

Notes 1/24/08

Today's work is on Vieta's formulas.

1. Find the sum and product of roots of the quadratic equation

$$x^2 - 2\sqrt{3}x + 2 = 0.$$

2. Write down Vieta's formulas for the quadratic equation

$$x^2 - x - 2 = 0.$$

and try to guess its roots.

3. Write down a quadratic polynomial whose roots are

(a) $a + 5$ and $a - 5$

(b) a/b and b/a

4. Find the geometric and arithmetic means of the roots of the quadratic equation

$$x^2 - 14x + 4.$$

5. Consider the parabola defined by the formula $y = ax^2 + bx + c$. Let x_1, x_2 be the roots of the quadratic equation $ax^2 + bx + c = 0$. Express the x -coordinate of the vertex of the parabola in terms of x_1 and x_2 .

6. Let p and q be prime numbers. It is known that both roots of

$$x^2 + px + q$$

are integers. Find p and q .

7. Find a quadratic polynomial whose roots are the reciprocals of the roots of

$$5x^2 - 7x - 13.$$

8. Consider the quadratic equation

$$x^2 + 4x + 1 = 0.$$

Without computing its roots x_1 and x_2 find

(a)

$$\frac{x_1 + x_2}{x_1 x_2}$$

(b)

$$x_1^2 + x_2^2$$

(c)

$$x_1 x_2^2 + x_1^2 x_2$$

(d)

$$x_1^3 + x_2^3$$

9. Derive Vieta's formulas for the cubic polynomial

$$ax^3 + bx^2 + cx + d.$$