1. **Problem 1.**
   a) Compute
   \[ \int_{|z|=r} x\,dz \]
   by use of a parameter.
   b) Compute the integral in a) by observing that
   \( x = (z + \bar{z})/2 = (z + r^2/z)/2 \) on the circle.
   c) Compute
   \[ \int_{|z|=2} \frac{dz}{z^2 - 1} \]
   for the positive sense of the circle.
   d) Compute
   \[ \int_{|z|=1} |z − 1||dz| \]

2. **Problem 2.**
   a) Suppose that \( f(z) \) is analytic on a closed curve \( \gamma \) (i.e., \( f \) is analytic in a region that contains \( \gamma \)). Show that
   \[ \int_{\gamma} \overline{f(z)} f'(z)\,dz \]
   is purely imaginary. (The continuity of \( f'(z) \) is taken for granted).
   b) Assume that \( f(z) \) is analytic and satisfies the inequality \(|f(z) - 1| < 1\) in a region \( \Omega \). Show that
   \[ \int_{\gamma} \frac{f'(z)}{f(z)}\,dz = 0 \]
   for every closed curve in \( \Omega \). (The continuity of \( f'(z) \) is taken for granted).
   c) If \( P(z) \) is a polynomial and \( C \) denotes the circle \(|z − a| = R\), what is the value of
   \[ \int_{C} P(z)\,dz? \]
   **Hint:** \(-2\pi i R^2 P'(a)\).
3. Problem 3.

a) Let \( f(z) \) be continuous in the domain \( \Omega = \{ z \in \mathbb{C} : |z| \geq R_0, \mathfrak{R}z \geq a \} \), \((a \text{ is a fixed real number})\), and in this domain \( f(z) \to 0 \) uniformly on arcs \( \{ z \in \Omega, |z| = R \} \), as \( z \to \infty \). Prove that for any \( m > 0 \),

\[
\lim_{R \to \infty} \int_{\Gamma_R} e^{imz} f(z) \, dz = 0,
\]

where \( \Gamma_R \) is an arc of the circle \( |z| = R \) in \( z \in \Omega \).

**Hint:** On the half-arc \( |z| = R, \mathfrak{R}z > 0 \), use \( \sin \theta \geq 2\theta / \pi, 0 \leq \theta \leq \pi/2 \). In the case \( a < 0 \) use the fact that the length of the corresponding arcs tends to \( |a| \) as \( R \to \infty \).

b) Let \( f(z) \) be continuous in the half-plane \( \mathfrak{R}z \geq \sigma \), \((\sigma \text{ is a fixed real number})\), and in this half-plane \( f(z) \to 0 \) uniformly on arcs \( \{ \mathfrak{R}z \geq \sigma, |z| = R \} \), as \( z \to \infty \). Prove that for any \( t < 0 \),

\[
\lim_{R \to \infty} \int_{\Gamma_R} e^{zt} f(z) \, dz = 0,
\]

where \( \Gamma_R \) is an arc of the circle \( |z| = R, \mathfrak{R}z \geq \sigma \). If \( f(z) \) is continuous in \( \mathfrak{R}z \leq \sigma \), then the statement is true, provided \( t > 0 \), and \( \Gamma_R \) is an arc of the circle \( |z| = R, \mathfrak{R}z \leq \sigma \).