TMM001 - COLLEGE ALGEBRA (Revised December 8, 2015; updated samples April 30, 2016)

Typical Range: 3-4 Semester Hours

Recommendation: This course should significantly reflect the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) subcommittee, Curriculum Renewal Across the First Two Years (CRAFTY), College Algebra Guidelines.

College Algebra provides students a college level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, where solutions to problems in real-world situations are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic and technology-based techniques as appropriate using a variety of mathematical notation. Students should develop a framework of problem-solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; determine the plausibility of and interpret solutions).

– Adapted from the MAA/CUPM CRAFTY 2007 College Algebra Guidelines

To qualify for TMM001 (College Algebra), a course must achieve all of the following essential learning outcomes listed in this document (marked with an asterisk). The Sample Tasks are recommendations for types of activities that could be used in the course.

1. Functions: Successful College Algebra students demonstrate a deep understanding of functions whether they are described verbally, numerically, graphically, or algebraically (both explicitly and implicitly). Students should be proficient working with the following families of functions: linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, and piecewise-defined functions (including absolute value).

The successful College Algebra student can:

1a. Analyze functions. Routine analysis includes discussion of domain, range, zeros, general function behavior (increasing, decreasing, extrema, etc.). In addition to performing rote processes, the student can articulate reasons for choosing a particular process, recognize function families and anticipate behavior, and explain the implementation of a process (e.g., why certain real numbers are excluded from the domain of a given function).*

Sample Tasks:

- The student determines the domain of a function described algebraically and gives reasons for domain restrictions.
- The student determines the range of a function given its graph.
- The student can explain to a peer how to evaluate a given piecewise-defined function.
- The student recognizes and accurately represents asymptotic behavior on the graph of an exponential function.

- The student can explain the difference between the quantities $f(x + h)$ and $f(x) + h$.
- The student can explain the difference between $f(x) + f(x)$ and $f(x) + f(y)$.

1b. Convert between different representations of a function.*

Sample Tasks:

- The student translates a function from a verbal description to an algebraic description to determine its domain.
- The student constructs a table or a graph to approximate the range of a function described algebraically.
- The student graphs a piecewise-defined function to determine intervals over which the function is increasing, decreasing, or constant.
- The student can formulate a possible equation for a function given a graph.
- The student can verbalize the function represented variously as: $y = x^2$, $h(t) = t^2$, and $\{(u, u^2) : u \text{ is a real number}\}$ as 'the squaring function.'
- The student uses knowledge of end behavior to adjust the viewing window of a graphing utility.
- The student can graph the equation $3u + 4t = 6$ both on the tu- and ut-axes and compare the slopes in each case.
- The student graphs $f(x) = x^4 + 2x^2 - 117$, suspects symmetry about the y-axis, and proves the symmetry analytically by showing $f(-x) = f(x)$.
- Discuss domain and range of functions defined implicitly: $y^3 + xy + x^{117} = 0$.

1c. Perform operations with functions including addition, subtraction, multiplication, division, composition, and inversion; connect properties of constituent functions to properties of the resultant function; and resolve a function into a sum, difference, product, quotient, and/or composite of functions.*

Sample Tasks:

- Given the formula for a rational function f, the student can simplify the difference quotient for f.
- Given the graph of f, the student constructs the graph of $\frac{1}{f}$.
- The student can verbally describe the relationship between the graph of $y = f(x)$, and each of $y = f(|x|)$ and $y = |f(x)|$.
- Given an algebraic description for f and a graph for g, the student can determine values for the sum, difference, product, quotient, and composition of f and g.
- Given the graph of f and g, the student can determine the domain of $\frac{f}{g}$ and $f \circ g$.
- Given formulas for $f(x)$ and $(f \circ g)(x)$, a student can create a formula for $g(x)$.
- The student can explain how to determine a formula for the composition of two piecewise-defined functions.
- Given the graph of a function, the student can determine if the function is invertible and, if so, graph the inverse.

- The student can find functions f, g, and h so that $F(t) = \sqrt{\frac{3t-4}{t+1}} = \left(f \circ \left(\frac{g}{h}\right)\right)(t)$.
- Given the graph of a function f, the student can graph $y = 3f(1-x) + 2$.
- Given the graphs of two functions, the student can determine if they appear to be related by a sequence of linear transformations.

2. Equations and Inequalities: Successful College Algebra students are proficient at solving a wide array of equations and inequalities involving linear, quadratic, higher-order polynomial, rational, exponential, logarithmic, radical, and piecewise-defined functions (including absolute value).

The successful College Algebra student can:

2a. Recognize function families as they appear in equations and inequalities and choose an appropriate solution methodology for a particular equation or inequality and can communicate reasons for that choice.*

Sample Tasks:

- The student can summarize a solution strategy for a given problem verbally, without actually solving the problem.
- The student can solve an equation by factoring and explain the connection to the zero product property of real (complex) numbers.
- The student can explain the steps taken to construct a sign diagram and use a sign diagram to solve an inequality.
- The student can solve an equation involving piecewise-defined functions.

2b. Use correct, consistent, and coherent notation throughout the solution process to a given equation or inequality.*

2c. Distinguish between exact and approximate solutions and which solution methodologies result in which kind of solutions.*

Sample Tasks:

- The student lists the exact values of the irrational zeros of a quadratic function and uses decimal approximations to sketch the graph.
- The student recognizes the need to approximate the solutions to $2 - x = e^x$ and uses a graphing utility to do so.

2d. Demonstrate an understanding of the correspondence between the solution to an equation, the zero of a function, and the point of intersection of two curves.*

Sample Tasks:

- The student solves an equation algebraically and verifies the solution using a graphing utility.
- Given the graphs of two functions f and g, the student can approximate solutions to $f(x) = g(x)$.

2e. Solve for one variable in terms of another.*

Sample Tasks:

- The student can solve for y: $2y = x(y-2)$.
- The student can write an equation for the volume of a box as a function of the height given relationships between the length, width, and height of the box.

2f. Solve systems of equations using substitution and/or elimination.*

3. Equivalencies: Successful College Algebra students are proficient in creating equivalencies in order to simplify expressions, solve equations and inequalities, or take advantage of a common structure or form.

The successful College Algebra student can:

3a. Purposefully create equivalences and indicate where they are valid.*

Sample Tasks:

- To graph $f(x) = \frac{(x^2-4)}{(x-2)}$, the student simplifies to $f(x) = x+2$, and graphs $y = x+2$ with a hole at (2,4).
- To solve $log(x) + log(x-2) = 1$, a student solves $log(x^2-2x) = 1$ and knows this procedure may result in extraneous solutions.
- A student solves $|2x-3| + 3x = 2$ by rewriting the left hand side as a piecewise-defined function.

3b. Recognize opportunities to create equivalencies in order to simplify workflow.*

Sample Tasks:

- A student recognizes $y = 2\,(x-2)^3 + 1$ as being related to $y = x^3$ via linear transformations and exploits this in order to sketch the graph.
- A student rewrites $x^2 - 4x + 4y^2 = 0$ in standard form in order to graph.

4. Modeling with Functions: Successful College Algebra students should have experience in using and creating mathematics which model a wide range of phenomena.

The successful College Algebra student can:

4a. Interpret the function correspondence and behavior of a given model in terms of the context of the model.*

Sample Tasks:

- Given a ‘doomsday’ model for population, the student can interpret the vertical asymptote as ‘doomsday.’
- Given a model that models the temperature of a cup of coffee, the student can determine the horizontal asymptote of the graph of the model and interpret it as the limiting temperature of the coffee.
- Given a model which models the height of a model rocket off the ground, the student can find and interpret the zeros of the model.

4b. Create linear models from data and interpret slope as a rate of change.*

Sample Tasks:

- The student can create the conversion between Fahrenheit and Celsius.
- The student can create a piecewise linear function which models the cost of a mobile phone data plan.
- The student can recognize linear trends in data and create a piecewise linear function to model behavior.

4c. Determine parameters of a model given the form of the model and data.*

Sample Tasks:

- The student can find and interpret the decay constant given the half-life of a radioactive element.
- Given the graph of what appears to be a parabola, the student can use knowledge of the vertex and intercepts to create a formula for the function in the form $f(x) = a(x-h)^2 + k$, or $f(x) = ax^2 + bx + c$, or $f(x) = a(x-r_1)(x-r_2)$.

4d. Determine a reasonable applied domain for the model as well as articulate the limitations of the model.*

Sample Tasks:

- Given the model: $h(t) = 640t - 32t^2$ gives height in feet of a model rocket off the ground t seconds after liftoff, a student determines a reasonable applied domain for the model can be obtained by solving $h(t) > 0$.

5. Appropriate Use of Technology: Successful College Algebra students are proficient at choosing and applying technology to assist in analyzing functions.

The successful College Algebra student can:

5a. Anticipate the output from a graphing utility and make adjustments, as needed, in order to efficiently use the technology to solve a problem.*

Sample Tasks:

- A student uses end behavior and a table of values to determine a reasonable window within which to locate the solution to an optimization problem.
- A student can use algebra and technology to produce a detailed graph of $f(x) = \frac{5(x^2-4)^{\frac{1}{2}}}{(x^2+1)}$.

5b. Use technology to verify solutions to equations and inequalities obtained algebraically.*

Sample Tasks:

- A student solves $x^2 - 3x > 1 + x$ and checks the reasonableness of the solution graphically.

5c. Use technology to obtain solutions to equations to equations and inequalities which are difficult to obtain algebraically and know the difference between approximate and exact solutions.*

Sample Tasks:

- A student decides to solve $e^x = 1 - x$ by graphing $Y1 = e^x$ and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.
- A student uses WolframAlpha to solve $e^x = 2 - x$, and, having no idea what $W(e^2)$ is, records the approximate answer of 0.44.

5d. Use technology and algebra in concert to locate and identify exact solutions.*

Sample Tasks:

- A student uses the Rational Zeros Theorem to help identify approximate zeros from a graphing utility, then uses the Factor Theorem to verify which real numbers are zeros.
- A student decides to solve $e^x = 1 - x$ by graphing $Y1 = e^x$ and $Y2 = 1 - x$ and observes the solution appears to be $x = 0$. The student then verifies the solution algebraically.

6. Reasons Mathematically: Successful college algebra students demonstrate a proficiency at reasoning mathematically.

The successful College Algebra student can:

6a. Recognize when a result (theorem) is applicable and use the result to make sound logical conclusions and provide counter-examples to conjectures.*

Sample Tasks:

- A student uses the factor theorem to partially factor a higher degree polynomial in order to help find the exact values of the remaining zeros.
- A student recognizes the equation $e^{2x} - e^{x} = 5$ as quadratic in form and uses the quadratic formula to solve for e^{x}.
- A student uses Descartes' Rule of Signs to prove $f(x) = x^3 + x - 3$ has no negative real zeros.
- A student realizes the vertex formula cannot be used to find the extreme values of a third degree polynomial.
- A student can find pairs of real numbers where $(a+b)^2$ and a^2+b^2 are different.

OHIO TRANSFER MODULE MATHEMATICS, STATISTICS, AND LOGIC: TMM001 COLLEGE ALGEBRA COURSE REVISION FACULTY PARTICIPANTS
Fall 2015

Ricardo Moena (Panel Lead)	University of Cincinnati
Carl Stitz (Subgroup Lead Expert)	Lakeland Community College
Lee Wayand (Subgroup Lead Expert)	Columbus State Community College
Jeffery Zeager (Subgroup Lead Expert)	Lorain County Community College
Pramod Kanwar (Subgroup Lead Expert)	Ohio University
Erin Susick	Belmont College
David Meel	Bowling Green State University
Pat Kan	Cleveland State University
Amanda Hanley	Cuyahoga Community College
Terry Calvert	Edison State Community College
Cheryl Mansky	Hocking College
Oana Mocioalca	Kent State University
Andrew Tonge	Kent State University
Patrick Dowling	Miami University
David Stott	Sinclair Community College
Karl Hess	Sinclair Community College
Matt Robinson	Stark State College
Michelle Younker	Terra Community College
Bill Husen	The Ohio State University
Irina Chernikova	The University of Akron
Guang-Hwa (Andy) Chang	Youngstown State University