Fundamental Concepts of Algebra
Spring 2008

Homework 4
due Thursday, February 14

1. Simplify

 (a)

$$\left(\frac{4x}{2\sqrt{x}-\sqrt{y}} \div \frac{12x\sqrt{x}}{4x-y}\right) \div \frac{2x}{6x-3\sqrt{xy}}$$

 (b)

$$\left(\frac{1}{\sqrt{y}} - \frac{2}{\sqrt{x}+\sqrt{y}}\right)\left(\sqrt{x} - \frac{x+y}{\sqrt{x}-\sqrt{y}}\right)$$

2. Prove

 (a)

$$\sqrt{2+\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{2}}$$

 (b)

$$\sqrt{2-\sqrt{3}}(2+\sqrt{3})(\sqrt{6}-\sqrt{2}) = 2$$

3. Use graphs to solve the inequalities

 (a)

$$|3-x| \leq 2$$

 (b)

$$|2x-10| \geq |x-2|$$

4. Explain why $|f(x)| \leq a$ with $a < 0$ has no solutions.
5. The inequality $|f(x)| \leq |g(x)|$ is equivalent to $f^2(x) \leq g^2(x)$, which is in turn equivalent to

$$\left(f(x)-g(x)\right)\left(f(x)+g(x)\right) \leq 0$$

 (will discuss in class). Use this to solve

 (a)

$$|5x+3| < |2x-1|$$

 (b)

$$|x^2-7x+3| < |2x^2+5x-10|$$

6. Show that $|f(x)| < f(x)$ has no solutions. (Consider the two cases $f(x) > 0$ and $f(x) < 0$ to get rid of the absolute value.)
7. Show that $|f(x)| > f(x)$ is equivalent to $f(x) < 0$. (Consider the two cases $f(x) > 0$ and $f(x) < 0$ to get rid of the absolute value.)

8. The inequality $|f(x)| < g(x)$ is equivalent to the system

$$\begin{cases} f(x) < g(x) \\ f(x) > -g(x) \end{cases}$$

(will discuss in class). Use this to solve

(a)

$$|5x + 7| < 8x - 11$$

(b)

$$|x^2 + x - 7| < 4x - 7$$

9. The perimeter of a rectangle is 28 inches. We construct squares on two adjacent sides. The sum of areas of these squares is 116 square inches. Find the sides of the rectangle.