Discrete Mathematics
Fall 2007

Hw9 - Answers and Solutions

Section 8.3

9 The characteristic polynomial here is $t^2 - 7t + 10 = 0$ with the roots $t = 5$ and $t = 2$. Hence the solution is of the form

$$b_k = c_1 5^k + c_2 2^k.$$

Plugging in the initial data we get

$$\begin{array}{rcccl} b_0 & = & c_1 + c_2 & = & 2 \\ b_1 & = & 5c_1 + 2c_2 & = & 2 \end{array}$$

Solving for c_1 and c_2 we get $c_1 = -2/3$, $c_2 = 8/3$ and

$$b_k = -\frac{2}{3}5^k + \frac{8}{3}2^k$$

15 The characteristic polynomial here is $x^2 - 6x + 9 = 0$ with the double root $x = 3$. Hence the solution is of the form

$$t_k = c_1 3^k + c_2 \cdot k \cdot 3^k.$$

Plugging in the initial data we get

$$\begin{array}{rcccl} t_0 & = & c_1 & = & 1 \\ t_1 & = & 3c_1 + 3c_2 & = & 3 \end{array}$$

Solving for c_1 and c_2 we get $c_1 = 1$, $c_2 = 0$ and

$$t_k = 3^k.$$

17 We find $c_1 = 1$ and $c_2 = 2$. To find c_k we think about our last step. It is either one stair or two stairs. The number of ways to climb the stairs with last step being 1 stair is c_{k-1}, and he number of ways to climb the stairs with last step being 2 stairs is c_{k-2}. Hence $c_k = c_{k-1} + c_{k-2}$. This is exactly Fibonacci numbers, see the formula on page 494.

24 (a) Cross-multiply

$$\frac{\phi}{1} = \frac{1}{\phi - 1},$$

get $\phi^2 - \phi - 1 = 0$ (b)

$$\phi_1 = \frac{1+\sqrt{5}}{2} \quad , \phi_2 = \frac{1-\sqrt{5}}{2}$$

(c) Using the formula 8.3.8 from page 494 we get:

$$F_n = \frac{\phi_1^{n+1}}{\sqrt{5}} - \frac{\phi_2^{n+1}}{\sqrt{5}}$$