Differential Geometry, Spring 2012.

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Assignment 11.

1. **Problem 1.**

   a) Find the normal curvature of the paraboloid $z = \frac{1}{2}(ax^2 + by^2)$ at the point $(0, 0)$ in the direction $(dx : dy)$.

   b) Prove that for any parametrization of the plane the second quadratic form of the plane is zero; the second quadratic form of the sphere is proportional to the first one.

   c) Find the asymptotic curves of the surface $z = \frac{x}{y} + \frac{y}{x}$.

   d) Find the asymptotic curves of the catenoid

   $$\boldsymbol{r}(u, v) = (\cosh u \cos v, \cosh u \sin v, u).$$

   e) Prove that on the helicoid

   $$\boldsymbol{r}(u, v) = (u \cos v, u \sin v, bv), \qquad b > 0,$$

   there are two families of the asymptotic curves: the straight lines and helixes.

2. **Problem 2.**

   a) Find the main curvatures of the paraboloid $z = a\,(x^2 + y^2)$ at the point $(0, 0, 0)$.

   b) Find the lines of curvature on the helicoid and the paraboloid $z = a\,xy$.

   c) Find the mean and the Gaussian curvature of the paraboloid $z = a\,xy$ at the point $x = y = 0$.

   d) Prove that the mean curvature of the helicoid is zero.

   e) Prove that the mean curvature of the catenoid $z = a \operatorname{arcosh} \frac{\sqrt{x^2+y^2}}{a}$ is zero.

3. **Problem 3.**

   Prove or give a counterexample:

   a) If a curve is both asymptotic curve and a line of curvature, then it must be planar.

   b) If a curve is planar and an asymptotic curve, then it must be a line.

4. **Problem 4.**

   a) Show that at a hyperbolic point, the principal directions bisect the asymptotic directions

   b) Show that if a surface is tangent to a plane along a curve, then the points of this curve are either parabolic or planar.