1. What is the sum $1 + 2 + 3 + \cdots + 201$ of the counting numbers from 1 to 201? 
  
**B)** $\frac{(201)(202)}{2}$. This is $1 + 2 + 3 + \cdots + N = \frac{(N)(N+1)}{2}$ for $N = 201$. 

2. What is the sum $222 + 220 + 218 + \cdots + 2$ of the even numbers from 222 down to 2? 
  
**D)** $(111)(112)$. This is $2(1 + 2 + 3 + \cdots + 111) = 2\left(\frac{(111)(112)}{2}\right)$ 

3. What is the sum $5 + 10 + 15 + \cdots + 1000$ of the multiples of five up to 1000? 
  
**C)** $5(100)(201)$. The sum is $5(1 + 2 + 3 + \cdots + 200) = 5\left(\frac{200(201)}{2}\right) = 5(100)(201)$ 

4. A student has 8 math books, 7 physics books and 5 engineering books. How many different ways can the student choose one math book and either a physics or engineering book for his bookbag? 
  
**B)** $8(7 + 5)$. The task is (Pick a math book) AND [(Pick a physics book) or (pick an engineering book.)]. There are $8 \times [7 + 5]$ ways to do this. 

5. If the sum of the the first $N$ multiples of ten, $10 + 20 + 30 + \cdots + 10N$, is 62,160 then what is the last term, $10N$? 
  
**A)** 1110. The sum $S$ is $S = 10\frac{N(N+1)}{2} = 5(N)(N+1)$. The given total divided by 5 is approximately $N^2$. So $N \sim \sqrt{\frac{62,160}{5}} = \sqrt{12,432} = 111.49\cdots$. So $N = 111$. We try this and get $10\frac{111(112)}{2} = 62,160$. The last term in the sum is $10N = 1110$. 

6. A restaurant has 8 appetizers, 3 soups, 2 salads, and 7 Dinners on special. Special meal number one consists of two different appetizers and either a soup or salad. How many ways are there to create Special meal number one? 
  
**D)** $(8)(7)(5)$. The task of creating a special meal breaks down as (Pick an appetizer) AND (Pick a different appetizer) AND [(Pick a soup) OR (Pick a salad)]. There are $8 \times [7 + 5]$ ways to do this. 

7. If $1 + 2 + 3 + \cdots + N = 3081$, then what is $N$? 
  
**A)** 78. We know that $1 + 2 + 3 + \cdots + N = \frac{(N)(N+1)}{2} = 3081$. So $N(N + 1) = 2(3081) = 6,162$. That is, 6,162 is approximately $N^2$. So, $N \sim \sqrt{6,162} = 78.49\cdots$. We check that $N = 78$ works since $\frac{(78)(79)}{2} = 3081.$
8. How many different ways can the group of letters \{A, B, C, D, E, F, G\} be arranged, in such a way that the vowels (A and E) are first?

   D) 240. The task breaks down as (Arrange 2 letters) AND (Arrange 5 letters). There 
   \((2!)(5!) = 2(120) = 240)\.

9. A license plate consists of 4 capital letters followed by 3 non-zero digits. How many license plates can be formed?

   A) \((26^4)(9^3)\). The task breaks down as
   (Pick a letter) AND (Pick a letter) AND (Pick a letter) AND (Pick a letter) AND
   (Pick a non-zero number) AND (Pick a non-zero number) AND (Pick a non-zero number).
   There are \((26)(26)(26)(26)(9)(9)(9) = (26^4)(9^3)\) ways to do this.

10. A password can use upper or lower case letters or digits or one of 8 special characters. How many 4 character passwords are possible?

   C) \(70^4\). There are 26 + 26 + 10 + 8 = 70 password characters. The task of making a
   4 character password can be completed \((70)(70)(70)(70) = 70^4\) ways.

11. A password can use upper or lower case letters or digits or one of 8 special characters. How many passwords are possible with 4 distinct characters?

   C) \((70)(69)(68)(67)\). This is the same as the previous problem except once you use
   a character, you cannot reuse it. So at each step there is one less possibility for the
   character you select.

12. A pairwise comparison election is settled by running each candidate head-to-head against all of his opponents. If an election has 25 candidates how many pairwise comparisons are needed to settle the election?

   A) 300. There are \(C(25, 2)\), 25 choose 2 pairs that can be made from the 25 candidates. That is, \(C(25, 2) = \frac{25!}{(25-2)!2!} = \frac{25!}{23!2!} = \frac{(25)(24)}{2} = (25)(12) = 300\). You could also
   get the total \(\frac{(25)(24)}{2}\) by noting that there are \(1 + 2 + \cdots + 24\) pairwise comparisons.
13. Jack has 5 pairs of pants, 10 T-shirts, 7 dress shirts and 3 neck ties. Jack always wears either a T-shirt or a dress shirt but never both, and he always wears a neck tie with a dress shirt. His outfit is always 1 pants and 1 top. How many different outfits are possible?

A) $5(10 + (7)(3))$. The task breaks down as (Pick pants) AND [(Pick a T-shirt) or (Pick a shirt) AND (Pick a tie)].

14. What is the sum $41 + 42 + 43 + \cdots + 110$ of the counting numbers from 41 to 110?

D) $\frac{(110)(111) - (41)(40)}{2}$. The answer is $T - S$ for $T = 1 + 2 + \cdots + 110 = \frac{(110)(111)}{2}$ and $S = 1 + 2 + \cdots + 40 = \frac{(40)(41)}{2}$.

15. A pairwise comparison election is settled by running each candidate head-to-head against all of his opponents. If an election requires 3,916 pairwise comparisons to settle, how many candidates were in the election?

C) 89. Let $N$ be the number of candidates. We are given $N$ choose 2, $C(N, 2)$ is 3,916. That is $\frac{(N)(N-1)}{2} = 3,916$. So, $N(N - 1) = 7,832$. So, $\sqrt{7,832} = 88.49$ is a little less than $N$. So $N = 89$. We check this $C(89, 2) = \frac{(89)(88)}{2} = 3,916$.

16. What is the sum $2 + 4 + 6 + \cdots + 1020$ of the even numbers up to 1020?

A) 260,610. This is $2(1 + 2 + \cdots + 510) = 2 \frac{(510)(511)}{2} = (510)(511) = 260,610$.

17. A pairwise comparison election is settled by running each candidate head-to-head against all of his opponents. If an election requires 55 pairwise comparisons to settle, how many candidates were in the election?

C) 11.

Let $N$ be the number of candidates. There are $1 + 2 + 3 + \cdots + N - 1$ pairwise comparisons, since candidates do not run against themselves. That is, $\frac{(N-1)(N)}{2} = 55$. This says $(N - 1)(N) = 110$. So, $\sqrt{110} = 10.48$ is a little less than $N$. Try $N = 11$. With 11 candidates there are $1 + 2 + 3 + \cdots + 10 = \frac{10(11)}{2} = 55$ pairwise comparisons.
18. A restaurant has 8 appetizers, 3 soups, 2 salads, and 7 Dinishs on special. Special meal number one consists of two (not necessarily different) appetizers and either a soup or salad. How many ways are there to create Special meal number one?

B) \((8)(8)(5)\).

The task of creating a special meal breaks down as

(Pick an appetizer) AND (Pick an appetizer) AND [(Pick a soup) OR (Pick a salad)].

There are \(8 \times 8 \times [3 + 2]\) ways to do this.

19. What is \(7! - 3!\), 7 factorial minus 3 factorial?

B) 5034. \(7! = (7)(6)(5)(4)(3)(2)(1) = 5040\) and \(3! = (3)(2)(1) = 6\). The difference is 5034.

20. A password character must be either a lower case letter, an upper case letter, a digit, or one of 8 special characters. How many 5 character passwords start with 3 digits?

B) \(10^3 \times 70^2\).

The task breaks down as (Pick a number) AND (Pick a number) AND (Pick a number) AND (Pick a character) AND (Pick a character). There are \(26 + 26 + 10 + 8 = 70\) characters. So there are \((10)(10)(70)(70) = (10^3)(70^2)\) ways to do this.