# Instructor Notes for Chapters 3 & 4

## Section 3.1 Complex Numbers

Goal for students:

- *perform computations involving complex numbers*

You might want to review the quadratic formula as you introduce complex numbers. Be sure to clarify that a negative discriminant implies no REAL solutions. The diagram on p. 236 is a good summary of subsets of the complex number system.

Mathematicians first came upon negatives under the radicals when working with Cardano's cubic formulas. As with most "discoveries," mathematicians ignored these numbers until eventually they just to accepted them.

| ***Suggested HW: MLP HW 3.1*** |
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## Section 3.2 Quadratic Equations, Functions and Models

Goals for students:

- Solve a quadratic equation by completing the square
- Understand that the quadratic formula is a result of completing the square on the general quadratic equation
- Use the quadratic formula to solve quadratic equations
- Relate the *x*-intercepts of the graph of a function, $f$, to the zeros of the function, to the solutions of the equation $f(x) = 0$
- Solve quadratic type equations

The quadratic formula is not in the "official" syllabus of the course, but experience has shown that students need a review of this fundamental tool. Please review completing the square (for a geometric approach, please see ( http://www.math.kent.edu/ebooks/FUNMATHV/conceptprep2_2.htm ) and then derive the formula.
You will need to spend significant time doing the "quadratic type" problems in #79 – 94.

| ***Suggested HW: MLP HW 3.2*** |
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## Section 3.3, Day 1: Analyzing Graphs of Quadratic Functions

*Goals for students:*

- *Find the vertex, axis of symmetry, and the max or min value of a quadratic function using the method of completing the square.*

- *Graph quadratic functions*

Similar to obtaining the quadratic formula by completing the square on a quadratic equation, we obtain a shortcut for finding the vertex by completing the square on a quadratic function. Be sure students understand that the maximum or minimum value of a function refers to its *output*, not the input. Spend today doing the procedure of completing the square and graphing. The next class will be devoted to applications.

***Suggested HW: MLP HW 3.3A***

## Section 3.3, Day 2: Analyzing Graphs of Quadratic Function and Modeling

Goals for students:

- Find the maximum or minimum value of a given quadratic function in applications

Pick up from where you left off yesterday, then emphasize finding the max or min in context. Students often have difficulty knowing which coordinate for which the problem is asking. You might encourage students to write the two changing quantities as an ordered pair. For example, for a falling object problem they might write (*time, height*). You might then work through example 6 in the text.

Modeling is another issue. As you probably know, modeling is difficult for students. Certainly emphasize the area type, like example 4 on p. 162 (back in section 2.1). The Instructor website has a worksheet with extra modeling problems. You might let students work in groups and try to figure it out. Another option is to guide them through it, then let them work on the rest of the problems on the sheet, offering help as needed. The goal of the sheet is *writing the models*.

***Suggested HW: MLP HW 3.3B***

## 4.1 Polynomial Functions and Models

Goals for students:

- Determine the end behavior of the graph of a polynomial function using the leading term test
- Find the zeros and their multiplicities of polynomial functions by factoring

After introducing basic terminology, you might do #1 – 10 odd on p. 306 together in class.

Discuss the characteristics of the graphs of polynomial functions. The graphing calculator is a great tool to illustrate the Leading-Term Test. You might try this example: (also available on the website as an investigative activity for students):

*In your graphing calculator, let* $Y_1 = 2x^3 - 7x^2 - 8x + 16$ *and let* $Y_2 = 2x^3$. Use the *viewing window* $[-5,5]$ *by* $[-25,25]$. *Compare the two graphs: How are they the same? How are they different?*
*Try changing the viewing window to* $[-40,40]$ *by* $[-5000,5000]$. *This is a "bird's eye view." How do the graphs compare now?*

Hopefully this visual approach will convince students that for large values of $|x|$, the leading term is sufficient for determining the behavior of the graph.

Review factoring and the connection between factors, zeros, and *x*-intercepts. Example 5 is good.

For even and odd multiplicities, you might try this:

*In your graphing calculator, let* $Y_1 = (x-2)^2(x+1)$
*What happens at the zeros of the function? Does the graph cross or touch? Why do you think this happens?*
*Repeat with* $Y_1 = (x+3)^2(x-2)$, $Y_1 = (x+5)^2(x+8)$, etc.
*Repeat with* $Y_1 = (x+5)^3(x+8)^2$, $Y_1 = (x+1)^2(x-4)^3$ (use viewing window [-5,5] by [-100,100] for this one), $Y_1 = (x+1)^4(x-3)$

Be sure students understand WHY the graph turns back at zeros with even multiplicities (*why does a parabola have a turning point?*). Then put these function rules in the Y= menu on your calculator and ask students to predict the zeros and whether the graphs cross or touch at these zeros. $Y_1 = (x+2)^2(x-3)$; $Y_1 = (x+1)^4(x-2)^3$; $Y_1 = (x+3)(x-2)^2$

Then you might put the graphs of these on the viewscreen one at a time and let students name the factors and a possible function rule. $Y_1 = (x+4)(x-1)^2$, $Y_1 = (x-1)^3(x-2)^2$ (use viewing window [-5,5] by [-10,10] for this one); and $Y_1 = (x+2)^2(x-2)^2$.

***Suggested HW: MLP HW 4.1***

## 4.2 Graphing Polynomial Functions

- Sketch by hand a graph of a polynomial function

You might begin class by asking students to do #1 – 12 in small groups or all together as a class.

Then, putting together the class discussion from yesterday, take your time as you work through 2 or 3 examples in detail. The sign chart will be new for some students.

Remind students that you will ask them to graph a polynomial by hand (with no graphing calculator) on a quiz or exam. Suggest that they work through the HW problems on MyLabsPlus BY HAND. Many of the MLP problems in this section are multiple choice.

The Intermediate Value Theorem is optional.

***Suggested HW: MLP HW 4.2***

## 4.3 Polynomial Division; Remainder and Factor Theorems

Goals for students:

- Perform long division with polynomials and determine whether one polynomial is a factor of another
- Use synthetic division to divide a polynomial by $x - c$.
- Understand and use the Remainder and Factor Theorems

Work through Example 1 using long division, relating the process to long division for numbers. Be sure to set up the checks by multiplication: DIVIDEND = DIVISOR x QUOTIENT + REMAINDER , i.e. $x^3 + 2x^2 - 5x - 6 = (x+1)(x^2 + x - 6) + 0$ (I wouldn't take the time to multiply this out, though); eventually use $P(x) = (x-c) \cdot Q(x) + R(x)$.

You might want to foreshadow the Remainder Theorem here by asking them the value of $P(-1)$.

Synthetic division is straightforward and easy for students, though they sometimes forget to put in a 0 as a placeholder for missing terms.

Example 6 from start to finish is a good way to end the lesson.

***Suggested HW: MLP HW 4.3***

## 4.4 Theorems about Zeros of Polynomial Functions

Goals for students:

- Given a polynomial function with integer coefficients, use the Rational Zero Test to name the possible rational zeros. Understand WHY the Rational Zero Test works (relate to factors of a quadratic).
- Given a polynomial with integer coefficients, find all the zeros.
- Write a polynomial function, given its zeros

In this section, we use the skills learned in the previous section and pull everything together.

Though the text begins with a statement of the Fundamental Theorem of Algebra, I'd save that for last…kind of a culmination of our work.

Intuitively develop the Rational Zeros Theorem. You might begin be recalling the process of factoring a quadratic such as $f(x) = x^2 - x - 6$.

*How do you find the factors?*
*In the factored form* $(x-a)(x-b)$*, what does the product ab =?*

Similarly for other easy quadratics such as $f(x) = x^2 - 2x - 15$, $f(x) = x^2 - 7x + 10$

Then you might ask them

*In the polynomial* $f(x) = x^4 + 5x^3 - 27x^2 + 31x - 10$*, how many (linear) factors do you think there would be?*

Then I'd write $(x-?)(x-??)(x-???)(x-????)$.

*What does the product of all the ?????????? have to be?*
*So what are the possible rational zeros?*

Similar questioning, then, with a polynomial with a leading coefficient not equal to 1, such as:

$f(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$ which would factor into the form:
$(ax-?)(bx-??)(cx-???)(dx-????)$
*How many linear factors (at most) would there be?*
*What does the product ?????????? have to be?*
*What are possible values for a, b, c, d?*
*How does this affect the zeros? How do you find a zero once you know a factor?*

Then state the Rational Zeros Theorem.

Pull everything together then and summarize the steps for finding zeros of a polynomial:

1. Use the Rational Zeros Theorem to list possible rational zeros
2. Use your graphing calculator to help narrow down the choices
3. Use synthetic division to find the zeros and list factored formula

Work through examples 5 and 6 with students. I insist that they show me the newly factored form every time they find a zero.

Descartes Rule of Signs is optional.

***Suggested HW: MLP HW 4.4***

## 4.5 Rational Functions

Goals for students:

- Graph by hand a rational function, identifying all asymptotes

You might begin by asking students to make a table of values for the simple rational function $y=\frac{1}{x}$, asking a student to put the table on the board and sketch the function. Your table might include these input values:

| $x$ | $y=\frac{1}{x}$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 10 | |
| 20 | |
| 100 | |

| $x$ | $y=\frac{1}{x}$ |
|---|---|
| $\frac{1}{2}$ | |
| $\frac{1}{4}$ | |
| $\frac{1}{5}$ | |
| $\frac{1}{10}$ | |
| $\frac{1}{100}$ | |
| $\frac{1}{1000}$ | |

and their negatives.

Discuss the shape of the graph, its domain, and why it looks like it does. Note the asymptotes and relate to the asymptotes of the exponential functions we studied in MATH 10036.
You might then ask students to make a table of values (perhaps on the calculator this time) and hand graph $y=\frac{1}{x-2}$, noting in particular the end behavior and the behavior close to $x=2$.
Be sure to suggest appropriate input values. *Start: t = 3;* $\Delta$ *= -0.1*. Then *Start: t = 2.5;* $\Delta$ *= -0.01*
Extend the idea to more complicated rational functions, emphasizing domains and asymptotes. Students will intuit the location of vertical asymptotes with proper questioning. Horizontal asymptotes are a little more subtle; the approach in example 4 on p. 346-7 foreshadows the limit concept.

You also might want to point out that we can graph the function *g* in example 6 on page 308 by writing it like this: $g(x)=2+\frac{1}{x^2}$, then using a vertical translation of the graph of $y=\frac{1}{x^2}$

Summarize and be sure to do a complete example from start to finish. Example 8 is good. Example 9 time permitting. You might note that a graph can cross a horizontal asymptote, but not a vertical one.

Oblique asymptotes are optional.

***Suggested HW: MLP HW 4.5***

## 4.5 Rational Functions, cont'd (Day 2)

You'll probably have many questions on the homework. You might use any left over time and by allowing students to work on some of the even exercises, or correct the ones on the homework they couldn't get.

Introduce oblique asymptotes if time. This topic provides is a good review of long division.

## 4.6 Polynomial and Rational Inequalities (2 days)

Goals for students:

- Solve polynomial and rational inequalities

After reviewing yesterday's homework, you might discuss #84 on page 358. Then as a motivation for section 4.6, problems #77 – 82 on pp. 369-70 are good. (You needn't solve these quite yet, just set up one or two).

Calc teachers assume that students know how to use sign charts, so teach carefully!

Please refer to the MATH 10035 e-book, available at http://www.math.kent.edu/ebooks/FUNMATHV/ch2_4.htm for ideas for teaching this section. Note especially the prep assignments at the beginning of the section – both the skill prep and concept prep. Feel free to print and use with your students.

Be sure that students can solve these both graphically and using sign charts. Reading the graph is essential, but not sufficient.

You might do polynomial inequalities on day 1 and rational inequalities on day 2, though our MATH 10035 students have seen quadratic inequalities. Another option is to begin with polynomials and finish with rational inequalities in one day, then use the second day as a work session. Conceptually the problems are the same, though of course the rational inequalities involve more algebraic manipulation.

***Suggested HW:***
**_Day 1: MLP HW 4.6A_** (polynomial inequalities – 15 questions)
**_Day 2: MLP HW 4.6B_**

**Exam Ch 3 and 4.2 – 4.4**

**MLP has a set of review problems and a practice exam. Please note that I did NOT include sections 4.5 and 4.6 on the exam. I'll quiz on these sections separately in class.**

**All in-class exams are paper and pencil exams and a sample is available for you. Please see Bev for access.**