Complex Analysis, Spring 2011.

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Assignment X.

1. **Problem 1.**

    a) Map the inside of the circle $r = a \cos \phi$, $a > 0$, on the inner part of cardioid $\rho = (1 + \cos \theta)/2$;

    b) Map the inside of the circle $r = a \cos \phi$, $a > 0$, on the inner part of the right branch of the lemniscate $\rho = \sqrt{\cos(2\theta)}$;

    c) Map $|z| < 1$ onto the inner part of the cardioid $\rho = A(1 + \cos \theta)$, $A > 0$, in such a way that $w(0) = A/8$, $w'(0) > 0$.

2. **Problem 2.**

    a) Find the image of $|z| < 1$ under $w = R(z + mz^2)$, $R > 0$, $0 \le m \le 1/2$. Find the image of the polar net of z-plane;

    b) Find the image of the semicircle $|z| < 1$, $\Re z > 0$, under $w = z + z^2$;

    c) Find the image of $|z| < 1$ under $w = R(z + z^n/n)$, $R > 0$, $n \in \mathbf{N}$;

    d) Find the image of $|z| > 1$ under $w = R(z + 1/(nz^n))$, $R > 0$, $n \in \mathbf{N}$.

3. **Problem 3.**

    a) Map $0 < \arg z < \pi\beta$, $0 < \beta < 2$, with a cut along the arc of a circle $|z| = 1$ from $z = 1$ to $z = e^{i\alpha}$, $0 < \alpha < \pi$ onto the upper half-plane;

    b) Map $|z| < 1$, with a cut along the radius $[-1, 0]$ and a segment $[a, 1]$, $0 < a < 1$, onto the upper half-plane;

4. **Problem 4.**

    a) Find the image of $|z| < 1$ under $w = z/(z^2 + 1)$;

    b) Find the image of the sector $-\pi/n < \arg z < \pi/n$, $|z| < 1$, under $w = z/(1 + z^n)^{2/n}$, $w(z) > 0$ for $z > 0$;

    Hint: Consider $z_1 = z^n$, $z_2 = z_1/(1 + z_1)^2$, $z_3 = z_2^{1/n}$.

    c)* Map the part of the z-plane to the left of the right-hand branch of the hyperbola $x^2 - y^2 = 1$ on a half-plane;

    Hint: Consider on one side the mapping of the upper half of the region by $w = z^2$, on the other side the mapping of a quadrant by $w = z^3 - 3z$.