Differential Geometry, Spring 2012.

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Assignment 2.

1. **Problem 1.**

   a) Find the arc length of *astroid* $x(t) = a\cos^3 t,\ y(t) = a\sin^3 t.$

   b) Find the curvature of the curve

   $$x(t) = t - \sin t, \qquad y(t) = 1 - \cos t, \qquad z(t) = 4\sin\frac{t}{2}.$$

   c) Write the equation of the tangent line to the curve given by the equation

   $$x^2 + y^2 + z^2 = 1, \qquad x^2 + y^2 = x,$$

   at the point $(0, 0, 1)$.

   d) Find the curvature of the line given implicitly by the equations

   $$x + \sinh x = \sin y + y, \qquad z + e^z = x + \ln(1 + x) + 1$$

   at the point $(0, 0, 0)$

   e) Find the curvature and the torsion of the curve

   $$x = a\ \cosh t, \qquad y = a\ \sinh t, \qquad z = a\,t.$$

   f) Find the equation of the parabola of the form $y = x^2 + ax + b$ that is tangent to the circle $x^2 + y^2 = 2$ at the point $(1, 1)$.

2. **Problem 2.** One often gives a plane curve in polar coordinates by $\rho = \rho(\theta)$, $\theta \in (a, b)$.

   a) Show that the arc length is

   $$\int_a^b \sqrt{\rho^2 + (\rho')^2}d\theta,$$

   where the prime denotes the derivative relative to θ.

   b) Show that the curvature is

   $$k(\theta) = \frac{2(\rho')^2 - \rho\rho'' + \rho^2}{((\rho')^2 + \rho^2)^{3/2}}.$$

3. **Problem 3.**

   Show that

$$\boldsymbol{\alpha}(s) = \Big(\frac{(1+s)^{3/2}}{3}, \frac{(1-s)^{3/2}}{3}, \frac{s}{\sqrt{2}}\Big)$$

   is a unit speed curve and compute its Frenet-Serret apparatus.

4. **Problem 4.**

   Assume that all normals of a regular curve pass through a fixed point. Prove that the trace of the curve is contained in a circle.

5. **Problem** 5^* **.** A unit speed curve $\boldsymbol{\alpha}(s)$ with $k \neq 0$ is a helix if and only if there is a constant c such that $\tau = c\,k$.