Math 6/71051
Minimal Guidelines for Writing Proofs

Writing a mathematical proof is more or less the same as writing any other essay or composition. In writing a proof, you are trying to convince the reader of the truth of a certain statement. This must be done in as clear and precise a manner as possible. The following specific guidelines should be followed:

1. Grammar

a. Be sure that your grammar, spelling, and punctuation are reasonably correct. The rules of grammar are at least as important in writing a mathematical proof as they are in any other form of writing, and probably more so because of the necessary precision of language in mathematics. A sloppy or grammatically incorrect proof is not correct, and will not receive full credit.

b. A proof *must* be written in complete sentences — that is, with subjects, verbs, etc. — and different ideas or parts of the proof should be separated into paragraphs. Run-on sentences should be avoided.

c. Use technical terms correctly. Mathematical language must be very precise and misuse of a technical word will usually make a statement incorrect.

2. Mathematical Symbols

a. Although mathematical symbols are permitted, and indeed usually necessary, it should be kept in mind that the symbols do stand for words. Sentences containing symbols should *make sense* as English sentences. A good test is to read the sentence aloud, substituting the correct words or phrases for the symbols.

b. Minimize the use of mathematical symbols. Any symbol standing for one or two words can easily be eliminated. For example, it is usually better to just write "there exists" instead of using the symbol "$\exists$." On the other hand, it would not be helpful to write "the integral of f of x from a to b with respect to x" when the expression "$\int_a^b f(x)\,dx$" is available.

c. Use mathematical symbols correctly and do not make up your own notation without explanation. Notational conventions have been developed over many years so that mathematics can be communicated easily and effectively. If someone makes up new notation without defining it, communication is difficult at best and is usually impossible.

d. A sentence must *NEVER* begin with a mathematical symbol. (Corollary: Every sentence must contain at least one word.)

3. Organization and Style

a. A proof must be well organized and follow as straight a line as possible from hypotheses to conclusions. It should not be a series of loosely related statements that must be tied together by the reader.

b. A proof should have a beginning and an end — the assumptions should be spelled out at the start, and the conclusion (i.e. what has been shown) should be indicated at the end. *These should not be left out simply because they appear in the statement of the proposition.*

c. It should be made clear which statements follow from which other statements. In fact, a *reason or justification* for each implication should be given. Even if the reason is completely obvious, however, it should still be clear that one statement follows from the previous statement or from some hypothesis. For example, write "if A is a symmetric matrix then $A^t = A$" and not "A is a symmetric matrix, $A^t = A$."

d. It should always be assumed that the reader knows less about the proof than the writer. Write as if your audience consists of fellow students and not simply the instructor. Do not use the fact that the instructor already knows the answer as an excuse to be vague.

e. It should *not* be assumed that the reader is a mind-reader. The proof should be complete and clear on its own, and should not require further interpretation on the part of the reader. After writing a proof, put it away for a while, and then try to read it yourself. If *you* don't understand what you've written, don't expect anyone else to understand it.

f. Avoid irrelevant statements and the "shotgun approach" to solving problems. In other words, do not simply try to write down everything you know about the subject with the hope that the instructor will find something relevant and correct.