Probability and Statistics

Quiz #7

Show all work in order to receive full credit.

1) Assume $A$ and $B$ are disjoint and that $P(A) + P(B) = 1$. Then if $C$ is any other event such that $P(C) \neq 0$ or $1$, then Baye’s theorem says

$$P(A|C) = \frac{P(C|A) \times P(A)}{P(C|A) \times P(A) + P(C|B) \times P(B)}$$

2a) The binomial coefficient $n \choose k$ is given by the formula

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

b) Evaluate $\binom{5}{2}$.

$$\binom{5}{2} = 10.$$ 

3a) If $X$ is binomially distributed with parameters $n$ and $p$, then the mean of $X$ is $np$ and the standard deviation of $X$ is $\sqrt{np(1-p)}$.

b) Assume that $\hat{p} = \frac{X}{n}$ where $X$ is distributed binomially with parameters $n$ and $p$. Find the variance $\hat{p}$ showing every step as you proceed.

$$\sigma_{\hat{p}}^2 = \frac{\sigma_X^2}{n} = \frac{1}{n^2} np(1-p) = \frac{n(1-p)}{n}.$$ 

4) Assume that $\mu_{X_i} = \mu$ and $\sigma_{X_i} = \sigma$ for all $i \in \{0,1,2,..,n\}$ where $\mu$ and $\sigma$ are the population mean and standard deviation respectively. What is $\mu_{\bar{X}}$ and $\sigma_{\bar{X}}$, where $\bar{X}$ is the sample mean (you do not need to show steps)?

$$\mu_{\bar{X}} = \mu$$ and $$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}.$$ 

5) The central limit theorem says that under certain conditions, the sample mean is approximately (put a distribution here with mean and standard deviation) $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ if $n$ is sufficiently large.