y - 3z = -2$

$$D = \begin{vmatrix} 2 & -1 & -1 \\ 3 & 2 & 2 \\ 1 & -5 & -3 \end{vmatrix} = 14$$

$$D_x = \begin{vmatrix} 2 & -1 & -1 \\ 10 & 2 & 2 \\ -2 & -5 & -3 \end{vmatrix} = 28$$

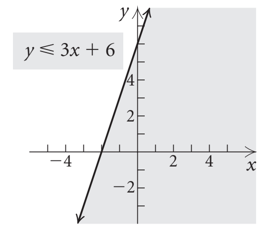
$$D_y = \begin{vmatrix} 2 & 2 & -1 \\ 3 & 10 & 2 \\ 1 & -2 & -3 \end{vmatrix} = -14$$

$$D_z = \begin{vmatrix} 2 & -1 & 2 \\ 3 & 2 & 10 \\ 1 & -5 & -2 \end{vmatrix} = 42$$

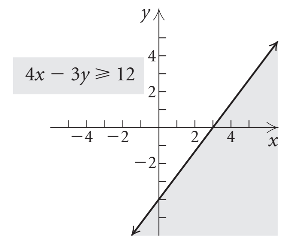
$$x = \frac{D_x}{D} = \frac{28}{14} = 2$$
$$y = \frac{D_y}{D} = \frac{-14}{14} = -1$$
$$z = \frac{D_z}{D} = \frac{42}{14} = 3$$

The solution is $(2, -1, 3)$.

**55.**

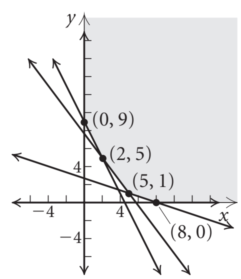



**56.**




**57.** Graph: $2x + y \geq 9,$
$4x + 3y \geq 23,$
$x + 3y \geq 8,$
$x \geq 0,$
$y \geq 0$

Shade the intersection of the graphs of the given inequalities.




We find the vertex $(0, 9)$ by solving the system
$2x + y = 9,$
$x = 0.$

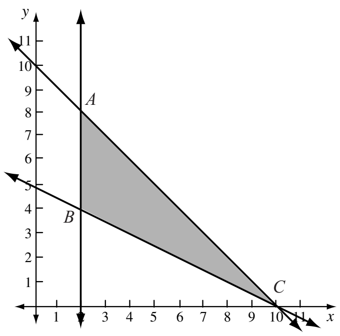
We find the vertex $(2, 5)$ by solving the system
$2x + y = 9,$
$4x + 3y = 23.$

We find the vertex $(5, 1)$ by solving the system
$4x + 3y = 23,$
$x + 3y = 8.$

We find the vertex $(8, 0)$ by solving the system
$x + 3y = 8,$
$y = 0.$

**58.** Find the maximum value and the minimum value of $T = 6x + 10y$ subject to
$x + y \leq 10,$
$5x + 10y \geq 50,$
$x \geq 2,$
$y \geq 0.$

Graph the system of inequalities and determine the vertices.




Vertex $A$:
We solve the system $x+y=10$ and $x=2$. The coordinates of point A are $(2,8)$.

Vertex $B$:
We solve the system $5x+10y=50$ and $x=2$. The coordinates of point B are $(2,4)$.

Vertex $C$:
We solve the system $5x+10y=50$ and $y=0$. The coordinates of point $C$ are $(10,0)$.

Evaluate the objective function $T$ at each vertex.

| Vertex | $T=6x+10y$ |
|---|---|
| $(2,8)$ | $6\cdot 2+10\cdot 8=92$ |
| $(2,4)$ | $6\cdot 2+10\cdot 4=52$ |
| $(10,0)$ | $6\cdot 10+10\cdot 0=60$ |

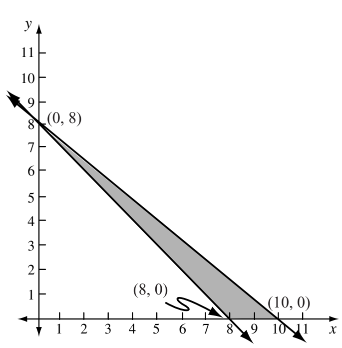
The maximum value of $T$ is 92 when $x=2$ and $y=8$.

The minimum value of $T$ is 52 when $x=2$ and $y=4$.

**59.** Let $x=$ the number of questions answered from group A and $y=$ the number of questions answered from group B. Find the maximum value of $S=7x+12y$ subject to

$$\begin{aligned} x+y&\ge 8,\\ 8x+10y&\le 80,\\ x&\ge 0,\\ y&\ge 0 \end{aligned}$$

Graph the system of inequalities, determine the vertices, and find the value of $T$ at each vertex.




| Vertex | $S=7x+12y$ |
|---|---|
| $(0,8)$ | $7\cdot 0+12\cdot 8=96$ |
| $(8,0)$ | $7\cdot 8+12\cdot 0=56$ |
| $(10,0)$ | $7\cdot 10+12\cdot 0=70$ |

The maximum score of 96 occurs when 0 questions from group A and 8 questions from group B are answered correctly.

**60.** $$\begin{aligned}\frac{5}{(x+2)^2(x+1)} &= \frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+1}\\ &= \frac{A(x+2)(x+1)+B(x+1)+C(x+2)^2}{(x+2)^2(x+1)}\end{aligned}$$

Equate the numerators.

$5=A(x+2)(x+1)+B(x+1)+C(x+2)^2 \quad (1)$

Let $x=-1: 5=0+0+C(-1+2)^2$

$5=C$

Let $x=-2:\quad 5=0+B(-2+1)+0$

$5=-B$

$-5=B$

To find $A$, choose any value for $x$ except $-1$ and $-2$ and replace $B$ with $-5$ and $C$ with 5. We let $x=0$.

$$\begin{aligned} 5&=A(0+2)(0+1)-5(0+1)+5(0+2)^2\\ 5&=2A-5+20\\ -10&=2A\\ -5&=A\end{aligned}$$

The decomposition is $-\dfrac{5}{x+2}-\dfrac{5}{(x+2)^2}+\dfrac{5}{x+1}$.

**61.** $$\begin{aligned}\frac{-8x+23}{2x^2+5x-12} &= \frac{-8x+23}{(2x-3)(x+4)}\\ &= \frac{A}{2x-3}+\frac{B}{x+4}\\ &= \frac{A(x+4)+B(2x-3)}{(2x-3)(x+4)}\end{aligned}$$

Equate the numerators.

$-8x+23=A(x+4)+B(2x-3)$

Let $x=\dfrac{3}{2}: -8\left(\dfrac{3}{2}\right)+23=A\left(\dfrac{3}{2}+4\right)+0$

$$\begin{aligned} -12+23&=\frac{11}{2}A\\ 11&=\frac{11}{2}A\\ 2&=A\end{aligned}$$

Let $x=-4: -8(-4)+23=0+B[2(-4)-3]$

$$\begin{aligned} 32+23&=-11B\\ 55&=-11B\\ -5&=B\end{aligned}$$

The decomposition is $\dfrac{2}{2x-3}-\dfrac{5}{x+4}$.

**62.** $2x + y = 7, \quad (1)$

$x - 2y = 6 \quad (2)$

Multiply equation (1) by 2 and add.

$$\begin{array}{rl} 4x + 2y &= 14 \\ x - 2y &= 6 \\ \hline 5x \quad &= 20 \\ x &= 4 \end{array}$$

Back-substitute to find $y$.

$$\begin{aligned} 2 \cdot 4 + y &= 7 \quad \text{Using equation (1)} \\ 8 + y &= 7 \\ y &= -1 \end{aligned}$$

The solution is $(4, -1)$, so answer C is correct.

**63.** Interchanging columns of a matrix is not a row-equivalent operation, so answer A is correct. (See page 765 in the text.)

**64.** Graph the system of inequalities. We see that B is the correct graph.

**65.** Let $x$, $y$, and $z$ represent the amounts invested at 4%, 5%, and $5\frac{1}{2}$%, respectively.

Solve: $x + y + z = 40,000,$

$0.04x + 0.05y + 0.055z = 1990,$

$0.055z = 0.04x + 590$

$x = \$10,000$, $y = \$12,000$, $z = \$18,000$

**66.** $\dfrac{2}{3x} + \dfrac{4}{5y} = 8,$

$\dfrac{5}{4x} - \dfrac{3}{2y} = -6$

Let $a = \dfrac{1}{x}$ and $b = \dfrac{1}{y}$. Then we have:

$$\frac{2}{3}a + \frac{4}{5}b = 8,$$
$$\frac{5}{4}a - \frac{3}{2}b = -6$$

Multiply the first equation by 15 and the second by 4 to clear the fractions.

$$\begin{aligned} 10a + 12b &= 120, \quad (1) \\ 5a - 6b &= -24 \quad (2) \end{aligned}$$

Multiply equation (2) by 2 and add.

$$\begin{array}{rl} 10a + 12b &= 120 \\ 10a - 12b &= -48 \\ \hline 20a \quad &= 72 \\ a &= \dfrac{18}{5} \end{array}$$

Back-substitute to find $b$.

$$\begin{aligned} 10 \cdot \frac{18}{5} + 12b &= 120 \quad \text{Using equation (1)} \\ 36 + 12b &= 120 \\ 12b &= 84 \\ b &= 7 \end{aligned}$$

Now find $x$ and $y$.

$$\begin{array}{ll} a = \dfrac{1}{x} & b = \dfrac{1}{y} \\ \dfrac{18}{5} = \dfrac{1}{x} & 7 = \dfrac{1}{y} \\ x = \dfrac{5}{18} & y = \dfrac{1}{7} \end{array}$$

The solution is $\left(\dfrac{5}{18}, \dfrac{1}{7}\right)$.

**67.** $\dfrac{3}{x} - \dfrac{4}{y} + \dfrac{1}{z} = -2, \quad (1)$

$\dfrac{5}{x} + \dfrac{1}{y} - \dfrac{2}{z} = 1, \quad (2)$

$\dfrac{7}{x} + \dfrac{3}{y} + \dfrac{2}{z} = 19 \quad (3)$

Multiply equations (2) and (3) by 3.

$$\begin{aligned} \frac{3}{x} - \frac{4}{y} + \frac{1}{z} &= -2 \quad (1) \\ \frac{15}{x} + \frac{3}{y} - \frac{6}{z} &= 3 \quad (4) \\ \frac{21}{x} + \frac{9}{y} + \frac{6}{z} &= 57 \quad (5) \end{aligned}$$

Multiply equation (1) by $-5$ and add it to equation (2).

Multiply equation (1) by $-7$ and add it to equation (3).

$$\begin{aligned} \frac{3}{x} - \frac{4}{y} + \frac{1}{z} &= -2 \quad (1) \\ \frac{23}{y} - \frac{11}{z} &= 13 \quad (6) \\ \frac{37}{y} - \frac{1}{z} &= 71 \quad (7) \end{aligned}$$

Multiply equation (7) by 23.

$$\begin{aligned} \frac{3}{x} - \frac{4}{y} + \frac{1}{z} &= -2 \quad (1) \\ \frac{23}{y} - \frac{11}{z} &= 13 \quad (6) \\ \frac{851}{y} - \frac{23}{z} &= 1633 \quad (8) \end{aligned}$$

Multiply equation (6) by $-37$ and add it to equation (8).

$$\begin{aligned} \frac{3}{x} - \frac{4}{y} + \frac{1}{z} &= -2 \quad (1) \\ \frac{23}{y} - \frac{11}{z} &= 13 \quad (6) \\ \frac{384}{z} &= 1152 \quad (9) \end{aligned}$$

Complete the solution.

$$\begin{aligned} \frac{384}{z} &= 1152 \\ \frac{1}{3} &= z \end{aligned}$$

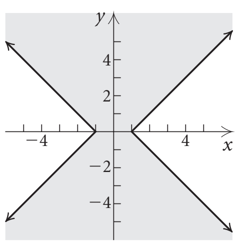
$$\begin{aligned}
\frac{23}{y} - \frac{11}{1/3} &= 13\\
\frac{23}{y} - 33 &= 13\\
\frac{23}{y} &= 46\\
\frac{1}{2} &= y
\end{aligned}$$

$$\begin{aligned}
\frac{3}{x} - \frac{4}{1/2} + \frac{1}{1/3} &= -2\\
\frac{3}{x} - 8 + 3 &= -2\\
\frac{3}{x} - 5 &= -2\\
\frac{3}{x} &= 3\\
1 &= x
\end{aligned}$$

The solution is $\left(1, \frac{1}{2}, \frac{1}{3}\right)$.

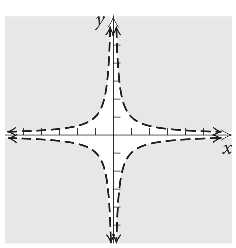
(We could also have solved this system of equations by first substituting $a$ for $\frac{1}{x}$, $b$ for $\frac{1}{y}$, and $c$ for $\frac{1}{z}$ and proceeding as we did in Exercise 66 above.)

**68.**




$|x| - |y| \leq 1$

**69.**




$|xy| > 1$

**70.** The solution of the equation $2x + 5 = 3x - 7$ is the first coordinate of the point of intersection of the graphs of $y_1 = 2x + 5$ and $y_2 = 3x - 7$. The solution of the system of equations $y = 2x + 5$, $y = 3x - 7$ is the ordered pair that is the point of intersection of $y_1$ and $y_2$.

**71.** In general, $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. $(\mathbf{AB})^2 = \mathbf{ABAB}$ and $\mathbf{A}^2\mathbf{B}^2 = \mathbf{AABB}$. Since matrix multiplication is not commutative, $\mathbf{BA} \neq \mathbf{AB}$, so $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$.

**72.** If $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$, then $a_1 = ka_2$ and $b_1 = kb_2$ for some number $k$. This means that the equations $a_1x + b_1y = c$, and $a_2x + b_2y = c_2$ are dependent, if $c_1 = kc_2$, or the system is inconsistent, if $c_1 \neq kc_2$.

**73.** If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ are parallel lines, then $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 \neq kc_2$, for some number $k$. Then $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$, $\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \neq 0$, and $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \neq 0$.

**74.** The graph of a linear equation consists of a set of points on a line. The graph of a linear inequality consists of the set of points in a half-plane and might also include the points on the line that is the boundary of the half-plane.

**75.** The denominator of the second fraction, $x^2 - 5x + 6$, can be factored into linear factors with real coefficients: $(x-3)(x-2)$. Thus, the given expression is not a partial fraction decomposition.

## Chapter 9 Test

**1.** $3x + 2y = 1$, (1)
$2x - y = -11$ (2)

Multiply equation (2) by 2 and add.

$$\begin{aligned}
3x + 2y &= 1\\
4x - 2y &= -22\\
\hline
7x \qquad &= -21\\
x &= -3
\end{aligned}$$

Back-substitute to find $y$.

$$\begin{aligned}
2(-3) - y &= -11 \quad \text{Using equation (2).}\\
-6 - y &= -11\\
-y &= -5\\
y &= 5
\end{aligned}$$

The solution is $(-3, 5)$. Since the system of equations has exactly one solution, it is consistent and the equations are independent.

**2.** $2x - y = 3$, (1)
$2y = 4x - 6$ (2)

Solve equation (1) for $y$.

$y = 2x - 3$

Subsitute in equation (2) and solve for $x$.

$$\begin{aligned}
2(2x - 3) &= 4x - 6\\
4x - 6 &= 4x - 6\\
0 &= 0
\end{aligned}$$

The equation $0 = 0$ is true for all values of $x$ and $y$. Thus the system of equations has infinitely many solutions. Solving either equation for $y$, we get $y = 2x - 3$, so the solutions are ordered pairs of the form $(x, 2x - 3)$. Equivalently, if we solve either equation for $x$, we get $x = \frac{y+3}{2}$, so the solutions can also be expressed as $\left(\frac{y+3}{2}, y\right)$. Since there are infinitely many solutions, the system of equations is consistent and the equations are dependent.

**3.** $x - y = 4,$ (1)

$3y = 3x - 8$ (2)

Solve equation (1) for $x$.

$x = y + 4$

Subsitute in equation (2) and solve for $y$.

$$3y = 3(y+4) - 8$$
$$3y = 3y + 12 - 8$$
$$0 = 4$$

We get a false equation so there is no solution. Since there is no solution the system of equations is inconsistent and the equations are independent.

**4.** $2x - 3y = 8,$ (1)

$5x - 2y = 9$ (2)

Multiply equation (1) by 5 and equation (2) by $-2$ and add.

$$\begin{array}{r} 10x - 15y = 40 \\ -10x + \ \ 4y = -18 \\ \hline -11y = 22 \\ y = -2 \end{array}$$

Back-substitute to find $x$.

$$2x - 3(-2) = 8$$
$$2x + 6 = 8$$
$$2x = 2$$
$$x = 1$$

The solution is $(1, -2)$. Since the system of equations has exactly one solution, it is consistent and the equations are independent.

**5.** $4x + 2y + z = 4,$ (1)

$3x - y + 5z = 4,$ (2)

$5x + 3y - 3z = -2$ (3)

Multiply equations (2) and (3) by 4.

$4x + 2y + z = 4$ (1)

$12x - 4y + 20z = 16$ (4)

$20x + 12y - 12z = -8$ (5)

Multiply equation (1) by $-3$ and add it to equation (4).

Multiply equation (1) by $-5$ and add it to equation (5).

$4x + 2y + z = 4$ (1)

$-10y + 17z = 4$ (6)

$2y - 17z = -28$ (7)

Interchange equations (6) and (7).

$4x + 2y + z = 4$ (1)

$2y - 17z = -28$ (7)

$-10y + 17z = 4$ (6)

Multiply equation (7) by 5 and add it to equation (6).

$4x + 2y + z = 4$ (1)

$2y - 17z = -28$ (7)

$-68z = -136$ (8)

Solve equation (8) for $z$.

$$-68z = -136$$
$$z = 2$$

Back-substitute 2 for $z$ in equation (7) and solve for $y$.

$$2y - 17 \cdot 2 = -28$$
$$2y - 34 = -28$$
$$2y = 6$$
$$y = 3$$

Back-substitute 3 for $y$ and 2 for $z$ in equation (1) and solve for $x$.

$$4x + 2 \cdot 3 + 2 = 4$$
$$4x + 8 = 4$$
$$4x = -4$$
$$x = -1$$

The solution is $(-1, 3, 2)$.

**6.** ***Familiarize***. Let $x$ and $y$ represent the number of student and nonstudent tickets sold, respectively. Then the receipts from the student tickets were $3x$ and the receipts from the nonstudent tickets were $5y$.

***Translate***. One equation comes from the fact that 750 tickets were sold.

$x + y = 750$

A second equation comes from the fact that the total receipts were \$3066.

$3x + 5y = 3066$

***Carry out***. We solve the system of equations.

$x + y = 750,$ (1)

$3x + 5y = 3066$ (2)

Multiply equation (1) by $-3$ and add.

$$\begin{array}{r} -3x - 3y = -2250 \\ 3x + 5y = 3066 \\ \hline 2y = 816 \\ y = 408 \end{array}$$

Substitute 408 for $y$ in equation (1) and solve for $x$.

$$x + 408 = 750$$
$$x = 342$$

***Check***. The number of tickets sold was $342 + 408$, or 750. The total receipts were $\$3 \cdot 342 + \$5 \cdot 408 = \$1026 + \$2040 = \$3066$. The solution checks.

***State***. 342 student tickets and 408 nonstudent tickets were sold.

**7.** ***Familiarize***. Let $x$, $y$, and $z$ represent the number of orders that can be processed per day by Tricia, Maria, and Antonio, respectively.

***Translate***.

Tricia, Maria, and Antonio can process 352 orders per day.

$x + y + z = 352$

Tricia and Maria together can process 224 orders per day.

$x + y = 224$

Tricia and Antonio together can process 248 orders per day.

$x+z=248$

We have a system of equations:

$$\begin{aligned} x+y+z&=352,\\ x+y\phantom{+z}&=224,\\ x\phantom{+y}+z&=248. \end{aligned}$$

***Carry out***. Solving the system of equations, we get $(120, 104, 128)$.

***Check***. Tricia, Maria, and Antonio can process $120+104+128$, or 352, orders per day. Together, Tricia and Maria can process $120+104$, or 224, orders per day. Together, Tricia and Antonio can process $120+128$, or 248, orders per day. The solution checks.

***State***. Tricia can process 120 orders per day, Maria can process 104 orders per day, and Antonio can process 128 orders per day.

**8.** $\mathbf{B}+\mathbf{C}=\begin{bmatrix}-5 & 1\\ -2 & 4\end{bmatrix}+\begin{bmatrix}3 & -4\\ -1 & 0\end{bmatrix}$

$$=\begin{bmatrix}-5+3 & 1+(-4)\\ -2+(-1) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -3\\ -3 & 4\end{bmatrix}$$

**9.** $\mathbf{A}$ and $\mathbf{C}$ do not have the same order, so it is not possible to find $\mathbf{A}-\mathbf{C}$.

**10.** $\mathbf{CB}=\begin{bmatrix}3 & -4\\ -1 & 0\end{bmatrix}\begin{bmatrix}-5 & 1\\ -2 & 4\end{bmatrix}$

$$=\begin{bmatrix}3(-5)+(-4)(-2) & 3(1)+(-4)(4)\\ -1(-5)+0(-2) & -1(1)+0(4)\end{bmatrix}$$

$$=\begin{bmatrix}-7 & -13\\ 5 & -1\end{bmatrix}$$

**11.** The product $\mathbf{AB}$ is not defined because the number of columns of $\mathbf{A}$, 3, is not equal to the number of rows of $\mathbf{B}$, 2.

**12.** $2\mathbf{A}=2\begin{bmatrix}1 & -1 & 3\\ -2 & 5 & 2\end{bmatrix}=\begin{bmatrix}2 & -2 & 6\\ -4 & 10 & 4\end{bmatrix}$

**13.** $\mathbf{C}=\begin{bmatrix}3 & -4\\ -1 & 0\end{bmatrix}$

Write the augmented matrix.

$$\left[\begin{array}{rr|rr}3 & -4 & 1 & 0\\ -1 & 0 & 0 & 1\end{array}\right]$$

Interchange rows.

$$\left[\begin{array}{rr|rr}-1 & 0 & 0 & 1\\ 3 & -4 & 1 & 0\end{array}\right]$$

Multiply row 1 by 3 and add it to row 2.

$$\left[\begin{array}{rr|rr}-1 & 0 & 0 & 1\\ 0 & -4 & 1 & 3\end{array}\right]$$

Multiply row 1 by $-1$ and row 2 by $-\dfrac{1}{4}$.

$$\left[\begin{array}{rr|rr}1 & 0 & 0 & -1\\ 0 & 1 & -\frac{1}{4} & -\frac{3}{4}\end{array}\right]$$

$$\mathbf{C}^{-1}=\begin{bmatrix}0 & -1\\ -\frac{1}{4} & -\frac{3}{4}\end{bmatrix}$$

**14.** a) $\mathbf{M}=\begin{bmatrix}0.95 & 0.40 & 0.39\\ 1.10 & 0.35 & 0.41\\ 1.05 & 0.39 & 0.36\end{bmatrix}$

b) $\mathbf{N}=\begin{bmatrix}26 & 18 & 23\end{bmatrix}$

c) $\mathbf{NM}=\begin{bmatrix}68.65 & 25.67 & 25.80\end{bmatrix}$

d) The entries of $\mathbf{NM}$ represent the total cost, in dollars, for each type of menu item served on the given day.

**15.** $\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 1\\ 1 & -5 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-8\\ 7\\ 3\end{bmatrix}$

**16.** $3x+2y+6z=2,$

$x+y+2z=1,$

$2x+2y+5z=3$

Write an equivalent matrix equation, $\mathbf{AX}=\mathbf{B}$.

$$\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 1\\ 3\end{bmatrix}$$

Then,

$$\mathbf{X}=\mathbf{A}^{-1}\mathbf{B}=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}2\\ 1\\ 3\end{bmatrix}=\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$$

The solution is $(-2, 1, 1)$.

**17.** $\begin{vmatrix}3 & -5\\ 8 & 7\end{vmatrix}=3\cdot 7-8(-5)=21+40=61$

**18.** We will expand across the first row.

$$\begin{vmatrix}2 & -1 & 4\\ -3 & 1 & -2\\ 5 & 3 & -1\end{vmatrix}$$

$$=2(-1)^{1+1}\begin{vmatrix}1 & -2\\ 3 & -1\end{vmatrix}+(-1)(-1)^{1+2}\begin{vmatrix}-3 & -2\\ 5 & -1\end{vmatrix}+$$

$$4(-1)^{1+3}\begin{vmatrix}-3 & 1\\ 5 & 3\end{vmatrix}$$

$$=2\cdot 1[1(-1)-3(-2)]+(-1)(-1)[-3(-1)-5(-2)]+$$
$$4\cdot 1[-3(3)-5(1)]$$

$$=2(5)+1(13)+4(-14)$$

$$=-33$$

**19.** $5x + 2y = -1,$
$7x + 6y = 1$

$$D = \begin{vmatrix} 5 & 2 \\ 7 & 6 \end{vmatrix} = 5(6) - 7(2) = 16$$

$$D_x = \begin{vmatrix} -1 & 2 \\ 1 & 6 \end{vmatrix} = -1(6) - (1)(2) = -8$$

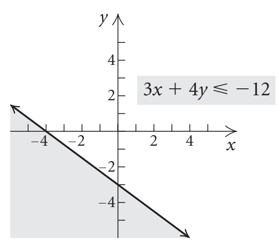
$$D_y = \begin{vmatrix} 5 & -1 \\ 7 & 1 \end{vmatrix} = 5(1) - 7(-1) = 12$$

$$x = \frac{D_x}{D} = \frac{-8}{16} = -\frac{1}{2}$$

$$y = \frac{D_y}{D} = \frac{12}{16} = \frac{3}{4}$$

The solution is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

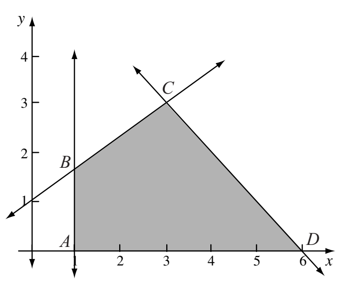
**20.**




**21.** Find the maximum value and the minimum value of $Q = 2x + 3y$ subject to

$$x + y \geq 6,$$
$$2x - 3y \geq -3,$$
$$x \geq 1,$$
$$y \geq 0.$$

Graph the system of inequalities and determine the vertices.




Vertex $A$:
We solve the system $x = 1$ and $y = 0$. The coordinates of point A are $(1, 0)$.

Vertex $B$:
We solve the system $2x - 3y = -3$ and $x = 1$. The coordinates of point B are $\left(1, \frac{5}{3}\right)$.

Vertex $C$:
We solve the system $x + y = 6$ and $2x - 3y = -3$. The coordinates of point $C$ are $(3, 3)$.

Vertex $D$:
We solve the system $x + y = 6$ and $y = 0$. The coordinates of point $D$ are $(6, 0)$.

Evaluate the objective function $Q$ at each vertex.

| Vertex | $Q = 2x + 3y$ |
|---|---|
| $(1, 0)$ | $2 \cdot 1 + 3 \cdot 0 = 2$ |
| $\left(1, \frac{5}{3}\right)$ | $2 \cdot 1 + 3 \cdot \frac{5}{3} = 7$ |
| $(3, 3)$ | $2 \cdot 3 + 3 \cdot 3 = 15$ |
| $(6, 0)$ | $2 \cdot 6 + 3 \cdot 0 = 12$ |

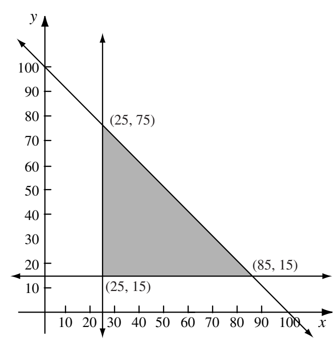
The maximum value of $Q$ is 15 when $x = 3$ and $y = 3$.

The minimum value of $Q$ is 2 when $x = 1$ and $y = 0$.

**22.** Let $x$ = the number of pound cakes prepared and $y$ = the number of carrot cakes. Find the maximum value of $P = 3x + 4y$ subject to

$$x + y \leq 100,$$
$$x \geq 25,$$
$$y \geq 15$$

Graph the system of inequalities, determine the vertices, and find the value of $P$ at each vertex.




| Vertex | $P = 3x + 4y$ |
|---|---|
| $(25, 15)$ | $3 \cdot 25 + 4 \cdot 15 = 135$ |
| $(25, 75)$ | $3 \cdot 25 + 4 \cdot 75 = 375$ |
| $(85, 15)$ | $3 \cdot 85 + 4 \cdot 15 = 315$ |

The maximum profit of \$375 occurs when 25 pound cakes and 75 carrot cakes are prepared.

**23.**
$$\frac{3x - 11}{x^2 + 2x - 3} = \frac{3x - 11}{(x - 1)(x + 3)} = \frac{A}{x - 1} + \frac{B}{x + 3} = \frac{A(x + 3) + B(x - 1)}{(x - 1)(x + 3)}$$

Equate the numerators.

$$3x - 11 = A(x + 3) + B(x - 1)$$

Let $x = -3 : 3(-3) - 11 = 0 + B(-3 - 1)$
$$-20 = -4B$$
$$5 = B$$

Let $x = 1 : 3(1) - 11 = A(1 + 3) + 0$
$$-8 = 4A$$
$$-2 = A$$

The decomposition is $-\dfrac{2}{x-1}+\dfrac{5}{x+3}$.

**24.** Graph the system of inequalities. We see that D is the correct graph.

**25.** Solve:

$$A(2)-B(-2)=C(2)-8$$
$$A(-3)-B(-1)=C(1)-8$$
$$A(4)-B(2)=C(9)-8$$

or

$$2A+2B-2C=-8$$
$$-3A+B-C=-8$$
$$4A-2B-9C=-8$$

The solution is $(1,-3,2)$, so $A=1$, $B=-3$, and $C=2$.