Calculus I - Test 2 Review - Spring 2019 - Dr. Smithies

Test Two is Friday March 1st. It will cover sections 2.6, 3.1, 3.3, 3.4 and 3.5 of our book. The assigned homework for this material was Short versions of the webassign section assignement. This is basically the problems with solutions in the back of your book from the problem sets:

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These questions and the worked examples from class and the book could be on the test.

Main Topics

- Make sure you are quick and accurate in calculating derivatives. This includes the Power Rule, Linearity, Product, Quotient and Chain Rules. Specifically,

<table>
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<th>Function</th>
<th>Derivative</th>
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<th>Derivative</th>
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<tr>
<td>( y = x^N )</td>
<td>( y' = N x^{N-1} )</td>
<td>( y = c f(x) )</td>
<td>( y' = c f'(x) )</td>
</tr>
<tr>
<td>( y = [f(x) + g(x)] )</td>
<td>( y' = f'(x) + g'(x) )</td>
<td>( y = u(x)v(x) )</td>
<td>( y' = u'(x)v(x) + v'(x)u(x) )</td>
</tr>
<tr>
<td>( y = \frac{N(x)}{D(x)} )</td>
<td>( y' = \frac{N'(x)D(x) - D'(x)N(x)}{(D(x))^2} )</td>
<td>( y = f \circ g(x) = f(g(x)) )</td>
<td>( y' = f'(g(x))g'(x) )</td>
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(2.6) Implicit Differentiation When the variables \( x \) and \( y \) are related by an equation like \( x^3 + y^3 = 6xy \) where it is not easy to express \( y \) as an explicit function of \( x \), we can still find the rate of change in \( y \) with respect to \( x \), \( y' = \frac{dy}{dx} \). The idea is recognize that \( y = y(x) \), that \( y \) is implicitly defined by \( x \). We take the derivative with respect to \( x \) of both sides of the equation using the chain rule:

\[
\frac{d}{dx} [f(y)] = \frac{d}{dy} [f(y)] \frac{dy}{dx} = \frac{d}{dy} [f(y)] y'.
\]

That is, the derivative of a function of \( y \) is the usual derivative with respect to \( y \) times the chain rule factor \( y' \). Using this we can solve for \( y' \) without ever solving for \( y \) in terms of \( x \).

(3.1) Extrema Let \( f(x) \) be a function defined on the interval \([a, b]\). We say \( f \) has a absolute maximum of \( f(c) \) at \( c \) in \([a, b]\) if \( f(c) \geq f(x) \) for all \( a \leq x \leq b \). We say \( f \) has a local maximum of \( f(c) \) at \( c \) if there is an interval \([w, z]\) in \([a, b]\) such that \( f(c) \geq f(x) \) for all \( w \leq x \leq z \). Similarly, we define a absolute minimum and a local minimum. Extrema means maxima or minima.
(3.1) Extrema Let $f(x)$ be a function defined on the interval $[a, b]$. The critical values of $f(x)$ on $[a, b]$ are the places where the derivative doesn’t exist or is zero. This always include the endpoints $a$ and $b$. To find the extrema of $f(x)$ on $[a, b]$

(i) Calculate $f'(x)$ and solve for where the derivative is zero or does not exist.

(ii) List these critical values in increasing order $a = c_1 \leq c_2 \leq \cdots \leq c_n = b$. Plug the critical values into the function to get $f(c_1), f(c_2), \cdots, f(c_n)$.

(iii) Classify each $c_i$ by comparing $f(c_i)$ to $f(c_{i-1})$ and $f(c_{i+1})$.

Remark: It is useful to recall that a function $f(x)$ is increasing if $x_1 < x_2$ implies that $f(x_1) < f(x_2)$. That is, plugging in a greater $x$ value, yields a greater output. Similarly $f(x)$ is decreasing if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$. That is, plugging in a greater $x$ value, yields a smaller output. A function is increasing whenever its derivative is positive and decreasing whenever its derivative is negative.

(3.3, 3.4) Graphing We learned how to sketch the graph of a function $f(x)$ by analyzing the following features when they are reasonably easy to find.

(a) Domain and if easy to see, Range;
(b) $x$ and $y$ intercepts;
(c) symmetry (odd or even) and periodicity (trig functions);
(d) asymptotes and holes;
(e) monotonicity (increasing or decreasing) and maxima and minima;
(f) concavity and inflection points.
(g) Shape of graph (combine (e) and (f)).

It is useful to recall that a function $f(x)$ is increasing whenever its derivative is positive and decreasing whenever its derivative is negative. Also a function $f(x)$ is concave upwards whenever its second derivative is positive and concave downwards whenever its second derivative is negative.

Remark: It is also useful to remember that the derivative does not always exist. The derivative is defined as the limit of the difference quotient and limits can fail to exists. In particular, the derivative does not exist at a place where the function is not continuous, or has a corner or has a vertical tangent. The derivative also does not exist at endpoints of the domain because the difference quotient does not have both both a right and left limit at these points.

(3.5) Optimization Optimization is finding the maximum or minimum values a function $f(x)$ will achieve. This is the same as section 3.1, in the context of a story problem. The method is

(1) Draw a diagram and define variables $x, x_1, \cdots x_n$.
(2) Describe the function to be optimized as $Q(x, x_1, \cdots, x_n)$.
(3) Use the conditions of the problems to express the variables $x_1, \cdots, x_n$ in terms of $x$ and to identify the bounds on $x$ such as $a \leq x \leq b$.
(4) From step (3) substitute out the variables $x_1, \cdots, x_n$. The problem is now optimize $Q(x)$ on $[a, b]$. This is done exactly as in section 3.1.