Quiz 4 - Take Home

Directions: This quiz is Due by the start of class on Wednesday February 3rd. To receive maximal credit, show all your work.

1) Calculate

\[ \int_{-\infty}^{\infty} xe^{x^3/3} \, dx \]

Using the substitution \( u = x \), \( du = dx \), and \( dv = e^{x^3/3} \, dx \), we have \( v = e^{x^3/3} \). Applying integration by parts,

\[ \lim_{t \to \infty} \int_{-t}^{t} xe^{x^3/3} \, dx = \lim_{t \to \infty} \left[ x e^{x^3/3} \right]_{-t}^{t} - \int_{-t}^{t} e^{x^3/3} \, dx \]

Applying l'Hopital's Rule:

\[ \lim_{t \to \infty} -3te^{t^3/3} = \lim_{t \to \infty} -3t \frac{e^{t^3/3}}{t^2} = 0 \]

\[ \lim_{t \to -\infty} -3te^{t^3/3} = \frac{9}{\infty} = 0 \]

\[ \lim_{t \to \infty} -3te^{t^3/3} = \frac{9}{\infty} = 0 \]

Therefore,

\[ \int_{-\infty}^{\infty} xe^{x^3/3} \, dx = 9e^2 \]

2) Calculate

\[ \int_{0}^{3} \frac{1}{\sqrt{x}} \, dx \]

\( \frac{1}{\sqrt{x}} \) has an infinite discontinuity at \( x = 0 \).

\[ \int_{0}^{3} \frac{3}{\sqrt{x}} \, dx = \lim_{t \to 0^+} \int_{t}^{3} \frac{1}{\sqrt{x}} \, dx = \lim_{t \to 0^+} 2\sqrt{x} \bigg|_{x=t}^{x=3} \]

\[ = \lim_{t \to 0^+} 2\sqrt{3} - 2\sqrt{t} \]

\[ = \sqrt{3} - 0 = \sqrt{3} \]