

Notes, 1/14/08

We start with high-school algebra review. Here are formulas that we will use for expression manipulation:

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 \\(a - b)^2 &= a^2 - 2ab + b^2 \\a^2 - b^2 &= (a - b)(a + b) \\(a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\(a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\a^3 - b^3 &= (a - b)(a^2 + ab + b^2)\end{aligned}$$

All these formulas can be obtained by opening the parentheses and collecting the terms. Notice that you that most of the formulas on the list come in pairs where b is replaced with $-b$ and you don't need to do the work twice. For example, after you have shown that

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

you can replace b with $-b$ to get

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3.$$

Here are our practice problems. We took up first five of them, the rest is for you to finish (do not hand in).

1. Simplify

$$\frac{x^2 - 1}{x^3 + 1} \div \frac{x^2 - 2x + 1}{x^2 - x + 1}$$

2. Simplify

$$\frac{a^2 - 6a + 9}{1 - b^3} \div \frac{2a - 6}{b^2 - 1}$$

3. Prove

$$\left(\frac{4x^2}{2x - y} \div \frac{12x^3}{4x^2 - y^2} \right) \cdot \frac{2x^2}{6x^2 + 3xy} = \frac{2}{9}$$

4. Simplify

$$(a - 2b)(a + 2b)(a^2 + 4b^2)$$

5. Simplify

$$(2m^2 + 3n)(4m^4 - 6m^2n + 9n^2) - 28n^3$$

6. Let $f(x) = \frac{1}{1-x}$. Find $f(f(f(f(x))))$.

7. Simplify

(a) $4\sqrt{2} - \sqrt{18}$

(b) $5\sqrt{3} - \sqrt{300} - \sqrt{27}$

(c) $(\sqrt{a} + \sqrt{b})^2$

(d) $(\sqrt{m} - \sqrt{n})(m + \sqrt{mn} + n)$

8. Simplify

(a) $\sqrt{2 + \sqrt{3}} \cdot \sqrt{2 - \sqrt{3}}$

(b) $\sqrt{2} \cdot \sqrt{2 + \sqrt{2}} \cdot \sqrt{2 - \sqrt{2}}$

(c) $\sqrt{2} \cdot \sqrt{2 + \sqrt{2 + \sqrt{2}}} \cdot \sqrt{2 - \sqrt{2 + \sqrt{2}}}$