1. **Problem 1.** Compute $1_{[0,1]} * 1_{[0,1]}(x), x \in \mathbb{R}$.

2. **Problem 2.** Let $f$ be a periodic (with period 1) integrable function on the real line, satisfying $\int_0^1 f(t)dt = 0$. Prove that $\forall \alpha \in (0,1)$ there exists a set $A_\alpha \subset \mathbb{R}$ such that $\int_{A_\alpha} f(t)dt = 0$, and $m(A_\alpha) = \alpha$.

   **Hint.** Where is Fubini? (periodicity is important!)

3. **Problem 3.** Prove that $(f * g) * h = f * (g * h)$, provided $f, g, h$ are integrable on the real line.

4. **Problem 4.** Put $\phi(t) = 1 - \cos t$ if $0 \leq t \leq 2\pi$, $\phi(t) = 0$ for all other real $t$. For $-\infty < x < \infty$, define

   $$ f(x) = 1, \quad g(x) = \phi'(x), \quad h(x) = \int_{-\infty}^x \phi(t)dt. $$

   Verify the following statements about convolutions of these functions:

   (i) $f * g(x) = 0$ for all $x$.

   (ii) $g * h(x) = \phi * \phi(x) > 0$ on $(0, 4\pi)$.

   (iii) Therefore $(f * g) * h = 0$, whereas $f * (g * h)$ is a positive constant. But convolution is supposedly associative, by Fubini Theorem (see Problem 2). What went wrong?

5. **Problem 5.**

   Use Fubini Theorem and the relation

   $$ \frac{1}{x} = \int_0^\infty e^{-xt}dt, \quad x > 0, $$

   to prove that

   $$ \lim_{A \to \infty} \int_0^A \frac{\sin x}{x}dx = \frac{\pi}{2}. $$

6. **Problem 6.** Suppose $A$ is a measurable subset of the real line, having finite positive measure. Show that the convolution $1_A * 1_A$ is continuous and not identically 0. Use this to prove that $A + A$ contains a segment.