Instructor Notes for Chapter 5

Section 5.1 One to One Functions (Day 1)

Goals for students

- Understand that an inverse relation undoes the original
- Understand why the line $y = x$ is a line of symmetry for the graphs of relations and their inverses
- Determine if a function is one-to-one

Discuss the idea of inverse relation. You might want to introduce the idea of inverses as "undoing" each other, perhaps by asking such questions as:

- *What operation undoes the operation of adding 3? of multiplying by 3?*
- *How can we write these using function notation?*

I then usually develop the idea that the inputs and the outputs are interchanged when we're dealing with inverse relations:

Let $f(x) = x + 3$ and $g(x) = x - 3$. Find $f(10)$, then $g(f(10))$.
Note that $f(10) = 13$ *so that the point* $(10,13)$ *is on the graph of the function f.*
Note also that $g(13) = 10$*, so that the point* $(13,10)$ *is on the graph of the function g. Notice that the x coordinate and y coordinate are interchanged.*

You might then work through examples 1 and 2 on pages 330-331 with your students. You might also give them a few problems like example 1 and develop with them intuitively that the line $y = x$ is the line of symmetry of the graph of these "inverse relations." You might then let them graph the inverses on the graphs in # 67 – 72 on p. 398.

I usually then ask my students to hand graph the function $y = x^2$ and try to sketch in its inverse.

- *What happens when you do this?*
- *Is there anything "wrong" with the result?*

Usually someone will note that the result is not a function because it doesn't pass the vertical line test.

- *How can we tell before we interchange the xs and ys if the result will be a function?*

Having students come up with the Horizontal Line test by themselves is quite cool.

Define one-to-one functions and if you're really brave, you might try the algebraic test in example 3 on p. 391. (Students usually have trouble understanding what they are doing, even if they can mimc you in these little proofs.)

Suggested HW: MLP HW 5.1A

Section 5.1 Inverse Functions (Day 2)

Goals for students:

- Use composition of functions to verify that functions are inverses of each other
- Given a function, find it's inverse

Build upon yesterday's lesson and show several examples verifying that functions are inverses of each other. Finding an inverse function, given a function, will follow quite naturally and is usually quite easy for students at this point.
Should spend some time discussing domains and ranges and also restricting the domain of a function to allow the inverse to be a function (p. 356 in text).

Suggested HW: MLP HW 5.1B

Section 5.2 Exponential Functions and Graphs

- Graph exponential functions using translations

For those students having just completed MATH 10024, this material will be review. For them, I'd suggest focusing on graphing exponentials using translations and also the modeling problems in the section.

If most of your students are directly from high school, you need to carefully develop the idea of exponential function, perhaps starting with a real world example such as the following:

> *Suppose over Christmas break you had a choice of two jobs. Company A will pay you $1,000,000 for 30 days work. Company B will be willing to pay you a penny on the first day, two cents on the second, and continue doubling the amount until the end of the month. On day 30 you would receive the final doubled amount (not the sum of all days). Which job should you take and why?*

Be sure to give a few counterexamples as well.

You'll need to discuss the domain, range, and asymptote of a simple exponential function, then move on to translations, the number *e*, and at least the compound interest application. A lot to cover in one day!

Suggested HW: MLP HW 5.2

Section 5.3 Logarithmic Functions and Graphs

Goals for students

- Convert between exponential and logarithmic equations
- Find the domain of a given log function
- Sketch by hand a graph of a given logarithmic function
- Understand the meaning of and use ln notation for logarithms to the base *e*.

One way of introducing logarithms is to ask students to find the inverse of an exponential function:

Given the function $y = 2^x$ *. Is this function one-to-one?*
Since it is one-to-one, it has an inverse. In small groups try to find its inverse.

Students will struggle with trying to solve for y. Take advantage of this teachable moment:

A LOGARITHM IS AN EXPONENT. Then give several examples of logarithms with different bases.
Reinforce the concept by asking students to convert between exponential and log equations.
Ask students to try to find $\log(-100)$, $\log(-10)$, $\log(0)$,etc., then discuss domains of log functions.
Graph of log functions require careful teaching, emphasizing the importance of finding the domain, asymptote, vertical intercept and one extra point.

Change of base formula is optional.

Suggested HW: MLP HW 5.3

Section 5.4 Properties of Logarithmic Functions

Goals for students

- Convert from logarithms of products, powers, and quotients to expressions of sums and differences of logs
- Express sums or differences of logs as a single logarithms
- Simplify expressions of the type $\log_a a^x$ and $a^{\log_a x}$ (and understand the *why* behind the *how*)

The authors of our text are not very intuitive here, choosing just to give the properties with no indication of *why* they might be true.

Logarithms to the base 10 were commonly used years ago to multiply and divide large numbers. Since logarithms are exponents, we can simply ADD them together rather than multiplying the original numbers. Of course, we would need a table or calculator to find the logarithms of the original numbers and then to switch back from the logarithms when we are done.

You might guide students to intuit the product rule of logarithms:

Find $\log(10^3 \cdot 10^5)$. (Most students will change this expression to $\log(10^8)$, then readily find the logarithm.)

Now find $\log 10^3$ *then* $\log 10^5$. *How are these logs related to the log of the product?*

You could try a slightly more formal approach:

$$10^3 \cdot 10^5 = 10^{3+5} = 10^8$$

Taking the log of all sides, we obtain:

$$\log 10^3 \cdot 10^5 = \log 10^{3+5} = \log 10^8 = 8 = \log 10^3 + \log 10^5$$

The Quotient rule follows naturally, as long as students are fully aware that A LOGARITHM IS AN EXPONENT.

You might introduce the Power Rule by example:

$$\log 10^3 = \log 10 \cdot 10 \cdot 10 = \log 10 + \log 10 + \log 10 \text{ (by the Product Rule)}$$
$$= 3\log 10$$

Students tend to have little difficulty applying the properties of logs. You might do #1 – 22 together orally in class.

Suggested HW: MLP HW 5.4

Section 5.5 Solving Exponential and Logarithmic Equations (2 DAYS!)

Goals for students

- Solve Exponential Equations (day 1)
- Solve Logarithmic Equations (day 2)

Solving exponential and logarithmic equations is generally difficult for students. The books use of a graphical approach to reinforce the algebraic approach is effective.

Suggested HW:
Day 1: MLP HW 5.5A
Day 2: MLP HW 5.5B

Section 5.6 Applications and Models: Growth and Decay

Goals for students

- Write a mathematical model for a real world scenario involving exponential growth or decay if given the growth or decay rate and initial value
- Find doubling time by solving an exponential equation
- Find the growth or decay rate given two ordered pairs in a real world context and then write an appropriate exponential model

There's no need for students to memorize the doubling time formulas on p. 454. Rather have them solve the exponential equation resulting from a given scenario.

Suggested HW:
Day 1: MLP HW 5.6A
Day 2: MLP HW 5.6B

Exam Ch 5

There is a set of review problems and a practice exam on MLP for the Exam.

All in-class exams are paper and pencil exams and a sample is available for you. Please see Bev for access.