Chapter 2 – weighted voting

Weighted Voting Only Vote on yes/no motion
Players have multiple votes

Form of weighted voting system is
\[ [q: \frac{w_1, w_2, \ldots, w_n}{}] \]
Convention: \( w_1 \geq w_2 \geq \cdots \geq w_n \). Player \( k \) has \( w_k \) votes.

Total Votes \( V = \frac{w_1 + w_2 + \cdots + w_n}{2} \)

Valid Quota \( q \) satisfies \( \frac{V}{2} < q \leq V \)
Example of Notation for Weighted Voting System

[10: 8, 4, 3, 2]

q = 10 = quota,
q or more Yes votes are needed for a motion to pass

Player 1 has 8 votes,
Player 2 has 4 votes,
Player 3 has 3 votes,
Player 4 has 2 votes

Total votes $V = 8 + 4 + 3 + 2 = 17$. 
Restriction on valid quota:
\[V = \text{total number of votes must have } \frac{V}{2} < q \leq V\]

If \( q \leq \frac{V}{2} \) you get ANARCHY – both Yes and No can meet the quota. Motion can both pass and fail

If \( q > V \) you get GRIDLOCK – can NEVER meet quota

Winning coalitions = Group of players with enough votes to meet or exceed the quota.

Losing coalition = Group of player whose votes total less than the quota.
Example of weighted Voting System [10: 8, 4, 3, 2]

Winning coalitions (Value):

4 member: \( \{P_1, P_2, P_3, P_4\} \) (17),
3 member: \( \{P_1, P_2, P_3\} \) (15), \( \{P_1, P_2, P_4\} \) (14), \( \{P_1, P_3, P_4\} \) (13),
2 member: \( \{P_1, P_2\} \) (12), \( \{P_1, P_3\} \) (11), \( \{P_1, P_4\} \) (10)

Notice Player 1 is in every winning coalition. Any Player that is in every winning coalition is said to have VETO POWER.

This happens when the sum of all other players is less than q. A motion will not pass unless the players with veto power all vote yes.
Player 1 is a **dictator** if Player 1 has enough votes to meet the quota alone.
Example: [5: 6, 2, 1]
Here a motion passes if Player 1 votes yes and fails if Player 1 votes no.

Players 2 and 3 are **dummies** they are never needed to form a winning coalition.
Example: U.S. Senate. A simple majority or 51 of the 100 votes are needed for a motion to pass. The number of party members are Republican = 49, Democrat = 48, Independents = 3

Assume each party votes as a block. This is the weighted voting system [51: 49,48,3]

Find all winning coalitions (value of coalition):
{R,D,I}(100), {R,D}(97), {R,I}(52), {D,I}(51)

Every group, R,D and I are in 3 of the 4 winning coalitions. Each has 33.3% of the power
Now assume the independents do not vote as a block. The weighted voting system is 
[51:49,48,1,1,1]
Call the players R, D, a, b, and c. Their votes are 
R = 49, D = 48, a = 1, b = 1, c = 1.
Winning coalitions = groups total 51 or more.

\{R, D, a, b, c\}(100) = \text{Grand coalition = unanimous}
\{R, D, a, b\}(99) \quad \{R, D, a\}(98) \quad \{R, D, b\}(98) \quad \{R, D, c\}(98)
\{R, D, a, c\}(99) \quad \{R, a, b\}(51) \quad \{R, a, c\}(51) \quad \{R, c, b\}(51)
\{R, D, b, c\}(99) \quad \{R, D\}(97)
\{R, a, b, c\}(52)
\{D, a, b, c\}(51)
We had R = D = I = 33.3\% \text{ of power when } I = \{a, b, c\} \text{ vote same. Now R has more power than D. D has more than a, b, or c. The independents all have the same power.}