Instructor Notes for Module 5

M5 I1 and I2 The Bottle Problem – Modeling Co-Varying Relationships

Learning outcomes: Student will be able to:

- Given bottles of various shapes, sketch a graph of the co-varying relationship between the height of the water in the bottle (dependent variable) vs the amount of water (volume) in the bottle (independent variable).
- Discuss changes in the average rate of change of height of water in a bottle vs volume of water in the bottle over different intervals. Explain how these changes are reflected in the graph of the co-varying relationship.
- Find the average rate of change of a function over a specified interval given its function rule.

Suggested order of lesson: This is going to be tough to do in one day. (understatement, right?)

- Use various sizes and shapes of bottles to perform the height vs volume experiment as performed in our College Teaching class.
- Show the “Bottle 1” animation available on Rational Reasoning site. The link is to the right of the Power Point opening slide for Module 5. Ask students to complete the table in #2a on p.181 as you do this.
- Rather than ask students to complete #2 as is, I would suggest asking the following question about the table of values they have completed: Calculate the average rates of change of the height of water in the bottle with respect to volume of water in the bottle for the following increases in volume of water.
  - As the volume of water in the bottle increases from 1 to 2.
  - As the volume of water in the bottle increases from 2 to 3.
  - As the volume of water in the bottle increases from 3 to 4.
- Comment/discuss about how the average rate of change changes over these intervals and how these changes are reflected in the graph.
- Discuss the definitions of concavity on p. 185, then work through #5 with students.
- As a large group, (or small groups) discuss #6, p. 185.
- Perhaps finish up with either #10 or #11.

HW due next class: HW 21 on iMathAS (There are quite a few problems on this assignment. You might ask them to complete 12 of them.) and PC 25 (The PCs appear to be mis-numbered in the iMathAS course. You might want to go into the settings and change the names so that they are sequential.)

M5 I3 Transformations of Polynomial Functions

The Pre-class assignment for this section (PC23) on IMathAS consists of problem #1 on p. 189 in the workbook and a discussion question. You might take student answers to the pre-class assignment and use them as a springboard for class discussion.

Day 1

Learning outcomes. Students will be able to

- Express a new function \( h \) in terms of a given function \( g \) if the output values of \( h \) are a constant value more or less than the output values of \( g \). Explain how the graph of \( h \) differs from the graph of \( g \).
- Express a new function \( k \) in terms of a given function \( g \) if the output values of \( k \) are a constant value times the output values of \( g \). Explain how the graph of \( k \) differs from the graph of \( g \).
- Express a new function \( w \) in terms of a given function \( g \) if the input values of \( w \) are a constant value more or less than the input values of \( g \). Explain how the graph of \( h \) differs from the graph of \( g \).
- Interpret the meaning of all of the above transformations in context.
- Express a new function that has a combination of the above transformations in terms of the original function.
Suggested order of lesson:
1. Quickly discuss #1 in the PC assignment (#1 p. 195). This problem is the basis for the next several problems in the investigation and also reviews the concept of average rate of change.
2. Ask students to work on #2 on p. 195 in small groups. Discuss. Be sure you discuss how the graphs differ. If you project the investigation onto the whiteboard, you can simply sketch the new graph over the original graph in #1. Be sure also to investigate if there are differences in the average rate of change of the functions.
3. Ask students to sketch the graphs in #1-#4 on the Pathways Transformation Practice worksheet, available on the Blackboard site.
4. Ask students to work on #3 on p. 190 in the workbook in small groups. Discuss. Be sure you discuss how the graphs differ. If you project the investigation onto the whiteboard, you can simply sketch the new graph over the original graph in #1. Be sure also to note any differences in the average rate of change of the functions.
5. Ask students to sketch the graphs in #5-#7 on the Pathways Transformation Practice worksheet,
6. Work through #4 on p. 191 in the workbook together and make a sketch.
7. You’ll need to provide some guidance in #4 since this is the first time they are given a horizontal translation. You might do something like the following, using the graphing calculator:
   \[ Y_1 = x^2 \text{ and } Y_2 = (x-2)^2 \]
   (a) Go to TABLE. Find the values \( x = 0 \) and \( Y_1 = 0 \) in the table. Where does \( Y_2 = 0 \)?
   (b) Find the values \( x = 2 \) and \( Y_1 = 4 \) in the table. For what values of \( x \) does \( Y_1 = 4 \)? How do you think this will affect the graph of \( Y_2 \)? Make a conjecture, then check on your graphing calculator.
   (c) Repeat the above for \( Y_2 = (x-3)^2 \), \( Y_2 = (x+4)^2 \), and \( Y_2 = (x+5)^2 \). What effect does the value \( b \) have on the graph of the function \( Y_2 = (x-b)^2 \)?
8. Discuss #6, time permitting, at least briefly. Correctly doing these types of problems is an important outcome for this section.

**HW due next class:** IMathAS: HW22.

**Day 2**

*Learning outcomes.* Students will be able to
- Accurately sketch the graph of a desired transformation of a function given by a formula or graph.
- Name the domain and range of a transformed function from its graph.

Suggested order of lesson:
1. With students, complete #8 - #12 on the Pathways Transformation Practice worksheet from yesterday.
2. Students work together on #1 - #10 on the “Practice in General I” worksheet, titled “Homework #49,” which is available on Blackboard. Ask students to name the domain and range of the transformed function in addition to sketching the graph.

**HW due next class:** “Practice in General II” worksheet, titled “Homework #30,” available on Blackboard. Ask students to name the domain and range of each transformation; PC 26
**MS 14 Quadratic Functions**
The Pre-class assignment for this section (PC24) on IMathAS consists of problem #1 on p. 193 and a discussion question. Problem #1 is an ideal segue from the previous lesson. Be sure to discuss it in class. Students may not be used to seeing transformations of a particular function \( i.e. g(x) = 3 \cdot f(x) \), so be sure to spend some time on this problem. Remember, we have TWO DAYS for this section.

**DAY 1**
*Learning outcomes.* Students will:
- Accurately sketch a hand graph of a transformed quadratic function
- Understand, define, and be able to use the terms roots, solutions, x-intercepts.
- Find the roots of a quadratic function in factored form and explain why these are the x-intercepts of the graph of the function
- Factor a given quadratic function to find its roots
- Understand that the roots of a quadratic function are horizontally equidistant from its vertex.
- Use the quadratic formula to find roots of a quadratic function

*Suggested order of lesson:*
1. Discuss #1 on p. 193 with students. You might ask a few students to come to the doc cam and show their sketches. Doing so would be a good review of the transformations.
2. Let students work on #2 in small groups. Discuss. Then #3 and #4. When finished, ask them to work on #5. Problem #5 asks good conceptual questions about the quadratic formula. Be sure students understand that 
\[
\pm \frac{\sqrt{b^2 - 4ac}}{2a}
\] is the horizontal distance of the roots from the vertex.
3. Students need practice finding roots. Problems #37, #39 (find the roots only), and #42 on pp. 208-209 in the workbook are good for this purpose. Be sure to work through at least a couple of these with your students before class ends.

**HW due next class:** Paper and Pencil: Finish #37, #29, and #42; perhaps #26 and #28 (p. 207) as well. Begin IMathAS: HW23. (HW28 has 19 problems, but it’s not due for 2 days. Students won’t be able to finish it in one sitting, however, so be sure to encourage them to begin early.) PC27

**DAY 2**
*Learning outcomes.* Students will:
- Find the max or min value of a given quadratic function and interpret it in context;
- Understand and use \( x = -\frac{b}{2a} \) to find the x-coordinate of the vertex.
- Given the roots of a quadratic function and one point on its graph, find the formula for the function.

*Suggested order of lesson:*
1. Build off your previous discussion of #5 on p. 195 and note that \( x = -\frac{b}{2a} \) is the x-coordinate of the vertex. Then let students work through (or you work through with them) #6 on p. 196.
2. Students work #32 and #35 on p. 207-208.
3. Work through #7 on p. 196 with students.
4. Let students try #40 & 41 on p. 209. Believe it or not, they usually don’t have too much trouble with these.

**HW due next class:** Paper and Pencil: #32 and #35 on p. 207-208 if not completed in class. IMathAS: HW23 PC 28
DAY 1
Learning outcomes. Students will:

- Identify the degree, leading term, leading coefficient of a given polynomial function;
- Determine if a function is a polynomial;
- Given a polynomial in factored form, identify its roots;
- Given the function rule for a polynomial, determine the end behavior of its graph;
- Use a sign chart to determine the intervals over which a polynomial is positive or negative;
- Make a rough hand sketch of a polynomial function without the use of technology.

Suggested order of lesson:

1. I like to begin by making two lists of functions – those that are polynomials and those that are not. Then I ask students to figure out the definition of polynomial. Follow with a bit of skill and drill about determining if a function is a polynomial or not. Also give vocabulary terms. The Rational Reasoning ebook is good here, starting on p. 21. Example 13 that follows is good for skill and drill. Also #48 on p. 209-210 in the workbook.

2. Finding the roots given the factored form of a polynomial should be easy for students. After briefly discussing #1 on p. 197 with students, you can ask them to list the roots of the polynomials listed in #49 and 50 on p. 210 in the workbook.

3. The concept of end behavior will be new to most students, but not too difficult for them to grasp. Let students work through #4 on p. 198 in the ebook. Discuss, reinforcing the meaning of the notation. The “Supplementary Materials” folder on Blackboard has a good investigative activity, “End Behavior of Polynomials” that will help students study the idea from the graphical, numeric, and algebraic point of view. Follow up with skill and drill: #56- #58 on p. 210 in the workbook.

4. The second page of the “End Behavior” worksheet discusses multiplicity. You might as well do that now as well.

5. Work through #1d) on p. 197 with students, suggesting a sign chart as an easy way to keep information organized, though you might want to let them struggle a bit as they try to find their own way. You might then suggest something like this:

   \[
   \begin{array}{ccc}
   (x+1) & - & + \\
   (x-3)^2 & + & + \\
   (x+1)(x-3)^2 & - & + \\
   \end{array}
   \]

   Of course, be sure to do it WITH students, not just FOR students. Then construct a rough sketch.

6. Time permitting, graph at least one of #56 (b or d), 57 (a – c) or 58(a, b) or #59-60 on pp. 210-211
   a. End behavior;
   b. Roots and multiplicities
   c. Sign chart for intervals of positive or negative values.

*HW due next class:* Paper and pencil: graph #57 (a and b), #58 (a and b); #59 on p.211.
*Begin IMathAS:* HW24 (This is not due next class.)
DAY 2
Learning outcomes. Students will:
   - Understand that knowing one factor of a polynomial, performing division gives another.
   - Perform long or synthetic division to find roots of polynomial functions, given one root.

Suggested order of lesson:
1. You might start by having students graph a polynomial by hand, perhaps sending students to the board to do this. Or perhaps give them a written quiz. My guess is they will need a little time here until they can accurately graph a polynomial by hand without the use of technology.

2. The Supplementary Materials folder, Module 5 on Blackboard, has a worksheet entitled, “The Factor Theorem and The Remainder Theorem.” You can use problems from this sheet to review long division of polynomials and teach synthetic division.

HW due next class: Paper and pencil: Factor Theorem Worksheet: #7, 9, 13, 15, 17, 31, 33, 35, 38 to hand in.
IMathAS: HW 24 (This is not due next class.)

DAY ?? OPTIONAL
Learning outcomes. Students will:
   - List the possible rational zeros of a given polynomial
   - Find all real zeros of a given polynomial function, using the rational zero theorem, synthetic division, and if necessary the quadratic formula.

Suggested order of lesson:
Intuitively develop the Rational Zeros Theorem. You might begin by recalling the process of factoring a quadratic such as \( f(x) = x^2 - x - 6 \)

How do you find the factors?
In the factored form \((x-a)(x-b)\) what does the product \(ab\) = ??

Similarly for other easy quadratics such as \( g(x) = x^2 - 2x - 15 \) and \( f(x) = x^2 - 7x - 10 \)
Then you might ask them
In the polynomial \( g(x) = x^4 + 5x^3 + 27x^2 + 31x - 10 \), how many \((linear)\) factors do you think there would be?
Then I’d write \((x-?) (x-?) (x-???) (x-????)\)
What does the product of all the ???? have to be?
So what are the possible rational zeros?

Similar questioning, then, with a polynomial with a leading coefficient not equal to 1, such as:
\( g(x) = 3x^4 - 4x^3 + x^2 + 6x - 2 \) which would factor into the form: \((ax-?) (bx-?) (cx-???) (dx-????)\)

How many linear factors (at most) would there be?
What does the product ????????? have to be?
What are possible values for \(a, b, c, d\)?
How does this affect the zeros? How do you find a zero once you know a factor?

Then state the Rational Zeros Theorem.
Pull everything together then and summarize the steps for finding zeros of a polynomial:
1. Use the Rational Zeros Theorem to list possible rational zeros
2. Use your graphing calculator to help narrow down the choices
3. Use synthetic division to find the zeros and list factored form.
Work through a couple examples with students. I insist that they show me the newly factored form every time they find a zero.

You will need to find examples from another text, or perhaps use a couple from the handout on Blackboard entitled, Higher Degree Polynomials – find zeros.

**HW due next class:** Paper and Pencil: Worksheet from Blackboard: Higher Degree Polynomials – find zeros, #61, 63, 65, 67

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**DAY 3**

Learning outcomes. Students will:

- Solve a polynomial inequality

**Suggested order of lesson:**

1. You might start by having students work some synthetic division problems at the board or give them a written quiz.

2. Students should have learned how to solve polynomial inequalities while graphing polynomials, so you might just give students several problems to work at the board, at their seats, or as a group quiz. You might also reinforce the reading of graphs, i.e. like #61 - #67 on pp. 211-212 in the workbook.

**HW due next class:** IMathAS: HW24