Fundamental Concepts of Algebra
Spring 2008

Homework 5
due Thursday, February 21

1. Solve the equations.
   (a)
   $$(|x-2|-14)(1-\sqrt{7-x})=0$$
   (b)
   $$(|x-2|-7)(1-\sqrt{7-|x|})=0$$
   (c)
   $$\frac{|2x-3|-1}{3x^3+5x-8}=0$$
   (d)
   $$|||x|-2|-2|=1$$
2. Prove the inequalities.
   (a)
   $$a^2d^2+\frac{b^2}{d^2}\geq 2ab$$
   (b)
   $$\frac{a^2+3}{\sqrt{a^2+2}}\geq 2$$
3. Let numbers a,b,c,d be greater or equal to zero. Prove the inequality
   $$\frac{a+b+c+d}{4}\geq\frac{\sqrt{ab}+\sqrt{cd}}{2}.$$
   Now use this inequality to prove
   $$\frac{a+b+c+d}{4}\geq\sqrt[4]{abcd}$$
4. Show that the product of any three consecutive integers is divisible by 6.
5. Show that the product of any five consecutive integers is divisible by 120.
6. Let $m,n\in\mathbb{Z}$. Show that $mn(m+n)$ is even.
7. Let a and b be digits. Given $7\mid a+b$ show that $7\mid\overline{aba}$.
8. Prove that
   $$4\mid\overline{a_d\dots a_1a_0}\quad\Leftrightarrow\quad 4\mid\overline{a_1a_0}.$$
9. Prove that
   $$9\mid\overline{a_d\dots a_1a_0}\quad\Leftrightarrow\quad 9\mid(a_d+a_{d-1}+\cdots+a_0).$$
10. For which values of p all three numbers p, $2p+1$, and $4p+1$ are prime?