Formula 1: \[ S = 1 + 2 + 3 + \ldots + N \]
\[ S = \frac{N(N+1)}{2} \]

Example: \[ 1 + 2 + 3 + \ldots + 75 \]
Total: \[ \frac{75 \cdot 76}{2} = 2850 \]

Example: \[ 5 + 10 + 15 + 20 + \ldots + 100 \]
\[ \frac{5 \cdot 20 \cdot 21}{2} = 1050 \]

Calculate \[ 4 + 5 + 6 + \ldots + 25 \]
\[ 1 + 2 + 3 + 4 + 5 + 6 + \ldots + 25 \text{ AND } 1 + 2 + 3 = 6 \]

So add 1 through 25 and subtract 6 from answer.
\[ 1 + 2 + 3 + 4 + \ldots + 25 = \frac{25 \cdot 26}{2} \]
\[ 4 + 5 + 6 + \ldots + 25 = \frac{25 \cdot 26}{2} - 6 = 319 \]
We want: \( T = 20 + 25 + 30 + 35 + \ldots + 100 = ? \)

We know how to get:

\[ S = 5 + 10 + 15 + 20 + 25 + \ldots + 100 \]

\[ S = T + 5 + 10 + 15 \]

\[ T = S - 30 \]

\[ S = 5(1 + 2 + 3 + 4 + \ldots + 20) \]

\[ S = \frac{5(20)(21)}{2} \]

SO \( T = S - 30 = 1050 - 30 = 1020 \)

What is \( N \)?

\[ S = 1 + 2 + 3 + \ldots + N = 3240 \]

\[ S = 3240 \]

, and \[ S = \frac{N(N + 1)}{2} \]

So we must have \[ \frac{N(N + 1)}{2} = 3240 \]

Multiply by 2: get \[ N(N + 1) = 6480 \]

Logic says: \( N^2 < N(N + 1) < (N + 1)^2 \)

So \( N^2 < 6480 < (N + 1)^2 \)
Take the square root:

\[ N < \sqrt{6480} < N + 1 \]

that is \[ N < 80.5 < (N+1) \]

Using logic, \[ N = 80 \]

Check it!!! \[ 1 + 2 + \ldots + 80 = \frac{80 \cdot 81}{2} = 3240 \], as expected.

Want \[ N \]

Given \[ S \], find the stopping point

" \[ N \] " by \[ N < \sqrt{2S} \]. We know \[ S = \frac{N(N+1)}{2} \], so we must have \[ 2S = N(N+1) \].

If we take \[ N < \sqrt{2S} \], find the square root and drop the decimal to get \[ N \].
What is $N$?

$S = 1 + 2 + 3 + \ldots + N = 5050$

From the formula, we know $\frac{N(N+1)}{2} = 5050$

Multiply both sides by 2: get $N(N+1) = 10100$

$N(N+1)$ is a little more than $N \times N$, so the number $N$ is a bit less than the square root of 10100. (as an example, Notice 6*7 is a bit more than 6*6)

$N^2 < N(N+1) = 10100$

$N < \sqrt{10100}$ (equals about) 100.5

So $N = 100$. Remember to check!!!

Variations:

$7 + 14 + 21 + 28 + \ldots + 140 = 7(1 + 2 + 3 + \ldots + 20)$
\[
\frac{7\cdot20\cdot21}{2} = 287
\]

\[25+26+27+\ldots+50\]

\[\sum (1+2+3+\ldots+50) - (1+2+3+\ldots+24)\]

\[
\frac{50\cdot51}{2} - \frac{24\cdot25}{2} = \frac{50\cdot51}{2} - \frac{24\cdot25}{2} = 975
\]

\[1+2+3+\ldots+N = 5151\]  \text{what is } N?  

\[5151 = \frac{N\cdot(N+1)}{2}\]

\[10302 = N\cdot(N+1) \approx N^2\]

\[101.5 \approx \sqrt{10302} \approx N\]

So \[N = 101\] (always check!!) \[
\frac{101\cdot102}{2} = 5151
\]